

Generative process

$$\begin{aligned}
F &= p(s|\mu_x)p(\mu'_x|\mu_x,\nu) \\
s &= \mu_{x_3} + \omega_s \\
\dot{\mathbf{x}} &= \begin{bmatrix} \mu_{x_2} \\ -\phi\mu_{x_1} \\ \mu_\nu\mu_{x_1} - \mu_{x_3} \end{bmatrix} + \boldsymbol{\omega}_x \\
\nu &= \mu_\nu + \omega_\nu
\end{aligned}$$

Gradients

$$\begin{aligned}
-\frac{\partial F}{\partial \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \mu_{x_3} \end{bmatrix}} &= \begin{bmatrix} -\frac{\mu_\nu^2\mu_{x_1} - \mu_\nu\mu_{x_3} - \mu_\nu d\mu_{x_3} + \mu_{x_1} - d\mu_{x_2}}{\sigma_x} \\ -\frac{\phi^2\mu_{x_2} + \phi d\mu_{x_1}}{\sigma_x} \\ -\frac{-\mu_\nu\mu_{x_1} + \mu_{x_3} + d\mu_{x_3}}{\sigma_x} - \frac{\mu_{x_3} - s}{\sigma_s^2} \end{bmatrix} \\
-\frac{\partial F}{\partial \begin{bmatrix} d\mu_{x_1} \\ d\mu_{x_2} \\ d\mu_{x_3} \end{bmatrix}} &= \begin{bmatrix} -\frac{\mu_{x_2}\phi + d\mu_{x_1}}{\sigma_x} \\ -\frac{-\mu_{x_1} + d\mu_{x_2}}{\sigma_x} \\ -\frac{\mu_\nu\mu_{x_1} + \mu_{x_3} + d\mu_{x_3}}{\sigma_x} \end{bmatrix} \\
-\frac{\partial F}{\partial a} &= -\frac{(-\mu_{x_3} + s(a)) \frac{d}{da}s(a)}{\sigma_s^2} \\
\frac{\partial F}{\partial \mu_\nu} &= -\frac{\mu_{x_1}(\mu_\nu\mu_{x_1} - \mu_{x_3} - d\mu_{x_3})}{\sigma_x}
\end{aligned} \tag{1}$$