UNIVERSITÀ di PARMA - INGEGNERIA GESTIONALE

ANALISI MATEMATICA 2 - SCHEDA N. 14

EQUAZIONI DIFFERENZIALI : SECONDO ORDINE

- A) i) Quante soluzioni ha un'equazione differenziale del 2° ordine?

  ii) Le soluzioni di un'eq.ne differenziale omogenea del 2° ordine possono essere date da $y(x) = c_1 e^{3x} + c_2 e^{3x}$ $(c_1, c_2 \in \mathbb{R})$? È da $y(x) = c_1 e^{-2x} + c_2 e^{4x}$ $(c_1, c_2 \in \mathbb{R})$?

  iii) La funzione $\bar{y}(x) = 3x^2 - 1$ può essere soluzione di un'eq.ne differenziale del tipo $y''(x) + a y'(x) = f(x)$ con $a \neq 0$ e $f(x)$ polinomio di 2° grado?

  iv) Le funzioni $y(x) = c_1 e^{-x} + c_2 e^{2x} + 3x^2 - 1$ $(c_1, c_2 \in \mathbb{R})$ di quale equazione differenziale sono le soluzioni?

- Risolvete le seguenti equazioni differenziali o problemi di Cauchy:

1) $y''(x) = e^{-\frac{1}{2}x}$

2) $y''(x) = 3x(x-1) - e^{-3x}$

3) $-8y''(x) = \pi \,\mathrm{sen}\left(\frac{\pi}{8}x\right)$

4) $\frac{1}{2}y''(x) - \frac{7}{2}y'(x) + 6y(x) = 0$

5) $y''(x) + 3y'(x) + 10y(x) = 0$

6) $\frac{2}{3}y''(x) - 4y'(x) + 6y(x) = 0$

7) $\begin{cases} \frac{1}{3}y''(x) - \frac{2}{3}y'(x) - y(x) = 0 \\ y(0) = 2 \\ y'(0) = 2 \end{cases}$

8) $\frac{1}{4}y''(x) - \frac{1}{2}y'(x) + \frac{17}{4}y(x) = 0$

9) $2y''(x) - 2y'(x) + \frac{1}{2}y(x) = 0$

10) $\begin{cases} y''(x) + \frac{1}{2}y'(x) - 3y(x) = 0 \\ y(0) = -1 \\ y'(0) = 2 \end{cases}$

11) $2y''(x) + 6y'(x) + \frac{13}{2}y(x) = 0$

12) $y''(x) + 3y'(x) - 10y(x) = x^2$

13) $\begin{cases} y''(x) + \frac{1}{2}y'(x) - \frac{3}{2}y(x) = \frac{3}{2}x + 1 \\ y(0) = 1 \quad y'(0) = 1 \end{cases}$

14) $\begin{cases} y''(x) - 2y'(x) = 3x^2 - 3x + 4 \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$

15) $y''(x) = \cos(\pi x) - x^2 + 3x$

16) $y''(x) + y'(x) - 2y(x) + 1 = 2\,\mathrm{sen}\,x$

17) $y''(x) + 2y'(x) + 5y(x) = 3x^2 - 1$

18) $y''(x) + y'(x) + 5y(x) = \mathrm{sen}\,x$

19) $\begin{cases} y''(x) + y'(x) - 2y(x) = \cos(2x) \\ y(0) = 1 \quad y'(0) = 4/5 \end{cases}$

20) $y''(x) + 2y'(x) - 8y(x) = -10\,\mathrm{sen}(2x) + 4x^2$

21) $\begin{cases} y''(x) + 9y(x) = 3x^2 - 9x + \frac{20}{3} \\ y(0) = 2 \quad y'(0) = 2 \end{cases}$

22) $y''(x) = x\,\mathrm{sen}(3x)$

23) $\begin{cases} y''(x) = \cos(2x) \\ y(0) = 3 \quad y'(0) = -1 \end{cases}$

24) $y''(x) - 4y'(x) + 4y(x) = e^{-x}$

25) $y''(x) + y'(x) - 2y(x) = 2x^3 - 5x^2$

26) $y''(x) + 4y'(x) = e^x + e^{-x}$

27) $\begin{cases} 4y''(x) + 10y'(x) + \frac{25}{4}y(x) = 0 \\ y(0) = 2 \quad y'(0) = -3 \end{cases}$

28) $\begin{cases} 2y''(x) - 5y'(x) - 3y(x) = 2x^2 + \frac{49}{6}x - \frac{1}{2} \\ y(0) = 0 \quad y'(0) = 5 \end{cases}$

29) $\begin{cases} 12y''(x) + 3y'(x) = 12x + \frac{3}{2}x^2 - 1 \\ y(0) = 3 \quad y'(0) = 0 \end{cases}$

30) $\begin{cases} y''(x) - 2y'(x) + y(x) = x \\ y(0) = 0 \quad y'(0) = 0 \end{cases}$

31) $y''(x) + 2y'(x) + 2y(x) = \mathrm{sen}\,x + \cos x$

32) $y''(x) - 4y'(x) + 5y(x) = \cos(2x) - 5\,\mathrm{sen}(2x)$

33) $\begin{cases} y''(x) + 4y'(x) + 4y(x) = 1 \\ y(0) = 0 \quad y'(0) = 0 \end{cases}$

34) $y''(x) + 5y(x) = e^x$

35) $y''(x) + 9y(x) = e^x + \mathrm{sen}(3x)$

36) $y''(x) - y'(x) = x \cdot e^x$

37) $\begin{cases} y''(x) = \dfrac{1}{1+x^2} \\ y(0) = 3 \quad y'(0) = -1 \end{cases}$

