Sved IBD recursion correction

$\Rightarrow$ L equilibrium is:

$$L_{eq} = 1/[1+(2N_e-1)(2c-c^2)] \approx 1/[1+4N_e c]$$

Sved (1971) states: $\mathbb{E}(r^2) \approx L$

↳ Argument: if genes are IBD then their correlation = 1

" " aren't IBD " " = 0

Sketch proof:

gene 1 $\sim$ Multinom (P_1, P_2, P_3, P_4)

if gene 2 == gene 1 (i.e. IBD) then $\mathrm{cor}(g^1, g_2) = 1$

else: gene 2 $\sim$ Multinom (P_1, P_2, P_3, P_4) indep. of gene 1

$\Rightarrow$ as r = correlation ?

~~You can extend to 2 loci:~~

~~$\mathbb{P}($~~

Island model

$L_W = \mathbb{P}($ 2 gametes in same island are IBD $)$

$L_B = \mathbb{P}($ " " different " " $)$

RmK : sample with replacement assumed

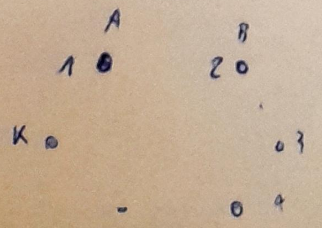
$$L'_W = \underbrace{\frac{1}{2Ne}}_{\substack{\text{sample} \\ \text{the same} \\ \text{gamete twice}}} + \underbrace{(1-c)^2}_{\substack{\text{no rec.} \\ \text{happens} \\ \text{to both}}} \left(1 - \frac{1}{2Ne}\right) * \Bigg\{ \Bigg[\underbrace{(1-m)^2}_{\substack{\text{both sampled} \\ \text{from this} \\ \text{island}}} + \underbrace{\frac{m^2}{K-1}}_{\substack{\text{both} \\ \text{sampled} \\ \text{from the} \\ \text{same different} \\ \text{island}}} \Bigg] L_W$$

$$+ \Bigg[\underbrace{2m(1-m)}_{\substack{\text{1 from same} \\ \text{island, 1} \\ \text{from diff. isl.}}} + \underbrace{\frac{m^2(K-2)}{K-1}}_{\substack{\text{from 2 diff.} \\ \text{external island}}} \Bigg] L_B \Bigg\}$$

$$L'_B = \underbrace{(1-C)^2}_{\substack{\text{no rec.} \\ \text{happens to} \\ \text{both}}} \Bigg\{ \Bigg[\underbrace{\frac{2m(1-m)}{K-1}}_{\substack{\mathbb{P}(\text{A stayed fixed,} \\ \text{B moved } 1\to 2) \\ \text{or viceversa}}} + \underbrace{\frac{m^2}{K-1}}_{} \Bigg] L_W$$

$m^2 \to \mathbb{P}($ A, B were migrating $)$
$\mathbb{P}($ A, B migrated from the same population $)$

(*) should be $m^2 \dfrac{K-2}{(K-1)^2}$



$$+ \Bigg[\underbrace{(1-m)^2}_{\substack{\text{they both} \\ \text{didn't migrate}}} + \underbrace{2m(1-m)\frac{(K-2)}{K-1}}_{\substack{\text{B migrated, A didn't} \\ \text{B didn't migrate from pop 1}}} + \underbrace{\frac{m^2(K-2)}{K-1}}_{\substack{\text{should} \\ \text{be (**)}}} \Bigg] L_B \Bigg\}$$

A B

1 • 2 •

K

$\mathbb{P}(A = B \text{ last gen}) = \mathbb{P}(B \text{ moved } 1 \to 2, A \text{ stayed})$

$\mathbb{P}(A, B \text{ were in } \{3, \dots, K\} \text{ together}, A \text{ moved to } 1, B \text{ moved to } 2)$

$=$ • • $1 + \{3, \dots, K\}$ so $K-1$

$\{2, 3, \dots, K\}$

$$\frac{K-2}{K-1} \cdot \frac{K-2}{K-1} \cdot \frac{1}{K-2} = \frac{K-2}{(K-1)^2}$$

$\mathbb{P}($ ~~...~~ A, B migrated, were in different sub pop.$)$

A come from $\{2, 3, \dots, K\}$ B come from $\{1\} \cup \{3, \dots, K\}$

$$\Rightarrow (**) \quad \underbrace{\left(1 - \frac{K-2}{K-1} \cdot \frac{K-2}{K-1}\right)}_{\text{A come from 2 or B come from 1}} + \underbrace{\left(\frac{K-2}{K-1}\right)^2}_{\text{A, B come from } \{3, \dots, K\}} \cdot \underbrace{\frac{K-3}{K-2}}_{\text{given, they come from diff. populations}}$$