GAL Buckle 95

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 $27~\mathrm{mai}~2014$

Table des matières

1	Init	tialisation
	1.1	Chargement des paquets
	1.2	Constantes et données
	1.3	
2	Esti	imation
	2.1	Données mises à l'échelle
	2.2	Première estimation par QEE
	2.3	Résultats de la première estimation par QEE
	2.4	Seconde estimation par QEE
	$\frac{2.1}{2.5}$	Résultats de la seconde estimation par QEE
	$\frac{2.6}{2.6}$	Estimation par GMM
	2.0	Estimation par Givivi
3	Cor	mparaison des résultats
	3.1	Fonction de répartition
		3.1.1 Graphiques
		3.1.2 Statistiques
	3.2	
	٥	3.2.1 Tests avec contraintes
		3.2.2 Vrais paramètres
	3.3	
	ა.ა	
		3.3.1 Données de base
		3.3.2 Paramètres neutres au risque
		3.3.3 Méthode de Epps
	3.4	Méthode de Carr-Madan

Chapitre 1

Initialisation

1.1 Chargement des paquets

```
> setwd("~/maitrise/GAL-Buckle95/")
> library(actuar)
> library(MASS)
> library(xtable)
> library(parallel)
> library(moments)
> library(FourierStuff)
> library(GeneralizedAsymmetricLaplace)
> library(GMMStuff)
> library(OptionPricingStuff)
> library(QuadraticEstimatingEquations)
```

1.2 Constantes et données

```
> #Nombre de décimales affichées
> options(digits=6)
> #Marge pour intervalles de confiance
> alpha.confint <- 0.05
> #Marge pour test d'hypothèses
> alpha.test <- 0.05
> #Chargement des données
> RETURNS <- head(read.csv("abbeyn.csv",sep="\t",header=TRUE)[,1],-1)
> #Taille de l'échantillon
> n <- length(RETURNS)
> #Nom de l'échantillon
> strData <- "Buckle95"</pre>
```

1.3 Test de normalité

```
> EppsPulley.test(RETURNS)

Epps-Pulley Normality test

T: 0.626033

T*: 0.635568
p-value: 0.007178

$Tstat
[1] 0.626033

$Tmod
[1] 0.635568
```

\$Zscore

[1] 2.44824

\$Pvalue

[1] 0.00717788

\$Reject
[1] TRUE

Chapitre 2

Estimation

2.1 Données mises à l'échelle

```
> sRET <- as.vector(scale(RETURNS))</pre>
```

2.2 Première estimation par QEE

```
> ## Point de départ
> pt.depart <- startparamGAL(sRET)</pre>
> ## Fonctions pour les moments
> meanQEE <- function(param) mGAL(param,1)</pre>
> varianceQEE <- function(param) cmGAL(param,2)</pre>
> sdQEE <- function(param) sqrt(cmGAL(param,2))</pre>
> skewnessQEE <- function(param) cmGAL(param,3)</pre>
> kurtosisQEE <- function(param) cmGAL(param,4)</pre>
> ## Fonctions pour les dérivées
> dmeanQEE <- function(param) dmGAL(param,1)</pre>
> dsdQEE <- function(param) dmGAL(param,2)</pre>
> ## Estimation gaussienne
> optim1 <- optim(pt.depart,obj.gauss,gr=NULL,sRET,
                   meanQEE, varianceQEE, dmeanQEE, dsdQEE)
> pt.optim1 <- optim1$par</pre>
> ## Estimation de crowder
> optim2 <- optim(pt.depart,obj.Crowder,gr=NULL,sRET,
                   meanQEE, varianceQEE, skewnessQEE, kurtosisQEE, dmeanQEE, dsdQEE)
> pt.optim2 <- optim2$par
> ## Estimation de crowder modifiée
> optim3 <- optim(pt.depart,obj.Crowder.Mod,gr=NULL,sRET,
                   meanQEE, varianceQEE, dmeanQEE, dsdQEE)
> pt.optim3 <- optim3$par</pre>
```

2.3 Résultats de la première estimation par QEE

```
LOWER ESTIMATE
                            UPPER
[1,] -0.780018 -0.726048 -0.672077
[2,] 0.436002 0.596316 0.756630
     0.262650 0.359186 0.455722
[4,] 1.994757 2.021370 2.047982
> confidence.interval.QEE(pt.optim2,cov.optim2,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.694457 -0.627404 -0.560351
[2,] 0.413764 0.640292 0.866820
[3,] 0.232650 0.334028 0.435405
[4,] 1.839966 1.878296 1.916626
> confidence.interval.QEE(pt.optim3,cov.optim3,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.765288 -0.711439 -0.657589
[2,] 0.455485 0.606642 0.757798
[3,] 0.264669 0.362932
                        0.461195
[4,]
     1.932691 1.960299
                         1.987906
```

