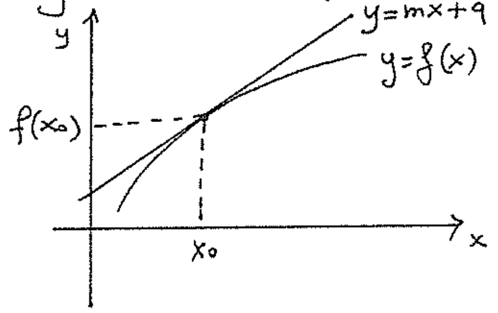
13) Riprendiamo il concetto di DERIVATA, il suo significato geometrico e le regole di CALCOLO

- La <u>DERIVATA</u> di una funzione f in un punto $x_0 \in \mathrm{dom} f$ <u>è un numero finito</u> che geometricamente rappresenta il <u>coefficiente angolare</u> della retta tangente al grafico di f (equazione $y=f(x)$) nel punto $(x_0, f(x_0))$



- La derivata di f in x_0 si indica con $f'(x_0)$, quindi il coefficiente angolare della retta tangente è $m=f'(x_0)$.

- Dal punto di vista analitico la derivata si può calcolare come limite del RAPPORTO INCREMENTALE di f in x_0

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \lim_{\substack{h \to 0 \\ (h=x-x_0)}} \frac{f(x_0+h)-f(x_0)}{h}$$

- Se in un punto $x_0 \in \mathrm{dom} f$ tale limite ESISTE ed È FINITO, allora f si dice DERIVABILE in x_0, altrimenti la funzione non è derivabile in quel punto.

- Si può costruire la FUNZIONE DERIVATA, cioè la funzione che ad ogni punto (in cui è possibile) fa corrispondere il valore della derivata in quel punto; tale funzione si indica con $f'(x)$

- FUNZIONI DERIVATE FONDAMENTALI

$f(x)$	$f'(x)$
K	0
x	1
x^2	$2x$
x^3	$3x^2$
x^α	$\alpha x^{\alpha-1}$
e^x	e^x
$\log x$	$\frac{1}{x}$
$\operatorname{sen} x$	$\cos x$
$\cos x$	$-\operatorname{sen} x$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
a^x	$a^x \cdot \log a$

ES.

$f(x) = 3x^2 - 5x + 2 + e^x$

$f'(x) = 6x - 5 + e^x$

(La derivata della somma è la somma delle derivate e lo stesso per la differenza)

- DERIVATA DEL PRODOTTO $D[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

ES. $f(x) = (\log x)\cdot(\operatorname{sen} x)$ $\quad f'(x) = \frac{1}{x}\cdot \operatorname{sen} x + \log x \cdot \cos x$

- DERIVATE DI FUNZIONI COMPOSTE $D[f(g(x))] =$

$= g'(x)\cdot f'(g(x))$

$f(x)$	$f'(x)$
$(g(x))^\alpha$	$\alpha\cdot(g(x))^{\alpha-1}\cdot g'(x)$
$e^{g(x)}$	$g'(x)\cdot e^{g(x)}$
$\log g(x)$	$g'(x)\cdot\frac{1}{g(x)}$
$\operatorname{sen} g(x)$	$g'(x)\cdot\cos(g(x))$
$\cos g(x)$	$-g'(x)\cdot\operatorname{sen}(g(x))$

ES.

$f(x) = e^{\operatorname{sen} x}$

$f'(x) = \cos x \cdot e^{\operatorname{sen} x}$

$f(x) = \log(1+x^2)$

$f'(x) = \dfrac{2x}{1+x^2}$

- DERIVATA DEL QUOZIENTE

$$D\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{(g(x))^2}$$

ES. $f(x)=\dfrac{x^2-3x+5}{x^3-4}$ $\quad f'(x)=\dfrac{(2x-3)(x^3-4)-(x^2-3x+5)\cdot 3x^2}{(x^3-4)^2}$

