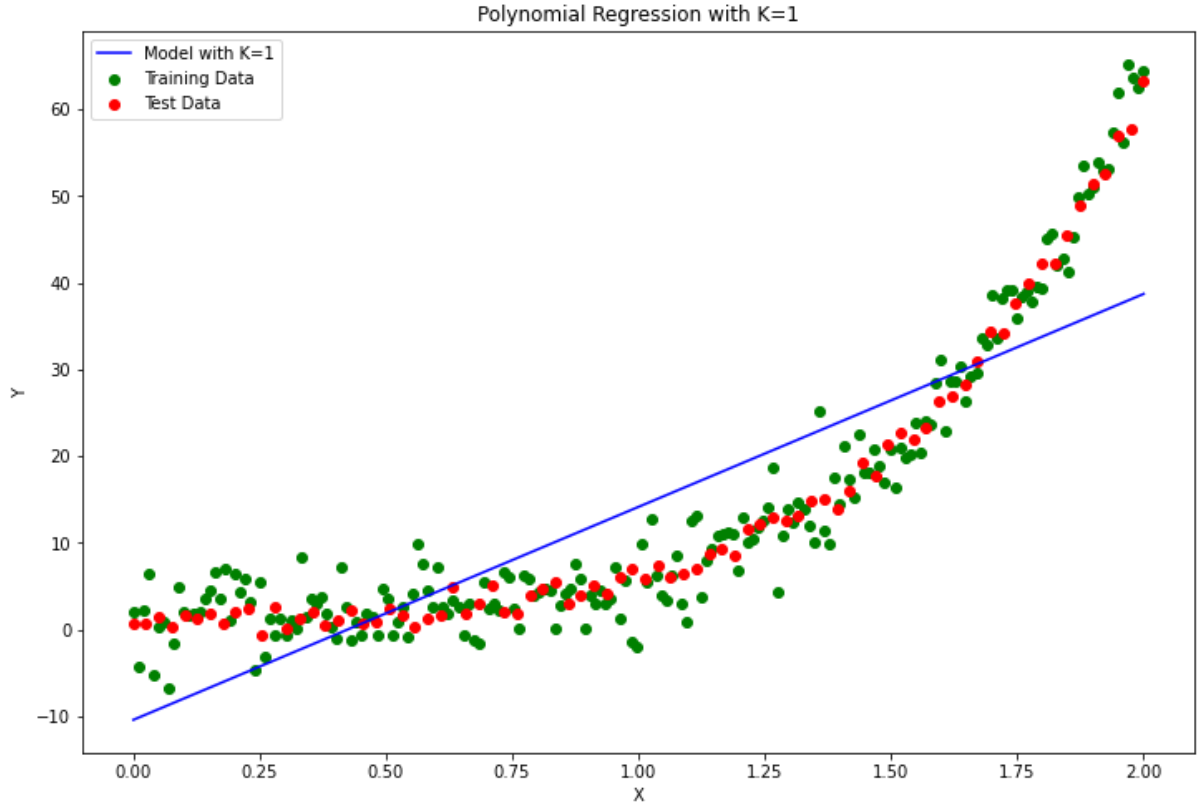
Generated test dataset for variance 1.0:

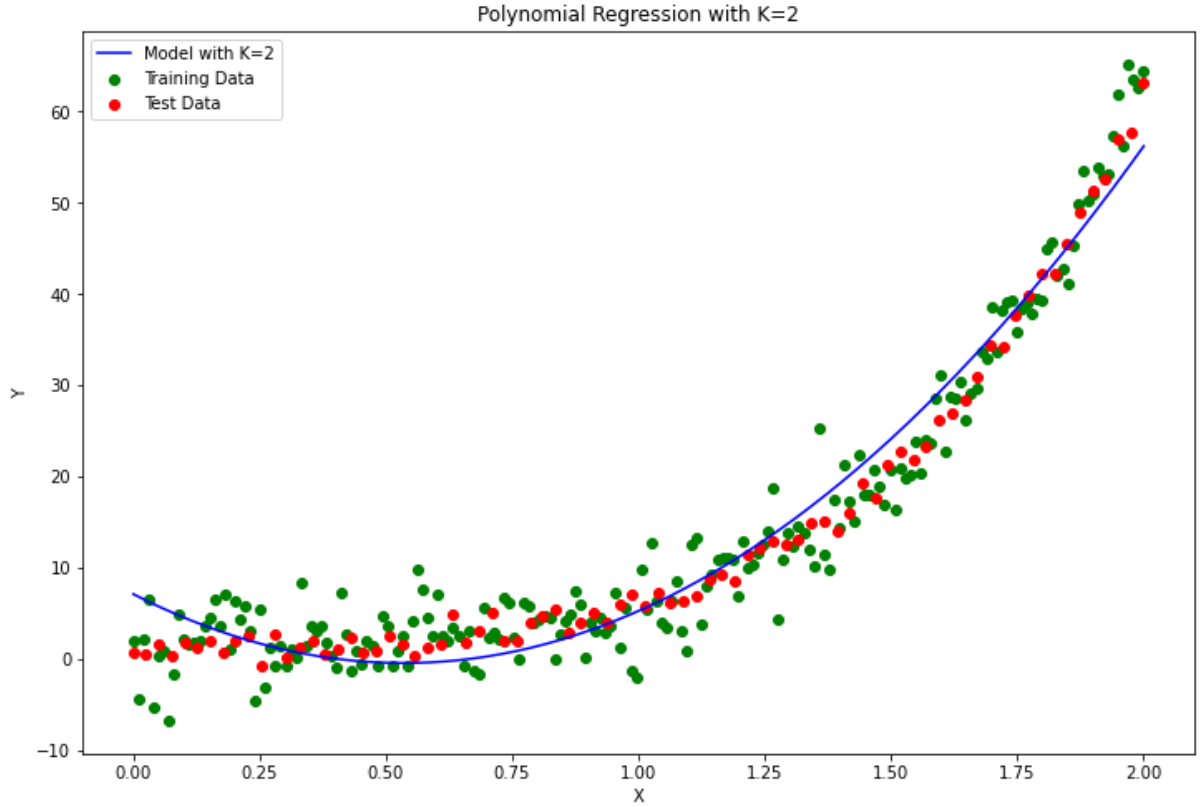
X: [0. 0.1010101 0.2020202 0.3030303 0.4040404 0.50505051
0.60606061 0.70707071 0.80808081 0.90909091 1.01010101 1.11111111
1.21212121 1.31313131 1.41414141 1.51515152 1.61616162 1.71717172
1.81818182 1.91919192 2.02020202 2.12121212 2.22222222 2.32323232
2.42424242 2.52525253 2.62626263 2.72727273 2.82828283 2.92929293
3.03030303 3.13131313 3.23232323 3.33333333 3.43434343 3.53535354
3.63636364 3.73737374 3.83838384 3.93939394 4.04040404 4.14141414
4.24242424 4.34343434 4.44444444 4.54545455 4.64646465 4.74747475
4.84848485 4.94949495 5.05050505 5.15151515 5.25252525 5.35353535
5.45454545 5.55555556 5.65656566 5.75757576 5.85858586 5.95959596
6.06060606 6.16161616 6.26262626 6.36363636 6.46464646 6.56565657
6.66666667 6.76767677 6.86868687 6.96969697 7.07070707 7.17171717
7.27272727 7.37373737 7.47474747 7.57575758 7.67676768 7.77777778
7.87878788 7.97979798 8.08080808 8.18181818 8.28282828 8.38383838
8.48484848 8.58585859 8.68686869 8.78787879 8.88888889 8.98989899
9.09090909 9.19191919 9.29292929 9.39393939 9.49494949 9.5959596
9.6969697 9.7979798 9.8989899 10.]

