Quantum variational learning for entanglement witnessing

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VCQ and AppQInfo 2022 Summer School

Concepts & Applications of Quantum Information

FULL PAPER

UNIVERSITÀ DI PAVIA
Dipartimento di Fisica

Machine Learning and Data
Different approaches

C: Classical
Q: Quantum

	Machine Learning	
Data	CC	QC
	CQ	QQ

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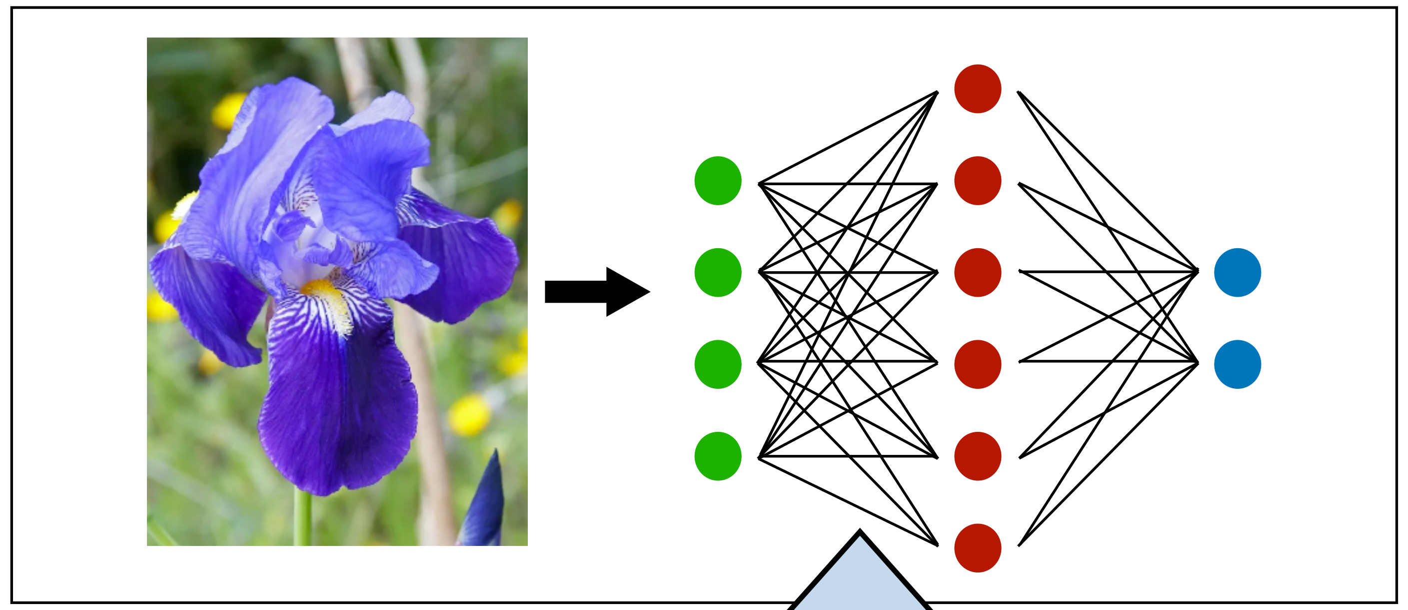
Machine Learning and Data
Different approaches



C: Classical
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	Machine Learning	
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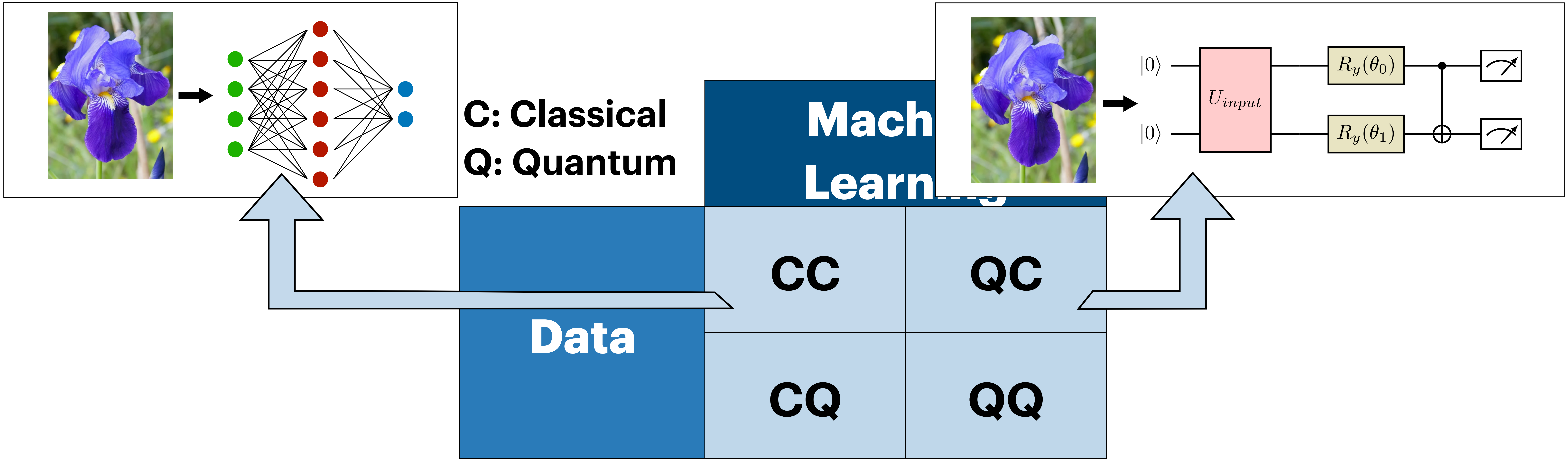
Machine Learning and Data
Different approaches



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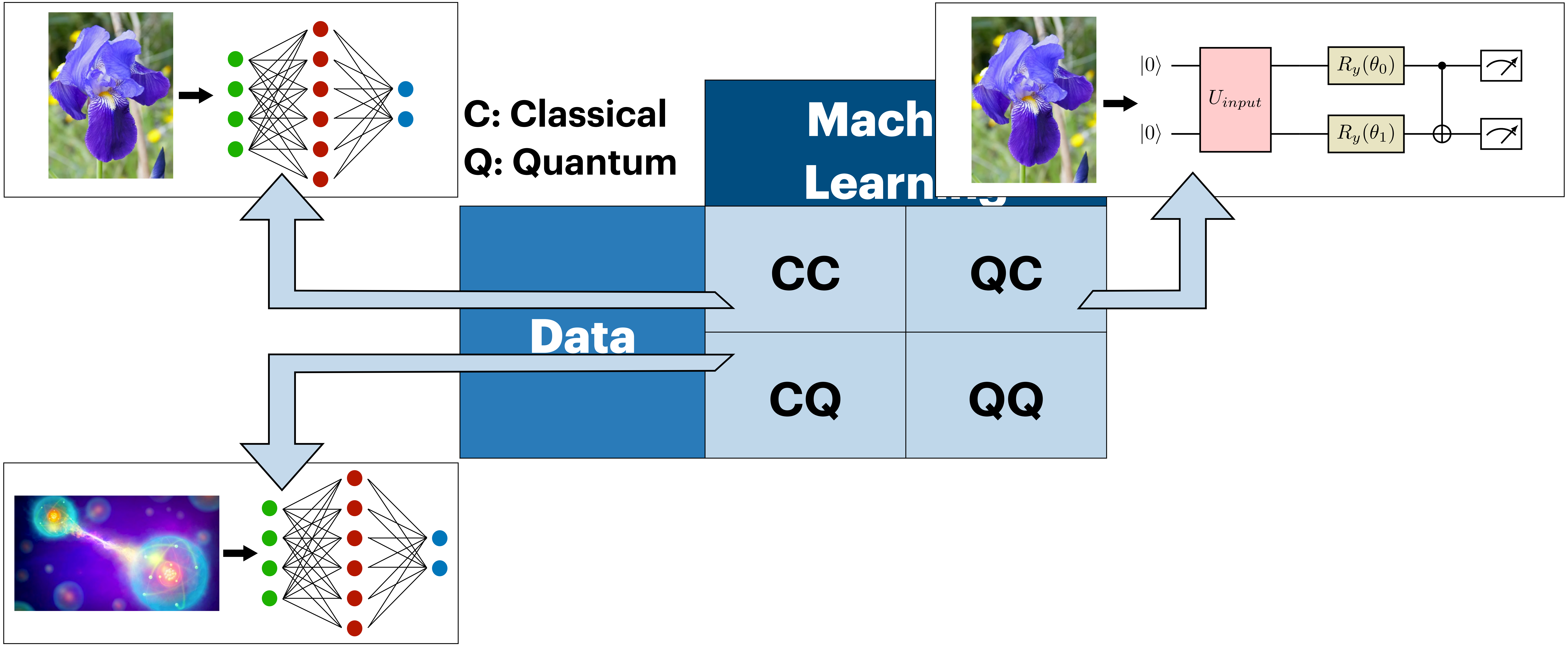
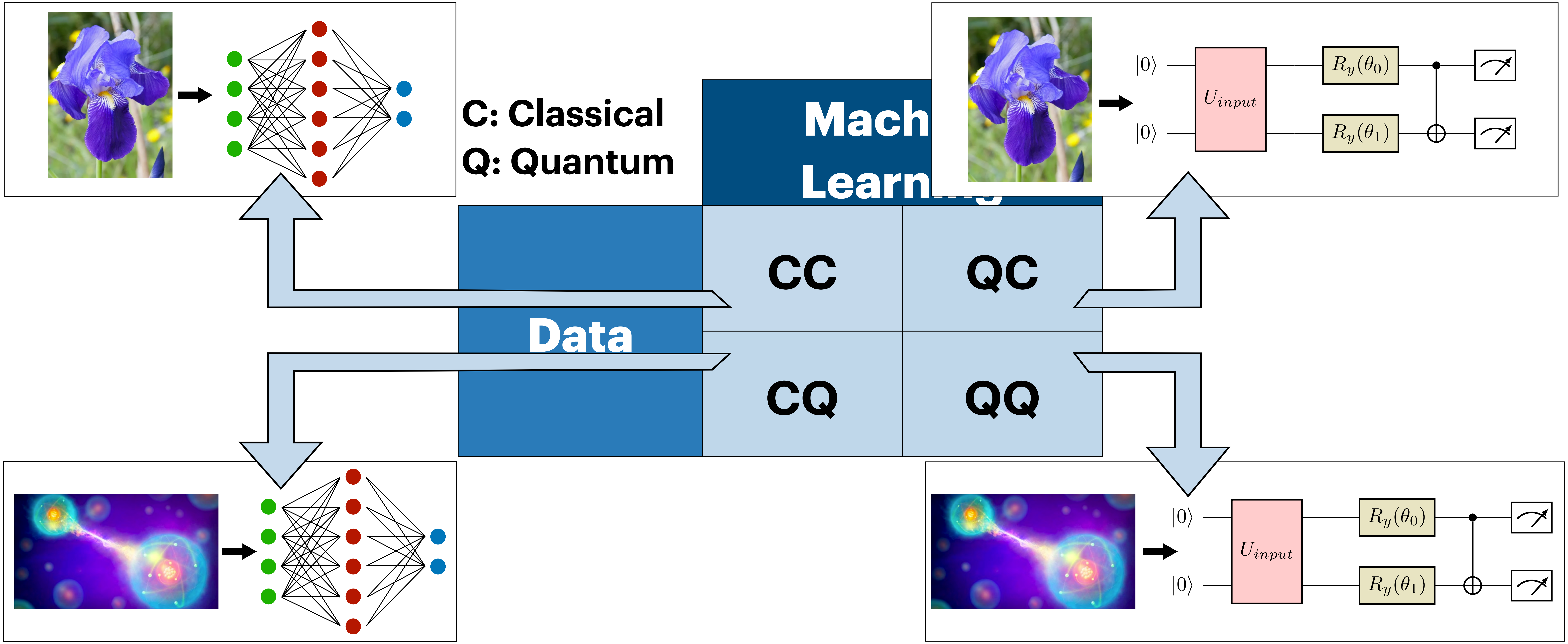
Machine Learning and Data
Different approaches



