Quantum Variational Learning for Entanglement Witnessing

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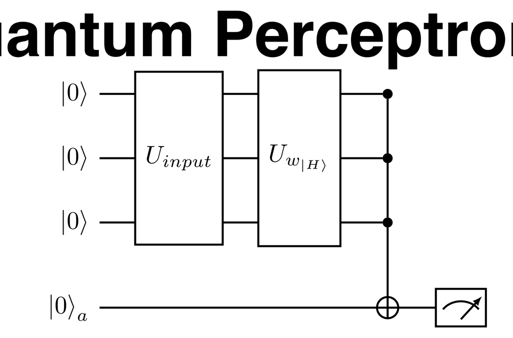
In this work we address the use of Quantum Machine Learning (QML) techniques for entanglement detection. In particular, we analyse a class of quantum states widely employed in quantum computing, the hypergraph states [1], exploiting the notion of entanglement witness. We show how to implement an entanglement witness and how to learn a witness thanks to the use of Quantum Neural Networks (QNNs) [2],[3]. The final outcome of this work is a quantum algorithm able to classify the entanglement of the input state, without any need to perform a measurement on the qubits encoding the state itself. With further improvements this result may pave the way to the use of more general entangled states in quantum information protocols.

Hypergraph States [1]

$$|\psi_f\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle$$

$f(x)$ is a Boolean function $f: \{0,1\}^n \rightarrow \{0,1\}$

Quantum Perceptron [2]



$$Tr[\rho_{input} |H\rangle\langle H|] = \langle H|\rho_{input}|H\rangle$$

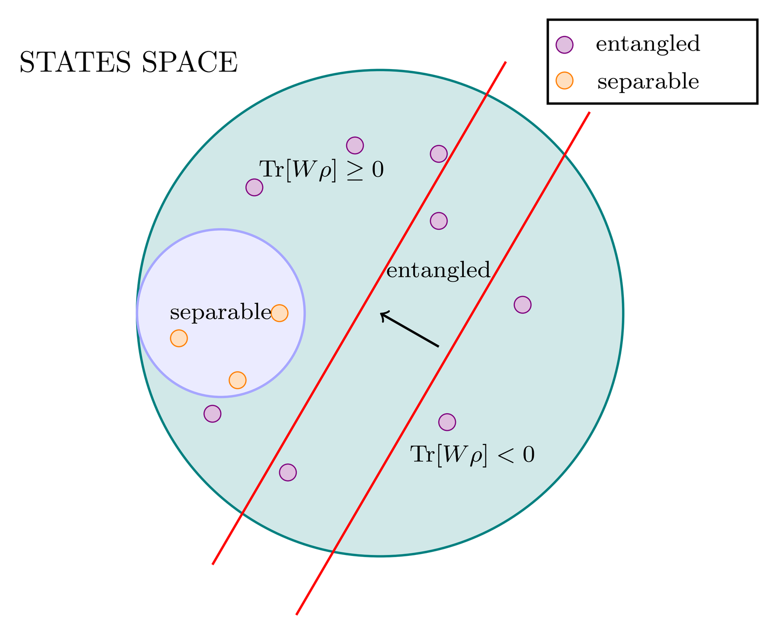
Entanglement Witness [3]

- ***Entanglement witness*** is an observable W such that $Tr[W\rho_s] \geq 0 \ \forall \rho_s$ separable and $Tr[W\rho_e] < 0$ for at least one entangled ρ_e.
- The entangled states for which $Tr[W\rho_e] < 0$ are the ones **detected** by the witness.
- A ***projective entanglement witness*** has the following form:

$$W = \alpha(|H\rangle)\mathbb{I} - |H\rangle\langle H|$$

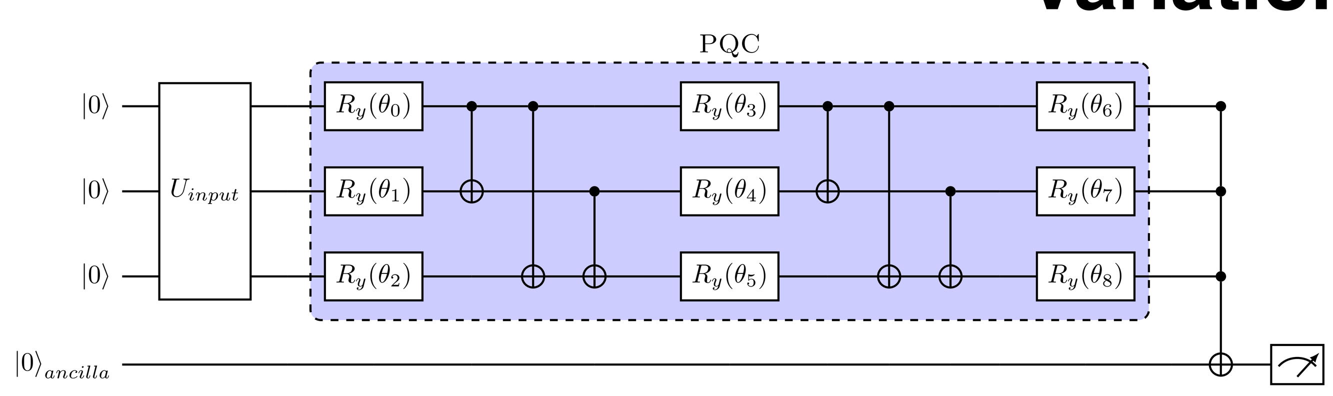
where $|H\rangle$ is an entangled state of reference, and $\alpha(|H\rangle)$ is a coefficient that can be calculated classically.
- It follows that: $Tr[\rho W] = \alpha(|H\rangle) - Tr[\rho|H\rangle\langle H|]$.

Entanglement witness action



The entanglement witness defines a hyperplane. Moving it may allow to detect more entangled states.

Variational Quantum Perceptron [4]



- Entanglement detection for hypergraph states of 3-qubits.
- Supervised learning with **COBYLA** optimizer.
- Moves the hyperplane as close as possible to the convex set of separable states without crossing it.
- Leaves on the other side as many entangled states as possible.

Known Witness Learning

- Input states labeled (y_i) with 1 only if they are recognised as entangled from the exact witness and 0 otherwise.
- Cost function (cross-entropy): $L(\mathbf{p}, \mathbf{y}) = -\sum_{i=0}^{m-1} y_i \log p_i + (1-y_i)\log(1-p_i)$ with p_i activations.

Unknown Witness Learning

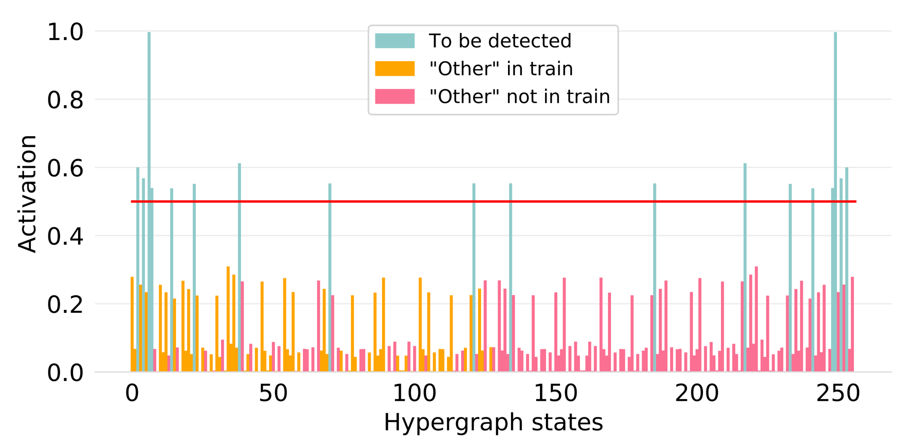
- Input states labeled with 1 if they are entangled and 0 if they are separable, meaning that we want to learn an (*a priori*) unknown witness.
- Problem inspired cost function [5]:

$$F_\beta = (1+\beta^2) \cdot \frac{\text{Precision} \cdot \text{Recall}}{\beta^2 \cdot \text{Precision} + \text{Recall}}$$

