UNIVERSITÀ DI PAVIA
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Symmetrizing Quantum Machine Learning for Quantum Field Theory

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ABSTRACT

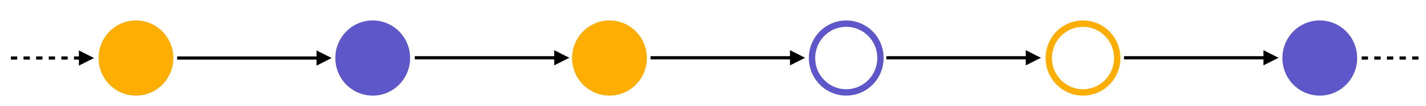
Research in the field of **Quantum Machine Learning** (QML) has highlighted issues in balancing trainability and expressivity of **Quantum Neural Networks** (QNNs). To tackle these issues, approaches have been identified to **exploit symmetries** of the data to create more efficient QNNs[1].

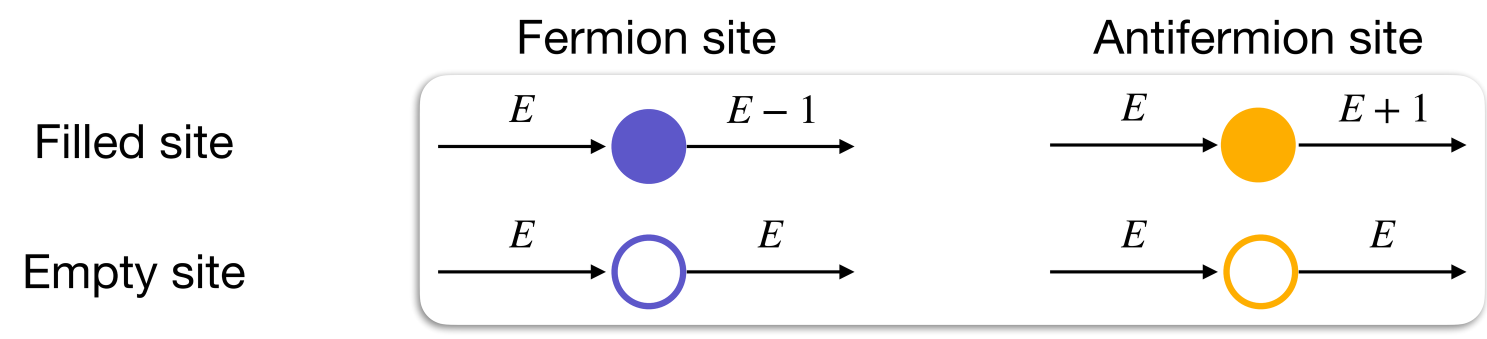
A framework that can offer both QML tasks and symmetries is the **Schwinger Model** [2] in 1+1 dimensions, a toy model of quantum electrodynamics. A case of interest in Quantum Simulations is the Schwinger Model with the spatial dimension discretized. The result is a N-sites chain, on which one usually imposes periodic boundary conditions, that evolves in time. A way to simplify the simulation is to consider only those eigenstates whose energy is lower than a selected threshold, an approach known as **truncation**.

The main goal of this work is the binary classification of the physical eigenstates of the Schwinger model based on truncation discrimination. This is done by exploiting the symmetries of the Schwinger chain in the variational ansatz of the QNN. The experiments are carried out using **noiseless numerical** .

PROBLEM

Schwinger Model on a chain with Periodic Boundary Conditions (PBC)

