Symmetrizing quantum machine learning for Quantum Field Theory

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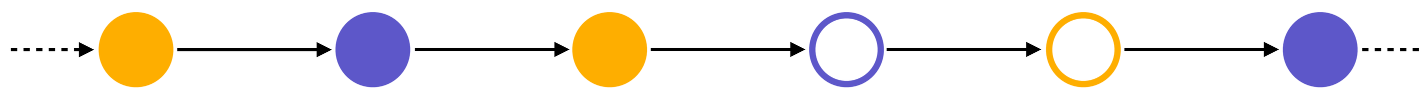
ABSTRACT

Research in the field of **Quantum Machine Learning** (QML) has highlighted issues in balancing trainability and expressivity of **Quantum Neural Networks** (QNNs). To tackle these issues, approaches have been identified to **exploit symmetries** of the data to create more efficient QNNs[1].
A framework that can offer both QML tasks and symmetries is the **Schwinger Model**, a simplification of the Standard Model. This is useful in its 1+1 version to perform quantum simulations. In particular, it is applied to recognize eigenstates with an energy lower than a selected threshold of a n-site chain with spatial periodic boundary conditions[2], an approach known as **truncation**.
This work looks at implementing the symmetries in the chain in order to improve the trainability and expressivity of a QNN model. The task is performing **binary classification** of the physical states of the Schwinger Model under a certain energy threshold. The experiments were carried out using **noiseless numerical simulations**.

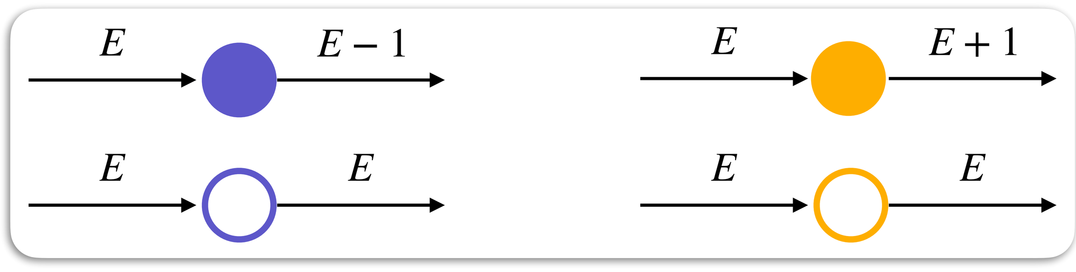
PROBLEM

Schwinger Model on a chain with periodic Boundary Conditions



Dots represent fermonic field sites

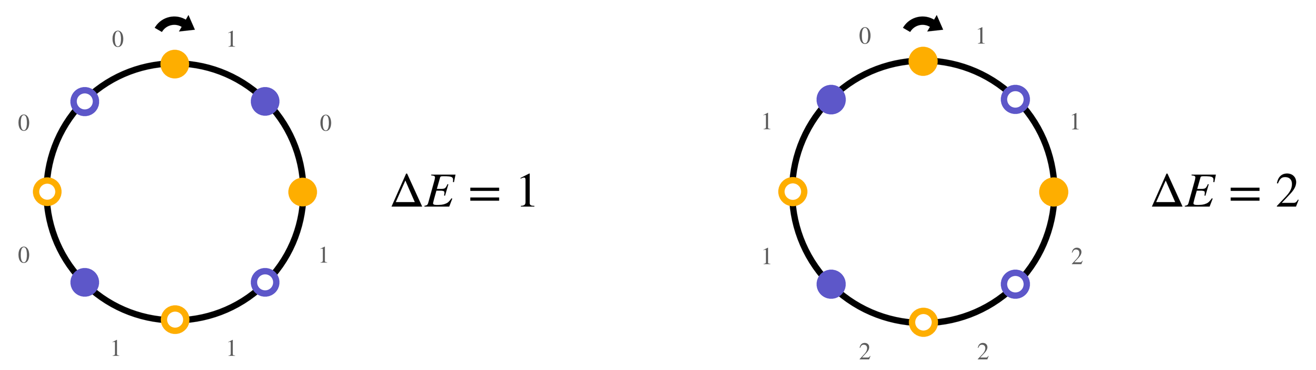
Links represent the electric field



- The energy of the system is proportional to $\Sigma_i E_i^2$, with i running over the sites.
- When interested in low energy states, the theory can be **truncated** defining

$$\Delta E \equiv E_{max} - E_{min}$$

And considering only the chains with ΔE under a certain value ε.



