A Gravity Model for Emergency Departments

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ABSTRACT

The problem of facility location holds significance in numerous complex systems, when resources have to be utilize efficiently. Gravity models, inspired by Newtonian physics, are commonly employed to address these problems and boast a long tradition of being used in healthcare. This paper aims to enhance the comprehension of patients' decision-making process in emergency healthcare by introducing an extension to existing gravity models, including two novel factors influencing emergency department choice: hospital sizes and patients' severity. The newly formulated gravity rule, which integrates these factors, demonstrated remarkable precision against real-world data in terms of overall hospital location and flows between cities and hospitals.

Please note: Abbreviations should be introduced at the first mention in the main text – no abbreviations lists. Suggested structure of main text (not enforced) is provided below.

1 Introduction

The facility location problems are relevant across various complex systems due to their critical role in the efficient utilization of resources (1; 2; 3). Usually, these problems can be solved by determining the better locations for a set of facilities in a given solution space, in such a way that maximizes users' accessibility and minimizes operational costs, thereby ensuring effective resource allocation (4). This decision-making process is crucial in different sectors such as logistics (5), urban planning (6), healthcare (7; 8; 9), and, in general, in every context where the facility placement affects the transportation costs, the level of services, and the operational effectiveness (10; 11).

Various methodologies have been developed to address location problems, each offering specific advantages (12; 13). For example, linear programming, a mathematical technique, is widely employed for solving location problems due to its effectiveness in handling large-scale linear models (14; 15) since it provides a structured framework to optimize the location of facilities while adhering to specific constraints (16). However, it cannot be always employed when the problem structure does not permit the representation of real-world features with linear functions. Metaheuristic optimization presents a more flexible alternative (17). Unlike linear programming, metaheuristics, which include techniques like genetic algorithms (18), and simulated annealing (19), are not confined to linear constraints and can be efficiently employed when the solution spaces are complex (20), at a much higher computational cost, which is not always feasible. Also, qualitative methodologies such as Delphi methods have also been employed (3).

A noteworthy specific case of the facility location problem arises when one or more locations should serve a large number of users distributed in a given space (21). This scenario appears in supermarkets (22), delivery centers (23), and hospitals (24). To effectively undertake such challenges, a possible approach involves leveraging the assumption that the potential customers' likelihood of visiting a facility is inversely proportional to their distance from it. This concept mirrors the principles of Newtonian mechanics, particularly the law of universal attraction (25), by suggesting that the attraction to a facility diminishes as the distance increases (26). Consequently, the models that employ this metaphor are aptly termed 'gravity models' (27). While this principle reflects the gravitational pull in physics, it can include other features (28).

Different fields employ gravity models. In economics, they can interpret the trade flow between two nations using GDPs and inter-country distances (29; 30). In logistics, they can support companies and policymakers in locating new logistics hubs (31). In marketing, they can assist the inference of retail facility attractiveness from secondary data regarding customers' buying power and sales volumes (32). Gravity models are also utilized in healthcare (33), offering insights for the decision-making process and policy formulation (34), with interesting potentialities for hospitals' improvement and application in various healthcare services' settings (35; 36; 37; 38; 39).

This paper endeavours to enhance the current understanding of the patients' decision-making process in emergency healthcare scenarios, improving state-of-the-art gravity models (39; 40; 41) and potentially using data available to policymakers (42). The contribution to the field regards the way two new factors, in addition to distance, influence the choice of emergency care department (43): the hospital size and the patients' perceived severity. The model relies on the assumption that a larger hospital, presumably with more extensive facilities and resources, is more likely to be the preferred choice for patients (44; 45), and that this preference is amplified by the patients' perceived severity of their condition. Especially, the more critical the condition, the greater the likelihood of a patient opting for a larger hospital. To the best of our knowledge, this hypothesis has never been investigated before. By including hospital size and condition severity in a gravity model, this work aims to increase the knowledge of patient decision-making processes.

The novel gravity rule underwent a calibration and validation phase on real-world data to gain relevancy to policy-makers (46), which yielded results that surpassed current benchmarks in the field. The outcome of this empirical testing shows that the mean error between real-world data and the results simulated with the gravity model is approximately 1.23%.