2.4 Seconde estimation par QEE

```
> ## Estimation gaussienne
> optim4 <- optim(pt.optim1,obj.gauss,gr=NULL,sRET,</pre>
                  meanQEE, varianceQEE, dmeanQEE, dsdQEE,
                  ginv(V.gauss(pt.optim1,sRET,meanQEE,
                                                     varianceQEE, skewnessQEE, kurtosisQEE,
                                                     dmeanQEE,dsdQEE)))
> pt.optim4 <- optim4$par</pre>
> ## Estimation de crowder
 optim5 <- optim(pt.optim2,obj.Crowder,gr=NULL,sRET,
                  meanQEE, varianceQEE, skewnessQEE, kurtosisQEE, dmeanQEE, dsdQEE,
                  ginv(V.Crowder(pt.optim2, sRET, varianceQEE, skewnessQEE,
                                                    kurtosisQEE,dmeanQEE,dsdQEE)))
> pt.optim5 <- optim5$par
> ## Estimation de crowder modifiée
 optim6 <- optim(pt.optim3,obj.Crowder.Mod,gr=NULL,sRET,
                  meanQEE, varianceQEE, dmeanQEE, dsdQEE,
                  ginv(V.Crowder.Mod(pt.optim3,sRET,varianceQEE,
                                                    dmeanQEE,dsdQEE)))
> pt.optim6 <- optim6$par
```

2.5 Résultats de la seconde estimation par QEE

```
LOWER ESTIMATE
                            UPPER
[1,] -0.779792 -0.725853 -0.671914
[2,] 0.436017 0.596319 0.756622
[3,] 0.262456 0.358969 0.455482
[4,] 1.995452 2.022048 2.048644
> confidence.interval.QEE(pt.optim5,cov.optim5,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.692712 -0.625874 -0.559036
[2,] 0.414139 0.640445 0.866750
[3,] 0.231568 0.332845 0.434122
[4,] 1.842116 1.880376 1.918636
> confidence.interval.QEE(pt.optim6,cov.optim6,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.766288 -0.712450 -0.658612
[2,] 0.455051 0.606193 0.757334
[3,] 0.264972 0.363196 0.461419
[4,] 1.934050 1.961614 1.989178
2.6
       Estimation par GMM
> ## GMM régulier
> optim7 <- optim.GMM(pt.depart,</pre>
                  conditions.vector=meanvariance.gmm.vector,
+
                  data=sRET, W=diag(2),
                 meanf=meanQEE, variancef=varianceQEE)
> pt.optim7 <- optim7$par
> cov.optim7 <- mean.variance.GMM.gradient.GAL(pt.optim7,sRET) %*%
                  covariance. GMM(pt.optim7, meanvariance.gmm.vector, sRET,
                                 meanf=meanQEE, variancef=varianceQEE) %*%
                  t(mean.variance.GMM.gradient.GAL(pt.optim7,sRET)) / n
> ## GMM itératif
> optim8 <- iterative.GMM(pt.depart,
                  conditions.vector=meanvariance.gmm.vector,
                  data=sRET, W=diag(2),
                  meanf=meanQEE, variancef=varianceQEE)
> pt.optim8 <- optim8$par
> cov.optim8 <- mean.variance.GMM.gradient.GAL(pt.optim8,sRET) %*%
                  optim8$cov %*%
                  t(mean.variance.GMM.gradient.GAL(pt.optim8,sRET)) / n
> confidence.interval.QEE(pt.optim7,cov.optim7,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.878702 -0.641646 -0.404589
[2,] -0.469225  0.625908  1.721040
[3,] -0.192234 0.326366 0.844965
[4,] 1.696121 1.965995 2.235869
> confidence.interval.QEE(pt.optim8,cov.optim8,n)
        LOWER ESTIMATE
                            UPPER
[1,] -0.874031 -0.636980 -0.399929
[2,] -0.473292  0.626346  1.725984
```

