Does CAPM Overestimate the Risk or Its Price More?

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Abstract

Empirical returns systematically depart from CAPM predictions, with deviations declining in asset betas. This paper decomposes this pattern into mismeasurement of risk and the risk premium, using a framework that accounts for leverage constraints and multiple risk factors. This spans and generalizes two previously separate explanations of the anomaly, showing how bid-ups for high-risk assets arise from funding tightness in the presence of risks beyond market exposure. Crucially, even with binding constraints, any factor model can be expressed as a single aggregate risk measure multiplying the expected market return. Funding tightness and exposure to omitted risks are then demonstrated to offset each other in explaining the beta-related departures, with their relative contributions quantifiable. GMM estimates show both channels are significant, with omitted risks accounting for a slightly larger share, and the spread generated by funding tightness at around 2% per year.

Keywords: Asset Pricing, CAPM, Leverage Constraints, Omitted Factors

JEL Codes: G11, G12, G20

1 Introduction

The Capital Asset Pricing Model (CAPM) is a foundational framework in finance and remains highly influential, as reflected by the nearly 70% of practitioners who rely on it to estimate the required return on equity (Pinto et al. 2019). Its appeal likely stems from its stylized nature, which however also leads the model to exhibit well-documented shortcomings in empirical applications. A prominent example, documented since Miller and Scholes (1972), is its tendency to overestimate the returns of high-risk assets and, conversely, underestimate the returns of low-risk assets – a pattern commonly referred to as the 'Low-Risk Anomaly' (LRA). Several explanations have been proposed, which can be broadly categorized as either a systematic misspecification of risk, arising from the CAPM's omission of specific risk sources, or a mispricing of the risk premium, driven by intermediaries' funding tightness.

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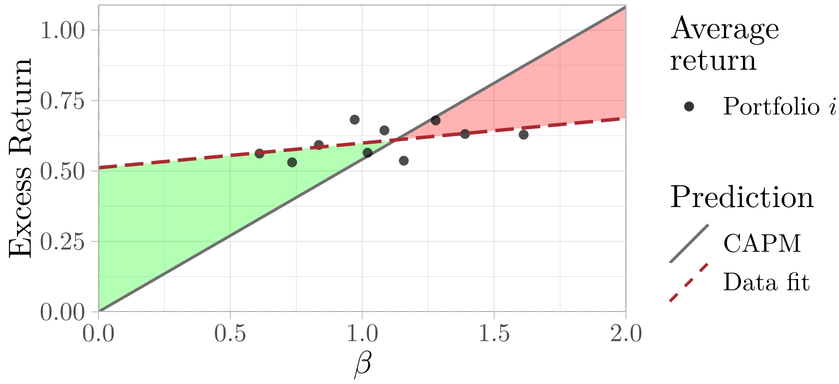

Figure 1: 10 US stocks portfolios. Monthly data, 1963-2019, source: K. French data library.

This paper considers leverage constraints and multiple priced risk factors within an integrated economy to make two contributions. First, it shows theoretically that the more 'true' risk deviates from the CAPM definition in explaining return anomalies, the less the 'true' risk premium can deviate from CAPM predictions. Second, it provides empirical evidence by quantifying the relative contribution of those two deviations/channels in the cross-section of U.S. stock portfolios. Both prove relevant with similar magnitudes, and funding tightness is associated with a 2% return spread per year.

CAPM states that the expected return of an asset with uncertain payoffs exceeds the risk-free rate in proportion to the asset's undiversifiable volatility – its *beta*, i.e., sensitivity to market returns – times a market-wide risk premium, corresponding to the expected excess return of the market.[1] CAPM predicts that cross-sectional average returns vary linearly with asset betas, with a slope equal to the market's average excess return, while assets uncorrelated with the market are expected to earn the risk-free rate. These predictions, however, have been extensively shown not to hold, as illustrated in Figure 1 and documented by Baker et al. (2014) and Frazzini and Pedersen (2014) for markets beyond the U.S., across geographies and asset classes. Setting aside statistical concerns,[2] Frazzini and Pedersen (2014), building on Black (1972), argue that high-beta assets underperform CAPM predictions because intermediaries, constrained in leverage, bid up the prices of assets requiring less leverage. In other words, high-beta assets are priced lower than low-beta ones, limiting the leverage needed to achieve a given expected payoff. Their key implications are: (1) heterogeneity in betas remains the primary driver of differences in average returns, as only

[1]This follows from assuming perfect markets and that variance is the only statistical property of wealth affecting agents' utility besides its first moment. Under these assumptions, the market portfolio is the optimal risky holding for any agent, as it is the most diversified possible. It follows that: (1) an asset's risk is measured by the increase in portfolio variance associated with holding more of it, which depends on its covariance with the market; and (2) this risk is compensated competitively, just as the variance of the optimal portfolio is, i.e., by the market's expected excess return.

[2]Black et al. (1972) first noted that the empirical security market line can appear flatter than CAPM predicts due to an errors-in-variables problem when estimating betas. They proposed grouping assets into portfolios to mitigate this issue, a method now widely used, but the anomaly persists. Other concerns remain, such as historical averages being a poor proxy for expectations (Elton (1999)) and the lack of inferential theory when the market portfolio is not directly observable (Guermat (2014)).

the risk premium can depart from CAPM predictions; and (2) the portfolio capturing these deviations – the so-called 'Betting Against Beta' (BAB) factor – earns positive returns reflecting intermediaries' leverage constraints. Conversely, among the risk sources ignored by CAPM, some systematically offset market exposure, making high-beta assets less risky than CAPM reflects and low-beta assets more risky.[3] These include residual coskewness (Schneider et al. 2020), idiosyncratic lottery-like payoffs (Bali, Brown, et al. 2017), size (Novy-Marx and Velikov 2022), and sensitivity to disagreement Hong and Sraer (2016).[4] The framework in this paper accommodates leverage constraints and virtually any systematic risk, though it does not directly capture purely diversifiable behavioral motives. Nonetheless, Schneider et al. (2020) show that the coskewness factor captures much of the idiosyncratic volatility dispersion, mitigating concerns from omitting such motives. The analysis is based on a representation of optimal pricing conditions predicting individual expected excess returns as a function of a liquidity-driven return spread, expected market returns, and a single risk measure, as in Frazzini and Pedersen (2014). This synthetic risk measure, however, aggregates *all* systematic risk sources in the economy, allowing explicit attribution of deviations from CAPM to either differences in risk considerations or to the priced risk premium, while illustrating that the stronger one effect, the weaker the other must be. Indeed, this framework generalizes the key insight from Frazzini and Pedersen (2014): leverage-constrained investors bidding up high-risk assets compress premia across all systematic risks, not just market exposure, so high-beta assets are less affected the more other risks offset market exposure, making them effectively safer.

Quantifying the contributions of funding constraints versus omitted risks matters beyond explaining the LRA. Baker et al. (2014), for instance, argue that stricter regulations raise banks' cost of capital, assuming the anomaly is entirely due to a low market risk premium and that banks' risk is low when the beta is low. However, if omitted risks are the main driver of the anomaly, this conclusion is challenged: banks and other low-beta equities may be riskier than CAPM suggests, making it crucial to move beyond CAPM to properly assess the impact of regulatory changes on discount rates. Firms' choices are also shaped by whether discount rate reductions for high-beta assets reflect embedded leverage alone or additional risk dimensions, such as idiosyncratic bankruptcy risk. For example, a firm can mechanically raise its beta by distributing to shareholders cash obtained at some interest rate. If this rate is lower than that available to the marginal investor and only embedded leverage matters, as in Frazzini and Pedersen (2014), firm value directly increases, rewarding the embedded leverage. If instead the discount rate cuts reflect risks associated with beta, firms may need to alter other attributes, with different managerial appeal than simply adding debt.