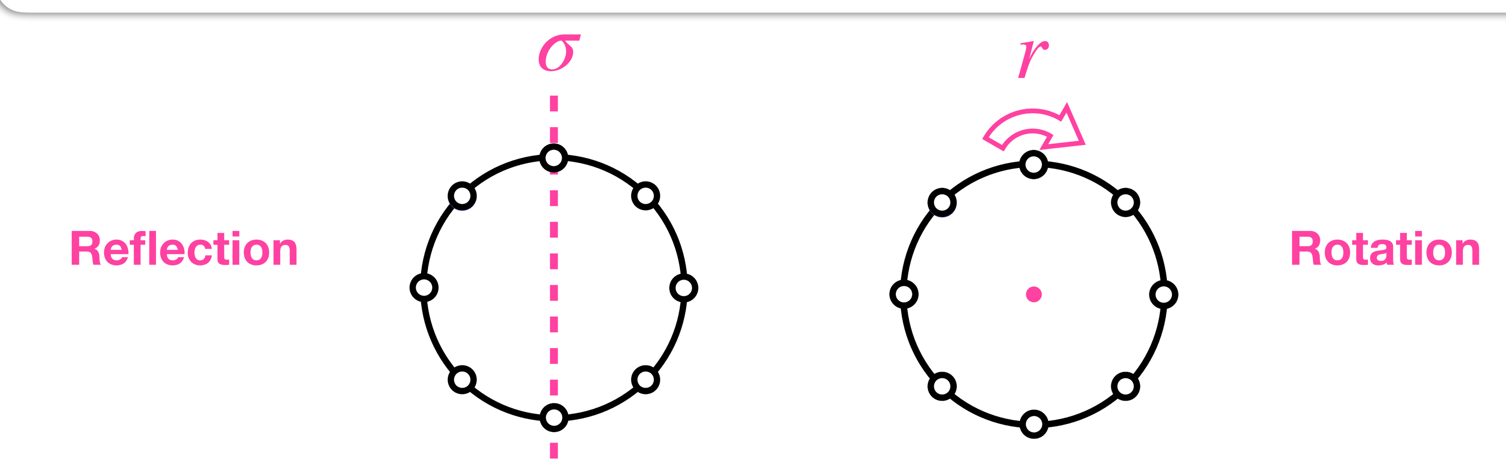
Symmetries

The value of ΔE associated with a configuration of a N-sites chain is invariant under the representation if the **N-Dihedral group**

$$D_N = \langle \sigma, r \,|\, \sigma^2 \cong r^N \cong e \rangle$$

group neutral element

The two generators of D_N act on the chain as a **reflection** and a **rotation** of $2\pi/N$



Classifying Schwinger states

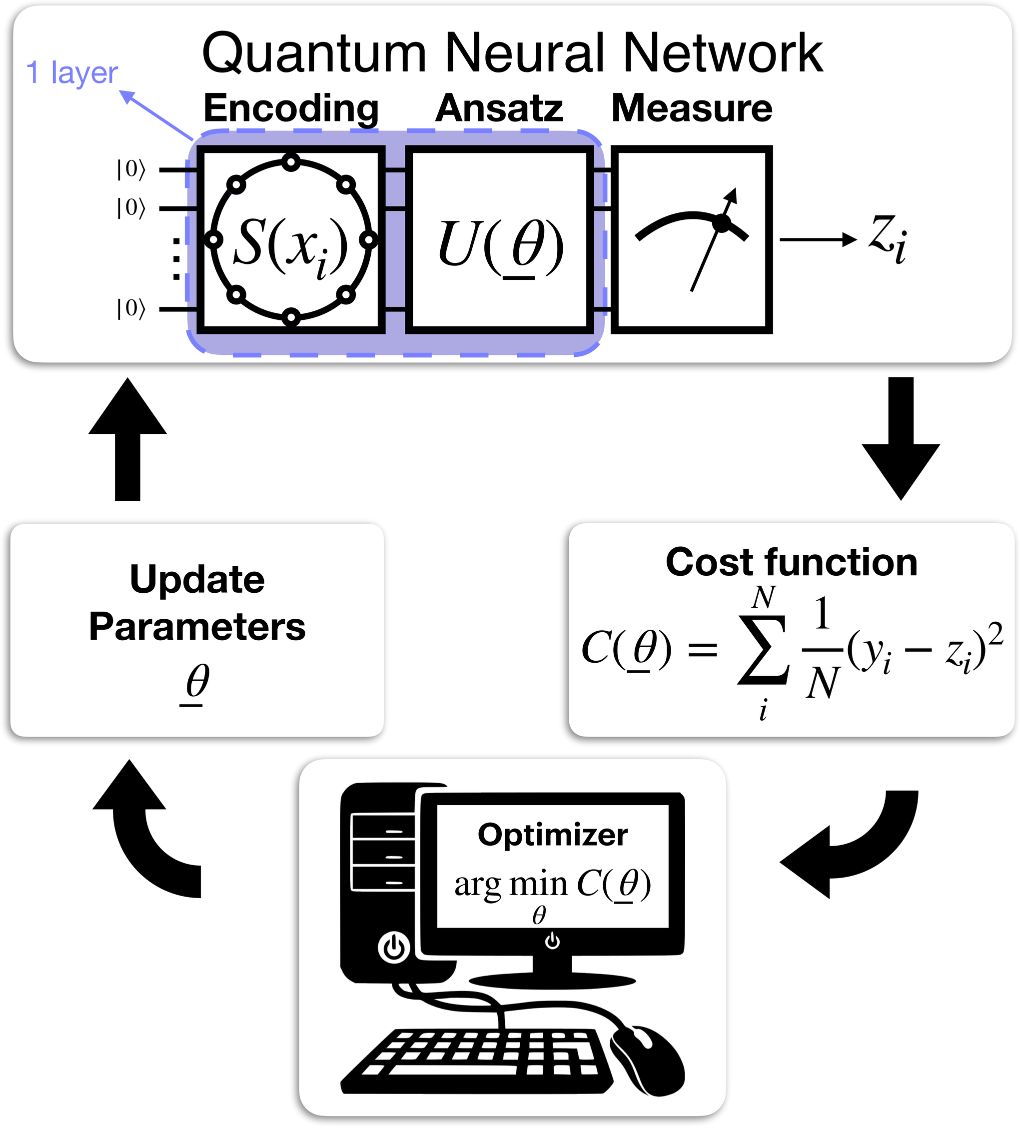
We focus on the Schwinger model discretized over an 8 sites chain, which has 69 different possible states. We set $\varepsilon = 2$, then our dataset is labelled in the following way:

- If $\Delta E < \varepsilon$, $y = 1$ (positive class) $\rightarrow 45$ states
- If $\Delta E \geq \varepsilon$, $y = -1$ (negative class) $\rightarrow 24$

Train set is composed of $70\,\%$ of the states, while we test over the remaining $30\,\%$.

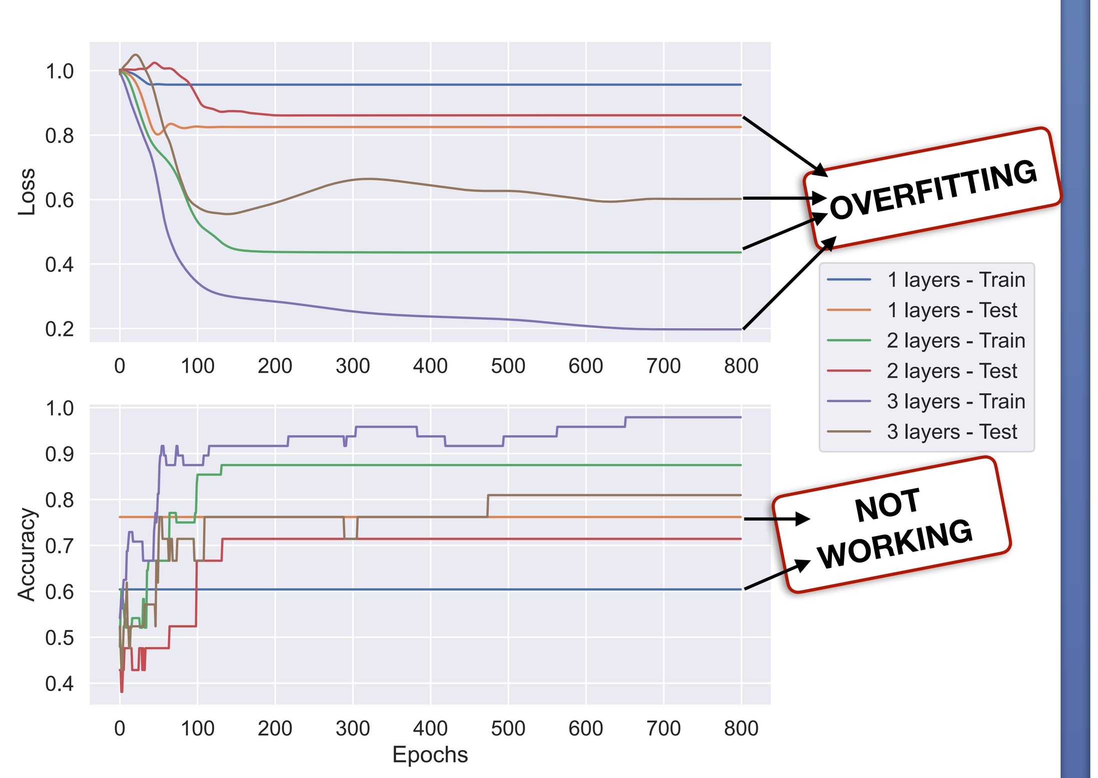
LEARNING APPROACH

Quantum Machine Learning Scheme



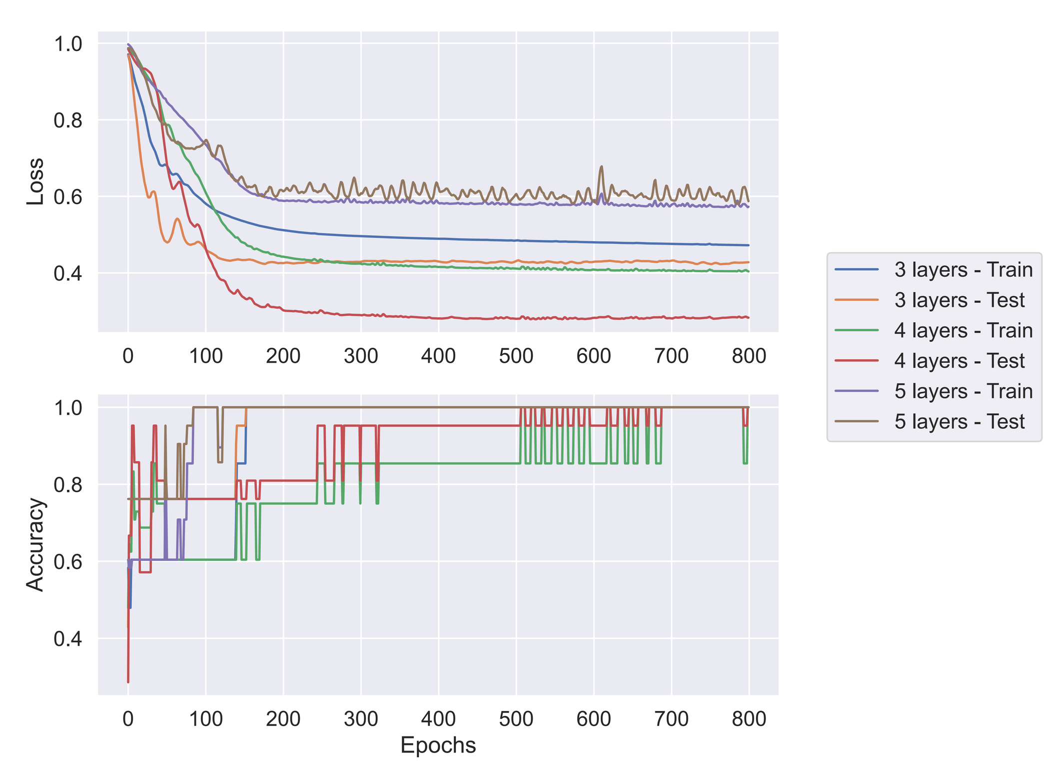
Standard Quantum Machine Learning

- **Encoding:** 1 site→1 qubit, $R_X(\pi \cdot x_i)$ rotations on each qubit
- **Ansatz:** R_X , R_Y , R_Z rotations on each qubit, followed by a compact circular layer of $C-Phase$ gates $\rightarrow 32$ parameters per layer



Symmetric Quantum Machine Learning

- **Encoding:** same
- **Ansatz:** R_X , R_Y , R_Z rotations on each qubit, followed by a compact circular layer of $C-Phase$ gates, but each kind of gate has the same parameter $\rightarrow$ 4 parameters per layer



CONCLUSIONS

We benchmark the theory of symmetric QNNs on the Schwinger model discretized over an 8 sites chain through numerical simulations. The performance of our model is compared to a Symmetry-free QNN, showing that exploiting symmetries has an impact on both the accuracy and the number of parameters needed to reach convergence. Further work would look at the application of the symmetries to tackle harder machine learning tasks and see how this algorithms work on NISQ hardware.

BIBLIOGRAFIA
[1] Meyer, J. J., Mularski, M., Gil-Fuster, E., Mele, A. A., Arzani, F., Wilms, A., & Eisert, J.“Exploiting Symmetry in Variational Quantum Machine Learning.” PRX Quantum, 4(1) (2023).
[2] Natalie Klco et al. “Quantum-classical computation of Schwinger model dynamics using quantum computers”. Physical Review A 98.3 (2018), p. 032331

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