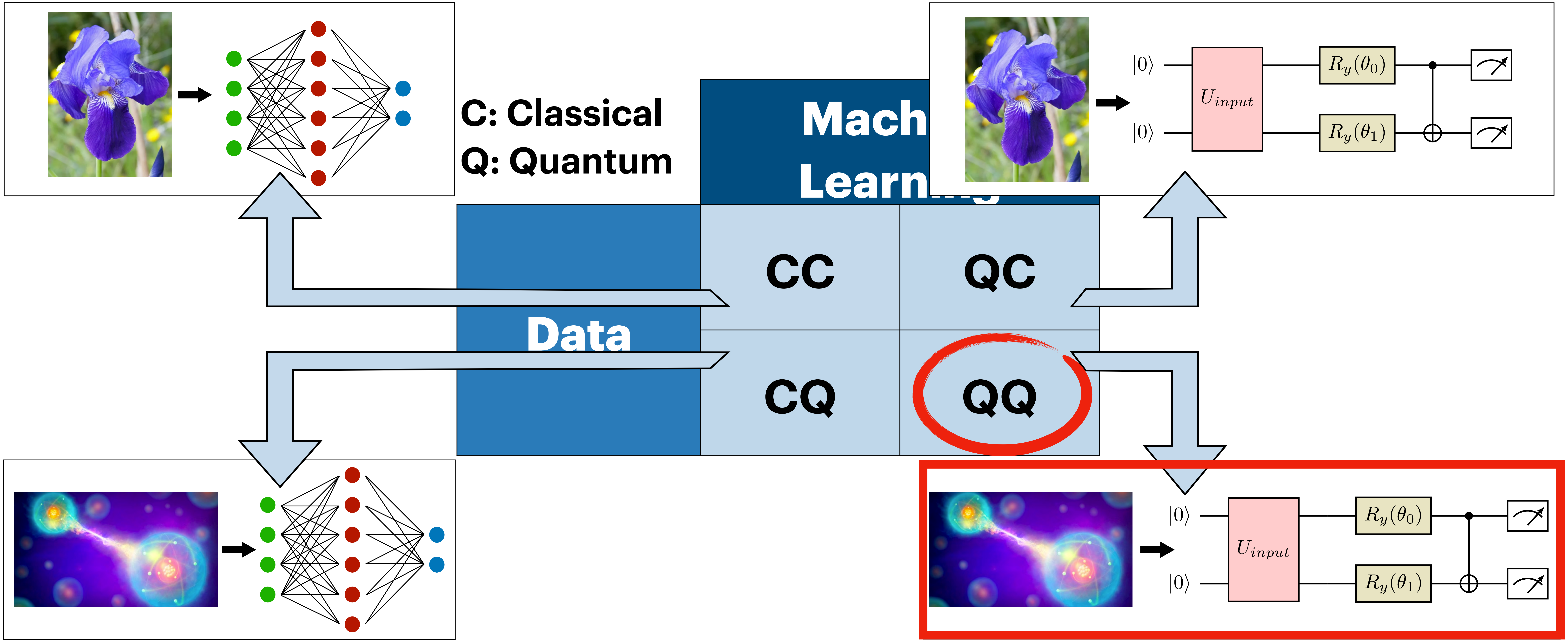
Machine Learning and Data

Different approaches

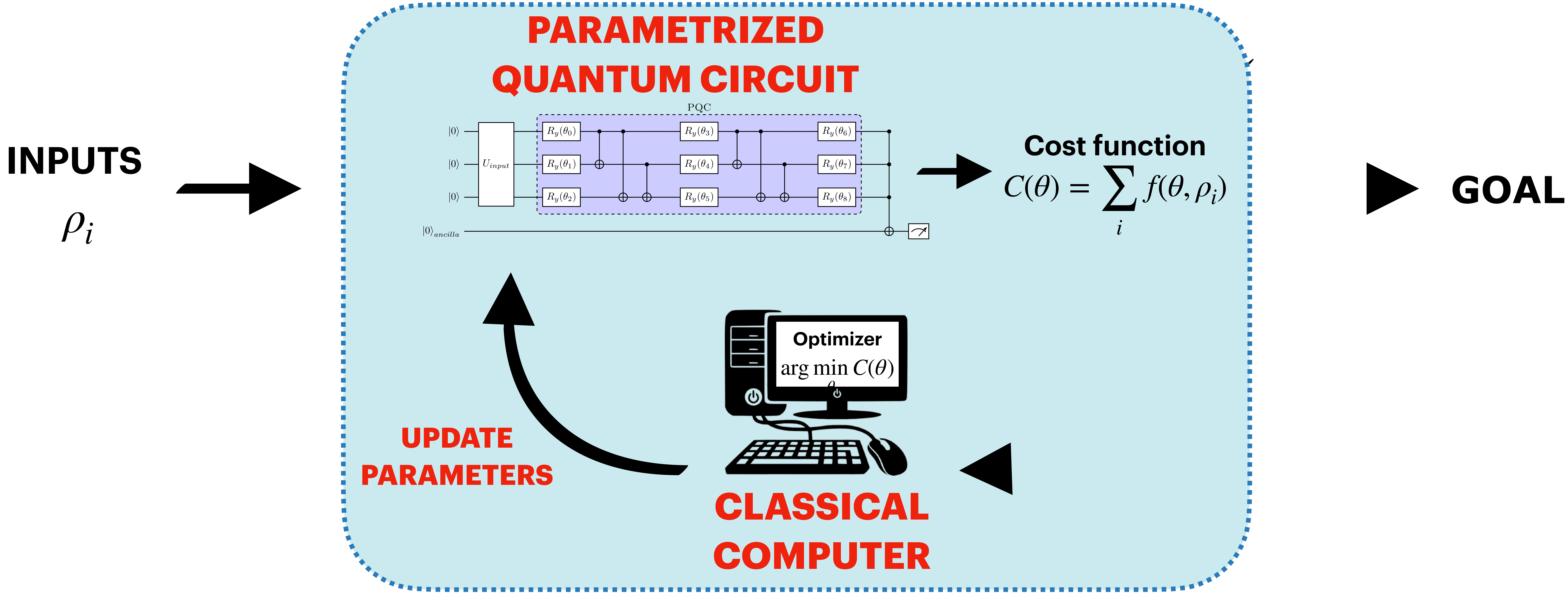


Machine Learning and Data

Different approaches



Variational Quantum Algorithms[1] (VQAs)
Are hybrid