The cornerstone of the analysis is the stock-level risk measure γ, defined as the ratio of an asset's covariance with the stochastic discount factor (SDF) to the market's covariance with the SDF. To estimate these covariances agnostically, the empirical strategy assumes a

[3]This relationship between additional asset characteristics and beta distinguishes the low-risk anomaly literature from the multi-factor literature, a review of which is in Feng et al. (2020).

[4](Bali, Brown, et al. 2017) integrates previous measures of idiosyncratic volatility, skewness, and past returns (Ang et al. 2006; Brunnermeier et al. 2007), showing that these explanations are closely related.

factor structure for returns. GMM is then used to concurrently estimate factor loadings and and the liquidity-spread employing a factor set that includes the market, the market squared (to account for coskewness), and principal components from a large asset pool. This agnostic approach, which sacrifices identification of the risk source for more comprehensive coverage, mitigates the risk of omitting relevant, yet unidentified, risk factors. This is important for two reasons: obtaining better pricing performance and, more critically, potentially capturing risk factors that are not yet known to affect the beta-return relation, but do. For instance, *sensitivity to* funding liquidity, considered in Lu and Qin (2021), is likely to enter this category, as high beta assets are expected to be positively related to funding shocks and vice-versa low beta stocks to be negatively related, which would make high beta stocks better hedges for funding shocks, lowering their discount rates, just like residual coskewness does. Nothing precludes other features from having a similar role despite not being yet discovered.

This work connects two literatures: studies using Principal Component Analysis in asset pricing, reviewed in Giglio et al. (2022), and those employing GMM to estimate factor loadings of the Stochastic Discount Factor (SDF), such as Croce et al. (2023). It is also related to research on imperfect financial markets and intermediaries' funding frictions, including Adrian et al. (2014) and He et al. (2017), and more specifically to work quantifying the cost of liquidity tightness, such as Jylhä (2018), Lu and Qin (2021), and Du et al. (2022). The peculiarity in this setting is that the estimates are obtained directly from the cross-section of equities, providing a new cross-sectional estimate of the liquidity-related premium spread, although an explicit test linking the zero-beta spread to intermediaries' funding tightness is not undertaken here. Finally, this paper is related to Asness et al. (2020), but differs by considering a broader set of systematic risks while integrating financial frictions with risk omitted by CAPM more explicitly.

The empirical results reveal a statistically significant liquidity-premium spread averaging over 2% annually, even when controlling for eight risk factors. This magnitude is four times that found in Lu and Qin (2021) and roughly one-quarter of the estimate in Di Tella et al. (2023). The decomposition further shows that, for the robust BAB factor, two-thirds of its returns are attributable to risk omission ($\gamma - \beta$), with the remaining third explained by the liquidity spread. By contrast, the returns of the original BAB portfolio of Frazzini and Pedersen (2014) are fully accounted for by risk omission. Overall, these findings suggest that while financial frictions matter, the omission of relevant risk factors is the quantitatively dominant source of the CAPM's mismatch with the data.

In section Section 2 I review the theoretical set-up and how to measure the contributions of deviation in the zero-beta rate from the risk-free rate and the omission of relevant risks in explaining the low-risk anomaly; in section Section 3 I show one way to make the analysis empirically feasible; in section Section 4 I show the estimates of the coefficients from a cross-sectional pricing exercise and the implied decomposition of the CAPM anomaly; Section 5 concludes.

2 Theoretical set-up

2.1 A simple model with leverage constraints

On the lines of Frazzini and Pedersen (2014), consider a simple Overlapping-Generations model where there is a representative agent that is born at time t with wealth W_t, invests it, and finally consumes all of the payouts at time $t+1$, right before dying. Wealth at time t can only be invested in a risk-less asset with price 1, held in the amount X_t^0, and a set of S risky securities, held in the amount X_t^s for $s \in \{1, \dots, S\}$, with budget constraint

$$W_t = X_t^0 + \sum_{s=1}^{S} X_t^s P_t^s. \tag{1}$$

Wealth consumed in the following period will then be determined by the known and exogenously set gross risk-free return R_t^f and risky securities' prices, P_{t+1}^s, and dividends, D_{t+1}^s, by:

$$W_{t+1} = X_t^0 R_t^f + \sum_{s=1}^{S} X_t^s (P_{t+1}^s + D_{t+1}^s). \tag{2}$$

The objective of the agent is then to maximise expected utility from final wealth:

$$\max_{\{X_t^s\}_0^S} \mathbb{E}_t \left[u\left(W_{t+1}\right)\right]. \tag{3}$$

Plugging constraints (1) and (2) in the objective function (3), the problem solved by the agent can be expressed as

$$\max_{\{X_t^s\}_1^S} \mathbb{E}_t \left[u \left(W_t R_t^f + \sum_{s=1}^{S} X_t^s (P_{t+1}^s + D_{t+1}^s - P_t^s R_t^f) \right) \right]. \tag{4}$$

The key addition this standard set-up, to rationalize the low-risk anomaly in the spirit of Frazzini and Pedersen (2014), is a leverage constraint:

$$W_t \geq \left(\sum_{s=1}^{S} X_t^s P_t^s \right) \cdot c_t, \tag{5}$$

which depends on the stylized and exogenously-set margin requirement c_t.

The first order condition with respect to holding X_t^s is then

$$\mathbb{E}_t \left[u'(W_{t+1})(R_{t+1}^s - R_t^f) \right] - \psi_t = 0 \tag{6}$$

where $R_{t+1}^s = \dfrac{P_{t+1}^s + D_{t+1}^s}{P_t^s}$ is the gross return on asset s and ψ_t is the Lagrange multiplier of (5), which is greater than 0 when the leverage constraint binds. As the representative investor in this set-up has to hold all of the assets, a binding constraint happens any time

$c_t > 1$. This condition can be interpreted as the assets holder, representing the intermediary sector, being asked to add holdings of the risk-free asset to simply force a safer portfolio onto it, considering that the R^f is not determined by market equilibrium and risky assets are in fixed supply in the short-term.

From (6) it follows that

$$\mathbb{E}_t \left[r^s_{t+1} \right] = \tilde{\psi}_t - \mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^s_{t+1} \right], \tag{7}$$

where $u'_{t+1} = u'(W_{t+1})$, $\tilde{x} = x / \mathbb{E}_t \left[u'_{t+1} \right]$, and $R^i_{t+1} - R^f_t = r^i_{t+1}$. Remember that R^f_t is exogenously set. (7) holds similarly for the market, meaning that

$$\mathbb{E}_t \left[r^M_{t+1} \right] = \tilde{\psi}_t - \mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^M_{t+1} \right]. \tag{8}$$

Combining these last two equation one obtains

$$\mathbb{E}_t \left[r^s_{t+1} \right] = \tilde{\psi}_t \left(1 - \frac{\mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^s_{t+1} \right]}{\mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^M_{t+1} \right]} \right) + \frac{\mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^s_{t+1} \right]}{\mathrm{Cov}_t \left[\tilde{u}'_{t+1}, r^M_{t+1} \right]} \cdot \mathbb{E}_t \left[r^M_{t+1} \right]. \tag{9}$$

Labelling $\frac{\mathrm{Cov}_t[\tilde{u}_{t+1}, r^s_{t+1}]}{\mathrm{Cov}_t[\tilde{u}_{t+1}, r^M_{t+1}]}$ as γ^s_t, (9) can be more simply stated as

$$\mathbb{E}_t \left[r^s_{t+1} \right] = \tilde{\psi}_t \left(1 - \gamma^s_t \right) + \gamma^s_t \cdot \mathbb{E}_t \left[r^M_{t+1} \right]. \tag{10}$$

Here γ is the only determinant of cross-sectional variation in expected returns and can be interpreted as as assets' comprehensive measure of risk, playing in fact the role that β has in a similar equation in Frazzini and Pedersen (2014). Also in a similar way, ψ measures the zero-beta spread that modifies how risk is rewarded with respect to a perfect market with no financing frictions.

