*Reviewer #1: The paper shows interesting results on lexicographic products of residuated ordered monoids. However, I do not think that this paper is suitable for this journal, since it is a residuated version of an accepted conference paper. In fact, I prefer the conference paper over the current paper, because it includes a section on the applicability of the main theorems.*

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**We thank the reviewer for the comment. However, we would like to point out that IPL has a limit of a few pages (which we completely used in our submission), since articles are used as short communications. Even if the conference paper has an applicability section, however it does not include the proofs of theorems: in our submission the goal is to report and show the methodological approach behind the paper, more than the application which, as correctly highlighted by the reviewer, is already shown in the conference paper.**

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*Reviewer #2: The lexicographic product of residuated partially ordered commutative monoids is discussed.  
  
Partially ordered commutative monoids offer a rich basis for expressing hard and soft constraints as well as preferences.  In previous work, the authors have solved the problem of how to combine these algebraic structures lexicographically by the introduction of collapsing elements [8].  They have also discussed the use of residuation, providing a weak inverse to a monoid's multiplication, for bipolar preferences [6].  Most recently, they have suggested a lexicographic combination of residuated partially ordered commutative monoids and their use for approximate soft constraint optimisation with mini-bucket elimination [11].  For this last work the present paper provides the technical details.  The main contribution is the extension of the use, or better avoidance, of collapsing elements in the definition of the residuation of lexicographic products, leading to another intriguing case distinction.  
  
The proof, however, that lexicographic products of residuated partially ordered commutative monoids are again residuated only starts on page 7 which is rather late.  If I am not mistaken, the presentation is mainly complicated by the fact that the result is shown for a k-fold lexicographic combination of a given residuated monoid, and not by proving the result for just two residuated monoids and extending this to the general case.  In particular, the arguments in Prop.s 2 to 4, covering the different cases, read somewhat repetitive.  I attach an alternative, substantially shorter presentation of the results that concentrates on the binary case.*

**\*\*\***

**We thank the reviewer for the additional proof related to the fact that a product of POMs is still residuated. We definitely agree such a proof is simpler than using a k-fold lexicographic combination, which we adopt in the paper. However, we would like to highlight that, as currently presented, the proof is easily extendible to the case of infinite tuples, which we show in Section 4.2. This second proof naturally follows the first one, and for this reason we opted for such a style.**

*\*\*  
  
The discussion of (finite) distributivity and (finite) completeness should be separated from the main line of arguments, as partially ordered commutative monoids do not satisfy such conditions in general; still, as already demonstrated, these additional properties provide good examples.*

**???** *p. 1: in order ``move'' -> in order to ``move''  
p. 1: extensions has -> extensions have  
p. 2: systematise and extending -> systematising and extending  
p. 2: we use the infix notation -> we use infix notation  
p. 3: in a SML -> in an SLM  
p. 3, Rem. 4, $\langle N \cup \{ infty \}, {\geq}, +, 0\rangle$: The $N$ should be $\mathbb{N}$?  
p. 4, Lem. 4: $C(A\_1) \times A\_2 \cup A\_2 \times C(A\_2)$  
-> $(C(A\_1) \times A\_2) \cup (A\_1 \times C(A\_2))$  
p. 4, Ex. 4, $C(\mathbb{N} \times \mathbb{N})$: Is $C((\mathbb{N} \cup \{ \infty \})^2)$ meant?  Better give the monoid a name, e.g., $N$ and use this.  
p. 5: $\leq\_1 = \leq$ -> ${\leq\_1} = {\leq}$  
p. 5: reflexivity and symmetry -> reflexivity and anti-symmetry  
p. 6, Lem. 7, last line: $I(A)\{ \bot \}^{n+1} \in \bigcup\_{i \leq n} I(A)^{i+1}A\{ \bot \}^{n-i}$ -> $I(A)\{ \bot \}^{n+1} \subseteq \bigcup\_{i \leq n} I(A)^{i+1}A\{ \bot \}^{n-i}$  
p. 7, Ex. 5, $\mathit{Lex}\_2(\mathbb{N})$: This should just be the set of natural numbers. You may refer to Ex. 4 and the name of the S/CLM introduced there.  
p. 7, Def. 7, and passim: $\mathit{min}$ -> $\min$  
p. 9: A straightforward adaptation of Proposition 1. -> This result is a straightforward adaptation of Proposition 1.  
p. 9: Also a straightforward adaptation, this time of Theorem 1. -> This result is a straightforward adaptation of Theorem 1.*

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**We thank the reviewer for spotting all the above typos. In this resubmission, we fixed all of them as suggested.**   
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*Reviewer #3: Partially ordered monoids are algebraic preference structures that are the basis for several approaches to soft constraint programming. This paper studies residuation of lexicographic orders over partially ordered monoids.  A novel elegant notion of the set of cancellative elements is defined and it is shown that for distributive structures both notions coincide. As main result, finite and infinite lexicographically ordered products of lexicographically are defined and it is shown that these product structures admit residuation if the underlying partially ordered monoid does so.   
The study of residuation of partially ordered monoids is of great practical interest in constraint programming (e.g. in case constraints or preferences have to be revised). The results are new and the construction of lexicographic order is mathematically elegant. The paper is nicely and clearly written. The related work is adequately described. I recommend acceptance of the paper.*

**\*\*\***

**We thank the reviewer for the positive comments.**

**\*\*\***

*Minor remarks:  
Def..3: Typo: "distributive is(?)"  
Def. 4: Give an example of the residuation function; e.g. of the powerset example.  
Remark 4: Typo: "sub-sets"  
Lemma 1: The residuation operator is missing in the tuple of the residuated POM. Which operation is monotone?  
Lemma 4: Typo: "… union A\_2  x C(A2)"?  
Section 4.1, line 2: Should "is a CLM" be "is a distributed CLM"?*

**\*\*\***

**We fixed all the minor comments and typos as suggested (we thank the reviewer for spotting them).**

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