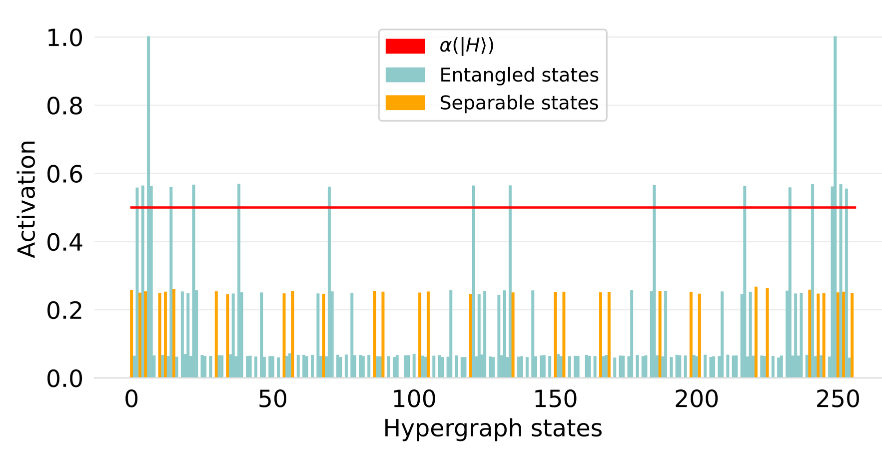
Precision $= \frac{tp}{tp+fp}$

Recall $= \frac{tp}{tp+fn}$

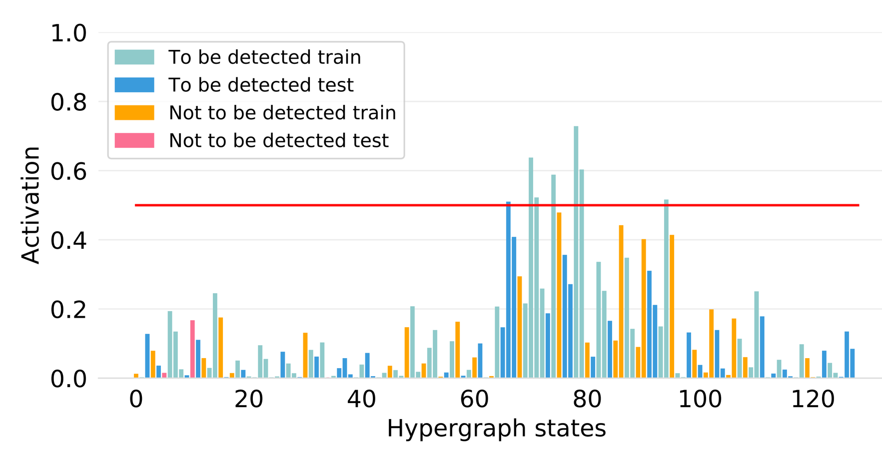
Known witness activations



Exact witness activations



Unknown witness activations



Conclusions and summary

- For all the possible 3-qubits hypergraph maximally entangled states of reference the VQA learns optimal values for the parameters
- One finds that the average cross entropy, between the exact activations and the ones obtained with the learnt witness, is equal to $0.26926 \pm 5.8 \cdot 10^{-4}$.

Conclusions and summary

- Variational unknown witness does not make any mistake on the separable states and it is able to detect also maximally entangled states.
- The learned reference state is not among the hypergraph states (no activation = 1).

FULL PAPER

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[5] D. M. W. Powers, arXiv: 2010.16061 (2020)