Optics Laboratory

Dispersion and Spectroscopy

1. OBJECTIVES

- Measure of the refractive index and spectral resolution of a prism.
- Measure of the period and spectral resolution of the diffraction grating.

2. THEORY

The main goal of a spectroscope is the analysis of a light source by decomposing its light in its different wavelength components. Hence, the key element is the dispersive medium. There are two fundamental optical principles that allow us to separate light in its wavelength components, namely, refraction and interference. These principles are used in spectroscopes that make use of, respectively, of prisms and diffraction gratings.

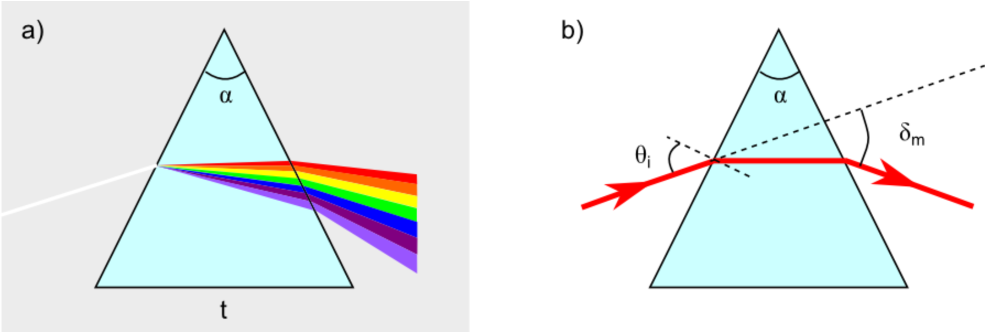
Independently of the design and the dispersive medium, the main feature of a spectroscope is its spectral resolution (R). This parameter describes the capability of the device to separate two nearby wavelengths and is formally given by the expression

$$R = \frac{\lambda}{\delta\lambda}, \tag{1}$$

where λ is the working wavelength and $\delta\lambda$ is the *spectral purity* or *instrumental profile*, namely, a measure of the width for each detected spectral line. This parameter renders an idea of the capability to "resolve" (distinguish) spectral lines with nearby wavelengths and, particularly, to observe fine details in their structure (e.g., if they are doublets, triplets, etc.). Following Rayleigh's criterion, a spectroscope is assumed to separate two spectral lines when the wavelength difference between their maxima ($\Delta\lambda$) is equal to or larger than the spectral purity ($\Delta\lambda \geq \delta\lambda$). Accordingly, spectroscopes with relatively large values of R will be able to resolve very close spectral lines.

2.1 Prisms

When white light travels across a dispersive medium (e.g., glass), every light component travels with a different speed (the medium refractive index is different for each component), which gives rise to their eventual separation. This effect is emphasized if there is refraction as each component will be deviated (refracted) at a different angle. In the case of a prism with normal dispersion, these deviations will be larger for shorter wavelength components, as seen in Fig. 1(a).



Figurea 1. (a) Chromatic dispersion produced by a prism. (b) Ray propagation under incidence conditions of minimum deviation.

An optical prism is a transparent homogeneous medium with two flat surfaces forming an *apex angle* α. The angle of deviation, δ, is the angle formed by the forward projection of the incident ray and the backward projection of the doubly refracted ray that reaches a minimum δ_m. At minimum deviation conditions, the expressions are simplified and once the angle of minimum deviation δ_m is known for a given wavelength, the refractive index of a prism can be determined for such wavelength by

$$n = \frac{\sin(\delta_m + \alpha/2)}{\sin(\alpha/2)}. \qquad (2)$$

On the other hand, within the visible spectrum, the dependence of the refractive index on the wavelength follows in good approximation Cauchy's formula:

$$n \approx A + \frac{B}{\lambda^2}. \qquad (3)$$

In the first part of this laboratory experience, the refractive index of the glass that the prism is made of will be determined from the experimental values of the angle of minimum deviation for several wavelengths, Eq. (2). This will allow us to determine later the parameters A and B by means of Eq. (3), since the theoretical values of the wavelength for the spectral lines observed are known. Finally, from these parameters, the spectral resolution of the prism spectroscope used here is obtained by [1-2]

$$R = \frac{\lambda}{\delta\lambda} = t\,\frac{dn}{d\lambda}, \qquad (4)$$

where t is the thickness of the prism (see Fig. 1). Accordingly, gaining resolution implies using more dispersive glasses (a higher variation of the refractive index with the wavelength), larger prisms, or a combination of both.

2.2 Diffraction gratings

A diffraction grating, in its simplest and most pedagogical version, can be understood as an array of parallel slits on an opaque substrate [see Fig. 2(a)]. When this array is illuminated by a coherence beam, the outgoing waves diffracted by each slit behave like a coherent superposition. Eventually, these waves will interfere, giving rise to a characteristic diffraction pattern of light spots that will be closer or further away one another depending on the number of slits in the array.

