INTERSTELLAR MEDIUM

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Contents

Introduction

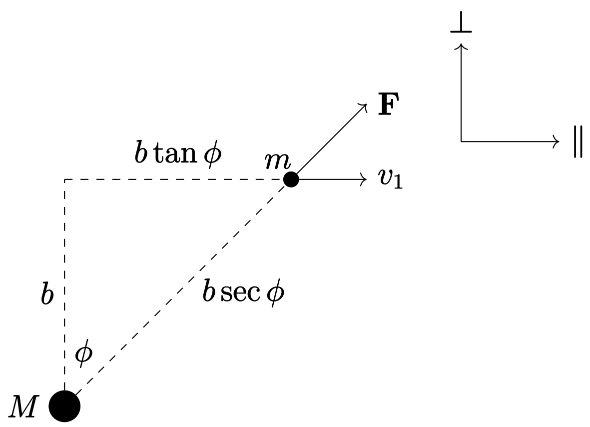
Studying the interstellar medium is fundamental to understand the stellar formuation. Molecular gas, cool enough, is needed to start to collapse and to compress. The condition for the gravitational collapse is given by the **Jeans length and mass**,

$$
\begin{aligned}
\lambda_J &= \left(\frac{\pi \, \overbrace{c_s}^{\text{Speed of sound}}{}^2}{G \, \underbrace{\rho}_{\text{Density of the cloud}}} \right)^{1/2}, \\
M_J &= \frac{4\pi}{3} \rho \frac{\lambda_J^3}{8},
\end{aligned}
\tag{1}
$$

The Jeans mass is the mass of a cloud that is stable against gravitational collapse. If the cloud is more massive than the Jeans mass, it will collapse.

Particle collisions

In **elastic collisions**, the total kinetic energy is conserved. Looking at the diagram, the target has a mass $M \gg m$ and the projectile has a mass m. The target is at rest and the projectile has an initial velocity $\mathbf{v}$. The impact parameter b is the distance of closest approach where there is no interaction between paticles.



The transfer of momentum is described with,

$$
\begin{aligned}
\Delta p_\perp &= \int_{-\infty}^{\infty} F_\perp \, dt \\
&= \int_{-\infty}^{\infty} \frac{Z_1 Z_2 e^2}{(b/\cos\phi)^2} \cos\phi \, dt \\
&= \frac{Z_1 Z_2 e^2}{b^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\phi \, \frac{d(b\tan\phi)}{v_1}, \quad \text{as } \frac{d|\mathbf{r}|}{dt} = v_1 \\
&= \frac{Z_1 Z_2 e^2}{b v_1} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\phi \, \frac{d\phi}{\cos^2\phi} \\
&= \frac{2 Z_1 Z_2 e^2}{b v_1}.
\end{aligned}
\tag{2}
$$

The classical approximation is still valid if $b \gg a_0$, where

Note (Bohr radius).

$$a_0 = \frac{\hbar^2}{m_e e^2} = 5.292 \times 10^{-9}\,\mathrm{cm}, \tag{3}$$

the wave function of the target electron will not be effected.
For de-excitation of ions due to electron collisions, the coefficient is given by

Definition .1 (De-excitation rate coefficient).

$$\langle \sigma v \rangle_{u \to l} \approx \pi a_0^2 \left(\frac{8kT}{\pi m_e} \right)^{1/2} \left(1 + \frac{Ze^2}{a_0 kT} \right), \tag{4}$$

which comes from the conservation of energy,

$$\frac{1}{2} m_e v_{\max}^2 = \frac{1}{2} m_e v + \frac{Ze^2}{r_{\min}}, \tag{5}$$

the momentum conservation,

$$v_{\max} r_{\min} = v b_{\mathrm{crit}}, \tag{6}$$

and defining the average cross section, using the velocity distribution field,

$$\langle \sigma v \rangle = \int_0^\infty \sigma(v) v f(v)\, dv, \quad \text{where } \sigma = \pi b_{\mathrm{crit}}^2 \tag{7}$$

Statistical mechanics

The partition function is defined as

$$Z(T) = \sum_i g_i e^{-E_i/kT}, \tag{8}$$

where g_i is the degeneracy of the energy level E_i. For an isothermal system,

$$\frac{P(s_1)}{P(s_2)} = \frac{\Omega(s_1)}{\Omega(s_2)} \quad S = k \ln \Omega. \tag{9}$$

The relative number of particles in energy level j with respect to the energy level i is given by

Definition .2 (Boltzmann equation).

$$\frac{N_j}{N_i} = \frac{g_j}{g_i} e^{-(E_j - E_i)/kT}. \tag{10}$$

The detailed balance is a statistical description of how many particles are in one state as opposed to the other. The ratio of number densities is proportional to the ratio of partition functions per unit volume,

$$\frac{\prod_j n(P_j)}{\prod_i n(P_i)} = \frac{\prod_j f(P_j)}{\prod_i f(P_i)}. \tag{11}$$

The **rate coefficient** per unit volume is,

$$R = n_A n_B \langle \sigma v \rangle_{AB} \quad \text{where } \langle \sigma v \rangle_{AB} = \int_0^\infty \sigma_{AB}(v) v f(v)\, dv. \tag{12}$$

The Saha equation is used to describe the ionization of a gas, it links the number of atoms in the ionization state versus the other. It is given by,

Definition .3 (Saha equation).

$$\underbrace{\frac{n_{i+1}}{n_i}}_{\text{Ionization ratio}} = \frac{2}{\underbrace{n_e}_{\text{Electron density}}} \overbrace{\frac{Z_{i+1}(T)}{Z_i(T)}}^{\text{Partition function ratio}} \left(\frac{2\pi m_e k T}{h^2}\right)^{3/2} e^{-\overbrace{E_i}^{\text{Ionization potential}}/kT}, \tag{13}$$

The last term assumes that the temperature is not too high, making the lowest energy term important and it is in the **local thermodynamic equilibrium**.

Quantum mechanics

Molecular transitions are important for the interstellar medium. The more symmetric the molecule, the weaker emission lines it will have, due to the lack of the permanent dipole moment. The dipole moment is given by,

$$\mu_L = \frac{e\omega r_e^2}{2c} \quad L = m\omega r_e^2 = \sqrt{j(j+1)}\hbar. \tag{14}$$

1 Spectral lines

The spectroscopic notation is a way to describe the energy levels of an atom or a molecule,

Definition 1.1 (Spectroscopic notation).

Term parity

$$ {}^{2S+1}L^{(p)}_{J}. \tag{1.1}$$

Spin multiplicity

Total angular momentum

Orbital angular momentum

The parity characterizes whether the wave function changes sign under reflection of all electron positions through the origin. It is blank for even parity and o for odd parity. The selection rules for the transitions are,

- Parity must change
- $\Delta L = 0, \pm 1$
- $\Delta J = 0, \pm 1$, but $\Delta J = 0 \to 0$ is forbidden
- Only one single electron wavefunction $n\ell$ can change with $\Delta \ell = \pm 1$
- $\Delta S = 0$ for electric dipole transitions

This approximation is valid when radiation involved has $\lambda \gg a_0$. Each electron is affected by the electric field produced by all other charges within the atom,

$$D_{ki} = \int \psi_k^*(\mathbf{r})\, \mathbf{D}\, \psi_i(\mathbf{r})\, d\mathbf{r}. \tag{1.2}$$

Dipole approximation

Forbidden lines are when one of the rules is not satisfied, while the **semi-forbidden lines** are where the spin selection rule is violated, and are much weaker than intersystem (permitted) transitions.