Optimal Redundancy Control of Robot Manipulators

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Optimal Control Project Report

Abstract – This project is about the optimal redundancy control of robot manipulators. This topic has been tackled by means of the *Pontryagin maximum principle*. Since only kinematics is considered, the optimal problem is reduced to minimal value searching in a space of as many dimensions as the degrees of redundancy. The formulation is: first to convert the redundancy resolution problem to an optimal control problem, and to obtain the optimal resolution of redundancy by using the necessary conditions of Pontryagin theory. As result, we get $2n$ first-order partial differential equations and the corresponding boundary conditions, solved by means of numerical tools. This work has been based on an article of Y. Nakamura and H. Hanafusa.

1. INTRODUCTION

Consider an open chain robot manipulator. With respect to a given task, it is said to be *redundant* if the dimension of the joint space is bigger than the dimension of the operational space. This property reflects into the fact that the *Jacobian matrix* is not square: its null space is non-trivial, and the ordinary operation of inversion is not possible. Thus, for a given end effector velocity there may exist different joint trajectories that realize it. In this work, we instantaneously exploit the null space of the Jacobian matrix in order to find the optimal joint trajectory that minimizes a given cost index. The problem is formulated by means of Pontryagin theory. The numerical algorithm proposed finds the optimal solution in a space of as many dimensions as the degree of redundancy. Some simulations are finally shown.

2. FORMULATION OF THE OPTIMAL REDUNDANCY PROBLEM

The optimal control problem of redundancy is formulated as follows. Suppose that the manipulation variable $\boldsymbol{r} \in \mathbb{R}^m$ represents the constrained variable, that is the position of the end effector, whose desired trajectory is described as a function of time. Let $\mathcal{J} \in \mathbb{R}$ represent a cost index of integral type, which evaluates the performance of redundancy utilization. The basic equations of the problem are:

$$\boldsymbol{r}(t) = \boldsymbol{dkf}\big(\boldsymbol{q}(t)\big), \tag{1}$$

$$\mathcal{J} = \int_{t_i}^{t_f} \mathcal{L}\big(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\big) dt, \tag{2}$$

where $\boldsymbol{q} \in \mathbb{R}^n$, $n > m$, is the joint variable vector of the manipulator, while t_i and t_f are the initial and final instants of time, respectively. For a given initial joint variable $\boldsymbol{q}(t_i) = \boldsymbol{q}_i$, the aim of the problem is to find the optimal joint trajectory $\boldsymbol{q}(t)$, $t_i \leq t \leq t_f$, that minimizes $\mathcal{J}$ realizing the desired operational trajectory $\boldsymbol{r}(t)$.

3. MANIPULATOR MODEL

The end effector position $\boldsymbol{r}(t)$ of an open chain manipulator can be described by the direct kinematics function, i.e. $\boldsymbol{dkf}: \mathbb{R}^n \to \mathbb{R}^m$. In order to describe the variables of the manipulator, the *Denavit-Hartenberg convention* has been considered. By differentiating (1) with respect to time, the differential kinematics function is obtained:

$$\dot{\boldsymbol{r}}(t) = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{3}$$

where $\boldsymbol{J}(\boldsymbol{q}) \in \mathbb{R}^{m\times n}$ is the Jacobian matrix of $\boldsymbol{dkf}(\boldsymbol{q})$. Suppose that the joint velocity vector $\dot{\boldsymbol{q}}$ satisfying (3) exists. Then for a given $\dot{\boldsymbol{r}}(t)$, the latter can be inverted obtaining:

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}(\boldsymbol{q})\dot{\boldsymbol{r}}(t), \tag{4}$$

where $\boldsymbol{J}^{\#}(\boldsymbol{q}) \in \mathbb{R}^{n\times m}$ is the pseudoinverse of the Jacobian matrix.
Let $\dot{\boldsymbol{q}}^{\dagger}$ be a solution of (3) and suppose that $\boldsymbol{\mathcal{P}} \in \mathbb{R}^{n\times n}$ is such that its range space (the *redundant space* of the manipulator) coincides with the null space of $\boldsymbol{J}$, $\mathcal{R}(\boldsymbol{\mathcal{P}}) \equiv \mathcal{N}(\boldsymbol{J})$. Then the joint velocity vector

