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Symmetrizing quantum machine learning for Quantum Field Theory

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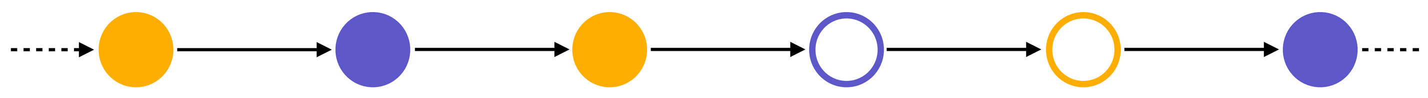
ABSTRACT

The field of **Quantum Machine Learning** (QML) looks at exploring if and how quantum computers can improve either accuracy or trainability of Machine Learning (ML) models. Despite their successful Implementations, **Quantum Neural Networks** (QNNs) show to have a tradeoff between trainability and expressivity of the model. A possible solution to this has been identified in **Symmetry driven** QNNs [1], which exploit symmetries in the problem at hand to improve the **trainability** of the model. The **Schwinger model** [2] is a simplification of the Standard Model, constructed to satisfy a few symmetry conditions. A case of interest in Quantum Simulations is the Schwinger model in 1+1 dimensions, with the space coordinates discretized on a N-sites chain with spatial periodic boundary conditions. Typically one is interested in the simulation of those eigenstates whose energy is lower than a selected threshold, an approach known as **truncation**. The main goal of this work is the binary classification of the physical states of the Schwinger model based on truncation discrimination. This is done by exploiting the symmetries of the Schwinger model, both in the data encoding and in the variational ansatz of the QNN.

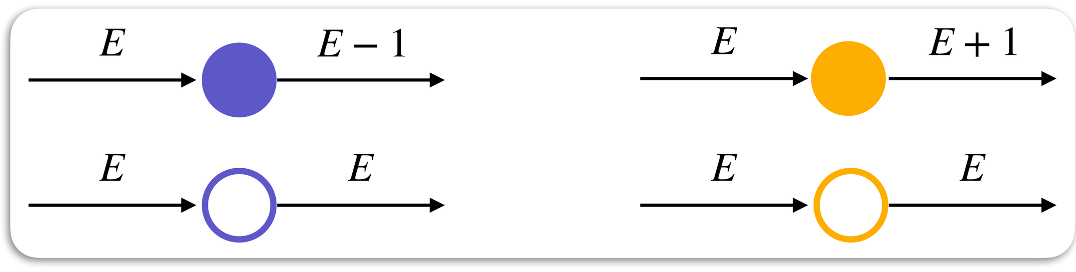
PROBLEM

Schwinger Model on a chain with periodic Boundary Conditions



Dots represent fermonic field sites

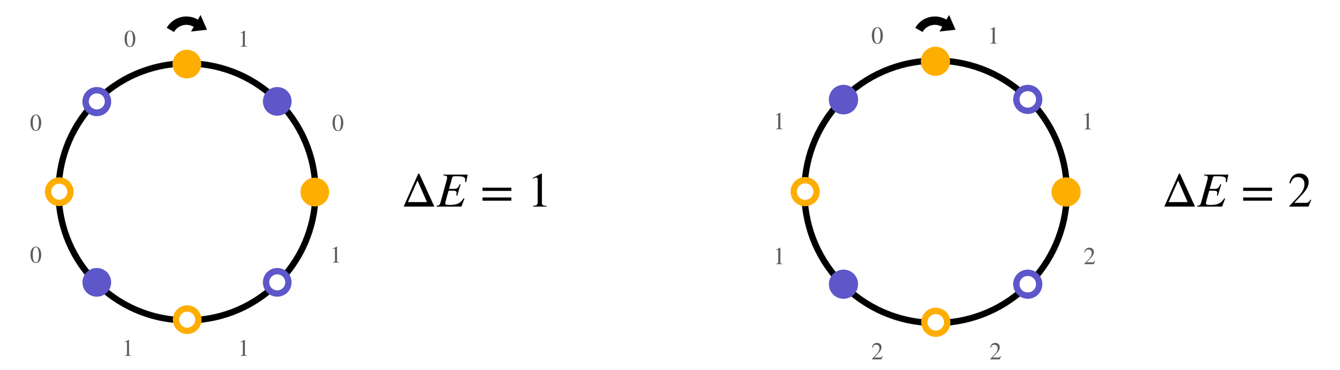
Links represent the electric field



- The energy of the system is proportional to $\Sigma_i E_i^2$, with i running over the sites.
- When interested in low energy states, the theory can be **truncated** defining

$$\Delta E \equiv E_{max} - E_{min}$$

And considering only the chains with ΔE under a certain value ε.