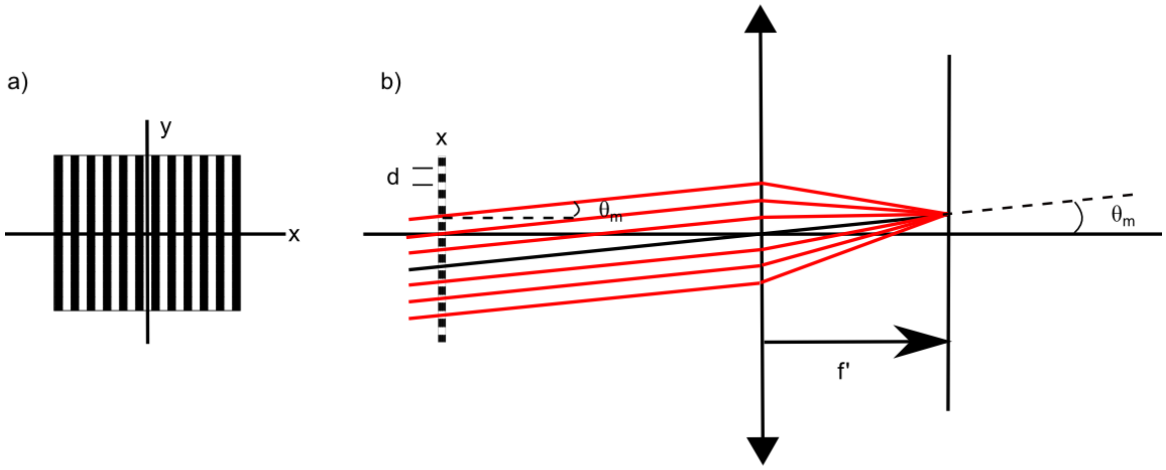


Figure 2. (a) Sketch of a diffraction grating. (b) Ray bundle illustrating the formation of an interference maximum with presence of a positive lens.

When a plane wave is incident perpendicularly on a diffraction grating, to observe an interference maximum on a point behind the grating, the path difference between waves coming from consecutive slits must be an integer number of wavelengths. Such waves will then be in phase and their interference will be constructive. Figure 2b illustrates this situation. The intensity maxima will appear in those directions, with respect to the incidence direction, such that the following condition is satisfied:

$$d \sin \theta_m = m\lambda \,, \tag{5}$$

where $m = 0, \pm 1, \pm 2, \ldots$ are integer numbers that label those maxima and are referred to as diffraction orders, d is the distance between any two consecutive slits or *period*, and λ is the wavelength of the light illuminating the grating. Typically, the grating period is indicated in terms of its inverse, i.e., the number of lines (slits) per milimiter (or centimeter). Following Eq. (5), for non-monochromatic beams, the same diffraction order for different wavelengths will appear at different angular positions. Therefore, a diffraction grating can be used to spatially separate the different wavelength components of the incident light.

The resolving power of these spectroscopes consists in the capability of a diffraction grating to separate nearby wavelengths. For a diffraction grating of length $L = dN$, with N being the total number of lines (slits) in the grating, the resolving power is given by the expression:

$$R = \frac{\lambda}{\delta\lambda} = mN \,, \tag{6}$$

where m is the diffraction order [the larger the diffraction order the larger the resolving power, because of the larger angular spread, according to Eq. (5)]. If d is small and L is large, then N will also be large and the grating will be able to separate very close spectral lines even in the first order.

3. EXPERIMENTAL METHOD

3.1 Description of the experimental setup

In order to perform accurate measurements, we shall use a spectroscope consisting of collimator, platform and telescope (see Fig. 3).

The collimator essentially consists of a single variable-width slit and a positive lens. It is used to produce an incident plane wave (a bundle of parallel rays), which is achieved by accommodating the slit at the front (object) focal plane of the lens. The prism or the diffraction grating are fixed on the central platform and are used to produce the light dispersion. This light is analyzed with the telescope (an objective and an eyepiece with crosshairs), which is attached to a goniometer that allows to accurately determine the outgoing angular deviation of each spectral line. If the system is aligned, when looking through the eyepiece, we observe the back (image) focal plane of the telescope objective. This means we have an image of the collimator slit, although in a different angular directions for each wavelength due to the dispersion caused by the prism or the diffraction grating. The spectrum observed is thus said to consist of well-defined spectral lines (see Fig. 4).

