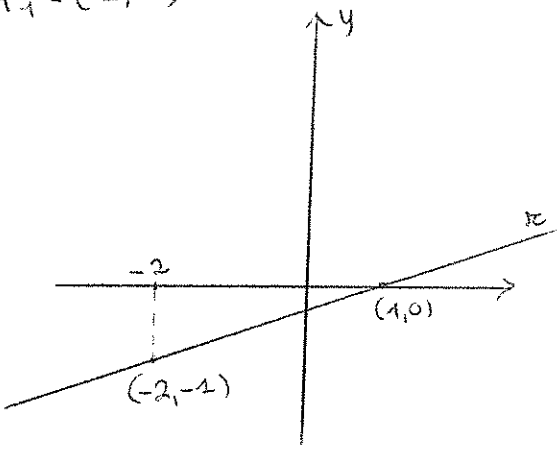
SOLUZIONE Scheda N.3 di ESERCIZI

1) a) <u>1° modo</u> retta per $P_0 = (-2,-1)$ e $P_1 = (1,0)$

eq.^{ne} cartesiana $y = \frac{1}{3}x - \frac{1}{3}$

eq.^{ni} param ① $\begin{cases} x = t \\ y = \frac{1}{3}t - \frac{1}{3} \end{cases}$ $t \in \mathbb{R}$

verso x crescenti



<u>2° modo</u> retta per $P_0 = (-2,-1)$ con direzione $\vec{v} = P_1 - P_0$

eq.^{ni} param. ② $\begin{cases} x = -2 + 3t \\ y = -1 + t \end{cases}$ $t \in \mathbb{R}$

verso x crescenti

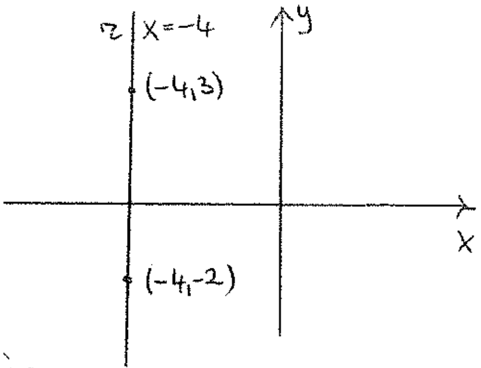
<u>3° modo</u> retta per $P_1 = (1,0)$ con direzione $\vec{v} = P_0 - P_1$

eq.^{ni} param. ③ $\begin{cases} x = 1 - 3t \\ y = -t \end{cases}$ $t \in \mathbb{R}$ verso x decrescenti

b) <u>1° modo</u> eq.^{ne} cartesiana $x = -4$

eq.^{ni} param ① $\begin{cases} x = -4 \\ y = t \end{cases}$ $t \in \mathbb{R}$

verso y crescenti



<u>2° modo</u> retta per $P_0 = (-4,-2)$ con direzione $\vec{v} = P_1 - P_0 = 5\vec{j}$

eq.^{ni} param ② $\begin{cases} x = -4 \\ y = -2 + 5t \end{cases}$ $t \in \mathbb{R}$ verso y crescenti

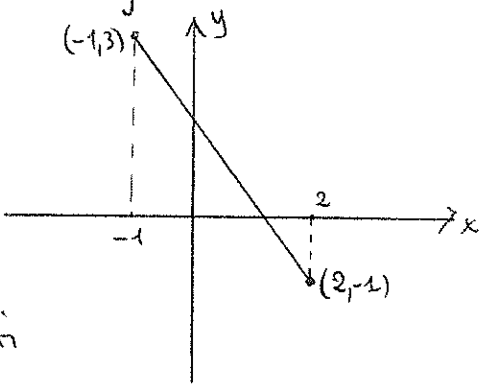
<u>3° modo</u> retta per $P_1 = (-4,3)$ con direz $\vec{v} = P_0 - P_1 = -5\vec{j}$

eq.^{ni} param ③ $\begin{cases} x = -4 \\ y = 3 - 5t \end{cases}$ $t \in \mathbb{R}$ verso y decrescenti

c) <u>1° modo</u> $P_0 = (-1,3)$ $P_1 = (2,-1)$

eq.^{ne} cartesiana $y = -\frac{4}{3}x + \frac{5}{3}$

eq.^{ni} param ① $\begin{cases} x = t \\ y = -\frac{4}{3}t + \frac{5}{3} \end{cases}$ $t \in [-1,2]$ verso x crescenti



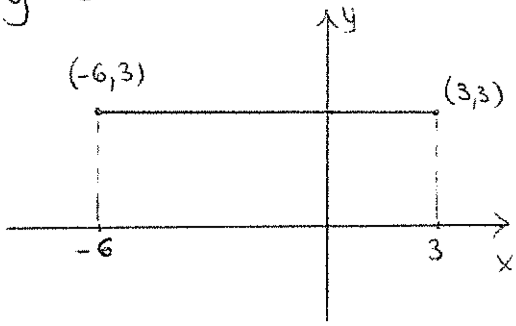
<u>2° modo</u> $P_0=(-1,3)$ $\vec{v}=P_1-P_0$

eq.^{ni} param ② $\begin{cases} x=-1+3t \\ y=3-4t \end{cases}$ $t\in[0,1]$ verso x crescenti

