Airpassengers

① AR1:

$$\log(\text{airp})_i \mid \chi, \Theta \sim N(\eta_i, \sigma^2) \quad \sigma^2 \to \frac{1}{2.37\times 10^4}$$

N

$$\eta_i = \beta_0 + \boxed{\beta\,\text{year}} + u_i$$

N N N

$$u_i \overset{d}{=} \rho u_{i-1} + \varepsilon_i \quad \therefore u_i \overset{d}{=} 0.748\, u_{i-1} + \varepsilon_i$$

$$u_1 \sim N\Big(0, \underbrace{[\tau^{\varepsilon}(1-\rho^2)]}_{K}{}^{-1}\Big) \therefore u_1 \sim N\left(0, \frac{1}{41}\right)$$

log(airp)i

② Seasonal:

$$\log(\text{airp})_i \mid \chi, \Theta \sim N(\eta_i, \sigma^2) \quad \sigma^2 \to 1/891$$

independent a priori but dependent a posteriori

$$\eta_i = \beta_0 + \beta\text{year} + u_i$$

$u_i = ??$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{144} \end{bmatrix} = \underline{u} \qquad \therefore \pi(\underline{u}\mid\tau) \propto \tau^{\frac{n-m+1}{2}} \exp(\underline{u}^T Q \underline{u})$$

$n = 144$, $m = 12$

where $Q = \tau \boxed{R}$ depends on m

$\tau \to 18068.39$

③ RW1: (CAR1 with $\rho = 1$)

$$\log(\text{airp})_i \mid \chi, \Theta \sim N(\eta_i, \sigma^2) \quad \sigma^2 \to \frac{1}{22038\ldots}$$

$$\eta_i = \beta_0 + \beta\text{year} + u_i$$

where $u_i - u_{i-1} \sim N(0, \tau^{-1})$ $\quad \tau \to 3.84$

$$\therefore u_i \overset{d}{=} u_{i-1} + \varepsilon_i \quad \varepsilon_i \sim N(0, \tau^{-1})$$

④ RW2: Does not work → smooths too much

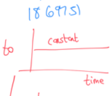
$$\log(\text{airp})_i \mid \chi, \Theta \sim N(\eta_i, \sigma^2) \quad \sigma^2 \to 1/54.34$$

$$\eta_i = \beta_0 + \beta\text{year} + u_i$$

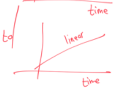
$$u_i - 2u_{i+1} + u_{i+2} \sim N(0, \tau^{-1})$$

$\tau \to 18\,6975\,1$

* RW1 $(\overset{\text{scale.model = TRUE}}{\tau \to \infty})$ reduces to



* RW2 $(\overset{\text{scale.model = TRUE}}{\tau \to \infty})$ reduces to



NC SIDS

Y = count of deaths due to SIDS

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\lambda_i = E_i \gamma_i \quad \leftarrow \text{relative risk}$$

$$\gamma_i = \underbrace{\exp(\eta_i)}$$

$\sim$ NWBIR74 + f( )

$$\eta_i = \underbrace{\beta_0 + \beta^T x}_{\text{fixed effects}} + \underbrace{\sum_{k=1}^{K} f(u_k)}_{\text{random effects}}$$

Calculate expected count if the risk was the same uniform over all "counties" units

* "Standardize for the difference in the "experimental units" number of births"

random effects
- temporal ✓
- $\boxed{\text{spatial}}$ u_k - discrete
- splines ✓
- iid
⋮

a) $Y_i \mid \eta_i \sim \text{Poisson}(\lambda_i)$

$$\lambda_i = E_i \times \underbrace{e^{\eta_i}}_{\gamma_i}, \quad \eta_i = \beta_0 + \beta_1 * \text{nwp\_hat74}_i$$

offset/given → E_i; γ_i ← fitted.values

Fitted model: $\hat{\eta}_i = -0.646 + 1.869(\text{nwp\_hat74}_i)$

b) $\eta_i = \beta_0 + \beta_1 * \text{nwp\_hat74}_i + u_i$ $\quad$ NOT SPATIAL

$$u_i \overset{iid}{\sim} N(0, \tau^{-1})$$

Fitted model: $\hat{\eta}_i = -0.644 + 1.872(\text{nwp\_hat74}_i) + u_i$

where $u_i \sim N\left(0, \frac{1}{15991.19}\right)$

c) $\eta_i = \beta_0 + \beta_1 * \text{nwp\_hat74}_i + \boxed{v_i}$ Besag

$$v_i \mid v_{-i}, \tau \sim N\left(\frac{1}{n_i}\sum_{i \sim j} v_j, \frac{1}{n_i \tau}\right)$$

τ ← Besag hyperparameter

Fitted model: $\hat{\eta}_i = -0.646 + 1.871 * \text{nwp\_hat74}_i + v_i$

where $v_i \mid v_{-i}, \tau \sim N\left(\frac{1}{n_i}\sum_{i \sim j} v_j, \frac{1}{n_i(18068.15)}\right)$

d) $\eta_i = \beta_0 + \beta_1 * \text{nwp\_hat74}_i + u_i + v_i$

$v_i \to$ Besag

$u_i \to$ iid

e) $\eta_i = \beta_0 + \beta_1 * \text{nwp\_hat74}_i + u_i^* + v_i^*$

$$u_i^* + v_i^* = \frac{1}{\sqrt{\tau}}\left\{\sqrt{1-\phi}\, u + \sqrt{\phi}\, v\right\}$$

Fitted model: $\hat{\eta}_i = -0.671 + 1.947 * \text{nwp\_hat74}_i + u_i^* + v_i^*$

where $u_i^* + v_i^* = \frac{1}{18.966}\left\{\sqrt{1-0.282}\, u_i + \sqrt{0.282}\, v_i\right\}$