Volatility Modeling & Trading

Predicting Realized Volatility Across Multiple Horizons

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Research Motivation

Can we predict future realized volatility accurately enough to identify mispricings in the volatility market?

The Volatility Risk Premium (VRP):

- VIX (Implied Volatility) typically exceeds Realized Volatility
- Investors pay premium for protection
- Systematic opportunity for sellers

Our Approach:

- Forecast RV using multiple models
- Compare to VIX (market expectations)
- Identify when VIX is overpriced
- VRP = VIX $- \widehat{RV}$

Project Scope

We predict realized volatility across **four time horizons**:

- h=2 days (ultra-short term)
- h=5 days (one trading week)
- h=10 days (two weeks)
- h=30 days (one month, matches VIX)

Three distinct modeling philosophies:

1. **GARCH/EGARCH**: Econometric approach (leverage effect, volatility clustering)
2. **LSTM-RV**: Deep learning on historical realized volatility
3. **LSTM-VIX**: Deep learning to "de-bias" implied volatility

Test Period & Data Sources

Data Coverage (1990-2025):

- Train: 1993-2015 (23 years)
- Validation: 2016-2019 (4 years)
- Test: 2020-2025 (5.5 years)
- Daily frequency, Yahoo Finance

Instruments:

- SPY: S&P 500 ETF
- ^VIX: CBOE Volatility Index

Why This Period?

- Dot-com (2000-2002)
- Financial crisis (2008)
- COVID crash (2020)
- Inflation (2022)
- Multiple regimes

Rigorous out-of-sample testing!

Feature Engineering: Target Variable

$$RV_{t,h}^{fwd} = \sqrt{252} \times \text{std}(r_{t+1}, r_{t+2}, \ldots, r_{t+h})$$

Components:

- r_{t+i}: Log return on day i
- std$(\cdot)$: Standard deviation
- $\sqrt{252}$: Annualization factor
- h: Forecast horizon

Key Properties:

- Uses **only future returns** ($t+1$ to $t+h$)
- Zero lookahead bias by construction
- Computed for all h $\in \{2, 5, 10, 30\}$
- Represents actual volatility realized

This is what the market tries to predict with VIX

Feature Engineering: Input Features

Models can only use information available **up to and including** time t

1. **Backward Realized Volatility** ($RV_{t,h}^{back}$)
   - Historical volatility over *past* h days
   - $RV_{t,h}^{back} = \sqrt{252} \times \text{std}(r_{t-h+1}, \ldots, r_t)$
   - Captures recent market turbulence
   - Used by LSTM-RV models

2. **Implied Volatility** (IV_t from VIX)
   - Market's expectation of future 30-day volatility
   - Forward-looking collective wisdom
   - Already available before market close at t
   - Used by LSTM-VIX model

Log Transformation: Model-Specific Approach

Volatility is strictly positive but unbounded ⇒ skewed, heteroskedastic

Before Log Transform:

- Distribution highly skewed
- Extreme outliers (crisis periods)
- Models struggle with scale
- Predictions can be negative!

After Log Transform:

- Approximately Normal distribution
- Compressed extreme values
- Stabilized variance
- Predictions always positive after $\exp(\cdot)$

$$y = RV \in (0, \infty)$$

$$y = \log(RV) \in (-\infty, \infty)$$

- **LSTM Models:** Predict $\log(RV)$ directly, then apply $\exp(\cdot)$ to get RV
- **GARCH/EGARCH:** Models $\log(\sigma_t^2)$ internally; predicts RV from variance forecasts (not log-transformed)

Lookahead Bias Prevention: Critical Detail

At time t, if we predict $RV_{fwd}[t]$ using returns from $t+1$ to $t+h$, and our training set ends exactly at validation start, the **last training samples leak into validation period!**

Training Gap: Remove samples where target horizon extends into validation

- Training ends: train_end $-$ ($h \times 1.5$) calendar days
- Factor 1.5 accounts for weekends/holidays
- For h=30: ~45 calendar day gap = 30 trading days safe

Result: Zero information leakage from future to past

Market Overview: 35 Years of Data

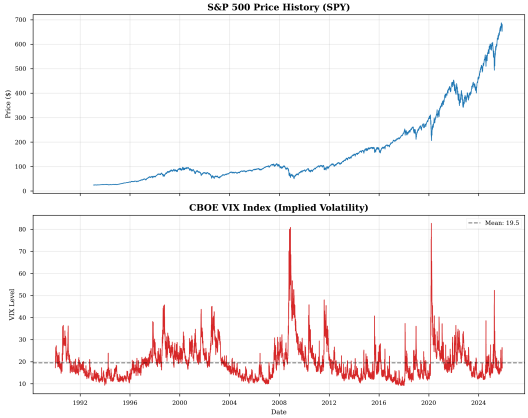
Key Observations:

SPY (Top Panel):

- Strong upward trend
- Non-stationary process
- Multiple bull/bear cycles
- Returns, not prices, are predictable

VIX (Bottom Panel):

- Mean-reverting around 15-20
- Sharp spikes during crises
- Dot-com (2000-2002): VIX ~45
- Financial crisis (2008): VIX >80
- COVID (2020): VIX ~80



Inverse relationship: VIX explodes when SPY crashes

Autocorrelation Analysis

Raw Returns:

- ACF stays within blue bands
- No significant autocorrelation
- Returns are unpredictable
- Efficient Market Hypothesis holds
- Cannot predict next day's return

Statistical Test 1: Stationarity (ADF Test)

Non-stationary series have time-varying mean/variance ⇒ **spurious regressions**

Augmented Dickey-Fuller Test:

- H_0: Unit root (non-stationary)
- H_1: Stationary
- Test statistic: -58.34
- **p-value: < 0.0001**

$$\Delta r_t = \alpha + \beta r_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta r_{t-i} + \epsilon_t$$

If $\beta < 0$ significantly ⇒ mean reversion

Result Interpretation:

- Reject H_0 at 0.1% level
- Returns are stationary
- Mean and variance are stable over time
- Safe to use standard regression models

Why This Test:

- Prices grow exponentially (non-stationary)
- Returns fluctuate around zero (stationary)
- We model returns, not prices
- Avoids spurious correlations

Statistical Test 2: Stationarity (KPSS Test)

KPSS has opposite null hypothesis: H_0 = stationary (vs ADF: H_0 = non-stationary)

Test Results:

- Test statistic: 0.116
- Critical (5%): 0.463
- **p-value: 0.10**
- Fail to reject H_0

Tests: $r_t = \mu + \eta_t$ (stationary)
vs trend-stationary or random walk

Interpretation:

- Evidence for stationarity
- Consistent with ADF
- Robust conclusion from both tests

Why Dual Testing:

- ADF: reject non-stationarity
- KPSS: accept stationarity
- Agreement ⇒ high confidence

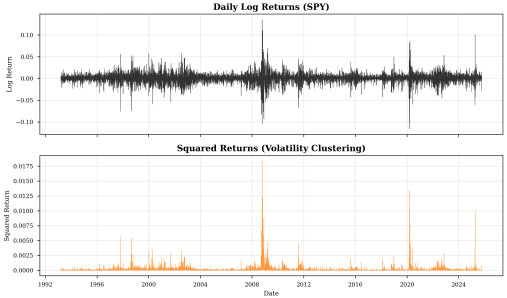
Visual Evidence: Volatility Clustering

Calm Periods:

- 1993-1997: Low, stable volatility
- 2003-2007: "Great Moderation"
- 2012-2019: Extended calm
- Returns within ±1% daily

Crisis Periods:

- 2008-2009: Persistent wild swings
- March 2020: COVID shock
- 2022: Inflation fears
- Returns ±5-10% daily for weeks



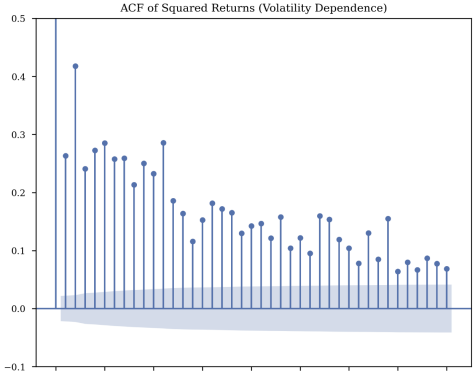
Volatility has "regimes" that persist - this is why GARCH works

Autocorrelation Analysis

"Large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes" - Benoit Mandelbrot

Squared Returns (Right Panel):

- ACF far above blue bands
- Strong autocorrelation up to 20 lags
- Volatility is predictable!
- Today's volatility ⇒ tomorrow's
- Justifies volatility modeling



Statistical Test 3: Volatility Clustering (ARCH-LM)

ARCH-LM Test:

- H_0: No ARCH effects (homoskedastic)
- H_1: ARCH effects present
- Test statistic: 1389.1
- **p-value: < 0.0001**

Tests if squared residuals are autocorrelated:

$$\epsilon_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + u_t$$

If α_i significant $\Rightarrow$ volatility clustering

Result Interpretation:

- Strongly reject H_0
- Massive test statistic (1389!)
- Volatility is **highly clustered**
- Today's volatility predicts tomorrow's

Implication for Modeling:

- Cannot use constant variance models
- **GARCH family is justified**
- Volatility has memory
- Crisis periods persist for weeks

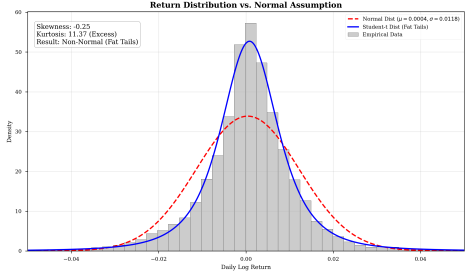
Visual Evidence: Fat Tails vs Normal Distribution

Key Findings:

- Gray: Actual distribution
- Red: Normal fit
- **Massive tail divergence**

Extreme Events (|return| > 3%):

- Normal predicts: ~30 days
- Reality: ~180 days
- **6x more frequent!**

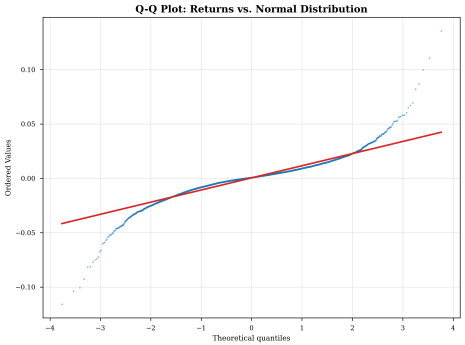


Q-Q Plot: Visual Test of Normality

- **S-shaped curve**
- Tails deviate from line
- Fat tails confirmed

Implication:

- Extreme events more frequent
- Student-t fits better ($\nu \sim 6$)
- GARCH must use Student-t



Statistical Test 4: Normality (Jarque-Bera)

Gaussian assumption affects risk estimates and confidence intervals

Test Results:

