

Symmetrizing quantum machine learning for Quantum Field Theory

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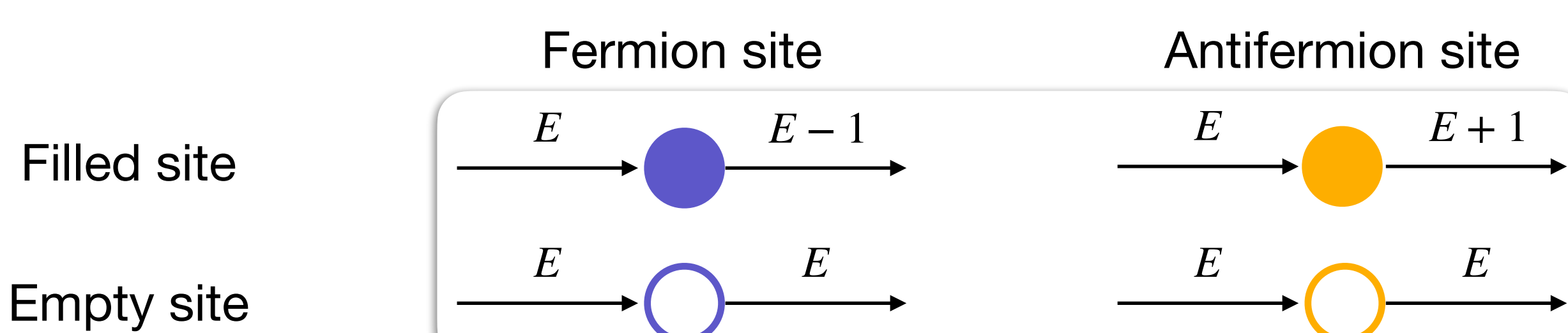
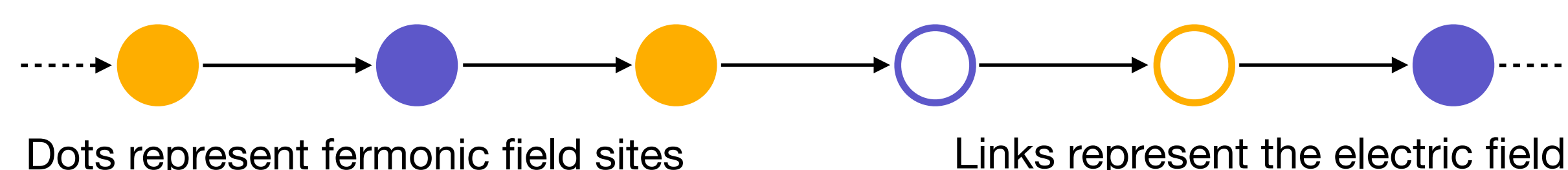
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ABSTRACT

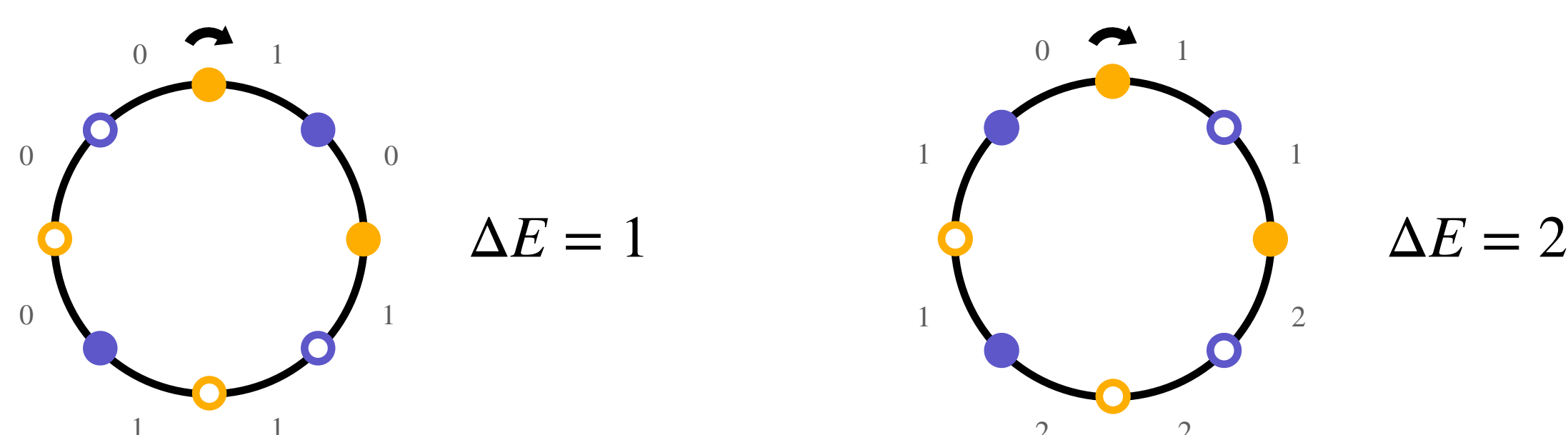
Research in the field of **Quantum Machine Learning** (QML) has highlighted issues in balancing trainability and expressivity of **Quantum Neural Networks** (QNNs). To tackle these issues, approaches have been identified to **exploit symmetries** of the data to create more efficient QNNs[1]. A framework that can offer both QML tasks and symmetries is the **Schwinger Model** [2], a toy model of the Standard Model. A case of interest in Quantum Simulations is the Schwinger model in 1+1 dimensions, one spatial and the other temporal, with the space coordinate discretised. The result is a N-sites chain, on which one usually imposes periodic boundary conditions, that evolves in time. A way to simplify the simulation is to consider only those eigenstates whose energy is lower than a selected threshold, an approach known as **truncation**. The main goal of this work is the binary classification of the physical eigenstates of the Schwinger model based on truncation discrimination. This is done by exploiting the symmetries of the Schwinger chain, both in the data encoding and in the variational ansatz of the QNN. The experiments are carried out using **noiseless numerical simulations**.

