The Innovation Long-run Risk Component

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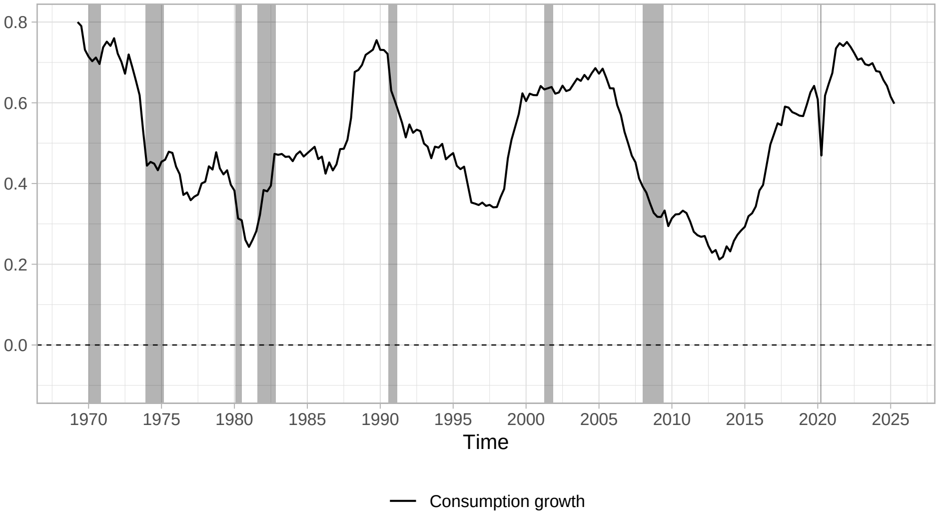
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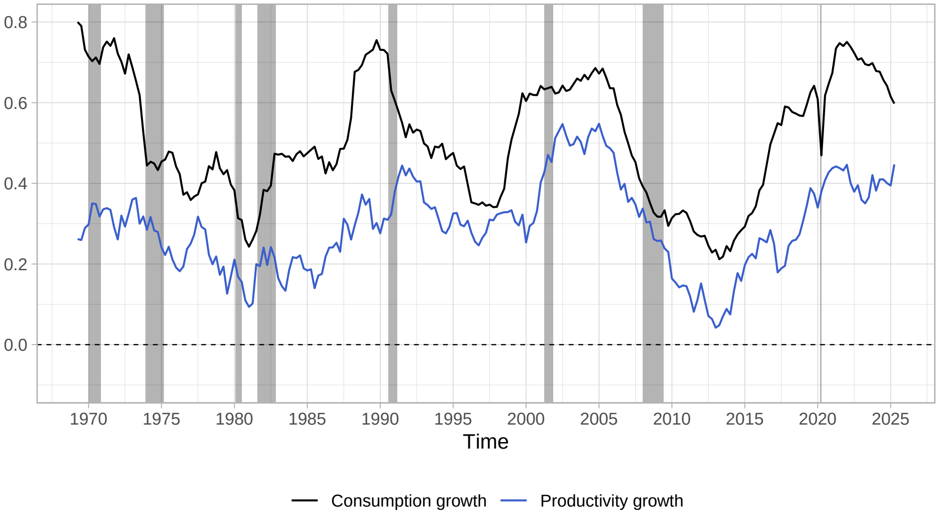
Time
Consumption growth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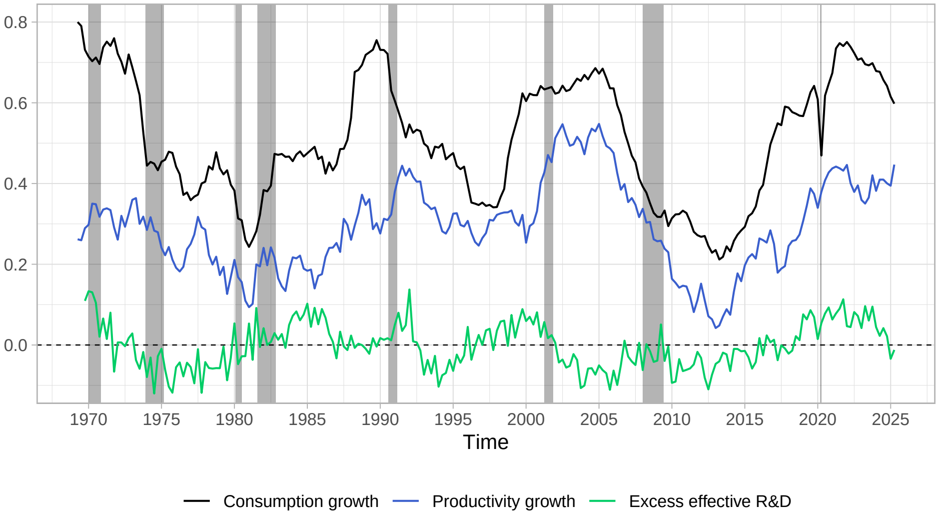
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Productivity growth
Excess effective R&D