$$\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}^{\dagger} + \boldsymbol{\mathcal{P}}\boldsymbol{u}, \tag{5}$$

with $\boldsymbol{u} \in \mathbb{R}^n$ as an arbitrary vector, is still a solution of (3). Thus, premultiplying both sides of (5) by $\boldsymbol{J}$, one gets:

$$\boldsymbol{J}\dot{\boldsymbol{q}} = \boldsymbol{J}\dot{\boldsymbol{q}}^{\dagger} + \boldsymbol{J}\boldsymbol{\mathcal{P}}\boldsymbol{u} = \boldsymbol{J}\dot{\boldsymbol{q}}^{\dagger} = \dot{\boldsymbol{r}},$$

since $\boldsymbol{J}\boldsymbol{\mathcal{P}} = \boldsymbol{O}_{m\times n}$. The effect of the arbitrary vector $\boldsymbol{u}$ is to generate *internal motions* of the robot that may allow a reconfiguration of the manipulator into more dexterous postures for the execution of a given task, without changing the position of the end effector. Then the (4) can be replaced by

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}\dot{\boldsymbol{r}}(t) + \boldsymbol{\mathcal{P}}\boldsymbol{u}(t).$$

A possible choice of $\boldsymbol{\mathcal{P}}$ is the matrix that allows the projection of the vector $\boldsymbol{u}$ onto the null space of $\boldsymbol{J}$ ([9] Siciliano et al., 2010), so as not to violate the constraint (3):

$$\boldsymbol{\mathcal{P}} = \boldsymbol{I}_n - \boldsymbol{J}^{\#}\boldsymbol{J}.$$

With this choice, the (4) finally becomes

$$\begin{aligned}\dot{\boldsymbol{q}} &= \boldsymbol{J}^{\#}\dot{\boldsymbol{r}}(t) + (\boldsymbol{I}_n - \boldsymbol{J}^{\#}\boldsymbol{J})\boldsymbol{u}(t) \\ &= \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{u}, t).\end{aligned} \tag{6}$$

4. APPLICATION OF PONTRYAGIN MAXIMUM PRINCIPLE

The control problem is to find the optimal arbitrary vector $\boldsymbol{u}^o$ and the optimal joint variable vector $\boldsymbol{q}^o$ such that:

- the dynamical constraint $\dot{\boldsymbol{q}}^o = \boldsymbol{f}(\boldsymbol{q}^o, \boldsymbol{u}^o, t)$ is satisfied;
- the final position condition $\boldsymbol{dkf}\left(\boldsymbol{q}(t_f)\right) - \boldsymbol{r}(t_f) = \boldsymbol{\chi}\left(\boldsymbol{q}(t_f)\right) = \boldsymbol{0}$ is satisfied;
- the performance index $\mathcal{J} = \int_{t_i}^{t_f} \mathcal{L}\big(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\big)dt$ evaluated in the optimum is maximized,

with the initial and final time instants t_i and t_f fixed and the initial joint variable $\boldsymbol{q}_i$ given. Thus, this problem can be addressed as an optimal control problem of a time-variant, nonlinear dynamical system whereas $\boldsymbol{q}$ is the state vector and $\boldsymbol{u}$ is the input vector. With this in mind, we can apply the Pontryagin maximum principle to the problem.
The Hamiltonian function is the following

$$\mathcal{H}(\boldsymbol{q}, \boldsymbol{u}, \lambda_0, \boldsymbol{\lambda}, t) = \lambda_0 \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{\lambda}^T(t)\boldsymbol{f}(\boldsymbol{q}, \boldsymbol{u}, t),$$

where $\lambda_0 < 0$ and $\boldsymbol{\lambda}(t) \in \mathbb{R}^n$ is the costate vector. In order to ensure that the problem does not admit singular solutions, the only joint variables considered are the ones that avoid singular configurations of the manipulator, i.e. the set of $\boldsymbol{q}(t)$ such that the Jacobian $\boldsymbol{J}\big(\boldsymbol{q}(t)\big)$ has full rank in the whole time interval. Then, the Hamiltonian can be written as:

$$\mathcal{H}(\boldsymbol{q}, \boldsymbol{u}, \boldsymbol{\lambda}, t) = -\mathcal{L} + \boldsymbol{\lambda}^T \boldsymbol{f}. \tag{7}$$

