Demonstration of the solution of the system of mass and energy balances for MS Birka Main Engines

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Abstract

1 Initial system

1.1 Mass balances

Mass balance on the split after the compressor, with one flow going to the engine inlet valves and the other flow going to the bypass valve

$$\dot{m}_{air,comp} = \dot{m}_{air,cyl} + \dot{m}_{air,bypass} \tag{1}$$

Mass balance on the mixer before the turbine, with one flow coming from the engine exhaust valves and the other flow coming from the bypass valve

$$\dot{m}_{eg,turb} = \dot{m}_{eg,cyl} + \dot{m}_{air,bypass} \tag{2}$$

Note that the exhaust gas mass flow from the cylinders is fully defined as follows:

$$\dot{m}_{eg,cyl} = \dot{m}_{air,cyl} + \dot{m}_{fuel,cyl} \tag{3}$$

1.2 Energy balances

Energy balance on the turbocharger: the power generated by the turbine must be equal to the power absorbed by the compressor

$$\dot{m}_{air,comp}\Delta h_{comp} = \dot{m}_{eg,turb}c_{p,eg}(T_{turb,in} - T_{turb,out})\eta_{mech} \tag{4}$$

Energy balance on the mixer between the air flow after the bypass valve and the exhaust gas flow after the exhaust valve

$$\dot{m}_{air,bypass}c_{p,air}(T_{comp,out}-T_0)+\dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out}-T_0) = \dot{m}_{eg,turb}c_{p,eg}(T_{turb,in}-T_0) \tag{5}$$

The system of four equations is to be solved in the four following unknowns:

- $\dot{m}_{eg,turb}$
- $\dot{m}_{air,bypass}$
- $\dot{m}_{air,comp}$
- $T_{turb,in}$ (In the case of the Main engines)
- $T_{cyl,out}$ (In the case of the Auxiliary engines)

2 Calculating the explicit system

To calculate what we need, we must make the system explicit on the variables we need to calculate. This process is different for the main and auxiliary engines, mostly because the temperature measurement of the exhaust gas before the turbine is positioned differently

2.1 Main Engines

In the case of the main engines, the temperature at the **cylinder outlet** is measured and, hence, known.

First, we make all equations explicit in $\dot{m}_{air,bypass}$ thus eliminating both $\dot{m}_{eg,turb}$ and $\dot{m}_{air,comp}$

$$(\dot{m}_{air,cyl}+\dot{m}_{air,bypass})\Delta h_{comp} = (\dot{m}_{eg,cyl}+\dot{m}_{air,bypass})c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}$$

$$\dot{m}_{air,bypass}c_{p,air}(T_{comp,out}-T_0)+\dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out}-T_0) = (\dot{m}_{eg,cyl}+\dot{m}_{air,bypass})c_{p,eg}(T_{turb,in}-T_0)$$

At this point, we use the first equation to simplify the system and make it explicit in the $\dot{m}_{air,bypass}$ variable alone:

$$\begin{aligned}
(\dot{m}_{eg,cyl} + \dot{m}_{air,bypass})c_{p,eg}(T_{turb,in} - T_{turb,out})\eta_{mech} &= (\dot{m}_{air,cyl} + \dot{m}_{air,bypass})\Delta h_{comp} \\
T_{turb,in} - T_{turb,out} &= \frac{\dot{m}_{air,cyl} + \dot{m}_{air,bypass}}{\dot{m}_{eg,cyl} + \dot{m}_{air,bypass}} \frac{\Delta h_{comp}}{c_{p,eg}\eta_{mech}} \\
T_{turb,in} &= T_{turb,out} + \frac{\dot{m}_{air,cyl} + \dot{m}_{air,bypass}}{\dot{m}_{eg,cyl} + \dot{m}_{air,bypass}} \frac{\Delta h_{comp}}{c_{p,eg}\eta_{mech}}
\end{aligned}$$

We can now substitute this expression in the second equation:

$$\begin{gathered}
\dot{m}_{air,bypass}c_{p,air}(T_{comp,out} - T_0) + \dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out} - T_0) = \\
(\dot{m}_{eg,cyl} + \dot{m}_{air,bypass})c_{p,eg}(T_{turb,out} + \frac{\dot{m}_{air,cyl} + \dot{m}_{air,bypass}}{\dot{m}_{eg,cyl} + \dot{m}_{air,bypass}} \frac{\Delta h_{comp}}{c_{p,eg}\eta_{mech}} - T_0)
\end{gathered} \tag{6}$$

We now have to simplify this equation in order to make the term $\dot{m}_{air,bypass}$ explicit.