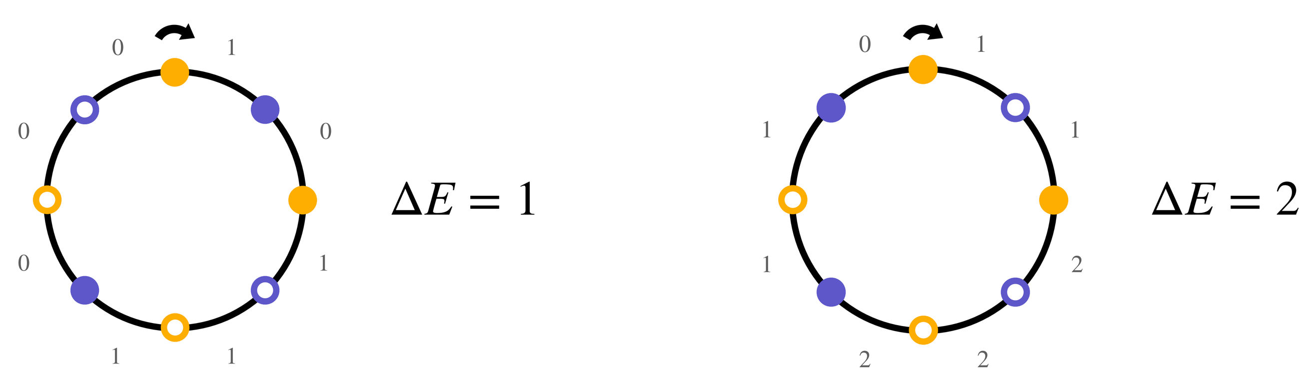




- The energy of the system is proportional to $\Sigma_i E_i^2$, with i running over the sites.
- When interested in low energy states, the theory can be **truncated** defining

$$\Delta E \equiv E_{max} - E_{min}$$

And considering only the chains with ΔE under a certain value ε.



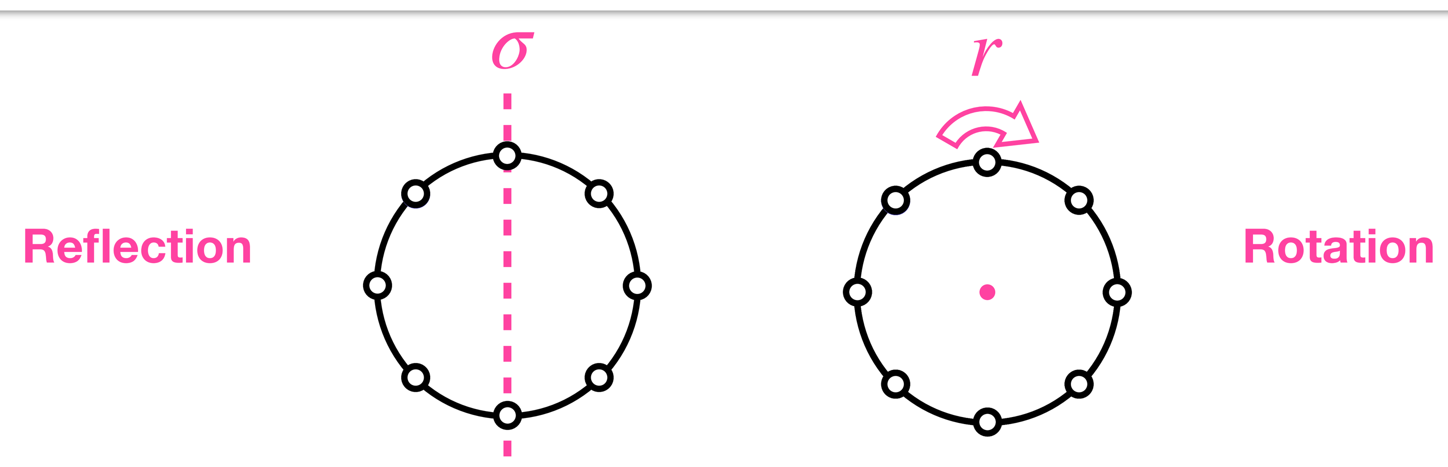
Symmetries

The value of ΔE associated with a configuration of a N-sites chain is invariant under the representation if the **N-Dihedral group**

$$D_N = \langle \sigma, r \,|\, \sigma^2 \cong r^N \cong e \rangle$$

group neutral element

The two generators of D_N act on the chain as a **reflection** and a **rotation** of $2\pi/N$