<u>3° modo</u> $P_1=(2,-1)$ $\vec{v}=P_0-P_1$

eq.^{ni} param. ③ $\begin{cases} x=2-3t \\ y=-1+4t \end{cases}$ $t\in[0,1]$ verso x decrescenti

d) <u>1° modo</u> $P_0=(-6,3)$ $P_1=(3,3)$ eq.^{ne} cartesiana $y=3$



eq.^{ni} param ① $\begin{cases} x=t \\ y=3 \end{cases}$ $t\in[-6,3]$ verso x crescenti

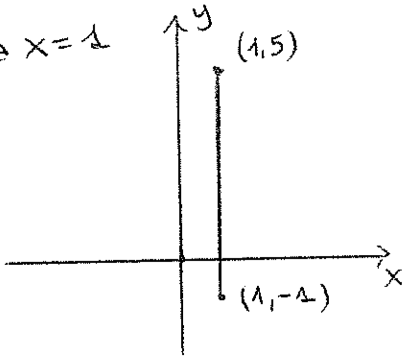
<u>2° modo</u> $P_0=(-6,3)$ $\vec{v}=P_1-P_0$

eq.^{ni} param ② $\begin{cases} x=-6+9t \\ y=3 \end{cases}$ $t\in[0,1]$ verso x crescenti

<u>3° modo</u> $P_1=(3,3)$ $\vec{v}=P_0-P_1$

eq.^{ni} param ③ $\begin{cases} x=3-9t \\ y=3 \end{cases}$ $t\in[0,1]$ verso x decrescenti

e) <u>1° modo</u> $P_0=(1,-1)$ $P_1=(1,5)$ eq.^{ne} cartesiana $x=1$



eq.^{ni} param ① $\begin{cases} x=1 \\ y=t \end{cases}$ $t\in[-1,5]$ verso y crescenti

<u>2° modo</u> $P_0=(1,-1)$ $\vec{v}=P_1-P_0$

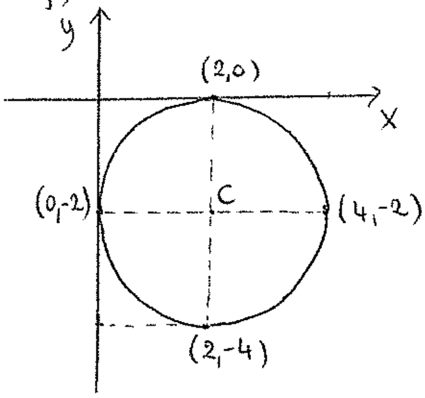
eq.^{ni} param ② $\begin{cases} x=1 \\ y=-1+6t \end{cases}$ $t\in[0,1]$ verso y crescenti

<u>3° modo</u> $P_1=(1,5)$ $\vec{v}=P_0-P_1$

eq.^{ni} param ③ $\begin{cases} x=1 \\ y=5-6t \end{cases}$ $t\in[0,1]$ verso y decrescenti

f) verso antiorario $\begin{cases} x=2+2\cos t \\ y=-2+2\operatorname{sen} t \end{cases}$

- $t\in[0,2\pi]$ 1 giro
- $t\in[\frac{\pi}{2},\frac{3}{2}\pi]$ $\frac{1}{2}$ giro
- $t\in[\pi,\frac{5}{2}\pi]$ $\frac{3}{4}$ giro oppure $t\in[-\pi,\frac{\pi}{2}]$



verso orario $\begin{cases} x=2+2\cos t \\ y=-2-2\operatorname{sen} t \end{cases}$

- $t\in[\frac{3}{2}\pi,\frac{7}{2}\pi]$ oppure $t\in[-\frac{\pi}{2},\frac{3}{2}\pi]$

Sol.^{ne}
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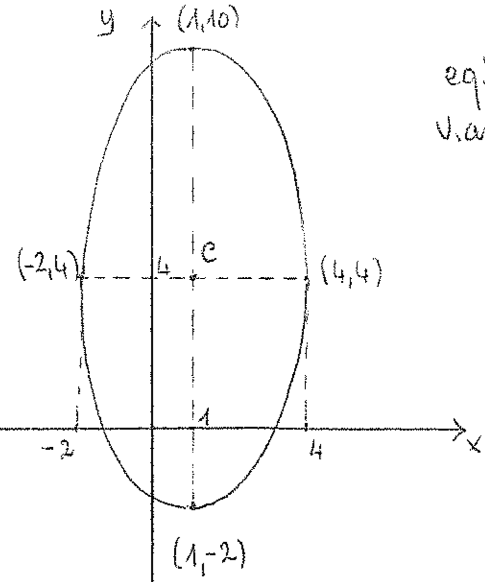
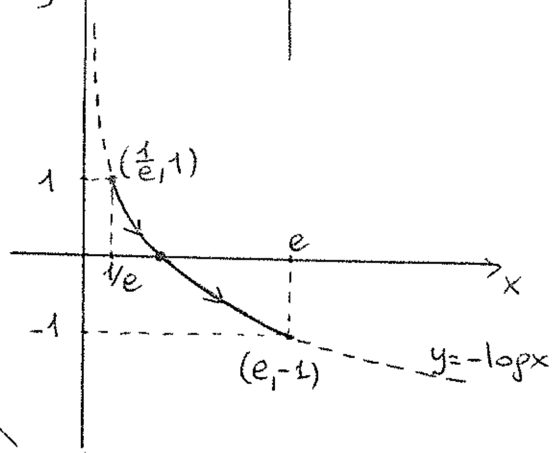
g)



eq.^{ni} v.antiorario $\begin{cases} x = 1 + 3\cos t \\ y = 4 + 6\,\text{sen}\,t \end{cases}$