2.2 Deviations from CAPM

Defining the expectations formed following (10) as $\mathbb{E}_t^{\mathrm{FULL}}[r^s_{t+1}]$ and the expectations formed following standard CAPM as

$$\mathbb{E}_t^{\mathrm{CAPM}}[r^s_{t+1}] = \beta^s_t \cdot \mathbb{E}_t \left[r^M_{t+1} \right] \qquad \beta^s_t = \frac{\mathrm{Cov}_t \left[R^M_{t+1}, R^s_{t+1} \right]}{\mathrm{Var}_t \left[R^M_{t+1} \right]}, \tag{11}$$

then deviations from CAPM predictions can be summarized by $\alpha^s_t = \mathbb{E}_t^{\mathrm{FULL}}[r^s_{t+1}] - \mathbb{E}_t^{\mathrm{CAPM}}[r^s_{t+1}]$, which takes value

$$\alpha^s_t = \underbrace{\tilde{\psi}_t (1 - \gamma^s_t)}_{\text{Liquidity deviation}} + \underbrace{(\gamma^s_t - \beta^s_t) \mathbb{E}_t \left[r^M_{t+1} \right]}_{\text{Omitted risk deviation}} \tag{12}$$

$$= \tilde{\psi}_t \left[(1 - \beta^s_t) - (\gamma^s_t - \beta^s_t) \right] + (\gamma^s_t - \beta^s_t) \mathbb{E}_t \left[r^M_{t+1} \right]. \tag{13}$$

(12) shows that the deviation of the expected return of a security from CAPM predictions can be split in a part due to a binding leverage constraint, or more generally a spread in the zero-beta rate, and a part due to a different assessment of 'total' risk from the beta measure. (13) makes an even clearer distinction between two stories put forward to explain the CAPM anomaly: Frazzini and Pedersen (2014) finds alpha equating $\tilde{\psi}(1-\beta)$, Schneider et al. (2020) relies on $\gamma - \beta$ where γs explicitly deviate from β because of the market squared in the marginal utility formation. Then, the counteracting effect of the two effects, liquidity motives versus the omitted risk factors, is synthesized by $\gamma - \beta$: the greater is the distance of 'total' risk from β, the more α will be due to risk remuneration of omitted risks, the smaller the term is, the more α will be due to a different market premium which gets captured by portfolios with simply different betas.

To understand in more practical terms the meaning of this, think first of the case in which the market really is the only relevant risk factor: $\gamma - \beta$ will be zeros and one would be exactly back to the pricing equation of Frazzini and Pedersen (2014) where it is the multiplier producing alpha as beta grows. Adding a second priced factor to which assets are sensitive in random way will not change much:[5] high-beta assets will be as riskier than low-beta assets as before, so α in the cross section would still decrease with β with the zero-beta spread driving it. Finally think of a second risk factor to which assets are sensitive to in an opposing way relative to how they are sensitive to the market, such as coskewness. Now high-beta assets are good hedges of market variance shocks and vice-versa low-beta assets are riskier in this second dimension. Bringing the counteracting effect to an extreme, γ could become a constant across assets; then, a liquidity spread would still be present in the market, but the relationship of alphas with beta would now be exclusively driven by the mis-measurement of how risky a security actually is.

Another way to synthesise how flatter the SML is relatively to the theoretical CAPM prediction, and possibly trade on it, is to look at the expected return of a BAB factor. Following Novy-Marx and Velikov (2022) observations, this can be formed holding a set of low CAPM-beta assets, shorting a set of high CAPM-beta ones, and making the portfolio beta-neutral by holding the market proportionally to the beta-tilt of the first two holdings, while financing this with the risk-free asset:

$$\begin{aligned} \mathbb{E}_t\left[r_{t+1}^{BAB}\right] &= \mathbb{E}_t\left[r_{t+1}^{L}\right] - \mathbb{E}_t\left[r_{t+1}^{H}\right] - (\beta_t^L - \beta_t^H)\mathbb{E}_t\left[r_{t+1}^{M}\right] && (14) \\ &= \tilde{\psi}_t\left(\gamma_t^H - \gamma_t^L\right) + \left[(\beta_t^H - \beta_t^L) - (\gamma_t^H - \gamma_t^L)\right]\mathbb{E}_t\left[r_{t+1}^{M}\right]. && (15) \end{aligned}$$

From here it is further clear that the origin of the CAPM mispricing critically depends on how different is the assets' 'real' total risk from CAPM assessment. Specifically, the smaller the difference in the γs of the beta-sorted portfolios, the greater the error by CAPM would be due to an omitted risk factor making more market-sensitive assets not so risky after all, and vice-versa. It should also be noted that adding coskewness, for example, one expects to lower the gamma-differential as high-beta assets are safer than low-beta ones in the coskewness

[5] Random in a cross-sectional sense, not in a time-varying fashion.

dimension, but there could even be risk factors with the opposite link that could counteract the counteracting effect of coskewness. On the other side, instead, if actually riskier assets, i.e. those with higher γ, also have high β, either because the market is the only risk priced or because the other risks are neutrally distributed along β dimension, funding-motives will more likely drive the anomaly and show up as a reduction in the premium.

2.3 Betting Against Gamma

With the synthetic risk measure γ capturing risk as β does in CAPM, to isolate the intercept-related spread, one can build a portfolio similar to BAB: long on low-γ (LG), short on high-γ (HG) and hedging the exposure with position $\gamma^{LG} - \gamma^{HG}$ in the market. I call this portfolio 'Betting Against Gamma' (BAG):

$$\begin{aligned} \mathbb{E}_t\left[r_{t+1}^{BAG}\right] &= \mathbb{E}_t\left[r_{t+1}^{LG}\right] - \mathbb{E}_t\left[r_{t+1}^{HG}\right] - (\gamma^{LG} - \gamma^{HG})\mathbb{E}_t\left[r_{t+1}^{M}\right] \quad (16) \\ &= \tilde{\psi}_t\left(\gamma_t^{HG} - \gamma_t^{LG}\right). \quad (17) \end{aligned}$$

This portfolio allows to investigate the sources of variation in the intercept, hypothesized to be related to funding motives, as can be seen through the lens of the decomposition from Campbell (1991),

$$r_t^{BAG} \approx \mathbb{E}_{t-1}\left[r_t^{BAG}\right] + \sum_{j=0}^{\infty} \rho^j \cdot (\mathbb{E}_t - \mathbb{E}_{t-1})[\Delta \ln D_{t+j}^{BAG}] - \sum_{j=1}^{\infty} \rho^j \cdot (\mathbb{E}_t - \mathbb{E}_{t-1})[r_{t+j}^{BAG}]. \quad (18)$$

Assuming, without great loss of generality, constant γs of the high and low portfolio; that shocks to the multiplier are i.i.d; and that shocks to cash flows expectations are independent from those to the leverage constraint, which is reasonable at high frequencies given the interpretation of the model; the contemporaneous relation between changes in ψ and BAG returns is

$$\frac{\partial\left(r_t^{BAG} - \mathbb{E}_{t-1}\left[r_t^{BAG}\right]\right)}{\partial\left(\tilde{\psi}_t - \mathbb{E}_{t-1}[\tilde{\psi}_t]\right)} = -\rho\left(\gamma_t^{HG} - \gamma_t^{LG}\right). \quad (19)$$

Therefore, a further testable implication is that a variable that captures funding tightness should covary negatively with the BAG returns, controlling for other factors.