PROBLEM

Schwinger Model on a chain with Periodic Boundary Conditions (PBC)



- The energy of the system is proportional to $\sum_i E_i^2$, with i running over the sites.
 - When interested in low energy states, the theory can be **truncated** defining $\Delta E \equiv E_{max} - E_{min}$
- And considering only the chains with ΔE under a certain value ε .

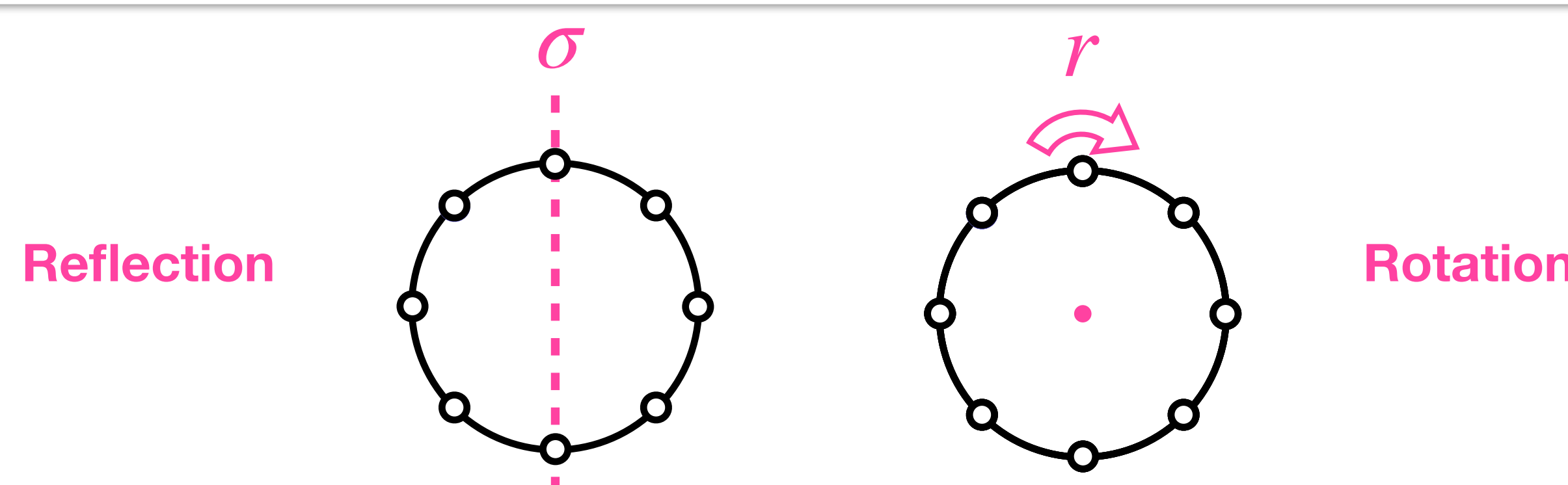


Symmetries

The value of ΔE associated with a configuration of a N-sites chain is invariant under the representation if the **N-Dihedral group**

$$D_N = \langle \sigma, r | \sigma^2 \cong r^N \cong e \rangle \rightarrow \text{group neutral element}$$

