Optics Laboratory

Fraunhofer Diffraction

1. OBJECTIVES

- To understand the diffraction phenomenon in the far-field or Fraunhofer regime using different types of apertures.
- Determining the width of a rectangular slit using diffraction and the object-image approach.

CAUTION: A laser source is used. To avoid serious injuries in your eyes, be careful not to look directly into the beam or to let a direct reflection of the beam enter your eyes.

2. THEORY

When a collimated, monochromatic light beam passes through an aperture, part of the light starts traveling in other directions than the original one. This phenomenon is called diffraction. In this laboratory experience, we are going to consider the Fraunhofer or far-field regime, because it allows us to establish a simpler connection between the diffraction pattern and the dimensions of the aperture.

In general, far-field conditions require observing the diffraction pattern at long distances from the aperture, which is an important constraint in a laboratory. However, if a positive lens is accommodated behind the aperture, the Fraunhofer conditions are obtained at the back (image) focal plane of the lens. That is, we are producing Fraunhofer conditions while reducing the lengths required to reach them when no lens is used. The experimental setup to observed Fraunhofer diffraction is sketched in Fig. 1.

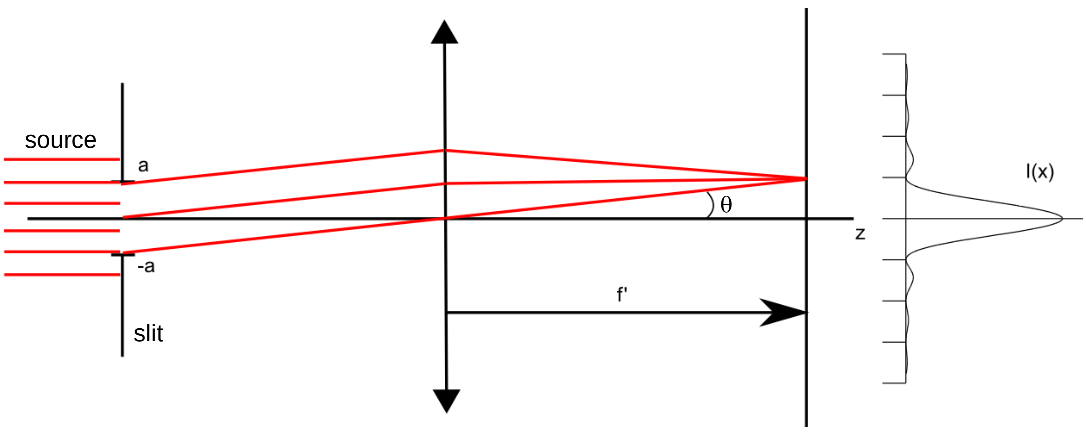

Figure 1. Sketch of the experimental setup aimed at observing Fraunhofer diffraction at the back (image) focal plane of a positive lens.

Rectangular slit

The Fraunhofer intensity distribution produced by a long rectangular slit **with a width $2a$** along the shortest side is given by the expression

$$I = I_0 \left[\frac{\sin(ka\sin\theta)}{ka\sin\theta}\right]^2, \tag{1}$$

where θ is the angle formed by the observation and the incidence directions (see Fig. 1), I_0 the maximum intensity, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength of the incident light and **a is the slit half-width**. Assuming the small-angle or paraxial approximation ($\theta \ll 1$, with the angle measured in radians), the relation

$$\sin\theta \approx \frac{x}{f'}, \tag{2}$$

is satisfied at the back focal plane, where f' is the lens focal length and x is the distance from the lens (image) focus to the observation point.

The diffraction pattern for a slit can be seen in Fig. 2(a), which corresponds to the intensity distribution described by Eq. (1). Note that a change in a gives rise to a change in the scale of the diffraction pattern. Therefore, by determining the dimensions of the diffraction pattern it is possible to also determine the slit half-width a.

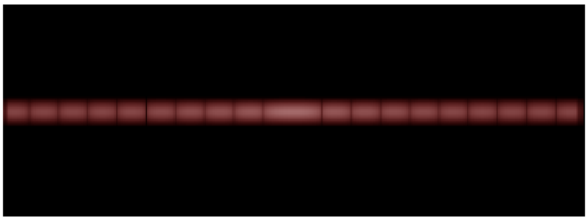

Figure 2. Fraunhofer diffraction patterns produced by long rectangular slit.

3. EXPERIMENTAL METHOD

3.1 Description of the Experimental Setup

A sketch of the experimental setup that is going to be used in this experience is shown in Fig. 3. It consists of a linearly polarized He-Ne laser ($\lambda = 632$ nm), with its beam passing, in this order, through a a polarizer and the aperture (a metal mask with a rectangular slit or a circular whole). The diffracted beam goes through a positive lens before reaching a CCD camera, with its chip lying on the back focal plane of the lens, at a distance f'. The focal length f' of this lens is 125 mm for setup C1 and 150 mm for setup C2. The polarizer allows us to control the intensity reaching the CCD and hence to avoid the saturation of the central diffraction maximum (which appears as a plateau in the 1D profile of the intensity).

