Quantum variational learning for entanglement witnessing

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FULL PAPER

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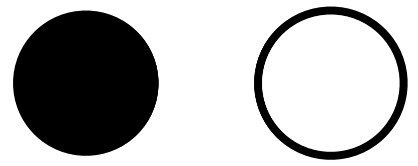
Dipartimento di Fisica

Qubits

Quantum Bits of Information

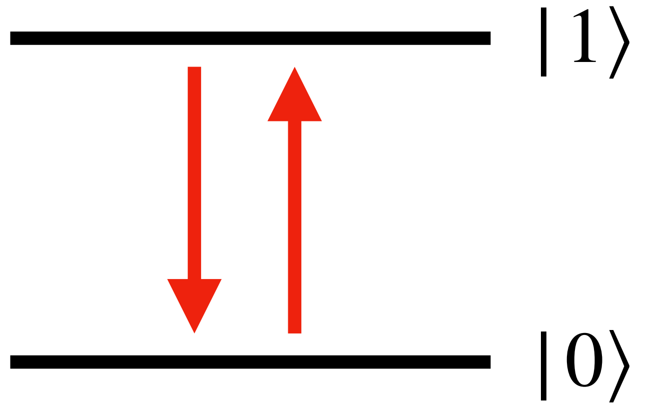
BIT



0 1

Vs

QUBIT



$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \ (\alpha^2 + \beta^2 = 1)$$

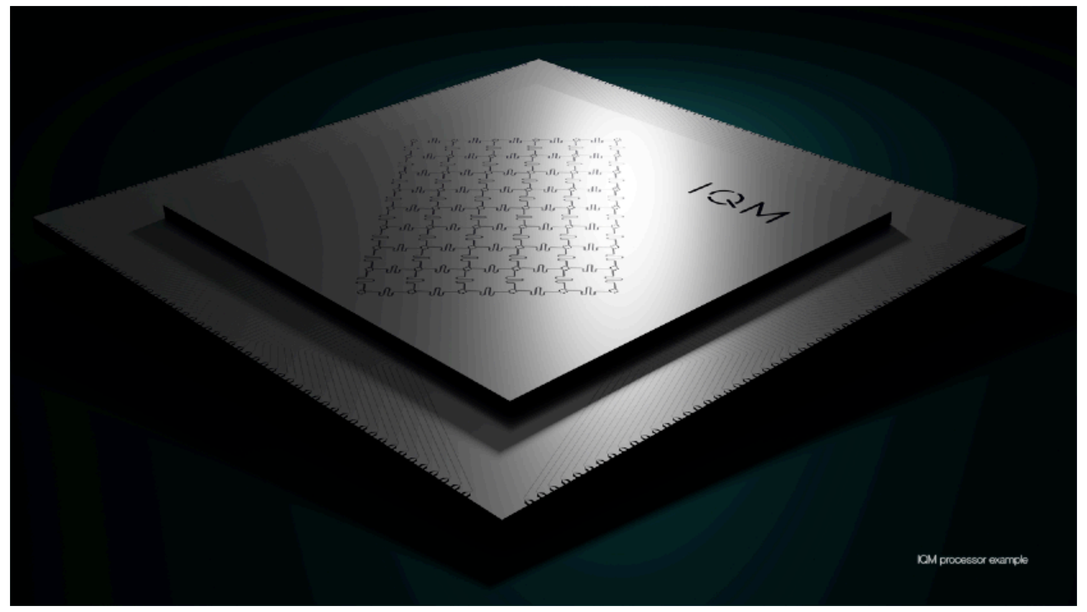
SUPERPOSITION

Superposition allows to have all the possible bit strings simultaneously

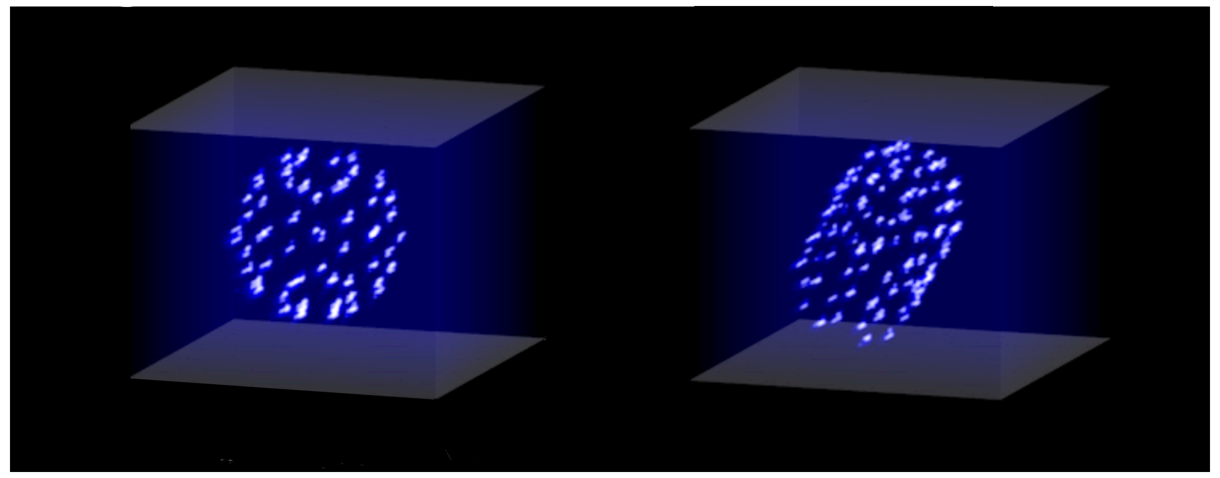
$$a_0|0...00\rangle + a_1|0...01\rangle + \ldots + a_{2^n}|1...11\rangle$$