2 Research objects and methods

This work proposes a systematic rule that explains decision-making in choosing an emergency care facility. So, this section presents the proposed gravity model and its calibration to a specific geographical area.

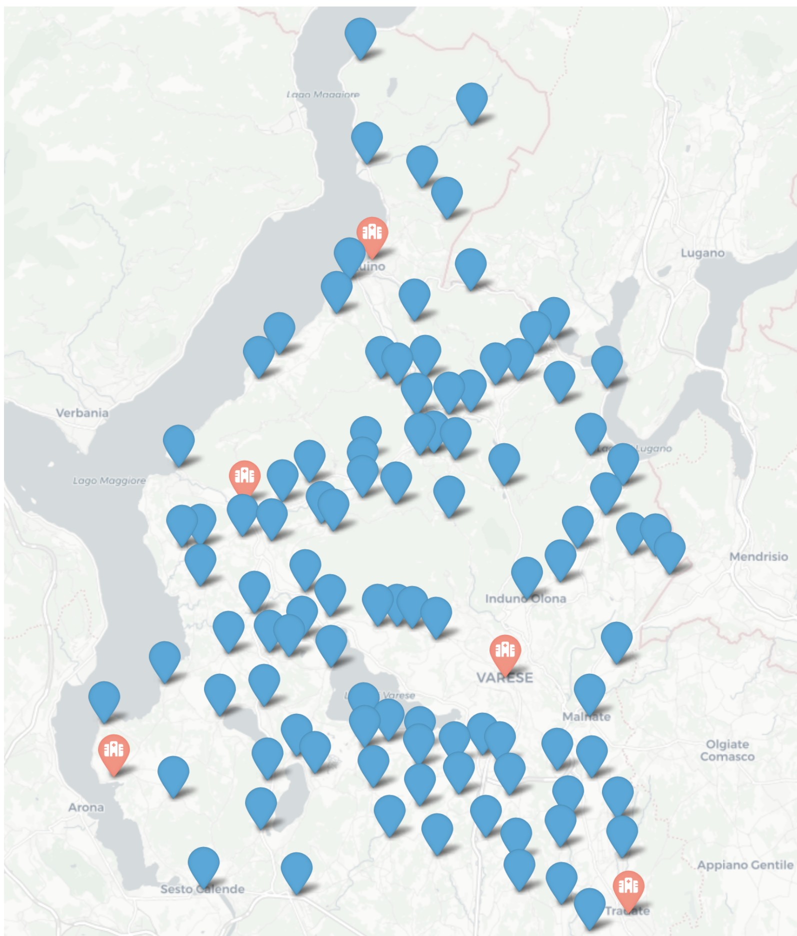

Figure 1. Geographical position of the towns in the considered area, divided for town where no emergency departments are present (red marker) or absent (blue marker)

This section is structured into two distinct subsections. The first part provides a formal description of our gravity meta-model for hospital selection, detailing the foundational modeling hypotheses, the variables incorporated into the model, and their interplay. This subsection is designed to be comprehensive, offering all necessary information for replicating this research in a different geographical context with varying data sets. The second part illustrates the application of this model in a specific context. It includes a thorough explanation of the methodologies and procedures employed in our experiments, ensuring scalability and generalizability of our findings. To facilitate this approach, the Python notebook utilized for data analysis, along with a representative subset of the data used, is publicly available upon reasonable request.

2.1 Gravity meta-model

In our research, we focus on developing a function that accurately models the patient likelihood of choosing a particular hospital, considering distance and various other determinants. Unlike existing models that often rely on general preference data, our approach is event-based specifically targets the final stage of the decision-making process (47). This stage captures the definitive choice a patient makes, either independently or through their transportation medium, in cases where the patient is not autonomous. Thus, the structure of the candidate class of functions is as follows:

$$p^*(H_i) = f(d_i, t, b_i) \tag{1}$$

where $p^*(H_i)$ represents the preference of an individual resident in the defined area for choosing hospital H_i among all the N hospitals available. The variables involved are the distance d_i from the patient to hospital i, the triage code t_k upon arrival, which represents the severity of patient k, and b_i, which is the average perceived size of hospital i, serving as a proxy for its emergency care department accessibility.