According to the Pontryagin theory, necessary conditions for an admissible point $(\boldsymbol{q}^*, \boldsymbol{u}^*)$ to be a maximum are:

$$\dot{\boldsymbol{\lambda}}^* = -\left.\frac{\partial \mathcal{H}}{\partial \boldsymbol{q}}\right|^{*T}, \tag{8}$$

$$\mathcal{H}(\boldsymbol{q}^*, \boldsymbol{\omega}, \boldsymbol{\lambda}^*, t) \leq \mathcal{H}(\boldsymbol{q}^*, \boldsymbol{u}^*, \boldsymbol{\lambda}^*, t) \;\; \forall \boldsymbol{\omega} \text{ admissible}, \tag{9}$$

$$\mathcal{H}|^{*T} = c \in \mathbb{R}, \forall t \in [t_i, t_f]. \tag{10}$$

Assuming that the cost index is such that the Hamiltonian is a continuous function of $\boldsymbol{u}$, the (9) can be replaced by

$$\left.\frac{\partial \mathcal{H}}{\partial \boldsymbol{u}}\right|^{*T} = \boldsymbol{0}. \tag{11}$$

5. BOUNDARY CONDITIONS

To solve (8) and (11), $2n$ boundary conditions are needed. The first n of them are trivial since the initial joint variable $\boldsymbol{q}_i$ is fixed. The remaining ones are obtained from the transversality conditions at the left-hand endpoint: $\boldsymbol{\lambda}(t_i)$ should intersect orthogonally with the manifold $\boldsymbol{r}(t_i) = \boldsymbol{dkf}(\boldsymbol{q}_i)$. The normal vectors of the manifold are the column vectors of $\boldsymbol{J}^T$. Then, the existence of a vector $\boldsymbol{\zeta} \in \mathbb{R}^n$ such that

$$\boldsymbol{J}^T(\boldsymbol{q}_i)\boldsymbol{\zeta} = \boldsymbol{\lambda}(t_i)$$

is guaranteed. A necessary and sufficient condition for the existence of a solution of the previous linear equation ([7] Rao and Mitra, 1971) is

$$[\boldsymbol{I}_n - \boldsymbol{J}^{\#}(\boldsymbol{q}_i)\boldsymbol{J}(\boldsymbol{q}_i)]\,\boldsymbol{\lambda}(t_i) = \boldsymbol{0}. \tag{12}$$

The latter implies that the initial value of the costate has to be an element of the complement of the redundant space.

6. PERFORMANCE INDICES

A set of representative performance indices is given by

$$\mathcal{J} = \int_{t_i}^{t_f} \{k_0 w(\boldsymbol{q}) + \dot{\boldsymbol{q}}^T\dot{\boldsymbol{q}}\}dt, \tag{13}$$

where k_0 is a nonnegative scalar that weights a secondary cost index. In addition, the second term of the integrand is included because it makes it easy to determine such $\boldsymbol{u}$ that maximizes the Hamiltonian. The optimal solution attempts to maximize the Lagrangian locally, compatible to the dynamical constraint. Two kinds of secondary cost indices are considered: the *manipulability measure* and the *distance from joint limits*.
The former is defined as

$$w_1(\boldsymbol{q}) = \sqrt{det\big(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\big)}, \tag{14}$$

which annihilates at singular configurations: by maximizing it, redundancy is exploited to move away from singularities.
The latter is defined as

$$w_2(\boldsymbol{q}) = -\frac{1}{2n}\sum_{i=1}^{n}\Big(\frac{q_i - \bar{q}_i}{q_{iM} - q_{im}}\Big), \tag{15}$$

where q_{iM} (q_{im}) denotes the maximum (minimum) joint limit of the i-th joint and $\bar{q}_i$ the middle value of the joint range; by maximizing it, redundancy is exploited to keep the joint variables as close as possible to the center of their ranges.