Available Quantum Computers
Main technologies

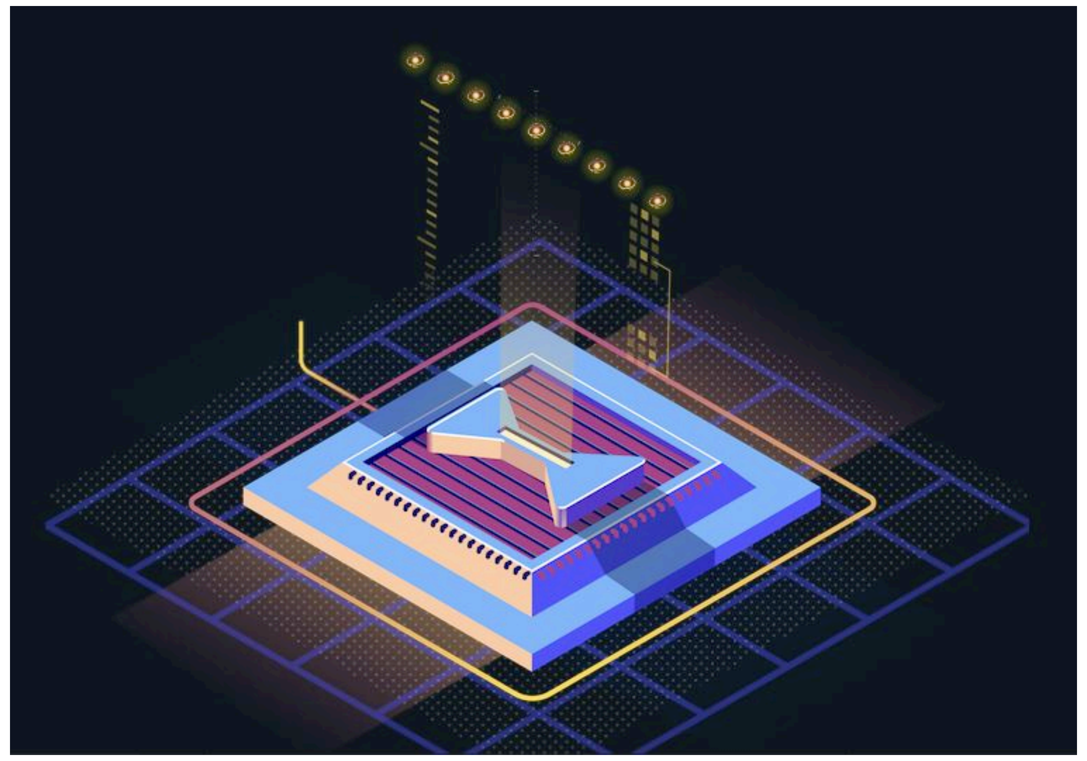
SUPERCONDUCTING



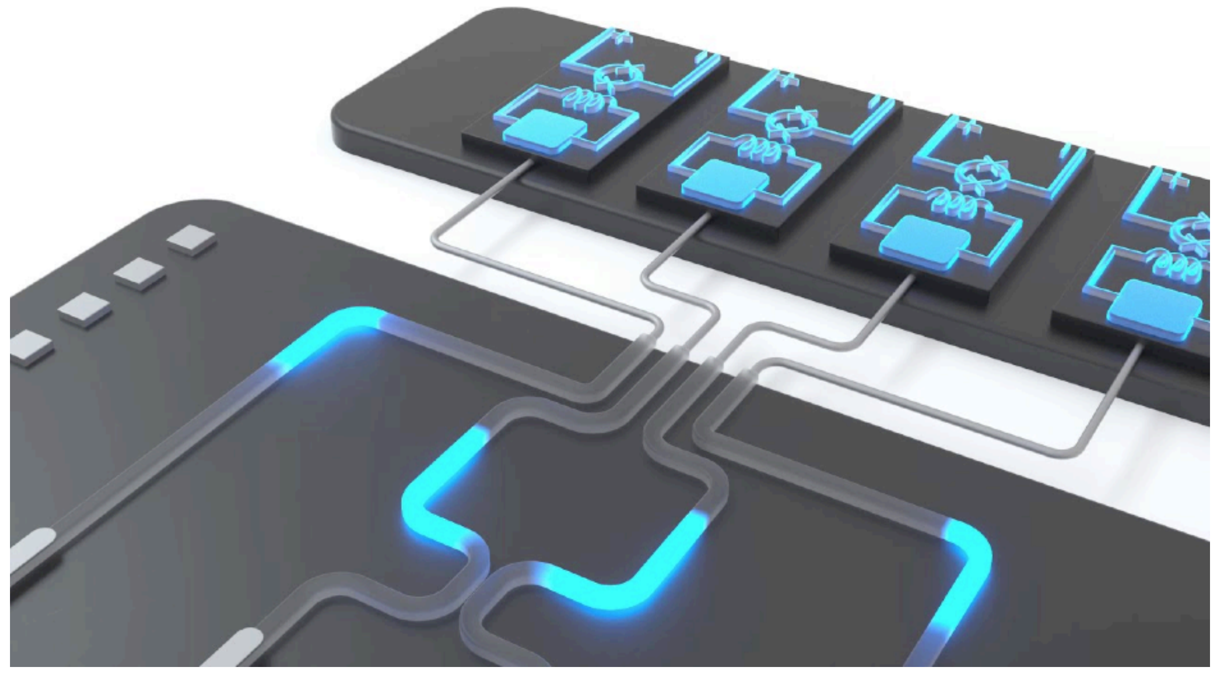
NEUTRAL ATOMS



TRAPPED IONS



PHOTONICS



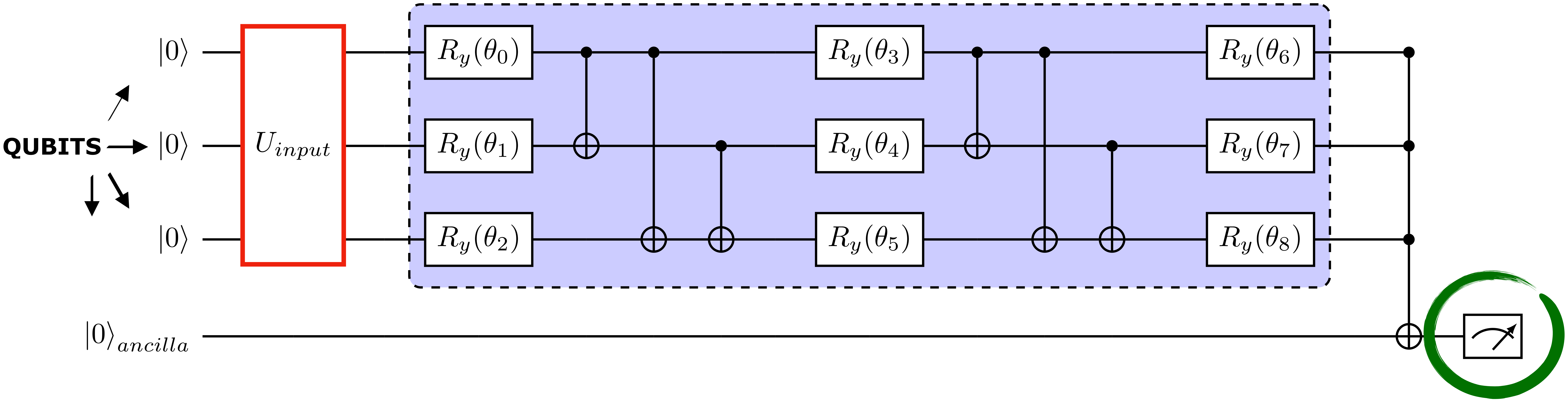
Quantum Computing

Computation using quantum mechanics laws

INPUT STATE

UNITARY OPERATIONS

MEASURE



Main goal: <u>**Quantum Advantage**</u>

Quantum Advantage

Requires entanglement

	Theoretical Quantum Advantage	Practical Quantum Advantage
Problem	Deutsch-Josza[1] Grover[1] Shor[1]	Sampling
Speed-up	Exponential (Deutsch-Josza, Shor) Quadratic (Grover)	Google[2] (2019): $200\ s$ vs 10000 years Xanadu[3] (2022): $36\ \mu s$ vs 9000 years

[1] D. Bruß and C. Macchiavello Phys. Rev. A 85, 049906 (2012)

[2] F. Arute, et al., Nature 574, 505–510 (2019)