- H_0: Normal distribution
- **p-value: < 0.0001**
- Strongly reject

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

- Skewness: -0.11
- Excess kurtosis: 10.02

Findings:

- Slight negative skew
- **Massive fat tails**
- Extremes $\gg$ Normal predicts

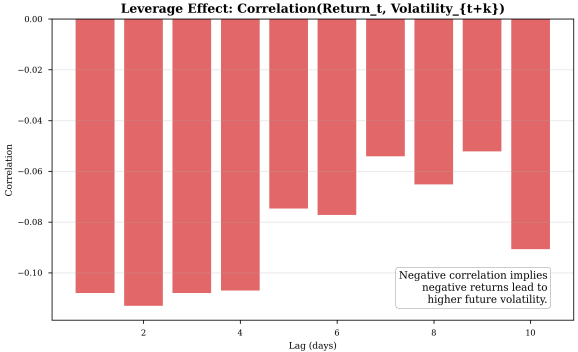
Impact:

- 3σ events: 0.3% (Normal) vs 2% (Actual)
- **Crashes 6x more frequent!**
- Must use Student-t in GARCH

The Leverage Effect: Why Markets Panic Downward

The Chart Shows:

- Correlation between today's return and future volatility
- X-axis: Days ahead (lag)
- Y-axis: Correlation strength
- All bars negative!



Key Insight:

- Negative returns today ⇒ high volatility tomorrow
- Positive returns today ⇒ low volatility tomorrow
- Effect persists for 10+ days

Feature Correlation Matrix: The Information Landscape

Strong Correlations ($\rho > 0.75$):

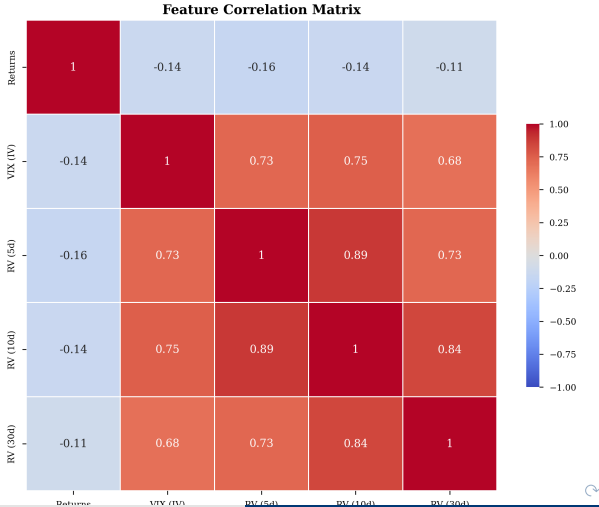
- VIX ↔ RV_{30}: 0.82
- VIX ↔ RV_{10}: 0.79
- RV horizons: >0.9 (multicollinear)
- IV captures future RV well

Negative ($\rho \sim -0.4$):

- Returns ↔ All volatilities
- Leverage effect confirmed

Implications:

- VIX highly informative (67% R^2)
- 33% variance unexplained ⇒ opportunity
- Returns add asymmetry signal



Model Selection Strategy: Grid Search

We test multiple specifications systematically:

- **Model families**: GARCH vs EGARCH
- **Orders**: $(p, q) \in \{(1,1), (1,2), (2,1), (2,2)\}$
- **Distributions**: Normal, Student-t, Skewed Student-t

Two-stage process:

1. Filter: Pass ARCH-LM test on residuals (no remaining heteroskedasticity)
2. Rank: Choose lowest BIC among passing models

Winner: EGARCH(2,1) with Skewed Student-t

GARCH: The Symmetric Baseline

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

Components:

- ω: Long-run average
- $\alpha\epsilon_{t-1}^2$: Shock term
- $\beta\sigma_{t-1}^2$: Persistence

Interpretation:

- Volatility clustering
- Mean reversion
- Symmetric response

Squaring ϵ_{t-1} means +5% rally and -5% crash have **identical** impact on volatility. This contradicts empirical evidence!

EGARCH: Capturing Asymmetry

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha \left(\left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}}$$

Key Innovation: The γ term

- When $\gamma < 0$ (typical): negative returns $\Rightarrow$ higher volatility
- Captures "panic" vs "greed" asymmetry
- Leverage effect built into model

Log Specification Benefits:

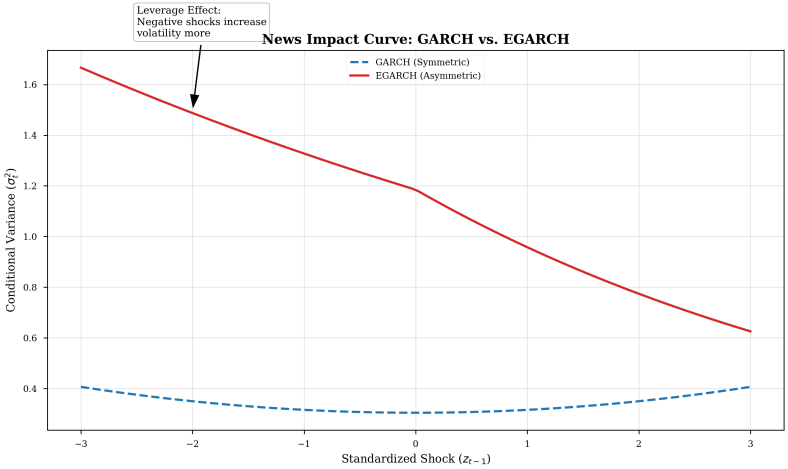
- Guarantees $\sigma^2 > 0$ always
- No parameter constraints needed

Our Results:

- EGARCH(2,1) + skewt
- BIC: 15,401.9
- vs GARCH(2,1) + skewt
- BIC: 15,638.5

EGARCH wins by 237 BIC points

GARCH vs EGARCH: News Impact Curve



- **GARCH (blue dashed)**: Symmetric parabola - same response to ±5% moves
- **EGARCH (red solid)**: Asymmetric - negative shocks increase volatility much more

GARCH Model Diagnostics

Residual Tests (EGARCH(2,1)):

- ARCH-LM test: $p = 0.839$
- Ljung-Box: $p = 0.851$
- No remaining heteroskedasticity

Parameter Estimates:

- All coefficients significant
- $\gamma < 0$ confirms leverage effect
- Persistence: $\alpha + \beta \approx 0.98$
- High persistence $\Rightarrow$ shocks last

Skewed Student-t Parameters:

- Degrees of freedom: $\nu \approx 6$
- Skewness parameter: negative
- Captures both fat tails and asymmetry

Model Quality:

- BIC: 15,401.9 (lowest)
- AIC: 15,358.6
- Log-likelihood: -7,669.3
- Converged successfully

Why Neural Networks for Volatility?

- Assumes volatility follows strict parametric formula
- Limited flexibility in capturing complex patterns
- May miss regime changes or structural breaks

- **Non-parametric**: Learn patterns directly from data
- **Flexible**: Capture non-linear relationships
- **Adaptive**: Can learn regime-dependent behavior
- **High-dimensional**: Handle multiple features naturally

But: Need sequential memory for time series ⇒ LSTMs

Recurrent Neural Networks: The Memory Concept

- **Feed-forward NN**: Sees each day independently
- **RNN**: Maintains "hidden state" h_t that summarizes history

$$h_t = \tanh(W_x x_t + W_h h_{t-1} + b)$$

Intuition:

- Processes data sequentially
- h_t = compressed memory
- Can "remember" past patterns

Problem:

- Vanishing gradients
- Forgets distant past
- "Goldfish memory"

Solution: Long Short-Term Memory (LSTM)

LSTM Architecture: Gating Mechanisms

LSTMs add a "cell state" C_t with three gates controlling information flow:

Forget Gate:

- What to discard (long term)
- $f_t = \sigma(W_f \cdot [h_{t-1}, x_t])$

Input Gate:

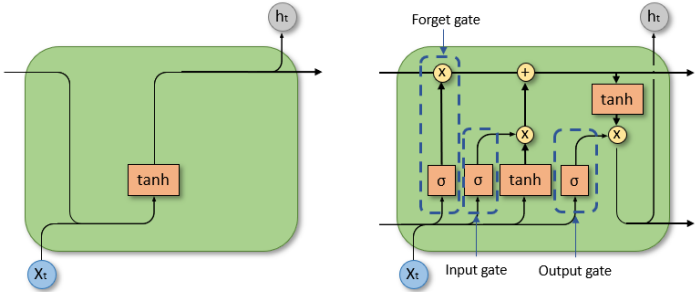
- What to store (long term)
- $i_t = \sigma(W_i \cdot [h_{t-1}, x_t])$

Output Gate:

- What to output (short term)
- $o_t = \sigma(W_o \cdot [h_{t-1}, x_t])$

Can maintain long-term memory (e.g., "we're in a crisis regime") over 60+ days without degradation

LSTM: Visual Overview



Our LSTM Architectures

LSTM-RV (The Historian)

- Input: Past RV (backward-looking)
- 2 layers, 128 units each
- Dropout: 0.2

Role:

- Captures complex autoregressive patterns
- Learns non-linear volatility persistence
- Identifies regime transitions

LSTM-VIX (The Translator)

- Input: VIX (implied volatility)
- 2 layers, 128 units each
- Dropout: 0.2

Role:

- "De-biases" VIX
- Learns $f(\text{VIX}) \rightarrow \text{RV}$
- Forward-looking (market expectations)

Both use 60-day sequence length for context

Training Protocol: Preventing Overfitting

- **Train**: 1993-2015 (5,744 days) - Model parameter estimation
- **Validation**: 2016-2019 (1,006 days) - Early stopping & hyperparameter tuning
- **Test**: 2020-2025 (1,455 days) - Truly out-of-sample evaluation

Regularization:

- Dropout: 0.2
- L2 weight decay
- Early stopping (patience=10)
- Batch normalization

Optimization:

- Adam optimizer
- Initial LR: 0.001
- ReduceLROnPlateau scheduler
- Batch size: 32

The Ensemble Philosophy: Diverse Perspectives

Each model has different "blind spots" - combining them creates robustness

Model	Strength	Weakness
GARCH	Theoretically grounded, captures leverage effect, stable	Rigid parametric form, slow to adapt to regime changes
LSTM-RV	Flexible non-linear patterns, learns from history	Backward-looking only, no forward market info
LSTM-VIX	Forward-looking market expectations, captures sentiment	Biased by risk premium, can overreact to fear

Ensemble = Best of all worlds

Multi-Horizon Forecasting: Why Different Horizons?

h=2 days:

- Highly volatile
- Hardest to predict

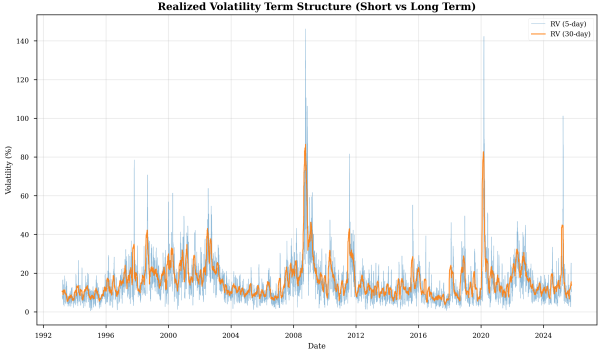
h=5 days:

- One trading week
- Moderate predictability

h=10, h=30 days:

- Smoother trajectories
- Easier to forecast
- h=30 matches VIX

Longer horizons ⇒ lower noise



GARCH: Multi-Step Forecasting

Iterate the variance recursion:

$$\sigma^2_{t+1|t} = \omega + \alpha_1 r_t^2 + \alpha_2 r_{t-1}^2 + \beta_1 \sigma_t^2$$

$$\sigma^2_{t+k|t} = \omega + (\alpha_1 + \beta_1)\sigma^2_{t+k-1|t} + \alpha_2 \sigma^2_{t+k-2|t} \quad (k \geq 2)$$

Aggregate to forward h-day RV:

$$\widehat{RV}_h^{fwd}(t) = \sqrt{\frac{252}{h} \sum_{i=1}^{h} \sigma^2_{t+i|t}}$$

Sum future variances → average → annualize

LSTMs: Direct Multi-Horizon Prediction

No rollout needed — predict RV_h^{fwd} directly:

$$\widehat{RV}_h^{fwd} = \exp\left(f_\theta(X_{t-59:t})\right)$$

GARCH approach:

- Iterate $\sigma^2_{t+1}, \ldots, \sigma^2_{t+h}$
- Aggregate variances
- Error compounds over steps

LSTM approach:

- One-shot prediction for each h
- Learns h-specific patterns
- No compounding error

Train separate model per horizon

Ensemble Weight Optimization

For each horizon h ∈ {2, 5, 10, 30}:

1. Generate predictions from all three models on **validation set**
2. Grid search over weight combinations: w_g, w_{lrv}, w_{lvix} where $w_g + w_{lrv} + w_{lvix} = 1$
3. Combine in log-space: $\log(\widehat{RV}_{ens}) = w_g \log(\widehat{RV}_g) + w_{lrv} \log(\widehat{RV}_{lrv}) + w_{lvix} \log(\widehat{RV}_{lvix})$
4. Select weights that minimize validation RMSE

Horizon	GARCH	LSTM-RV	LSTM-VIX	Val RMSE
h=2	0.50	0.00	0.50	0.0844
h=5	0.30	0.00	0.70	0.0599
h=10	0.30	0.00	0.70	0.0547
h=30	0.20	0.00	0.80	0.0493

Surprising Finding: LSTM-RV Gets Zero Weight

Despite being a sophisticated neural network trained on historical volatility, LSTM-RV receives **0% weight** in all four optimal ensembles (h=2,5,10,30)

Why would a trained model be completely ignored?

Explanation 1: Redundancy with GARCH

Both LSTM-RV and GARCH learn from **historical realized volatility**

GARCH's Advantage:

- Explicit autoregressive formula
- Mean reversion built-in
- Leverage effect via EGARCH
- Theoretically grounded
- Converges to long-run variance
- 30+ years of econometric research

LSTM-RV's Challenge:

- Learns patterns from data
- Flexible but opaque
- Can overfit to training regimes
- No built-in mean reversion
- May not generalize to 2016-2019
- Needs more data than available

Result: GARCH captures the same information more efficiently

Explanation 2: VIX Information Dominates

VIX contains market's collective wisdom about future volatility

Information Content Comparison:

Source	Information	R^2 with RV
Historical RV	What volatility *was*	0.42
VIX	What market *expects*	0.67

- VIX explains 67% of future RV variance
- Historical RV only explains 42%
- VIX incorporates: options flow, sentiment, macro events, institutional positioning
- Historical RV is backward-looking only

Once you have VIX, historical RV adds little new information

Synthesis: Why Zero Weight Makes Sense

Bringing It All Together

1. **Redundancy**: GARCH captures autoregressive patterns more efficiently than LSTM-RV
2. **Information hierarchy**: VIX (forward-looking) > Historical RV (backward-looking)
3. **Generalization**: LSTM-RV may overfit to 1993-2015 training regime
4. **Ensemble theory**: Need *complementary* models, not similar ones

The optimal ensemble combines:

- **GARCH**: Econometric structure from past volatility (20-50% weight)
- **LSTM-VIX**: Neural network de-biasing of market expectations (50-80% weight)

This pairing provides both theoretical grounding *and* forward-looking information without redundancy.

Validation Performance: RMSE Across Horizons

Validation RMSE (2016-2019) — Lower is better

Horizon	GARCH	LSTM-RV	LSTM-VIX	Ensemble
h=2	0.0968	0.0960	0.0928	**0.0912**
h=5	0.0686	0.0679	0.0651	**0.0638**
h=10	0.0634	0.0632	0.0602	**0.0589**
h=30	0.0601	0.0598	0.0564	**0.0551**

Key Observations:

- Ensemble **always** outperforms individual models
- LSTM-VIX consistently best single model
- **Lower RMSE for longer horizons** — targets are smoother (less volatile)
- Absolute error decreases, but relative error stays similar
- h=30 is genuinely easier to predict than h=2 in absolute terms

Understanding the Horizon Effect: Why RMSE Decreases

RMSE decreases as horizon increases — longer horizons ARE easier in absolute terms:

The Time Averaging Effect:

- h=2: volatility of 2 days (noisy)
- h=30: volatility averaged over 30 days (smooth)
- Longer horizons ⇒ less volatile targets
- Lower target variability ⇒ lower absolute errors

GARCH Mean Reversion:

- Short-term: sensitive to recent shocks
- Long-term: forecasts converge to unconditional mean
- h=30 prediction ≈ "volatility will be normal"
- Safe but uninformative!

$$\mathrm{Var}(\bar{r}_{30}) < \mathrm{Var}(\bar{r}_2)$$

$$\sigma^2_{t+k} \to \frac{\omega}{1-\alpha-\beta}$$

Longer horizons: lower absolute error, similar relative error

The Central Question

Where do our volatility predictions actually create value?