In a nutshell

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- Effective R&D: scaling to account for spillovers and product variety effects

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- Multivariate approach: persistent effects of R&D and shocks identification

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- Effective R&D: scaling to account for spillovers and product variety effects
- Multivariate approach: persistent effects of R&D and shocks identification
- Significant and robust cross-sectional risk premium, key role of cash-flow channel

Theoretical framework

Minimal ingredients

Assumptions:

$$Z_t = I_t^{\xi} \cdot e^{a_t} \tag{1}$$

$$\tag{2}$$

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$$I_t = (1-\phi) I_{t-1} + \chi \cdot S_{t-1}^{\eta} I_{t-1}^{\psi} Q_{t-1}^{-\omega} \tag{2}$$

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Key prediction:

$$\Delta \ln Z_{t+1} \approx \gamma_0 + \gamma_1 \left(\ln S_t - \frac{1-\psi}{\eta} \ln I_t - \frac{\omega}{\eta} \ln Q_t \right) + \Delta a_{t+1} \tag{3}$$

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"Effective R&D":

$$s_t \equiv \ln S_t - \frac{1-\psi}{\eta} \ln I_t - \frac{\omega}{\eta} \ln Q_t \tag{4}$$

Stationary TFP and effective R&D:

$$\tilde{s}_t = s_t - \bar{s} \tag{5}$$

The Innovation Component of Productivity Growth

Stationary TFP and effective R&D:

$$\tilde{s}_t = s_t - \bar{s} \tag{5}$$

$$E_t\left[\Delta \ln Z_{t+1}\right] \approx \mu + \gamma_1 \cdot \tilde{s}_t \tag{6}$$

Stationary TFP and effective R&D:

$$\tilde{s}_t = s_t - \bar{s} = \rho_s \tilde{s}_{t-1} + \tilde{\varepsilon}_t \tag{5}$$

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Long-run impact of R&D shocks:

$$E_{t+1} - E_t \left(\sum_{j=0}^{\infty} \Delta \ln Z_{t+1+j} \right) = \frac{\rho_s}{1 - \rho_s} \tilde{\varepsilon}_{t+1} \tag{7}$$

(8)

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$$\propto \{E_{t+1} - E_t\} \sum_{j=1}^{\infty} \Delta \ln C_{t+j} \tag{8}$$

The Innovation Long-Run Risk premium

Fundamental asset pricing equation:

$$E\left[R^i_{t+1}\right] - R^f_t = -R^f_t \cdot \text{Cov}\left[M_{t+1}, R^i_{t+1}\right]$$

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Recursive preferences:

$$\ln M_{t+1} = E_t\left[\ln M_{t+1}\right] - b_c\, \varepsilon_{c,t+1} - b_x\, \varepsilon_{x,t+1}$$

where

$$\varepsilon_{c,t+1} = \ln C_{t+1} - E_t\left[\ln C_{t+1}\right], \qquad \varepsilon_{x,t+1} = \{E_{t+1} - E_t\} \sum_{j=1}^{\infty} \kappa_x^j \Delta \ln C_{t+1+j}$$

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Result:

$$E_t\left[R_{t+1}^{i}\right] - R_t^f = \lambda_c \beta_c^i + \lambda_x \beta_x^i \tag{9}$$