38) $y''(x) - 6y'(x) + 13y(x) = 13x^2 + 14x$

39) $y''(x) + 3y'(x) + 2y(x) = 4\cos x - 2\,\mathrm{sen}\,x$

40) $y''(x) + y(x) = x^2$

41) $\begin{cases} y''(x) + 5y'(x) - 6y(x) = 10\cos(2x) \\ y(0) = 0 \quad y'(0) = 5 \end{cases}$

42) $y''(x) + 4y(x) = 3\cos(2x)$

43) $y''(x)+4y'(x)+4y(x)=4x^3$

44) $y''(x)+\frac{1}{4}y(x)=2\,\text{sen}\left(\frac{x}{2}\right)$

45) $2y''(x)+\frac{1}{2}y(x)=4\cos\left(\frac{x}{2}\right)$

46) $\begin{cases} y''(x)-6y'(x)+10y(x)=6\cos(2x)+2\,\text{sen}(2x) \\ y(0)=0 \quad y'(0)=0 \end{cases}$

47) $y''(x)+y'(x)+2y(x)=e^x(x^2+1)$

48) $y''(x)+y(x)=x^2+2e^x$

49) $y''(x)+y'(x)-12y(x)=2e^{3x}$

50) $y''(x)-6y'(x)+9y(x)=2e^{3x}$

51) $y''(x)+6y'(x)+9y(x)=3xe^{-3x}$

52) $\begin{cases} y''(x)+y'(x)-2y(x)=e^x+1 \\ y(0)=-1 \quad y'(0)=\frac{22}{3} \end{cases}$

53) $\begin{cases} y''(x)-4y(x)=-12x^5+68x^3-12x \\ y(0)=2 \quad y'(0)=0 \end{cases}$

54) $y''(x)-2y'(x)=-8x^3$

55) $y''(x)-4y'(x)+4y(x)=x^4-3x^3+\frac{5}{2}x^2+x$

56) $y''(x)+2y'(x)+10y(x)=5x^2+\frac{8}{5}$

57) $\begin{cases} y''(x)+2y'(x)+2y(x)=\text{Sen}x+x^2 \\ y(0)=0 \quad y'(0)=1 \end{cases}$

$\boxed{\text{RISPOSTE}}$

A) i) Un'eq.^{ne} del 2° ordine ha sempre INFINITE SOLUZIONI al variare di 2 costanti arbitrarie

ii) $y(x)=c_1e^{3x}+c_2e^{3x}$ sono le soluzioni dell'eq.^{ne} $y'(x)-3y(x)=0$ del <u>1° ordine</u>

perchè $y(x)=(c_1+c_2)e^{3x}=ce^{3x}$ $(c\in\mathbb{R})$ quindi la risposta è <u>NO</u>.

$y(x)=c_1e^{-2x}+c_2e^{4x}$ Sono le sol.^{ni} di $y''(x)-2y'(x)-8y(x)=0$

iii) <u>NO</u> : $\bar{y}'=6x$ $\bar{y}''(x)=6$ $\Rightarrow$ $\bar{y}''(x)+a\bar{y}'(x)=6+6ax$ non può essere un POLINOMIO di 2° grado. Infatti in questo caso per determinare la soluzione particolare si deve prendere $\bar{y}(x)=x\cdot(Ax^2+Bx+C)=Ax^3+Bx^2+Cx$ di grado 3.

iv) $y''(x)-y'(x)-2y(x)=-6x^2-6x+8$

1) $y(x) = 4e^{-\frac{1}{2}x} + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

2) $y(x) = \frac{1}{4}x^4 - \frac{1}{2}x^3 - \frac{1}{9}e^{-3x} + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

3) $y(x) = \frac{8}{\pi}\,\mathrm{sen}\left(\frac{\pi}{8}x\right) + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

4) $y(x) = c_1 e^{3x} + c_2 e^{4x} \quad (c_1, c_2 \in \mathbb{R})$

5) $y(x) = c_1 e^{-\frac{3}{2}x}\,\mathrm{sen}\,\frac{\sqrt{31}}{2}x + c_2 e^{-\frac{3}{2}x}\cos\frac{\sqrt{31}}{2}x \quad (c_1, c_2 \in \mathbb{R})$

6) $y(x) = c_1 e^{3x} + c_2 x e^{3x} \quad (c_1, c_2 \in \mathbb{R})$

7) $y(x) = e^{-x} + e^{3x}$

8) $y(x) = c_1 e^{x}\,\mathrm{sen}(4x) + c_2 e^{x}\cos(4x) \quad (c_1, c_2 \in \mathbb{R})$

9) $y(x) = c_1 e^{\frac{1}{2}x} + c_2 x e^{\frac{1}{2}x} \quad (c_1, c_2 \in \mathbb{R})$

10) $y(x) = -e^{-2x}$

11) $y(x) = c_1 e^{-\frac{3}{2}x} \cdot \mathrm{sen}\,x + c_2 e^{-\frac{3}{2}x} \cdot \cos x \quad (c_1, c_2 \in \mathbb{R})$

12) $y(x) = c_1 e^{2x} + c_2 e^{-5x} - \frac{1}{10}x^2 - \frac{3}{50}x - \frac{19}{500} \quad (c_1, c_2 \in \mathbb{R})$

