es 1) a) $\int \operatorname{sen}(2x)\,dx = \frac{1}{2}\cdot\int 2\operatorname{sen}(2x)\,dx = -\frac{1}{2}\cos(2x)+c$

(oppure $2x=t$)

$$\int_0^{\pi/3}\operatorname{sen}(2x)\,dx = \left[-\frac{1}{2}\cos(2x)\right]_0^{\pi/3} = -\frac{1}{2}\left[\cos(2x)\right]_0^{\pi/3} = -\frac{1}{2}\left[\cos\left(\frac{2}{3}\pi\right)-\cos 0\right] =$$

$$= -\frac{1}{2}\left(-\frac{1}{2}-1\right) = -\frac{1}{2}\cdot\left(-\frac{3}{2}\right) = \boxed{\frac{3}{4}}$$

$\cos = -\frac{1}{2}$

b) $\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\cdot\int -2x\cdot(4-x^2)^{1/2}\,dx = -\frac{1}{2}\cdot\frac{(4-x^2)^{3/2}}{3/2}+c =$

$= -\frac{1}{3}(4-x^2)^{3/2}+c$ (oppure $4-x^2=t$)

$$\int_0^2 x\sqrt{4-x^2}\,dx = -\frac{1}{3}\left[(4-x^2)^{3/2}\right]_0^2 =$$

$$= -\frac{1}{3}\left[0-4^{3/2}\right] = -\frac{1}{3}\left[-8\right] = \boxed{\frac{8}{3}}$$

ricordiamo che $a^{3/2} = \sqrt{a^3} = (\sqrt{a})^3 = a\sqrt{a}$ definito solo se $a \geq 0$

c) $\int 2x^2\cdot(x^3-2)^2\,dx = \frac{2}{3}\int 3x^2\cdot(x^3-2)^2\,dx =$

$= \frac{2}{3}\cdot\frac{(x^3-2)^3}{3}+c = \boxed{\frac{2}{9}(x^3-2)^3+c}$ (oppure $x^3-2=t$)

d) $$\int_0^4 x^{3/2}\,dx = \left[\frac{x^{5/2}}{\frac{5}{2}}\right]_0^4 = \frac{2}{5}\left[x^{5/2}\right]_0^4 = \frac{2}{5}\left[4^{5/2}-0\right] = \frac{2}{5}\cdot 32 = \boxed{\frac{64}{5}}$$

e) $\int (1+2x)^4\,dx = \frac{1}{2}\int 2(1+2x)^4\,dx = \frac{1}{2}\cdot\frac{(1+2x)^5}{5}+c = \frac{1}{10}(1+2x)^5+c$

(oppure $1+2x=t$)

$$\int_{-1}^1 (1+2x)^4\,dx = \frac{1}{10}\left[(1+2x)^5\right]_{-1}^1 = \frac{1}{10}\left[3^5-(-1)^5\right] = \frac{1}{10}\left[243+1\right] = \frac{244}{10} = \boxed{\frac{122}{5}}$$

f) $\int x\cos x\,dx \underset{\text{PER PARTI}}{=} x\operatorname{sen}x - \int \operatorname{sen}x\,dx = \boxed{x\operatorname{sen}x+\cos x+c}$

$f(x)=x$
$f'(x)=1$
$g'(x)=\cos x \quad g(x)=\operatorname{sen}x$

g) $\int \operatorname{sen}^3 x\cdot\cos x\,dx = \frac{\operatorname{sen}^4 x}{4}+c$

(oppure $\operatorname{sen}x=t$)

$$\int_{-\pi/4}^0 \operatorname{sen}^3 x\cdot\cos x\,dx = \frac{1}{4}\left[\operatorname{sen}^4 x\right]_{-\pi/4}^0 = \frac{1}{4}\left[0-\left(-\frac{\sqrt{2}}{2}\right)^4\right] = \frac{1}{4}\left[-\frac{1}{4}\right] = \boxed{-\frac{1}{16}}$$