Macroeconometric framework

A feasible measure

Ideas limit sample timespan and fragile to misspecification:

$$\tilde{s}_t = S_t - \frac{1-\psi}{\eta} I_t - \frac{\omega}{\eta} Q_t - \bar{s} \tag{10}$$

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$$\tilde{s}_t = S_t - \alpha_Z(\ln Z_t - a_t) - \alpha_L L_t - \bar{s} \tag{10}$$

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New object, “gross” effective R&D:

$$\hat{s}_t = \tilde{s}_t - \alpha_Z a_t \tag{11}$$

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$\tilde{s}$ not necessary to get $\varepsilon_{s,t+1}$, but bonus ($\kappa_a \equiv 1 - \alpha_Z \gamma_1$):

$$\tilde{s}_t = \alpha_Z \left(\sum_{j=0}^{t-1} \kappa_a^j (\Delta \ln Z_{t-j} - \mu) \right) + \sum_{j=0}^{t-1} \kappa_a^j \Delta \hat{s}_{t-j} + \kappa_a^t \tilde{s}_0 \tag{12}$$

Adding feedback effects:

$$a_{t+1} = \theta_s \tilde{s}_t + \rho_a a_t + b_{aa} \varepsilon_{a,t+1} \tag{13a}$$

$$\Delta \ln Z_{t+1} = (\gamma_1 + \theta_s) \tilde{s}_t + (\rho_a - 1) a_t + b_{aa} \varepsilon_{a,t+1} \tag{13b}$$

$$\tilde{s}_{t+1} = \rho_s \tilde{s}_t + \theta_a a_t + b_{as} \varepsilon_{a,t+1} + b_{ss} \varepsilon_{s,t+1} \tag{13c}$$

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VARMA from taking a_t out and $\hat{s}$ in:

$$\Delta \ln Z_{t+1} = \quad \bar{\gamma}_{11} \hat{s}_t + \rho_a \Delta \ln Z_t - \bar{\gamma}_{12} \hat{s}_{t-1} \quad + \bar{b}_{aa} \varepsilon_{a,t+1} - \bar{b}_{aa} \varepsilon_{a,t} \tag{14a}$$

$$\hat{s}_{t+1} = \quad \rho_s \hat{s}_t - \bar{\gamma}_{21} \Delta \ln Z_t + \bar{\gamma}_{22} \hat{s}_{t-1} \quad + \bar{b}_{as,1} \varepsilon_{a,t+1} + b_{ss} \varepsilon_{s,t+1} + \bar{b}_{as,2} \varepsilon_{a,t} \tag{14b}$$

The empirical innovation component

The gross effective R&D

$$S_t = \alpha_0 + \alpha_Z \ln Z_t + \alpha_L \ln L_t + \hat{s}_t$$

	Baseline	S: Tot. R&D	Z: Raw TFP	Q: N.F. Empl.	Est. Meth.: IM
α_Z	3.526***	4.197***	3.655***	3.349***	2.821***
α_L	0.909***	−0.354	0.956***	0.953***	1.387***
T	309	309	309	309	309
κ	3.4×10^6	3.4×10^6	3.3×10^6	3.5×10^6	1.1×10^8
			$\hat{s}_t$		
$\sigma_{\hat{s}}$	0.130	0.144	0.128	0.129	0.253
tt	0.00	0.00	0.00	0.00	0.00
tt^2	0.00	0.00	0.00	0.00	0.00
ADF	−2.57**	−2.45**	−2.92***	−2.45**	−9.18***
KPSS	0.09	0.09	0.09	0.10	0.29
AR(1)	0.96	0.96	0.95	0.97	0.15
HL low	2.6	2.7	2.1	2.7	0.1
HL high	21.0	23.6	12.0	25.1	0.1