3 Empirical implementation

The key object of this study, as in most asset pricing models, is the marginal utility, whose covariance with assets' returns determines everything shown so far. It is important to recover its process in the most agnostic way to avoid the risk of omitting variables, which is high in mapping it on arbitrary factors. To do so, I look at its minimum-variance projection on all the assets returns, in the style of Hansen and Jagannathan (1991), and assume a latent factor structure in returns. I then proceed with the contemporaneous estimation of the average zero-beta spread and the marginal utility loadings of the first few principal components only, in a standard GMM procedure.[6]

3.1 A sparse and agnostic approach

As an empirical counterpart of the standardized marginal utility $\tilde{u}'_{t+1}$, consider its linear projection $\tilde{u}'^{*}_{t+1}$ on N returns, with coefficients $\boldsymbol{\beta}_{u\mathbf{r}}$,

$$\tilde{u}'^{*}_{t+1} - 1 = (\mathbf{r}_{t+1} - \mathbb{E}[\mathbf{r}_{t+1}])^{\top} \boldsymbol{\beta}_{u\mathbf{r}}, \tag{20}$$

where $\mathbf{r}_t$ is the vector of N returns at time t. Then, the coefficients can then be retrieved by substituting $\tilde{u}'^{*}_{t+1}$ in the stacked unconditional expectations of all excess returns, as in (6), scaled by $\mathbb{E}[u'_{t+1}]$, i.e.

$$\mathbb{E}[\tilde{u}'^{*}_{t+1}\mathbf{r}_{t+1}] = \mathbb{E}\left[\tilde{\psi}_t\right] \mathbf{1}_N. \tag{21}$$

Subtracting $\mathbb{E}[\mathbf{r}_{t+1}]$ from both sides, one obtains

$$\mathbb{E}[(\tilde{u}'^{*}_{t+1} - 1)(\mathbf{r}_{t+1} - \mathbb{E}[\mathbf{r}_{t+1}])] = \mathbb{E}\left[\tilde{\psi}_t\right] \mathbf{1}_N - \mathbb{E}[\mathbf{r}_{t+1}], \tag{22}$$

from which $\boldsymbol{\beta}_{u\mathbf{r}}$ can be obtained by plugging the definition of $\tilde{u}'^{*}_{t+1} - 1$ in,

$$\boldsymbol{\beta}_{u\mathbf{r}} = \Sigma_{\mathbf{rr}}^{-1}(\mathbb{E}[\psi_t] \mathbf{1}_N - \mathbb{E}[\mathbf{r}_{t+1}]), \tag{23}$$

where $\Sigma_{\mathbf{rr}}$ is the covariance matrix of the N returns. The resulting process of the marginal utility projection process is

$$u'^{*}_{t+1} - 1 = (\mathbf{r}_{t+1} - \mathbb{E}[\mathbf{r}_{t+1}])^{\top} \Sigma_{\mathbf{rr}}^{-1}(\mathbb{E}[\psi_t] \mathbf{1}_N - \mathbb{E}[\mathbf{r}_{t+1}]). \tag{24}$$

This involves estimating a great number of parameters and inverting a huge matrix, both concerning $\Sigma_{\mathbf{rr}}$, so this formulation is highly impractical.

Looking for a formulation of the problem that maintains as much information as possible while keeping the problem empirically feasible, I make the further assumption of a perfect factor structure for returns innovations such as

$$\mathbf{r}_{t+1} - \mathbb{E}[\mathbf{r}_{t+1}] = \mathbf{e}_{t+1} \tag{25}$$

$$= \mathrm{B} \cdot \mathbf{f}_{t+1} + \boldsymbol{\epsilon}_{t+1}, \tag{26}$$

where B is the $N \times K$ matrix of factor loadings. Factors $\mathbf{f}$ and residuals $\boldsymbol{\epsilon}$ are independent and both zero mean, with $\mathbb{E}[\mathbf{f}_{t+1}\mathbf{f}_{t+1}^{\top}] = I_K$, and $\mathbb{E}[\boldsymbol{\epsilon}_{t+1}\boldsymbol{\epsilon}_{t+1}^{\top}]$ being a diagonal matrix filled with the vector $\boldsymbol{\sigma}^2_{\boldsymbol{\epsilon}}$, as most standard applications of factor structures.

[6]To see how realistic is for the first few principal components to capture most of the relevant information *pricing-wise*, an extension of Kozak et al. (2018) is needed, which will likely result in some bound for ψ on the lines of Jiang and Richmond (2022) and Cochrane and Saa-Requejo (2000).

The marginal utility projection process can then be expressed as

$$\tilde{u}'^{*}_{t+1} = 1 + (\mathbf{f}^{\top}_{t+1}\mathrm{B}^{\top} + \boldsymbol{\epsilon}^{\top}_{t+1})\Sigma^{-1}_{\mathbf{rr}}(\mathbb{E}\left[\psi_t\right]\mathbf{1}_N - \mathbb{E}\left[\mathbf{r}_{t+1}\right]), \tag{27}$$

or, more succinctly,

$$\tilde{u}'^{*}_{t+1} = 1 - \mathbf{f}^{\top}_{t+1}\boldsymbol{\beta}_{u\mathbf{f}} - \boldsymbol{\epsilon}^{\top}_{t+1}\boldsymbol{\beta}_{u\boldsymbol{\epsilon}}, \tag{28}$$

where $\boldsymbol{\beta}_{u\mathbf{f}}$ are the loadings of the pervasive factors in marginal utility and $\boldsymbol{\beta}_{u\boldsymbol{\epsilon}}$ is the linear mapping of the marginal utility on the individual innovations residuals.

Using the projected marginal utility, the pricing equation (7) becomes

$$\mathbb{E}_t\left[r^s_{t+1}\right] = \tilde{\psi}_t + \boldsymbol{\beta}^{\top}_{u\mathbf{f}} \cdot \mathrm{Cov}_t\left[\mathbf{f}_{t+1},\ r^s_{t+1}\right] + \boldsymbol{\beta}^{\top}_{u\boldsymbol{\epsilon}} \cdot \mathrm{Cov}_t\left[\boldsymbol{\epsilon}_{t+1},\ r^s_{t+1}\right]. \tag{29}$$

If the factor structure approximates well the returns distribution, the variance of idiosyncratic residuals $\mathrm{Cov}_t\left[\boldsymbol{\epsilon}_{t+1},\ r^s_{t+1}\right] = \sigma^2_{\epsilon,s}$ will be minimal, although not zero. While I have no formal guarantee that $\boldsymbol{\beta}_{u\boldsymbol{\epsilon}}$ tends to 0 in any way at this stage, assuming $\boldsymbol{\beta}^{\top}_{u\boldsymbol{\epsilon}} \cdot \mathrm{Cov}_t\left[\boldsymbol{\epsilon}_{t+1},\ r^s_{t+1}\right] \approx 0$ has the advantage that the following condition involves only a few unknown constants, $\mathbb{E}\left[\tilde{\psi}_t\right]$ and $\boldsymbol{\beta}_{u\mathbf{f}}$, to make meaningful asset pricing predictions and obtain estimates of γs:

$$\mathbb{E}\left[r^s_{t+1}\right] = \mathbb{E}\left[\tilde{\psi}_t\right] + \boldsymbol{\beta}^{\top}_{u\mathbf{F}} \cdot \mathrm{Cov}\left[\mathbf{f}_{t+1}, r^s_{t+1}\right]. \tag{30}$$

A sense of how much information that is relevant to pricing has gotten lost is offered by ex-post performance measures. Again, a model that tends to have no pricing errors would be ideal, but a ‘wrong’ model can still be good enough to leave no room for flipping the results with a better, or even perfectly, performing model. Therefore, the exercise is likely meaningful even with errors as long as these are reasonably small.