As it is mentioned above, the CCD chip lies on the lens focal plane (see Fig. 3), which means that the intensity distribution recorded by the camera will correspond to the Fraunhofer diffraction pattern of the aperture. This pattern is displayed in the laptop, with its x and y axes describing a horizontal and vertical positions (in pixel units) on the CCD chip. The real size of the pattern (or any distance between two points) can readily be obtained from Eq (1) if we know the physical size of each pixel. The CCD chip consists of square pixels, with pixel size of 3.75 μm for C1 camera or 4.65 μm for C2 camera respectively.

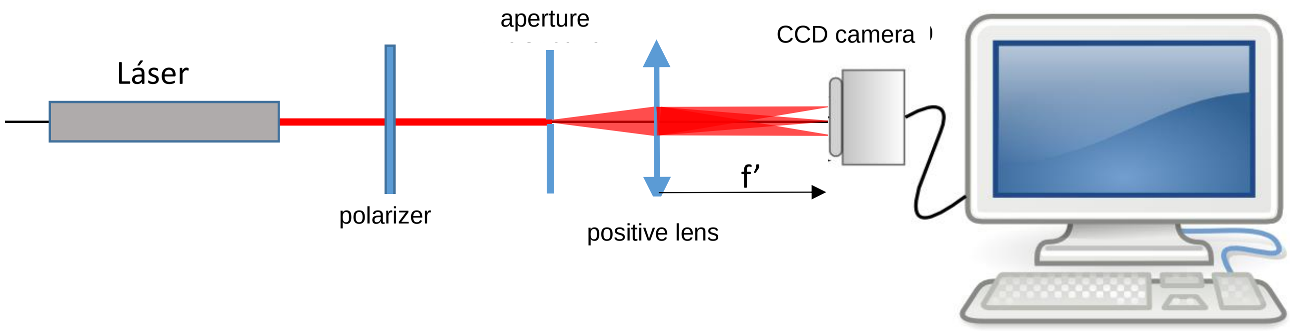

Figure 3. Sketch of the experimental setup, which includes laser source, beam expander, polarizer, aperture, positive lens and CCD camera (connected to the laptop).

The setup used in the laboratory has an additional positive lens just before the aperture (not shown in Fig. 3) that can be removed if desired. This additional lens will be used to capture an image of the slip by the CCD in the object-image approach. Note that if we remove this additional lens, then we will observe the diffraction pattern in the CCD. This additional lens has a focal distance of 50 mm or 60 mm.

3.2 Instructions for Data Acquisition and Analysis

The CCD camera is controlled by means of a Matlab executable file that you will see on the laptop desktop. When the laser is on, as soon as the program opens, there is a picture in real time of the intensity detected by the CCD. This picture can be frozen by clicking on **CCD:Stop** (which will become **CCD:Start**), which can be saved by clicking on **Save**. Saved images should be kept in a folder on the desktop.

To carry out measurements on the frozen figure (or a saved figure), click on **Profile**. This action opens a window with a new menu. Lines can be directly drawn on this figure, which is done by clicking on the left mouse button to set the initial point and on the right one to set the final point of the line. This opens a new window with a 1D intensity profile (along the marked line) that allows us to measure distances in pixel units. The image coordinates (in pixel units) can also be seen by clicking on the  button. In case you wish to upload a new image, close the actual one and click again on **Profile** and repeat again the same measurement process.

4. EXPERIMENTS

In this experience, the size of a rectangular slit is going to be determined by two different approaches: diffraction and the object-image approach. In the diffraction method, we will find through Eqs. (1) and (3), a relation between the corresponding Fraunhofer diffraction pattern and its dimension parameter (width). In the object-image approach, we will relate the width detected in the image plane with the actual slit width. To proceed, consider that the laser wavelength is 632 nm and the lens focal length is 125 mm for setup 1 and 150 mm for setup 2 (consider a vanishing uncertainty in both cases). The CCD chip consists of square pixels, with pixel size of 3.75 μm for C1 camera or 4.65 μm for C2 camera respectively. Please follow the following steps:

1) In addition to this manual you have included in the virtual campus several videos about the objectives, the experimental setup and the measurements, including the operation of the Matlab program. You also have the Matlab program that you will use in the laboratory.