All ECTs plot

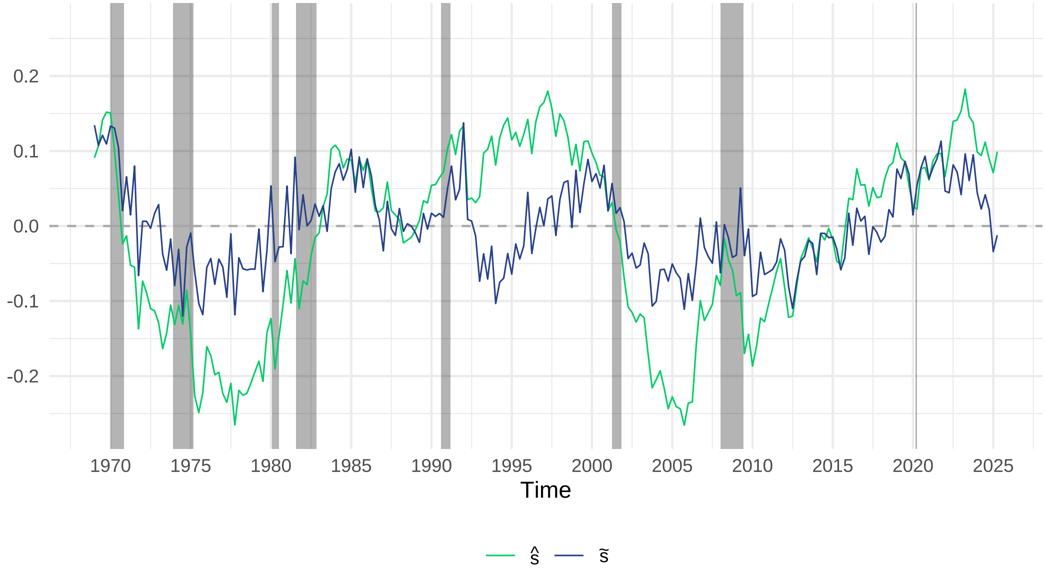
Recovering the effective R&D

▸ All $\tilde{s}$ plot

▸ κ decay

$$\Delta \ln Z_{t+1} = b_0 + b_s \hat{s}_t + \mathbf{b}_f' \mathbf{f}_t + u_{t+1}$$

	Baseline		S: Tot. R&D		Z: Raw TFP		Q: N.F. Empl.	
	BS	LN	BS	LN	BS	LN	BS	LN
b_s (%)	1.558***	1.549***	1.066***	1.223***	0.997***	0.794**	1.507***	1.520***
T	292	261	292	261	291	260	292	261
R^2 (%)	9.5	12.4	7.4	11.8	21.8	41.7	9.1	12.2
$W(k)$	79.97***	61.00***	64.56***	50.27***	287.53***	2775.10***	73.09***	61.99***
κ_a	0.964	0.971	0.950	0.949	0.945	0.945	0.955	0.949
	(0.014)	(0.013)	(0.016)	(0.012)	(0.017)	(0.012)	(0.014)	(0.012)
	$\tilde{s}_t \quad (\kappa_a^t < 0.01)$							
$T_{\tilde{s}}$	226	225	207	220	183	151	219	219
$\sigma_{\tilde{s}}$	0.058	0.057	0.068	0.065	0.060	0.062	0.055	0.055
ADF	−3.91***	−3.95***	−3.17***	−3.49***	−3.95***	−3.50***	−3.56***	−3.56***
KPSS	0.09	0.09	0.18	0.13	0.22	0.19	0.14	0.14
AR(1)	0.71	0.70	0.72	0.68	0.68	0.70	0.70	0.70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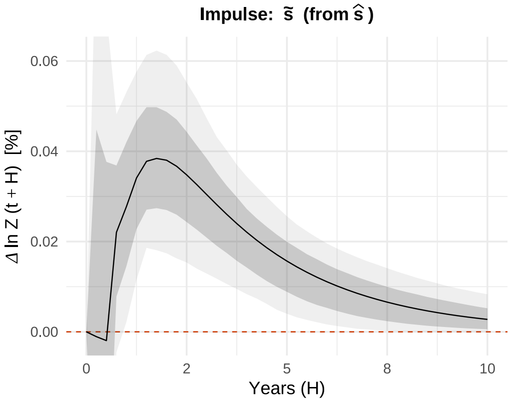
The long-run risk from innovation

