



# Degree in Physics

## Physics Laboratory III

Year 2022–2023 1<sup>st</sup> semester

## Optics Laboratory

### Polarization Experiments

**CAUTION: A laser source is used. To avoid serious injuries in your eyes, be careful not to look directly into the beam or to let a direct reflection of the beam enter your eyes.**

#### 1. OBJECTIVES

- Study of the Malus law for a linear polarizer. Measurement of  $k_1$  and  $k_2$  coefficients for a linear polarizer.
- Determination of the transmission axis of a linear polarizer.
- Determination by the polarization method of the refractive index and Brewster angle of a dielectric material.

#### 2. THEORY

##### 2.1 Polarizers

Polarizers are optical elements that act like filters, ideally letting only electric fields vibrating along a given direction to pass through, while electric fields vibrating in other directions are blocked (partially or totally). The simplest type is the linear polarizer, which only allows electric fields vibrating along their transmission or polarization axis to go through, thus, producing linearly polarized light with a specific polarization state. This is the type of polarizer to be used here.

Consider linearly polarized light incident onto a linear polarizer, with the electric field vibrating at an angle  $\varphi$  with respect to the polarization axis. After having crossed the polarizer, only the projection of the incident electric field along the transmission direction of the polarizer is allowed to pass through. Thus, field amplitude undergoes a reduction by a factor  $\cos \varphi$ . Since the light intensity has a quadratic dependence on the field amplitude, the power is also going to present the same dependence. Therefore, the transmitted power, which is the quantity to be measured (detected) here, will be

$$P(\varphi) = P_0 \cos^2 \varphi, \quad (1)$$

where  $P_0$  is the input power. The expression is known as the Malus law and applies to any ideal linear polarizer.

In real polarizers, though, the material that the polarizer is made of might partly absorb of the incident field, thus diminishing the amplitude of the transmitted field parallel to the polarization axis. Moreover, the field vibrating along the perpendicular direction might also be transmitted partially. This

means that the Malus law (1) needs to be generalized to include these situations. The general expression for Malus law reads as

$$P(\varphi) = P_0(k_1 \cos^2 \varphi + k_2 \sin^2 \varphi) = P_0(k_1 - k_2) \cos^2 \varphi + P_0 k_2, \quad (2)$$

where  $k_1$  and  $k_2$  are coefficients that account for the reduction of the transmitted power along the directions parallel and perpendicular to the polarization axis, respectively. Typically, for good quality polarizers,  $k_1 \gg k_2$ , with  $k_1$  close to 1. For an ideal linear polarizer,  $k_1 = 1$  and  $k_2 = 0$ , which turns Eq. (2) into Eq. (1).

## 2.2 Reflections at interfaces

When an electric field propagating through a medium with refractive index  $n$  is incident onto a medium with refractive index  $n' \neq n$ , light is partly reflected and partly transmitted. As seen in Fig. 1(a), the behavior of the field at the interface between the two media depends on the incidence angle,  $\theta$ , and the azimuth,  $\alpha$ . The azimuth is the angle formed by the plane in which light is vibrating (polarization plane) and incidence planes, i.e., the plane that contains the incident and transmitted wavevectors as well as the normal to the surface. The incidence angle and transmission (refraction) one,  $\theta'$ , are related by the Snell law,  $n \sin \theta = n' \sin \theta'$ , while the relationship between the incident and reflected amplitudes,  $A$  and  $R$ , respectively, are given by the Fresnel formulae:

$$R_{\parallel} = \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')} A_{\parallel}, \quad (3)$$