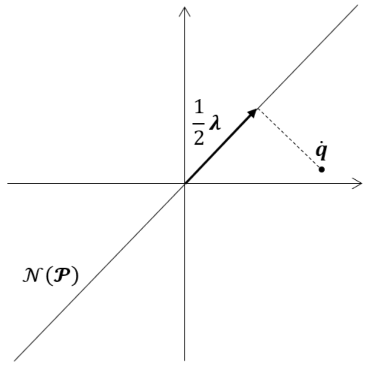

FIG. 1 - Suppose that the joint space has dimension $n = 2$, while the operational space has dimension $m = 1$. Thus, the redundant space is the line defined as the null space of the matrix $\boldsymbol{\mathcal{P}}$. For any given joint velocity, the costate is two times the projection of it onto the redundant space.

With the cost index defined as in (13), the Hamiltonian function becomes

$$\begin{aligned}\mathcal{H}(\boldsymbol{q},\boldsymbol{u},\boldsymbol{\lambda},t) &= -k_0 w(\boldsymbol{q}) - \dot{\boldsymbol{q}}^T\dot{\boldsymbol{q}} + \boldsymbol{\lambda}^T\dot{\boldsymbol{q}} \\ &= -\left(\boldsymbol{f} - \frac{1}{2}\boldsymbol{\lambda}\right)^T\left(\boldsymbol{f} - \frac{1}{2}\boldsymbol{\lambda}\right) + \frac{1}{4}\boldsymbol{\lambda}^T\boldsymbol{\lambda} - k_0 w(\boldsymbol{q}).\end{aligned} \tag{16}$$

Then equation (8) can be written as:

$$\dot{\boldsymbol{\lambda}}^* = k_0 \left.\frac{\partial w}{\partial \boldsymbol{q}}\right|^{*T} + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}}\right|^{*T} (2\dot{\boldsymbol{q}}^* - \boldsymbol{\lambda}^*). \tag{17}$$

In (16) the second and third terms of the right-hand side are independent on $\boldsymbol{u}$, because $\boldsymbol{u}$ is included only in $\boldsymbol{f}$. Therefore, the $\boldsymbol{u}^*$ that minimizes $\left\|\boldsymbol{f} - \frac{1}{2}\boldsymbol{\lambda}\right\|$ will maximize the Hamiltonian. Since $\boldsymbol{\lambda} \in \mathcal{N}(\boldsymbol{\mathcal{P}})$ (see eq. 12), it occurs when

$$\boldsymbol{\lambda}^* = 2proj_{\mathcal{N}(\boldsymbol{\mathcal{P}})}(\dot{\boldsymbol{q}}^*), \tag{18}$$

where $proj_{\mathcal{V}}(\boldsymbol{x})$ represents the orthogonal projection of the vector $\boldsymbol{x}$ onto the vector space $\mathcal{V}$ (FIG. 1). Recalling (6), the optimal arbitrary vector is

$$\boldsymbol{u}^* = (\boldsymbol{I}_n - \boldsymbol{J}^\#\boldsymbol{J})^\# \left(\frac{1}{2}\boldsymbol{\lambda}^* - \boldsymbol{J}^\#\dot{\boldsymbol{r}}\right).$$

Using the properties of the pseudoinverse[1], it becomes

$$\boldsymbol{u}^* = \frac{1}{2}(\boldsymbol{I}_n - \boldsymbol{J}^\#\boldsymbol{J})\boldsymbol{\lambda}^*. \tag{19}$$

This equation means that the half of the orthogonal projection of the optimal costate vector $\boldsymbol{\lambda}^*$ onto the redundant space of the manipulation variable $\boldsymbol{r}$ is the optimal arbitrary vector for the problem.

[1] Let $\boldsymbol{A}$ be a matrix of real numbers. The following properties of the pseudoinverse have been considered: $(\boldsymbol{I} - \boldsymbol{A}^\#\boldsymbol{A})^\# = \boldsymbol{I} - \boldsymbol{A}^\#\boldsymbol{A}$ and $(\boldsymbol{I} - \boldsymbol{A}^\#\boldsymbol{A})\boldsymbol{A}^\# = \boldsymbol{O}$.