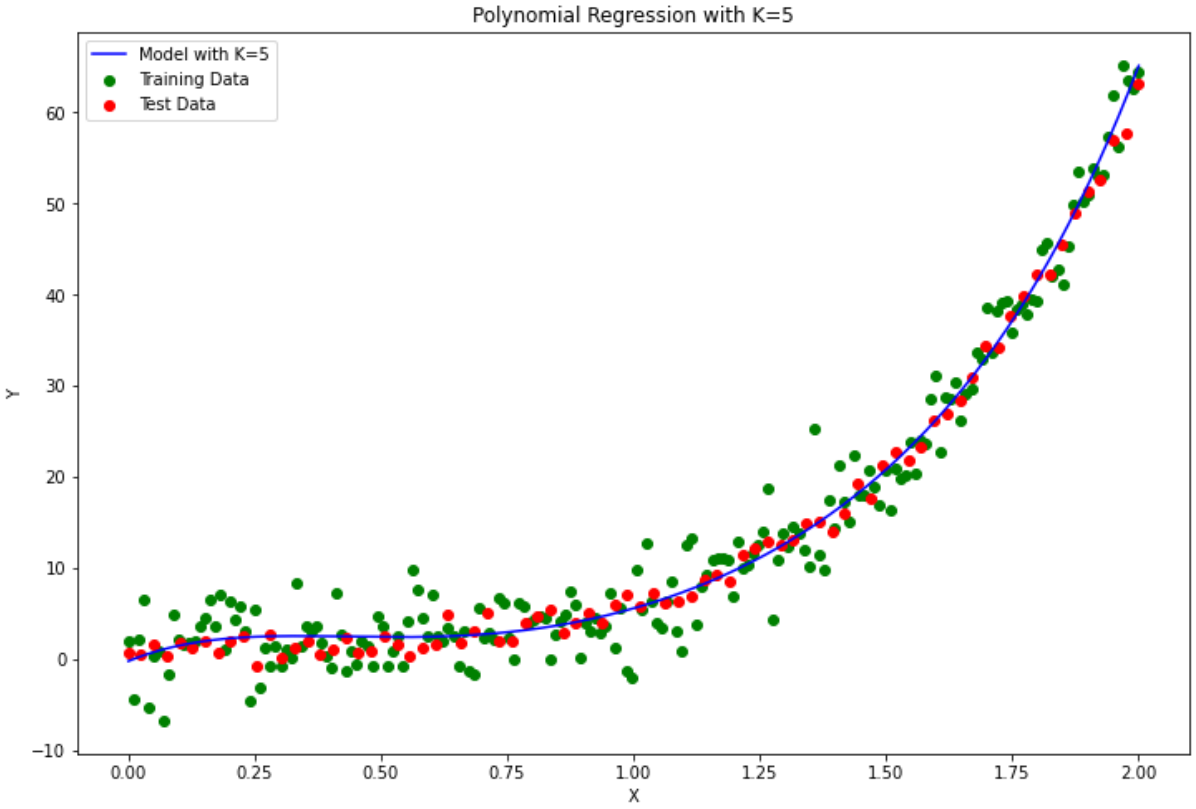
Y: [-2.48444078e-02 3.92689369e-01 1.79632889e+00 3.04224336e+00
2.94489212e-02 1.07651074e+00 2.50812627e+00 5.08111995e+00
3.66880371e+00 5.95151068e+00 6.56301077e+00 9.28173070e+00
9.02817518e+00 1.30400077e+01 2.01045530e+01 2.21888175e+01
2.67949343e+01 3.50334796e+01 4.38535992e+01 5.31779606e+01
6.49417279e+01 8.08122220e+01 9.57604931e+01 1.18053625e+02
1.44318377e+02 1.69868851e+02 2.02206651e+02 2.38107949e+02
2.80637338e+02 3.26452679e+02 3.80697209e+02 4.42691114e+02
5.09554080e+02 5.86666746e+02 6.75885987e+02 7.68666234e+02
8.77262845e+02 9.94687974e+02 1.12880381e+03 1.27172897e+03
1.42958700e+03 1.60570593e+03 1.79755829e+03 2.00812823e+03
2.23589591e+03 2.48882767e+03 2.76000778e+03 3.05555490e+03
3.37709066e+03 3.72381602e+03 4.09676692e+03 4.50242936e+03
4.93745254e+03 5.40768792e+03 5.91220118e+03 6.45332748e+03
7.03444596e+03 7.65768101e+03 8.32339382e+03 9.03397743e+03
9.79223129e+03 1.06010445e+04 1.14633374e+04 1.23801297e+04
1.33572597e+04 1.43915611e+04 1.54937921e+04 1.66575723e+04
1.78932137e+04 1.92024556e+04 2.05836192e+04 2.20447125e+04
2.35894729e+04 2.52191511e+04 2.69390644e+04 2.87480824e+04
3.06559369e+04 3.26625214e+04 3.47720926e+04 3.69912531e+04
3.93225623e+04 4.17699393e+04 4.43387940e+04 4.70299498e+04
4.98511310e+04 5.28070716e+04 5.59024051e+04 5.91395290e+04
6.25269345e+04 6.60676832e+04 6.97671194e+04 7.36282108e+04
7.76611127e+04 8.18673048e+04 8.62570095e+04 9.08312371e+04
9.55993815e+04 1.00561040e+05 1.05732989e+05 1.11112131e+05]

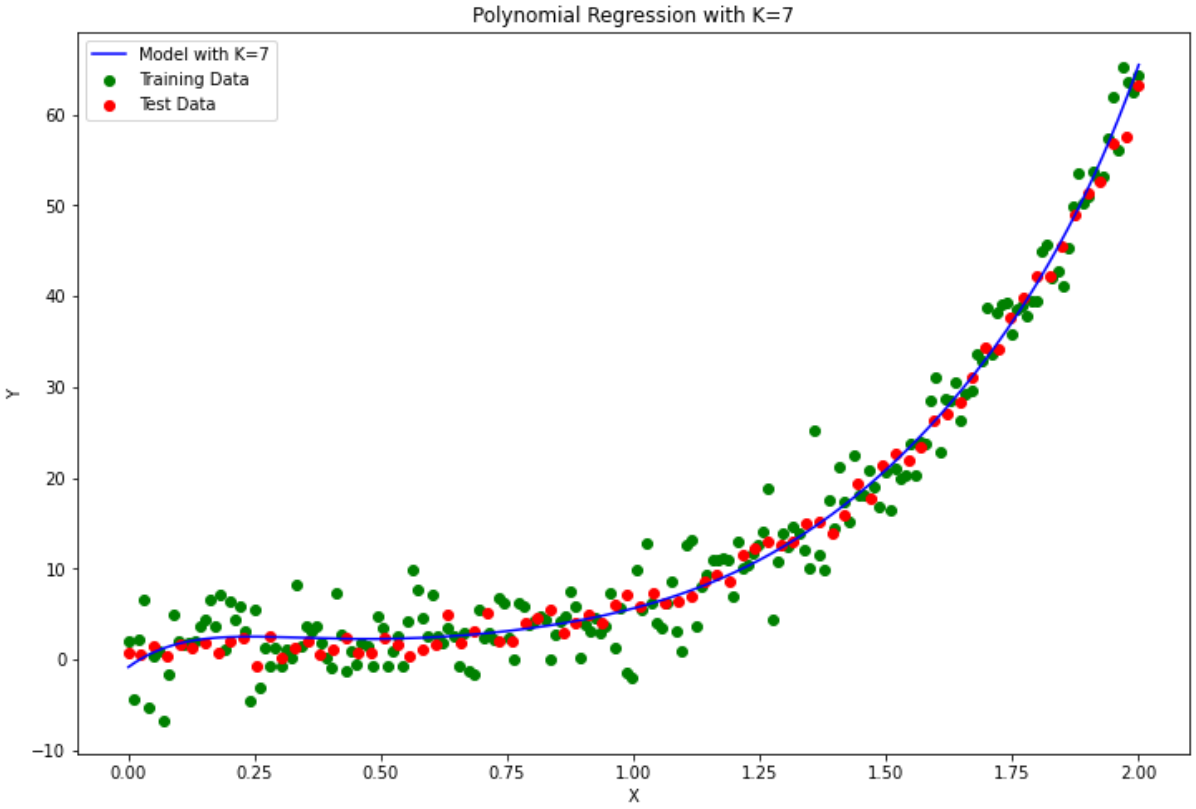
```
Shape of X_train: (200,)
Shape of theta_true: (6,)
Shape of Y_train: (200,)

Coefficients MLE: [ -0.21246239  22.90340539 -66.81677559  81.01931661 -40.0385832
```