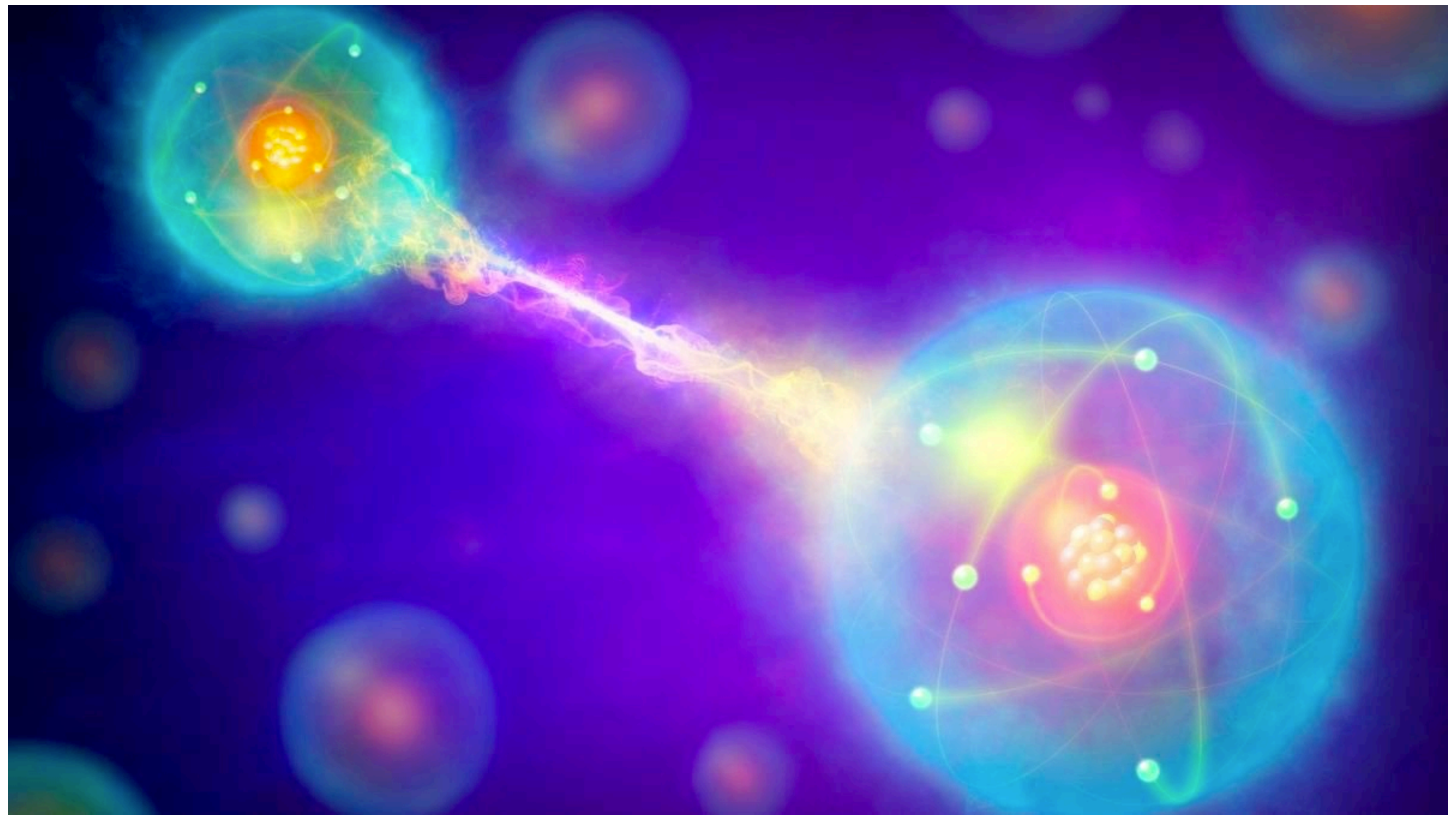
[3] L.S. Madsen, et al. Nature 606, 75–81 (2022)

Quantum Entanglement

A non-classical correlation

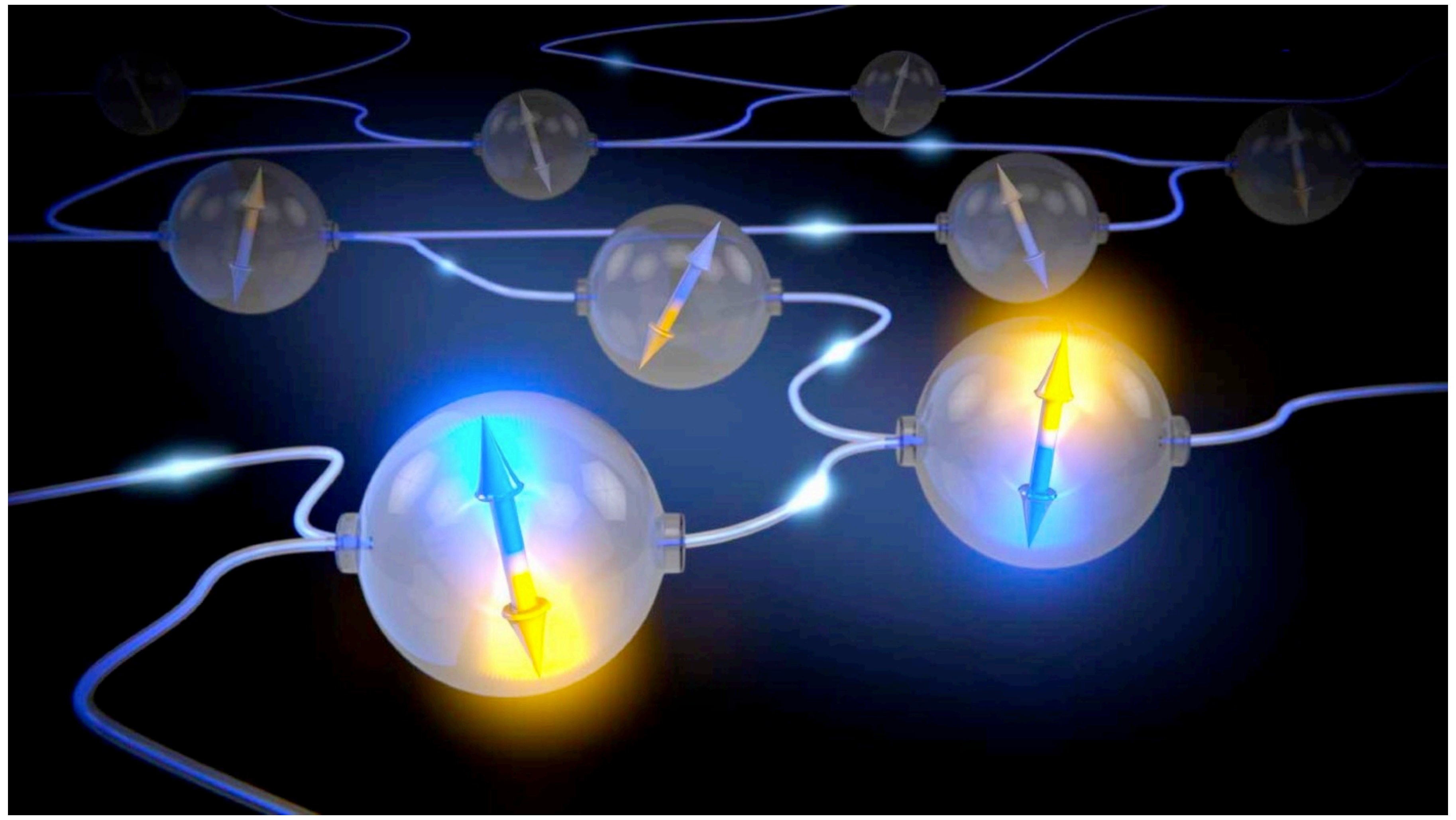
A genuine quantum mechanical property of physical systems