The distance d_i is computed using the public API from the Open Source Routing Machine using Python 3.9 on the 13th September, 2022. The metric was selected under the hypothesis that the preference towards a specific hospital is affected by the duration of the travel and not by the actual routing distance. Thus, the traveling time between every city in the area of analysis was collected, measured in seconds. The underlying assumption here is that, for an inhabitant of the area, the residence town is a sufficiently good approximation of the town in which they are located (48). The triage code t_k is a value assigned to each patient k at the entrance of an emergency care department. The code embodies the evaluation of patients' severity made by nurses when a patient arrives at the emergency department of reference. In this work, the code was used as a proxy for the seriousness of the condition of a patient at the moment they leave their residence. The Italian healthcare system categorizes the severity of a patient's condition using color codes, in ascending order of severity: white, green, yellow, and red, which are respectively encoded in a non-ordinal numerical variable whose value can be 1, 2, 3, and 4. A further code, the black one, also exists to include patients that are deceased upon arrival at the emergency department. These patients were not included in the analysis. This classification was later updated, but it was employed at the time of data collection.

The parameter b_i stands for the size perceived by a patient regarding a specific hospital H_i during the decision, given that data regarding individual preferences are not available. The rationale is that a hospital with more beds could be considered to have a "higher" quality than a hospital with fewer beds. We use the term "perceived" because patients, in a situation of bounded rationality, are not supposed to know the exact services provided by each hospital; even so, they do not have the time and the resources to process this information. Also, even if a survey were conducted, the results could be affected by two elements. First, the answer would not be given in a moment of stress, such as the one in which a patient is choosing which emergency care department to direct to. Second, it would not include the decision-making of rescue vehicle drivers, which should be obtained by means of a second survey. Since the goal of this work is to find a unique and simple rule that can simulate the distribution of individuals in different areas, we used as a proxy of the perceived accessibility parameter the number of beds in the emergency care department of the hospital H_i. This metric takes into consideration both the perception of private individuals and rescue vehicle drivers, since all of them are affected in their location selection process by the size of the hospital. Moreover, it has also high adherence to reality, since the dimension of the hospital is likely connected to the number of sanitary services that it is able to offer, representing a direct measure of the accessibility to the healthcare system.

Given these hypotheses and assumptions, the specific preference function proposed in this work is the following.

$$p^*(H_i) = \frac{b_i^{\alpha t}}{d_i^{\beta}} \tag{2}$$

The preference function combines b_i, t, and d_i by raising the perceived accessibility to the power of t and dividing it by the distance. α and β are two parameters that regulate the effect of each factor. This equation implies that as the perceived accessibility increases or the severity of the patient's condition intensifies (higher b_i and t values), the preference for a particular hospital ($p^*(H_i)$) also increases. Conversely, there is an inverse relationship between the distance from a hospital and the preference toward it, so that the longer it takes to get to the hospital, the less likely it is for a patient to direct there. The strength of this relationship is regulated by the parameter α.

From all the N preferences $p^*(H_i)$, a probability for a patient to head to the H_i structure can be derived as

$$p(H_i) = \frac{p^(H_i)}{\sum_{i=1}^{N} p^(H_i)} \tag{3}$$

Maintaining the metaphor of gravity, the probability for a patient to go to the hospital H_i in a given condition is proportional to the attractiveness of H_i

2.2 Model specification

This section describes how the meta-model presented in the previous paragraph was adapted to a specific area. The selected area for this study encompasses the region served by the hospitals associated with the Sette Laghi Territorial Social and Healthcare Organizations (ASST), a healthcare facility located in the northern part of Lombardy, Italy. Specifically, this study focuses on six hospital facilities, each corresponding to an emergency department included in the analysis. These emergency departments are integral components of the healthcare infrastructure within the ASST Sette Laghi , providing critical emergency services to the surrounding population. It provides comprehensive medical aids to the population, including specialized treatments, diagnostic services, and emergency care. The ASST includes six different emergency departments, summarized in Table ??. The number of beds provides the values for b_i later in the calibration model. One could argue that the patients' perception regarding a hospital depends not on the size of the emergency care department, but from the overall number of beds of the whole hospital. Table ?? also reports the total number of beds, which strongly linearly correlates with the number of beds in the emergency care department. Thus, for the purpose of this research, we have considered the beds in the emergency care department, which is more in scope with the purpose of the model.