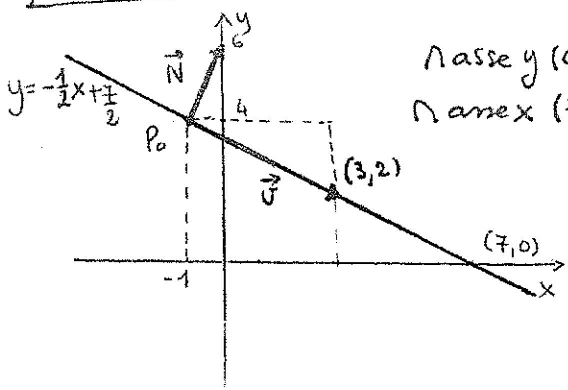
es.2) Sol.^{ni} a pag. 127-130 (VETTORI su ELLY)

es.3) a) $y = 4 - \frac{1}{2}(x+1)$ $\boxed{y = -\frac{1}{2}x + \frac{7}{2}}$ per $x = -1 \rightarrow P_0 = (-1, 4)$

per $x = 3 \rightarrow P_1 = (3, 2)$

quindi il vettore $\vec{v} = P_1 - P_0 = (4, -2)$ dirige la retta che si può dunque scrivere $P = P_0 + t\vec{v}$ $t \in \mathbb{R}$, cioè $\boxed{P = (-1, 4) + t(4, -2) \quad t \in \mathbb{R}}$

(da $\vec{v} = (4, -2)$ si deduce $m = \frac{-2}{4} = -\frac{1}{2}$ che è corretto)



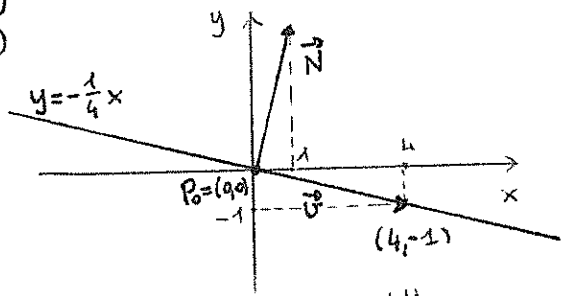
$\cap$ asse y $(0, \frac{7}{2})$
$\cap$ asse x $(7, 0)$

Dall'eq.^{ne} $x + 2y - 7 = 0$ si deduce che un vettore normale alla retta è $\vec{N} = (1, 2)$ da cui

$(P - P_0) \cdot \vec{N} = 0$ cioè la retta si può anche scrivere come $\boxed{(x+1, y-4) \cdot (1, 2) = 0}$

b) $m = \frac{m_a}{2} = -\frac{1}{4}$ $\boxed{y = -\frac{1}{4}x}$ $x = 4 \rightarrow y = -1$ (P_1) $\vec{v} = (4, -1) = P_1 - P_0$

$P_0 = (0, 0)$



$\boxed{P = t(4, -1) \quad t \in \mathbb{R}}$

eq.^{ne} $x + 4y = 0 \rightarrow \vec{N} = (1, 4)$

$\boxed{(x, y) \cdot (1, 4) = 0}$

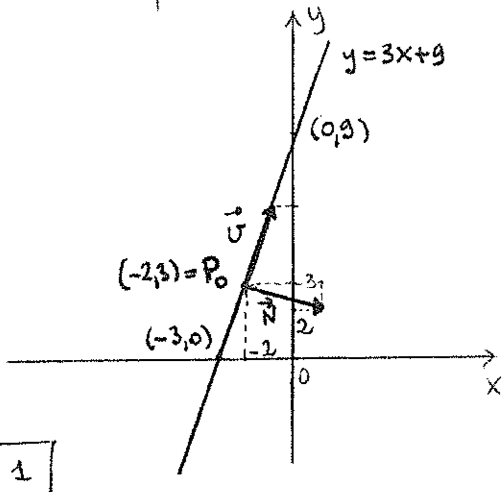
c) $\boxed{P = (-2, 3) + t(1, 3) \quad t \in \mathbb{R}}$ $m = \frac{v_2}{v_1} = 3$