[3,] -0.193600 0.322895 0.839390 [4,] 1.704166 1.972716 2.241265

Chapitre 3

Comparaison des résultats

3.1 Fonction de répartition

```
> # Points d'évaluation
> xi <- seq(2*min(sRET),2*max(sRET),length.out=2^6)</pre>
> # Fonction de répartition par intégration de la fonction caractéristique
> dist1 <- cbind(cftocdf(xi,cfGAL,param=pt.optim1),</pre>
                  cftocdf(xi,cfGAL,param=pt.optim2),
                  cftocdf(xi,cfGAL,param=pt.optim3),
                  cftocdf(xi,cfGAL,param=pt.optim4),
                  cftocdf(xi,cfGAL,param=pt.optim5),
                  cftocdf(xi,cfGAL,param=pt.optim6),
                  cftocdf(xi,cfGAL,param=pt.optim7),
                  cftocdf(xi,cfGAL,param=pt.optim8))
> # Fonction de répartition par point de selle
> dist2 <- cbind(psaddleapproxGAL(xi,pt.optim1),</pre>
                  psaddleapproxGAL(xi,pt.optim2),
                  psaddleapproxGAL(xi,pt.optim3),
                  psaddleapproxGAL(xi,pt.optim4),
                  psaddleapproxGAL(xi,pt.optim5),
                  psaddleapproxGAL(xi,pt.optim6),
                  psaddleapproxGAL(xi,pt.optim7),
                  psaddleapproxGAL(xi,pt.optim8))
> # Fonction de répartition par intégration de la fonction de densité
> dist3 <- cbind(pGAL(xi,pt.optim1),</pre>
                  pGAL(xi,pt.optim2),
                  pGAL(xi,pt.optim3),
                  pGAL(xi,pt.optim4),
                  pGAL(xi,pt.optim5),
                  pGAL(xi,pt.optim6),
                  pGAL(xi,pt.optim7),
                  pGAL(xi,pt.optim8))
```

3.1.1 Graphiques

```
> for (i in 1:8)
+ {
          file<-paste(strData,"-repart-",i,".pdf",sep="")</pre>
          pdf(file=file,paper="special",width=6,height=6)
          plot.ecdf(sRET,main=paste("Fonction de répartition ",i))
          lines(xi,dist1[,i],col="green")
          lines(xi,dist2[,1],col="red")
          lines(xi,dist3[,1],col="pink")
          lines(xi,pnorm(xi),type="l",col="blue")
          dev.off()
          cat("\\includegraphics[height=4in,width=4in]{",
                          file,"}\n",sep="")
+ }
Buckle95-repart-1.pdf
                                                         Buckle95-repart-2.pdf
```

Buckle95-repart-3.pdf	Buckle95-repart-4.pdf
Buckle95-repart-5.pdf	Buckle95-repart-6.pdf

Buckle95-repart-7.pdf

Buckle95-repart-8.pdf

3.1.2 Statistiques

```
Test du \chi^2, Méthode avec intégration
```

	chisquare.stat	df	p.value
pt.optim1	5.473824	6.000000	0.484626
pt.optim2	5.329673	6.000000	0.502277
pt.optim3	5.388158	6.000000	0.495076
pt.optim4	5.474310	6.000000	0.484567
pt.optim5	5.337004	6.000000	0.501372
pt.optim6	5.390662	6.000000	0.494769
pt.optim7	5.454256	6.000000	0.487003
pt.optim8	5.476963	6.000000	0.484245

Test du χ^2 , Méthode avec point de selle

	chisquare.stat	df	p.value
pt.optim1	9.293574	6.000000	0.157728
pt.optim2	8.345592	6.000000	0.213862
pt.optim3	9.050625	6.000000	0.170751
pt.optim4	9.292836	6.000000	0.157767
pt.optim5	8.344140	6.000000	0.213959
pt.optim6	9.062381	6.000000	0.170100
pt.optim7	8.616379	6.000000	0.196330
pt.optim8	8.610490	6.000000	0.196698

	statistic	p.value	alternative	method	data.name
pt.optim1	0.158220	0.171912	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim2	0.140346	0.289345	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim3	0.156772	0.179751	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim4	0.158159	0.172235	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim5	0.139916	0.292753	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim6	0.156960	0.178718	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim7	0.141230	0.282437	two-sided	One-sample Kolmogorov-Smirnov test	X
pt.optim8	0.140016	0.291954	two-sided	One-sample Kolmogorov-Smirnov test	X