13) $y(x) = 2e^{x} - x - 1$

14) $y(x) = -\frac{3}{2} + \frac{5}{2}e^{2x} - \frac{1}{2}x^3 - 2x$

15) $y(x) = -\frac{1}{\pi^2}\cos(\pi x) - \frac{x^4}{12} + \frac{1}{2}x^3 + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

16) $y(x) = c_1 e^{x} + c_2 e^{-2x} + \frac{1}{2} - \frac{3}{5}\mathrm{sen}\,x - \frac{1}{5}\cos x \quad (c_1, c_2 \in \mathbb{R})$

17) $y(x) = c_1 e^{-x}\,\mathrm{sen}(2x) + c_2 e^{-x}\cos(2x) + \frac{3}{5}x^2 - \frac{12}{25}x - \frac{31}{125} \quad (c_1, c_2 \in \mathbb{R})$

18) $y(x) = c_1 e^{-\frac{1}{2}x}\,\mathrm{sen}\left(\frac{\sqrt{19}}{2}x\right) + c_2 e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{19}}{2}x\right) + \frac{4}{17}\mathrm{sen}\,x - \frac{1}{17}\cos x \quad (c_1, c_2 \in \mathbb{R})$

19) $y(x) = e^{x} + \frac{3}{20}e^{-2x} + \frac{1}{20}\mathrm{sen}(2x) - \frac{3}{20}\cos(2x)$

20) $y(x) = c_1 e^{2x} + c_2 e^{-4x} + \frac{3}{4}\mathrm{sen}(2x) + \frac{1}{4}\cos(2x) - \frac{1}{2}x^2 - \frac{1}{4}x - \frac{3}{16} \quad (c_1, c_2 \in \mathbb{R})$

21) $y(x) = \mathrm{sen}(3x) + \frac{4}{3}\cos(3x) + \frac{1}{3}x^2 - x + \frac{2}{3}$

22) $y'(x) = -\frac{1}{3}x\cos(3x) + \frac{1}{9}\mathrm{sen}(3x) + c_1$

$y(x) = -\frac{1}{9}x\,\mathrm{sen}(3x) - \frac{2}{27}\cos(3x) + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

23) $y(x) = -\frac{1}{4}\cos(2x) - x + \frac{13}{4}$

24) $y(x) = c_1 e^{2x} + c_2 x e^{2x} + \frac{1}{9}e^{-x} \quad (c_1, c_2 \in \mathbb{R})$

25) $y(x) = c_1 e^{x} + c_2 e^{-2x} - x^3 + x^2 - 2x \quad (c_1, c_2 \in \mathbb{R})$

26) $y(x) = c_1 + c_2 e^{-4x} + \frac{1}{5}e^{x} - \frac{1}{3}e^{-x} \quad (c_1, c_2 \in \mathbb{R})$

27) $y(x) = 2e^{-\frac{5}{4}x} - \frac{1}{2}xe^{-\frac{5}{4}x}$

28) $y(x) = \frac{14}{9}e^{3x} - \frac{5}{3}e^{-\frac{1}{2}x} - \frac{2}{3}x^2 - \frac{1}{2}x + \frac{1}{9}$

29) $y(x) = \frac{13}{3} - \frac{4}{3}e^{-\frac{1}{4}x} + \frac{1}{6}x^3 - \frac{1}{3}x$

30) $y(x) = -2e^{x} + xe^{x} + x + 2$

31) $y(x)=c_1e^{-x}\operatorname{sen}x+c_2e^{-x}\cos x+\frac{3}{5}\operatorname{sen}x-\frac{1}{5}\cos x \quad (c_1,c_2\in\mathbb{R})$

32) $y(x)=c_1e^{2x}\operatorname{sen}x+c_2e^{2x}\cos x-\frac{13}{65}\operatorname{sen}(2x)-\frac{39}{65}\cos(2x) \quad (c_1,c_2\in\mathbb{R})$

33) $y(x)=-\frac{1}{4}e^{-2x}-\frac{1}{2}xe^{-2x}+\frac{1}{4}$

34) $y(x)=c_1\operatorname{sen}(\sqrt{5}x)+c_2\cos(\sqrt{5}x)+\frac{1}{6}e^{x}$
$(c_1,c_2\in\mathbb{R})$

35) $y(x)=c_1\operatorname{sen}(3x)+c_2\cos(3x)+\frac{1}{10}e^{x}-\frac{1}{6}x\cos(3x) \quad (c_1,c_2\in\mathbb{R})$

36) $y(x)=c_1+c_2e^{x}+(\frac{1}{2}x^2-x)e^{x} \quad (c_1,c_2\in\mathbb{R})$

37) $y(x)=x\cdot\arctan x-\frac{1}{2}\log(1+x^2)-x+3$

38) $y(x)=c_1e^{3x}\cos(2x)+c_2e^{3x}\operatorname{sen}(2x)+x^2+2x+\frac{10}{13} \quad (c_1,c_2\in\mathbb{R})$

39) $y(x)=c_1e^{-x}+c_2e^{-2x}+\operatorname{sen}x+\cos x$
$(c_1,c_2\in\mathbb{R})$

40) $y(x)=c_1\operatorname{sen}x+c_2\cos x+x^2-2$
$(c_1,c_2\in\mathbb{R})$

41) $y(x)=e^{x}-\frac{1}{2}e^{-6x}+\frac{1}{2}\operatorname{sen}(2x)-\frac{1}{2}\cos(2x)$

42) $y(x)=c_1\operatorname{sen}(2x)+c_2\cos(2x)+\frac{3}{4}x\operatorname{sen}(2x) \quad (c_1,c_2\in\mathbb{R})$

