HomeWork 1_2

```
My machine epsilon eps is approximately: 2.220446049250313e-16
Numpy machine epsilon eps is: 2.220446049250313e-16
2^-52 in scientific notation: 2.2e-16

Calculated Epsilon: 2.2204460492503130808472633361816406250000000000000e-16
Machine Epsilon:     2.2204460492503130808472633361816406250000000000000e-16
The error is: 0.0000000000000000000000000000000000000000000000000e+00
```

```
n = 10: an = 2.5937424601000023, Error = 0.12453936835904278
n = 100: an = 2.7048138294215285, Error = 0.01346799903751661
n = 1000: an = 2.7169239322355936, Error = 0.0013578962234515046
n = 10000: an = 2.7181459268249255, Error = 0.000135901634119584
n = 100000: an = 2.7182682371922975, Error = 1.359126674760347e-05
[                   1                  10                 104
                 1072               10974              112332
              1149756            11768119           120450354
           1232846739         12618568830        129154966501
        1321941148466      13530477745798     138488637139387
     1417474162926807   14508287784959432  148496826225446944
  1519911082952939264 -9223372036854775808 -9223372036854775808
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[2.         2.59374246 2.70532731 2.71701506 2.71815799 2.71826973
 2.71828065 2.71828171 2.71828179 2.71828219 2.71828511 2.71830238
 2.71843506 2.71951912 2.75873325 2.57082657 1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.        ]
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[7.18281828e-01 1.24539368e-01 1.29545216e-02 1.26677215e-03
 1.23840661e-04 1.20992518e-05 1.18226919e-06 1.13953295e-07
 3.40240249e-08 3.64556589e-07 3.28563973e-06 2.05546877e-05
 1.53232381e-04 1.23729539e-03 4.04514182e-02 1.47455254e-01
 1.71828183e+00 1.71828183e+00 1.71828183e+00 1.71828183e+00
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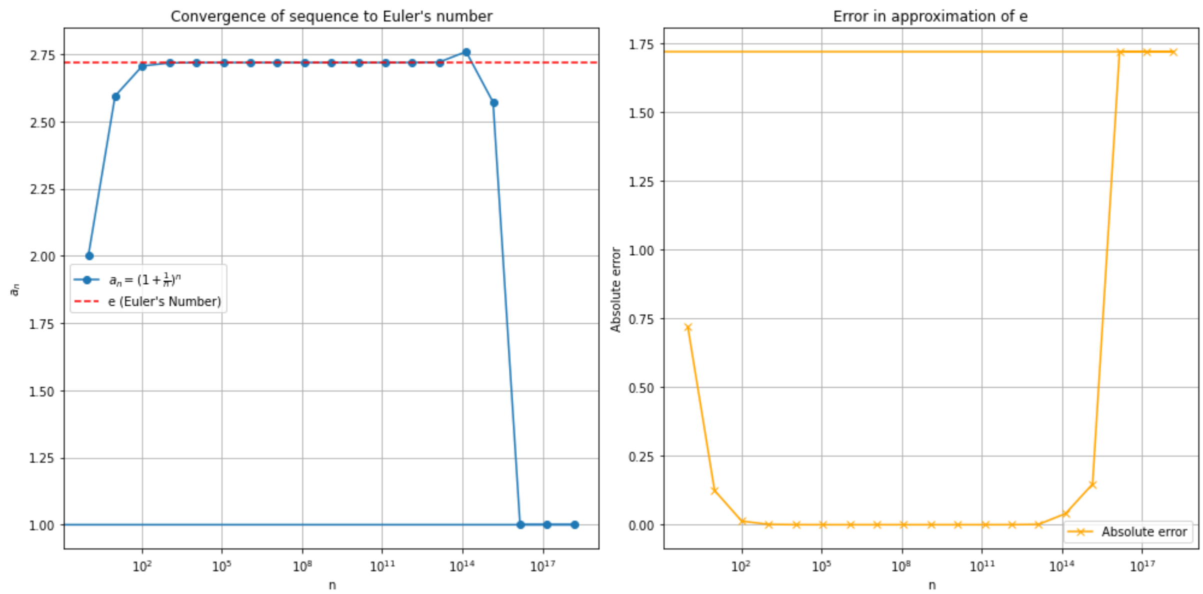

Analysis of the Sequence Convergence to Euler's Number

The sequence a_n = 1 + 1/n is known to converge to Euler's number e as n approaches infinity. We conducted a numerical analysis to observe this convergence and the behavior of the sequence for large values of n.

Results

- For small to moderately large values of n, the sequence values a_n approach e, as expected theoretically.
- As n becomes very large, the sequence values initially continue to approximate e more closely.
- However, beyond a certain point, due to the limitations of floating-point arithmetic, the values of a_n deviate from e and converge to 1 instead. This is because the term 1/n becomes too small to be represented accurately in floating-point, effectively turning the expression into 1^n that is equal to 1 for every n.
- The absolute error |a_n - e| decreases exponentially with increasing n until it reaches the floating-point precision limit. Once this limit is reached, the error no longer decreases but instead increases significantly, stabilizing around the value e - 1.

Conclusion

The observed behavior underscores the impact of floating-point precision on numerical computations. It also illustrates the practical limitations when approaching theoretical limits in computational mathematics. The error's increase for extremely large n values is a manifestation of the precision threshold inherent to the machine representation of real numbers

Rank of matrix A: 2
Eigenvalues of matrix A: [5. 2.]
Rank of matrix B: 1
Eigenvalues of matrix B: [5. 0.]

```
For a square matrix, being full rank implies that all its eigenvalues are non-zero,
as a non-zero determinant (which is the product of its eigenvalues) indicates that
the matrix is invertible and thus full rank. However, the presence of non-zero
eigenvalues alone does not guarantee that the matrix is full rank. The rank of a
matrix is fundamentally determined by the linear independence of its rows or
columns. In other words, while non-zero eigenvalues are necessary for a square
matrix to be full rank, they are not sufficient on their own to establish this
condition.
```

.0]

	Matrix	Shape	Eigenvalues	Rank	Is full Rank
Diagonal full rank matrix	[[1, 0, 0], [0, 2, 0], [0, 0, 3]]	(3, 3)	[1.0, 2.0, 3.0]	3	True
Diagonal not full rank matrix	[[1, 0, 0], [0, 0, 0], [0, 0, 1]]	(3, 3)	[1.0, 0.0, 1.0]	2	False
Symmetric full rank matrix	[[2, -1], [-1, 2]]	(2, 2)	[3.0, 1.0]	2	True
Rows Linear dependent	[[1, 2], [2, 4]]	(2, 2)	[0.0, 5.0]	1	False
Random full rank	[[0.29226352204833683, 0.06745841938386976, 0....	(5, 5)	[(2.5238839892056837+0j), (-0.1548711031368265...	5	True
Not full rank	[[1, 2, 3], [4, 5, 6], [7, 8, 9]]	(3, 3)	[16.116843969807043, -1.1168439698070427, -1.3...	2	False