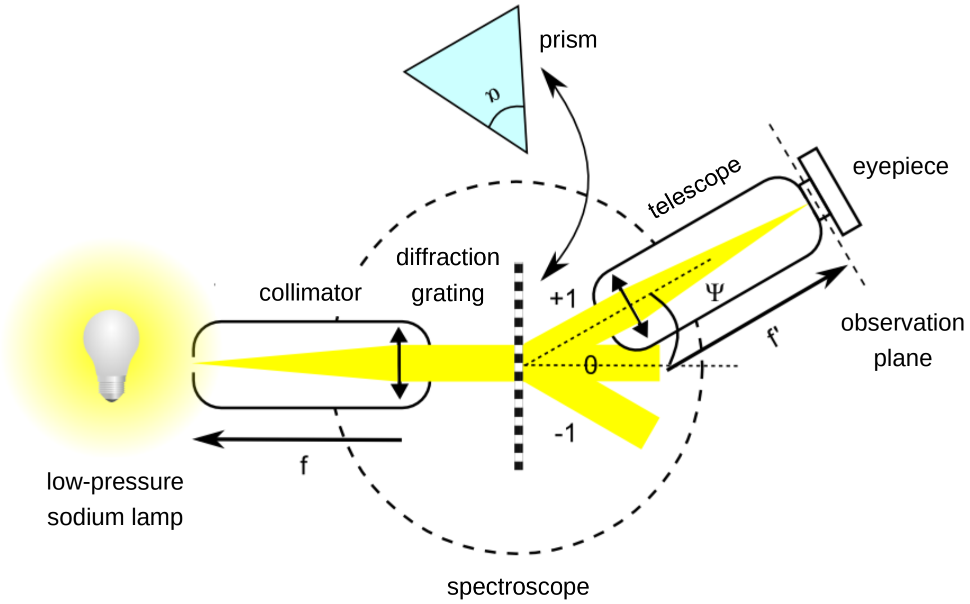


Figure 3. Sketch of the spectroscope with collimator and telescope. The prism or the diffraction grating are accommodated at the center of the platform.

Nonetheless, the spectroscope might not be aligned. In such a case, before proceeding with the experience, it is important to align it properly, which requires the following steps:

1. Gently move back and forth the telescope eyepiece until you can clearly observe the crosshairs (a piece of white paper in front of the objective might help you).
2. Direct the telescope to a distance object in the room (~10 m) and act on the side (telescope) screw until you observe a focused image of such an object through the eyepiece.
3. Without making any change, direct the telescope to the collimator lens until you observe the image of the illuminated slit. Get a clear image by acting on the collimator screw. At this point, the collimator is producing plane wavefronts. Adapt the slit width to your needs, widening or narrowing it depending on whether you need more or less light (for fainter spectral lines you may need to increase the width).
4. Before starting, make sure that you know how to read angles with the goniometer and the Vernier scale (nonius). It is advisable to make a few tries before starting collecting data. Notice that this is a very accurate apparatus. Moreover, keep in mind that measures have always to be done with the same nonius (there are two, on opposite sides).

3.2 Determination of the refractive index

An equilateral prism is going to be used in this experience, so the apex is $\alpha = 60°$. In order to determine the refractive index, first the angle of minimum deviation must be experimentally obtained for each wavelength. In order to get a significant sample of wavelengths, but not too much, a low-pressure sodium lamp is used, which emits light with a convenient set of wavelengths (see Table 1).

Color	**λ (nm)**
Red (doublet)	616.1 – 615.4
Yellow (doublet)	589.6 – 589.0
Light green (doublet)	568.8 – 568.3
Green	515
Blue (doublet)	498.3 – 497.8

Table 1. Wavelengths of the emission spectral lines for a low-pressure sodium lamp.

3.3 Determination of the angle of minimum deviation

To determine the angle of minimum deviation, first the prism is accommodated on the central platform of the spectroscope. As a starting point, you can use any of the two configurations displayed in Fig. 4.

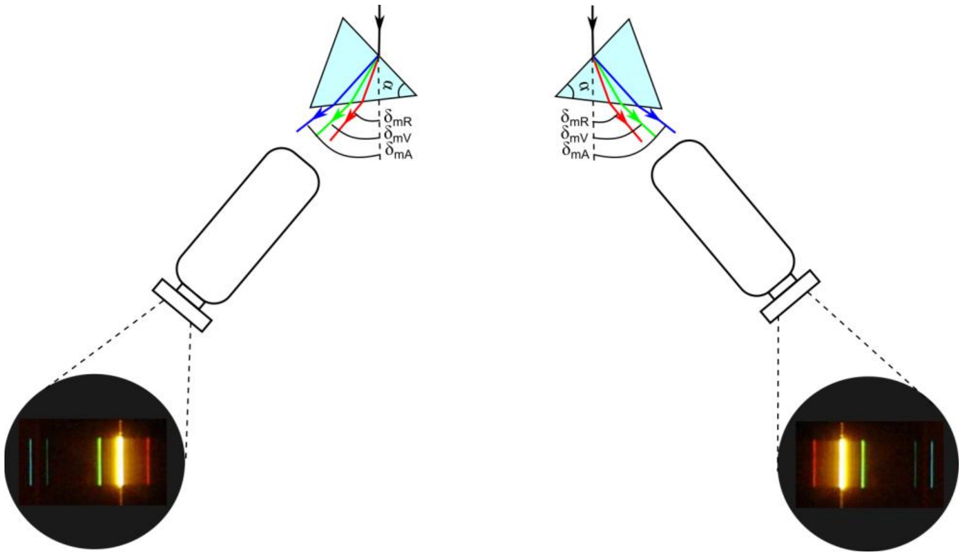


Figure 4. Emission spectrum of a low-pressure sodium lamp observed with the two positions for the prism.