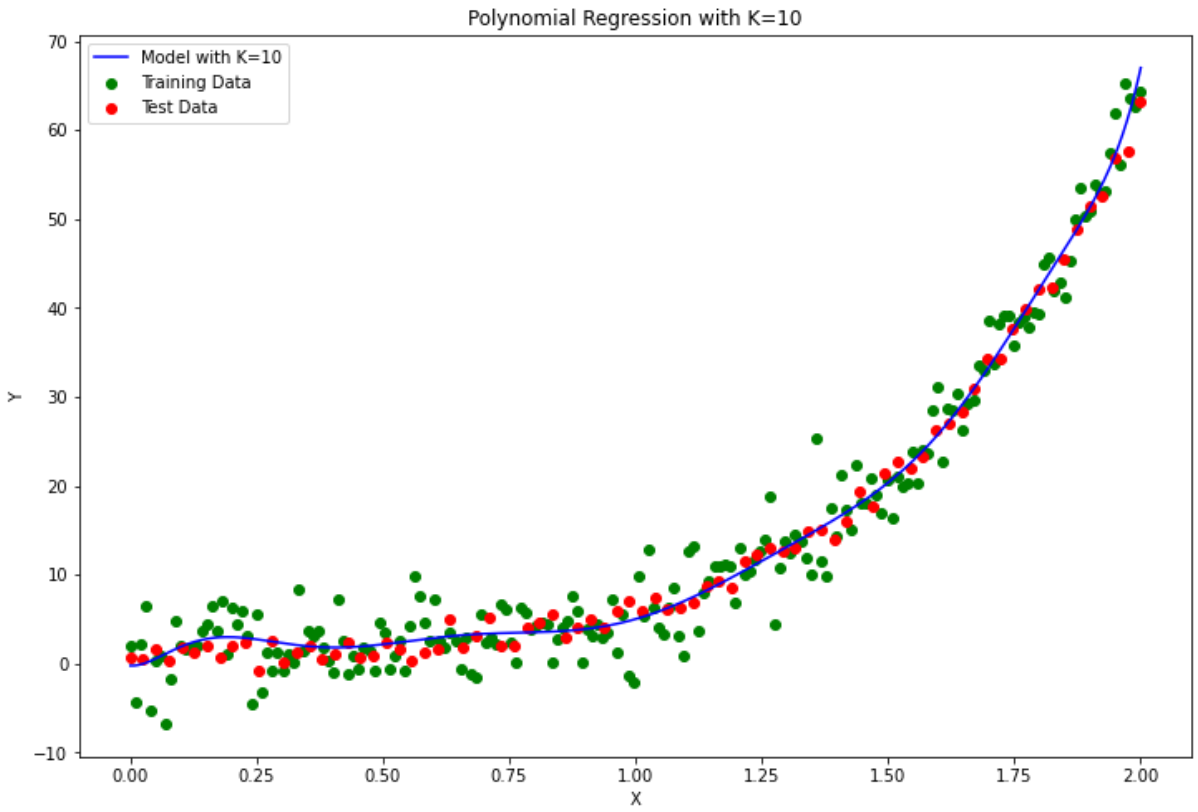
```
   8.72728899]
Average Absolute Error: 0.7816272840329866
Average Relative Error: 1.4763435911169718
```

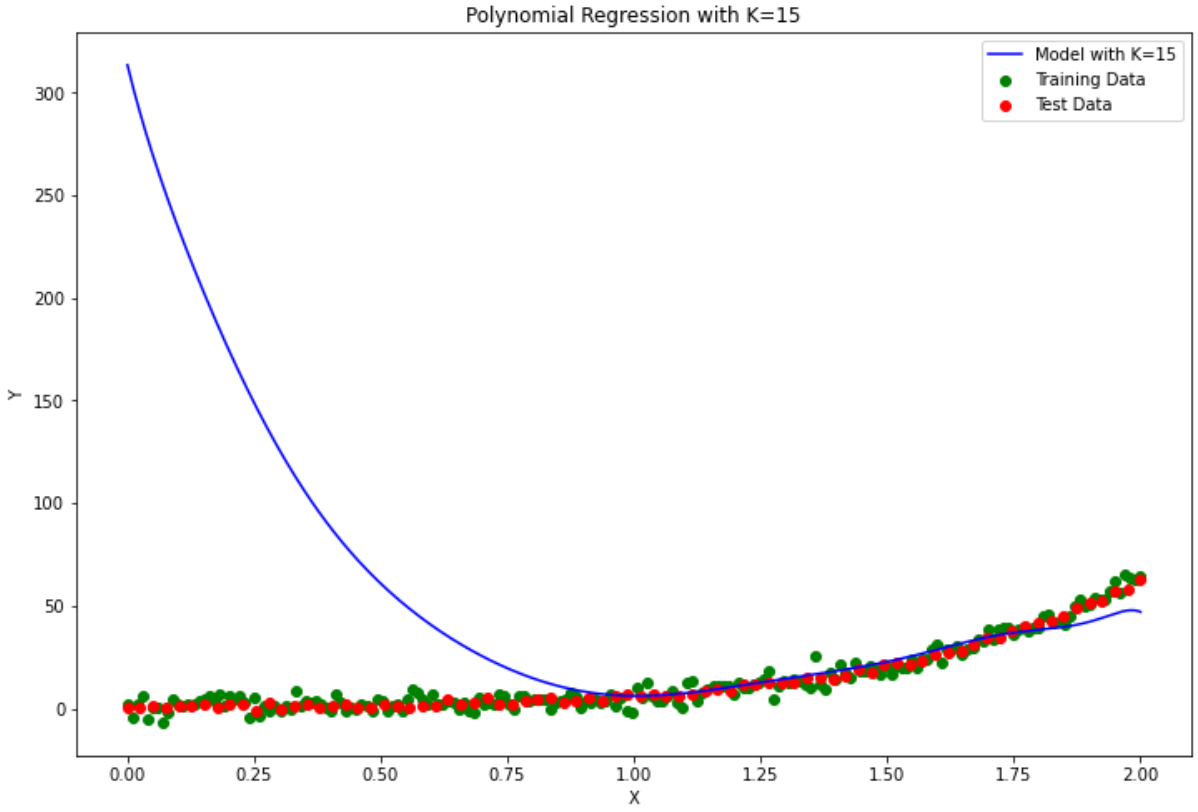


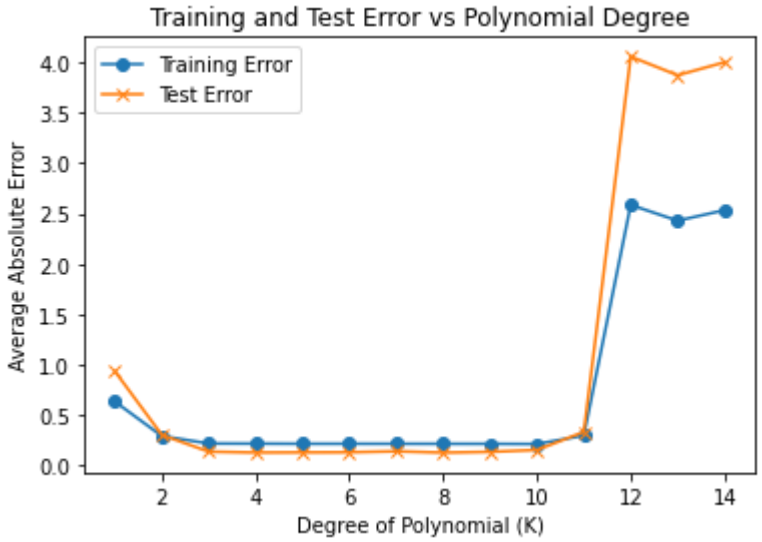


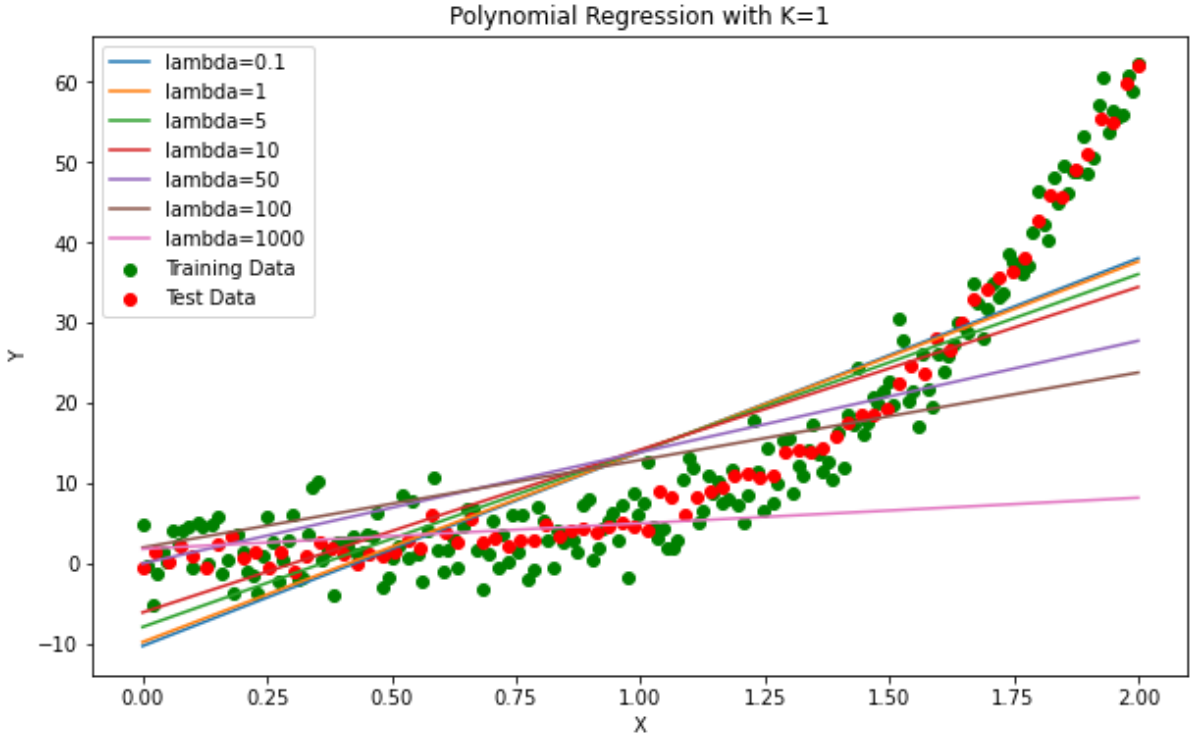

Polynomial Regression with K=5
Model with K=5
Training Data
Test Data
Y
X
60
50
40
30
20
10
0
−10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