3.2 Fonction de densité

Intégration de la fonction de densité approximée avec le point de selle, pour la normaliser en fonction qui intègre à 1.

```
> f_integrale_saddle <- function(param,f,lower,upper)</pre>
          integrate(f,lower,upper,param)$value
> norm_int_saddle <- sapply(1.pts.estim,f_integrale_saddle,
                  f=dsaddleapproxGAL,lower=-Inf,upper=Inf)
   Séquence de points pour les graphiques
> x_sRET <- seq(min(sRET)-sd(sRET), max(sRET)+sd(sRET), length.out=50)
  Graphique de la fonction de densité
> colors2=c("black", "red", "green", "blue", "grey")
> for (i in 1:dim(pts.estim)[2])
+ {
          file=paste(strData,"-densite-", i, ".pdf", sep="")
          pdf(file=file, paper="special", width=6, height=6)
          plot(density(sRET),ylim=c(0,.7),type="1",
                           main=paste("Densité de", strData, i), xlab=strData,
                           ylab="f", lwd=2, lty=1)
          points(x_sRET,
                           dGAL(x_sRET,pts.estim[,i]),
                           type="b",ylim=c(0,4),col="red",pch=19,lwd=2,lty=2)
          points(x_sRET,
                           dsaddleapproxGAL(x_sRET,pts.estim[,i])/norm_int_saddle[i],
                           type="b",ylim=c(0,4),col="green",pch=20,1wd=2,1ty=3)
          lines(x_sRET, dnorm(x_sRET), type="b", col="blue",
                           pch=21, lwd=2, lty=4)
          points(seq(-2,4,length.out=1000)[seq(40,1000,by=40)],
                           cftodensity.fft(cfGAL,1000,-2,4,pts.estim[,i])$dens[seq(40,1000,by=40)],
                           type="b",col="grey",pch=23,1ty=6)
          legend(quantile(sRET,0.9),0.7, c("emp","est.GAL","pt.selle","appx.nrm","fft"),
                           cex=0.8, col=colors2, pch=c(NA,19:23), lty=1:6, title="Courbes")
          dev.off()
          cat("\\includegraphics[height=4in, width=4in]{"
                           ,file, "}\n", sep="")
+ }
```

md.stat	df	p.value
0.000422	21.000000	0.000000
0.120174	21.000000	0.000000
0.001384	21.000000	0.000000
0.000388	21.000000	0.000000
0.123295	21.000000	0.000000
0.001451	21.000000	0.000000
0.007980	21.000000	0.000000
0.010416	21.000000	0.000000
	0.000422 0.120174 0.001384 0.000388 0.123295 0.001451 0.007980	0.000422 21.000000 0.120174 21.000000 0.001384 21.000000 0.00388 21.000000 0.123295 21.000000 0.001451 21.000000 0.007980 21.000000

Buckle95-densite-1.pdf	Buckle95-densite-2.pdf
Buckle95-densite-3.pdf	Buckle95-densite-4.pdf

Buckle95-densite-5.pdf	Buckle95-densite-6.pdf
Buckle95-densite-7.pdf	Buckle95-densite-8.pdf

3.2.1 Tests avec contraintes

Test de Wald

```
caption="Test de Wald", digits=2)
% latex table generated in R 3.1.0 by xtable 1.7-3 package
% Tue May 27 23:05:26 2014
\begin{table}[ht]
\centering
\begin{tabular}{rrrrrrrr}
 \hline
& pt.optim1 & pt.optim2 & pt.optim3 & pt.optim4 & pt.optim5 & pt.optim6 & pt.optim7 & pt.optim8 \\
  \hline
wald.stat & 1861.21 & 1796.75 & 1690.26 & 1865.01 & 1814.81 & 1690.62 & 2111.08 & 2175.45 \\
 p.value & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
 reject & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \
  \hline
\end{tabular}
\caption{Test de Wald}
\end{table}
```

3.2.2 Vrais paramètres

Comme nous avons estimé avec des données centrées et réduites, nous utilisons une propriété de la distribution GAL qui nous permet d'obtenir les paramètres des rendements non réduits.

	pt.optim1	pt.optim2	pt.optim3	pt.optim4	pt.optim5	pt.optim6	pt.optim7	pt.optim8
1	-0.0092	-0.0080	-0.0090	-0.0092	-0.0079	-0.0091	-0.0081	-0.0081
2	0.0078	0.0083	0.0079	0.0078	0.0083	0.0079	0.0081	0.0081
3	0.0033	0.0031	0.0033	0.0033	0.0031	0.0033	0.0030	0.0030
4	2.0214	1.8783	1.9603	2.0220	1.8804	1.9616	1.9660	1.9727

Table 3.1 – Paramètres des données non centrées et réduites

3.3 Prix d'options

3.3.1 Données de base

```
> #Taux sans risque
> rfrate <- .05/365
> #Échéance
> T <- 30
> #Pas de discrétisation courbe des prix
> pas <- 0.005
> #Prix initial
> stock0 <- 299
> #Prix d'exercice dans le cours (put)
> strike1 <- stock0*seq(0.98,1,pas)
> #Prix d'exercice hors le cours (put)
> strike2 <- stock0*seq(1+pas,1.02,pas)</pre>
> #Prix d'exercice combinés
> strike <- c(strike1,strike2)</pre>
> #Damping parameter
> alpha <- 3
```