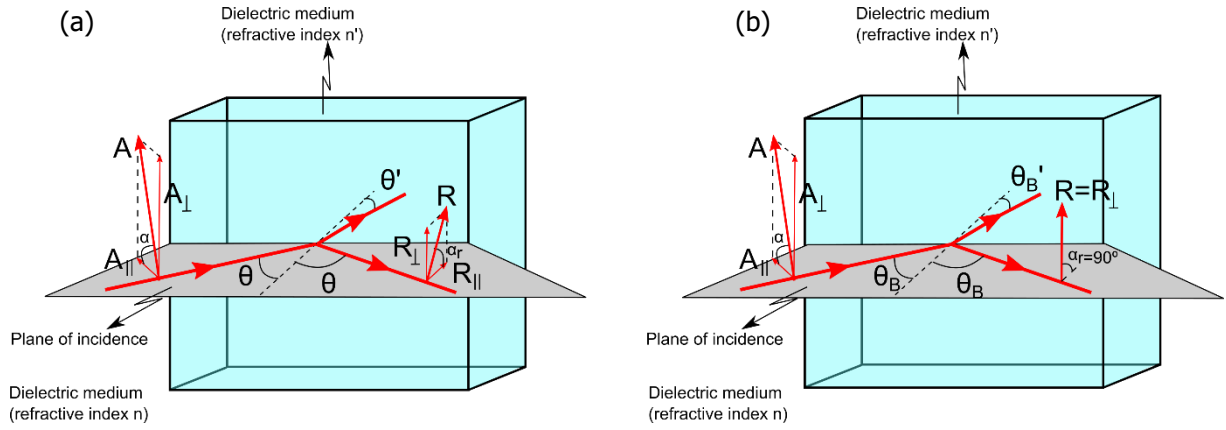
$$R_{\perp} = -\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')} A_{\perp}, \quad (4)$$

where  $\parallel$  and  $\perp$  denote, respectively, the field components parallel and perpendicular to the incidence plane. The power associated with the field vibrating along each direction is proportional to the square of these quantities (see below).

It can be noticed that in the particular case where  $\theta + \theta' = \pi/2$ , the denominator of Eq. (3) becomes infinity and the coefficient  $R_{\parallel}$  vanishes. This particular value for the incidence angle is called the polarization or Brewster angle,  $\theta_B$  (sometimes also denoted as  $\theta_P$ ). In this case, the reflected light is totally polarized perpendicular to the incidence plane regardless of the polarization state of the incident light [see Fig. 1(b)], which is readily seen by substituting the condition  $\theta_B + \theta' = \pi/2$  in Eq. (5). This renders an azimuth  $\alpha_r = \pi/2$ , independent of the value of the incidence azimuth,  $\alpha$ . Substituting  $\theta' = \pi/2 - \theta_B$  into Snell's law, it is also found:

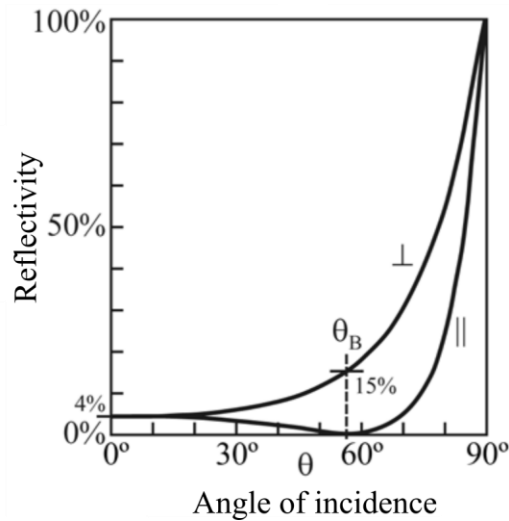
$$\tan \theta_B = \frac{n'}{n}, \quad (5)$$

which relates the Brewster angle with the refractive indices of both media.



**Figure 1.** Incident and reflection directions with the respective azimuths for an arbitrary incidence angle  $\theta$  (a) and for  $\theta = \theta_B$  (b). In case (b), the reflected light is linearly polarized perpendicular to the incidence plane.

The reflected fields for an arbitrary angle of incidence and for  $\theta = \theta_B$  are schematically plotted in Fig. 1. Fig. 2 shows the reflectance or reflectivity of the parallel  $\parallel$  and perpendicular  $\perp$  components of the field as a function of the angle of incidence. Note that the reflectance or reflectivity refers to the reflected beam power divided by incident beam power for the  $\parallel$  and  $\perp$  cases. It is observed that the perpendicular component of the reflectance does not cancel out for any angle, while the parallel component cancels just at the Brewster angle.