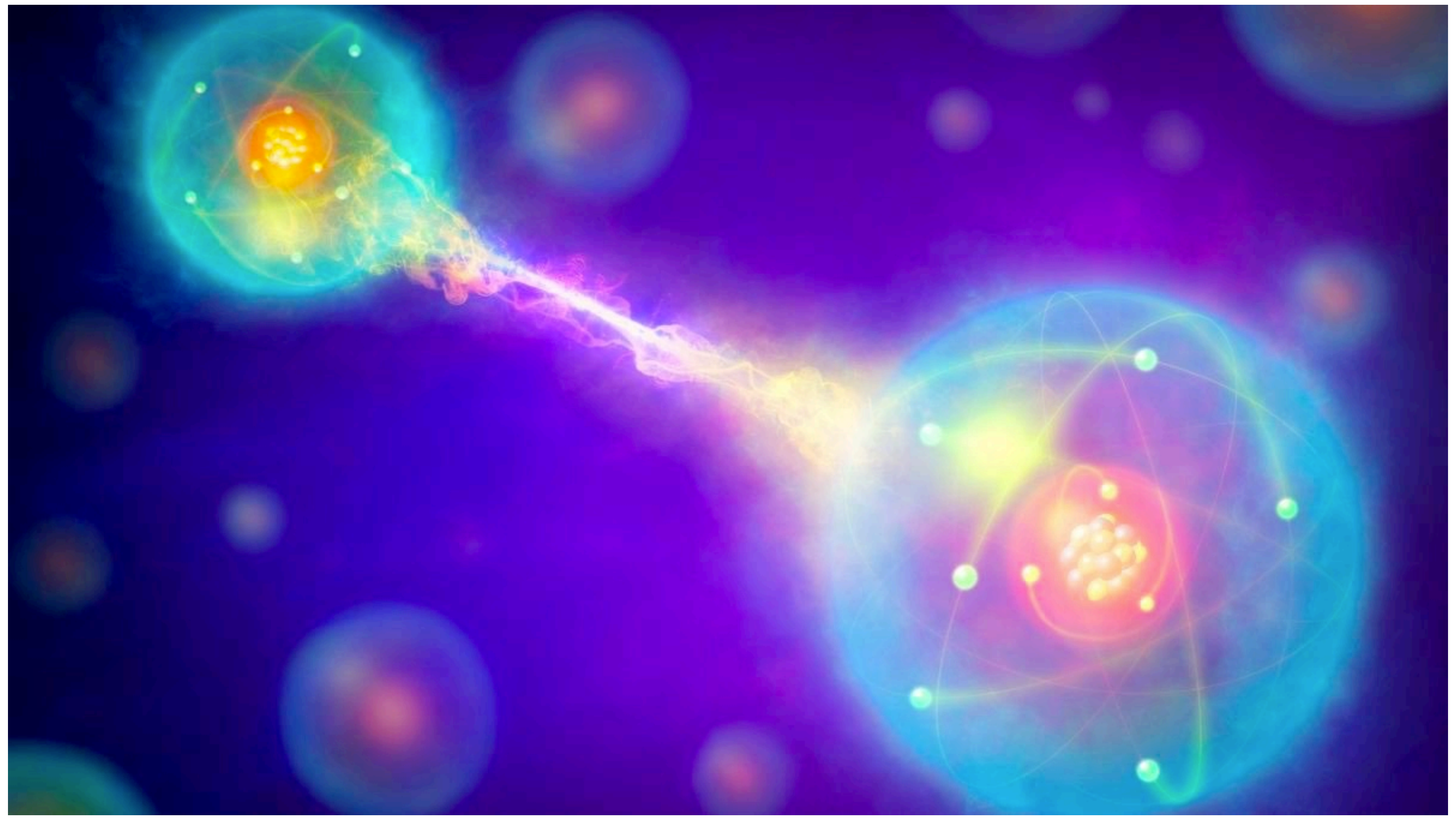


[1] M. Cerezo, et al. Nat Rev Phys 3, 625–644 (2021)

Quantum Entanglement
A non-classical correlation

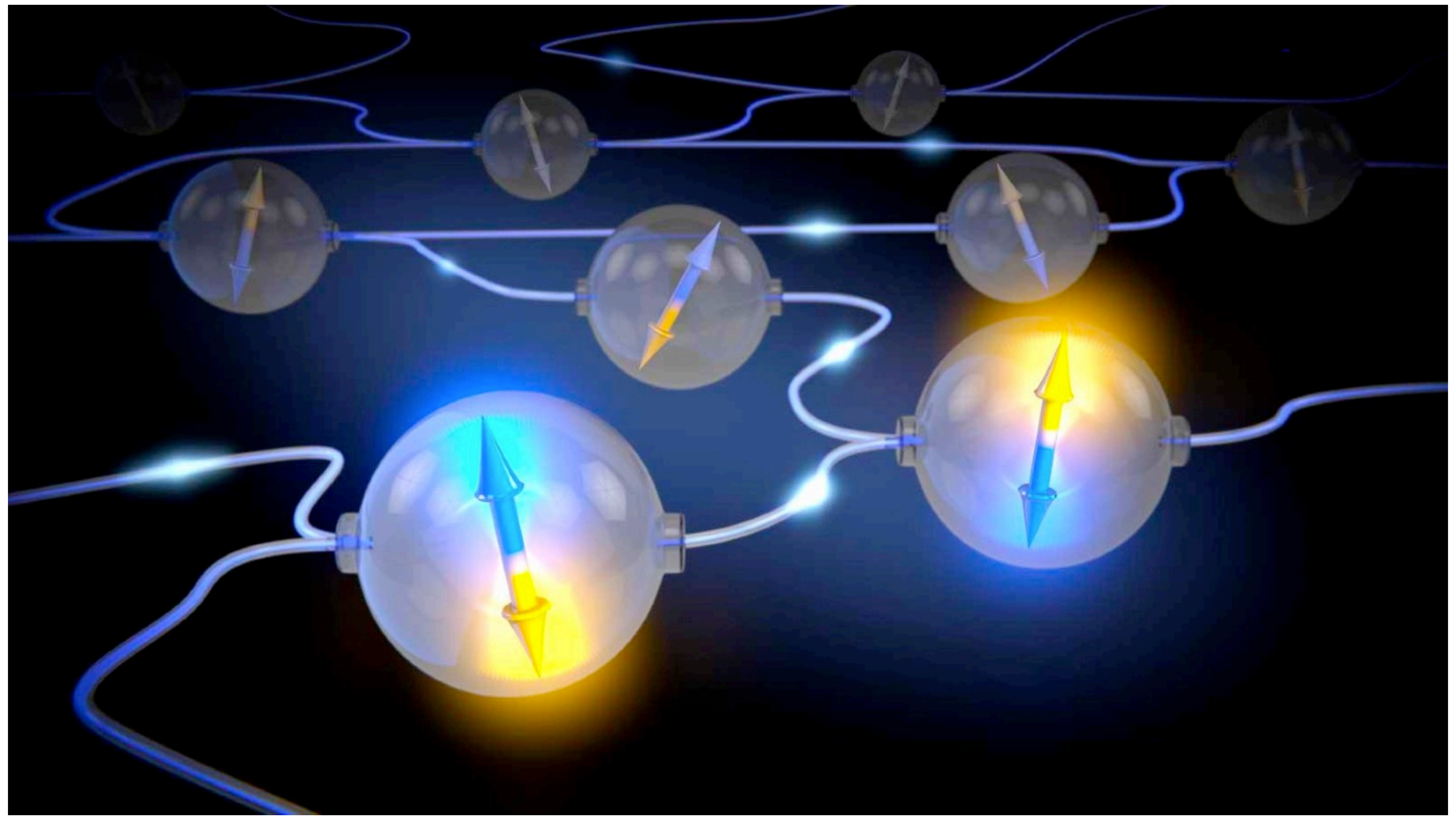
A genuine quantum mechanical property of physical systems