4 Empirical Analysis

4.1 Empirical strategy and test assets

The analysis verges around the estimates of $\mathbb{E}\left[\tilde{\psi}_t\right]$ and $\boldsymbol{\beta}_{u\mathbf{F}}$, to obtain an empirical counterpart of $\tilde{u}'_t$. Assuming a constant ψ_t, this implies estimating the coefficients $\{a, \mathbf{b}\}$ of the following moment condition, given by the pricing equation (30):

$$\mathbb{E}\left[r^s_t\right] - a - \mathbb{E}\left[\mathbf{b}^{\top}\hat{\mathbf{f}}_t \cdot (r^s_t - \mathbb{E}\left[r^s_t\right])\right] = 0. \tag{31}$$

This can be easily used in a GMM estimation, after which the estimates of unconditional γs amount to

$$\gamma^s = \frac{\mathbf{b}^{\top}\mathrm{Cov}\left[\hat{\mathbf{f}}_t, r^s_t\right]}{\mathbf{b}^{\top}\mathrm{Cov}\left[\hat{\mathbf{f}}_t, r^M_t\right]}, \tag{32}$$

with standard errors obtainable using the Delta method and GMM estimates of the covariances.

Before moving to GMM the procedure needs an earlier step to form the factors. Indeed, these are composed by (1) risk factors previously known to be relevant for the anomaly and (2) pervasive latent factors of a wide set of test assets. More precisely, the market factor and the market squared excess returns, for coskewness, are the first two. The rest of the factors are the first few Principal Components (PCs) the test assets, whose computation procedure exactly aims at minimizing the assets' residuals' variance. Before performing PCA, the test assets are orthogonalized with respect to the market and the market squared to avoid redundant information.[7] .

The market factor is the monthly CRSP value-weighted market index in excess of the 1-month risk-free, again from CRSP. The test assets pool from which PCs are extracted is mostly populated by the 153 monthly stock portfolios used in Jensen et al. (2021),[8] all of which are available from November 1971 to December 2022. The original BAB portfolio from Frazzini and Pedersen (2014) is among these portfolios. Then, I add 10 beta-sorted stocks portfolios and the key asset of the analysis: the BAB factor, based on a 3-fold split of beta-sorted US stocks, robust to the criticisms of Novy-Marx and Velikov (2022).

To keep the dimensionality of the GMM problem reasonable and the estimation well behaved, the test assets actually used in the GMM step of the analysis are a 'condensed' version of the ones used to obtain $\hat{\mathbf{f}}$. Precisely, this second set of test assets is formed by the market excess returns, the BAB factor, 3 beta-sorted stocks portfolios, and 13 'themed' portfolios build by clustering the 153 portfolios of Jensen et al. (2021), provided by the same authors. Statistics for these portfolios are in Table 6, in the appendix. The clustering technique used to create these 13 theme-portfolios has the precise intent of keeping a great dispersion across-theme and a high degree of within-theme correlation and economic concept similarity. The original BAB portfolios is among those clustered in the 'low risk'-themed portfolio, but I also explicitly add it to the test assets pool of the second step for consistency with the other BAB factor.

4.2 Beta-sorted portfolios

I construct the beta-sorted portfolios using monthly data from CRSP, correcting for delisting returns depending on nature of the delisting, as suggested by Bali, Engle, et al. (2016). Following common practice in the literature, I only consider stocks of share types 10 and 11, traded on NYSE, Nasdaq or AMEX exchanges. To form portfolios, every month all the stocks are ranked based on the rolling betas estimated on a 5-year window ending the month before. They are then split into 10 or 3 portfolios with weights corresponding to the relative capitalization. The BAB portfolio is built following Novy-Marx and Velikov (2022), i.e. subtracting returns of the top-third beta portfolio from bottom-third beta portfolio and subtracting to this the excess returns of the market, proportionally to the estimated beta of

[7] To do so, I simply regress every test asset on the market and the market squared, keeping the residuals.
[8] Available at https://jkpfactors.com.

Table 1: Beta-sorted portfolios statistics. In parenthesis, HAC standard errors obtained as suggested in Lazarus et al. (2018). Monthly annualized returns from Nov 1971 to Nov 2022.

	Bottom	Top	BAB	Orig. BAB
Avg. ret (%)	11.935	12.635	3.881	10.083
SD (%)	43.274	80.635	42.549	40.741
CAPM α (%)	7.283	3.179	4.079	10.52
	(1.219)	(1.446)	(1.986)	(2.773)
CAPM β	0.684	1.391	-0.029	-0.064
	(0.048)	(0.041)	(0.065)	(0.099)
Res. coskew. ($\times 10^3$)	0.217	0.429	-0.111	-1.659
	(0.199)	(0.291)	(0.449)	(0.447)

the low-minus-high portfolio in the 5 years ending the month before. Summary statistics of the bottom and top thirds, BAB portfolio and a BAB portfolio as originally formed by Frazzini and Pedersen (2014) are shown in Table 1. The timespan of the analysis is from November 1971 to November 2022, dictated by the availability of portfolios from Jensen et al. (2021).

As expected, the 'top' portfolio earns a higher return than the 'bottom' one, but its CAPM alpha is significantly lower. Top portfolio also shows a higher residual coskewness, obtained regressing CAPM residuals on the market squared, following Schneider et al. (2020). Both BAB portfolios earn a significant alpha with respect to CAPM while having no significant exposure to the market. However, they significantly differ in the magnitude of alphas and the residual coskewness, with the 'original' BAB portfolio earning a higher return unexplained by CAPM and having a lower residual coskewness, which suggests the value-weighting mitigating the coskewness risk. Statistics covering the full sample of the portfolios are in Table 5, while the statistics for the middle third portfolio and the 10-split portfolios over the same time-frame are in Table 7, both showing a similar pattern.

4.3 Estimation results

4.3.1 PCs factors

Figure 2 shows the scree plot relative to the Principal Component Analysis of the 164 test assets. At this stage no formal test is conducted to choose the number of components to keep. Rather, I arbitrarily choose to keep 4 because adding them to the market factor and the squared market factor results in a total of 6 factors, which is a number comparable with the other *sparse* models at the frontier at the time of this work, such as Fama and French (2018). This also seems a sensible choice because the following component, the 5$^{\text{th}}$, would explain almost half the variance explained by the 4$^{\text{th}}$ one, less than 5%. The first 4 PCs end up explaining 60% of test assets' residual variance after orthogonalization to market and market squared; the first 6 ones explain a total of 67%. To control for the gains from adding factors, I also use a specification keeping 6 components, resulting in 8 factors, although it does not change results in a significant way.

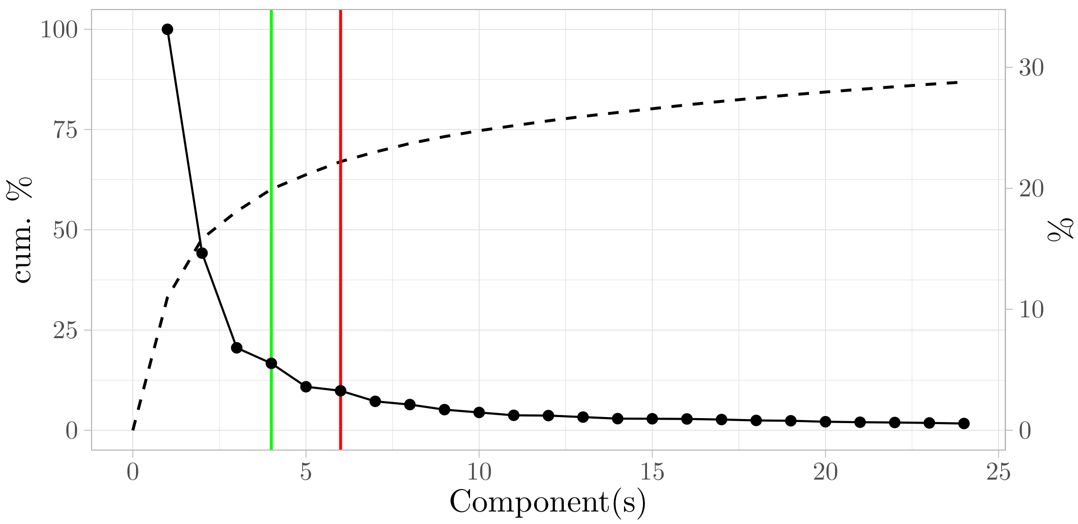

Figure 2: Scree plot of principal component analysis. On the horizontal axis the number of component considered, on the vertical axis the share of variance explained by the relative component (solid line) and the cumulative share of variance explained (dotted line). The green vertical line marks the 4^{th} component while the red one the 6^{th}.

