

Laboratory 06

Finite Element method for non vector-valued problems

Exercise 1.

Let $\Omega = (0, 1)^3$ be the unit cube and let us consider the following linear elasticity problem: find a displacement field $\mathbf{u} : \Omega \rightarrow \mathbb{R}^3$ such that

$$\begin{cases} -\nabla \cdot \sigma(\mathbf{u}) = \mathbf{f} & \text{in } \Omega, \\ \mathbf{u} = \mathbf{g} & \text{on } \Gamma_0 \cup \Gamma_1, \\ \sigma(\mathbf{u})\mathbf{n} = \mathbf{0} & \text{on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5, \end{cases} \quad (1)$$

where

$$\begin{aligned} \sigma(\mathbf{u}) &= \mu \nabla \mathbf{u} + \lambda (\nabla \cdot \mathbf{u}) I, \\ \Gamma_0 &= \{x = 0, y \in (0, 1), z \in (0, 1)\}, \\ \Gamma_1 &= \{x = 1, y \in (0, 1), z \in (0, 1)\}, \\ \Gamma_2 &= \{x \in (0, 1), y = 0, z \in (0, 1)\}, \\ \Gamma_3 &= \{x \in (0, 1), y = 1, z \in (0, 1)\}, \\ \Gamma_4 &= \{x \in (0, 1), y \in (0, 1), z = 0\}, \\ \Gamma_5 &= \{x \in (0, 1), y \in (0, 1), z = 1\}, \end{aligned}$$

$\mu = 1$, $\lambda = 10$, $\mathbf{g}(\mathbf{x}) = (0.25x, 0.25x, 0)^T$ and $\mathbf{f}(\mathbf{x}) = (0, 0, -1)^T$.

1.1. Write the weak formulation of problem (1).

1.2. Implement in `deal.II` a finite element solver for problem (1).