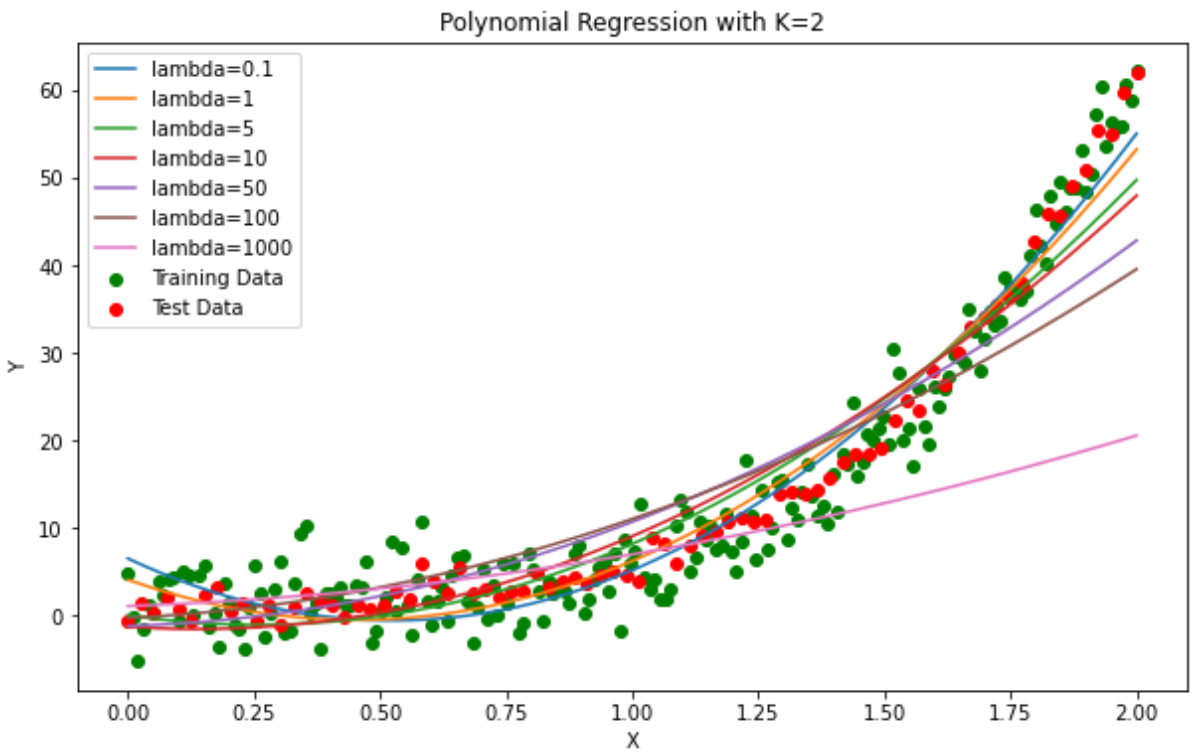

Polynomial Regression with K=7
Model with K=7
Training Data
Test Data
Y
X
60
50
40
30
20
10
0
−10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

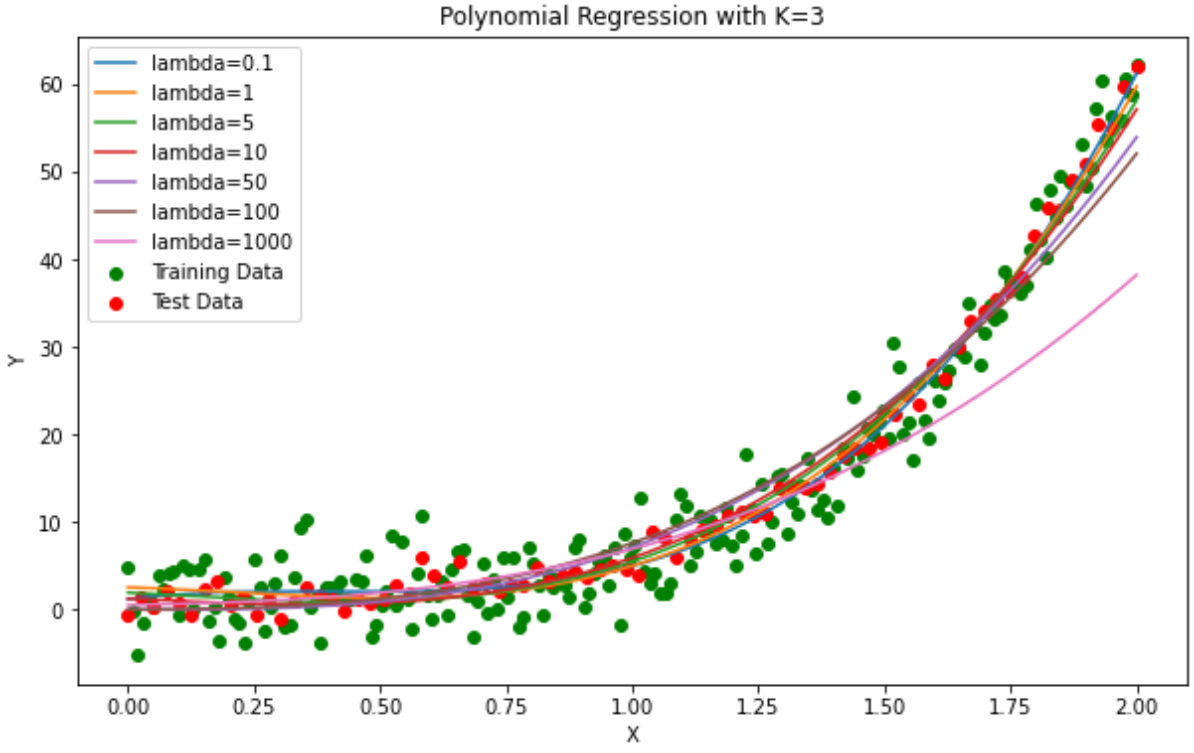

Polynomial Regression with K=10
Model with K=10
Training Data
Test Data
Y
X
70
60
50
40
30
20
10
0
−10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