Hospital code	City	# Total beds	# beds in ED
1	A	112	25
2	B	40	10
3	C	86	15
4	D	44	13
5	E	469	84
6	E	139	15

Table 1. Hospitals with emergency department in the ASST Sette Laghi, with the number of beds places in the hospital

The dataset utilized in this study comprises patient behavior records in the specified region, encompassing patient arrivals at an emergency department during 2019, the second half of 2021, and the first half of 2022. This dataset documents a total of $325,886$ arrivals. Each entry details relevant information, such as the severity level and city of residence of the patients. For the purpose of this analysis, patients not residing in towns within the ASST Sette Laghi area were excluded, resulting in a consideration of $256,701$ emergency department arrivals. The primary objective of this research is to understand the decision-making process undertaken by patients or emergency transport services in selecting an appropriate emergency department relative to their current location. It is important to clarify that while the model is designed for adaptability and potential applicability in various settings, its universality cannot be asserted at this stage due to the lack of validation in different scenarios.

The process of model calibration involves determining the optimal values of parameters α and β to minimize the discrepancy between the observed and simulated distributions of resident arrivals at emergency departments within the study area. The simulation of the distribution is executed as follows: A random subset of $25,000$ records, each representing a patient arrival at an emergency unit, is extracted from the dataset. For each record, a selection probability $p(H_i)$ is calculated for each hospital H_i. Subsequently, a destination hospital H_i^* is chosen based on this probability distribution, and the patient is assigned to that hospital. Upon allocating all $25,000$ patients, a specific metric is utilized to evaluate the divergence between the actual and simulated distributions for each hospital.

$$e_i = |f_i{}^r - f_i{}^s| \tag{4}$$

where $f_i{}^r$ and $f_i{}^s$ respectively the relative frequency of H_i for real data and simulated data. A genetic algorithm is then employed to estimate α and β minimizing $max(e_i)$. The algorithm runs for 250 generations, with a population of $5,000$ possible solutions.

3 Results

This section delineates the outcomes derived from applying the gravity model, as detailed in the preceding section, to the ASST Sette Laghi dataset, along with the tests conducted on these results. The implementation of the gravity model has produced results that are both significant and consistent, conforming to anticipated patterns. It effectively elucidates the observed behavioral patterns, offering valuable insights into the underlying dynamics of patient distribution among emergency departments.

The analysis is divided into two distinct parts. The first part encompasses an aggregate analysis, which underscores the efficacy of the model in accurately allocating the population to the appropriate emergency care departments. The second part of the analysis focuses on how this allocation is achieved, emphasizing the retention of the unique characteristics inherent in the allocation network. This dual approach provides a comprehensive understanding of the performance of the methodology.