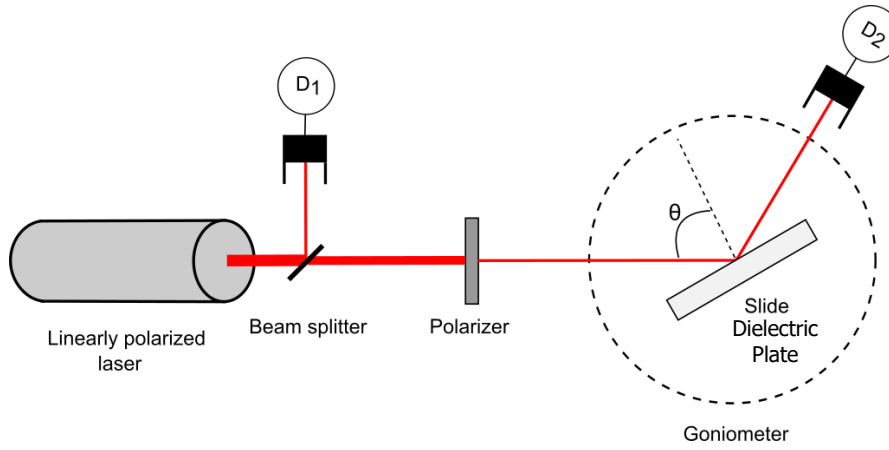
**Figure 2.** Reflectivity as a function of  $\theta$  for the parallel and perpendicular components to the incidence plane of incidence at an air-glass interface ( $n' = 1.5$ ).

We can easily obtain mathematically the expression of the reflectivity or reflectance versus incident angle for the  $\perp$  case from Eq. 4, the Snell's law and assuming  $n=1$  as

$$\left(\frac{R_{\perp}}{A_{\perp}}\right)^2 = \left[\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}\right]^2 = \left(\frac{\cos \theta - \sqrt{n'^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n'^2 - \sin^2 \theta}}\right)^2 \quad (6)$$

### 3. EXPERIMENTAL METHOD

The experiment will be carried out using the setup shown in Fig. 3. A linearly polarized He-Ne laser with an azimuth of approximately  $45^\circ$  passes through a beam splitter. One of the beams is deviated by reflection towards detector  $D_1$ . The beam transmitted through the beam splitter goes across a linear polarizer before reaching polarizer  $D_2$ , which can be either fixed to the setup or attached to the mobile arm of a digital goniometer, depending of the experience to be performed. Before starting the experiment, you must verify that, in absence of laser light, both detectors register a zero signal (zero power). At the center of the goniometer there is also a holder to accommodate a dielectric plate, which will be used in the second experience.



**Figure 3.** Sketch of the full experimental setup used in this practice. More specifically, this depiction shows the configuration used to determine the Brewster angle and refractive index of a material.

An important consideration must be considered to proceed correctly. Although it is rather stable, the power emitted by the He-Ne laser source, it might undergo slight random fluctuations over time. To minimize the effect in the data recorded, two simultaneous measurements of the power are going to be taken during the performance of the experiment, one with  $D_1$  ( $P_1$ ) and the other with  $D_2$  ( $P_2$ ). Thus, rather than using absolute powers in the calculations and their analysis, the relative power will always be considered instead, defined as the ratio  $P = P_2/P_1$ . This ratio should be considered in every case, even when determining the incident amplitudes in the second part. In this way, if  $P_2$  undergoes a variation related to the stability of the laser, the same variation will also be present in  $P_1$  in the same, which will be removed when computing the abovementioned ratio. Therefore, make sure that you measure both  $P_1$  and  $P_2$  systematically along the experiment to avoid errors related to the stability of the source.

## 4. EXPERIMENTS

### 3.1 Measurement of the coefficients $k_1$ and $k_2$ for a dichroic linear polarizer

In this first experience, the setup sketched in Fig. 3 will be used without the dielectric plate and with the detector  $D_2$  fixed and aligned with the incident beam. To align the system, you can try to make the incident beam on  $D_2$  to get back to the laser following the same way where it came from. Note that the polarizer scale does not directly indicate the  $\varphi$  value.

Once the experimental setup is ready and you have checked that the detectors measure a nearly constant power, you can proceed to determine the incidence power as well as the maximum and minimum powers when the polarizer is inserted, which will be used to determine the factors  $k_1$  and  $k_2$ , according to Eq. (2). In order to minimize errors, you can make several measures for each value of the power considering different angles  $\varphi$ . Annotate in every case the angle and the corresponding values of the power measured by each detector.

