EXERCISE 1

A company is choosing where to open new warehouses to serve an area with 10 cities.

Six different possible locations for the warehouses have been identified. The following matrix contains a "1" if opening the warehouse at a certain location (row) covers the demand of a city (column).

The cost for opening the warehouses are: [150 180 80 150 100 80] for the six possible locations.

Write an LP model to minimize the total cost of opening the warehouses, to serve at least 8 of the 10 cities.

Loc\Cities	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	0	0	1	0	0	1	1	0	0
2	0	0	1	0	0	1	0	1	0	1
3	0	1	0	0	1	0	0	0	0	0
4	0	0	1	0	1	0	1	0	0	1
5	0	1	0	1	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	1	0

SOLUTION 1

$$x_i = \{0,1\} - Opening\ warehouse\ i\ or\ not.$$
$$z_j = \{0,1\} - Covering\ city\ i\ or\ not$$
$$d_{ij} = value\ in\ above\ matrix\ (warehouse\ i - city\ j)$$
$$c_i = cost\ of\ opening\ warehouse\ i$$

$$min \sum_i c_i x_i$$
$$\sum_i d_{ij} x_i \geq z_j \qquad \forall j$$
$$\sum_j z_j \geq 8$$

Typical mistake: forcing a city to be covered by one and only one warehouse (written nowhere).

EXERCISE 2

A company manufactures four products (1,2,3,4) on two machines (A and B). The profit per unit for each product (1,2,3,4) is \$10, \$12, \$17 and \$8 respectively.

The time (in minutes) to process one unit of each product on each machine is shown below.

PRODUCT	MACHINE A	MACHINE B
1	10	27
2	12	19
3	13	33
4	8	23

Product 1 must be produced on ***both*** machines A and B (hence, for instance, producing one unit of product 1 requires using 10 minutes on A **AND** 27 minutes on B), while products 2, 3 and 4 needs only one machine and they can be produced on both A or B.

The factory is very small and this means that floor space is very limited. Only one week's production is stored in 50 square metres of floor space where the floor space taken up by each product is 0.1, 0.15, 0.5 and 0.05 (square metres) for products 1, 2, 3 and 4 respectively.

A customer requirement means that only one between products 3 and 4 must be produced, with a minimum quantity of 30 units.

During the week, machine A is out of action (for maintenance/because of breakdown) 5% of the time and machine B 10% of the time.

Assuming a working week 35 hours long formulate the problem of planning the manufacturing as a linear program with the goal of maximizing the profit.

SOLUTION 2

X_1 = PROD 1 $\in \mathbb{Z}^+$

X_{pm} = PRODUCT p (= 2,3,4) ON MACHINE m (= A, B) $\in \mathbb{Z}^+$

$y_3, y_4 \in \{0,1\}$

$$\text{MAX} \quad 10X_1 + 12(X_{2A} + X_{2B}) + 17(X_{3A} + X_{3B}) + 8(X_{4A} + X_{4B})$$

$$10X_1 + 12X_{2A} + 13X_{3A} + 8X_{4A} \leq 35 \cdot 60 \cdot 0{,}95$$

$$27X_1 + 19X_{2B} + 33X_{3B} + 23X_{4B} \leq 35 \cdot 60 \cdot 0{,}9$$

$$0{,}1X_1 + 0{,}15(X_{2A} + X_{2B}) + 0{,}5(X_{3A} + X_{3B}) + 0{,}05(X_{4A} + X_{4B}) \leq 50$$

$$(X_{3A} + X_{3B}) \leq M y_3 \qquad y_3 + y_4 \leq 1$$

$$(X_{4A} + X_{4B}) \leq M y_4$$

$$X_{3A} + X_{3B} + X_{4A} + X_{4B} \geq 30$$

EXERCISE 3

A large organization is run by teams of administrators. In the course of the week these administrators attend several meetings where decisions are taken.

Suppose that: in a particular week there are 10 different meetings to be scheduled; we have only 5 different time slots available during the week to hold the meetings; there are 5 administrators (A,B,C,D,E).

If two meetings require the attendance of the same administrator, they cannot both be scheduled in the same time slot, because that will create a conflict for that administrator. On the other hand if there is no common administrator that is required to attend two meetings, both of them can be scheduled in the same time slot.

As an additional constraint, at least one between meeting 5 OR meetings 7 and 9 must be scheduled.

The problem is to schedule as many of the 10 meetings as possible in the 5 available time slots.

Meeting	1	2	3	4	5	6	7	8	9	10
Who has to attend	A B	A C	B D E	B C	A D E	C D	A D	A C	D E	B C E

SOLUTION 3

$$x_{ij} \in \{0,1\} = \begin{cases} 1 & \text{MEETING } i \text{ SCHEDULED IN SLOT } j \\ 0 & \text{OTHERWISE} \end{cases}$$

$$\text{MAX} \sum_{i=1}^{10} \sum_{j=1}^{5} x_{ij}$$

$$\text{ADMIN. A} > \quad x_{1j} + x_{2j} + x_{5j} + x_{7j} + x_{8j} \leq 1 \quad \forall j = 1..5$$

$$\text{'' B} > \quad x_{1j} + x_{3j} + x_{4j} + x_{10j} \leq 1 \quad \forall j = 1..5$$

$$\text{'' C} > \quad x_{2j} + x_{4j} + x_{6j} + x_{8j} \leq 1 \quad \forall j = 1 \ldots 5$$

$$(\ldots)$$

$$\sum_{j=1}^{5} x_{ij} \leq 1 \qquad \forall i = 1 \ldots 10 \qquad \left(\text{EACH MEETING SCHEDULED ONCE}\right)$$

$$2 \cdot \sum_{j=1}^{5} x_{5j} + \sum_{j=1}^{5} x_{7j} + \sum_{j=1}^{5} x_{9j} \geq 2$$