- $t \in [0, 2\pi]$ 1 giro
- $t \in [0, \pi]$ $\frac{1}{2}$ giro da $(4,4)$
- $t \in [\pi, 2\pi]$ opp $t \in [-\pi, 0]$ $\frac{1}{2}$ giro da $(-2,+4)$
- $t \in [\frac{3}{2}\pi, 3\pi]$ opp $t \in [-\frac{\pi}{2}, \pi]$ $\frac{3}{4}$ giro

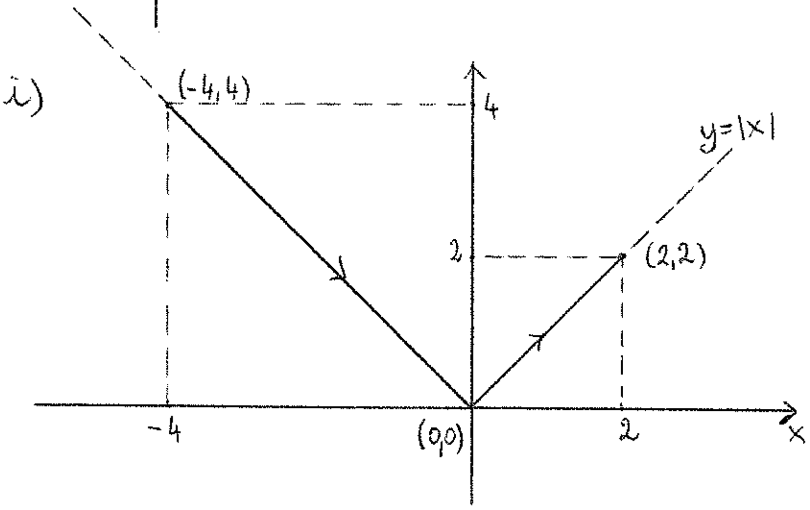
h)



$y = -\log x$ è il simmetrico di $y = \log x$ rispetto all'asse x

$\begin{cases} x = t \\ y = -\log t \end{cases}$ $t \in [\frac{1}{e}, e]$ verso x crescenti

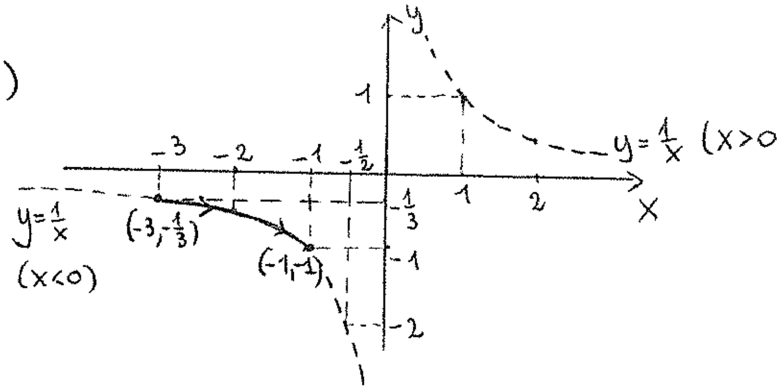
i)



$\begin{cases} x = t \\ y = |t| \end{cases}$ $t \in [-4, 2]$

verso x crescenti

j)



$\begin{cases} x = t \\ y = \frac{1}{t} \end{cases}$ $t \in [-3, -1]$

verso x crescenti

2) (E_1) $x^2+y^2=1$ circonf. $C(0,0)$ $R=1$ $1 \le x^2+y^2 \le 16$ CORONA CIRCOLARE

$x^2+y^2=16$ " " $R=4$

bisettrice 2° e 4° q

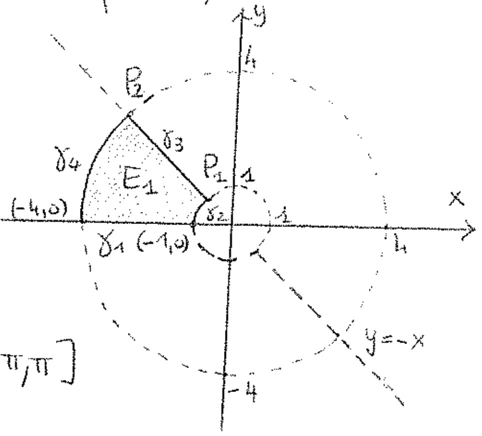
$y \le -x$ al di sotto della retta $y=-x$, retta compresa, $y \geq 0$

$y=-x \cap x^2+y^2=1 \rightarrow \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)=P_1$