With the naked eye, find the image of the collimating slit on the surface of the prism opposite to the collimator (be aware of not being looking at the rough, translucent face of the prism!). Once this image is observed, turn the telescope until it points along the same direction. If you look now through the eyepiece, you will immediately see the set of spectral lines, something similar to the sets displayed in Fig. 4. Finally, in order to determine the angle of minimum deviation, slowly rotate the support platform where the prism lies on. Through the eyepiece, you will see that the set of spectral lines moves in the same direction of the rotation, until, at some point, they start moving in the opposite direction. The turn indicates the angular position of minimum deviation. Find it with accuracy and do not move the prism platform during the performance of this experiment. At this prism position write down the angular measures obtained by the goniometer for each wavelength of the spectrum $\psi_1(\lambda)$. When all measurements are obtained for one side of the prism, place the prism in the symmetric position (see Fig. 4). Find again the prism position of minimum deviation and write again the minimum deviation angles for the different wavelengths $\psi_2(\lambda)$. With these measurements, we can obtain the minimum deviation angle as $\delta_{\mathrm{m}} = (\psi_1 - \psi_2)/2$ and the refractive index for each wavelength given in Table 1 by Eq. (2).

3.4 Determination of the period of a diffraction grating

Once the prism has been substituted by the diffraction grating, assuming the spectroscope is correctly aligned, it must be ensured that the grating is perpendicular to the incident beam. To this end, first turn the telescope until it forms a 90º angle with the collimator. Turn now the grating until you can see the eyepiece crosshairs at the center of the reflected image of the slit. In this configuration, the incident beam makes a 45º angle with the grating. If the latter is now rotated 45º, only moving the platform with the Vernier (never the support platform), then the grating becomes perpendicular to the incident beam.

The spectral lines will be observed on both sides with respect to the zeroth order (direct propagation). The image of the slit for each wavelength appears at a different angle, as follows form Eq. (5). Annotate the angles for each wavelength in the first order (both sides, and then compute the mean value) and only for the yellow doublet (most intense line) in the second order. The period can be obtained now from Eq. (5).

4. BIBLIOGRAPHY

[1] A. Ghatak, *Optics* (McGraw-Hill, 6th Ed., 2017).

[2] F. L. Pedrotti, L. M. Pedrotti and L. S. Pedrotti, *Introduction to Optics* (Pearson Int'l Edition, 2006).

[3] J. F. James, *An Introduction to Practical Laboratory Optics* (Cambridge University Press, 2014).

QUESTIONNAIRE

- **In this practice, <u>uncertainties are optional</u>.**
- **For an easier identification of the answers, please, specify in a visible place the number of the question responded (unless you have included explicitly the question).**

Determination of the angle of minimum deviation for a prism

1. Build a table and write down the results from all measurements, both raw data (direct angles measured with the spectroscope) and processed data (final angles of minimum deviation to be used in the calculations). Complete the table including the corresponding wavelengths (see Table 1 in the practice manual).
2. From the angles reported in the table of point 2, obtain the value of the refractive index corresponding to each wavelength. Include these values in the table.
3. Determine the parameters A and B of the Cauchy equation, Eq. (3). Indicate the procedure followed to obtain the value of these parameters.
4. Calculate the spectral resolution of the prism.

Measurement of the angular position of diffraction orders

5. Build a table and write down the results from all measurements, both raw data (direct angles measured with the spectroscope) and processed data (final diffraction angles for all the orders to be used in the calculations: orders +1 and –1 for all wavelengths and only +2 and –2 for the yellow doublet). Complete the table including the corresponding wavelengths (see Table 1 in the practice manual).
6. From the measures reported in the table of point 6, determine the period of the grating, d, and the number of lines (slits) per millimeter, $N = 1/d$. Indicate the procedure followed to obtain the value of these parameters.
7. Determine the spectral resolution of the diffraction grating.

Comparison Spectral resolution

8. With the above data, compare the spectral resolution of both the prism and the diffraction grating. For which wavelength it is observed a larger dispersion?

Additional comments

9. If you have observed or thought about something else that has not been previously considered in any of the above points, you can add it here.