43) $y(x)=c_1e^{-2x}+c_2xe^{-2x}+x^3-3x^2+\frac{9}{2}x-3 \quad (c_1,c_2\in\mathbb{R})$

44) $y(x)=c_1\operatorname{sen}(\frac{x}{2})+c_2\cos(\frac{x}{2})-2x\cos(\frac{x}{2})$
$(c_1,c_2\in\mathbb{R})$

45) $y(x)=c_1\operatorname{sen}(\frac{x}{2})+c_2\cos(\frac{x}{2})+2x\operatorname{sen}(\frac{x}{2})$
$(c_1,c_2\in\mathbb{R})$

46) $y(x)=\frac{5}{3}e^{3x}\operatorname{sen}x-\frac{1}{3}e^{3x}\cos x-\frac{1}{3}\operatorname{sen}(2x)+\frac{1}{3}\cos(2x)$

47) $y(x)=c_1e^{-\frac{1}{2}x}\operatorname{sen}(\frac{\sqrt{7}}{2}x)+c_2e^{-\frac{1}{2}x}\cos(\frac{\sqrt{7}}{2}x)+(\frac{1}{4}x^2-\frac{3}{8}x+\frac{13}{32})e^{x} \quad (c_1,c_2\in\mathbb{R})$

48) $y(x)=c_1\operatorname{sen}x+c_2\cos x+x^2-2+e^{x}$
$(c_1,c_2\in\mathbb{R})$

49) $y(x)=c_1e^{3x}+c_2e^{-4x}+\frac{2}{7}xe^{3x} \quad (c_1,c_2\in\mathbb{R})$

50) $y(x)=c_1e^{3x}+c_2xe^{3x}+x^2e^{3x}$
$(c_1,c_2\in\mathbb{R})$

51) $y(x)=c_1e^{-3x}+c_2xe^{-3x}+\frac{1}{2}x^3e^{-3x}$
$(c_1,c_2\in\mathbb{R})$

52) $y(x)=2e^{x}-\frac{5}{2}e^{-2x}-\frac{1}{2}+\frac{1}{3}xe^{x}$

53) $y(x)=e^{2x}+e^{-2x}+3x^5-2x^3$

54) $y(x)=c_1+c_2e^{2x}+x^4+2x^3+3x^2+3x$
$(c_1,c_2\in\mathbb{R})$

55) $y(x)=c_1e^{2x}+c_2xe^{2x}+\frac{1}{4}x^4+\frac{1}{4}x^3+\frac{5}{8}x^2+\frac{9}{8}x+\frac{13}{16} \quad (c_1,c_2\in\mathbb{R})$

56) $y(x)=c_1e^{-x}\operatorname{sen}(3x)+c_2e^{-x}\cos(3x)+\frac{1}{2}x^2-\frac{1}{5}x+\frac{1}{10} \quad (c_1,c_2\in\mathbb{R})$

57) $y(x)=\frac{17}{10}e^{-x}\operatorname{sen}x-\frac{1}{10}e^{-x}\cos x+\frac{1}{5}\operatorname{sen}x-\frac{2}{5}\cos x+\frac{1}{2}x^2-x+\frac{1}{2}$

Soluzione di <u>alcuni esercizi</u> della $\boxed{\text{Scheda N. 14}}$

2) $y''(x) = 3x^2 - 3x - e^{-3x} \rightarrow y'(x) = x^3 - \frac{3}{2}x^2 + \frac{1}{3}e^{-3x} + c_1$

<u>SOL.ne</u> $y(x) = \frac{1}{4}x^4 - \frac{1}{2}x^3 - \frac{1}{9}e^{-3x} + c_1 x + c_2 \quad (c_1, c_2 \in \mathbb{R})$

5) eq.ne caratt. $t^2 + 3t + 10 = 0 \quad t_{1,2} = \frac{-3 \pm \sqrt{9-40}}{2} = -\frac{3}{2} \pm \frac{\sqrt{31}}{2} i \quad \alpha = -\frac{3}{2} \quad \beta = \frac{\sqrt{31}}{2}$

<u>SOL.ne</u> $y(x) = c_1 e^{-\frac{3}{2}x} \operatorname{sen}\left(\frac{\sqrt{31}}{2}x\right) + c_2 e^{-\frac{3}{2}x} \cos\left(\frac{\sqrt{31}}{2}x\right) \quad (c_1, c_2 \in \mathbb{R})$

14) eq.ne omogenea $y''(x) - 2y'(x) = 0$ eq.ne caratt. $t^2 - 2t = 0 \quad t_1 = 0 \quad t_2 = 2$

Sol.ni FOND $y_1(x) = e^{0x} = 1 \quad y_2(x) = e^{2x}$

Sol.ni omogenea $y(x) = c_1 + c_2 e^{2x} \quad (c_1, c_2 \in \mathbb{R})$

Sol.ne particolare $\bar{y}(x) = x(Ax^2 + Bx + C) = Ax^3 + Bx^2 + Cx$ perchè il 2° membro è un polinomio di grado 2 e nell'eq.ne NON compare $y(x)$, ma $y'(x)$ sì