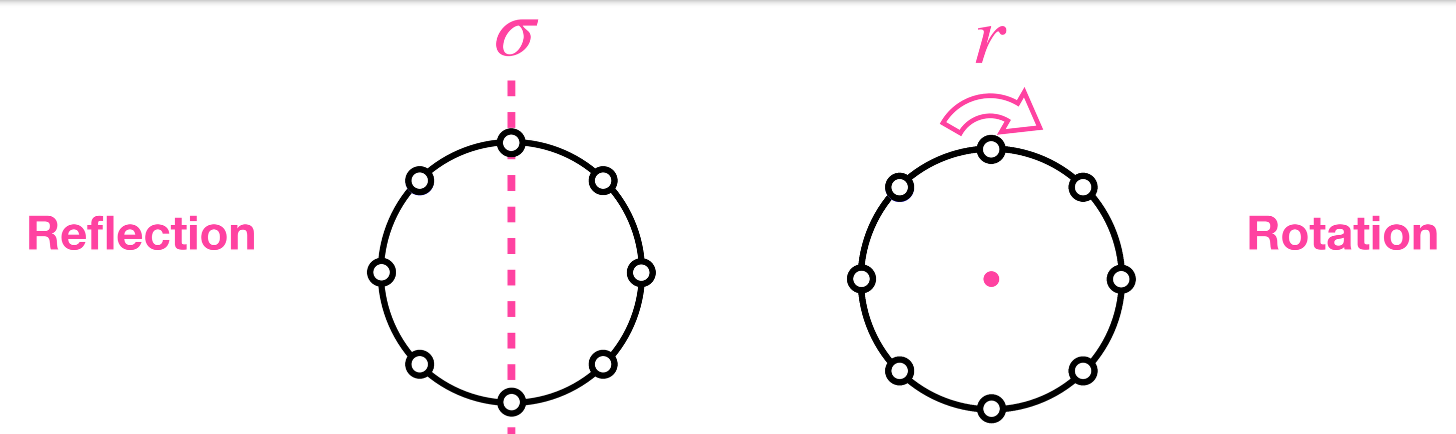
Symmetries

The value of ΔE associated with a configuration of a N-sites chain is invariant under the representation if the **N-Dihedral group**

$$D_N = \langle \sigma, r \,|\, \sigma^2 \cong r^N \cong e \rangle$$

group neutral element

The two generators of D_N act on the chain as a **reflection** and a **rotation** of $2\pi/N$



