Programming Ontological Generators

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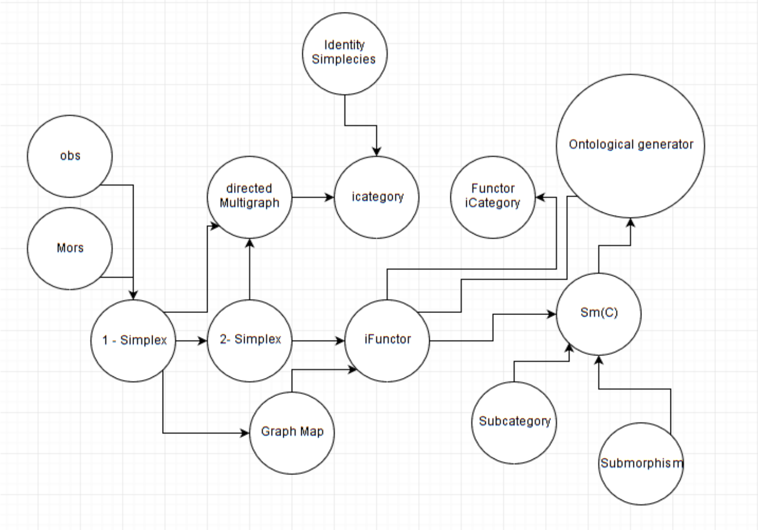
Olog Golog



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-create multigraph class multigraph.morphisms = [morphisms] multigraph.objects = [objects]

Graph Maps

Graph Map

Let G, H be two graphs, A pair
$F = (F^0 : ob(G) \to ob(H), F^1 : mor(G) \to mor(H))$ is a graph map, if $F : G \to H$ satisfies:
i)$F^1(f).dom = F^0(f.dom)$
ii)$F^1(f).codom = F^0(f.codom)$

G-Simplecies/ simplicial graphs

Consider the following multigraph Δ^2:
(tikz)

G-simplex

A G-simplex is a graph Map $F : \Delta^2 \to G$ which we write as $(f, g, gof) = (F(01), F(12), F(02))$
A the set of G-simplecies is GSimp, and comes with two maps$\pi_f, \pi_g : GSimp \to mor^2(G)$ by

$$\pi_f(F) = F_1(01) \text{ and } \pi_g(F) = F_1(12)$$

Consider a subset of G-simplecies $Simp \subseteq GSimp$
Simp is called coherant, if $\forall F \in simp$ F is the only simplex such that $F(01) = f$ and $F(12) = g$

$$(\text{i.e. } |[\pi_0 1 \times \pi_1 2]^{-1}(mor(G)^2)| \leq 1.)$$

simplicial graphs

simplicial graph

A simplicial graph C $=$ (G,Simp) is a multigraph G with a coherant set of simplecies Simp.

simplicial map

Let C, D be two simplicial graphs, a graph Map $F : G \to H$ is a simplicial map if the following equations hold:
i)$F_2(simp).ob = F_0(simp.ob)$
ii)$F_2(simp).mor = F_1(simp.mor)$

identities

Identity

Let C and D be two simplicial graphs, a morphism $I : o \rightarrow o$ is an identity if $\forall f : A \rightarrow C, g : C \rightarrow B$ there is a simplex (f,I,f) and (I,g,g).

iCategory

an icategory is a simplicial graph such that every object o has an identity morphism $I(o) = I_o$, where $I : ob \rightarrow mor$

functors, subcategories

iFunctor

an iFunctor $F : C \to D$ between icategories, is a simplicial map such that:
$F^1(I(o)) = I(F^0(o))$
Q: is this redundant? Can this be derived via simplecies?
A: yes, the simplecies (I_o, I'_o, I_o) and (I_o, I'_o, I'_o) results in $I_o = I'_o$ by uniqueness.

Sm(C)

subcategory

a subcategory $S \leq C$ is three subsets $ob_S \leq ob_C$, $mor_S \leq mor_C$, $simp_S \leq simp_C$ such that the inclusion induces a functor $i : S \to C$ (this means that the inclusion maps (i^1, i^2, i^3) satisfy the graph map, simplicial map and iFunctor conditions)

submorphism

Let $Hom_C(S, S')$ be the collection of maps $f : o \to o'$ such that $o, o' \in obj(S, S')$
A submorphism $\mathcal{F} : S \to S'$ is a subset $\mathcal{F} \subseteq Hom_C(S, S')$

Sm(C)

Sm(C)

Let Sm(C) be the icategory of subcategories and submorphisms.

of course we have to define the simplecies (and identities):
let

$$\mathcal{G} \circ \mathcal{F} = \{g \circ f | f : o \to o', g : o' \to o'' g.dom = f.codom\}$$

and define the simplecies $(\mathcal{F}, \mathcal{G}, \mathcal{G} \circ \mathcal{F})$ then,
$I_S = \{Id_o | o \in S\}$

Ontological Generators

Ontological Generator

An ontological generator is a Functor $OG : C \rightarrow sm(C)$