Bipartite entanglement



Shared by two particles

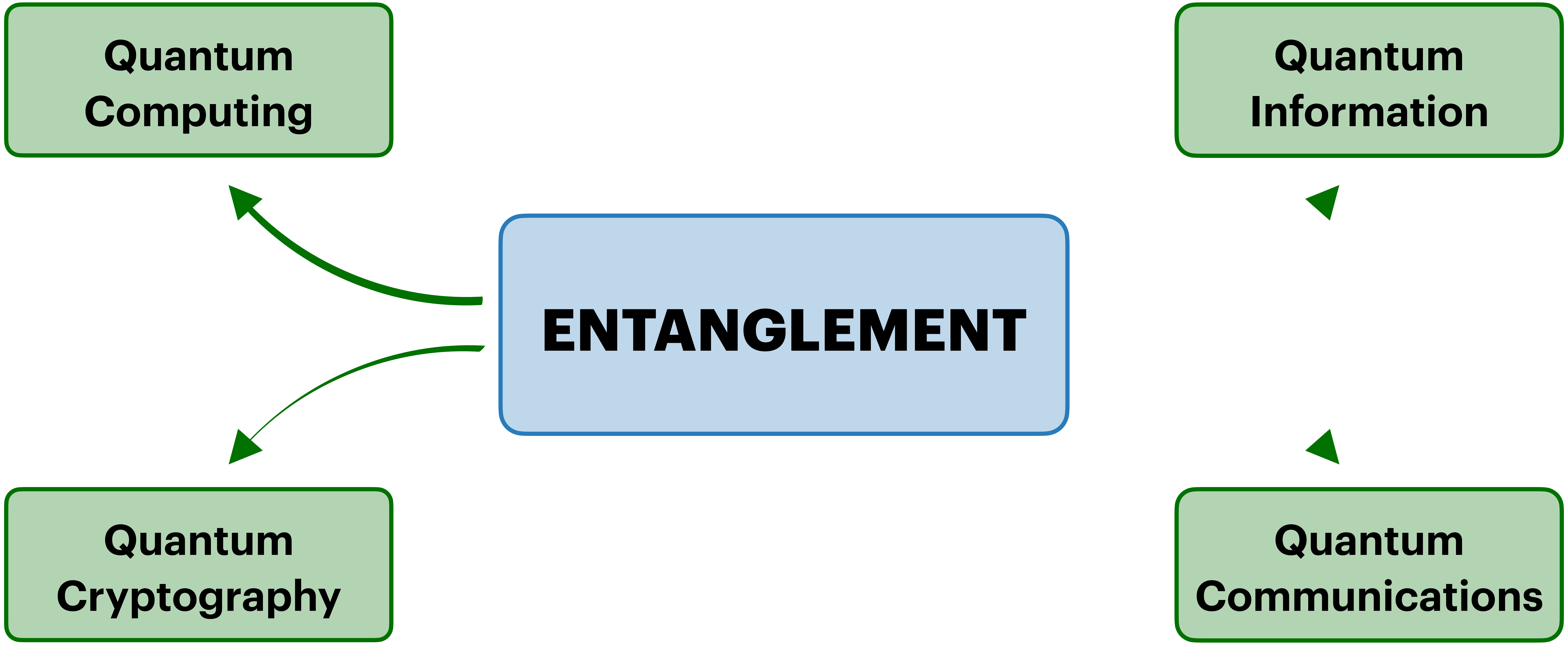
Multipartite entanglement



Shared by multiple particles

Why seeking for entanglement?

It is a fundamental resource in Quantum Technologies



How to detect entanglement?

Exponential difficulties

Classical methods:

HARD to simulate large quantum systems → $\dim(\mathscr{H}) = 2^N$ for N qubits

Quantum methods:

Quantum State Tomography → 3^N quantum measurements for N qubits

Machine Learning and Data

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C: Classical
Q: Quantum

		Machine Learning	
		C	Q
Data	C	CC	QC
	Q	CQ	QQ

Machine Learning and Data

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C: Classical
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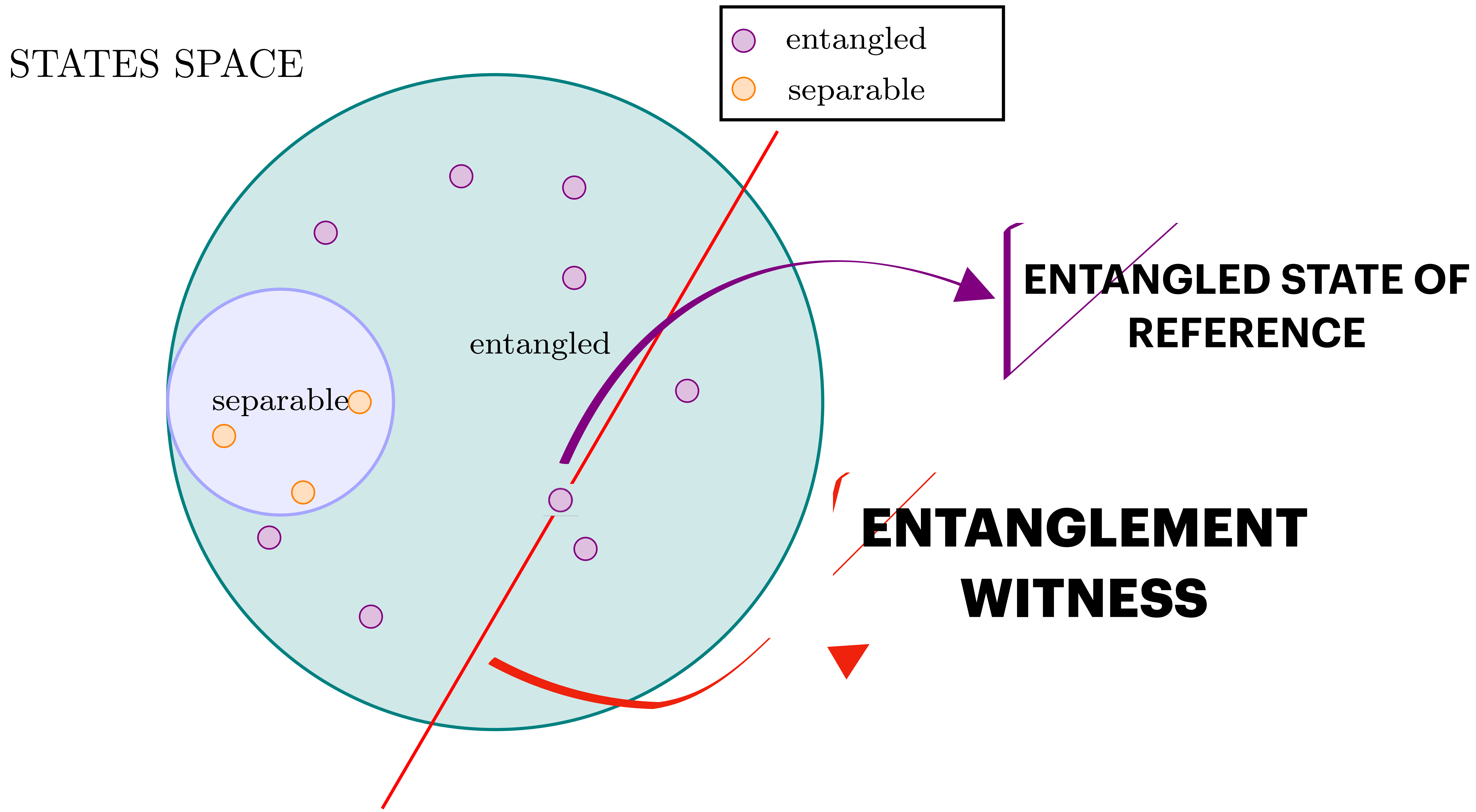
		Machine Learning C	Machine Learning Q
Data	C	CC	QC
Data	Q	CQ	QQ