### 3.2 Measurement of the Brewster angle and the refractive index of a dielectric plate

**WARNING: One of the faces of the dielectric plate is shining, while the other is matte. This experiment should be performed with the shining face (that is, the laser should be incident on and reflected off the shining face).**

The goal of this second experience is to explore two methods to determine the refractive index of a dielectric plate, namely, from the property displayed by the reflectivity of the parallel component at the Brewster angle, and from a nonlinear least-square fitting of the reflectivity for the perpendicular component as a function of the incidence angle. To this end, the full experimental setup is going to be used, which allows to gradually vary the incidence angle and the polarization of the incident beam.

The incidence plane coincides with the plane of the workbench (see Fig. 1). The laser beam is linearly polarized with an azimuth of nearly  $45^\circ$ , so the polarizer studied before can select light polarized parallel or perpendicular with respect to the incidence plane. In order to determine these polarization directions, you can make use of the property of the parallel reflectivity at the Brewster angle either with the aid of the plate (observing its reflection on the wall) or just removing the polarizer from its holder and determine its transmission axis by looking at the reflection off the workbench of a lamp.

Next, you need to ensure that the zero for the angles in the goniometer is correct. To that end, check that the system is correctly realigned before taking any measure of the incidence angle. Then, you can start measuring the reflected power (make sure that previously you have measured the incident one), which can be done by accommodating detector  $D_2$  in the mobile holder. Once this is settled:

1. For **parallel incidence**, briefly explain which value of  $P_2$  we should seek in order to obtain  $\theta_B$  and, from it, to determine the refractive index of the plate using Eq. (6). Note that it is not necessary to measure the power with the detector, but only to observe the reflected light.
2. For **perpendicular incidence**, plot the experimental measurements of the reflectivity or reflectance as a function of the incidence angle along a rather wide angular range. Remember that we obtain always

relative measurements between D2 and D1. Thus, the reflectivity is obtained as  $(R_{\perp}/A_{\perp})^2$ . Where  $R_{\perp}^2$  and  $A_{\perp}^2$  are computed dividing the values obtained from D2 and D1. Obtain a nonlinear best-fit curve by using Eq. (6), from which the value of  $n'$  is straightforwardly determined. Compare this value with the one calculated with the parallel component.

## 5. BIBLIOGRAPHY

- [1] A. Ghatak, *Optics* (McGraw-Hill, 6th Ed., 2017).
- [2] F. L. Pedrotti, L. M. Pedrotti and L. S. Pedrotti, *Introduction to Optics* (Pearson Int'l Edition, 2006).
- [3] J. F. James, *An Introduction to Practical Laboratory Optics* (Cambridge University Press, 2014).

## QUESTIONNAIRE

- **In this practice, all quantities must be given with their corresponding uncertainties.**
- **For an easier identification of the answers, please, specify in a visible place the number of the question responded (unless you have included explicitly the question).**

### Answer before performing any measurement

1. Briefly explain the advantage of using two photodetectors and, therefore, of working with relative quantities instead of absolute ones.

### Measurement of the coefficients $k_1$ and $k_2$ for a dichroic linear polarizer

2. Briefly explain the experimental procedure considered to determine the coefficients  $k_1$  and  $k_2$  of the linear polarizer used here. Write down all the measurements and final values. Include the corresponding uncertainties.

### Measurement of the Brewster angle and the refractive index of a dielectric plate

3. Explain how the polarizer transmission axis has been determined. Note that this procedure is necessary to produce light polarized perpendicular or parallel with respect to the incidence plane. Given that the laser azimuth is approximate, comment on any further operation necessary to determine with more accuracy the azimuth and, therefore, the abovementioned directions of the polarizer transmission axis
4. Explain how the zero angle has been set in the goniometer before making any angular measurement.
5. Briefly and clearly explain the two experimental procedures that can be used to determine the refractive index  $n'$  of the dielectric plate.
6. Write down the results of all the measurements performed. Represent the data of the relative power for the perpendicular component as a function of the incidence angle. To determine the refractive index of the dielectric plate, consider a nonlinear fitting based on Eq. (8) (you can use Origin or SciDAVis, for instance). Include the uncertainties and correlation coefficient rendered by the nonlinear fitting.

7. Give the values obtained for the Brewster angle and the refractive index of the dielectric plate with the two methods considered, including the corresponding uncertainties.
8. Briefly comment on the pros and cons of the two methods considered to determine  $n'$ , properly justifying your arguments.

**Additional Comments**

9. If you have observed or thought about something else that has not been previously considered in any of the above points, you can add it here.