Bipartite entanglement



Shared by two systems

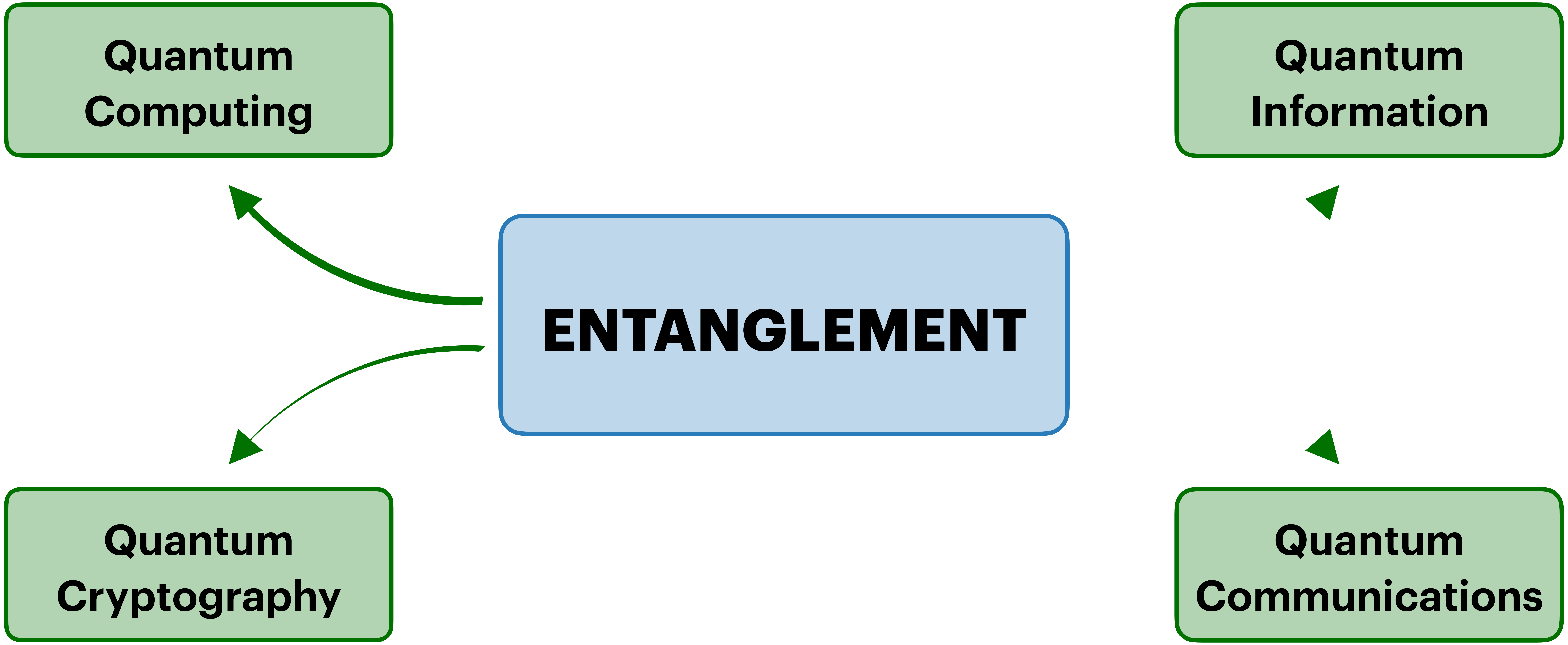
Multipartite entanglement



Shared by multiple systems

Why seeking for entanglement?

It is a fundamental resource



How to detect entanglement?

Exponential difficulties

Classical methods:

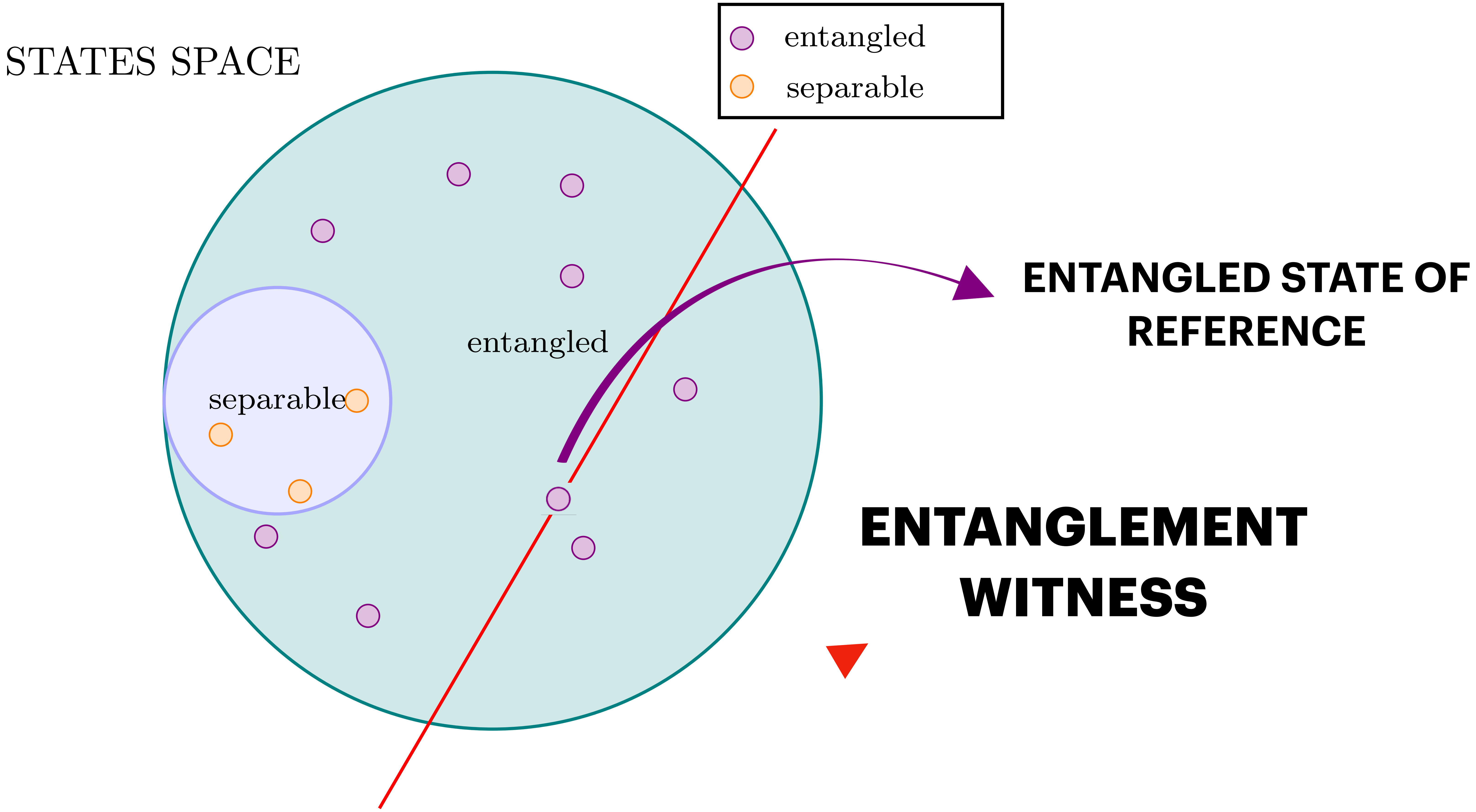
HARD to simulate large quantum systems → $\dim(\mathscr{H}) = 2^N$ **for** N **qubits**

Quantum methods:

Quantum State Tomography → 3^N **quantum measurements for** N **qubits**

How to detect entanglement?