$y=-x \cap x^2+y^2=16 \rightarrow (-2\sqrt{2}, 2\sqrt{2})$

$\gamma_1 \begin{cases} x=t \\ y=0 \end{cases} t \in [-4,-1]$ $\gamma_2 \begin{cases} x=\cos t \\ y=\operatorname{sen} t \end{cases} t \in \left[\frac{3}{4}\pi, \pi\right]$

$\gamma_3 \begin{cases} x=t \\ y=-t \end{cases} t \in \left[-2\sqrt{2}, -\frac{\sqrt{2}}{2}\right]$ $\gamma_4 \begin{cases} x=4\cos t \\ y=4\operatorname{sen} t \end{cases} t \in \left[\frac{3}{4}\pi, \pi\right]$



(E_2) $\frac{x^2}{4/3}+\frac{y^2}{4} \le 1$ INTERNO + BORDO dell'ellisse di $C(0,0)$

e semiassi $a=\frac{2}{\sqrt{3}}$ $b=2$, $y \ge x$ sopra la retta

$a \simeq 1,15$

$y=x$, retta compresa ($y=x$ bisettrice 1°-3° q)

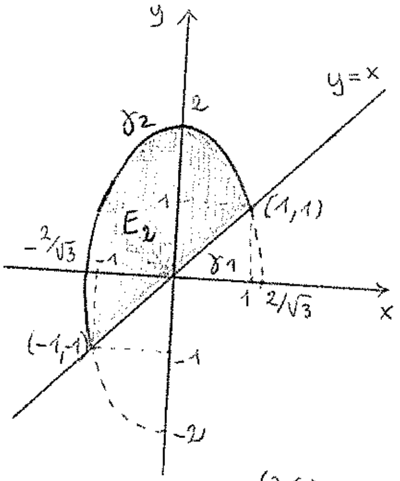
$y=x \cap \frac{x^2}{4/3}+\frac{y^2}{4}=1 \rightarrow x^2=1 \rightarrow x=\pm 1$

$(-1,1)$ e $(1,1)$

$\gamma_1 \begin{cases} x=t \\ y=t \end{cases} t \in [-1,1]$ $\gamma_2 \begin{cases} x=\frac{2}{\sqrt{3}}\cos t \\ y=2\operatorname{sen} t \end{cases} t \in \left[\frac{\pi}{6}, \frac{7}{6}\pi\right]$

$\begin{cases} -1=\frac{2}{\sqrt{3}}\cos t \\ -1=2\operatorname{sen} t \end{cases} \begin{cases} \cos t=-\frac{\sqrt{3}}{2} \\ \operatorname{sen} t=-\frac{1}{2} \end{cases} \rightarrow t_{(-1,-1)}=\frac{7}{6}\pi$

$\begin{cases} 1=\frac{2}{\sqrt{3}}\cos t \\ 1=2\operatorname{sen} t \end{cases} \begin{cases} \cos t=\sqrt{3}/2 \\ \operatorname{sen} t=1/2 \end{cases} \rightarrow t_{(1,1)}=\frac{\pi}{6}$



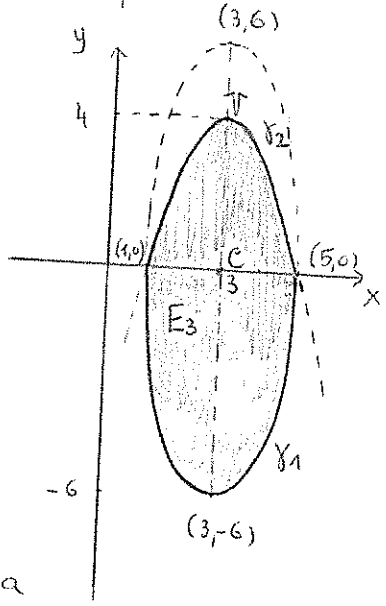
(E_3) $\frac{(x-3)^2}{4}+\frac{y^2}{36} \le 1$ INTERNO + BORDO dell'ellisse

di $C(3,0)$ e semiassi $a=2$ $b=6$

$y=-x^2+6x-5$ è una parabola di $V(3,4)$

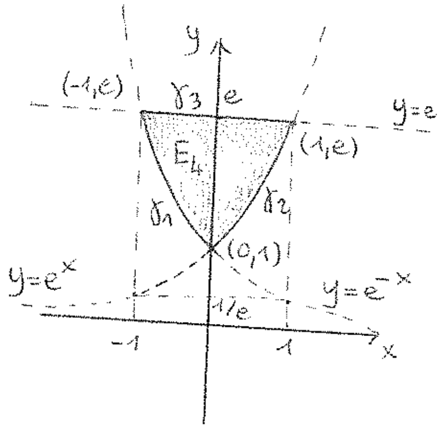
verso il basso, $\cap\, y=0 \rightarrow x=1, x=5$

si deve stare sotto la parabola, parabola compresa



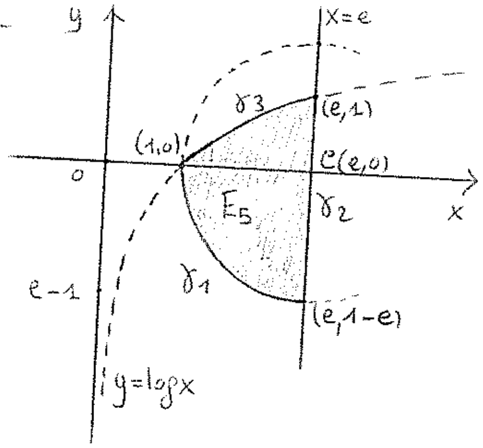
$\gamma_1 \begin{cases} x = 3 + 2\cos t \\ y = 6\,\text{sen}\,t \end{cases}$ $t \in [\pi, 2\pi]$ $\gamma_2 \begin{cases} x = t \\ y = -t^2 + 6t + 5 \end{cases}$ $t \in [1, 5]$