EXERCISE 4

A medical institute must plan the purchases of a drug for the next 3 month. At the beginning of the first month there are 30 doses of the drug in stock. The requested quantity of this drug for the next three month is *d(t)*, according to the following table:

Month 1	Month 2	Month 3
42 doses	73 doses	56 doses

The drug can be acquired from different suppliers at the beginning of each month. Each supplier sells boxes with a different price and drug doses:

Supplier	Cost	Doses in a box
S1	22 Euro/dose (220 a box)	10 doses
S2	30 Euro/dose (30 a box)	1 dose
S3	15 Euro/dose (525 a box)	35 doses

In case the drug is not used during the month, it must be preserved into a specific refrigerated inventory with a limited space for 55 doses. (Note: if economically convenient, it is possible to buy more doses and throw away the surplus).

Moreover, for commercial reasons, it is not possible to buy in the same month from F2 and F3 together.

Formulate the LP for minimizing the total cost of the drug provisioning.

SOLUTION 4

S_t = STOCK AT BEGINNING OF MONTH t

X_{st} = BOXES FROM s AT BEGINNING OF MONTH t

NOTE: $\text{COST}(s) = [220 \quad 30 \quad 525]$

$$\text{MIN} \sum_s \sum_t \text{COST}(s) \cdot X_{st}$$

$$S_1 = 30$$

$$S_t + 10X_{1t} + X_{2t} + 35X_{3t} - d(t) \geq S_{t+1} \qquad \forall t: 1..3$$

$$S_t \leq 55 \qquad \forall t: 1..3$$

$$X_{2t} \leq My \qquad \forall t: 1..3$$

$$X_{3t} \leq M(1-y)$$

EXERCISE 5 (No solution given)

A steel factory produces two different types of steel: standard and premium, using 3 different machines A, B and C. The machines can be activated in blocks of hours, and their productivity in one hour is the following:

machine A: 3 slabs of standard and 1 of premium.
machine B: 1 slabs of standard and 2 of premium.
machine C: 2 slabs of standard and 1 of premium.

The daily minimum request is: 130 standard and 90 premium slabs.

The machines can work for 20 hours a day. The production costs of the machines are: 90 euro/hour for machine A, 80 euro/hour for machine B and 60 euro/hour for machine C.

It is possible to use the machines in extra-time for 4 more hours, paying an additional cost of 200 Euros for each machine used in extra-time.

Write the model of the linear program to determine the daily production at minimum cost.

EXERCISE 6 (No solution given)

A company manufactures small toys for children. For the next week the company plans to produce 900 toys. Each toy is assembled from three parts which can be bought externally or produced by the company itself. Parts produced by the company itself can be processed in departments A or B. Activating a production in a department means using some time and money in order to setup the productions lines for that production. In the following table it is given the time consumption (in minutes) for the components in the different departments, the setup time for each part and department (in hours), the machine capacities (time available) of the departments for the next week (in hours), the unitary costs of production (same for any department), the unitary cost of opening a setup (same for any department, paid once independently on the quantity of production of the part in the departments) or for buying components from external suppliers. As an additional constraint, department B can't activate more than one setup during the week. The company wants to minimize the costs (sum of production costs, externally purchased parts costs and setup costs). Write a linear programming model.

PRODUCT	DEPT A	DEPT B	Production Cost	Setup Cost	External Bought Cost
Part 1	7 (min) (setup 1h)	5 (min) (setup 2h)	7 $/part	1000 $	15 $/part
Part 2	3 (min) (setup 3h)	4 (min) (setup 5h)	2 $/part	5000 $	9 $/part
Part 3	2 (min) (setup 5h)	3 (min) (setup 2h)	1 $/part	3000 $	5 $/part
Available time in one week	50 (h)	60 (h)			

EXERCISE 7 (No solution given)

A company manufactures two products P1 and P2, which are obtained by assembling together two components (C1,C2).

In order to produce P1, we need 2 parts of C1 and 3 parts of C2.

In order to produce P2, we need 2 parts of C1 and 1 part of C2.

The selling cost of P1 is 100$, the selling cost of P2 is 50$.

The components can be produced on two machines (M1 and M2).

The production costs of one component (any type) on the machines are: 3$ (machine M1) and 2$ (machine M2).

Machine M2 is able to produce a mix of both the components, while machine M1 is constrained to run the production of only one of the two components.

The available productive time is 50 hours on each machine, for the next week. It is possible to add 5 additional overtime hours to only one of the two machines.

The time to process one unit of each part on each machine is:

Component 1 = 5 minutes for machine A, 10 minutes for machine B.

Component 2 = 15 minutes for machine A, 20 minutes for machine B.

Write a linear programming model for maximizing the company profit.