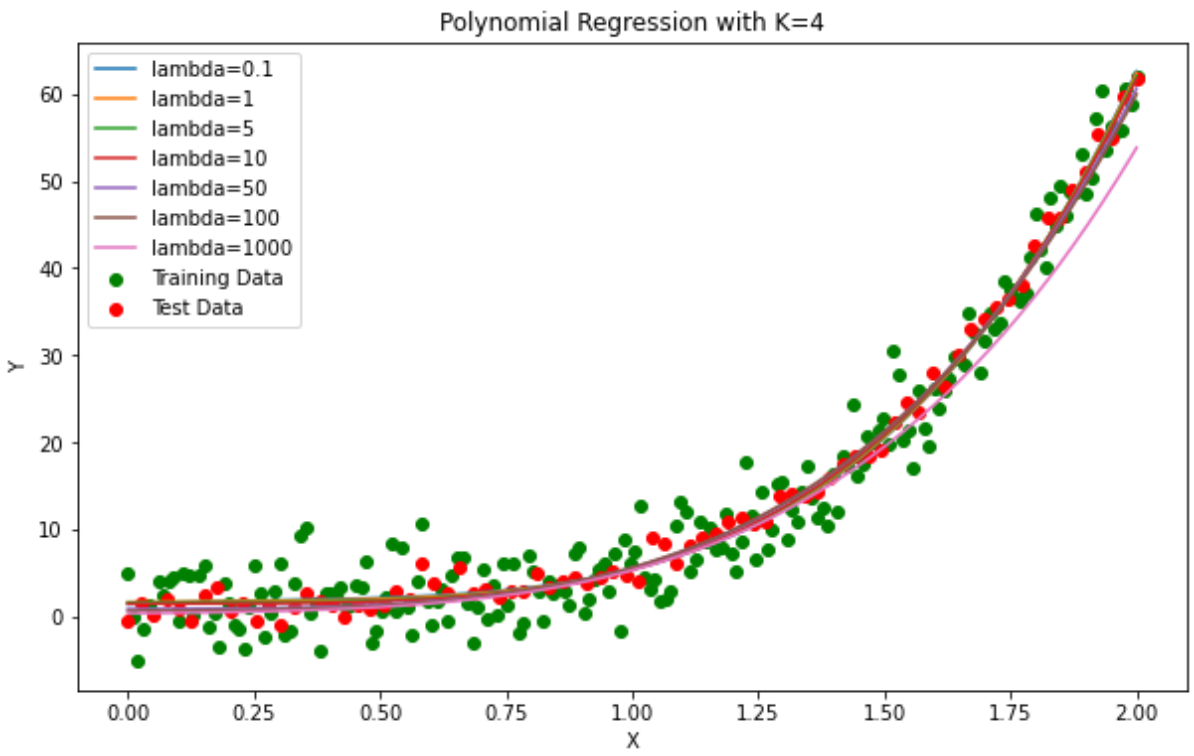

Polynomial Regression with K=15
Model with K=15
Training Data
Test Data
Y
X
300
250
200
150
100
50
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

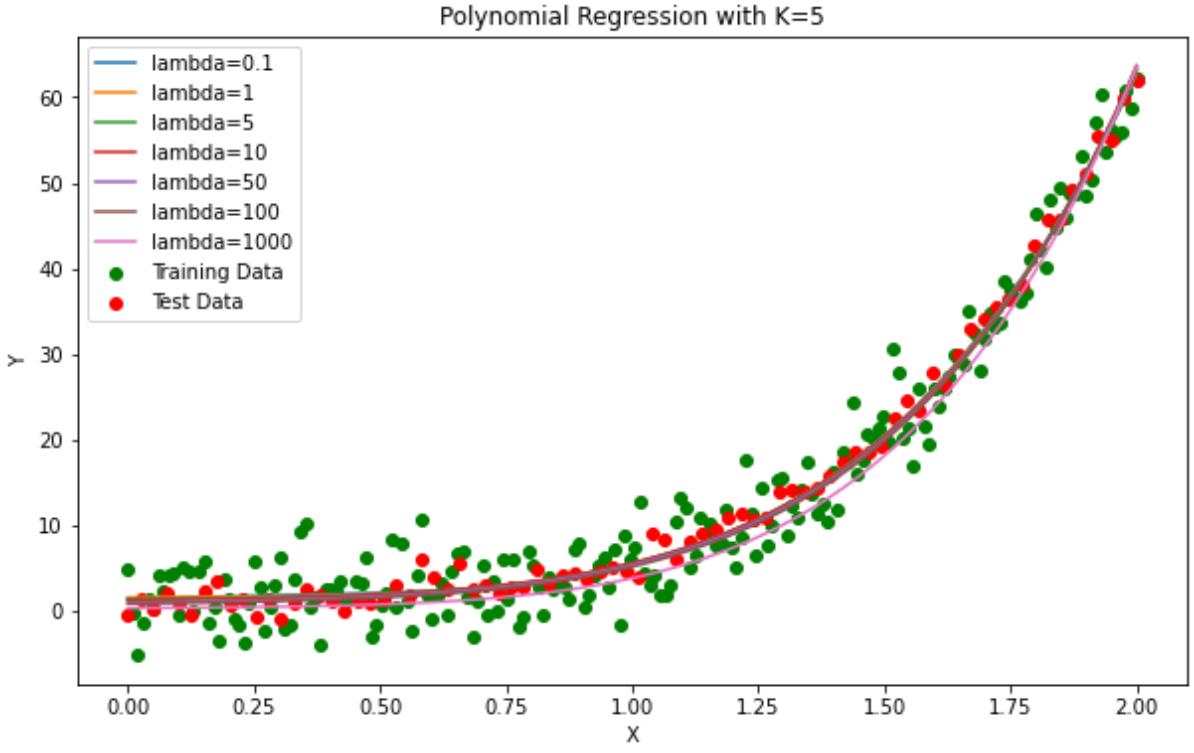

Training and Test Error vs Polynomial Degree
Training Error
Test Error
Average Absolute Error
Degree of Polynomial (K)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
2
4
6
8
10
12
14

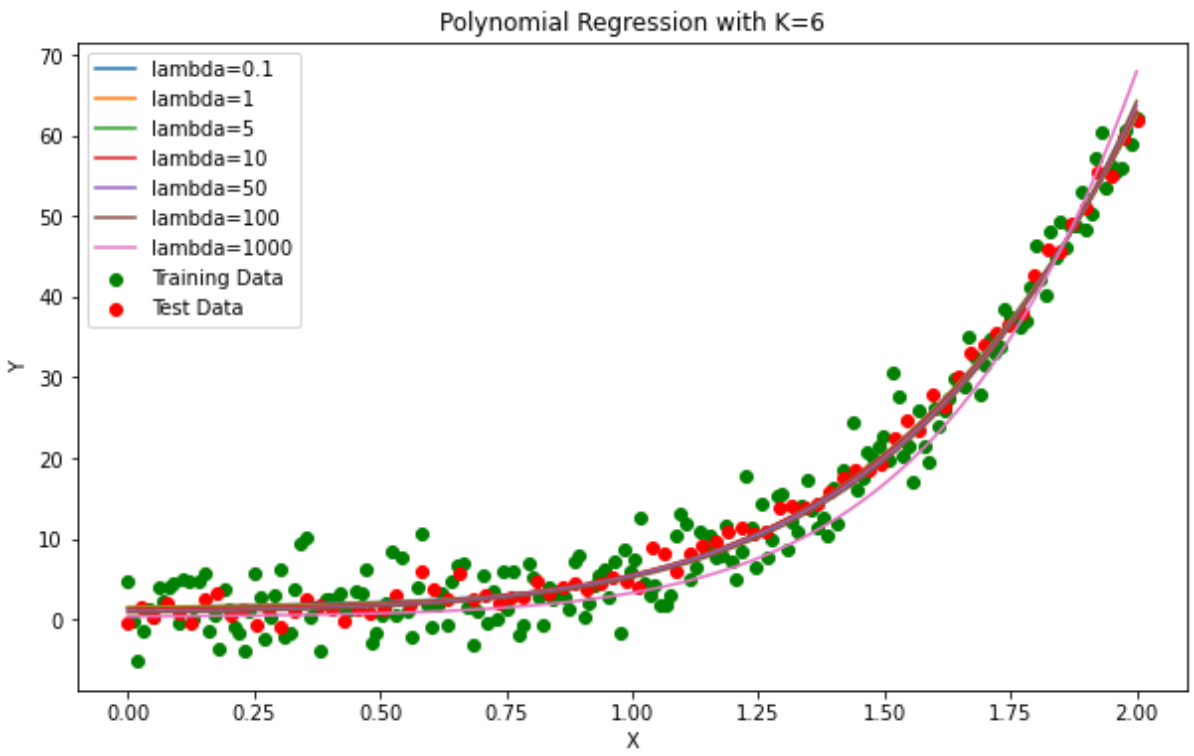

Polynomial Regression with K=1
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
60
50
40
30
20
10
0
−10
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