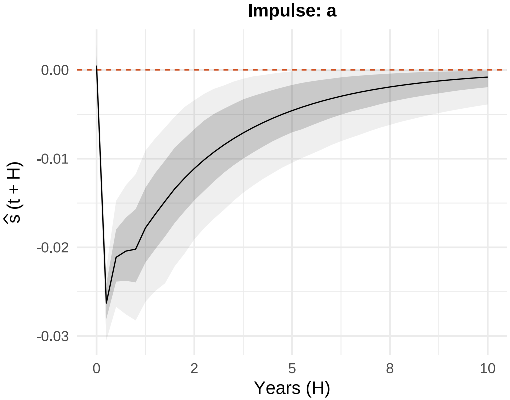
VAR estimates

	Baseline	S: Tot. R&D	Z: Raw TFP	Q: N.F. Empl.	$\tilde{s}$
N. Obs.	305	305	306	305	223
N. Lags	3	3	2	3	2
$R^2_{\Delta Z}$ (%)	5.3	4.4	4.8	5.3	2.8
$\max\lvert\text{roots}\rvert$	0.92	0.90	0.93	0.92	0.93
κ	2.2×10^3	5.7×10^3	2.7×10^3	2.2×10^3	2.8×10^3
H-LM$(z, 4)$	3.2	2.3	22.6***	3.3	4.9
H-LM$(s, 4)$	30.2***	27.0***	33.2***	29.4***	6.4
AC-LM(1)	5.8	6.3	6.2	5.1	7.2
AC-LM(8)	29.4	37.9	41.9	26.7	37.5
AC-LM(16)	69.6	68.6	69.3	69.6	69.1
AC-LM(40)	169.4	155.7	166.9	165.9	154.7
F-GC(s)	5.3***	6.5***	6.1***	5.5***	1.7