Projective entanglement witness



F. Scala, et al. *arXiv:2205.10429* (2022)

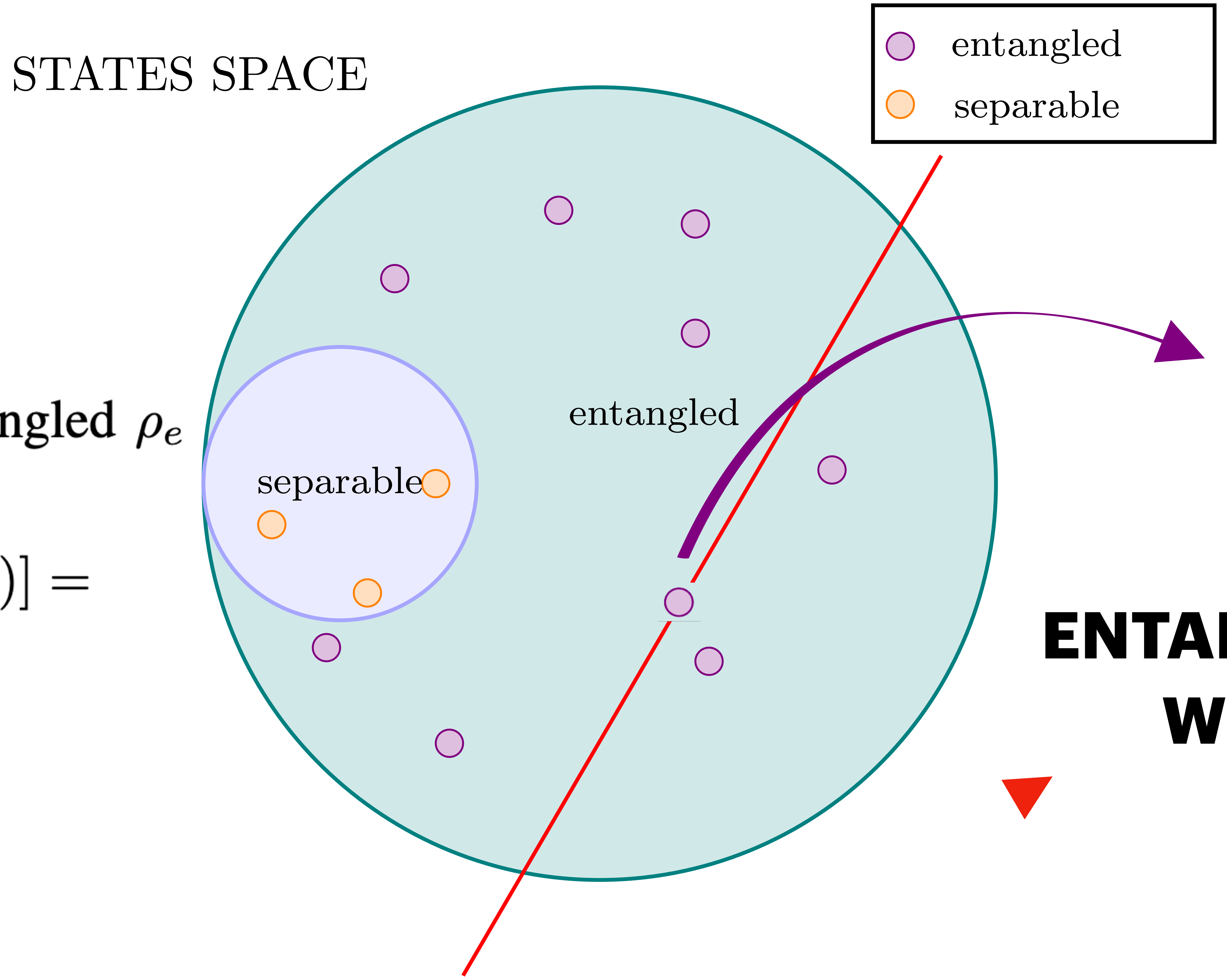
How to detect entanglement?

Projective entanglement witness

$$W = \alpha(\,|\,(H)\rangle \mathrm{I} - |\,H\rangle\langle H\,|$$

$$\begin{cases} \mathrm{Tr}[W\rho_s] \geq 0 & \forall \rho_s \text{ separable,} \\ \mathrm{Tr}[W\rho_e] < 0 & \text{for at least one entangled } \rho_e \end{cases}$$

$$\begin{aligned} \mathrm{Tr}[\rho W] &= \mathrm{Tr}[\rho(\alpha(|H\rangle)\mathbb{I} - |H\rangle\langle H|)] = \\ &= \alpha(|H\rangle) - \mathrm{Tr}[\rho\,|H\rangle\langle H|] \end{aligned}$$



ENTANGLED STATE OF REFERENCE

ENTANGLEMENT WITNESS

F. Scala, et al. *arXiv:2205.10429* (2022)

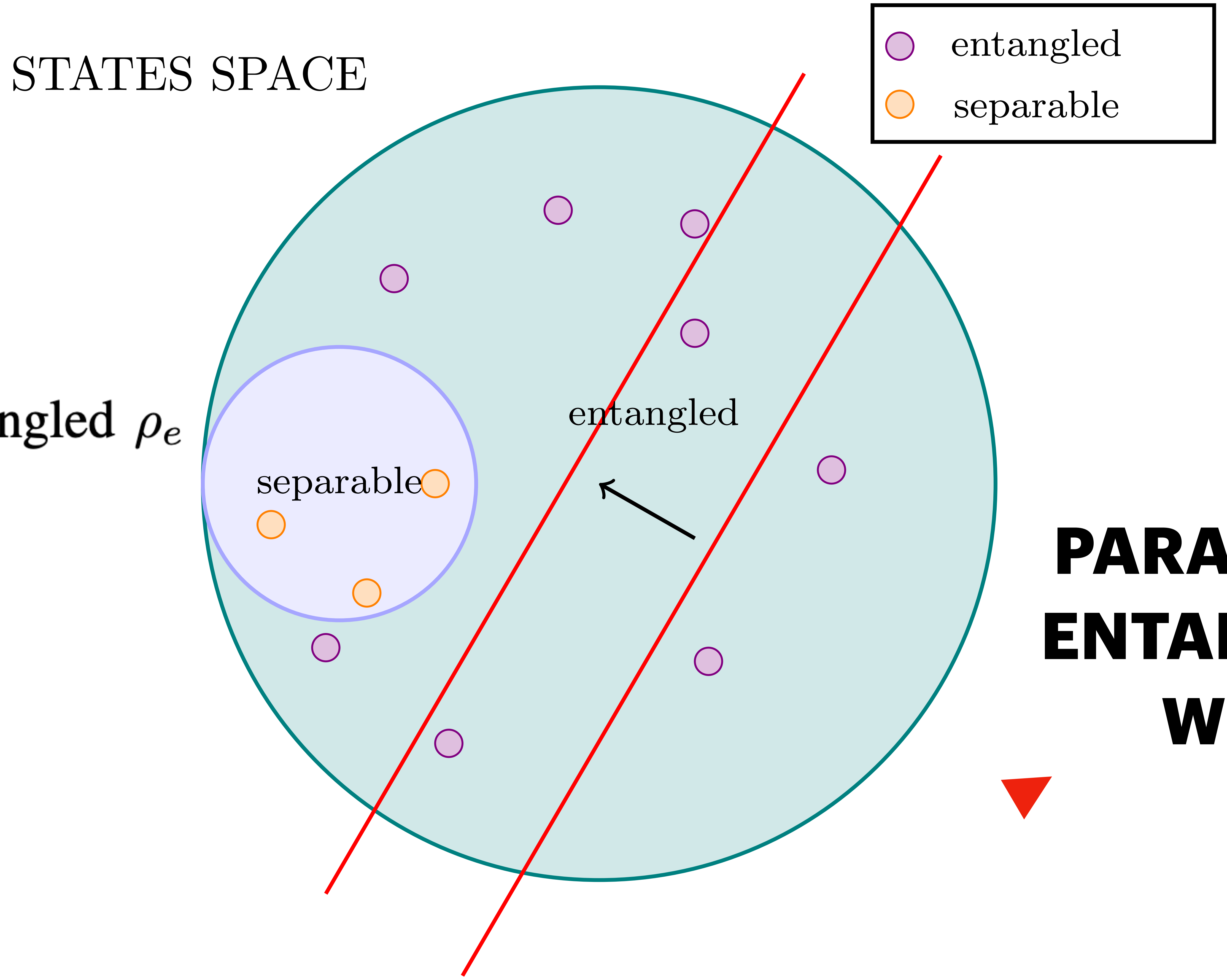
Witness Optimization

No state of reference

$$W = \alpha \mathrm{I} - |V(\theta)\rangle\langle V(\theta)|$$

$$\begin{cases} \mathrm{Tr}[W\rho_s] \geq 0 & \forall \rho_s \text{ separable,} \\ \mathrm{Tr}[W\rho_e] < 0 & \text{for at least one entangled } \rho_e \end{cases}$$

$$\mathrm{Tr}[\rho W] = \alpha - \mathrm{Tr}[\rho\,|V(\theta)\rangle\langle V(\theta)|\,]$$