2) Using equation (1) determine a method to determine the width of the slit from the diffraction pattern. Please consider the following questions when planning your method:

- Will you measure maxima or minima?
- Is it a good idea to measure the position of the center of the figure?
- Measure the two maxima/minima farthest from the center on each side?
- Measure the two maxima/minima closest to the center on each side?
- Measure several maxima/minima and make a fit?

3) Before doing any measurement, check if the CCD plane is placed at the image focal length of the lens. You can do this by moving the lens without placing any aperture until the laser spot shows a minimum size at the CCD. Obtain a measurement of the slit width with its uncertainty from the diffraction pattern.

4) To use the object-image approach, insert the additional lens in the setup without moving the other lens. Move the additional lens until you could see a clear image in the CCD. Register this image and measure the slit width in the image plane. Obtain the actual slit width from the previous measure and the focal distances of the two lenses. Repeat this measurement several times to improve the result. Tip: Consider that the slit must be at the object focal plane of the first (additional) lens (justify it) and trace a ray leaving the object from the optical axis to apply the Lagrange-Helmholtz invariant to obtain the lateral magnification.

5) Compare the results of the slit width by the two methods (diffraction and object-image). Include in the comparison a hypothesis test as illustrated in the appendix.

6) Pause for thought (<u>not necessary to include the answer in the report</u>): Why is diffraction not seen in the object-image method if the slit is the same?

7) Localize in the laboratory the color leds. Holding different apertures in front of your eye, look each color through it. Is the intensity distribution observed similar to the one expected in the Fraunhofer regime? How does this distribution change with the wavelength of the source? Why? (<u>Not necessary to include the answer in the report</u>).

8) Pick up a hair of yours, put it in the place of the aperture and observe the diffraction pattern displayed in the laptop. Explain the result observed by the Babinet principle. (<u>Not necessary to include the answer in the report</u>).

5. BIBLIOGRAPHY

[1] A. Ghatak, *Optics* (McGraw-Hill, 6th Ed., 2017).

[2] F. L. Pedrotti, L. M. Pedrotti and L. S. Pedrotti, *Introduction to Optics* (Pearson Int'l Edition, 2006).

[3] J. F. James, *An Introduction to Practical Laboratory Optics* (Cambridge University Press, 2014).

6. APPENDIX

The question of determining whether the results of two measurements of the same magnitude are consistent or not arises very often. These notes are a summary of a way to answer this and similar questions according to the theory of hypothesis testing. More details can be found for example in Estadística básica para estudiantes de Ciencias, by Gorgas, Cardiel and Zamorano, 2009, available at the link:

http://www.ucm.es/info/Astrof/users/jaz/ESTADISTICA/libro_GCZ2009.pdf

Contrasting the equality of means of two populations

Suppose that we have a series of measurements of two quantities X, Y and we want to know if they are compatible. In our case, these quantities are the width of the slit measured by two different methods. We formulate the hypothesis $X = Y$, called the null hypothesis H_0 and construct the statistic:

$$t = \frac{\overline{X}-\overline{Y}}{\sqrt{\Delta^2 X+\Delta^2 Y}} ,$$

whose distribution is of Student's t-type (Welch's approximation). Roughly speaking, the smaller *t*, the closer the null hypothesis is to being true, considering uncertainties that determine what is close or far. Given enough measurements, the *t* statistic can be approximated by a Gaussian distribution that we will consider in this practice.

$$p(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} .$$

We choose a significance level, for example α =0.05 (meaning 5% probability of rejecting H_0 being true). We determine the critical value T such that the probability that $|t| > T$ is α. In the case that the Gaussian approximation is valid, we have that T=1.96. We accept the null hypothesis if $|t| \leq T$ and reject it if $|t| > T$.

QUESTIONNAIRE

- **In this practice, <u>all quantities must be given with their corresponding uncertainties</u>.**
- **For an easier identification of the answers, please, specify in a visible place the number of the question responded (unless you have included explicitly the question).**

Determination of the width of a rectangular slit with the diffractive method

1. Explain and justify the method that is going to be followed to determine the slit width. Include the expressions involved in the method as well as those required to determine the corresponding uncertainties.
2. Clearly show all the measurements performed as well as the corresponding uncertainties.
3. Clearly show the final results with the corresponding uncertainties justifying how have been obtained.

Determination of the width of a rectangular slit with the object-image method

4. Explain and justify the method that is going to be followed to determine the slit width from the image obtained. Justify the expression used for the lateral magnification.
5. Clearly show all the measurements performed as well as the corresponding uncertainties.
6. Clearly show the final results with the corresponding uncertainties justifying how have been obtained.

Comparison between the methods

7. Compare the results obtained including a hypothesis contrast analysis.
8. Which method is more accurate? Comment on any possible deviation between the results obtained with each method.

Additional comments

9. If you have observed or thought about something else that has not been previously considered in any of the above points, you can add it here.