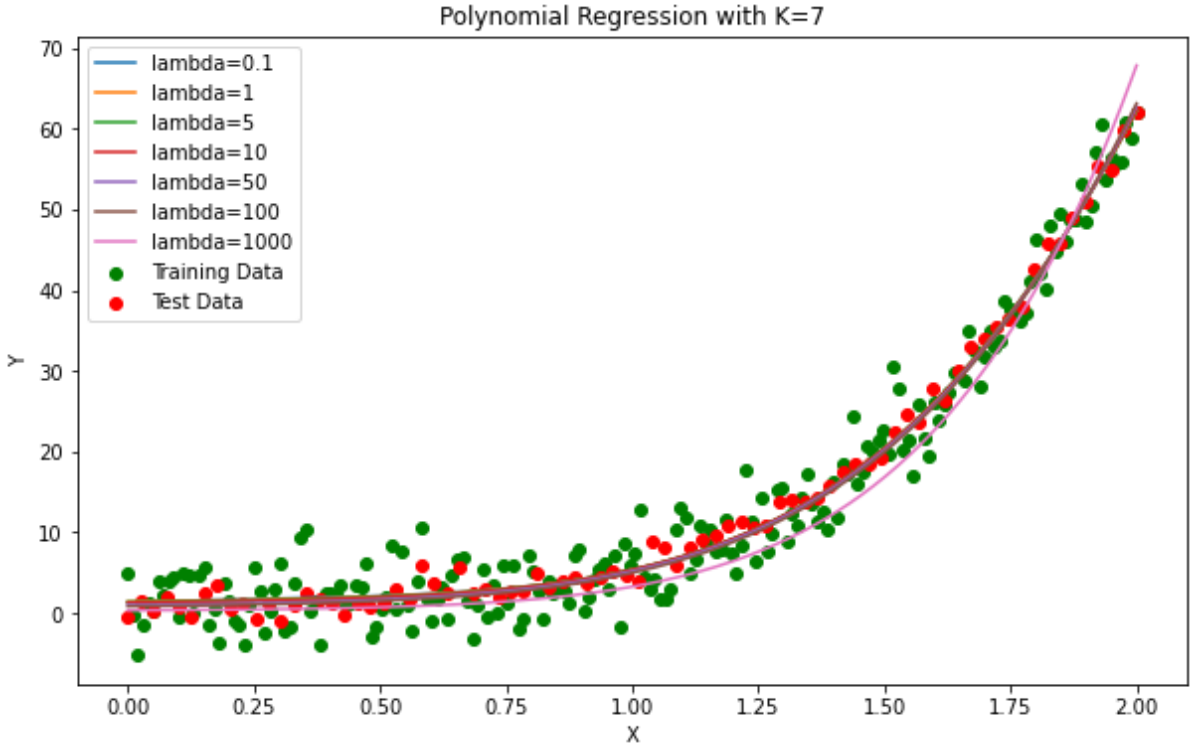

Polynomial Regression with K=2
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

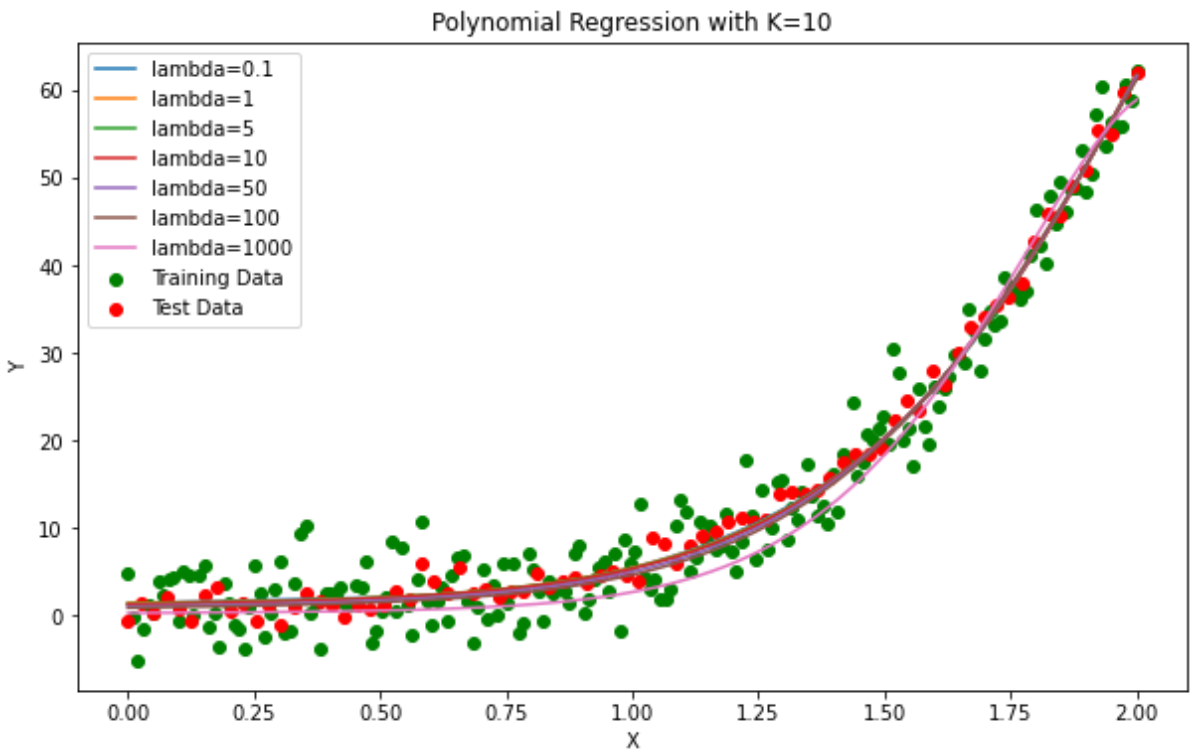

Polynomial Regression with K=3
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

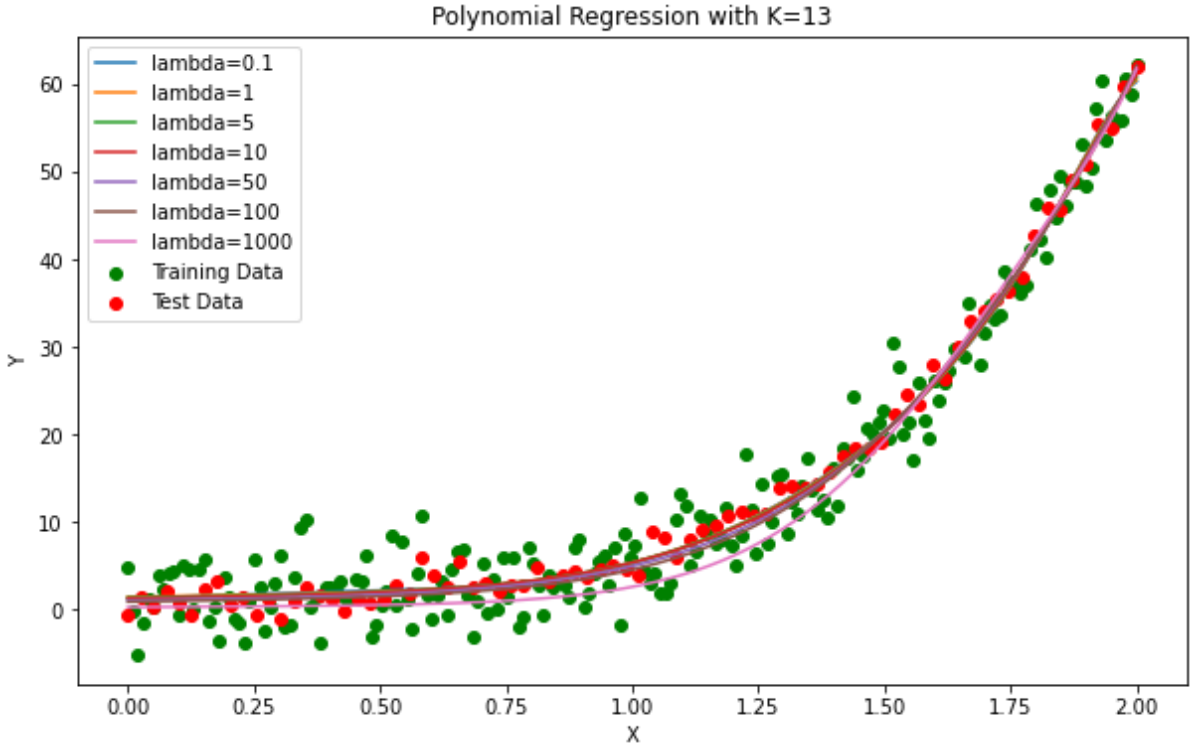

Polynomial Regression with K=4
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

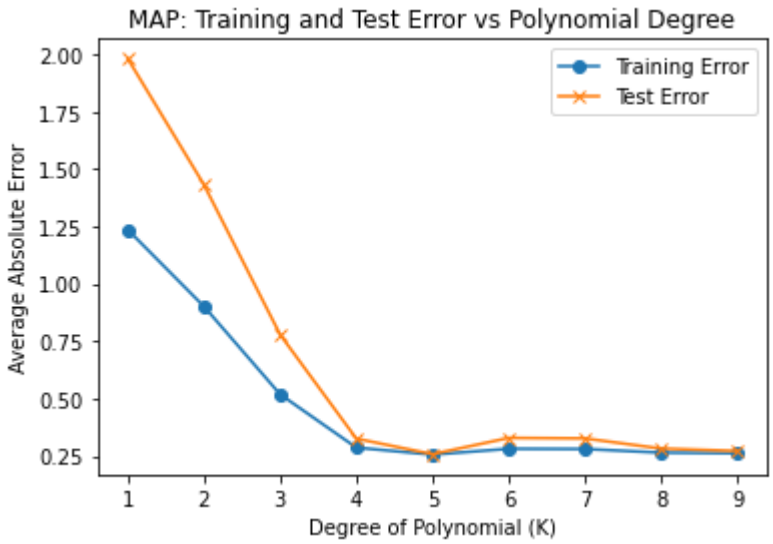

Polynomial Regression with K=5
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