PARAMETRIZED ENTANGLEMENT WITNESS

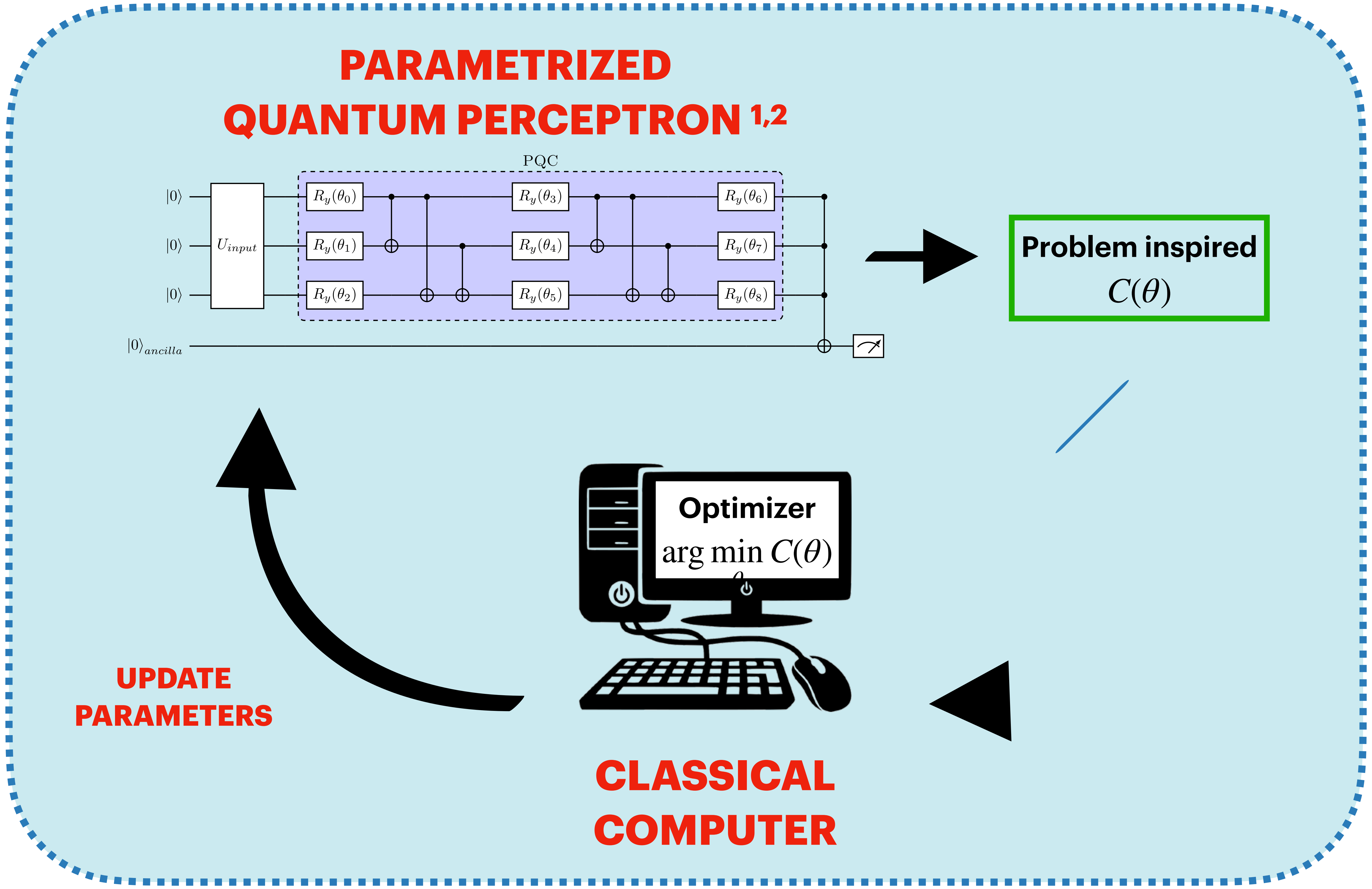
J. Roik, et al., Phys. Rev. Applied, vol. 15, no. 5, 2021.
L. T. Wu, et al., in Conference on Lasers and Electro-Optics, Optical Society of America, 2021, FW3N.1

Variational Quantum Algorithm (VQA)

For entanglement witnessing

HYPERGRAPH STATES

ρ_i



GENERALIZED ENTANGLEMENT WITNESS

[1] F. Tacchino et al. , npj Quantum Inf 5, 26 (2019)
[2] F. Tacchino et al. , IEEE Transactions on Quantum Engineering, vol. 2, pp. 1–10 (2021)

Input states

Separable-entangled proportions

3 qubits hypergraph states[1]:

$$|\psi_f\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle$$

256 states

(scales as 2^{2^n})

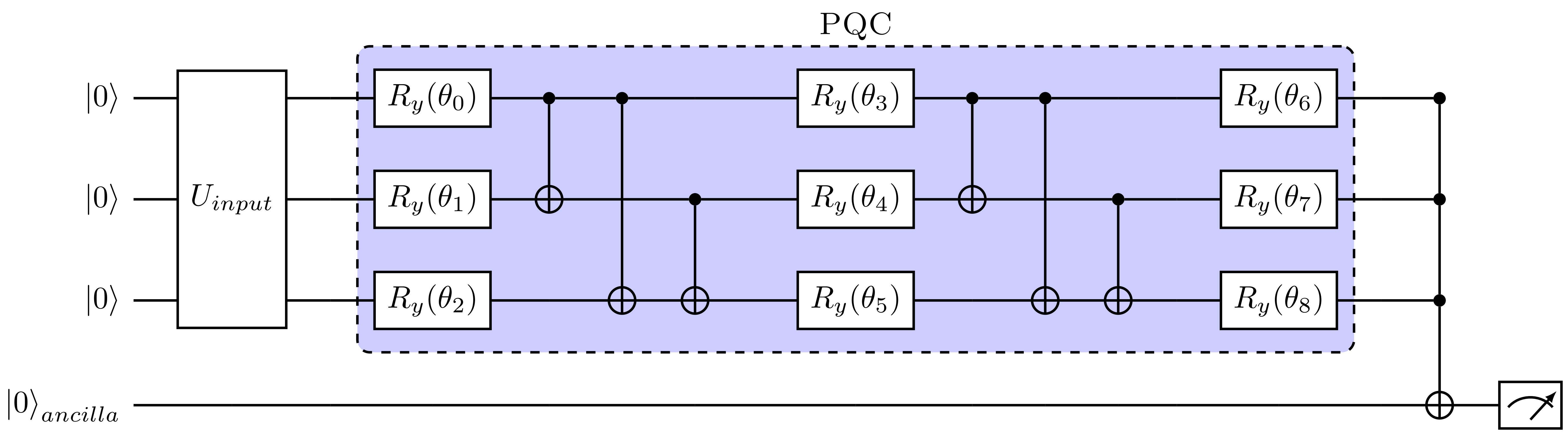
64 bi-separable

192 entangled
(64 maximally entangled)

[1] M. Rossi *et al.*, New J. Phys. 15 113022 (2013)

Parametrized Quantum Model

Parametrized Quantum Perceptron[1]



Only one measurement setting required to estimate

$\mathrm{Tr}[\rho\,|V(\theta)\rangle\langle V(\theta)|\,]$

- **2 layers Parametrized Quantum Model** (R_y **rotations**)
- **COBYLA optimizer**
- **Activation threshold** $\alpha = 0.5$

[1] F. Tacchino et al. , IEEE Transactions on Quantum Engineering, vol. 2, pp. 1–10 (2021)

Problem inspired cost function

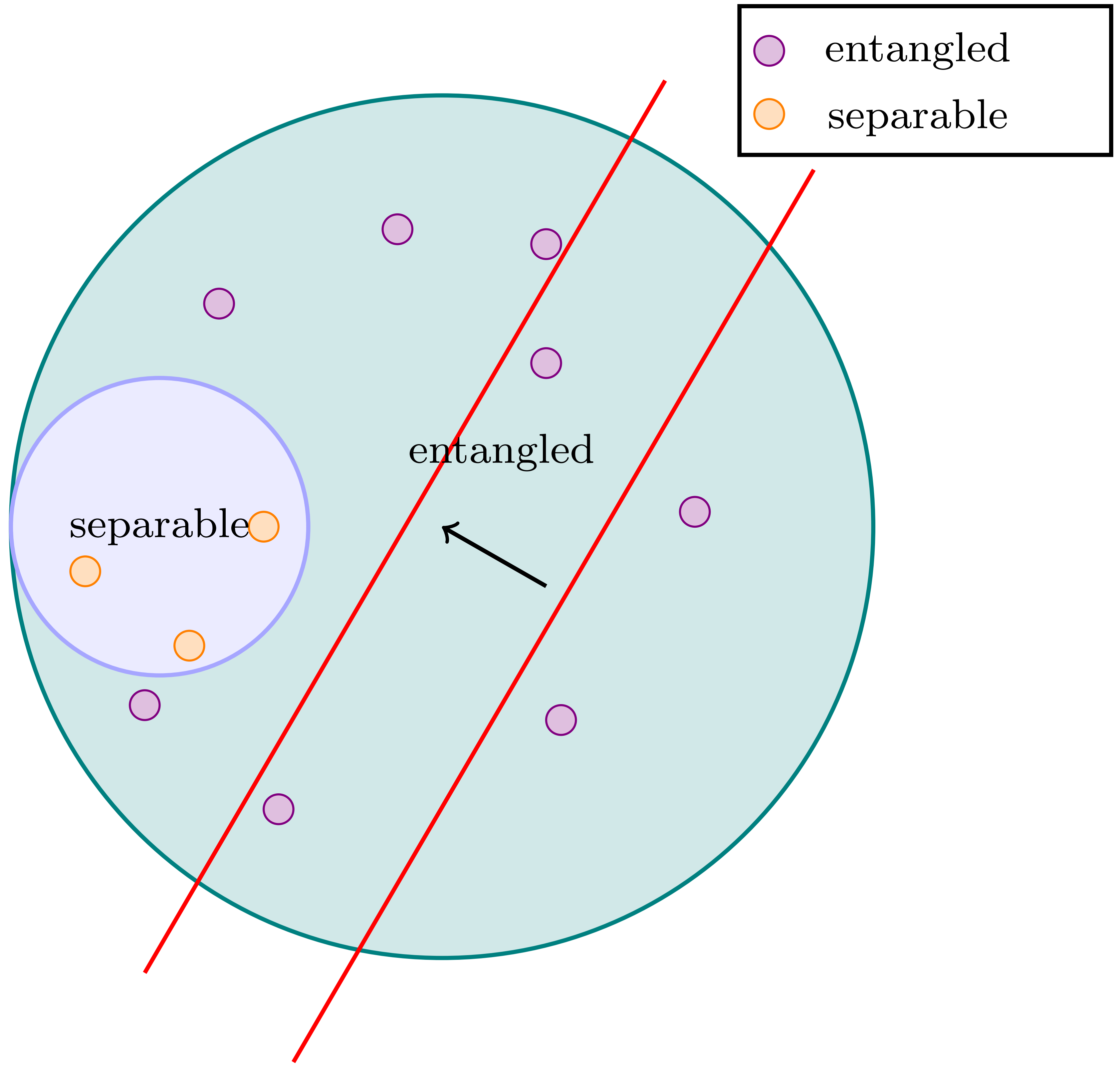
$$\text{Precision} = \frac{\text{True Positive}}{\text{All Positive}}$$

$$\text{Recall} = \frac{\text{True Positive}}{\text{Positive Class}}$$

$$F_\beta = (1+\beta^2) \cdot \frac{\text{Precision} \cdot \text{Recall}}{\beta^2 \cdot \text{Precision} + \text{Recall}}$$