$\bar{y}'(x) = 3Ax^2 + 2Bx + C \quad \bar{y}''(x) = 6Ax + 2B$ nell'eq.ne

$6Ax + 2B - 2(3Ax^2 + 2Bx + C) = 3x^2 - 3x + 4 \quad \forall x \in \mathbb{R}$

$-6Ax^2 + (6A - 4B)x + (2B - 2C) = 3x^2 - 3x + 4 \quad \forall x \in \mathbb{R}$

principio di IDENTITÀ dei POLINOMI $\begin{cases} -6A = 3 \\ 6A - 4B = -3 \\ 2B - 2C = 4 \end{cases} \quad \begin{cases} A = -\frac{1}{2} \\ 4B = -3 + 3 = 0 \rightarrow B = 0 \\ C = -2 \end{cases} \quad \bar{y}(x) = -\frac{1}{2}x^3 - 2x$

Tutte le sol.ni $y(x) = c_1 + c_2 e^{2x} - \frac{1}{2}x^3 - 2x \quad (c_1, c_2 \in \mathbb{R})$

$y'(x) = 2c_2 e^{2x} - \frac{3}{2}x^2 - 2$

$\begin{cases} y(0) = 1 \\ y'(0) = 3 \end{cases} \quad \begin{matrix} y(0) = c_1 + c_2 = 1 & c_1 = 1 - c_2 & c_1 = -\frac{3}{2} \\ y'(0) = 2c_2 - 2 = 3 & \rightarrow 2c_2 = 5 & c_2 = \frac{5}{2} \end{matrix}$

$\boxed{\text{SOL.ne}}$ $y(x) = -\frac{3}{2} + \frac{5}{2}e^{2x} - \frac{1}{2}x^3 - 2x$

19) Eq.ne omog. $y''(x) + y'(x) - 2y(x) = 0$ eq.ne caratt. $t^2 + t - 2 = 0 \quad t_1 = 1 \quad t_2 = -2$

Sol.ni FOND. $y_1(x) = e^x \quad y_2(x) = e^{-2x}$

Sol.ni eq.ne omogenea $y(x) = c_1 e^x + c_2 e^{-2x} \quad (c_1, c_2 \in \mathbb{R})$

Sol.ne particolare $\bar{y}(x) = A \operatorname{sen}(2x) + B\cos(2x)$ (perchè le due funzioni $\operatorname{sen}(2x)$ e $\cos(2x)$ <u>NON</u> sono le soluzioni fondamentali dell'eq.ne omogenea)

$\bar{y}'(x) = 2A\cos(2x) - 2B\,\text{sen}(2x)$

$\bar{y}''(x) = -4A\,\text{sen}(2x) - 4B\cos(2x)$ nell'eq.ne

$-4A\,\text{sen}(2x) - 4B\cos(2x) + 2A\cos(2x) - 2B\,\text{sen}(2x)) - 2(A\,\text{sen}(2x) + B\cos(2x)) =$

$= \cos(2x) \quad \forall x \in \mathbb{R}$

$(-6A - 2B)\,\text{sen}(2x) + (-6B + 2A - 1)\cos(2x) = 0 \quad \forall x \in \mathbb{R}$

$\begin{cases} -6A - 2B = 0 \\ -6B + 2A - 1 = 0 \end{cases}$ $\quad 2B = -6A \quad B = -3A \quad 20A = 1$ $\quad \begin{cases} A = \frac{1}{20} \\ B = -\frac{3}{20} \end{cases}$ $\quad \bar{y}(x) = \frac{1}{20}\text{sen}(2x) - \frac{3}{20}\cos(2x)$

Tutte le sol.ni $y(x) = c_1 e^{x} + c_2 e^{-2x} + \frac{1}{20}\text{sen}(2x) - \frac{3}{20}\cos(2x) \quad (c_1, c_2 \in \mathbb{R})$

$y'(x) = c_1 e^{x} - 2c_2 e^{-2x} + \frac{1}{10}\cos(2x) + \frac{3}{10}\text{sen}(2x)$

$\begin{cases} y(0) = c_1 + c_2 - \frac{3}{20} = 1 \\ y'(0) = c_1 - 2c_2 + \frac{1}{10} = \frac{4}{5} \end{cases}$ $\quad \begin{cases} c_1 + c_2 = \frac{23}{20} \\ c_1 - 2c_2 = \frac{7}{10} \end{cases}$ $\quad c_2 = \frac{23}{20} - c_1 \quad c_1 - \frac{23}{10} + 2c_1 = \frac{7}{10}$

$\begin{cases} 3c_1 = 3 \\ c_2 = \frac{23}{20} - c_1 \end{cases}$ $\quad \begin{cases} c_1 = 1 \\ c_2 = \frac{3}{20} \end{cases}$ $\quad \boxed{\text{SOL.}^{ne}} \; y(x) = e^{x} + \frac{3}{20}e^{-2x} + \frac{1}{20}\text{sen}(2x) - \frac{3}{20}\cos(2x)$

21) eq.ne omog. $y'' + 9y = 0$ eq.ne caratt. $t^2 + 9 = 0$ $t = \pm 3i$ $\alpha = 0$ $\beta = 3$

SOL.ni FONDAM. $y_1(x) = \text{sen}(3x)$ $y_2(x) = \cos(3x)$

Sol.ni omogenea $y(x) = c_1\,\text{sen}(3x) + c_2\cos(3x) \quad (c_1, c_2 \in \mathbb{R})$

Sol.ne particolare $\bar{y}(x) = Ax^2 + Bx + C$ (il 2° m è un polinomio di 2° grado e nell'eq.ne compare $y(x)$)

$\bar{y}'(x) = 2Ax + B \quad \bar{y}''(x) = 2A$

Nell'eq.ne $2A + 9(Ax^2 + Bx + C) = 3x^2 - 9x + \frac{20}{3} \quad \underline{\forall x \in \mathbb{R}}$