- RETTA TANGENTE

La retta tangente al grafico di una funzione f in un punto $(x_0,f(x_0))$ è la retta per tale punto avente come coefficiente angolare $f'(x_0)$. Pertanto l'equazione è

$$y-f(x_0)=f'(x_0)(x-x_0) \longrightarrow y=f(x_0)+f'(x_0)(x-x_0)$$

ES. $f(x)=2x^3-7x^2+8x-3$ eq.ne della retta tangente nel punto di ascissa -1

dom $f=\mathbb{R}$ eq.ne del grafico $y=2x^3-7x^2+8x-3$

$x_0=-1$ $\quad f(x_0)=f(-1)=-20$ $\quad P_0=(-1,-20)$

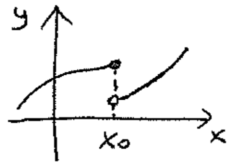
$f'(x)=6x^2-14x+8$ $\quad f'(-1)=28$

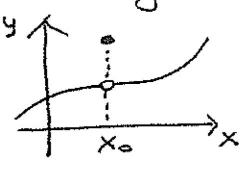
retta tangente $y=-20+28(x+1) \rightarrow y=28x+8$

- DERIVABILITÀ

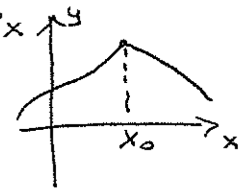
Una funzione NON è DERIVABILE in x_0 nei seguenti casi:

se è DISCONTINUA in x_0

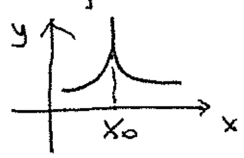


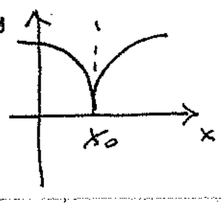


se presenta in x_0 un PUNTO ANGOLOSO in cui la derivata destra è diversa dalla derivata sinistra

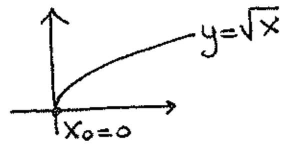


se presenta in x_0 una CUSPIDE





in un punto a TANGENTE VERTICALE



14) Delle seguenti funzioni calcoliamo il dominio e la derivata

a) $f(x)= 4x^4+3x-3$ b) $y(t)= 3t^6-2t^2$ c) $g(r)=r^{1000}$

d) $f(x)= 7x^5-8x^9$ e) $f(x)= 3m^2 x^{m-1}$, $m\in\mathbb{N}, m\geq 1$ f) $f(x)=\pi^3$

g) $f(x)=-\frac{1}{x}$ h) $f(x)=\frac{3}{x^2}$ i) $g(t)=-\frac{5}{t^4}$

j) $y(t)=e^4$ k) $f(x)=5x^{\frac{1}{5}}$ l) $f(x)=6\sqrt[4]{x}+x^2-\frac{3}{x^3}$

m) $f(x)=\frac{1}{x^{2/5}}$ n) $f(x)=5e^x+3$ o) $g(t)=\sqrt{t}-2e^t$

p) $V(r)=\frac{4}{3}\pi r^3$ q) $f(x)=\frac{x^2+4x+3}{\sqrt{x}}$ r) $y(t)=\frac{t^2-2\sqrt{t}}{t^3}$

s) $f(x)=\log(1+x^4)$ t) $g(x)=e^{-x}+x\cdot e^{-x}$

14bis) Calcolate $f'(x), f''(x), f'''(x)$ per le seguenti funzioni

a) $f(x)=x^3-x$ b) $f(x)=\frac{4}{x}$ c) $f(x)=\sqrt[3]{x}$ d) $f(x)=e^{-x^2}$

e) $f(x)=x^4-2x^2$ f) $f(x)=\sqrt{3-5x}$ (solo fino a f'') g) $f(x)=\frac{4-x}{3+x}$ (solo $\to f''$)

h) $f(x)=2xe^x$ i) $f(x)=\frac{\sqrt{x}}{x+1}$ (solo f'') j) $f(x)=e^x\cos x$

k) $f(x)=\frac{\mathrm{sen}(5x)}{x}$ (solo f'') l) $f(x)=\pi\,\mathrm{sen}\frac{1}{x}$ (solo f'')