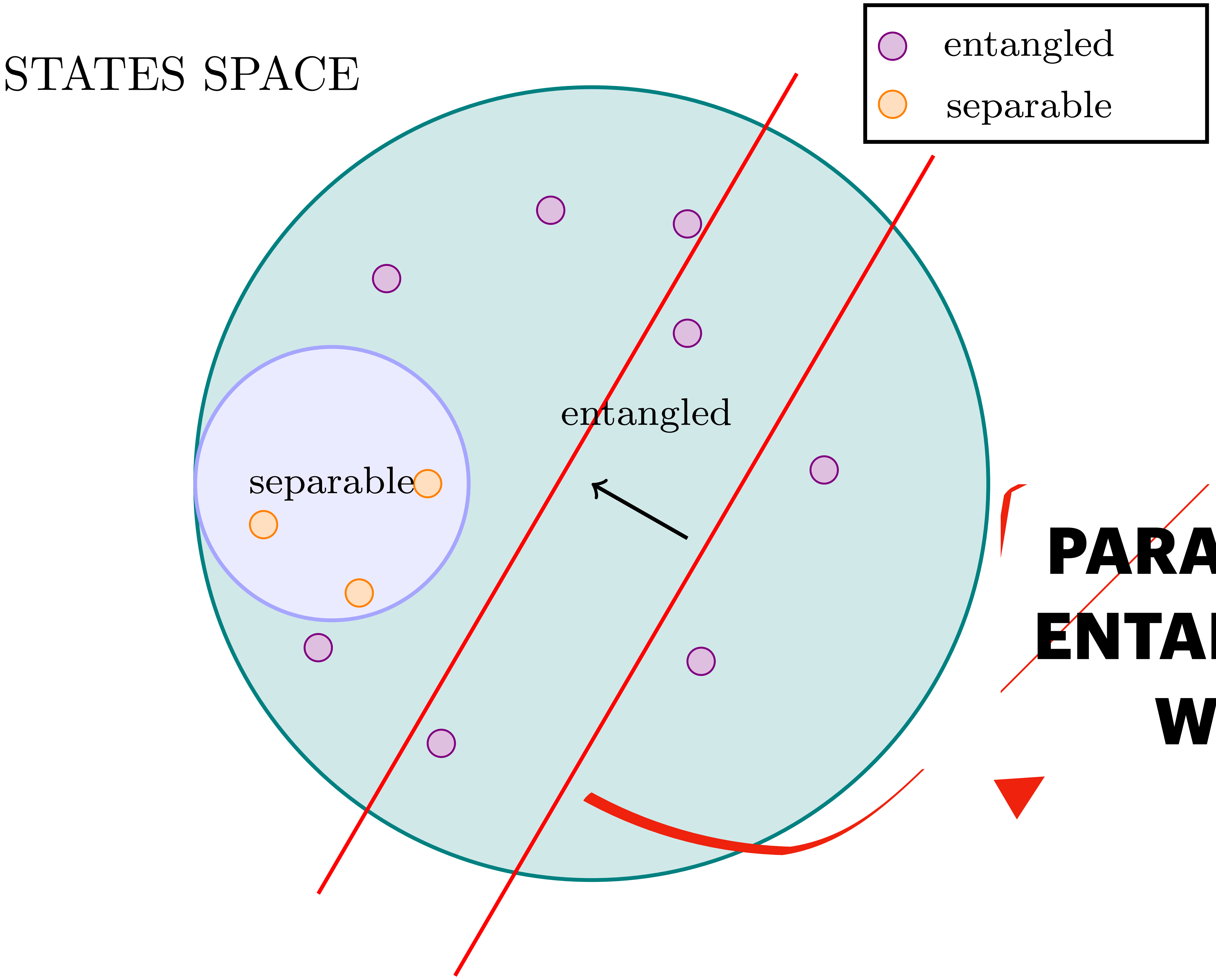
How to detect entanglement?

Projective entanglement witness



F. Scala, et al. *arXiv:2205.10429* (2022)

Witness Optimization
No state of reference



PARAMETRIZED ENTANGLEMENT WITNESS

J. Roik, et al., Phys. Rev. Applied, vol. 15, no. 5, 2021.
L. T. Wu, et al., in Conference on Lasers and Electro-Optics, Optical Society of America, 2021, FW3N.1

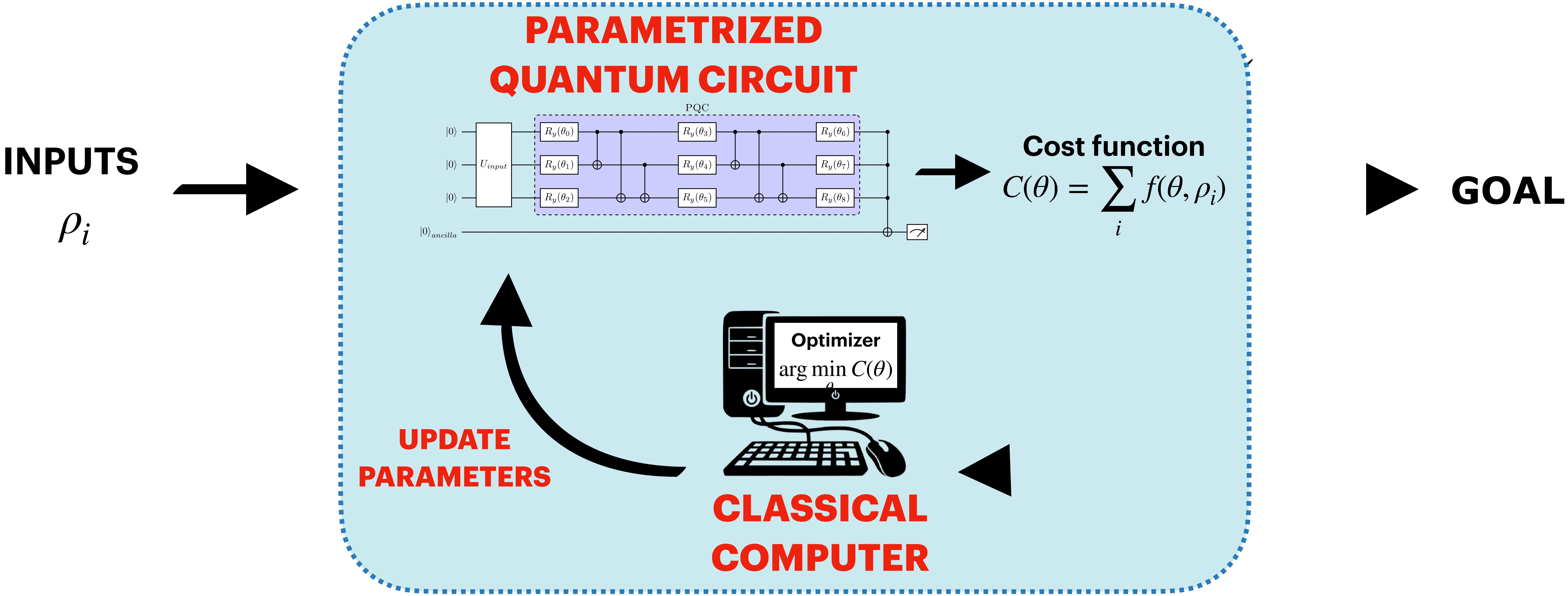
Aim of the work

Detecting multipartite entanglement
with a
Variational Quantum Algorithm (VQA)

F. Scala, et al. *arXiv:2205.10429* (2022)

Variational Quantum Algorithms[1] (VQAs)

Are hybrid



[1] M. Cerezo, et al. Nat Rev Phys 3, 625–644 (2021)

Problem inspired cost function

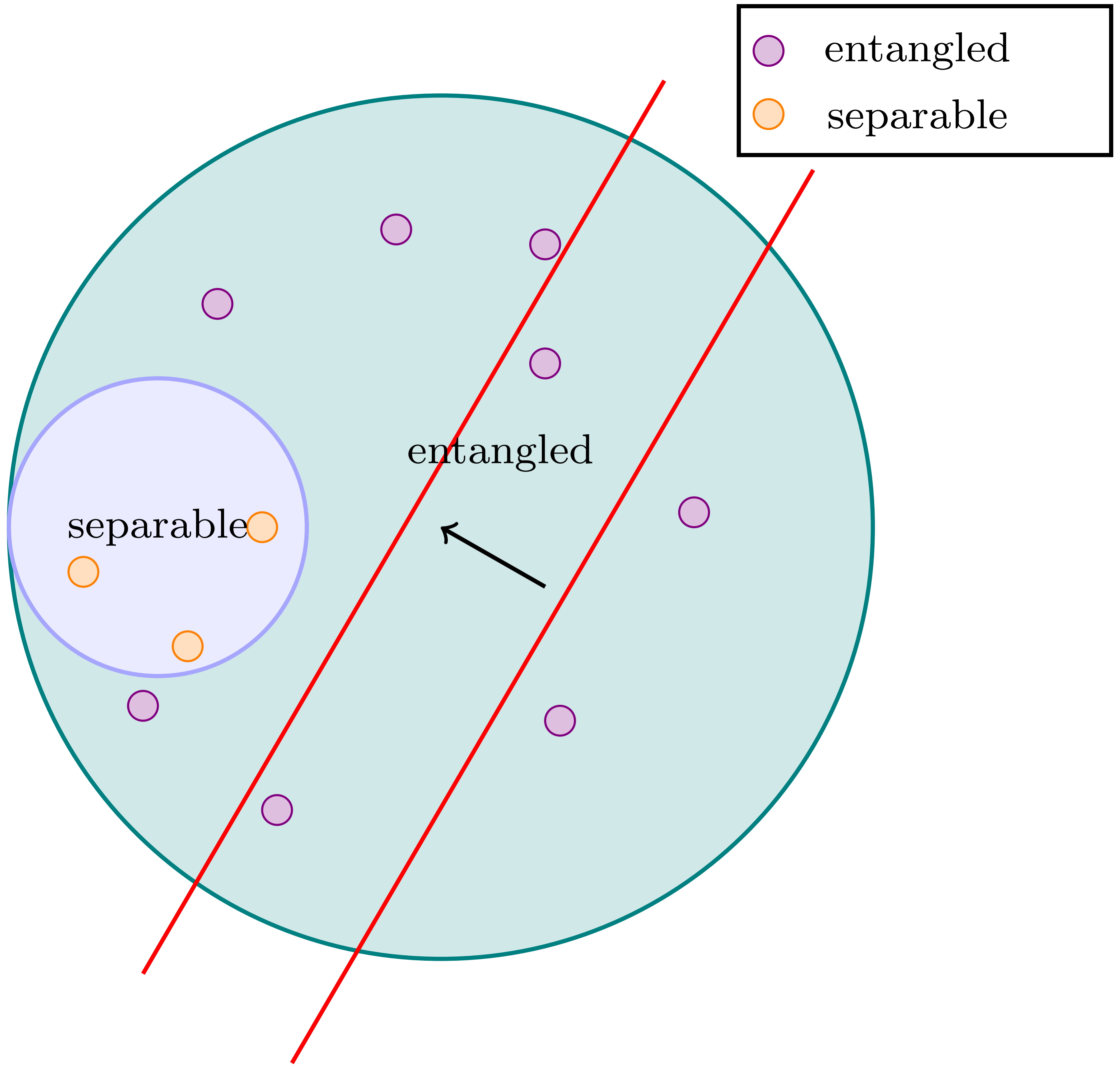
Precision = $\frac{\text{True Positive}}{\text{All Positive}}$