$9Ax^2 + 9Bx + (2A + 9C) = 3x^2 - 9x + \frac{20}{3} \quad \forall x \in \mathbb{R}$

per il principio di IDENTITÀ dei POLINOMI $\begin{cases} 9A = 3 \\ 9B = -9 \\ 2A + 9C = \frac{20}{3} \end{cases}$ $\quad \begin{cases} A = \frac{1}{3} \\ B = -1 \\ 9C = 6 \end{cases}$ $\quad C = \frac{2}{3}$ $\quad \bar{y}(x) = \frac{1}{3}x^2 - x + \frac{2}{3}$

Tutte le sol.ni $y(x) = c_1\,\text{sen}(3x) + c_2\cos(3x) + \frac{1}{3}x^2 - x + \frac{2}{3}$

$y'(x) = 3c_1\cos(3x) - 3c_2\,\text{sen}(3x) + \frac{2}{3}x - 1$

$\begin{cases} y(0) = c_2 + \frac{2}{3} = 2 \\ y'(0) = 3c_1 - 1 = 2 \end{cases}$ $\quad \begin{cases} c_1 = 1 \\ c_2 = 4/3 \end{cases}$ $\quad \underline{\text{SOL.}^{ne}} \; y(x) = \text{sen}(3x) + \frac{4}{3}\cos(3x) + \frac{1}{3}x^2 - x + \frac{2}{3}$

27) eq.^{ne} caratt. $4t^2+10t+\frac{25}{4}=0$ $\Delta=0$ $t_1=-\frac{5}{4}$ con molt 2

SOL.^{ni} FOND. $y_1(x)=e^{-5/4 x}$ $y_2(x)=x\cdot e^{-\frac{5}{4}x}$

Sol.^{ni} eq.^{ne} (che è OMOG) $y(x)=c_1e^{-5/4x}+c_2xe^{-\frac{5}{4}x}$ $(c_1,c_2\in\mathbb{R})$

$$y'(x)=-\frac{5}{4}c_1e^{-5/4x}+c_2e^{-5/4x}-\frac{5}{4}c_2xe^{-5/4x}$$

$$\begin{cases} y(0)=c_1=2 \\ y'(0)=-\frac{5}{4}c_1+c_2=-3 \end{cases} \quad \begin{cases} c_1=2 \\ c_2=-3+\frac{5}{2}=-\frac{1}{2} \end{cases}$$

Sol.^{ne} $y(x)=2e^{-\frac{5}{4}x}-\frac{1}{2}xe^{-\frac{5}{4}x}$

32) eq.^{ne} omog: $y''(x)-4y'(x)+5y(x)=0$ eq.^{ne} caratt. $t^2-4t+5=0$

$t_{1,2}=\frac{2\pm\sqrt{4-5}}{1}=2\pm i$ $\alpha=2$ $\beta=1$ Sol.^{ni} FOND. $y_1(x)=e^{2x}\operatorname{sen}x$ $y_2(x)=e^{2x}\cos x$

Sol.^{ni} omog $y(x)=c_1e^{2x}\operatorname{sen}x+c_2e^{2x}\cos x$ $(c_1,c_2\in\mathbb{R})$

Sol.^{ne} particolare $\bar{y}=A\operatorname{sen}(2x)+B\cos(2x)$ perché il 2° m è una combinazione lineare di sen(2x) e cos(2x) $(M=-5, N=1, \omega=2)$ ma sen(2x) e cos(2x) NON sono le sol.^{ni} fondamentali dell' eq.^{ne} omogenea.

$\bar{y}'(x)=2A\cos(2x)-2B\operatorname{sen}(2x)$ $\bar{y}''(x)=-4A\operatorname{sen}(2x)-4B\cos(2x)$

nell' eq.^{ne}
$$-4A\operatorname{sen}(2x)-4B\cos(2x)-4(2A\cos(2x)-2B\operatorname{sen}(2x))+$$
$$+5(A\operatorname{sen}(2x)+B\cos(2x))=\cos(2x)-5\operatorname{sen}(2x) \quad \forall x\in\mathbb{R}$$

$$(A+8B+5)\operatorname{sen}(2x)+(B-8A-1)\cos(2x)=0 \quad \forall x\in\mathbb{R}$$

$$\begin{cases} A+8B+5=0 \\ B-8A-1=0 \end{cases} \quad \begin{cases} A+64A+13=0 \\ B=8A+1 \end{cases} \quad \begin{cases} A=-\frac{1}{5} \\ B=-\frac{3}{5} \end{cases}$$

$$\bar{y}(x)=-\frac{1}{5}\operatorname{sen}(2x)-\frac{3}{5}\cos(2x)$$

Sol.^{ni} $y(x)=c_1e^{2x}\operatorname{sen}x+c_2e^{2x}\cos x-\frac{1}{5}\operatorname{sen}(2x)-\frac{3}{5}\cos(2x)$

$(c_1,c_2\in\mathbb{R})$

36) eq.^{ne} omog $y''(x)-y'(x)=0$ eq.^{ne} caratt $t^2-t=0$

$t(t-1)=0 \quad t_1=0 \quad t_2=1$ SOL.^{ni} FOND $y_1(x)=1$
$y_2(x)=e^x$

Sol.^{ni} omog $y(x)=c_1+c_2e^x \quad (c_1,c_2\in\mathbb{R})$

Sol.^{ne} part.