$y = 3 + 3(x+2)$ $\boxed{y = 3x + 9}$ $\cap$ asse y $(0, 9)$

$\cap$ asse x $(-3, 0)$

$3x - y + 9 = 0 \rightarrow \vec{N} = (3, -1)$

$\boxed{(x+2, y-3) \cdot (3, -1) = 0}$

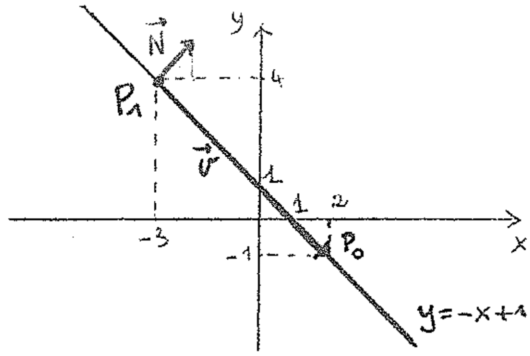


d) $m = \frac{\Delta y}{\Delta x} = \frac{5}{-5} = -1$ $y = -1 - (x-2)$ $\boxed{y = -x + 1}$

$\vec{v} = P_0 - P_1 = (5, -5)$ $\boxed{P = (2, -1) + t(5, -5) \quad t \in \mathbb{R}}$

$\cap$ asse y $(0, 1)$
$\cap$ asse x $(1, 0)$

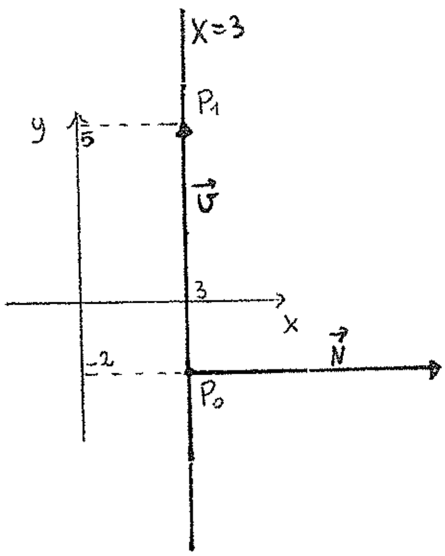
$x+y-1=0 \to \vec{N}=(1,1)$



$$\boxed{(x-2,y+1)\cdot(1,1)=0}$$

e) $x_{P_0}=x_{P_1}=3 \Rightarrow$ la retta è VERTICALE di eq.^{ne} $\boxed{x=3}$

m non è definito



$\vec{v}=P_1-P_0=(0,7)$ oppure $\vec{v}=(0,b)$ qualunque b va bene purché $b\neq 0$

$$\boxed{P=(3,-2)+t(0,7) \quad t\in\mathbb{R}}$$

Risulta normale alla retta un qualunque vettore ORIZZONTALE $\vec{N}=(b,0)$, ad es. $\vec{N}=(7,0)$

$$\boxed{(x-3,y+2)\cdot(7,0)=0}$$

$7\cdot(x-3)=0$

$(x-3)=0$

$\boxed{x=3}$

f) $\vec{N}=(-2,4)$ $P_0=(3,1)$

$$\boxed{(x-3,y-1)\cdot(-2,4)=0}$$

da cui, se si vuole, $-2(x-3)+4(y-1)=0$

$-2x+4y+6-4=0$ $4y=2x-2$ $\boxed{y=\frac{1}{2}x-\frac{1}{2}}$

altro modo: $\vec{N}$ dirige la retta $\perp$ a quella che stiamo cercando (r)

$\vec{N}=(-2,4) \Rightarrow m_\perp=\frac{4}{-2}=-2 \Rightarrow m_r=\frac{1}{2} \Rightarrow y=1+\frac{1}{2}(x-3) \Rightarrow y=\frac{1}{2}x-\frac{1}{2}$

per $x=1 \to y=0$ $P_1=(1,0)$

$\vec{v}=P_0-P_1=(2,1)$

$$\boxed{P=(3,1)+t(2,1) \quad t\in\mathbb{R}}$$