<u>SOL.^{ni}</u> 14) a) $\mathbb{R}$ $f'(x)=16x^3+3$ b) $\mathbb{R}$ $y'(t)=18t^5-4t$

c) $\mathbb{R}$ $g'(r)=1000r^{999}$ d) $\mathbb{R}$ $f'(x)=35x^4-72x^8$

e) $\mathbb{R}$ $f'(x)=3m^2(m-1)x^{m-2}$ f) $\mathbb{R}$ $f'(x)=0$ g) $x\neq 0$ $f'(x)=\frac{1}{x^2}$

h) $x\neq 0$ $f'(x)=-\frac{6}{x^3}$ i) $t\neq 0$ $g'(t)=\frac{20}{t^5}$ j) $\mathbb{R}$ $y'(t)=0$

k) $\mathbb{R}$ f è derivabile su $\mathbb{R}\setminus\{0\}$ e $f'(x)=\frac{1}{\sqrt[5]{x^4}}$ $f'(0)=+\infty$ quindi f non è der. in $x=0$

xii) $x>0$ f è derivabile su $]0,+\infty[$ e $f'(x) = \frac{3}{2\sqrt[4]{x^3}} + 2x + \frac{9}{x^4}$

xiii) $x \neq 0$ $f'(x) = -\frac{2}{5x^{7/5}}$ xiv) $\mathbb{R}$ $f'(x) = 5e^x$

o) dom $t \geq 0$ $g'(t) = \frac{1}{2\sqrt{t}} - 2e^t$ per $t>0$

p) $\mathbb{R}$ $V'(r) = 4\pi R^2$ q) $x>0$ $f'(x) = \frac{(2x+4)\sqrt{x} - (x^2+4x+3)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} =$

$= \frac{4x^2+8x-x^2-4x-3}{2x\sqrt{x}} = \frac{3x^2+4x-3}{2x\sqrt{x}}$

r) $t>0$ $y'(t) = \frac{(2t-\frac{1}{\sqrt{t}})t^3-(t^2-2\sqrt{t})3t^2}{(t^3)^2} = \frac{t^2(2t^2-\sqrt{t}-3t^2+6\sqrt{t})}{t^6} =$

$= \frac{5\sqrt{t}-t^2}{t^4}$

s) $\mathbb{R}$ $f'(x) = \frac{4x^3}{1+x^4}$ t) $\mathbb{R}$ $g'(x) = -e^{-x} + e^{-x} - x\cdot e^{-x} = -x\cdot e^{-x}$

14bis)

a) $f'(x) = 3x^2-1$ $f''(x) = 6x$ $f'''(x) = 6$

b) $f'(x) = -\frac{4}{x^2}$ $f''(x) = \frac{8}{x^3}$ $f'''(x) = -\frac{24}{x^4}$

c) $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ $f''(x) = -\frac{2}{9}x^{-5/3} = -\frac{2}{9x^{5/3}}$ $f'''(x) = \frac{10}{27}x^{-8/3}$

d) $f'(x) = (-2x)e^{-x^2}$ $f''(x) = -2e^{-x^2} + (-2x)^2 e^{-x^2} = (4x^2-2)e^{-x^2}$

$f'''(x) = 8x e^{-x^2} + (-2x)(4x^2-2)e^{-x^2} = e^{-x^2}(-8x^3+12x) \left[= 4x(3-2x^2)e^{-x^2}\right]$

e) $f'(x) = 4x^3-4x$ $f''(x) = 12x^2-4$ $f'''(x) = 24x$

f) $f'(x) = -\frac{5}{2\sqrt{3-5x}} = -\frac{5}{2}(3-5x)^{-\frac{1}{2}}$ $f''(x) = -\frac{25}{4}(3-5x)^{-3/2} = -\frac{25}{4\sqrt{(3-5x)^3}}$