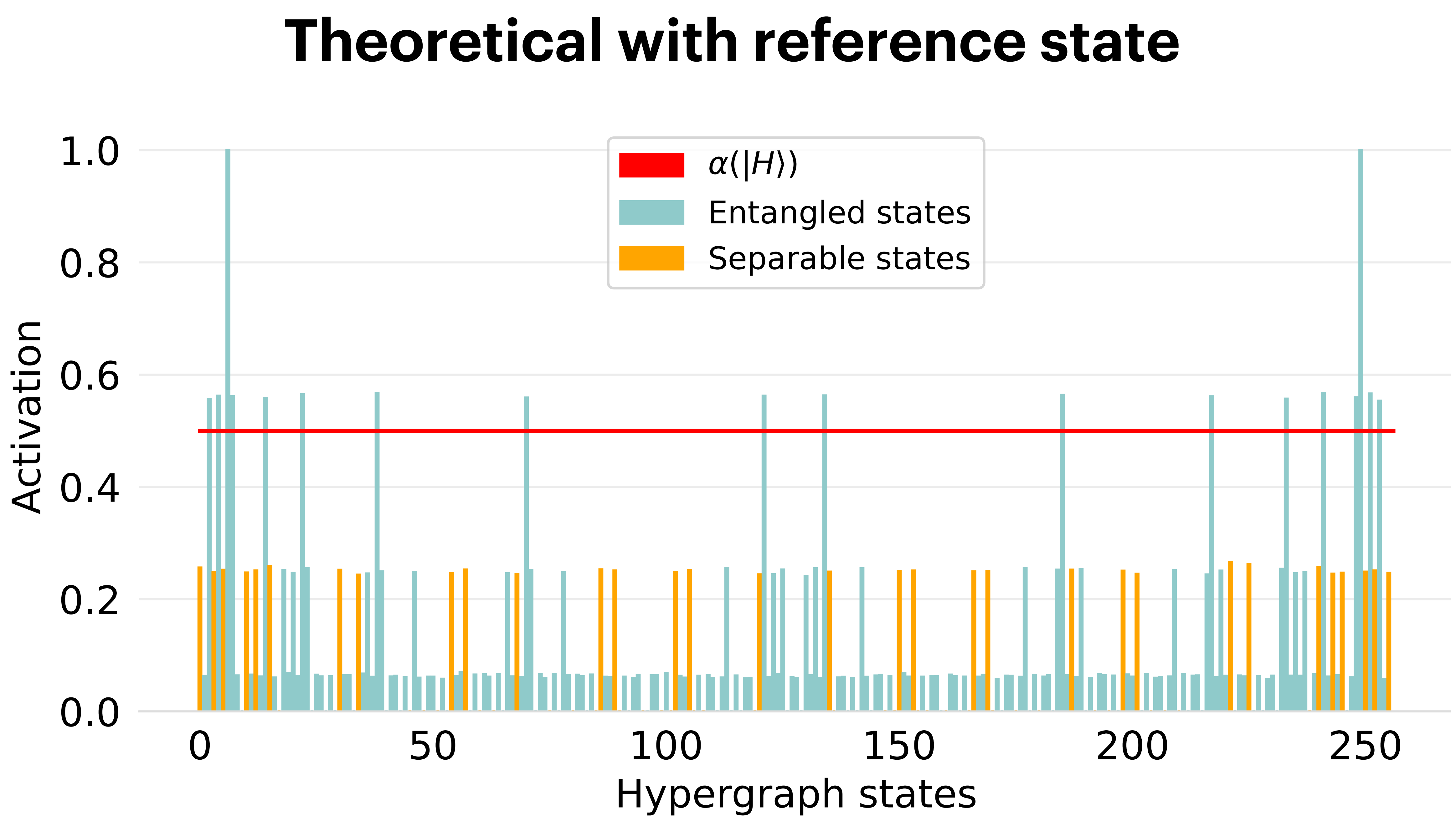
Favor precision over recall $0 < \beta < 1$

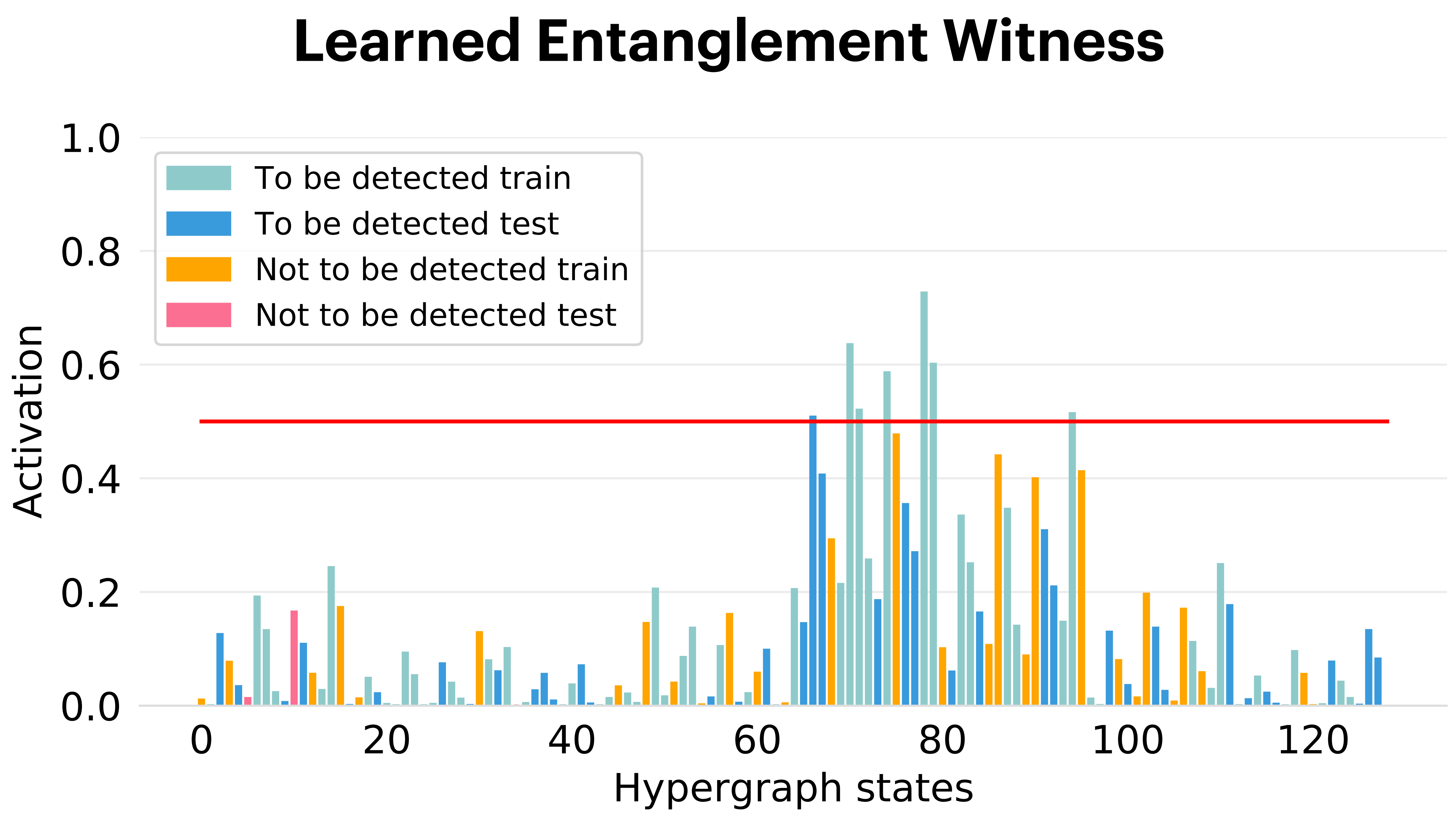
$$C(\theta) = 1 - F_\beta$$



Simulation results

python™ Qiskit





F. Scala, et al. *arXiv:2205.10429* (2022)

Summary and outlooks

Entanglement detection is hard in general

➡ VQAs are able to learn an **entanglement witness**

➡ VQAs are more suited to work with **quantum data**

$\rightarrow$ **direct state manipulation**

Future works

Generalization:

- Inputs and model
- Optimizer
- Entanglement measures
- Real quantum hardware

Collaborators


Stefano Mangini


Chiara Macchiavello


Daniele Bajoni


Dario Gerace

UNIVERSITÀ DI PAVIA

Dipartimento di Fisica

Dipartimento di Ingegneria Industriale e dell'Informazione

For further details:

arXiv:2205.10429 (2022)

FULL PAPER

Quantum Entanglement

A non-classical correlation

DEFINITION

A n-parties **pure state** $|\psi\rangle$ is called ***fully entangled*** **if and only if** it is not bi-separable with respect to any bipartition:

$$|\psi\rangle = |A\rangle \otimes |B\rangle,$$

where $|A\rangle$ is an m parties state and $|B\rangle$ is an n−m parties state.