Classifying Schwinger states

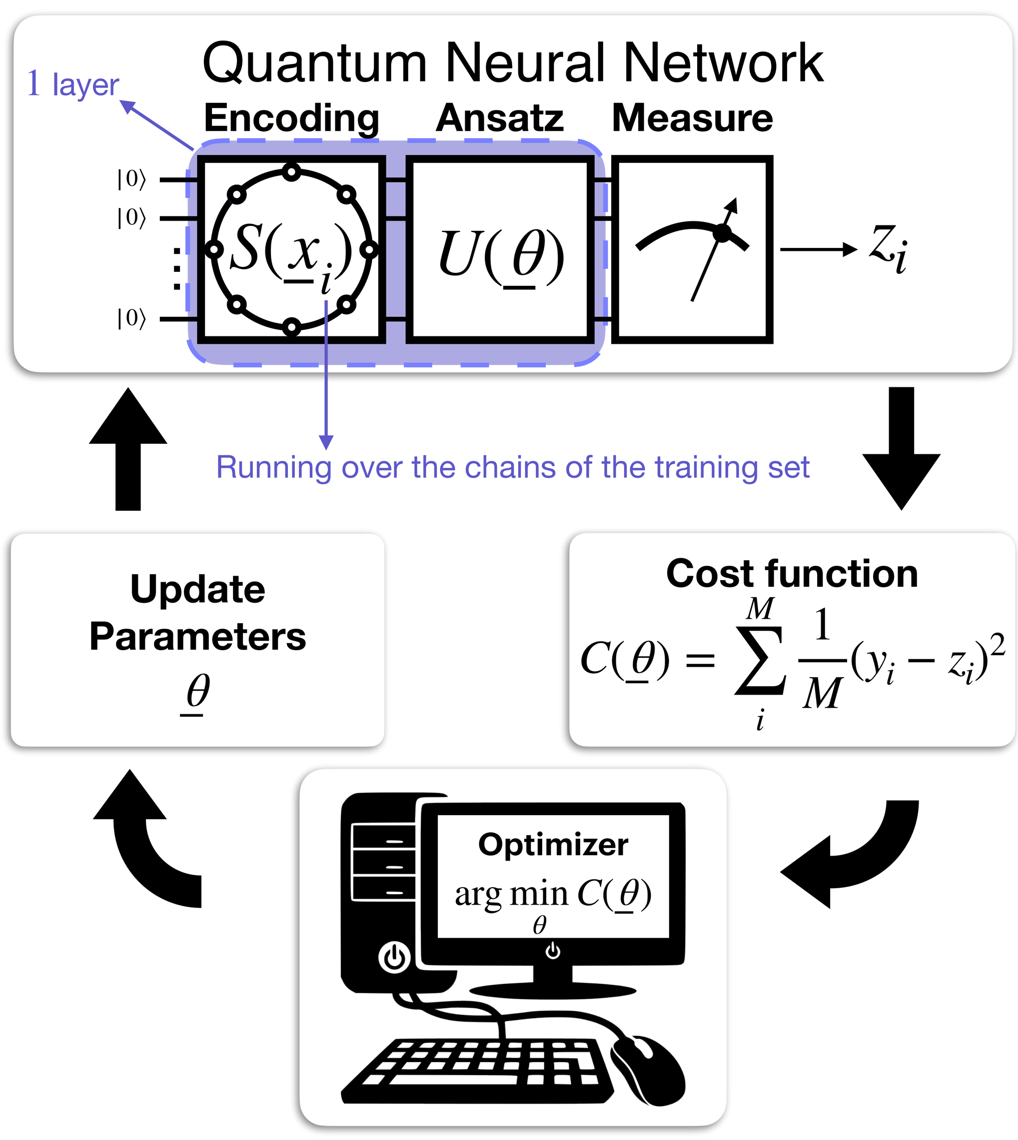
We focus on the Schwinger model discretized over an 8 sites chain with PBC, which has 70 different possible configurations. Each chain $\underline{x}$ has 8 features, representing the occupation number of its sites. We set $\varepsilon = 2$, then our dataset is labelled in the following way:

- If $\Delta E < \varepsilon$, $y = 1$ (positive class) $\rightarrow 46$ states
- If $\Delta E \geq \varepsilon$, $y = -1$ (negative class) $\rightarrow 24$ states