g) $f'(x) = \frac{-(3+x)-(4-x)}{(3+x)^2} = \frac{-7}{(3+x)^2}$ $f''(x) = \frac{7\cdot 2(3+x)}{(3+x)^4} = \frac{14}{(3+x)^3}$

h) $f'(x) = 2e^x + 2xe^x = 2(1+x)e^x$ $f''(x) = 2e^x + 2(1+x)e^x = 2(2+x)e^x$

$f'''(x) = 2(x+3)e^x$

i) $f'(x)=\dfrac{\frac{1}{2\sqrt{x}}\cdot(x+1)-\sqrt{x}}{(x+1)^2}=\dfrac{x+1-2x}{2\sqrt{x}\,(x+1)^2}=\dfrac{1-x}{2\sqrt{x}(x+1)^2}$

$f''(x)=\dfrac{-2\sqrt{x}(x+1)^2-(1-x)\left[\frac{1}{\sqrt{x}}(x+1)^2+4\sqrt{x}(x+1)\right]}{4x(x+1)^4}=\dfrac{3x^2-6x-1}{4x\sqrt{x}\,(x+1)^3}$

j) $f'(x)=e^x(\cos x-\operatorname{sen}x)$ $\quad f''(x)=e^x(\cos x-\operatorname{sen}x)+e^x(-\operatorname{sen}x-\cos x)=-2e^x\operatorname{sen}x$

$f'''(x)=-2e^x\operatorname{sen}x-2e^x\cos x=-2(\operatorname{sen}x+\cos x)e^x$

K) $f'(x)=\dfrac{5\cos(5x)-\operatorname{sen}(5x)}{x^2}$ $\quad f''(x)=\dfrac{\left[-25\operatorname{sen}(5x)-5\cos(5x)\right]\cdot x^2-(5\cos(5x)-\operatorname{sen}(5x))\cdot 2x}{x^4}$

l) $f'(x)=-\dfrac{\pi}{x^2}\cos\dfrac{1}{x}$

$f''(x)=\dfrac{2\pi}{x^3}\cos\dfrac{1}{x}-\dfrac{\pi}{x^2}\left(-\operatorname{sen}\dfrac{1}{x}\right)\left(-\dfrac{1}{x^2}\right)=\dfrac{2\pi}{x^3}\cos\dfrac{1}{x}-\dfrac{\pi}{x^4}\operatorname{sen}\dfrac{1}{x}$

15) Delle seguenti funzioni calcoliamo il dominio e la derivata

a) $f(x) = e^{\frac{x}{2}} \cdot \sqrt{27 - 2x^2 + 3x} + \log(3x^2 - 4x + \frac{4}{3}) + \frac{\operatorname{sen} x}{x^2+1}$

b) $f(x) = e^{4x} \cdot \sqrt{5 - \frac{1}{5}x^2} + \log(8x^2+2) - \frac{\operatorname{sen}(3x)}{3x^2 - 5x - 12} + \log(9-2x)$

c) $f(x) = -\frac{1}{x^2-1} \cdot \sqrt{5 - \frac{2}{5}x^2 - x} + \log(8 - \frac{1}{2}x^2) + \frac{\cos(4x)}{e^{6x}} - \log(3-2x)$

d) $f(x) = \sqrt{3x - \frac{1}{3}x^2} - \frac{1}{3}x^5 + \frac{\pi - 2}{x}$

e) $f(x) = \log(6x + 1 + 9x^2)$ ← (calcolate anche l'equazione della retta tangente nel punto di ascissa $x_0 = \frac{1}{3}$)

f) $f(x) = x \cdot \cos\left(\frac{x^2}{3}\right) + e^{\frac{x}{2}}$

g) $f(x) = 3 \operatorname{sen} x + 3x \cdot \log(1 + \frac{x}{2})$ (calcolate anche $f''(x)$)