Multipartite Entanglement measure

$$E(|\psi_n\rangle) := \min_{AB} E^{AB}(|\psi_n\rangle) =$$
$$= 1 - \max_{|\phi\rangle_A |\phi\rangle_B, AB} |\langle\phi|_A \langle\phi|_B \psi_n\rangle|^2 =$$
$$= 1 - \alpha(|\psi_n\rangle)$$

the $\max$ is taken over ALL the possible biseparable states $|\phi_k\rangle_A, |\phi_{n-k}\rangle_B$

$$\alpha(|\psi\rangle) = \max_{AB} \alpha^{AB}(|\psi\rangle)$$

2

$$\alpha^{AB}(|\psi\rangle) = \max_{k=1,\ldots,R} s_k^{AB}(|\psi\rangle)^2$$

where s_k^{AB} are the Schmidt coefficients, and R is the Schmidt rank

[1] M. Ghio et al., J. Phys. A: Math. Theor. 51 045302 (2018)
[2] M. Bourennane, *et al.*, Phys. Rev. Lett., vol. 92, p. 087 902, 8 (2004)

Hypegraph states

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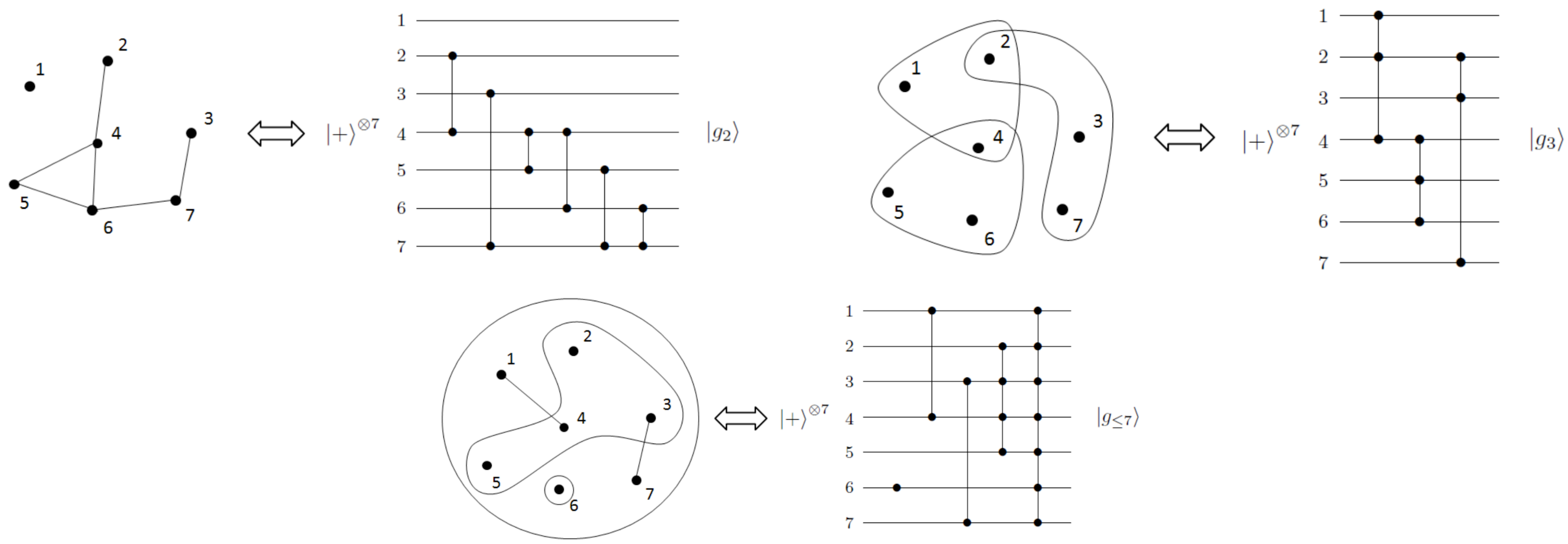
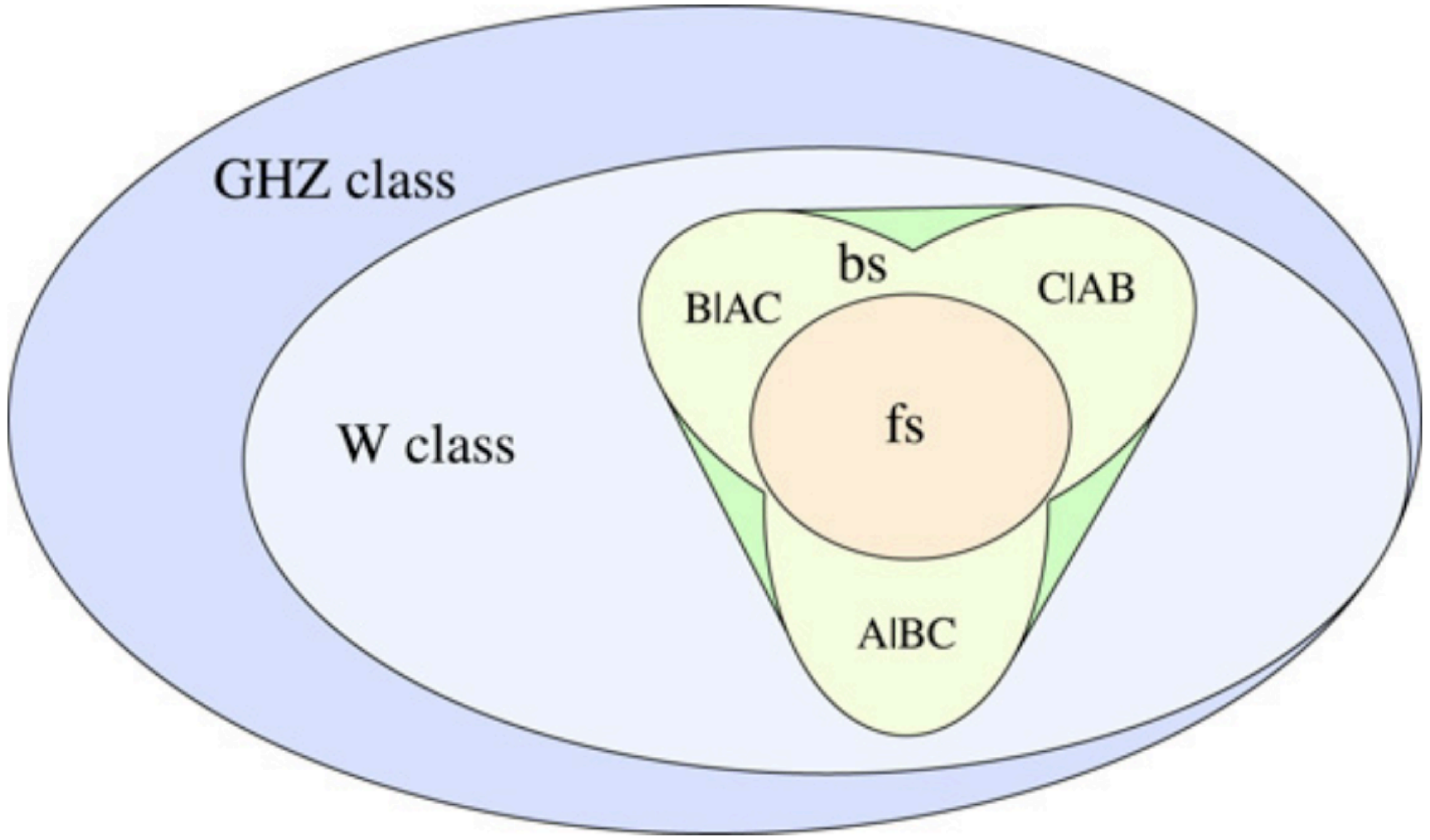


M. Rossi *et al.*, New J. Phys. 15 113022 (2013)

States space structure
3 qubits



$$|GHZ_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|W_3\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

$$|\mathrm{H}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle - |111\rangle)$$

Ghune *et al.*, Physics Reports 474 (2009) 1–75