

Assignment 4

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Task 1

- Prior probabilities:

$$P(c1) = 3 / 5 = 0.6$$

$$P(c2) = 2 / 5 = 0.4$$

- Conditional probabilities:

$$P(\text{happy}|c1) = P(\text{year}|c1) = (2 + 1) / (7 + 7) = 3 / 14 = 0.2143$$

$$P(\text{new}|c1) = (1 + 1) / (7 + 7) = 2 / 14 = 0.1429$$

$$P(\text{celebrations}|c1) = (0 + 1) / (7 + 7) = 0.0714$$

$$P(\text{happy}|c2) = P(\text{new}|c2) = P(\text{year}|c2) = P(\text{celebrations}|c2) = (0 + 1) / (4 + 7) = 0.0909$$

- Classification:

$$P(c1|\text{doc}) = 0.6 * (0.2143)^2 * 0.1429 * 0.0714 = 0.00028$$

$$P(c2|\text{doc}) = 0.4 * (0.0909)^4 = 0.00003$$

The model assigns the document to class 1.

Task 2

Subtask 1

$$p(\text{SPAM}|x1) =$$

$$= 0.2 * 1 - 0.1 * 0 + 0.5 * 1 - 0.2 * 0 - 0.1 * 0 + 0.4 * 0 + 0.1 * 1 + 0.0 * 0 =$$

$$= 0.2 + 0.5 + 0.1 =$$

$$= 0.8$$

$$p(\text{HAM} | x_1) =$$

$$= 0.2 * 0 - 0.1 * 1 + 0.5 * 0 - 0.2 * 1 - 0.1 * 0 + 0.4 * 0 + 0.1 * 0 + 0.0 * 1 =$$

$$= -0.1 - 0.2 =$$

$$= -0.3$$

$$\exp^{\text{sum}(\text{weight} * \text{features}(\text{SPAM}, x_1))} \rightarrow e^{0.8} = 2.23$$

$$\exp^{\text{sum}(\text{weight} * \text{features}(\text{HAM}, x_1))} \rightarrow e^{0.3} = 0.74$$

$$p(\text{SPAM} | x_1) = 2.23 / (2.23 + 0.74) = 2.23 / 2.97 = 0.75 = 1$$

$$p(\text{HAM} | x_1) = 0.74 / (2.23 + 0.74) = 0.74 / 2.97 = 0.25 = 0$$

Subtask 2

$f_6 \rightarrow y = \text{you in HAM}$

$\text{Weight}_6 \rightarrow 0.4$

$$\text{Derivative A} = 0 + 0 + 0 + 0 + 1 + 1 = 2$$

Derivative B =

$$P(\text{HAM} | x_3) = e^{(0.4 - 0.1)} / e^{(0.4 - 0.1)} + e^{(0.2 - 0.1 + 0.1)} = 0.525$$

$$p(\text{HAM} | x_5) = e^{(0.4 - 0.2)} / e^{(0.4 - 0.2)} + e^{(-0.1 + 0.5 + 0.1)} = 0.426$$

$$p(\text{HAM} | x_6) = e^{(0.4 - 0.1)} / e^{(0.4 - 0.1)} + e^{(-0.1 + 0.2 + 0.1)} = 0.525$$

$$A - B = 2 - (0.525 + 0.426 + 0.525) = 0.524 = 0.52$$