Recall = $\frac{\text{True Positive}}{\text{Positive Class}}$

$$F_\beta = (1+\beta^2)\cdot\frac{\text{Precision}\cdot\text{Recall}}{\beta^2\cdot\text{Precision}+\text{Recall}}$$

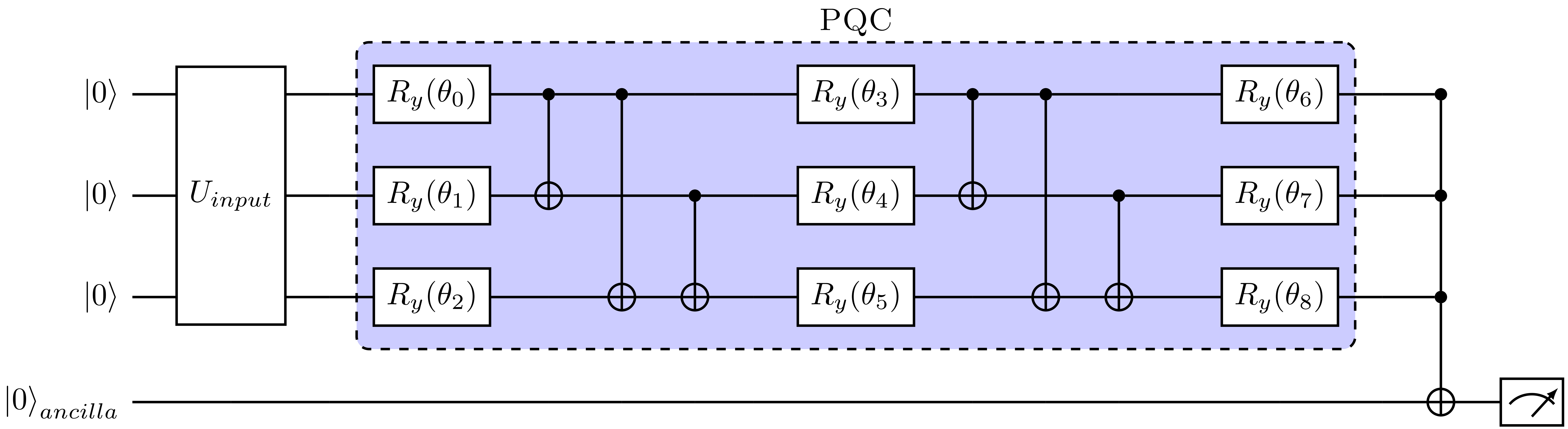
Favor precision over recall $0 < \beta < 1$

$$C(\theta) = 1 - F_\beta$$



Parametrized Quantum Model

Parametrized Quantum Perceptron[1]

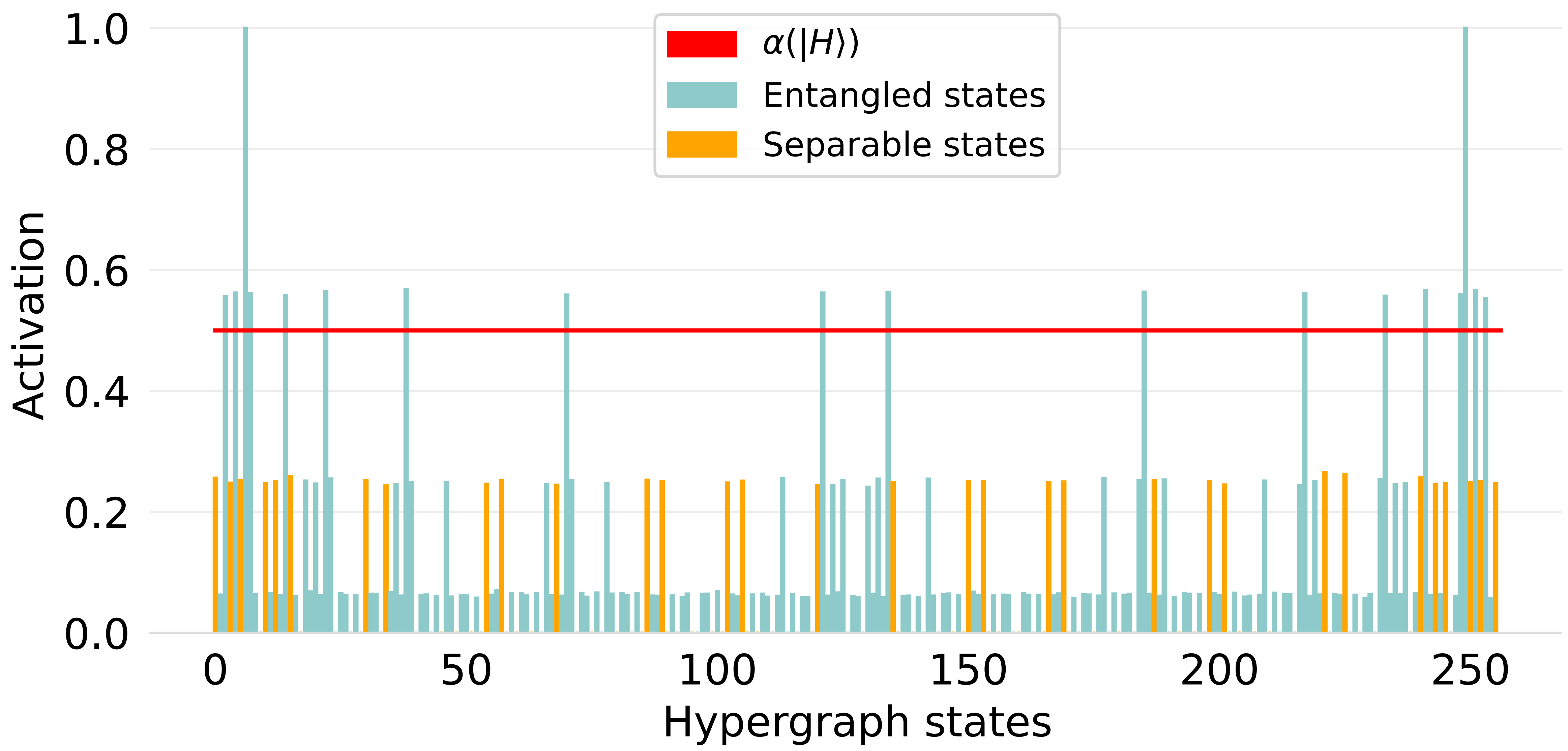


Only one measurement setting required!