$\bar{y}(x)=x(Ax+B)\,e^x$ perché il 2° m è prodotto di un polinomio di 1° grado (x) per un'esponenziale e $\alpha=1$ e sol.^{ne} dell'eq.^{ne} caratt. con molt. 1

$(Ax^2+Bx)\,e^x$

$\bar{y}'(x)=(2Ax+B)e^x+(Ax^2+Bx)e^x$

$=(Ax^2+(2A+B)x+B)e^x$

$\bar{y}''(x)=(2Ax+2A+B)e^x+$

$+(Ax^2+(2A+B)x+B)e^x=(Ax^2+(4A+B)x+2A+2B)e^x$

Nell'eq.^{ne} $(Ax^2+(4A+B)x+2A+2B)e^x-(Ax^2+(2A+B)x+B)e^x=xe^x \quad \underline{\forall x\in\mathbb{R}}$

$[(2A-1)x+(2A+B)]\cancel{e^x}=0 \quad \forall x\in\mathbb{R} \qquad e^x\neq 0 \quad \forall x\in\mathbb{R}$

$(2A-1)x+(2A+B)=0 \quad \forall x\in\mathbb{R}$ principio IDENT polinomi $\begin{cases}2A-1=0 & A=\frac{1}{2}\\ 2A+B=0 & B=-1\end{cases}$

$\bar{y}(x)=\left(\frac{1}{2}x^2-x\right)e^x$ $\underline{\text{SOL.}^{ni}}$ $y(x)=c_1+c_2e^x+\left(\frac{1}{2}x^2-x\right)e^x \quad (c_1,c_2\in\mathbb{R})$

37) $\int\frac{1}{1+x^2}\,dx=\arctan x+c_1 \qquad y'(x)=\arctan x+c_1$

$\int\arctan x\,dx=\int 1\cdot\arctan x\,dx = x\cdot\arctan x-\int\frac{x}{1+x^2}\,dx=$ PER PARTI

$g'=1\Rightarrow g=x$
$f=\arctan x \quad f'=\frac{1}{1+x^2}$

$=x\cdot\arctan x-\frac{1}{2}\log(1+x^2)+\text{cost}$

$\rightarrow y(x)=x\cdot\arctan x-\frac{1}{2}\log(1+x^2)+c_1x+c_2 \qquad (c_1,c_2\in\mathbb{R})$

$y'(x)=\arctan x+c_1$

$\begin{cases}y(0)=c_2=3\\ y'(0)=c_1=-1\end{cases}$ $\arctan 0=0$
$\log 1=0$

$\underline{\text{SOL.}^{ne}}$ $y(x)=x\cdot\arctan x-\frac{1}{2}\log(1+x^2)-x+3$

42) Eq.^{ne} omog. $y''(x)+4y(x)=0$ eq.^{ne} caratt. $t^2+4=0$ $\Delta<0$

$t=\pm 2i$ $\alpha=0$ $\beta=2$ Sol.^{ni} FOND. $y_1(x)=\operatorname{sen}(2x)$

$y_2(x)=\cos(2x)$

Sol.^{ni} omog. $y(x)=c_1\operatorname{sen}(2x)+c_2\cos(2x)$ $(c_1,c_2\in\mathbb{R})$

Sol.^{ne} part. $\bar{y}(x)=x(A\operatorname{sen}(2x)+B\cos(2x))$ perchè il 2° m

è una combinazione lineare di $\operatorname{sen}(2x)$ e $\cos(2x)$ $(M=0, N=3, \omega=2)$

e queste due funzioni sono proprio le soluzioni fondamentali dell' eq.^{ne} omogenea

$\bar{y}'(x)=A\operatorname{sen}(2x)+B\cos(2x)+x(+2A\cos(2x)-2B\operatorname{sen}(2x))$

$\bar{y}''(x)=4A\cos(2x)-4B\operatorname{sen}(2x)+x(-4A\operatorname{sen}(2x)-4B\cos(2x))$

Nell' eq.^{ne} $4A\cos(2x)-4B\operatorname{sen}(2x)+x(-4A\operatorname{sen}(2x)-4B\cos(2x))+$

$+4x(A\operatorname{sen}(2x)+B\cos(2x))=3\cos(2x)\quad\forall x\in\mathbb{R}$

$(4A-3)\cos(2x)-4B\operatorname{sen}(2x)=0\quad\forall x\in\mathbb{R}$

$$\begin{cases}4A-3=0\\-4B=0\end{cases}\quad\begin{cases}A=\frac{3}{4}\\B=0\end{cases}\quad \bar{y}(x)=\frac{3}{4}x\operatorname{sen}(2x)$$

Sol.^{ni} $y(x)=c_1\operatorname{sen}(2x)+c_2\cos(2x)+\frac{3}{4}x\operatorname{sen}(2x)$ $(c_1,c_2\in\mathbb{R})$

46) eq.^{ne} omog. $y''(x)-6y'(x)+10y(x)=0$ eq.^{ne} caratt. $t^2-6t+10=0$

$t_{1,2}=\dfrac{3\pm\sqrt{9-10}}{1}=3\pm i$ $\alpha=3$ $\beta=1$ Sol.^{ni} FOND. $y_1(x)=e^{3x}\operatorname{sen}x$

$y_2(x)=e^{3x}\cos x$

Sol.^{ni} omog. $y(x)=c_1e^{3x}\operatorname{sen}x+c_2e^{3x}\cos x$ $(c_1,c_2\in\mathbb{R})$