Substituting (18) into (6) and taking in mind the properties of the pseudoinverse, the optimal joint velocity is

$$\dot{\boldsymbol{q}}^* = \boldsymbol{J}^\# \dot{\boldsymbol{r}} + \frac{1}{2}(\boldsymbol{I}_n - \boldsymbol{J}^\# \boldsymbol{J})\boldsymbol{\lambda}^*. \tag{20}$$

Finally, the optimal redundancy control problem of a robot manipulator can be summarized as follows:

$$\begin{aligned}
&\dot{\boldsymbol{q}}^* = \boldsymbol{J}^\# \dot{\boldsymbol{r}} + \frac{1}{2}(\boldsymbol{I}_n - \boldsymbol{J}^\# \boldsymbol{J})\boldsymbol{\lambda}^*,\\
&\dot{\boldsymbol{\lambda}}^* = k_0 \frac{\partial w}{\partial \boldsymbol{q}}\bigg|^{*T} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}}\bigg|^{*T} (2\dot{\boldsymbol{q}}^* - \boldsymbol{\lambda}^*),\\
&\boldsymbol{q}^*(t_i) = \boldsymbol{q}_i,\\
&\boldsymbol{\lambda}^*(t_i) = 2proj_{\mathcal{N}(\boldsymbol{\mathcal{P}})}\big(\dot{\boldsymbol{q}}^*(t_i)\big).
\end{aligned}$$

7. DISCRETIZATION OF THE PROBLEM

In order to numerically simulate the problem, Euler integration method has been applied to discretize the differential equations (17) and (20). Let Δt be the sampling time and s the number of samples, the k-th samples of the optimal joint variable and the optimal costate are obtained by

$$\boldsymbol{q}_k^* = \Delta t\left\{[\boldsymbol{J}^\#(\boldsymbol{q}_{k-1}^*)\dot{\boldsymbol{r}}_k] + \frac{1}{2}[\boldsymbol{I}_n - \boldsymbol{J}^\#(\boldsymbol{q}_{k-1}^*)\boldsymbol{J}(\boldsymbol{q}_{k-1}^*)]\boldsymbol{\lambda}_{k-1}^*\right\} + \boldsymbol{q}_{k-1}^*, \tag{21}$$

$$\boldsymbol{\lambda}_k^* = \Delta t\left\{k_0 \frac{\partial w}{\partial \boldsymbol{q}}\bigg|_{\boldsymbol{q}_{k-1}^*}^{T} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}}\bigg|_{\boldsymbol{q}_{k-1}^*,\boldsymbol{\lambda}_{k-1}^*}^{T} \left(2\frac{\boldsymbol{q}_k^* - \boldsymbol{q}_{k-1}^*}{\Delta t} - \boldsymbol{\lambda}_{k-1}^*\right)\right\} + \boldsymbol{\lambda}_{k-1}^*, \tag{22}$$

with $k = 1, 2, \dots, s$ and the boundary conditions described above. Calling $\boldsymbol{q}_0 = \boldsymbol{q}(t_i)$, the cost index in (13) takes the form

$$\mathcal{J} = \sum_{k=0}^{s}\left\{k_0 w(\boldsymbol{q}_k^*) + \left(\frac{\boldsymbol{q}_k^* - \boldsymbol{q}_{k-1}^*}{\Delta t}\right)^T \left(\frac{\boldsymbol{q}_k^* - \boldsymbol{q}_{k-1}^*}{\Delta t}\right)\right\}. \tag{23}$$

8. SIMULATION RESULTS

Numerical examples of the optimal redundancy control problem have been performed on a planar 3R manipulator, where the length of each link is 1 m (FIG. 2). In this particular case the dimension of the joint space is $n = 3$, while the dimension of the operational space is

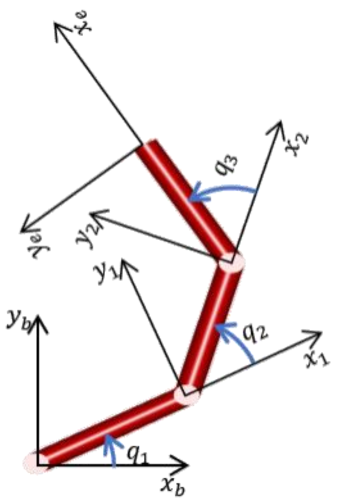

FIG. 2 - The planar 3R manipulator and its Denavit-Hartenberg frames assignment

$m = 2$: this implies that the degree of redundancy is 1. In every simulation, the following data are fixed:

- the initial and final instants are $t_i = 0\ s$ and $t_f = 1\ s$, respectively;
- the initial joint configuration is $\boldsymbol{q}_i = [0 \quad \pi/2 \quad 0]^T\ [rad]$;
- the end effector velocity is the constant vector $\boldsymbol{v}_e = [-1 \quad 0]^T\ [m/s]$;
- the sampling time $\Delta t = 0.01\ s$.