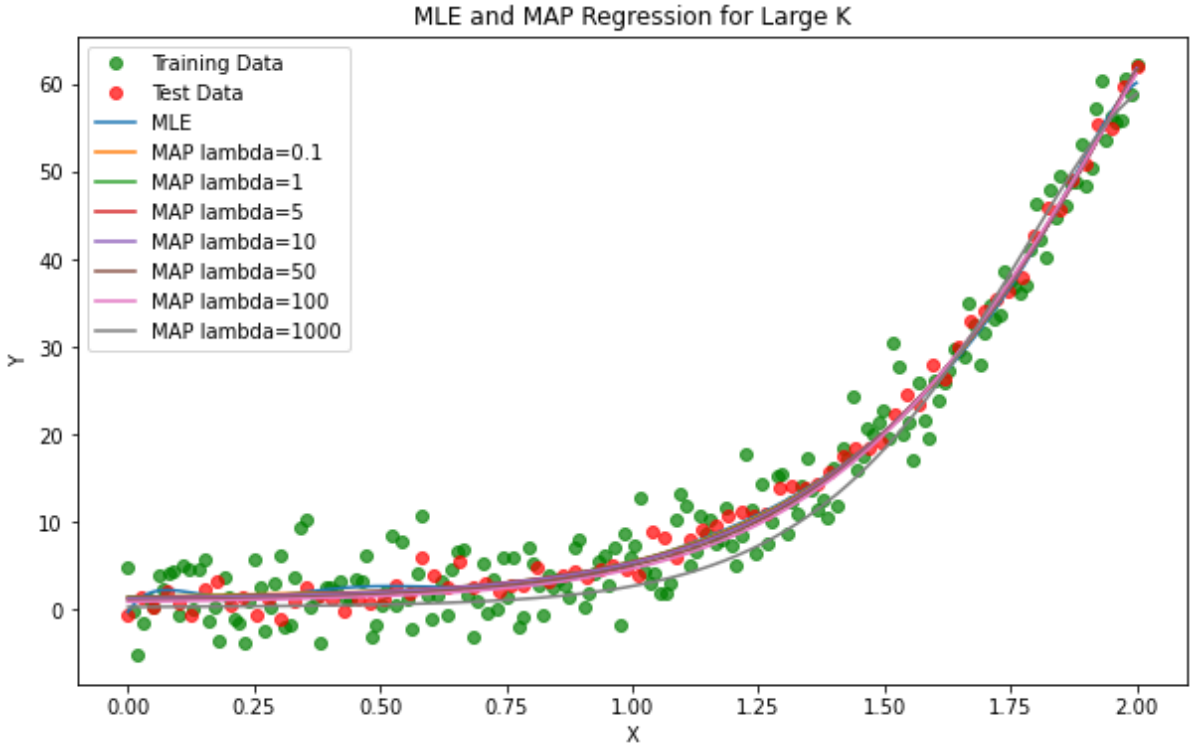

Polynomial Regression with K=6
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
70
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

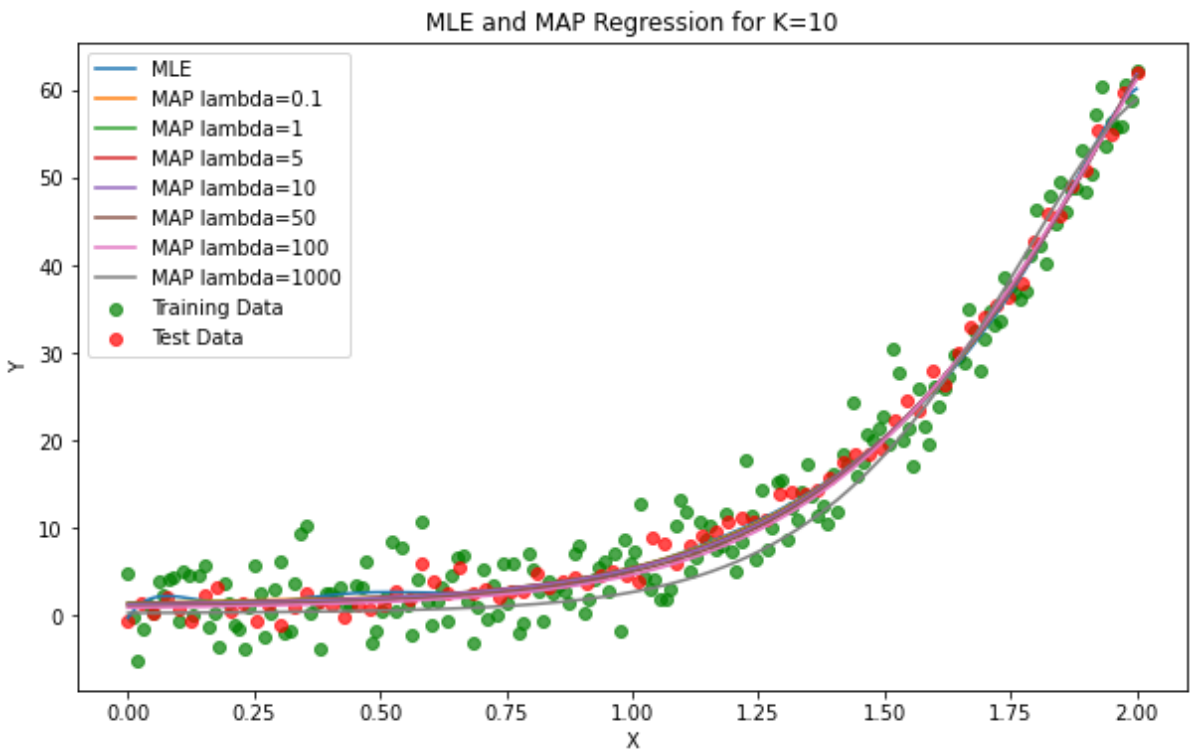

Polynomial Regression with K=7
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
70
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

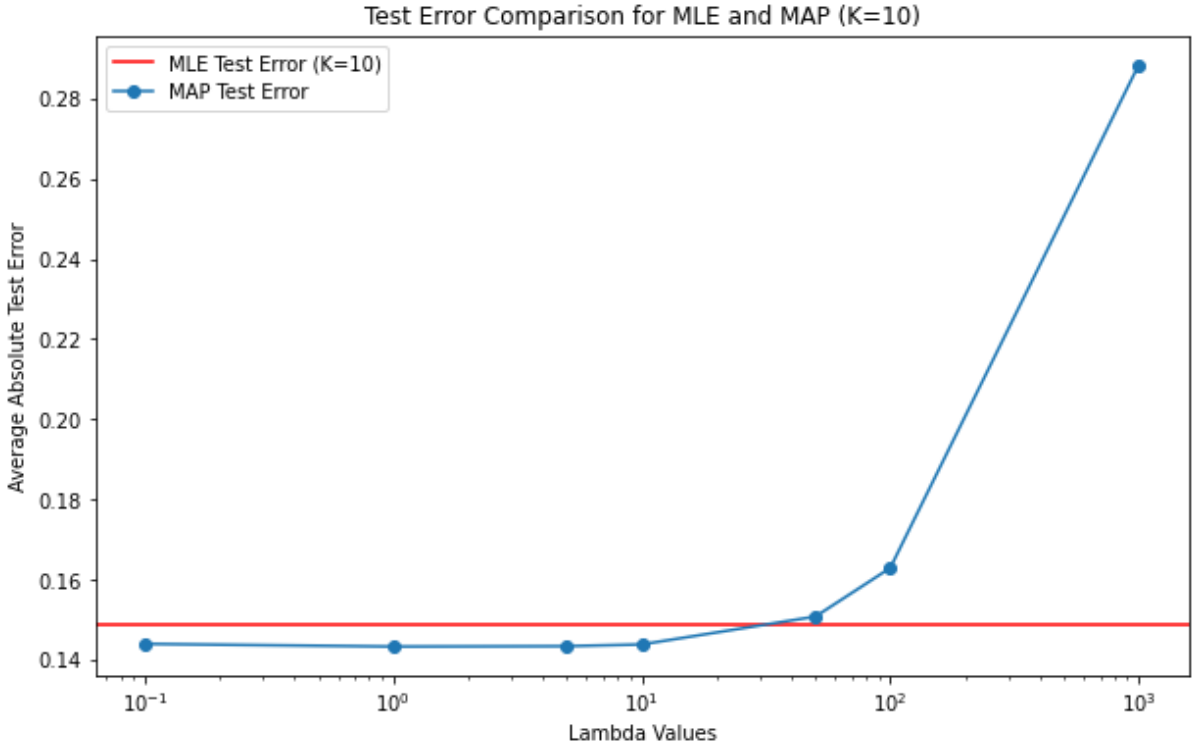

Polynomial Regression with K=10
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