h) $f(x) = 4e^{-x^2}$ i) $f(x) = \frac{x}{x^2-1}$

j) $f(x) = e^{-3x} - 3x$ k) $f(x) = \cos(3x) - \frac{2}{x^3}$

l) $f(x) = (e^4 - e^x) \cdot \log(x+1) + \sqrt{5-3x}$

m) $f(x) = \frac{4x-1}{2x+1} + \sqrt{5x - 4x^2 + 6}$

n) $f(x) = -\frac{1}{2}\operatorname{sen}(3x)$ (calcolate anche $f'(\frac{\pi}{4}), f'(\frac{\pi}{2}), f'(\frac{\pi}{3}), f'(\frac{4}{3}\pi)$)

o) $f(x) = x^3 \cdot e^{6x - 2x^2}$

p) $f(x) = \frac{\cos(3x)}{\operatorname{sen} x}$ (con anche $f'(\frac{\pi}{3}), f'(\frac{\pi}{4}), f'(\frac{\pi}{2})$)

j) $\mathbb{R}$ $f'(x) = -3e^{3x} - 3$

K) $x \neq 0$ ($\text{dom} f = \mathbb{R} \setminus \{0\}$) $f'(x) = -3\,\text{sen}(3x) + \frac{6}{x^4}$

l) $\text{dom} f \begin{cases} x+1>0 \\ 5-3x \geq 0 \end{cases} \begin{cases} x>-1 \\ x \leq \frac{5}{3} \end{cases}$ $\text{dom} f =]-1, \frac{5}{3}]$

$$f'(x) = -e^x \cdot \log(x+1) + \frac{e^4 - e^x}{x+1} + \frac{-3}{2\sqrt{5-3x}}$$

m) $\text{dom} f \begin{cases} 2x+1 \neq 0 \quad x \neq -\frac{1}{2} \\ 5x - 4x^2 + 6 \geq 0 \end{cases} \begin{cases} x \neq -\frac{1}{2} \\ 4x^2 - 5x - 6 \leq 0 \end{cases}$ $\text{dom} f = [-\frac{3}{4}, -\frac{1}{2}[\cup$
$\cup]-\frac{1}{2}, 2]$

$$f'(x) = \frac{6}{(2x+1)^2} + \frac{5-8x}{2\sqrt{5x-4x^2+6}}$$

n) $\text{dom} f = \mathbb{R}$ $f'(x) = -\frac{3}{2}\cos(3x)$ $f'(\frac{\pi}{4}) = -\frac{3}{2}\cos(\frac{3}{4}\pi) = \frac{3\sqrt{2}}{4}$

$f'(\frac{\pi}{2}) = -\frac{3}{2}\cos(\frac{3}{2}\pi) = 0$ $f'(\frac{\pi}{3}) = -\frac{3}{2}\cos\pi = \frac{3}{2}$ $f'(\frac{4}{3}\pi) = -\frac{3}{2}\cos 4\pi = -\frac{3}{2}$

o) $\text{dom} f = \mathbb{R}$ $f'(x) = 3x^2 \cdot e^{6x-2x^2} + x^3(6-4x) \cdot e^{6x-2x^2}$

p) $\text{dom} f = \mathbb{R} \setminus \{0, \pi, -\pi, 2\pi, -2\pi, \dots\} = \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$

$$f'(x) = \frac{-3\,\text{sen}(3x) \cdot \text{sen}\,x - \cos(3x) \cdot \cos x}{(\text{sen}\,x)^2}$$

$$f'(\frac{\pi}{3}) = \frac{-3\,\cancel{\text{sen}\,\pi} \cdot \text{sen}\frac{\pi}{3} - \cos\pi \cdot \cos\frac{\pi}{3}}{(\text{sen}\frac{\pi}{3})^2} = \frac{\frac{1}{2}}{(\frac{\sqrt{3}}{2})^2} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

$$f'(\frac{\pi}{4}) = \frac{-3\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{(\frac{\sqrt{2}}{2})^2} = \frac{-1}{\frac{1}{2}} = -2 \quad f'(\frac{\pi}{2}) = \frac{-3 \cdot (-1) \cdot 1}{1^2} = 3$$