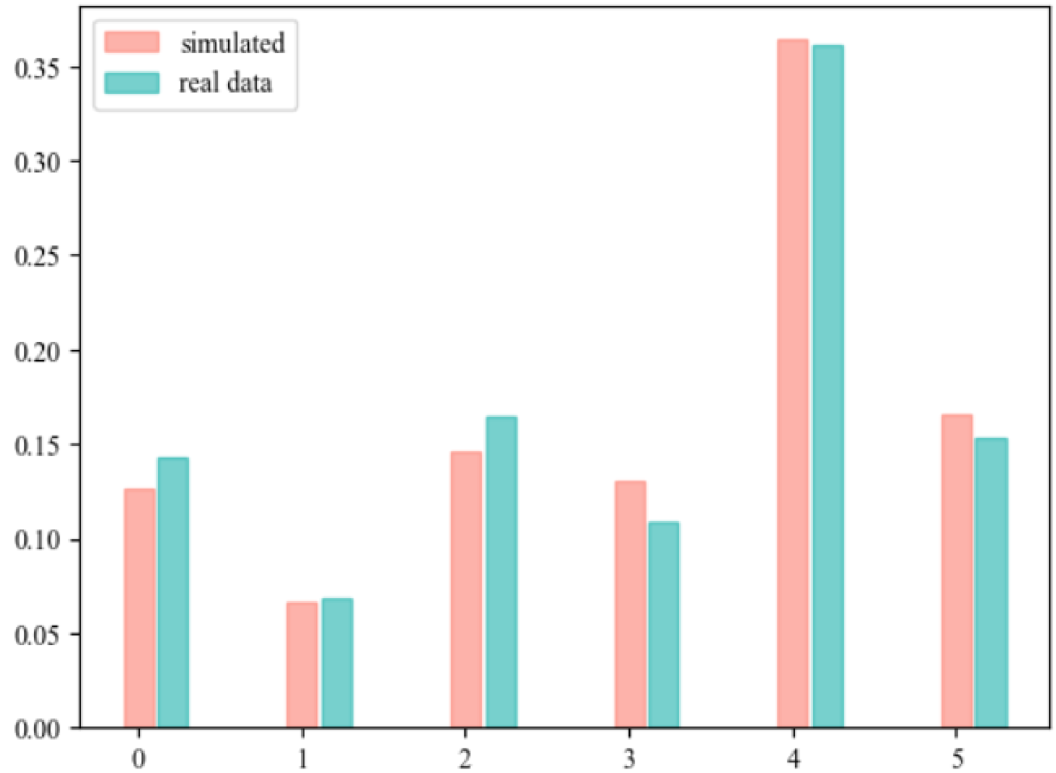

Figure 2. Comparative analysis of the relative distributions of patient arrivals at each hospital level

The principal findings of this study are illustrated in Figure 2, which displays the relative frequency of allocating the population from a city to a hospital, as delineated in the model specification section. The experimental results indicate that the maximum discrepancy between any two elements in the relative distribution is $max(e_i) = 0.0215$, while the average error is quantified at $E[e_i] = 0.0123$. This implies that the calibrated model can assign each patient, based on their triage code and city of residence, to the appropriate hospital with an average precision of 0.9877.

Figure 3 presents a network visualization of this complex system (49; 50), where each node represents a city, positioned according to its geographical coordinates (longitude and latitude). Each link denotes a movement of inhabitants from city j to an emergency department in another city k. The color of each link indicates the city of the emergency department to which the majority of a city's population travels, and the node size corresponds to the population size of the city. The comparative analysis of these networks elucidates two observations. Firstly, employing travel time as a metric, as opposed to Euclidean distance or travel distance, preserves geographical proximity, affirming the model's adherence to real-world spatial relationships. Secondly, the model not only accurately allocates individuals to hospitals but also precisely reconstructs the network of connections between cities of residence and hospital locations (51). This dual capability of the model highlights its effectiveness in both individual allocation and in mapping the broader network of healthcare access.

To enhance the reliability of the results, a final analysis was conducted to assess the precision of the model's allocation of each city's population to a hospital, in comparison with actual data. Specifically, for each city j, the discrepancy in the proportion of individuals attending hospital i between the real data and the outcomes predicted by the gravity model was calculated. The distribution of these discrepancies is illustrated in Figure 4. Although the distribution is right-skewed, indicating some higher values, the average error across all cities is 1.982%, while the median error is at 0.897%.