The two generators of D_N act on the chain as a **reflection** and a **rotation** of $2\pi/N$



Classifying Schwinger states

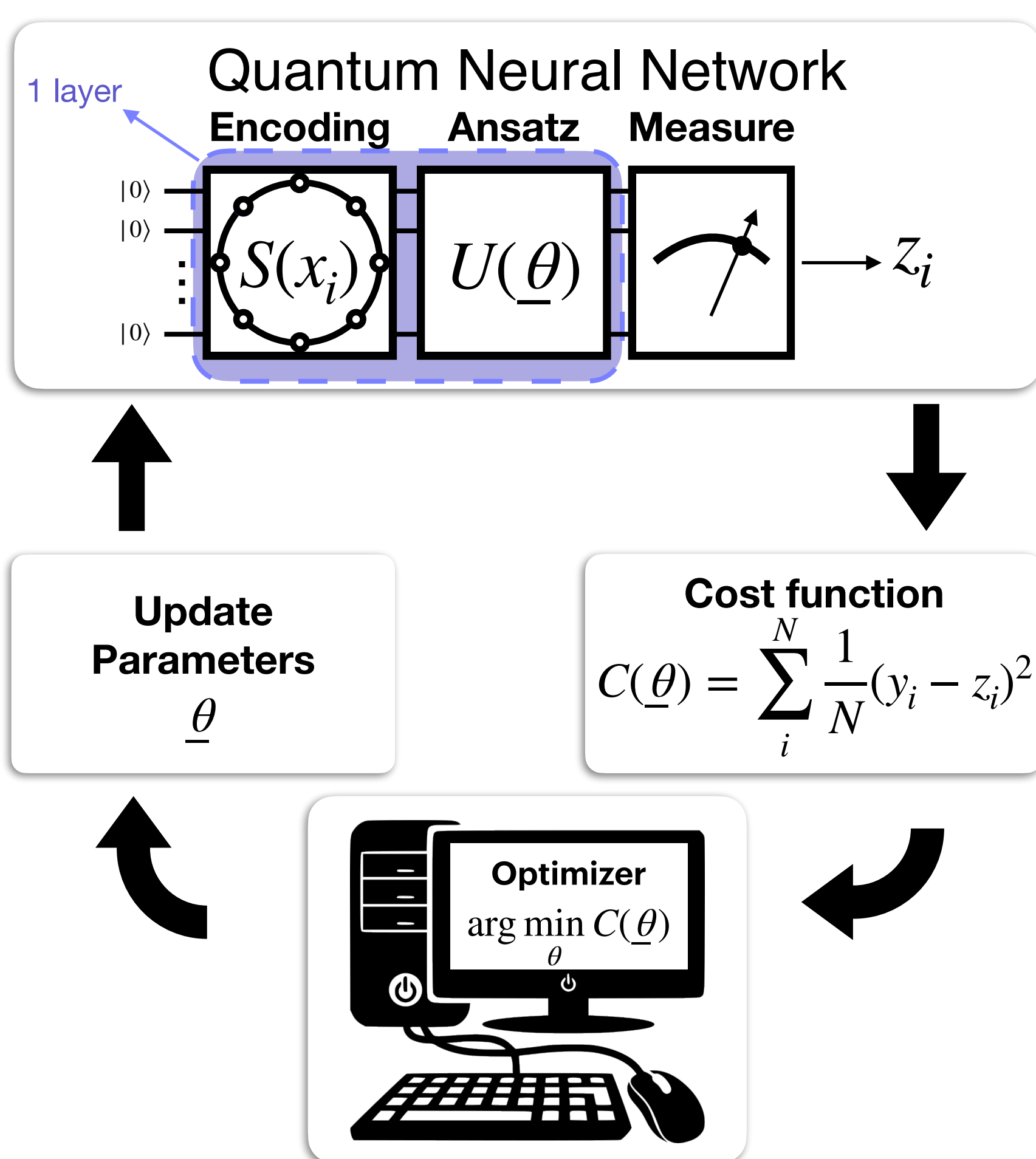
We focus on the Schwinger model discretized over an 8 sites chain with PBC, which has 70 different possible states. Each state x has 8 features (x_i), one for each site of the chain, representing its occupation number. We set $\varepsilon = 2$, then our dataset is labelled in the following way:

- If $\Delta E < \varepsilon$, $y = 1$ (positive class) $\rightarrow 45$ states
- If $\Delta E \geq \varepsilon$, $y = -1$ (negative class) $\rightarrow 24$ states