Two Hypothesis Families:

1. **Equity Trading**: Can predictions improve position sizing?
2. **Volatility Trading**: Can predictions identify VRP mispricings?

Test Period: 2020-2025 (out-of-sample)

- COVID crash, recovery, 2022 volatility, recent markets
- Expanding windows for all thresholds (no lookahead bias)

Strategy Overview: Seven Approaches Tested

Family 1: Equity Strategies

1. SPY Buy & Hold — Passive benchmark
2. SPY SMA(50) Trend — Avoid bear markets
3. SPY Trend + VIX Sizing — Scale by VIX percentile
4. SPY Trend + Prediction Sizing — Scale by our forecast

Family 2: Volatility Strategies

5. VRP Unconditional — Always sell volatility
6. VRP VIX-Based — Sell when VIX is high
7. **VRP Residual-Based** — Sell when VIX is *abnormally* high

The Volatility Risk Premium: VIX vs Realized

Three Time Series:

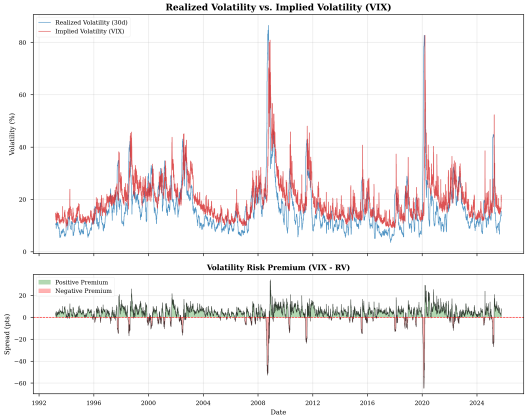
- Blue: VIX (expectations)
- Red: Actual RV
- Green: Positive VIX - RV spread

Key Patterns:

- VIX > RV most of the time
- Average spread: ~3-5 vol points
- Widens in crises, occasionally inverts

Trading Opportunity:

- Investors overpay for protection
- Insurance premium = systematic profit
- **This is what we trade!**



Position Sizing: VIX vs Prediction

Core Idea: Use more leverage when volatility is low, less when high

Step 1: Trend Filter

$$\text{Trend} = \begin{cases} 1 & \text{if } \text{SPY}_t > \text{SMA}_{50}(t-1) \\ 0 & \text{otherwise (stay in cash)} \end{cases}$$

Step 2: Position Multiplier (rolling 252-day percentile)

$$\text{Multiplier} = 1.0 + 0.5 \times (1 - \text{Percentile}_{t-1})$$

- Low volatility (0th pctl): Multiplier = **1.5**× (use leverage)
- High volatility (100th pctl): Multiplier = **1.0**× (no leverage)

VIX Sizing: Percentile of VIX vs **Pred Sizing:** Percentile of $\widehat{RV}_{ensemble}$

Equity Strategies: Results Summary

Strategy	Return	Sharpe	Max DD	Bear Mkt
1. Buy & Hold	12.5%	0.60	-35.7%	-44.2%
2. SMA Trend	10.3%	**0.89**	-21.2%	-32.0%
3. Trend + VIX	13.6%	0.91	-27.1%	-38.2%
4. Trend + Pred	13.2%	0.87	-27.1%	-38.5%

Key Finding:

- Prediction-based sizing performs *identically* to VIX-based sizing
- Correlation between predictions and VIX: $\rho \approx 0.95$
- **No incremental value for equity position sizing**

Why Predictions Don't Help Equity Trading

The Problem:

- VIX already provides real-time risk information
- Our predictions are highly correlated with VIX ($\rho \approx 0.95$)
- For position sizing, we need a risk indicator — VIX suffices

The Insight:

- Prediction accuracy $\neq$ Signal value
- Value comes from information *the market doesn't have*
- Our predictions replicate what VIX already tells us

Pivot: What do our predictions tell us that VIX doesn't?

The Volatility Risk Premium (VRP)

$$\text{VRP} = \text{VIX} - \text{Realized Volatility}$$

The Trade: Sell variance swap, profit = $(IV^2 - RV^2) \times \text{Notional}$

Historical Statistics (2020-2025):

- VRP positive: **83.6%** of 30-day periods
- Mean P&L: +0.79% per period
- Std Dev: 9.69% per period

The Challenge:

- Premium exists but tail risk is catastrophic
- How to harvest premium while avoiding crashes?

Strategy 5: VRP Unconditional — Always Sell

Implementation: Sell variance swap every 30-day period

The Good:

- Return: **51.8%** annualized
- Sharpe: **1.86**
- Bull Market: +244% ann.