E_4 $y = e^x$ grafico dell'esponenziale, $x = 1 \to y = e$
$x = -1 \to y = 1/e$
(si deve stare al di sopra), $y = e$ retta ORIZZONTALE (si deve stare sotto)
$y = e^{-x}$ simmetrico del grafico di e^x rispetto all'asse y (si deve stare al di sopra)
$x = -1 \to y = e$, $x = 1 \to y = \frac{1}{e} \simeq 0{,}37$



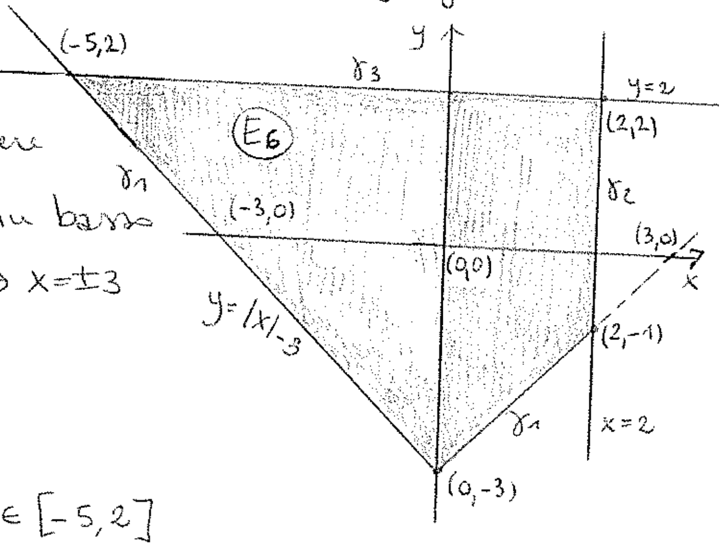
$\gamma_1 \begin{cases} x = t \\ y = e^{-t} \end{cases}$ $t \in [-1, 0]$ $\gamma_2 \begin{cases} x = t \\ y = e^t \end{cases}$ $t \in [0, 1]$ $\gamma_3 \begin{cases} x = t \\ y = e \end{cases}$ $t \in [-1, 1]$

E_5 $x = e$ retta verticale, si deve stare a sinistra retta compresa
$y = \log x$ grafico del logaritmo in base e
si deve stare al di sotto
$(x-e)^2 + y^2 \leq (e^2 - 2e + 1) = (e-1)^2$
cerchio (interno + bordo) di $C(e, 0)$
e $R = e - 1$



$\gamma_1 \begin{cases} x = e + (e-1)\cos t \\ y = (e-1)\,\text{sen}\,t \end{cases}$ $t \in [\pi, \frac{3}{2}\pi]$ $\gamma_2 \begin{cases} x = e \\ y = t \end{cases}$ $t \in [1-e, 1]$ $\gamma_3 \begin{cases} x = t \\ y = \log t \end{cases}$ $t \in [1, e]$

E_6 $x = 2$ retta verticale, si deve stare a sinistra
$y = 2$ retta orizzontale, si deve stare sotto
$y = |x| - 3$ grafico del $|x|$ spostato in basso di 3 (si deve stare sopra), $y = 0 \to x = \pm 3$



$\gamma_1 \begin{cases} x = t \\ y = |t| - 3 \end{cases}$ $t \in [-5, 2]$

$\gamma_2 \begin{cases} x = 2 \\ y = t \end{cases}$ $t \in [-1, 2]$ $\gamma_3 \begin{cases} x = t \\ y = 2 \end{cases}$ $t \in [-5, 2]$

$\boxed{\text{LUNGH}}$ $\boxed{L(\partial E_1)} = \int_{-4}^{1} 1\,dt + \int_{3/4\pi}^{\pi} 1\,dt + \int_{-2\sqrt{2}}^{-\frac{\sqrt{2}}{2}} \sqrt{2}\,dt + \int_{3/4\pi}^{\pi} 4\,dt =$

$\|\gamma_1'\| = 1$ $\|\gamma_2'\| = 1$ $\|\gamma_3'\| = \sqrt{2}$ $\|\gamma_4'\| = 4$

$\gamma_1' = (1,0)$ $\gamma_2'(t) = (-\text{sen}t, \cos t)$ $\gamma_3'(t) = (1,-1)$ $\gamma_4'(t) = (-4\,\text{sen}t, 4\cos t)$

$$= 3 + \frac{\pi}{4} + \underbrace{\sqrt{2}\left(-\frac{\sqrt{2}}{2} + 2\sqrt{2}\right)}_{-1+4} + \underbrace{4\cdot\frac{\pi}{4}}_{\pi} = \boxed{6 + \frac{5}{4}\pi}$$