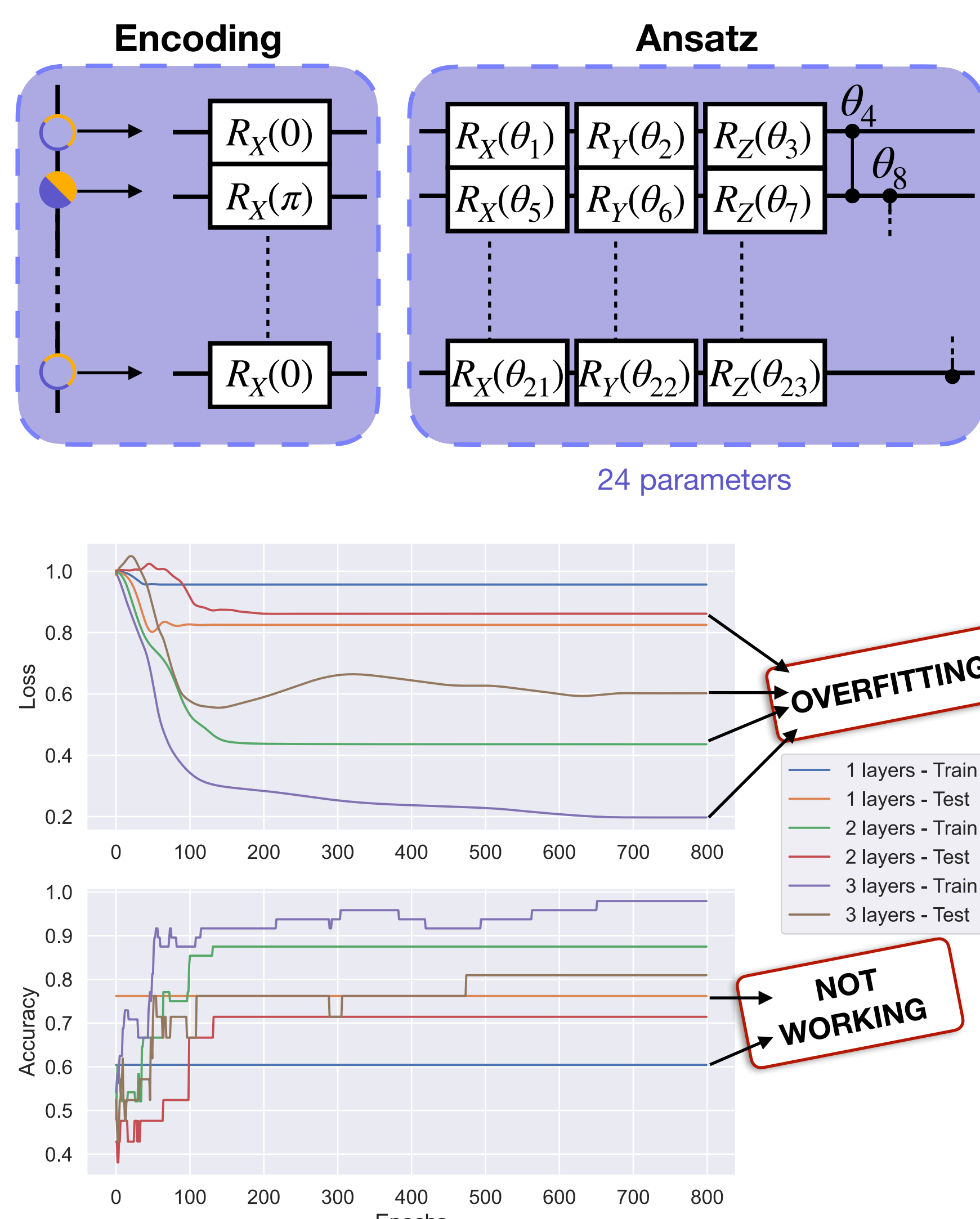
Train set is composed of 70 % of the states, while we test over the remaining 30 %.