Classifying Schwinger states

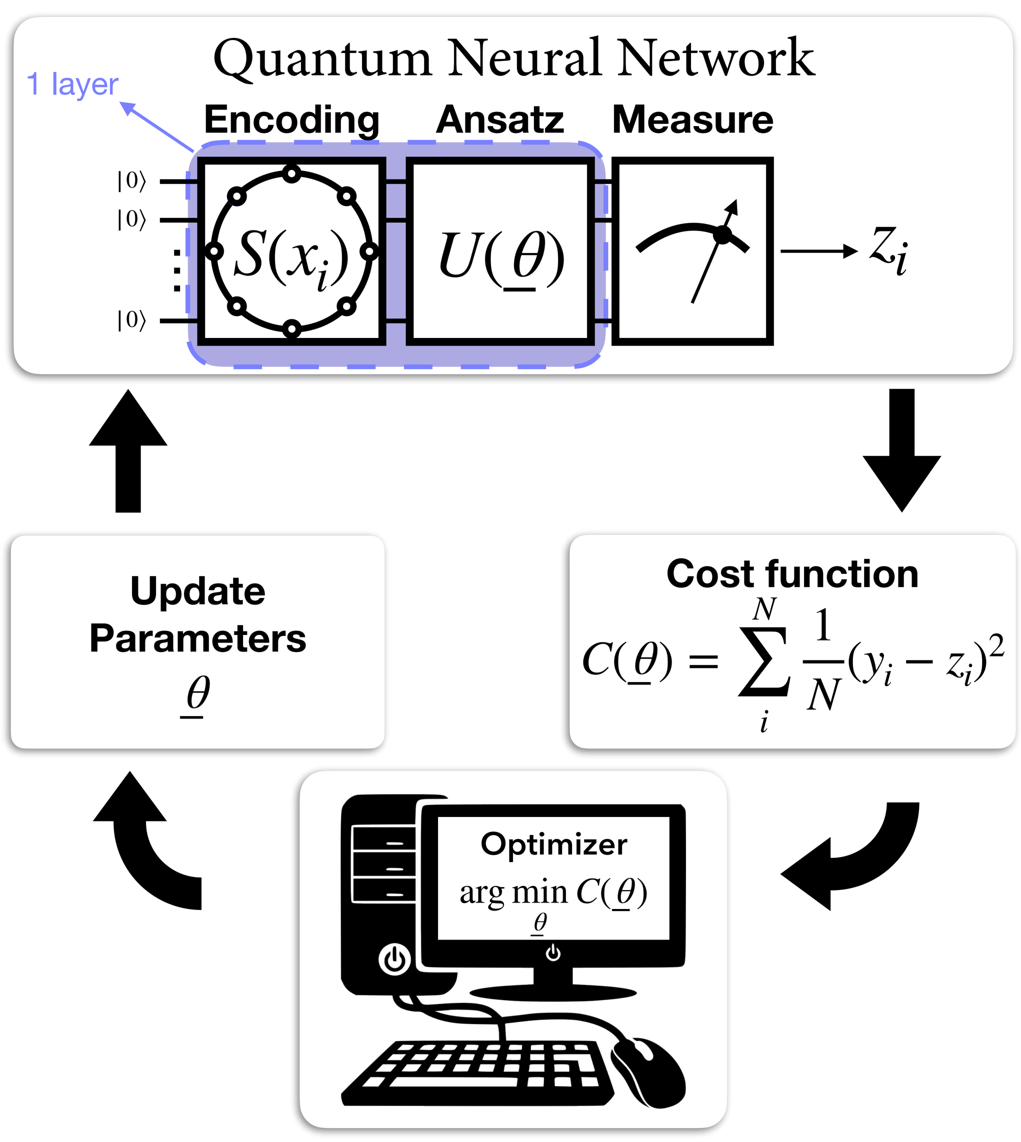
We focus on the Schwinger model discretized over an 8 sites chain, which has 69 different possible states. We set $\varepsilon = 2$, then our dataset is labelled in the following way:

- If $\Delta E < \varepsilon$, $y = 1$ (positive class) $\rightarrow$ 45 states
- If $\Delta E \geq \varepsilon$, $y = -1$ (negative class) $\rightarrow$ 24

Train set is composed of 70 % of the states, while we test over the remaining 30 % .