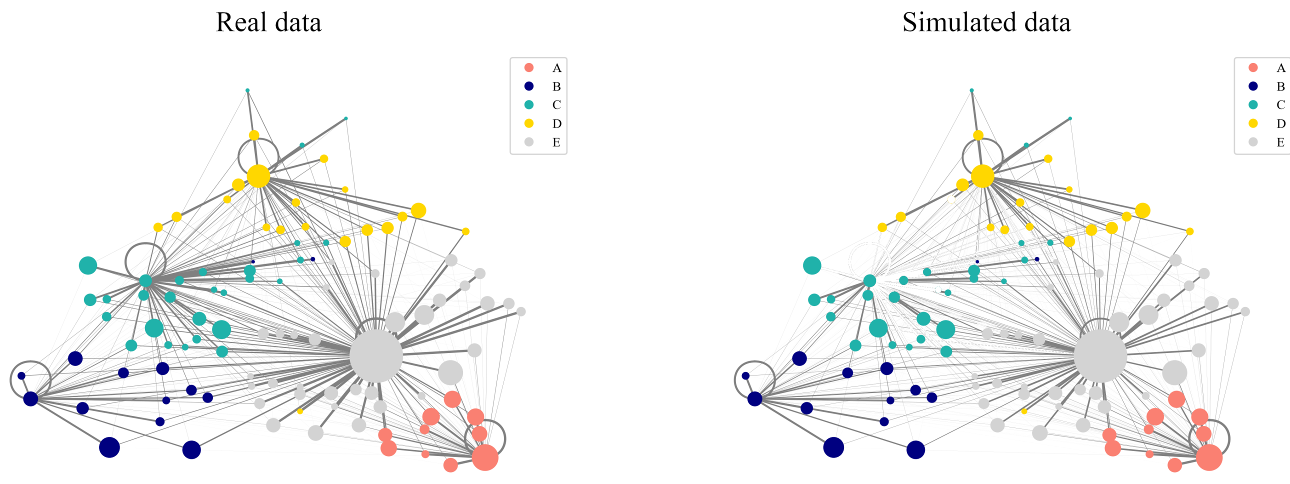

Figure 3. Comparison of the network of distributions between real data and the results of the simulated data. The color stands from the targeted city in which most of the population has gone, while the size embody the population of the corresponding city.

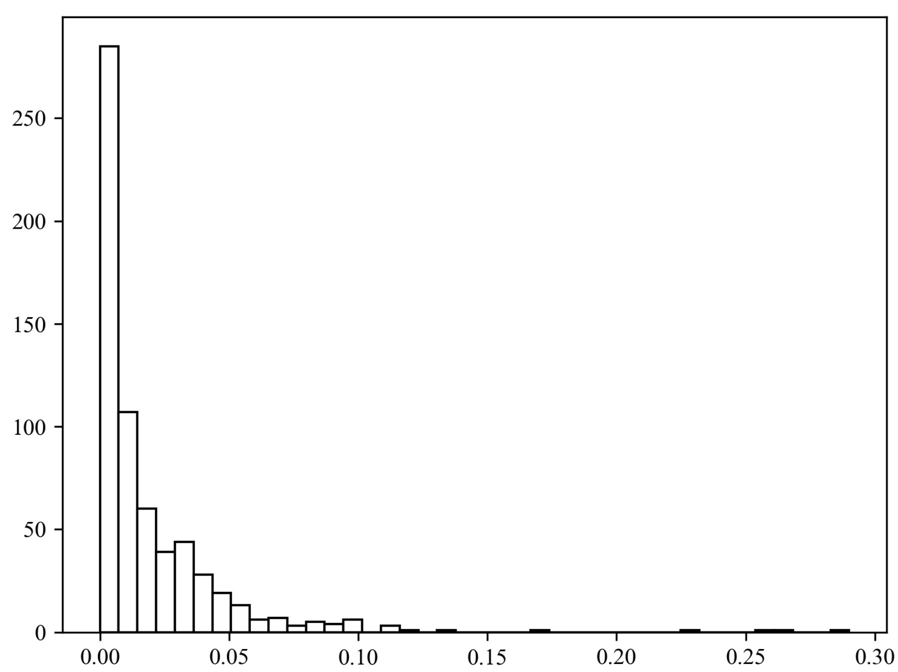

Figure 4. Distribution of the error of evaluation for each combination of city of origin an city of the hospital

4 Discussion

The gravity model introduced in this paper demonstrates precision in two key aspects: the specific allocation of individuals to hospitals, which is executed with high accuracy (as illustrated in Figure 2), and in determining the contribution of cities, denoted as j, to specific hospitals, labelled as i, in terms of patient flow (see 3 and 4).