4.3.2 Pricing the cross-section

Table 2 shows the estimation results for a few specifications: 'CAPM', where the only risk factor is the market and ψ is fixed at 0; 'F(Market)', where the market is the only factor, but $\mathbb{E}[\psi]$ is freely estimated; 'F(cskw)', which adds the market squared among the factors; 'F(6)', which adds 4 cross-sectional principal components as factors to the previous specification; and finally, 'F(8)', which adds 6 components instead.

Comparing the first two columns, it can be seen that introducing the spread $\mathbb{E}[\psi]$ significantly improves the pricing of the cross-section, cutting the Mean Absolute Pricing Error (MAPE) in half. Indeed, the spread is significantly different from 0 in all specifications and has a substantial magnitude, between 2% and 3.5% annually. The average unexplained return of the 'robust' BAB portfolio immediately becomes statistically and economically insignificant, while the unexplained excess return of the 'original' BAB factor does not. This further remarks the difference in the two and highlights how much do the portfolio formation details matter. The market factor is never significant apart from the CAPM specification.

The market squared enters the marginal utility negatively and quite significantly in all specifications including it. Also, consistently with Schneider et al. (2020), adding the market squared among the factors makes the pricing error in the original BAB portfolio insignificant too. This additional explanatory power, however, appears to be more related to the non-standard practices used to form the original BAB factor,[9] rather than to an intrinsic mechanism of the CAPM anomaly, since the robust BAB portfolio does not experience such a reduction in pricing error. Employing the returns' principal components as factors further reduces MAPEs, decreasing them to less than half of the MAPE in the model with

[9] As highlighted by Novy-Marx and Velikov (2022), these tilt holdings towards small and illiquid stocks possibly making the portfolio load more on coskewness.

Table 2: Model estimation results. HAC standard errors in parenthesis. Monthly sample from 1971 to 2022.

	CAPM	F(Market)	F(cskw)	F(6)	F(8)
$\mathbb{E}[\psi]$		3.583***	3.184***	2.050***	2.038***
		(0.423)	(0.532)	(0.256)	(0.258)
Market factors	0.100*	0.060	0.000	0.030	0.000
	(0.046)	(0.046)	(0.085)	(0.080)	(0.103)
Market squared			−0.516°	−0.485°	−0.719°
			(0.281)	(0.288)	(0.385)
J-test	330.140***	351.514***	131.472***	42.699***	30.007***
MAPE	3.431	1.625	1.505	0.709	0.570
BAB a.p.e.	4.149*	0.501	0.591	−0.433	−0.276
	(2.005)	(2.016)	(2.733)	(2.991)	(3.173)
Orig. BAB a.p.e.	10.526***	6.800**	2.230	1.145	1.298
	(2.722)	(2.640)	(3.642)	(3.562)	(4.402)

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{\circ}p < 0.1$

coskewness. Indeed, deviations on both BAB portfolios further decrease too.

All the over-restricting conditions of the models are not valid at the 0.1% of confidence level, although F(8) has a p-value that is only slightly lower than that. This, however, is not too much of a concern: the goal is to see through the lenses of a better reality approximation the sources of CAPM failures and any better-performing model can be used to this aim. Obviously, the more information a model leaves unexplained, the higher the chances of the decomposition being twisted with an even better model. Anyway, despite not achieving a perfect fit, over these test assets with average mean return of around 4%,[10] F(6) and F(8) produce MAPEs that are less than 1%, a fifth of CAPM's starting 3.5%. This certainly leaves room for additional information to improve the analysis, but it seems a rather small space to completely reverse the main takeaways of this exercise. This is further supported in the next sub-section by the uniformity of the decomposition pattern development when increasing the model complexity.

4.3.3 Revisiting the low-risk anomaly

Estimates of $\mathbf{b}$ imply estimates of γs too, which are shown in Table 3. The first pattern to emerge is that, when more risks are considered, the spread in the synthetic risk measure decreases, which can be seen by observing the γ of the high-minus-low beta portfolio 'HmL'. This, again, is in line with Schneider et al. (2020), which shows high-beta stocks being safer than what the betas would suggest once coskewness is taken into account, and extends it further to more unidentified risks. On the other hand, the BAB portfolio gets riskier and riskier as the number of risks considered, and arguably the 'realism' of the model, increases, although not significantly. To understand why, the definition in (15) is useful: assuming the extreme case in which all actual risks in the economy makes the high-beta and the low-beta

[10] See Table 6 in the appendix.

Table 3: Portolios γ. HAC standard errors in parenthesis, obtained through Delta method from previous GMM estimation. Monthly sample from 1971 to 2022.

	Bottom	Top	HmL	BAB	Orig. BAB
γ CAPM	0.688^{***}	1.395^{***}	0.707^{**}	-0.030	-0.064
	(0.026)	(0.034)	(0.359)	(0.051)	(0.058)
γ Market	0.688^{***}	1.395^{***}	0.707	-0.030	-0.064
	(0.026)	(0.034)	(0.672)	(0.051)	(0.058)
γ Coskew.	0.549^{***}	1.109^{***}	0.560	0.048	1.082
	(0.157)	(0.210)	(0.488)	(0.277)	(0.702)
γ All (4)	0.881^{***}	1.037^{***}	0.156	0.411	1.211^{**}
	(0.177)	(0.223)	(0.429)	(0.328)	(0.551)
γ All (6)	0.786^{***}	1.047^{***}	0.261	0.363	1.116^{**}
	(0.177)	(0.215)	(0.431)	(0.320)	(0.497)

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

portfolios having the same total risk, then, any holding of the market originally taken to hedge the beta of the low-minus-high beta portfolio will distort the total-risk neutrality. Such risk in this formulation is rewarded with $\mathbb{E}\left[r^M\right]$ per unit of γ, which would be the source of the BAB expected return. In this case, none of the BAB return would be due to liquidity considerations because having the high-beta and the low-beta portfolios the same γ, they would provide cash-flows with the same discount rate, and no embedded leverage would be enjoyed by agents. Note that this would not mean that the liquidity motive is irrelevant: the BAG portfolio defined in (16) would still be entirely determined by the liquidity compensation,[11] just like the BAB return is in Frazzini and Pedersen (2014).

The last example also shows why the CAPM anomaly is not a 'plain' case of omitted factors: to make high-beta assets have the same total risk of low-beta assets, assets have to be riskier in a second dimension inversely proportional to their beta, otherwise the difference in total risk levels of high-beta and low-beta will persist. This means that it does not suffice for the additional factor to have a specific correlation with the market, but that there is a deeper link in the way in which sensitivity to the market relates to sensitivity to this second factor. Notice also that additional risks could also make it *harder* to explain the anomaly, in case the risk pattern relates to beta in the opposite way as coskewness does. However, this does not seem the case, as γs converge towards 1 increasing the risks. All in all, as from F(cskw) and F(6) the difference in risk between high-beta and low-beta decreases and BAB risk increases, the evidence points towards the existence of omitted factors relevant to the anomaly, although not yet specifically identified in the literature.

The presence of risks not orthogonally distributed with respect to betas is extremely clear in the original BAB, where the synthetic risk measure is significant. Any BAB portfolio is characterized by market-risk neutrality, so a significant γ, interpreted through (12), suggests the risk component being relevant in explaining the CAPM anomaly, even considering the liquidity spread. It is important however to understand the extent to which the apparent 'mispricing' of CAPM is due to risks remuneration or funding provision, in

[11] At the current state, the BAG portfolio has not been studied, but it will be included in this work.