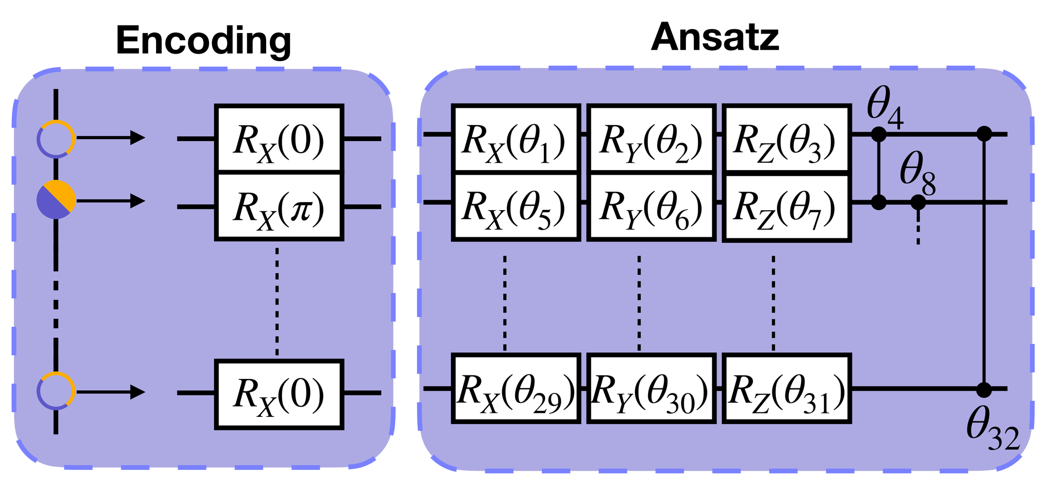
Train set is composed of $70\,\%$ of the states, while we test over the remaining $30\,\%$.