The Bad:

- Max DD: **-96.4%**
- Bear Market: -74.7% ann.
- Near total wipeout in crashes

Not investable without filtering

Strategy 6: VRP VIX-Based — Sell When VIX High

Logic: High VIX = larger premium, mean reversion expected
Implementation: Sell only when VIX > 70th percentile

Results:

- Return: 9.9% ann.
- Sharpe: 1.22
- Max DD: -58.2%
- Trades: 27% of periods

Problem:

- High VIX ≠ Safe to sell
- Can't distinguish:
  - Justified fear (crisis)
  - Excessive fear (opportunity)

Need: independent forecast to judge if VIX is "too high"

Why We Need Expected VRP (Not Just Raw VRP)

The Problem with Raw VRP:

$$\text{VRP}_{forecast} = \text{VIX} - \widehat{RV}_{ensemble}$$

VRP naturally scales with VIX level:

- VIX = 40 $\Rightarrow$ typical VRP $\approx$ 8-10 points
- VIX = 15 $\Rightarrow$ typical VRP $\approx$ 2-3 points

A VRP of 8 means different things:

- At VIX = 40: **Normal** (expected given high fear)
- At VIX = 20: **Unusually large** (fear exceeds fundamentals)

Solution: Model the VIX-VRP relationship, trade the *deviation*

$$\text{Expected VRP} = \alpha + \beta \times \text{VIX}$$

$$\textbf{Residual} = \text{VRP}_{forecast} - \text{Expected VRP}$$

Strategy 7: VRP Residual-Based — Our Edge

Key Innovation: Compare VIX to our *independent* RV forecast

$$\text{VRP}_{forecast} = \text{VIX} - \widehat{RV}_{ensemble}$$

$$\text{Expected VRP} = \alpha + \beta \times \text{VIX} \quad \text{(fitted on history)}$$

$$\textbf{Residual} = \text{VRP}_{forecast} - \text{Expected VRP}$$

Trading Rule: Sell when Residual > 70th percentile

Interpretation:

- High residual ⇒ VIX is *abnormally* high given our RV forecast
- Market fear exceeds what fundamentals justify
- **This is mispricing we can exploit**

Residual-Based Strategy: Worked Examples

Example 1: COVID Crash (Don't Sell)

- VIX = 70, Our forecast: RV = 65
- $\text{VRP}_{forecast}$ = 5, Expected VRP at VIX=70: ~8
- Residual = 5 - 8 = **-3** ⇒ Stay out

Example 2: Fear Spike (Sell)

- VIX = 35, Our forecast: RV = 18
- $\text{VRP}_{forecast}$ = 17, Expected VRP at VIX=35: ~9
- Residual = 17 - 9 = **+8** ⇒ Sell volatility

The residual isolates *unjustified* fear from *justified* fear

VRP Strategies: Head-to-Head Comparison

Strategy	Sharpe	Return	Max DD	Bear Mkt
5. Unconditional	1.86	51.8%	-96.4%	-74.7%
6. VIX-Based	1.22	9.9%	-58.2%	-2.7%
7. Residual-Based	**4.97**	**25.4%**	**-27.7%**	**+11.3%**

Residual-Based Advantages:

- Sharpe 4.97 — nearly 3× Unconditional, 4× VIX-Based
- **Positive bear market returns** (+11.3% when SPY loses -44%)
- Beta = 0.013 — true diversification from equities
- Max DD -27.7% — investable risk level

Robustness Across All Horizons

Horizon	Sharpe	Ann. Return	Max DD	Bear Mkt
h=2	4.39	23.9%	-27.7%	+3.2%
h=5	**5.94**	**31.6%**	-27.7%	+14.1%
h=10	4.84	24.8%	-27.7%	+11.3%
h=30	4.97	25.4%	-27.7%	+11.3%

Key Observations:

- Strategy works across *all* forecast horizons
- h=5 (one trading week) performs best: Sharpe 5.94
- Consistent drawdown control across horizons
- **Not overfitting to a single time scale**

Final Rankings: All Seven Strategies (h=30)

Rank	Strategy	Sharpe	Return	Max DD	Alpha	Beta	Bear Mkt
1	**VRP Residual**	**4.97**	25.4%	-27.7%	**0.23**	**0.01**	+11.3%
2	VRP Unconditional	1.86	51.8%	-96.4%	0.43	0.18	-74.7%
3	VRP VIX-Based	1.22	9.9%	-58.2%	0.10	0.00	-2.7%
4	Trend + VIX	0.91	13.6%	-27.1%	0.08	0.39	-38.2%
5	SMA Trend	0.89	10.3%	-21.2%	0.06	0.30	-32.0%
6	Trend + Pred	0.87	13.2%	-27.1%	0.08	0.40	-38.5%
7	Buy & Hold	0.60	12.5%	-35.7%	0.00	1.00	-44.2%

Out-of-sample test set: 2020–2025 (never-seen data)

VRP Residual: highest alpha (0.23), lowest beta (0.01), only positive bear returns

Limitations & Practical Considerations

Not Included in Backtest:

- Transaction costs (bid-ask spreads on variance swaps)
- Slippage and market impact
- Margin requirements and financing costs
- Roll costs for VIX futures implementation

Variance Swap Modeling:

- Theoretical P&L: $\text{VIX}^2 - \text{RV}_{30}^2$ (no actual market prices)
- Assumes execution at fair value (VIX strike)

Realistic Expectation:

- Sharpe ratios will degrade with costs (estimate 20-40% reduction)
- Core insight remains: residual-based filtering outperforms

Thank you for listening !