Table 4: Deviations' contribution. 'Prd/real' is the ratio of predicted return of the portfolio over the actual average return. HAC standard errors in parenthesis, obtained through Delta method from previous GMM estimation. Monthly sample from 1971 to 2022.

	BAB				Orig. BAB			
	$\psi(1-\gamma)$	$\gamma \cdot \mathbb{E}\left[r^M\right]$	Prd/real%	Δ%	$\psi(1-\gamma)$	$\gamma \cdot \mathbb{E}\left[r^M\right]$	Prd/real%	Δ%
Market	3.69***	−0.21	87.4	111.8***	3.81***	−0.45	33.1	126.4***
	(0.49)	(0.36)		(21.0)	(0.54)	(0.42)		(26.3)
Coskew.	3.03**	0.33	85.2	80.3	−0.26	7.49	78.1	−107.2*
	(1.16)	(1.92)		(109.1)	(2.24)	(5.42)		(59.5)
All (4)	1.21*	2.85	110.9	−40.4	−0.43	8.39*	88.7	−110.9***
	(0.67)	(2.45)		(57.8)	(1.14)	(4.67)		(25.2)
All (6)	1.30*	2.52	106.9	−31.9	−0.24	7.73*	87.2	−106.3***
	(0.67)	(2.36)		(63.8)	(1.02)	(4.25)		(25.2)

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

order to improve assessments such as that in Baker et al. (2014). To do this, I report in Table 4 the measurements of the liquidity component and the risk component, with standard errors obtained via Delta method using previous GMM estimates. I also compute a measure of relative contribution, the share of return prediction that is associated to pure liquidity motives or to pure risk mis-measurement, defined as

$$\Delta = \frac{\text{Liq. component} - \text{Risk component}}{\text{Tot. prediction}} = \frac{\mathbb{E}\left[\psi\right]\left(1-\gamma^{BAB}\right) - \gamma^{BAB} \cdot \mathbb{E}\left[r^M\right]}{\mathbb{E}\left[\psi\right]\left(1-\gamma^{BAB}\right) + \gamma^{BAB} \cdot \mathbb{E}\left[r^M\right]}. \tag{33}$$

This is positive when contribution due to the zero-beta spread is greater than that due to γ-risk, relative to the total return that the model is able to predict, and vice-versa. Once again, remember that BAB portfolios have theoretically and empirically 0 β, so any measure of γ different from 0 is a reflection of mis-measurement of risk in CAPM.

It can be seen that the liquidity spread play a major role when the market risk only is considered: the liquidity component contributes 112% more than the risk component in the prediction of the BAB return and 126% more of the original BAB. Here, the risk component even contributes negatively, with a negative return prediction originated in the residual market risk of the portfolios. However, as more risks are considered, the two components switch roles. Considering coskewness, it can be seen that for the robust BAB portfolio the liquidity component stops significantly contributing more than the risk component, while for the original BAB the relative contribution flips significantly already, again supporting the strong risk mis-measurement motive behind it. Moving to full-fledged models, it can be seen that for both specifications F(6) and F(8) the percentage of average return explained by the omission of factors, which makes a BAB portfolio risky, is higher with no statistical significance for the robust BAB, but with high statistical significance for the original BAB. A Δ not statistically different from zero means that an equal contribution of the liquidity and the risk components cannot be excluded. At the same time, the standard errors of Δ for F(6) and F(8) do not exclude all of the BAB return being due to risk omission, while they essentially rule out the possibility of being all of it imputed to the liquidity component.

Overall, despite possibly suffering of low estimation accuracy, the results support the existence of a funding tightness spread as well as the prominence of omitted risks in explaining the CAPM low-risk anomaly. This could be further studied with a betting-against-gamma portfolio, which can also provide information on the liquidity spread dynamics. The flexibility of the formulation also allows for more complex methods, possibly even those outlined in Didisheim et al. (2023), to be applied.

5 Conclusion

The remuneration of risk, as defined by the CAPM – the most fundamental model in financial economics, is not as high in the data as it is expected to be from theory. This has been hypothesized to be due either because financial frictions reduce such remuneration or because assets do not bring as much risk as they are expected to. In one case acting on financial frictions has an impact on the cost of capital of firms, in the other does not. Also, in one case a firm can expect to gain from exploiting a better funding than investors to leverage up and harvest the zero-beta spread, while in the other case the only effect is that it would become riskier. A formulation of the optimal pricing behaviour of an agent with both a leverage constraint and no specific preferences, in order to accommodate different degrees of realism, illustrates how antagonist to each other the two effects are. In this formulation an inclusive measure of risk can be compared to β, the CAPM risk measure, and inform on how the two differ. It is shown that increasing the risks considered, the return of BAB portfolios are more likely to be compensation for risks omitted by CAPM, despite also supporting an extremely significant role of the spread generated in the zero-beta asset by the financial frictions. This relation can be further tested with the formation of a Betting-Against-Gamma portfolio, which would also be instrumental in assessing contemporaneous relations between the zero-beta spread and funding liquidity measures.

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References

Adrian, Tobias, Erkko Etula, and Tyler Muir (2014). 'Financial Intermediaries and the Cross-Section of Asset Returns'. In: *The Journal of Finance* 69.6, pp. 2557–2596.

Ang, Andrew et al. (2006). 'The Cross-Section of Volatility and Expected Returns'. In: *The Journal of Finance* 61.1, pp. 259–299.

Asness, Cliff et al. (2020). 'Betting against correlation: Testing theories of the low-risk effect'. In: *Journal of Financial Economics* 135.3, pp. 629–652.

Baker, Malcolm, Brendan Bradley, and Ryan Taliaferro (1, 2014). 'The Low-Risk Anomaly: A Decomposition into Micro and Macro Effects'. In: *Financial Analysts Journal* 70.2, pp. 43–58.

Bali, Turan G., Stephen J. Brown, et al. (2017). 'A Lottery-Demand-Based Explanation of the Beta Anomaly'. In: *Journal of Financial and Quantitative Analysis* 52.6, pp. 2369–2397.

Bali, Turan G., R. F. (Robert F.) Engle, and Scott Murray (2016). *Empirical asset pricing: the cross section of stock returns.* Hoboken, New Jersey: Wiley. 494 pp. ISBN: 978-1-118-09504-1.

Black, Fischer (1972). 'Capital Market Equilibrium with Restricted Borrowing'. In: *The Journal of Business* 45.3, pp. 444–455.

Black, Fischer, Michael C. Jensen, and Myron Scholes (1972). 'The Capital Asset Pricing Model: Some Empirical Tests'. In: *Studies in the Theory of Capital Markets.* Vol. 81, pp. 79–121.

Brealey, Richard et al. (25, 2022). *Principles of Corporate Finance.* 14th ed. McGraw Hill. ISBN: 978-1-264-08094-6.

Brunnermeier, Markus K., Christian Gollier, and Jonathan A. Parker (2007). 'Optimal Beliefs, Asset Prices, and the Preference for Skewed Returns'. In: *American Economic Review* 97.2, pp. 159–165.

Campbell, John Y. (1991). 'A Variance Decomposition for Stock Returns'. In: *The Economic Journal* 101.405, p. 157.

Chamberlain, Gary and Michael Rothschild (1983). 'Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets'. In: *Econometrica* 51.5, pp. 1281–1304.

Cochrane, John H. and Jesus Saa-Requejo (2000). 'Beyond Arbitrage: Good-Deal Asset Price Bounds in Incomplete Markets'. In: *Journal of Political Economy* 108.1, pp. 79–119.

Croce, Mariano M, Tatyana Marchuk, and Christian Schlag (1, 2023). 'The Leading Premium'. In: *The Review of Financial Studies* 36.8, pp. 2997–3033.