Polynomial Regression with K=13
lambda=0.1
lambda=1
lambda=5
lambda=10
lambda=50
lambda=100
lambda=1000
Training Data
Test Data
Y
X
0
10
20
30
40
50
60
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


MAP: Training and Test Error vs Polynomial Degree
Training Error
Test Error
Average Absolute Error
Degree of Polynomial (K)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
1
2
3
4
5
6
7
8
9


MLE and MAP Regression for Large K
Training Data
Test Data
MLE
MAP lambda=0.1
MAP lambda=1
MAP lambda=5
MAP lambda=10
MAP lambda=50
MAP lambda=100
MAP lambda=1000
Y
X
60
50
40
30
20
10
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

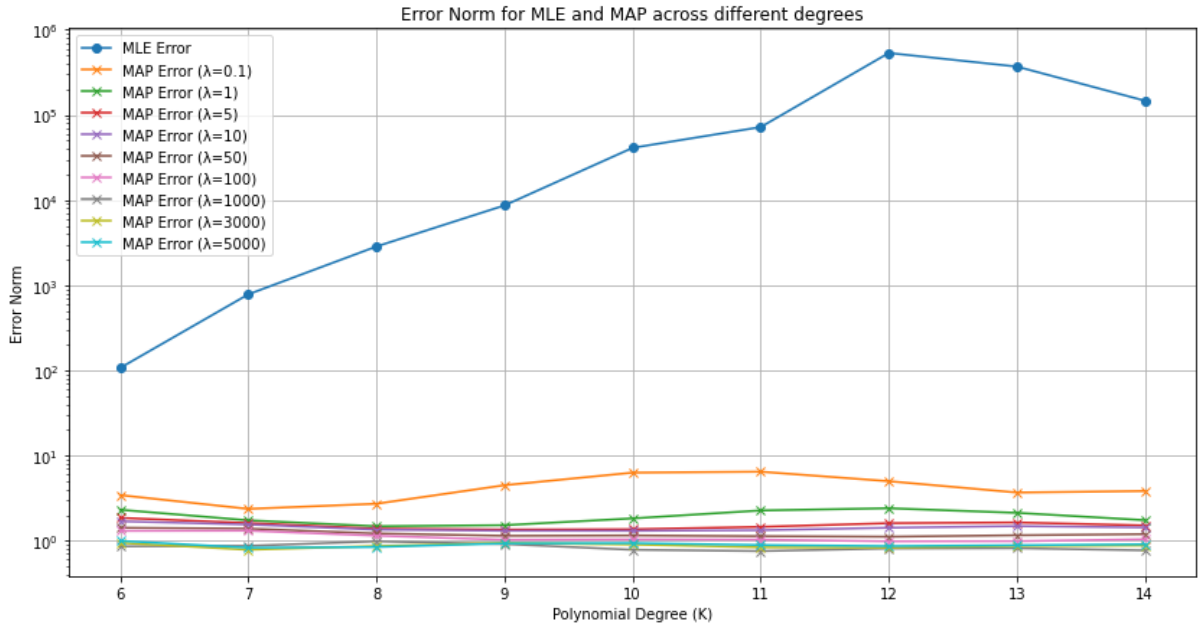




MLE Test Error (K=10): 0.14903818890970505
MAP Test Errors for different lambda values: [0.1440406279018708, 0.14337969424150437, 0.14346957108718705, 0.14390139851734787, 0.15084703927024282, 0.16296529610438992, 0.2883703543044513]

MLE Errors for increasing K:
[107.01969789758388, 781.8043423224105, 2850.5554206500224, 8660.113231883792, 41066.14861143392, 72014.5113719006, 533970.7035754803, 369642.0082560581, 146793.52767459187]

```
MAP Errors for K=6 with different lambda values:
[3.433641517075752, 2.3111736183929783, 1.857053043689216, 1.6919878187668567,
1.4209380688502775, 1.3053578773421988, 0.8602328151102215, 0.9457237256691506,
0.9965068597931639]
MAP Errors for K=7 with different lambda values:
[2.3681156158395633, 1.7378334387227723, 1.6147859617016413, 1.5516716131074269,
1.3874994515655945, 1.311542299282677, 0.8712753694934489, 0.7837936017650216,
0.8287929973095957]
MAP Errors for K=8 with different lambda values:
[2.7187334309120645, 1.479993455343047, 1.3855900602743307, 1.3505877745110082,
1.2194845533607934, 1.1463710671099205, 0.9843753742279644, 0.8667275373206963,
0.8444163866061989]
MAP Errors for K=9 with different lambda values:
[4.4881692298996025, 1.523246112813558, 1.3532147637832723, 1.3103873349049482,
1.1430216229254941, 1.0257890426730327, 0.9147710372923494, 0.9427762222567934,
0.930648538124112]
MAP Errors for K=10 with different lambda values:
[6.305161540010761, 1.8294897208009682, 1.3639767398166587, 1.3093602723721196,
1.1531911516659068, 1.0355581929594697, 0.7834159365874133, 0.9023917793585048,
0.9376309414231712]
MAP Errors for K=11 with different lambda values:
[6.470279148503957, 2.2678646805733043, 1.4562126322976088, 1.3298340429181976,
1.13159970261865, 1.0310080781739555, 0.7562095170960388, 0.8380271114961688,
0.8913258313942498]
MAP Errors for K=12 with different lambda values:
[5.003015627271819, 2.4080032423032818, 1.6079497285354012, 1.4217523719659613,
1.1155999952708624, 0.9841981523113704, 0.810944035680662, 0.8316229504315852,
0.8667517546042512]
MAP Errors for K=13 with different lambda values:
[3.683503442098944, 2.1271654937738784, 1.640360226917092, 1.4862524965123918,
1.159911781275386, 0.9887824181876556, 0.8178477995279222, 0.8684690842792635,
0.885671880738728]
MAP Errors for K=14 with different lambda values:
[3.8361290739664855, 1.745127778285735, 1.5132180638614146, 1.4332543400843634,
1.1954288536397808, 1.041575801121263, 0.7715208399128174, 0.8798481926856347,
0.9090598224225402]
```



N=100, MLE Error: 137.08649422876798
N=200, MLE Error: 189.370997585205
N=500, MLE Error: 275.4255233762785
N=1000, MLE Error: 8.727945648137085
N=3000, MLE Error: 34.4465518161456
N=5000, MLE Error: 26.22559870594238
N=10000, MLE Error: 8.247491401423565
N=200000, MLE Error: 2.8207955561485183
MAP Errors for lambda=0.1: [14.2847324987021, 9.43823176421249, 14.529363865925207, 1.4272214109912102, 7.089406024395362, 9.388590006795036, 5.192351508062064, 2.6503458567627365]
MAP Errors for lambda=1: [4.83619171418296, 2.4201143818080824, 3.273728278151219, 1.2855174312050945, 1.1911074065341989, 3.767161791198318, 3.362240271431573, 1.6730846450655015]
MAP Errors for lambda=5: [2.168697769871992, 1.6208338162431077, 1.4256563132544227, 1.1216889207572598, 0.7535077502603836, 1.8943399119523037, 2.266620968269517, 0.5445006615084735]
MAP Errors for lambda=10: [1.5350293291840642, 1.4895135010916043, 1.024786864066331, 0.9954700608609433, 0.7005018935826788, 1.2554240958824343, 1.7395726643010514, 0.3554827174855888]
MAP Errors for lambda=50: [0.896303773117829, 1.0510009549488801, 0.5599880360855753, 0.6962039494481724, 0.608924979849243, 0.5834704988553174, 1.022819790970546, 0.4383726782499045]
MAP Errors for lambda=100: [0.8060909251331317, 0.8670925485849535, 0.5169808068975359, 0.6311220280102863, 0.5815783981270675, 0.5090686322749085, 0.869450592789142, 0.4521451610778539]
MAP Errors for lambda=1000: [0.7612123721957391, 0.7280307485693347, 0.5688944373769138, 0.6281050343133635, 0.5555388790451259, 0.5010991315978189, 0.5486097209719575, 0.44903674264991306]
MAP Errors for lambda=3000: [0.7624030670648959, 0.7476083010847256, 0.6589217772715049, 0.6802870200167446, 0.5941772966603271, 0.5307083366303084, 0.5106662256725295, 0.45204845238417435]
MAP Errors for lambda=5000: [0.7685255508681521, 0.756205942312604, 0.6927431986249202, 0.7036718907098386, 0.6242559644033853, 0.555273942520726, 0.5125342920023227, 0.45660555159988914]