We first execute the multiplication in the term on the right of the equal:

$$\begin{aligned}\dot{m}_{air,bypass}c_{p,air}(T_{comp,out} - T_0) + \dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out} - T_0) = \\ \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,out} - T_0) + \\ \dot{m}_{air,bypass}c_{p,eg}(T_{turb,out} - T_0) + \\ \frac{\Delta h_{comp}}{c_{p,eg}\eta_{mech}}(\dot{m}_{air,cyl} + \dot{m}_{air,bypass})c_{p,eg}\end{aligned} \tag{7}$$

Now, we can group together the elements in the variable of interest:

$$\begin{aligned}\dot{m}_{air,bypass}\left[c_{p,air}(T_{comp,out} - T_0) - c_{p,eg}(T_{turb,out} - T_0) - \frac{\Delta h_{comp}}{\eta_{mech}}\right] = \\ \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,out} - T_0) - \dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out} - T_0) + \frac{\Delta h_{comp}}{\eta_{mech}}\dot{m}_{air,cyl}\end{aligned} \tag{8}$$

This can be further simplified into:

$$\begin{aligned}\dot{m}_{air,bypass}\left[c_{p,air}(T_{comp,out} - T_0) - c_{p,eg}(T_{turb,out} - T_0) - \frac{\Delta h_{comp}}{\eta_{mech}}\right] = \\ \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,out} - T_{cyl,out}) + \frac{\Delta h_{comp}}{\eta_{mech}}\dot{m}_{air,cyl}\end{aligned} \tag{9}$$

Finally, we can assume the specific heat at constant pressure of air and exhaust gas to be sufficiently similar for considering them equal in the term at the numerator (we ignored changes in the properties of the mixture in other points as well):

$$\begin{aligned}\dot{m}_{air,bypass}\left[c_{p,eg}(T_{comp,out} - T_{turb,out}) - \frac{\Delta h_{comp}}{\eta_{mech}}\right] = \\ \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,out} - T_{cyl,out}) + \frac{\Delta h_{comp}}{\eta_{mech}}\dot{m}_{air,cyl}\end{aligned} \tag{10}$$

From which we can derive the final form:

$$\dot{m}_{air,bypass} = \frac{\dot{m}_{eg,cyl}c_{p,eg}(T_{turb,out} - T_{cyl,out}) + \frac{\Delta h_{comp}}{\eta_{mech}}\dot{m}_{air,cyl}}{c_{p,eg}(T_{comp,out} - T_{turb,out}) - \frac{\Delta h_{comp}}{\eta_{mech}}} \tag{11}$$

Which can be rewritten, in order to have both numerator and denominator positive, as follows:

$$\dot{m}_{air,bypass} = \frac{\dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out} - T_{turb,out}) - \frac{\Delta h_{comp}}{\eta_{mech}}\dot{m}_{air,cyl}}{c_{p,eg}(T_{turb,out} - T_{comp,out}) - \frac{\Delta h_{comp}}{\eta_{mech}}} \tag{12}$$

2.2 Auxiliary Engines

In the case of the auxiliary engines, what we know instead is the temperature of the exhaust gas **before the turbine**, hence after the merging between the exhaust gas and the bypass.

In this case, however, things appear to be simple: the energy balance on the turbocharger can be calculated as there is only one variable unknown: the total mass flow in the compressor, and hence the bypass flow:

$$(\dot{m}_{air,cyl}+\dot{m}_{air,bypass})\Delta h_{comp} = (\dot{m}_{eg,cyl}+\dot{m}_{air,bypass})c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}$$

$$\dot{m}_{air,cyl}\Delta h_{comp}+\dot{m}_{air,bypass}\Delta h_{comp} = \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,in}-T_{turb,out})+\dot{m}_{air,bypass}c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}$$

$$\dot{m}_{air,bypass}(\Delta h_{comp}-c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}) = \dot{m}_{eg,cyl}c_{p,eg}(T_{turb,in}-T_{turb,out})-\dot{m}_{air,cyl}\Delta h_{comp}$$

$$\dot{m}_{air,bypass} = \frac{\dot{m}_{eg,cyl}c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}-\dot{m}_{air,cyl}\Delta h_{comp}}{\Delta h_{comp}-c_{p,eg}(T_{turb,in}-T_{turb,out})\eta_{mech}} \quad (13)$$

Now that we have the value for the bypass flow, it is easy to calculate backwards the temperature of the exhaust gas after the cylinder outlet valves, before the merging with the bypass flow:

$$\dot{m}_{air,bypass}c_{p,air}(T_{comp,out}-T_0)+\dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out}-T_0) = \dot{m}_{eg,turb}c_{p,eg}(T_{turb,in}-T_0)$$

$$\dot{m}_{eg,cyl}c_{p,eg}(T_{cyl,out}-T_0) = \dot{m}_{eg,turb}c_{p,eg}(T_{turb,in}-T_0)-\dot{m}_{air,bypass}c_{p,air}(T_{comp,out}-T_0)$$

$$T_{cyl,out} = T_0 + \frac{\dot{m}_{eg,turb}c_{p,eg}(T_{turb,in}-T_0)-\dot{m}_{air,bypass}c_{p,air}(T_{comp,out}-T_0)}{\dot{m}_{eg,cyl}c_{p,eg}} \quad (14)$$