Sol.^{ne} part. $\bar{y}(x)=A\operatorname{sen}(2x)+B\cos(2x)$ perchè il 2° m è combin. lineare di $\operatorname{sen}(2x)$ e $\cos(2x)$ che NON sono le sol.^{ni} fondam. dell' omogenea

$\bar{y}'(x)=2A\cos(2x)-2B\operatorname{sen}(2x)$

$\bar{y}''(x)=-4A\operatorname{sen}(2x)-4B\cos(2x)$

nell' eq.^{ne} $-4A\operatorname{sen}(2x)-4B\cos(2x)-6(2A\cos(2x)-2B\operatorname{sen}(2x))+$

$+10(A\operatorname{sen}(2x)+B\cos(2x))=6\cos(2x)+2\operatorname{sen}(2x)\quad\forall x\in\mathbb{R}$

$(6A+12B-2)\,\text{sen}(2x) + (6B-12A-6)\cos(2x) = 0 \quad \forall x \in \mathbb{R}$

$$\begin{cases} 6A+12B-2=0 & 6A+24A+12-2=0 & 30A=-10 & A=-\frac{1}{3} \\ 6B-12A-6=0 & 6B=12A+6 & & B=\frac{1}{3} \\ & B=2A+1 & & \end{cases}$$

$\bar{y}(x) = -\frac{1}{3}\,\text{sen}(2x) + \frac{1}{3}\cos(2x)$

Tutte le sol.^{ni} $y(x) = c_1 e^{3x}\,\text{sen}\,x + c_2 e^{3x}\cos x - \frac{1}{3}\,\text{sen}(2x) + \frac{1}{3}\cos(2x) \quad (c_1, c_2 \in \mathbb{R})$

$y'(x) = 3c_1 e^{3x}\,\text{sen}\,x + c_1 e^{3x}\cos x + 3c_2 e^{x}\cos x - c_2 e^{3x}\,\text{sen}\,x - \frac{2}{3}\cos(2x) - \frac{2}{3}\,\text{sen}(2x)$

$$\begin{cases} y(0) = c_2 + \frac{1}{3} = 0 \\ y'(0) = c_1 + 3c_2 - \frac{2}{3} = 0 \end{cases} \qquad \begin{cases} c_1 = \frac{2}{3} - 3c_2 = \frac{5}{3} \\ c_2 = -\frac{1}{3} \end{cases}$$

SOL.^{ne} $y(x) = \frac{5}{3} e^{3x}\,\text{sen}\,x - \frac{1}{3} e^{3x}\cos x - \frac{1}{3}\,\text{sen}(2x) + \frac{1}{3}\cos(2x)$

47) eq.^{ne} omog. $y''(x) + y'(x) + 2y(x) = 0$ eq.^{ne} caratt. $t^2 + t + 2 = 0$

$t_{1,2} = \dfrac{-1 \pm \sqrt{1-8}}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2} i \qquad \alpha = -\frac{1}{2} \quad \beta = \frac{\sqrt{7}}{2}$

SOL.^{ni} FOND. $y_1(x) = e^{-\frac{1}{2}x}\,\text{sen}\left(\frac{\sqrt{7}}{2}x\right)$

$y_2(x) = e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right)$

Sol.^{ni} omog.

$y(x) = c_1 e^{-\frac{1}{2}x}\,\text{sen}\left(\frac{\sqrt{7}}{2}x\right) + c_2 e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right)$

$(c_1, c_2 \in \mathbb{R})$

Sol.^{ne} particolare $\bar{y}(x) = (Ax^2 + Bx + C)e^x$

($\alpha = 1$ non è sol.^{ne} dell'eq.^{ne} caratt. $\Delta < 0$)

$\bar{y}'(x) = (2Ax + B)e^x + (Ax^2 + Bx + C)e^x = (Ax^2 + (2A+B)x + (B+C))e^x$

$\bar{y}''(x) = (2Ax + 2A + B)e^x + (Ax^2 + (2A+B)x + (B+C))e^x = (Ax^2 + (4A+B)x + 2A + 2B + C)e^x$

Nell'eq.^{ne} $(Ax^2 + (4A+B)x + 2A + 2B + C)e^x + (Ax^2 + (2A+B)x + (B+C))e^x +$

$+ 2(Ax^2 + Bx + C)e^x = (x^2+1)e^x \quad \forall x \in \mathbb{R}$

$[(4A-1)x^2 + (6A+4B)x + (2A+3B+4C-1)]e^x = 0 \quad \forall x \in \mathbb{R} \qquad e^x \neq 0 \quad \forall x \in \mathbb{R}$

$(4A-1)x^2 + (6A+4B)x + (2A+3B+4C-1) = 0 \quad \forall x \in \mathbb{R}$ IDENTITÀ POLINOMI

$$\begin{cases} 4A-1=0 & A = 1/4 \\ 6A+4B=0 & B = -3/8 \\ 2A+3B+4C-1=0 & 4C = 1 - \frac{1}{2} + \frac{9}{8} = \frac{13}{8} \\ & C = \frac{13}{32} \end{cases}$$

$\bar{y}(x) = \left(\frac{1}{4}x^2 - \frac{3}{8}x + \frac{13}{32}\right)e^x$

SOL.^{ni} $y(x) = c_1 e^{-\frac{1}{2}x}\,\text{sen}\left(\frac{\sqrt{7}}{2}x\right) + c_2 e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right) + \left(\frac{1}{4}x^2 - \frac{3}{8}x + \frac{13}{32}\right)e^x \quad (c_1, c_2 \in \mathbb{R})$