LEARNING APPROACH

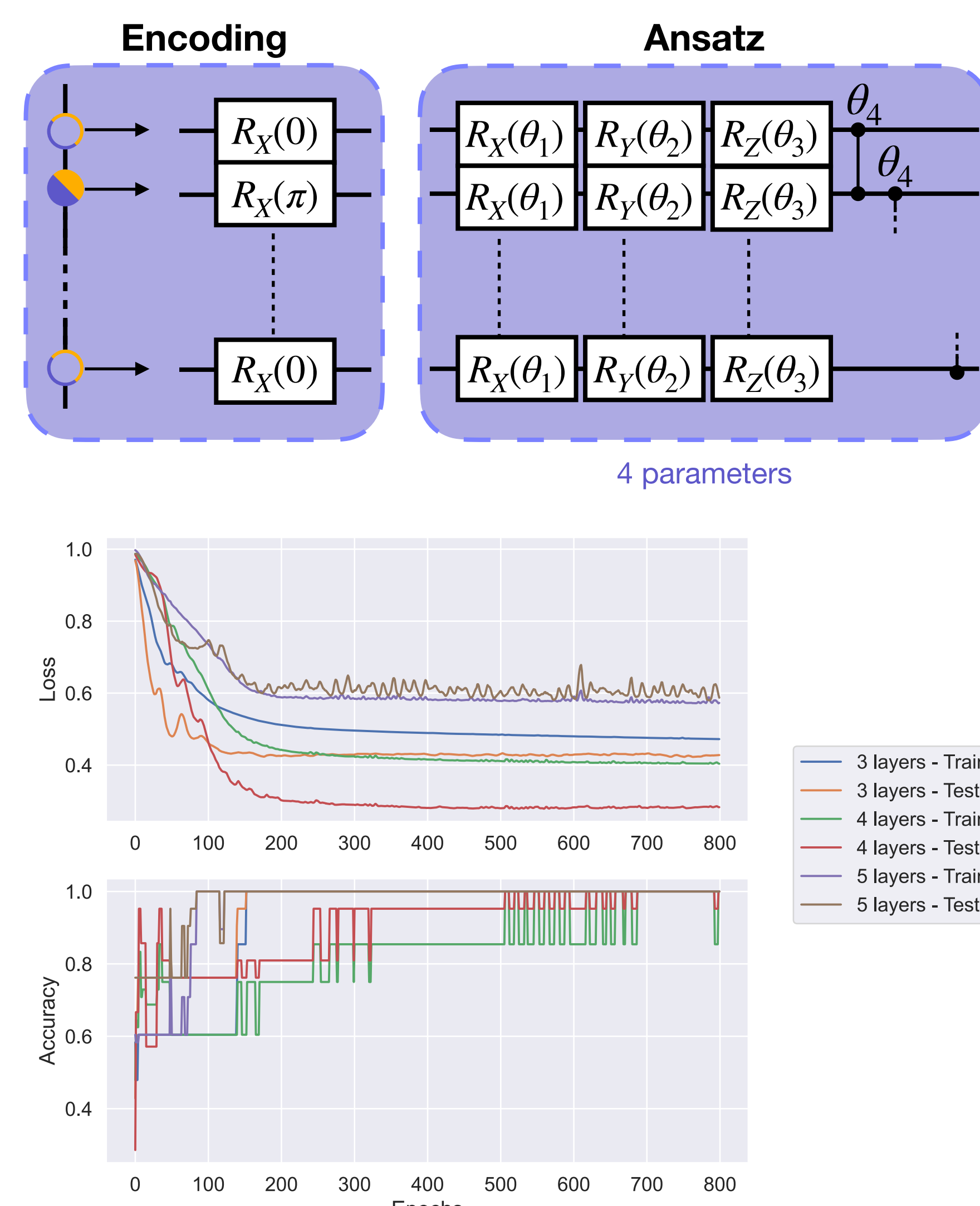
Quantum Machine Learning Scheme



Standard Quantum Machine Learning



Symmetric Quantum Machine Learning

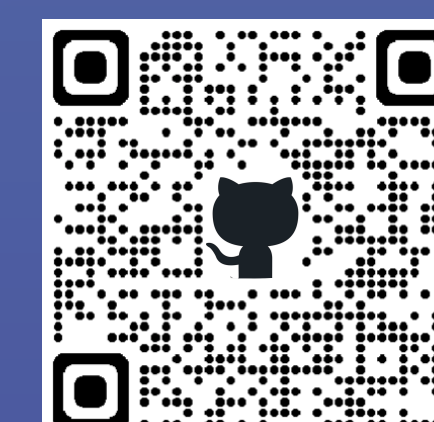


CONCLUSIONS

We benchmark the theory of symmetric QNNs on the Schwinger model discretized over an 8 sites chain through numerical simulations. The performance of our model is compared to a Symmetry-free QNN, showing that exploiting symmetries has an impact on both the accuracy and the number of parameters needed to reach convergence. Further work would look at the application of the symmetries to tackle harder machine learning tasks and see how this algorithms work on NISQ hardware.

BIBLIOGRAPHY

- [1] Meyer, J. J., Mularski, M., Gil-Fuster, E., Mele, A. A., Arzani, F., Wilms, A., & Eisert, J. "Exploiting Symmetry in Variational Quantum Machine Learning." PRX Quantum, 4(1) (2023).
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