In modo elementare : $L(\gamma_1) = \text{segmento} = 3 = L(\gamma_3)$

$L(\gamma_2) = \frac{1}{8}(2\pi \overset{1}{R}) = \frac{\pi}{4}$ $\quad L(\gamma_4) = \frac{1}{8}(2\pi \overset{4}{R}) = \pi$

$$\boxed{L(\partial E_6)} = \int_{-5}^{0} \sqrt{2}\,dt + \int_{0}^{2} \sqrt{2}\,dt + \int_{-1}^{2} 1\,dt + \int_{-5}^{2} 1\,dt = 5\sqrt{2} + 2\sqrt{2} + 3 + 7 = \boxed{10 + 7\sqrt{2}}$$

spezzando γ_1 in due tratti di classe C^1: $\gamma_{1,1} \begin{cases} x = t \\ y = -t-3 \end{cases}$ $t \in [-5,0]$

$\gamma_{1,2} \begin{cases} x = t \\ y = t-3 \end{cases}$ $t \in [0,2]$ $\begin{matrix} t \geq 0 \\ |t| = t \end{matrix}$ $\qquad t \leq 0$ $|t| = -t$

$\|\gamma_{1,1}'\| = \sqrt{2}$ $\|\gamma_{1,2}'\| = \sqrt{2}$ $\|\gamma_2'\| = 1$ $\|\gamma_3'\| = 1$

$\gamma_{1,1}' = (1,-1)$ $\gamma_{1,2}' = (1,1)$ $\gamma_2' = (0,1)$ $\gamma_3' = (1,0)$

In modo elementare sono 4 segmenti : $L(\gamma_1) = L(\gamma_{1,1}) + L(\gamma_{1,2}) = 7\sqrt{2} + 2\sqrt{2}$
$L(\gamma_2) = 3$ $L(\gamma_3) = 7$.

Teorema Tutte le curve di cui è stata calcolata la lunghezza sono definite su un intervallo chiuso e limitato ($[a,b]$) e sono di classe C^1, pertanto si può applicare il teorema che permette di calcolare la $L(\gamma)$ tramite un integrale.

Ad es. in $L(\partial E_1)$ γ_1 è definita su $[-4,1]$ e $\gamma_1' = (1,0)$ quindi γ_1 è derivabile e le derivate $x_1' = 1$ $y_1' = 0$ sono continue, γ_2 è definita su $[\frac{3}{4}\pi, \pi]$ e $\gamma_2' = (-\text{sen}t, \cos t)$ quindi γ_2 è derivabile e le derivate $x_2' = -\text{sen}t$ $y_2' = \cos t$ sono continue e così via.

3) γ_1 $\gamma_1'(t)=(e^t\cos t-e^t\,\text{sen}\,t,\ e^t\,\text{sen}\,t+e^t\cos t)$

L'intervallo è $[0,2\pi]$ chiuso e limitato, γ_1 è continua e derivabile in quanto $x(t)$ e $y(t)$ sono prodotto di funzioni continue e derivabili e $\gamma_1'(t)$ è continua in quanto $x'(t)$ e $y'(t)$ sono somma, prodotto e differenze di funzioni continue. Allora γ_1 è di classe C^1.

$$\|\gamma_1'(t)\|=\sqrt{(e^t(\cos t-\text{sen}\,t))^2+(e^t(\text{sen}\,t+\cos t))^2}=$$

$$=\sqrt{e^{2t}\cdot(\underbrace{\cos^2 t+\text{sen}^2 t}_{1}-\cancel{2\,\text{sen}\,t}\cos t+\underbrace{\text{sen}^2 t+\cos^2 t}_{1}+2\cancel{\text{sen}\,t}\cos t)}$$

$$=\sqrt{2e^{2t}}=\sqrt{2}\,|e^t|\underset{\substack{\downarrow\\ e^t>0\ \forall t}}{=}\sqrt{2}\cdot e^t$$

Quindi per il teorema sulla lunghezza di una curva di classe C^1

$$L(\gamma_1)=\int_0^{2\pi}\sqrt{2}\cdot e^t\,dt=\sqrt{2}\,\left[e^t\right]_0^{2\pi}=\sqrt{2}\,(e^{2\pi}-e^0)=\boxed{\sqrt{2}\,(e^{2\pi}-1)}$$

$t_0=\frac{\pi}{2}\to P_0=(0,e^{\pi/2})$ $\gamma'_{1,P_0}=\gamma_1'(\frac{\pi}{2})=\vec{v}_{P_0}=(-e^{\pi/2},e^{\pi/2})=-e^{\pi/2}\,\vec{\imath}+e^{\pi/2}\,\vec{\jmath}$

γ_2 L'intervallo è $[0,\pi]$ chiuso e limitato, γ_2 è continua e derivabile in quanto $x(t)$ e $y(t)$ sono prodotto di funzioni continue e derivabili $(\text{sen}\,t,\cos t)$.