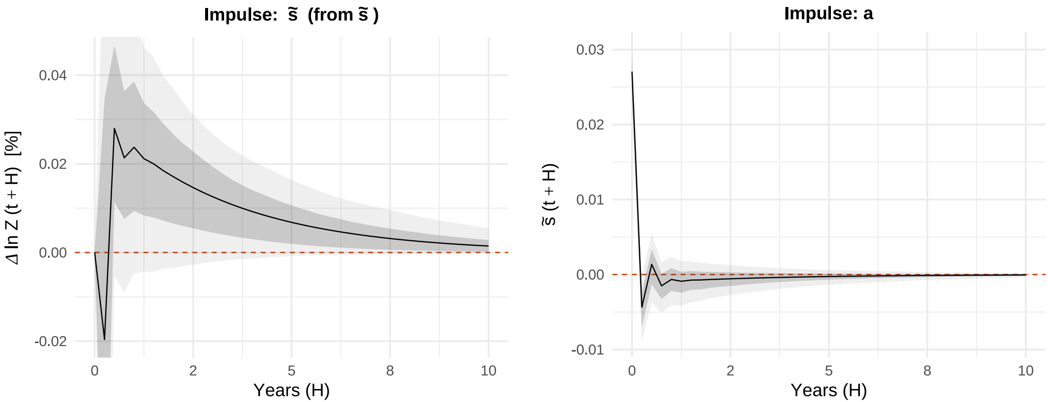
All ε_S plot

VAR IRF's – gross effective R&D

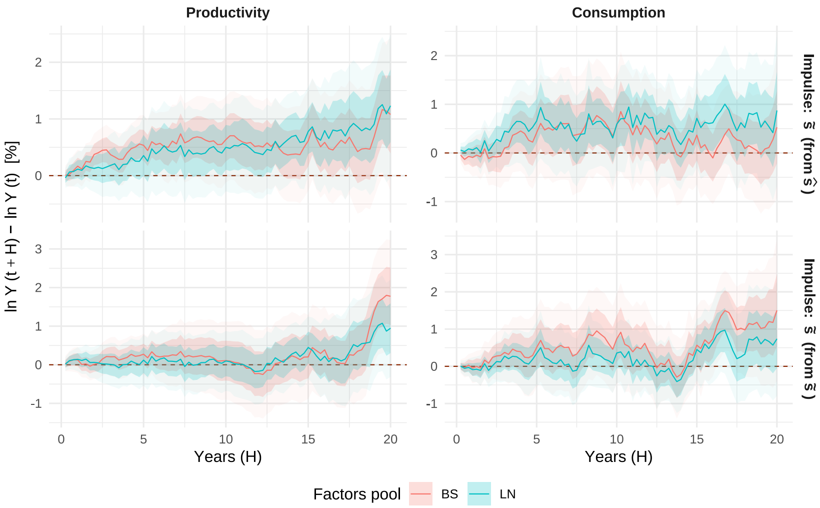




VAR IRF's – net effective R&D



LP IRF's



Productivity
Consumption
ln Y (t + H) − ln Y (t) [%]
Impulse: s̃ (from ŝ)
Impulse: s̃ (from s̃)
Years (H)
Years (H)
Factors pool
BS
LN

The cross-sectional risk premium

Estimates robust to omitted risk factors (Giglio, Xiu (2021, JPE), 183 test assets)

	Baseline	S: Tot. R&D	Z: Raw TFP	Q: N.F. Empl.	$\tilde{s}$
Horizon: 1 quarter					
p=6	0.01	0.02	0.02	0.02	0.02
	$[0.93]$	$[0.90]$	$[1.22]$	$[1.03]$	$[1.28]$
p=14	0.04	0.03	0.05	0.04	0.04
	$[1.32]$	$[0.74]$	$[1.35]$	$[1.12]$	$[1.08]$
p=22	-0.01	-0.09	0.01	-0.02	-0.04
	$[-0.21]$	$[-1.30]$	$[0.09]$	$[-0.38]$	$[-0.55]$
Horizon: 4 years					
p=6	0.08	0.07	0.09	0.08	0.11
	$[1.33]$	$[1.11]$	$[1.33]$	$[1.36]$	$[0.97]$
p=14	0.48^{***}	0.34^{**}	0.52^{***}	0.45^{***}	0.69^{***}
	$[3.28]$	$[2.50]$	$[3.52]$	$[3.11]$	$[2.75]$
p=22	0.54^{***}	0.43^{**}	0.61^{***}	0.48^{**}	0.80^{**}
	$[2.80]$	$[2.23]$	$[3.17]$	$[2.52]$	$[2.28]$
Num.Obs.	213	213	213	213	213

Fundamentals channel: cash flows sensitivities

Portfolio	Cons.		Raw TFP		Adj. TFP		$\tilde{s}$: shock		$\tilde{s}$: level	
Horizon	*1*	*8*	*1*	*8*	*1*	*8*	*1*	*8*	*1*	*8*
RD(1-small)	0.09	0.18	0.05	0.39	−0.07	0.03	0.01	0.00	−0.59	−0.58
RD(2-small)	0.06	0.04	0.02	0.46	−0.11	0.19	0.04	−0.02	−1.16	−0.73
RD(3-small)	0.75	−0.68	0.59	1.17	−0.35	0.45	0.54	1.09	−1.55	−3.37
RD(1-big)	0.01	0.03	0.00	0.08	−0.02	−0.01	0.00	−0.03	−0.27	−0.29
RD(2-big)	0.05	0.12	0.01	0.23	−0.03	0.00	0.03	−0.07	−0.36	−0.28
RD(3-big)	0.03	0.09	−0.01	0.24	−0.07	−0.09	−0.02	−0.17	−1.20	−1.20
To(1-small)	0.05	0.13	0.05	0.27	−0.01	0.03	0.02	0.06	−0.25	−0.76
To(2-small)	0.10	0.29	−0.01	0.81	−0.13	0.22	0.01	0.28	0.20	0.99
To(3-small)	0.38	1.06	0.23	1.85	−0.15	0.08	0.12	0.75	2.64	3.53
TQ(1-small)	0.35	0.87	0.05	2.43	−0.50	0.78	0.18	0.27	−1.51	1.92
TQ(2-small)	0.14	0.65	0.01	1.10	−0.12	−0.15	0.06	0.20	0.36	0.63
TQ(3-small)	0.06	0.14	0.05	0.24	−0.01	0.00	−0.01	−0.06	−0.13	−0.37