Common sense and previous studies suggest that the workload of a hospital is influenced by the population size and the availability of alternative healthcare options in the vicinity (52). This gravity model provides a means to account for these factors. Moreover, successive fine-tuning with other factors such as the attractiveness of the hospital could help to better fit the data (52). Nevertheless, these fine-tuning operations usually complicate the model, and even when they have a great fit with data, they become less explainable with every new assumption (53).

Drawing from the previous outcomes, discernible spatial patterns become evident concerning the possible geographical accessibility to emergency care departments within the specified territories examined in the analysis, when the conditions of patients are taken into account. One significant inquiry arises: what are the underlying reasons for the distinctive spatial pattern observed in terms of potential access to emergency departments? (36) While factors such as topography and historical settlement patterns have certainly influenced the current distribution of the population (54), it is evident that these spatial patterns cannot be solely attributed to them. Another crucial explanatory factor is the decision-making process in healthcare services, which has decided both the position of the emergency department and the assignation of a fourfold progressive code to classify the severity of a patient's condition. These systemic factors are likely to contribute to the spatial patterns identified through the modified gravity model. Nevertheless, their precise role remains uncertain at present. To address this knowledge gap, it would be necessary to engage in consultations with decision-makers responsible for resource allocation, system design, and other key functions (36).

5 Conclusion

This paper aims to advance the understanding of patient decision-making in emergency healthcare by refining gravity models, focusing on how hospital size and patient condition severity, beyond just distance, influence emergency department choice. A new gravity rule, surpassing current benchmarks, was developed and tested, showing a mean error of 1.23% when applied to real-world data, highlighting its precision and potential in predictive modelling.

Even if yielding promising results, this study is subject to certain limitations that merit consideration. Firstly, there is the need for further validation in diverse geographical areas. Designed with generality in mind, the model is theoretically applicable to any region where Geographic Information System data are available and patient arrivals at the emergency department are encoded into a hierarchical classification system. However, the current research does not assert that the relationships identified are universally applicable across different contexts or areas. Consequently, further empirical studies are needed to assess the model's adaptability and efficacy in other settings.

Another notable limitation pertains to the distinction between patients' residential locations and the actual departure point. The present model solely accounts for the former, lacking data on the latter. This absence of information about the actual starting location of emergency care journeys may limit the accuracy of the model. The gravity model demonstrates robust results when analyzing patient allocation based on residential areas, but a more precise fit to the data might remain impracticable without the integration of additional information. It highlights the need for an even more comprehensive data collection.

In light of the findings of this study, several avenues for future research have been identified to further substantiate and expand upon the current results.

A primary development involves designing a survey aimed at directly confirming the cause-effect relationship between individual decision-making processes and the variables identified in this study as influential factors in patient behaviour. This survey would provide qualitative insights, complementing the quantitative data to validate the theoretical underpinnings of this paper.

Second, replicating the current analysis across a broader and more diverse range of territories could validate the results obtained in this study and test the applicability and robustness of this gravity model extension in different geographic and demographic contexts, increasing the scientific relevance of the work. The goal could be to assess whether the patterns and relationships identified are consistent across various settings or are unique to the areas with specific features.

Finally, developing a simulation model could additionally confirm the results of this study For example, an agent-based model simulating the interactions of individual agents (in this case, patients) within a defined environment (the healthcare emergency departments system) could offer a powerful tool for testing hypotheses and observing emergent behaviours under varying conditions (55).

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Author contributions statement

E.F. secured the founding and the data access. F.S and F.A. collected and pre-processed the data. F.B. analyzed the data, conceived the experiment, conducted the experiment, and analysed the results. F.B. and F.S. wrote the first draft of the manuscript. All authors reviewed the manuscript.

Additional information

A code and a subsection of the data used for the experiments are available in the following repository: https://anonymous.4open.science/r/gravity submission. The authors do not have any compelling interests.

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