$\gamma_2'(t)=(\cos^2 t-\text{sen}^2 t,\ 2\,\text{sen}\,t\cos t)\longrightarrow$ γ_2 è di classe C^1 perché $x'(t)$ e $y'(t)$ sono continue (prodotto, differenza di funzioni continue)

$$\|\gamma_2'(t)\|=\sqrt{(\cos^2 t-\text{sen}^2 t)^2+(2\,\text{sen}\,t\cos t)^2}=\sqrt{\cos^4 t+\text{sen}^4 t-2\,\text{sen}^2 t\cos^2 t+4\,\text{sen}^2 t\cos^2 t}=\sqrt{\cos^4 t+2\,\text{sen}^2 t\cos^2 t+\text{sen}^4 t}=$$

$$=\sqrt{(\underbrace{\cos^2 t+\text{sen}^2 t}_{=1})^2}=1\quad\forall t$$

Stesso risultato osservando che per le formule di duplicazione

$$\gamma_2'(t)=(\cos(2t),\text{sen}(2t))\longrightarrow\|\gamma_2'(t)\|=\sqrt{\underbrace{(\cos(2t))^2+(\text{sen}(2t))^2}_{1}}=1$$

Quindi $L(\gamma_2)=\int_0^{\pi} 1\,dt = \boxed{\pi}$.

<u>OSS.</u> $P_{in}=P_{fin}=(0,0)$ $\quad x^2+(y-\frac{1}{2})^2 = \text{sen}^2t\cos^2t+(\text{sen}^2t-\frac{1}{2})^2=$

$= \text{sen}^2t\cdot\cos^2t+\text{sen}^4t+\frac{1}{4}-\text{sen}^2t = \text{sen}^2t(\cos^2t\cancel{+\text{sen}^2t-1})+\frac{1}{4}=\frac{1}{4}$

$\rightarrow \quad x^2+(y-\frac{1}{2})^2=\frac{1}{4}=(\frac{1}{2})^2$

La curva percorre la circonferenza di $C(0,\frac{1}{2})$ e $R=\frac{1}{2}$ in verso antiorario per 1 giro partendo da $(0,0)$ - $L(\gamma_2)$ in modo elementare $=2\pi R=2\pi\frac{1}{2}=\pi$

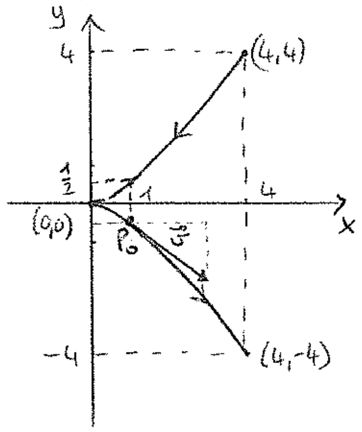
$P_0=(\frac{1}{2},\frac{1}{2}) \rightarrow t_0=\frac{\pi}{4}$ $\quad \left(\begin{cases} \frac{1}{2}=\text{sen}t\cos t \\ \frac{1}{2}=\text{sen}^2t \\ t\in[0,\pi] \end{cases} \rightarrow \begin{cases} \cos t \text{ deve essere } =\pm\frac{\sqrt{2}}{2} \text{ e concorde con sen}t \\ \text{sen}t=\pm\frac{\sqrt{2}}{2} \end{cases}\right.$

2 possibilità $\begin{cases} \text{sen}t=\frac{\sqrt{2}}{2} \\ \cos t=\frac{\sqrt{2}}{2} \end{cases}$ $t_0=\frac{\pi}{4}$ accettabile $\quad \begin{cases} \text{sen}t=-\frac{\sqrt{2}}{2} \\ \cos t=-\frac{\sqrt{2}}{2} \end{cases}$ $t_0=\frac{5}{4}\pi \notin[0,\pi]$ non accett. $\left.\right)$

$\gamma'_{2,P_0}=\gamma'_2(\frac{\pi}{4})=\vec{v}_{P_0}=\left(\frac{1}{2}-\frac{1}{2},\ 2\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)=\vec{j}$

Ⓡ γ_3 $\quad P_{in}=(4,4) \quad P_{fin}=(4,-4)$

la curva percorre il grafico di $y=\frac{1}{2}x^{3/2}$ da $(4,4)$ a $(0,0)$ nel verso delle x decrescenti (o delle y decrescenti) e poi il grafico di $y=-\frac{1}{2}x^{3/2}$ da $(0,0)$ a $(4,-4)$ nel verso delle x crescenti (o delle y decrescenti)



($t^2=x$ se $t\in[0,2]\rightarrow t=\sqrt{x}$ questo vale per $x\geq0$ e $y\geq0$, ma se $t\in[-2,0]\rightarrow t=-\sqrt{x}$ e questo vale per $x\geq0$ e $y\geq0$)

$t=-1\rightarrow(1,+\frac{1}{2})$, $t=1\rightarrow(1,-\frac{1}{2})$

$t=0\rightarrow(0,0)$

$P_0=(1,-\frac{1}{2})\overset{(*)}{\rightarrow} t_0=1 \quad \gamma'(t)=(2t,-\frac{3}{2}t^2)$