Fundamentals channel: cash flows risk premium

	Cons.	Raw TFP	Adj. TFP	$\tilde{s}$: shock	$\tilde{s}$: level
		Horizon: 1 quarter			
Ext. pool	1.72**	1.97***	−2.47**	2.28**	0.06*
	[2.58]	[3.14]	[−2.12]	[2.31]	[1.68]
R^2 (%)	24.85	20.44	17.95	16.77	4.89
MAPE (%)	0.41	0.44	0.43	0.46	0.48
		Horizon: 2 years			
Ext. pool	0.32	0.40**	1.17**	0.71**	0.04
	[1.65]	[2.15]	[2.15]	[2.14]	[1.63]
R^2 (%)	3.06	23.66	29.24	20.32	4.58
MAPE (%)	0.48	0.41	0.41	0.44	0.48

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Conclusions

Key Takeaways

- Endogenous growth models provides a synthetic and informative measure of aggregate R&D
- R&D has persistent effects on TFP growth, accumulating through system interactions
- R&D is significantly priced in stock markets

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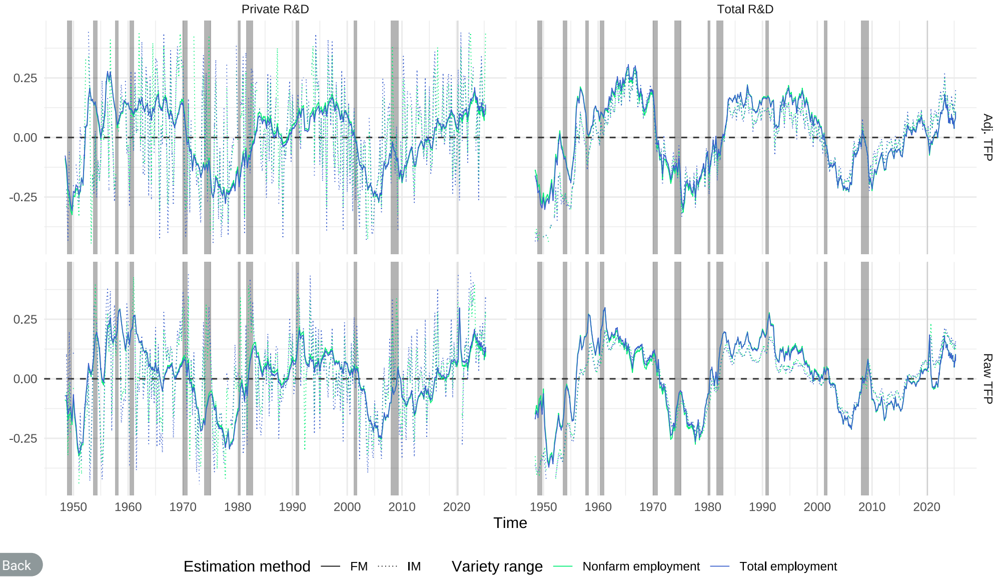
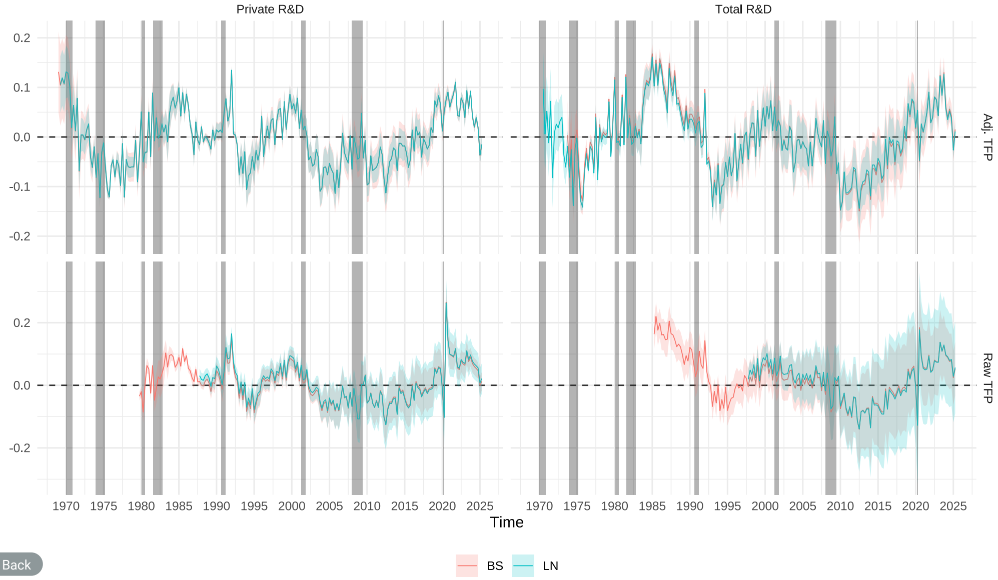
Additional Figures