3.3.2 Paramètres neutres au risque

```
> pts.estim.ns.rn <- apply(pts.estim.ns,2,riskneutralparGAL,rfrate)
> l.pts.estim.ns.rn <- as.list(data.frame(pts.estim.ns.rn))</pre>
```

> xtable(pts.estim.ns.rn,caption="Paramètres neutres au risque",digits=4)

	pt.optim1	pt.optim2	pt.optim3	pt.optim4	pt.optim5	pt.optim6	pt.optim7	pt.optim8
1	-0.0066	-0.0057	-0.0065	-0.0066	-0.0057	-0.0065	-0.0058	-0.0058
2	0.0078	0.0083	0.0079	0.0078	0.0083	0.0079	0.0081	0.0081
3	0.0033	0.0031	0.0033	0.0033	0.0031	0.0033	0.0030	0.0030
4	2.0214	1.8783	1.9603	2.0220	1.8804	1.9616	1.9660	1.9727

Table 3.2 – Paramètres neutres au risque

3.3.3 Méthode de Epps

```
> f_putEpps <- function(param,strikeprice,char.fn,eval.time,expiry.time,rate,...)
+ putEpps(strikeprice,char.fn,param,eval.time,expiry.time,rate,...)
> prix_put_Epps <- as.data.frame(cbind(strike/stock0,sapply(l.pts.estim.ns.rn,f_putEpps,strike/stock0,sapply(l.pts.estim.ns.rn,f_putEpps,caption="Prix unitaire de l'option de vente, Méthode de Epps",digits=6)
> xtable(prix_put_Epps,caption="Prix unitaire de l'option de vente, Méthode de Epps",digits=6)
```

	V1	pt.optim1	pt.optim2	pt.optim3	pt.optim4	pt.optim5	pt.optim6	pt.optim7	pt.optim8
1	0.980000	0.015415	0.015785	0.015431	0.015417	0.015794	0.015427	0.015792	0.015819
2	0.985000	0.017360	0.017737	0.017377	0.017362	0.017746	0.017372	0.017742	0.017770
3	0.990000	0.019456	0.019838	0.019474	0.019458	0.019847	0.019469	0.019843	0.019871
4	0.995000	0.021705	0.022090	0.021724	0.021707	0.022099	0.021719	0.022093	0.022122
5	1.000000	0.024107	0.024492	0.024126	0.024109	0.024501	0.024121	0.024495	0.024523
6	1.005000	0.026661	0.027044	0.026681	0.026663	0.027053	0.026676	0.027046	0.027074
7	1.010000	0.029365	0.029745	0.029385	0.029367	0.029753	0.029381	0.029745	0.029773
8	1.015000	0.032217	0.032591	0.032238	0.032219	0.032600	0.032233	0.032591	0.032617
9	1.020000	0.035214	0.035580	0.035235	0.035216	0.035589	0.035231	0.035579	0.035605

Table 3.3 – Prix unitaire de l'option de vente, Méthode de Epps

3.4 Méthode de Carr-Madan

	V1	pt.optim1	pt.optim2	pt.optim3	pt.optim4	pt.optim5	pt.optim6	pt.optim7	pt.optim8
1	0.980000	0.044899	0.045144	0.044912	0.044900	0.045150	0.044909	0.045145	0.045163
2	0.985000	0.041847	0.042096	0.041860	0.041848	0.042102	0.041857	0.042096	0.042114
3	0.990000	0.038895	0.039148	0.038909	0.038896	0.039153	0.038906	0.039147	0.039165
4	0.995000	0.036045	0.036300	0.036059	0.036046	0.036306	0.036056	0.036299	0.036317
5	1.000000	0.033297	0.033555	0.033312	0.033298	0.033560	0.033309	0.033553	0.033571
6	1.005000	0.030653	0.030912	0.030668	0.030654	0.030918	0.030665	0.030910	0.030928
7	1.010000	0.028114	0.028375	0.028130	0.028115	0.028381	0.028126	0.028372	0.028390
8	1.015000	0.025681	0.025943	0.025697	0.025682	0.025949	0.025694	0.025940	0.025958
9	1.020000	0.023356	0.023618	0.023373	0.023357	0.023624	0.023369	0.023615	0.023633

Table 3.4 – Prix unitaire de l'option d'a chat, Méthode de Carr-Madan