$x(t)$ sono cont.
$y(t)$ derivabili
con $x'(t), y'(t)$ cont
$\Rightarrow \gamma$ è di classe C^1

$\vec{v}_{P_0}=2\vec{i}-\frac{3}{2}\vec{j}$

$\|\gamma'(t)\|=\sqrt{4t^2+\frac{9}{4}t^4}=|t|\sqrt{4+\frac{9}{4}t^2}$

$L(\gamma)=\int_{-2}^{0}(-t)\underbrace{\sqrt{4+\frac{9}{4}t^2}}_{(f(t))^{\frac{1}{2}}}dt+\int_0^2 t\sqrt{4+\frac{9}{4}t^2}\,dt = -\frac{2}{9}\int_{-2}^{0}\frac{9}{2}t(4+\frac{9}{4}t^2)^{1/2}dt+$

$f'(t)=\frac{9}{2}t$

$+\frac{2}{9}\int_0^2\frac{9}{2}t\,(4+\frac{9}{4}t^2)^{1/2}dt=$

(*) $t^2=1$
$\rightarrow t=\pm1 \quad -\frac{1}{2}t^3=-\frac{1}{2}\rightarrow t=1$

$$= -\frac{2}{9}\left[\frac{(4+\frac{9}{4}t^2)^{3/2}}{3/2}\right]_{-2}^{0} + \frac{2}{9}\left[\frac{(4+9/4\,t^2)^{3/2}}{3/2}\right]_{0}^{2} =$$

$$= -\frac{4}{27}\left[8-13\sqrt{13}\right] + \frac{4}{27}\left[13\sqrt{13}-8\right] = \boxed{\frac{8}{27}\left[13\sqrt{13}-8\right]}$$

(alternativa al calcolo dell'integrale: cambiamento di variabile $s = 4+\frac{9}{4}t^2$

Semplificazione del calcolo di $L(\gamma)$: osservare che

$$\int_{-2}^{2}|t|\sqrt{4+\frac{9}{4}t^2}\,dt \underset{\substack{\downarrow \\ \text{per simmetria}}}{=} 2\int_{0}^{2} t\sqrt{4+\frac{9}{4}t^2}\,dt\ ).$$

γ_4 $x(t), y(t)$ sono continue e derivabili (somma, differenza e prodotto di funzioni continue e derivabili)

$$\gamma'(t) = \left(2\cdot(-\mathrm{sen}\,t+\mathrm{sen}\,t+t\cos t),\ 2(\cos t-\cos t+t\,\mathrm{sen}\,t)\right) =$$
$$= (2t\cos t,\ 2t\,\mathrm{sen}\,t)$$

essendo $x'(t), y'(t)$ continue $\Rightarrow \gamma$ è di classe C^1

$$\|\gamma'(t)\| = \sqrt{4t^2\cos^2 t + 4t^2\mathrm{sen}^2 t} = \sqrt{4t^2(\underbrace{\cos^2 t+\mathrm{sen}^2 t}_{=1})} = \sqrt{4t^2} = 2|t|$$

Essendo $[0,2\pi]$ un intervallo chiuso e limitato, per il Teorema sulla lunghezza

$$L(\gamma) = \int_0^{2\pi} 2|t|\,dt \underset{|t|=t}{=} \int_0^{2\pi} 2t\,dt = \left[t^2\right]_0^{2\pi} = \boxed{4\pi^2}$$

$P_0 \to t_0 = \frac{3}{2}\pi$ (per tentativi)
$\vec{v}_{P_0} = -3\pi\vec{j}$

γ_5 $\gamma'(t) = (-2\,\mathrm{sen}\,t\cos t,\ 2\,\mathrm{sen}\,t\cos t)$ γ è di classe C^1, $[0,\pi]$ chiuso e lim.

$$\|\gamma'(t)\| = \sqrt{(-2\,\mathrm{sen}\,t\cos t)^2+(2\,\mathrm{sen}\,t\cos t)^2} = \sqrt{8\,\mathrm{sen}^2 t\cos^2 t} = 2\sqrt{2}\,|\mathrm{sen}\,t\cos t|$$

$$L(\gamma) = \int_0^{\pi} 2\sqrt{2}\,|\mathrm{sen}\,t|\,|\cos t|\,dt \underset{|\mathrm{sen}\,t|=\mathrm{sen}\,t}{=} \sqrt{2}\left(\underset{|\cos t|=\cos t}{\int_0^{\pi/2} 2\,\mathrm{sen}\,t\cos t\,dt} + \underset{|\cos t|=-\cos t}{\int_{\pi/2}^{\pi} -2\,\mathrm{sen}\,t\cos t\,dt}\right) =$$

$$= \sqrt{2}\left(\left[\mathrm{sen}^2 t\right]_0^{\pi/2} + \left[-\mathrm{sen}^2 t\right]_{\pi/2}^{\pi}\right) = \sqrt{2}\,(1-0+0+1) = \boxed{2\sqrt{2}}$$

$t_0 = \frac{3}{4}\pi \to P_0 = \left(\frac{1}{2}, \frac{1}{2}\right)$ $\vec{v}_{P_0} = \vec{i}-\vec{j}$