All gross effective R&D series



◂ Back

All effective R&D series



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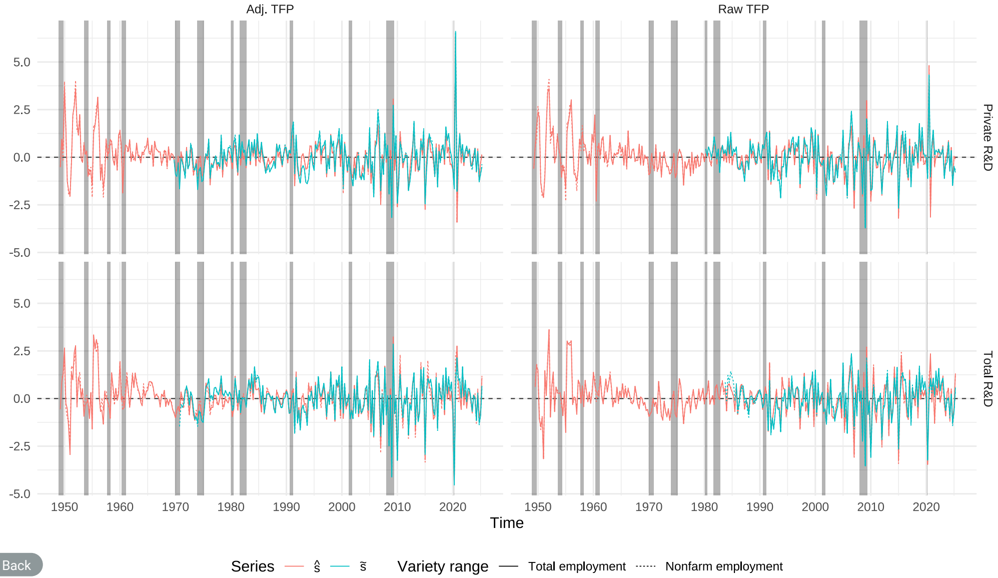
All effective R&D structural shocks series



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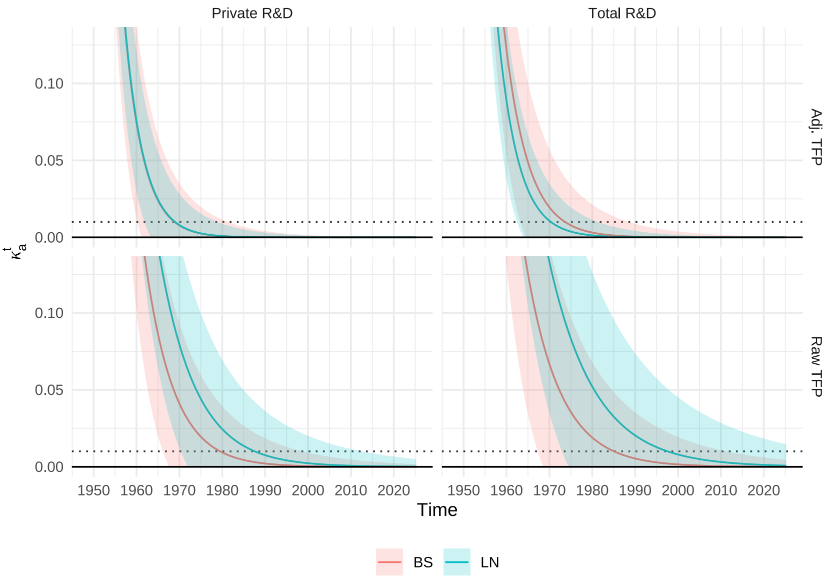
Recovery approximation accuracy



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Recovery uncertainty - lower bound