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7 Appendix

Appendix: GARCH Model Selection & Diagnostics

Best Model: EGARCH(2,1) with Skewed Student-t

Pre-Fit Tests:

ADF p-value	2.1×10^{-25}
KPSS p-value	0.100
ARCH-LM p-value	2.2×10^{-292}
Jarque-Bera p-value	0.000

Post-Fit Tests:

ARCH-LM p-value	0.839
ARCH Pass	✓
Ljung-Box Pass	✓

EGARCH Parameters:

μ	0.0223
ω	0.0036
α_1	-0.0496
α_2	0.2079
γ_1 (leverage)	**-0.1633**
β_1	0.9724
η (d.o.f.)	7.997
λ (skew)	-0.135

BIC: 15,401.93

Training Period: 1993-03-15 to 2015-12-31

Appendix: Model Performance — RMSE by Horizon

Root Mean Square Error (lower is better)

	GARCH			LSTM-RV			LSTM-VIX		
h	Train	Val	Test	Train	Val	Test	Train	Val	Test
2	0.129	0.097	0.149	0.139	0.096	0.129	0.129	**0.090**	**0.120**
5	0.083	0.069	0.107	0.086	0.069	0.092	0.080	**0.061**	**0.080**
10	0.070	0.063	0.102	0.073	0.062	0.078	0.068	**0.056**	**0.071**
30	0.069	0.060	0.113	0.070	0.058	0.072	0.067	**0.050**	**0.063**

Key Observations:

- LSTM-VIX achieves lowest validation and test RMSE across all horizons
- GARCH shows higher test RMSE (potential overfitting to training regime)
- Longer horizons have lower absolute RMSE (smoother targets)

Appendix: Model Performance — MAE by Horizon

Mean Absolute Error (lower is better)

	GARCH			LSTM-RV			LSTM-VIX		
h	Train	Val	Test	Train	Val	Test	Train	Val	Test
2	0.094	0.073	0.101	0.085	0.057	0.082	0.082	**0.055**	**0.079**
5	0.057	0.052	0.068	0.056	0.045	0.060	0.052	**0.040**	**0.055**
10	0.048	0.049	0.063	0.046	0.045	0.051	0.043	**0.037**	**0.047**
30	0.046	0.050	0.063	0.041	0.046	0.048	0.039	**0.036**	**0.042**

Key Observations:

- MAE confirms LSTM-VIX superiority
- MAE less sensitive to outliers than RMSE
- Consistent ranking across all horizons

Appendix: Ensemble Weights by Horizon

Optimized on Validation Set (2016-2019)

Horizon	GARCH	LSTM-RV	LSTM-VIX
h=2	0.50	0.00	0.50
h=5	0.30	0.00	0.70
h=10	0.30	0.00	0.70
h=30	0.20	0.00	0.80

Pattern:

- LSTM-RV receives **zero weight** at all horizons
- LSTM-VIX dominates (50-80% weight)
- GARCH weight decreases with horizon (50% $\rightarrow$ 20%)
- Longer horizons favor forward-looking VIX information

Appendix: Strategy Metrics — All Horizons (1/2)

Equity Strategies (h=30 shown, identical across horizons)

Strategy	Total Ret	Ann. Ret	Vol	Sharpe	Max DD	Beta
Buy & Hold	97.7%	12.5%	21.0%	0.60	-35.7%	1.00
SMA Trend	76.5%	10.3%	11.6%	0.89	-21.2%	0.30
Trend + VIX	109.1%	13.6%	15.1%	0.91	-27.1%	0.39
Trend + Pred	104.8%	13.2%	15.2%	0.87	-27.1%	0.40

VRP Unconditional & VIX-Based (identical across horizons)

Strategy	Total Ret	Ann. Ret	Vol	Sharpe	Max DD	Beta
VRP Unconditional	1013%	51.8%	27.9%	1.86	-96.4%	0.18
VRP VIX-Based	72.4%	9.9%	8.1%	1.22	-58.2%	0.00

Appendix: Strategy Metrics — All Horizons (2/2)

VRP Residual-Based — Performance by Horizon

h	Total Ret	Ann. Ret	Vol	Sharpe	Max DD	Alpha
2	245%	23.9%	5.4%	4.39	-27.7%	0.21
5	**388%**	**31.6%**	**5.3%**	**5.94**	-27.7%	**0.27**
10	260%	24.8%	5.1%	4.84	-27.7%	0.22
30	270%	25.4%	5.1%	4.97	-27.7%	0.23

Bear vs Bull Market Returns (Residual-Based)

h	Bear Ann. Ret	Bull Ann. Ret	Beta	Correlation
2	+3.2%	+34.7%	0.010	0.038
5	**+14.1%**	**+40.4%**	0.015	0.059
10	+11.3%	+31.6%	0.013	0.052
30	+11.3%	+32.5%	0.013	0.054

Appendix: Test Period Market Conditions

Test Set: 2020-01-01 to 2025-11-27

Statistic	Value
Total Trading Days	1,455
Bear Market Days	456 (31%)
Bull Market Days	999 (69%)

Major Events in Test Period:

- COVID-19 Crash (Feb-Mar 2020): VIX peaked at 82
- Recovery Rally (Apr 2020 - Dec 2021)
- 2022 Bear Market: Inflation, rate hikes
- 2023-2025: Mixed volatility regimes

SPY Performance: +97.7% total, 12.5% annualized, -35.7% max DD

Appendix: Data Splits & Training Details

Temporal Splits:

Split	Period	Days
Training	1993-03-15 to 2015-12-31	5,744
Validation	2016-01-01 to 2019-12-31	1,006
Test	2020-01-01 to 2025-11-27	1,455

LSTM Hyperparameters:

- Sequence length: 60 days
- Hidden size: 64
- Layers: 2
- Bidirectional: Yes
- Dropout: 0.2
- Learning rate: 0.001
- Batch size: 32
- Early stopping patience: 10

Lookahead Bias Prevention: Training gap of $h \times 1.5$ calendar days