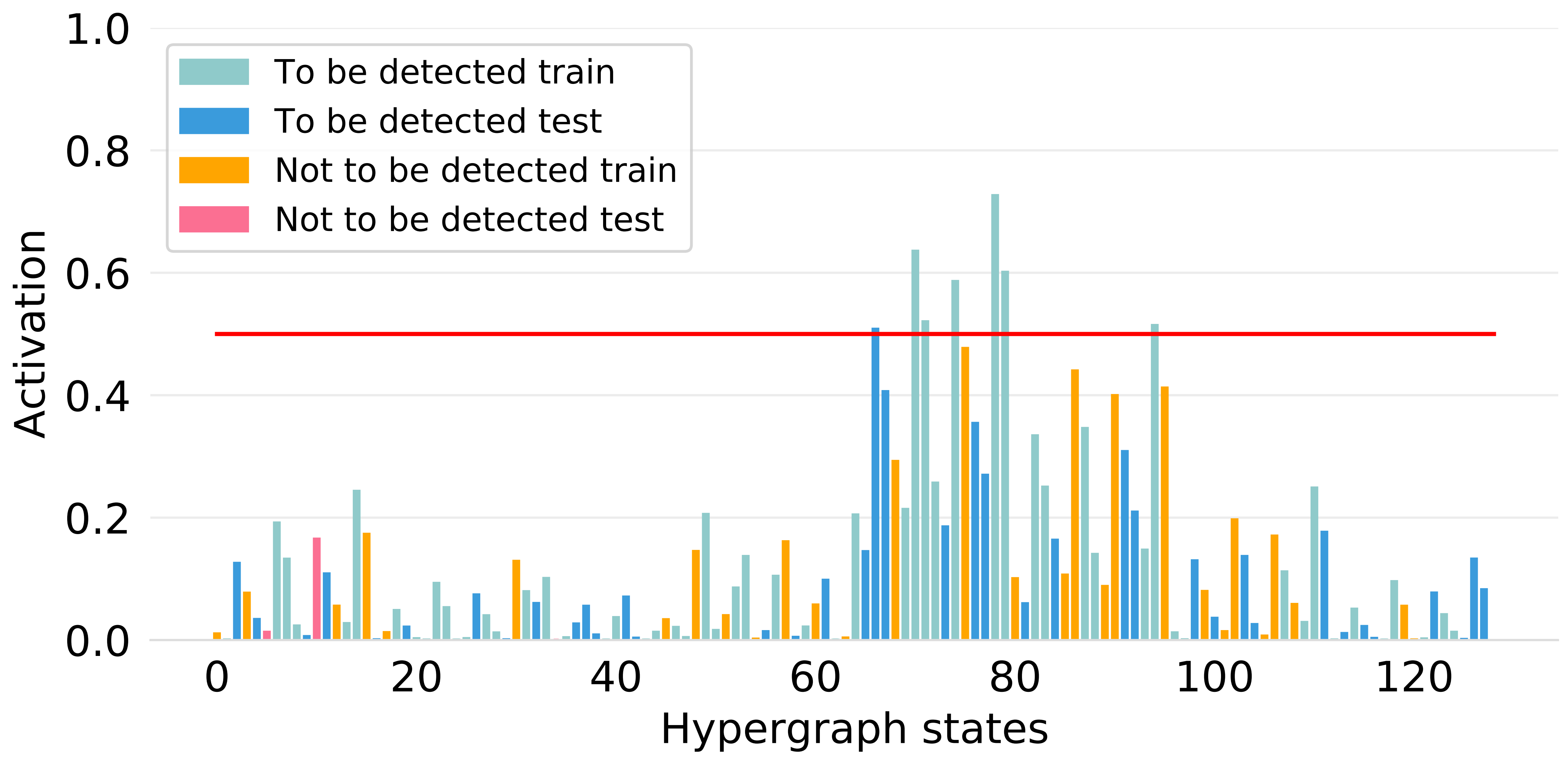
[1] F. Tacchino et al. , IEEE Transactions on Quantum Engineering, vol. 2, pp. 1–10 (2021)

Results

Theoretical with reference state



Learned Entanglement Witness



F. Scala, et al. ***arXiv:2205.10429*** **(2022)**

Summary and outlooks

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Entanglement is non-classical correlation typical of quantum systems

➡ **VQAs are able to learn an entanglement witness**

➡ **VQAs more suited to work with quantum data → direct state manipulation**

Future works

Generalization:

- Inputs and model
- Optimizer
- Entanglement measures

Collaborators

Stefano Mangini

Chiara Macchiavello

Daniele Bajoni

Dario Gerace

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Dipartimento di Fisica

Dipartimento di Ingegneria Industriale e dell'Informazione

For further details:
arXiv:2205.10429 (2022)

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Quantum Entanglement

A non-classical correlation

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DEFINITION

A n-parties state $|\psi\rangle$ is called ***fully entangled*** **if and only if** it is not bi-separable with respect to any bipartition:

$$|\psi\rangle = |A\rangle \otimes |B\rangle,$$

where $|A\rangle$is an m parties state and $|B\rangle$ is an n−m parties state.

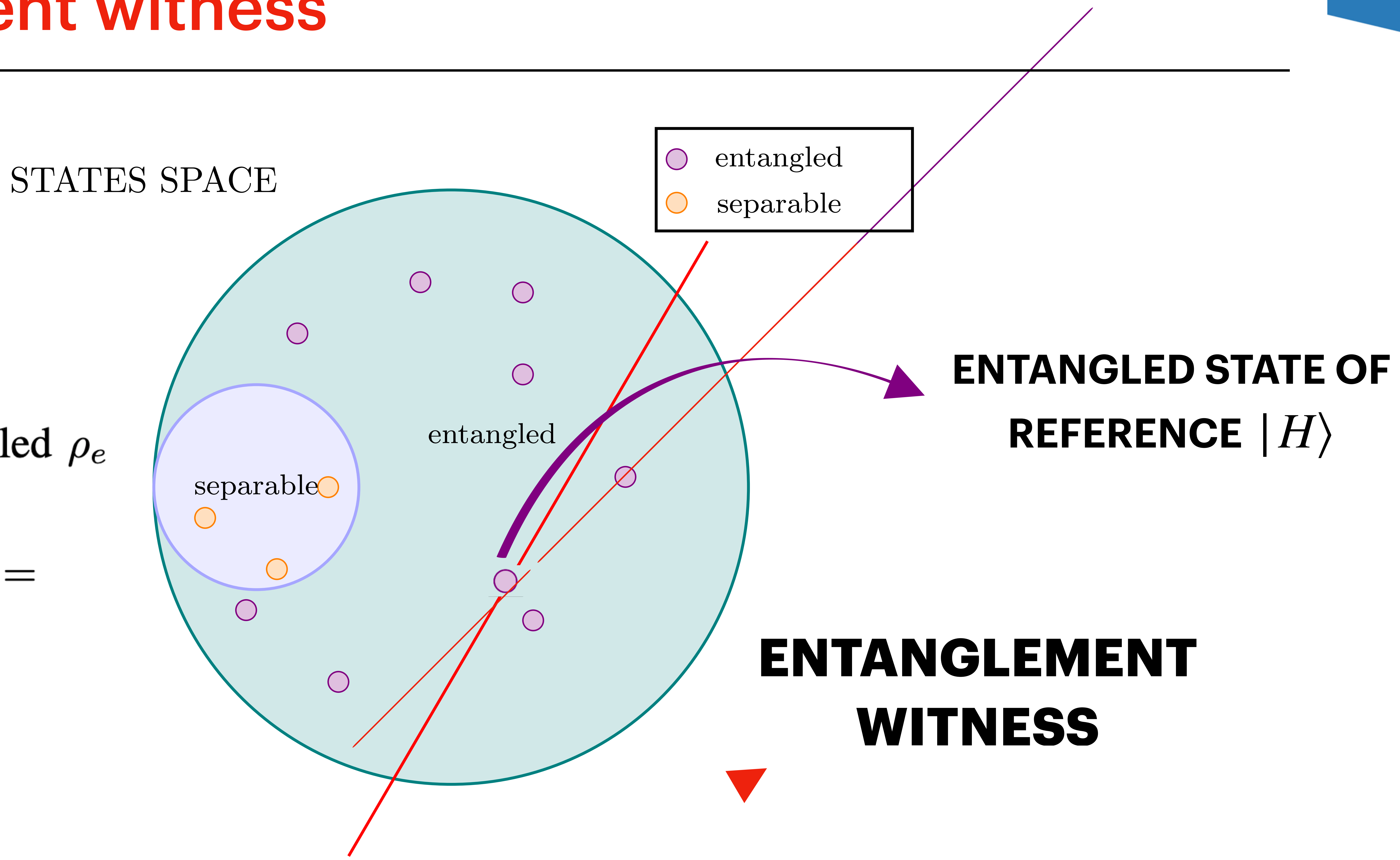
How to detect entanglement?