The direct kinematics function for the described robot is

$$\boldsymbol{dkf}(\boldsymbol{q}) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_{123} + c_{12} + c_1 \\ s_{123} + s_{12} + s_1 \end{bmatrix}$$

and the Jacobian matrix is

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{bmatrix} -s_{123} - s_{12} - s_1 & -s_{123} - s_{12} & -s_{123} \\ c_{123} + c_{12} + c_1 & c_{123} + c_{12} & c_{123} \end{bmatrix},$$

where $s_{ijk} = \sin(q_i + q_j + q_k)$ and, similarly, $c_{ijk} = \cos(q_i + q_j + q_k)$.
Note that, since $\boldsymbol{v}_e$ is constant, the dynamics of the system $\dot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{u}, t)$ is time-invariant. Moreover, since there is no feedback control action, the error generated by the discretization would increase with time. Four different simulations are shown.

Case 1: no secondary cost index. The considered performance index is

$$\mathcal{J} = \int_{t_i}^{t_f} \dot{\boldsymbol{q}}^T \dot{\boldsymbol{q}}\, dt,$$

without exploiting the presence of the redundancy. The solution can be obtained in closed form by the inverse differential kinematics equation, since the pseudoinverse aims to minimize the norm of the joint velocity at each step. Anyway, the proposed algorithm gives the joint variable evolution in (FIG. 3a). (FIG. 3b) shows the evolution of the position error: at the end of the simulation it has a magnitude of about $0.0026\ m$.

Case 2: manipulability measure as secondary cost index. In order to have more dexterous joint variable evolution, the manipulability measure has been considered weighted by $k_0 = 10$:

$$\mathcal{J} = \int_{t_i}^{t_f} \left\{ k_0 \sqrt{det\big(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\big)} + \dot{\boldsymbol{q}}^T \dot{\boldsymbol{q}} \right\} dt,$$

The joint variables in (FIG. 4a) moves the end effector as specified trying to move away from singular configurations. This leads to an error that is larger than in the previous case, and that it is shown in (FIG. 4b).

Case 3: distance from joint limits as secondary cost index. The joint limits considered are

$$-180° \leq q_1 \leq 180°,$$

$$-95° \leq q_2 \leq 95°,$$

$$-180° \leq q_3 \leq 180°,$$

while the cost function has been weighted by $k_0 = 1000$:

$$\mathcal{J} = \int_{t_i}^{t_f} \left\{ k_0 \left[-\frac{1}{2n} \sum_{i=1}^{n} \left(\frac{q_i - \bar{q}_i}{q_{iM} - q_{im}} \right) \right] + \dot{\boldsymbol{q}}^T \dot{\boldsymbol{q}} \right\} dt,$$

The optimization algorithm does not ensure that a joint variable does not exceed its limit, but it tries to keep it close to the mean value of the admissible range; this causes an error that is one order bigger than before (FIG. 5b). The weight of the secondary cost index is big enough to generate joint variables that do not exceed their limits (FIG. 5a).

Case 4: hybrid secondary cost index. The aim of this simulation is to produce the joint variables that try to minimize the norm of the joint velocities and to maximize the sum of the manipulability measure and the distance from the joint limits. The joint limits considered are the same as the previous case. The weight of the secondary cost index is $k_0 = 200$:

$$\mathcal{J} = \int_{t_i}^{t_f} \left\{ k_0 \left[\sqrt{det\big(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\big)} - \frac{1}{2n} \sum_{i=1}^{n} \left(\frac{q_i - \bar{q}_i}{q_{iM} - q_{im}} \right) \right] + \dot{\boldsymbol{q}}^T \dot{\boldsymbol{q}} \right\} dt$$

The obtained joint variables are shown in (FIG. 6a). As it is shown in (FIG. 6b), the error is relevant since its magnitude is about $0.06\,m$. As it has been said above, the absence of a feedback controller can not recover the error produced by the discretization. An easy way to bypass this problem is to reduce the sampling time. For instance, choosing $\Delta t = 0.001\,s$ the error decreases of one order of magnitude (FIG. 7b). The corresponding joint variables are shown in (FIG. 7a).