LEARNING APPROACH

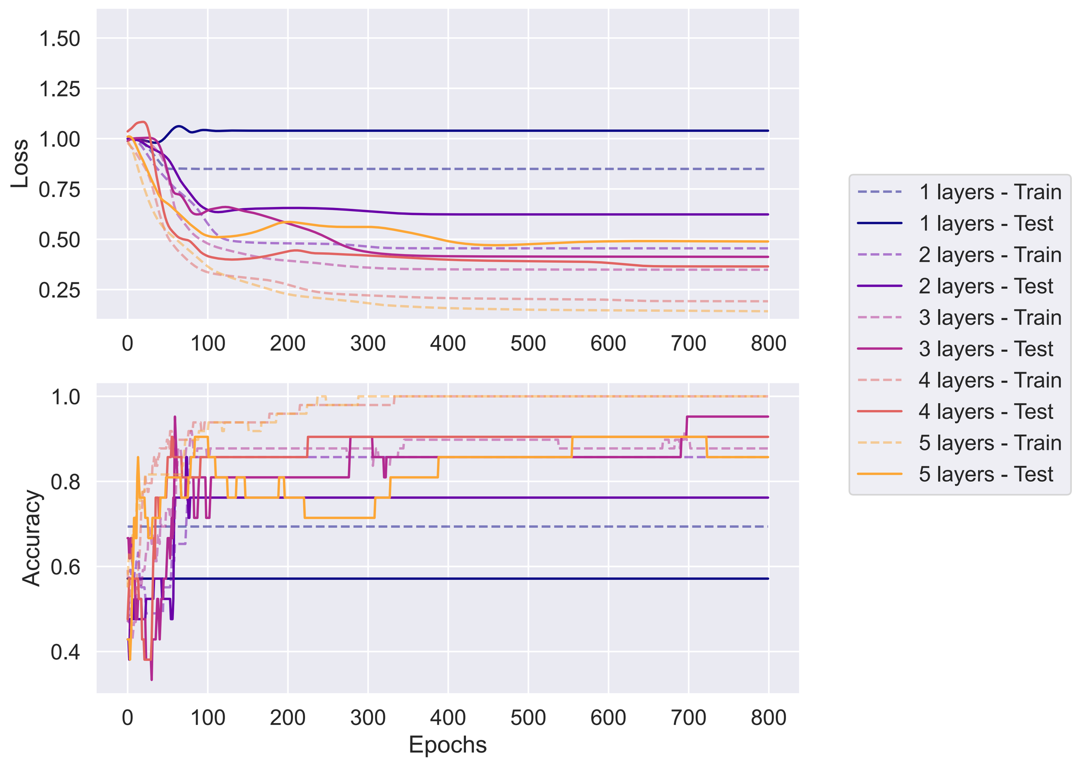
Quantum Machine Learning Scheme



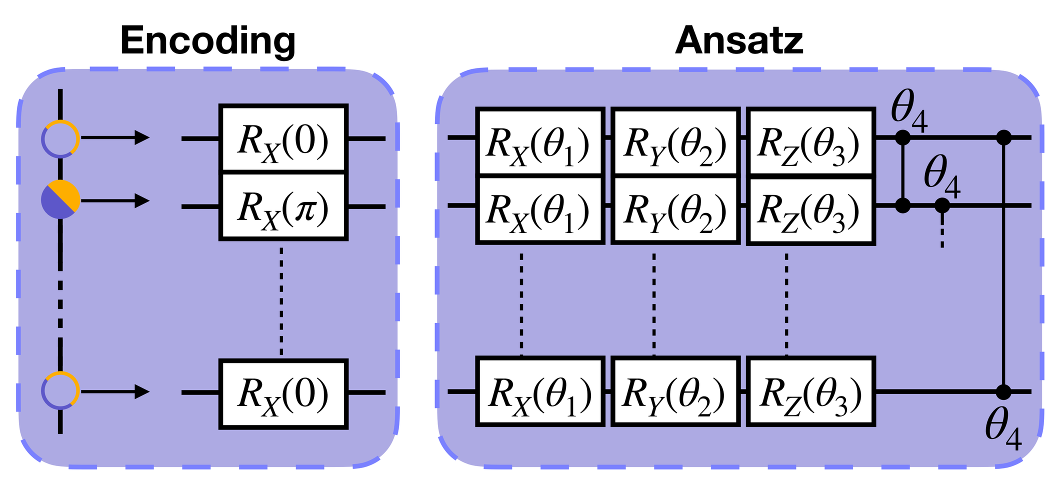
Standard Quantum Machine Learning



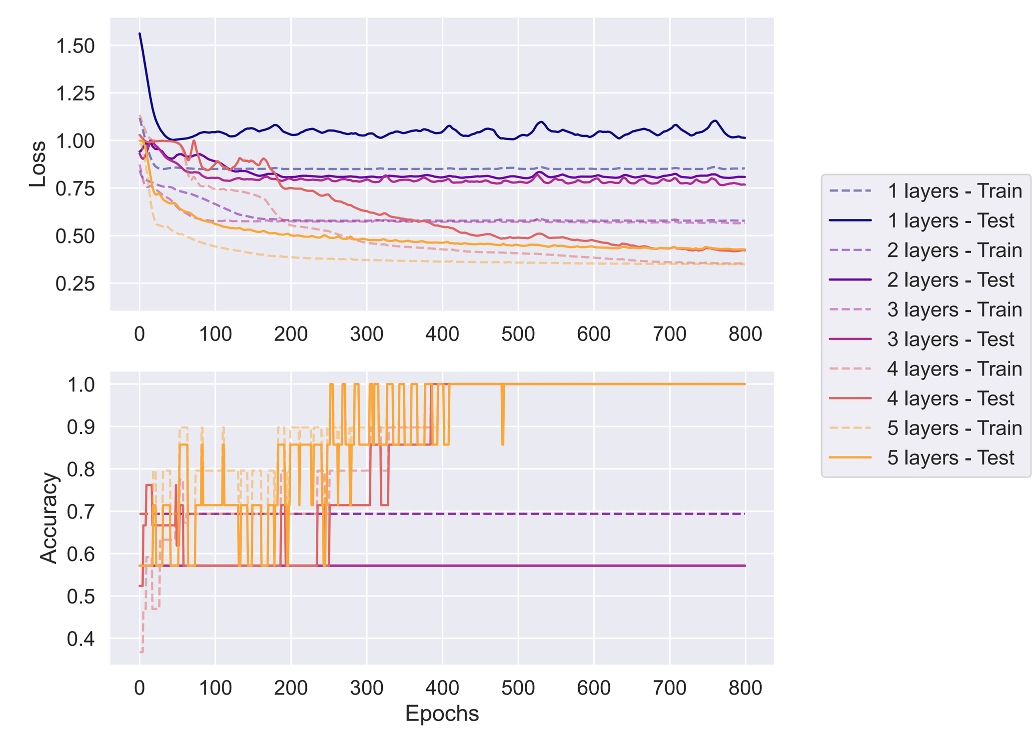
32 parameters



Symmetry-based Quantum Machine Learning



4 parameters



CONCLUSIONS

We benchmark the theory of symmetry-based QNNs on the Schwinger model discretized over a 8 sites chain through numerical simulations. The performance of our model is compared to a Symmetry-free QNN, showing that exploiting symmetries has an impact on both the accuracy and the number of parameters needed to reach convergence. Further work would look at the application of the symmetries to tackle harder machine learning tasks and see how these algorithms work on NISQ hardware.

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