Projective entanglement witness

$$W = \alpha(|H\rangle)\mathrm{I} - |H\rangle\langle H|$$

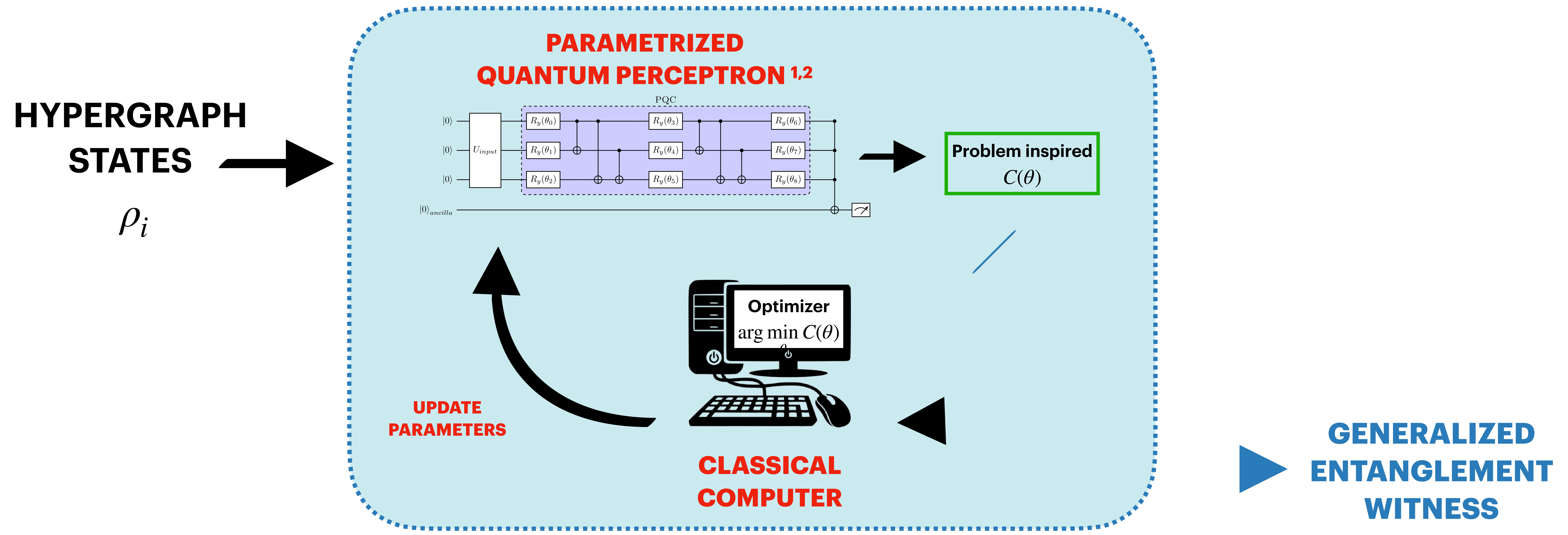
$$\begin{cases} \mathrm{Tr}[W\rho_s] \geq 0 & \forall \rho_s \text{ separable,} \\ \mathrm{Tr}[W\rho_e] < 0 & \text{for at least one entangled } \rho_e \end{cases}$$

$$\begin{aligned} \mathrm{Tr}[\rho W] &= \mathrm{Tr}[\rho(\alpha(|H\rangle)\mathbb{I} - |H\rangle\langle H|)] = \\ &= \alpha(|H\rangle) - \mathrm{Tr}[\rho\,|H\rangle\langle H|] \quad . \end{aligned}$$



F. Scala, et al. *arXiv:2205.10429* (2022)

Variational Quantum Algorithm (VQA)
For entanglement witnessing



[1] F. Tacchino et al. , npj Quantum Inf 5, 26 (2019)

[2] F. Tacchino et al. , IEEE Transactions on Quantum Engineering, vol. 2, pp. 1–10 (2021)

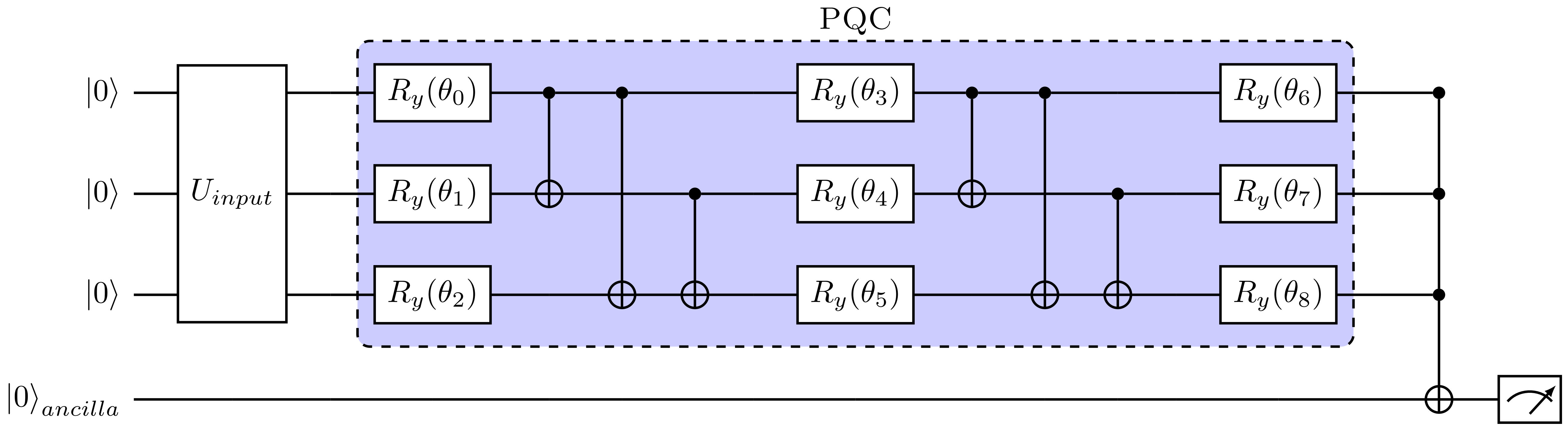
Model

Separable-entangled proportions of hypergraph states (3 qubits)

256 states : 64 bi-separable, 192 are entangled (64 have maximally entangled)

Train-Test proportions

- Train: 60% of the entangled states and 90% of the separable ones



- **2 layers Parametrized Quantum Model**
- **COBYLA optimizer**
- **Activation threshold 0.5**