Di Tella, Sebastian et al. (2023). *The Zero-Beta Interest Rate.*

Didisheim, Antoine et al. (2023). *Complexity in Factor Pricing Models.*

Du, Wenxin, Benjamin Hébert, and Amy Wang Huber (9, 2022). 'Are Intermediary Constraints Priced?' In: *The Review of Financial Studies.*

Elton, Edwin J. (1999). 'Presidential Address: Expected Return, Realized Return, and Asset Pricing Tests'. In: *The Journal of Finance* 54.4, pp. 1199–1220.

Fama, Eugene F. and Kenneth R. French (1, 2018). 'Choosing factors'. In: *Journal of Financial Economics* 128.2, pp. 234–252.

Feng, Guanhao, Stefano Giglio, and Dacheng Xiu (2020). 'Taming the Factor Zoo: A Test of New Factors'. In: *The Journal of Finance* 75.3, pp. 1327–1370.

Frazzini, Andrea and Lasse Heje Pedersen (2014). 'Betting against beta'. In: *Journal of Financial Economics* 111 (1), pp. 1–25.

Giglio, Stefano, Bryan Kelly, and Dacheng Xiu (1, 2022). 'Factor Models, Machine Learning, and Asset Pricing'. In: *Annual Review of Financial Economics* 14.1, pp. 337–368.

Guermat, Cherif (1, 2014). 'Yes, the CAPM is testable'. In: *Journal of Banking & Finance* 46, pp. 31–42.

Hansen, Lars Peter and Ravi Jagannathan (1991). 'Implications of Security Market Data for Models of Dynamic Economies'. In: *Journal of political economy* 99 (2), pp. 225–262.

He, Zhiguo, Bryan Kelly, and Asaf Manela (2017). 'Intermediary asset pricing: New evidence from many asset classes'. In: *Journal of Financial Economics* 126 (1), pp. 1–35.

Hong, Harrison and David A. Sraer (2016). 'Speculative Betas'. In: *The Journal of Finance* 71.5, pp. 2095–2144.

Jensen, Theis Ingerslev, Bryan T. Kelly, and Lasse Heje Pedersen (2021). 'Is There a Replication Crisis in Finance?' In: *SSRN Electronic Journal*.

Jiang, Zhengyang and Robert Richmond (12, 2022). *Convenience Yields and Asset Pricing Models*. Rochester, NY.

Jylhä, Petri (2018). 'Margin Requirements and the Security Market Line'. In: *The Journal of Finance* 73.3, pp. 1281–1321.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh (2018). 'Interpreting Factor Models'. In: *The Journal of Finance* 73 (3), pp. 1183–1223.

Lazarus, Eben et al. (2018). 'HAR Inference: Recommendations for Practice'. In: *Journal of Business and Economic Statistics* 36 (4), pp. 541–559.

Lu, Zhongjin and Zhongling Qin (1, 2021). 'Leveraged Funds and the Shadow Cost of Leverage Constraints'. In: *The Journal of Finance* 76.3, pp. 1295–1338.

Miller, Merton H. and Myron Scholes (1972). 'Rates of return in relation to risk: A re-examination of some recent findings'. In: *Studies in the theory of capital markets* 23, pp. 47–48.

Novy-Marx, Robert and Mihail Velikov (2022). 'Betting against betting against beta'. In: *Journal of Financial Economics* 143 (1), pp. 80–106.

Pinto, Jerald E., Thomas R. Robinson, and John D. Stowe (2019). 'Equity valuation: A survey of professional practice'. In: *Review of Financial Economics* 37.2, pp. 219–233.

Pukthuanthong, Kuntara, Richard Roll, and Junbo L. Wang (3, 2021). *An Agnostic and Practically Useful Estimator of the Stochastic Discount Factor*. Rochester, NY.

Schneider, Paul, Christian Wagner, and Josef Zechner (2020). 'Low-Risk Anomalies?' In: *The Journal of Finance* 75 (5), pp. 2673–2718.

Sharpe, William F. (1964). 'Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*'. In: *The Journal of Finance* 19.3, pp. 425–442.

6 Additional tables and figures

Table 5: Beta-sorted portfolios statistics. In parenthesis, HAC standard errors obtained as suggested in Lazarus et al. (2018). Monthly annualized returns from Dec 1934 to Nov 2022.

	Bottom	Top	BAB	Orig. BAB
Avg. ret (%)	11.808	13.781	3.837	8.46
SD (%)	43.801	85.02	41.325	36.795
CAPM α (%)	5.801	1.597	3.75	8.943
	(0.983)	(1.159)	(1.598)	(1.808)
CAPM β	0.72	1.461	0.01	-0.058
	(0.034)	(0.056)	(0.05)	(0.064)
Res. coskew.$\times 10^3$	-0.027	0.667	-0.93	-1.335
	(0.155)	(0.228)	(0.487)	(0.332)

Table 6: Test assets statistics. In parenthesis, HAC standard errors obtained as suggested in Lazarus et al. (2018), a part from 'AVG' column: this reports averages of the statistic across portfolios and in parenthesis are standard deviations of such statistics. Monthly annualized returns from Nov 1971 to Nov 2022.

Stat	accruals	debt_iss	invest	low_lev	low_risk	mom	prof_gr	profitab	quality	seasonal	s_t_rev	size	value	AVG
Avg. ret (%)	2.627	2.522	3.544	-0.661	2.168	4.256	1.736	3.235	3.34	1.58	1.366	1.165	4.661	4.112
SD (%)	13.248	8.857	26.877	38.781	47.207	39.366	13.574	27.288	18.644	7.071	15.365	25.89	41.577	32.484
CAPM α (%)	2.799	2.526	5.085	-2.659	6.473	5.476	1.477	4.186	3.522	1.818	1.329	0.4	6.515	3.271
	(0.946)	(0.624)	(1.48)	(1.824)	(1.767)	(1.325)	(0.522)	(1.409)	(0.815)	(0.419)	(0.591)	(1.366)	(2.293)	(1.312)
CAPM β	-0.022	-0.001	-0.198	0.256	-0.552	-0.157	0.033	-0.122	-0.023	-0.031	0.005	0.098	-0.238	0.108
	(0.033)	(0.01)	(0.066)	(0.097)	(0.091)	(0.079)	(0.028)	(0.058)	(0.03)	(0.012)	(0.015)	(0.033)	(0.101)	(0.053)
Res. cskw.$\times 10^3$	0.032	0.116	0.17	0.319	-0.285	-0.868	-0.157	0.215	0.179	-0.053	0.147	-0.263	0.073	-0.09
	(0.176)	(0.066)	(0.271)	(0.343)	(0.412)	(0.381)	(0.088)	(0.228)	(0.115)	(0.073)	(0.169)	(0.183)	(0.376)	(0.242)

Table 7: Additional beta-sorted portfolios statistics.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	BAB10
Avg. ret (%)	11.217	11.41	12.561	12.594	12.241	12.268	11.895	12.364	12.381	16.188	3.389
SD (%)	44.274	43.824	48.733	52.167	57.676	61.593	67.967	74.618	88.647	115.511	83.726
CAPM α (%)	7.647	7.078	7.338	6.806	5.628	5.151	3.953	3.652	2.364	4.207	3.702
	(1.55)	(1.486)	(1.218)	(1.163)	(1.101)	(1.189)	(1.016)	(1.027)	(1.885)	(3.159)	(3.877)
CAPM β	0.525	0.637	0.768	0.851	0.972	1.047	1.168	1.281	1.473	1.762	-0.046
	(0.056)	(0.046)	(0.054)	(0.06)	(0.055)	(0.048)	(0.038)	(0.032)	(0.057)	(0.098)	(0.114)
Res. coskew.$\times 10^3$	-0.267	0.302	0.545	0.199	0.438	0.211	0.144	0.107	0.474	1.543	-1.724
	(0.285)	(0.252)	(0.217)	(0.24)	(0.263)	(0.213)	(0.174)	(0.238)	(0.342)	(0.811)	(1.012)