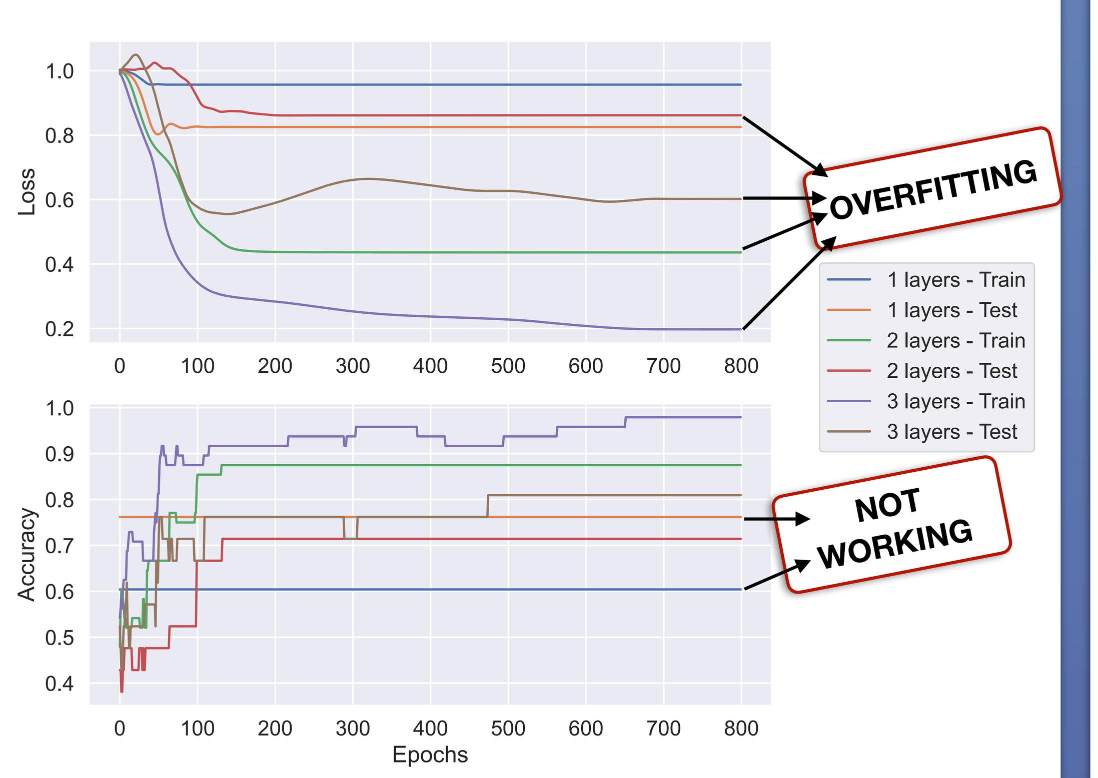
LEARNING APPROACH

Quantum Machine Learning Scheme



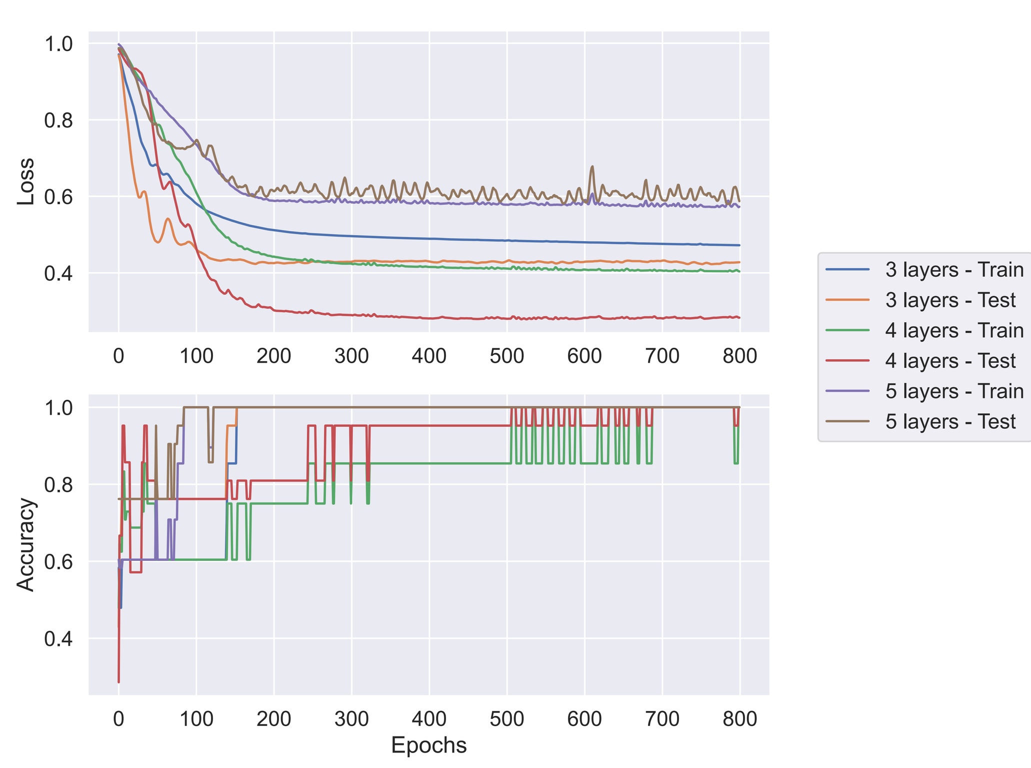
Standard Quantum Machine Learning

- **Encoding:** 1 site→1 qubit, $R_X(\pi \cdot x_i)$ rotations on each qubit
- **Ansatz:** R_X , R_Y , R_Z rotations on each qubit, followed by a compact circular layer of $C-Phase$ gates $\rightarrow$ 32 parameters per layer



Symmetric Quantum Machine Learning

- **Encoding:** same
- **Ansatz:** R_X , R_Y , R_Z rotations on each qubit, followed by a compact circular layer of $C-Phase$ gates, but each kind of gate has the same parameter $\rightarrow$ 4 parameters per layer



CONCLUSIONS

We benchmark the theory of symmetric QNNs on the Schwinger model discretized over an 8 sites chain through numerical simulations. The performance of our model is compared to a Symmetry-free QNN, showing that exploiting symmetries has an impact on both the accuracy and the number of parameters needed to reach convergence. Further work would look at the application of the symmetries to tackle harder machine learning tasks and see how this algorithms work on NISQ hardware.

BIBLIOGRAFIA

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