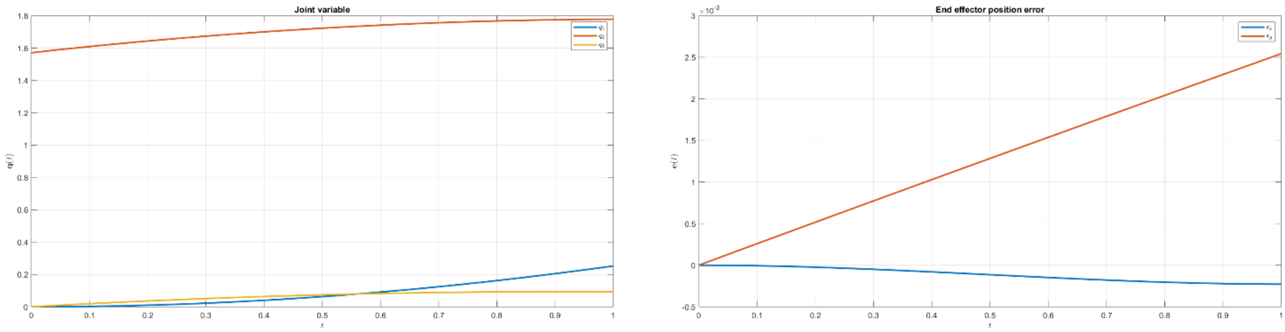

FIG. 3 - The joint variable (a) and the position error (b) in Case 1

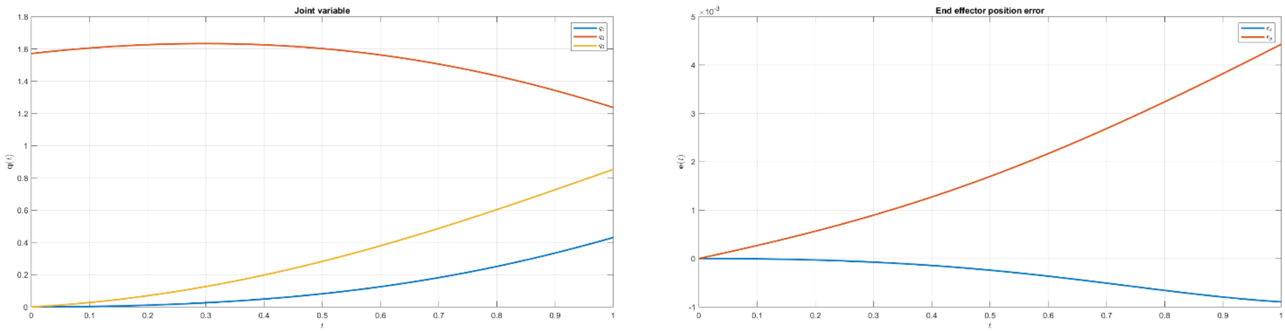

FIG. 4 - The joint variable (a) and the position error (b) in Case 2

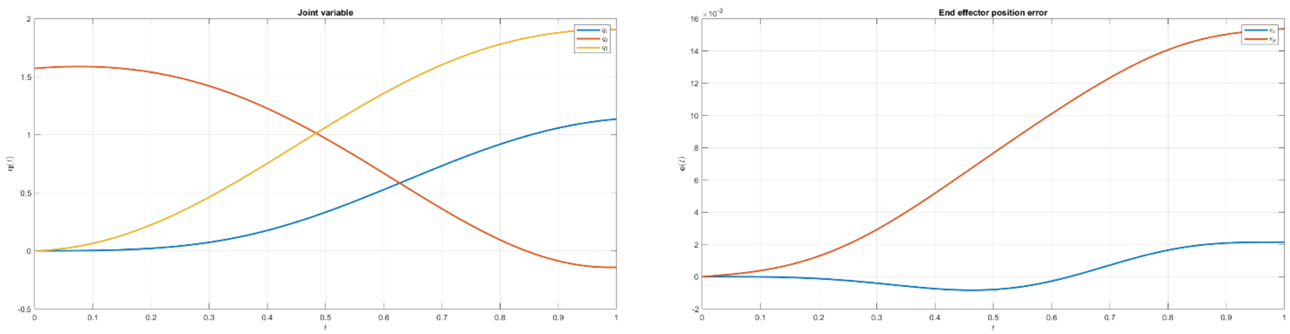

FIG. 5 - The joint variable (a) and the position error (b) in Case 3

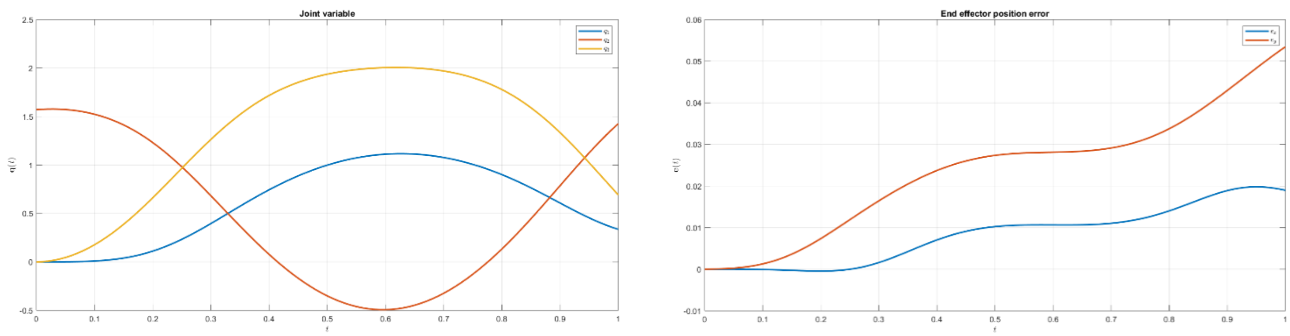

FIG. 6 - The joint variable (a) and the position error (b) in Case 4a

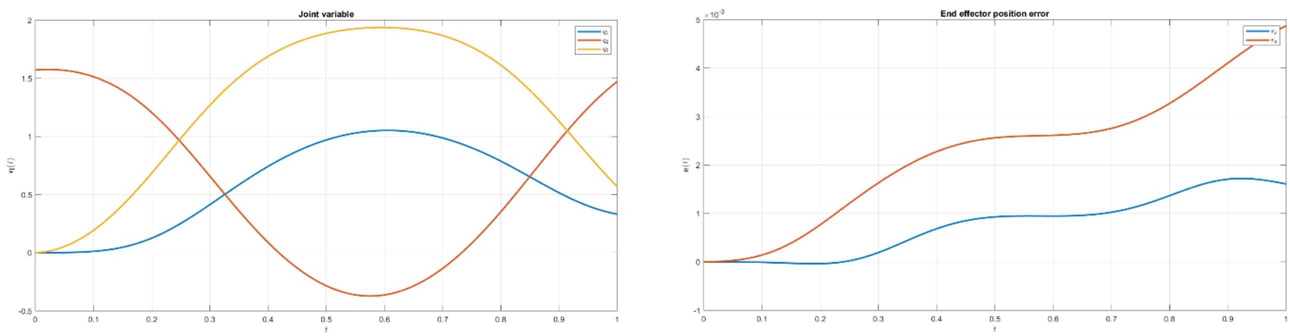

FIG. 7 - The joint variable (a) and the position error (b) in Case 4b

9. CONCLUSION

We have presented the optimal kinematic control for the optimal redundancy problem of robot manipulators. The main idea is that that the kinematic resolution problem is considered as an optimal control problem. From the necessary conditions of the Pontryagin theory, the redundancy is solved. We have achieved the optimal solution described by $2n$ first order partial differential equations with their boundary conditions.

To verify the effectiveness of the proposed algorithm, four simulations were shown: the first minimized only the joint velocity without the presence of a secondary cost index, while the others took into account the manipulability measure, the distance from the joint limits and their combination, respectively. Some remarks were also made concerning the position error.

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