SOL.^{NE}

a) $\mathrm{dom} f = \begin{cases} 27-2x^2+3x \geq 0 \\ 3x^2-4x+4/3 > 0 \\ x^2+1 \neq 0 \end{cases} \quad \begin{cases} -3 \leq x \leq \frac{9}{2} \\ x \neq 2/3 \\ \forall x \in \mathbb{R} \ (x^2 \neq -1 \text{ è sempre vera}) \end{cases}$

$\mathrm{dom} f = \left[-3, \frac{2}{3}\right[\cup \left]\frac{2}{3}, \frac{9}{2}\right]$

$$f'(x) = \frac{1}{2} e^{\frac{x}{2}} \cdot \sqrt{27-2x^2+3x} + e^{\frac{x}{2}} \cdot \frac{(-4x+3)}{2\sqrt{27-2x^2+3x}} + \frac{6x-4}{3x^2-4x+\frac{4}{3}} +$$

$$+ \frac{\cos x \cdot (x^2+1) - \operatorname{sen} x \cdot (2x)}{(x^2+1)^2}$$

b) $\mathrm{dom} f = \begin{cases} 5-\frac{1}{5}x^2 \geq 0 \\ 8x^2+2 > 0 \\ 3x^2-5x-12 \neq 0 \\ 9-2x > 0 \end{cases} \quad \begin{cases} -5 \leq x \leq 5 \\ \forall x \quad (\Delta < 0) \\ x \neq -4/3 \quad x \neq 3 \\ x < \frac{9}{2} \end{cases}$

$\mathrm{dom} f = \left[-5, -\frac{4}{3}\right[\cup \left]-\frac{4}{3}, 3\right[\cup \left]3, \frac{9}{2}\right[$

$$f'(x) = 4 \cdot e^{4x} \cdot \sqrt{5-\frac{1}{5}x^2} + e^{4x} \cdot \frac{-\frac{2}{5}x}{\sqrt{5-\frac{1}{5}x^2}} + \frac{16x}{8x^2+2}$$

$$- \frac{3 \cdot \cos(3x) \cdot (3x^2-5x-12) - \operatorname{sen}(3x) \cdot (6x-5)}{(3x^2-5x-12)^2} + \frac{-2}{9-2x}$$

c) $\mathrm{dom} f \begin{cases} x^2-1 \neq 0 \\ 5-\frac{2}{5}x^2-x \geq 0 \\ 8-\frac{1}{2}x^2 > 0 \\ e^{6x} \neq 0 \\ 3-2x > 0 \end{cases} \quad \begin{cases} x \neq \pm 1 \\ -5 \leq x \leq \frac{5}{2} \\ -4 < x < 4 \\ \forall x \in \mathbb{R} \\ x < 3/2 \end{cases}$

$\mathrm{dom} f = \left]-4, -1\right[\cup \left]-1, 1\right[\cup \left]1, \frac{3}{2}\right[$

$$f'(x)= \frac{2x}{(x^2-1)^2}\cdot\sqrt{5-\frac{2}{5}x^2-x} - \frac{1}{x^2-1}\cdot\frac{(-\frac{4}{5}x-1)}{2\sqrt{5-\frac{2}{5}x^2-x}} +$$

$$+ \frac{-x}{8-\frac{1}{2}x^2} + \frac{-4\,\mathrm{sen}(4x)\cdot e^{6x} - \cos(4x)\cdot 6e^{6x}}{(e^{6x})^2} - \frac{-2}{3-2x}$$

d) dom f $\begin{cases} 3x-\frac{1}{3}x^2 \geq 0 \\ x \neq 0 \end{cases}$ $\begin{cases} \frac{1}{3}x^2-3x \leq 0 \\ x\neq 0 \end{cases}$ $\begin{cases} x^2-9x\leq 0 \\ x\neq 0\end{cases}$ $\begin{cases} x\in[0,9] \\ x\neq 0\end{cases}$

dom f = $]0,9]$

$$f'(x)= \frac{3-\frac{2}{3}x}{2\sqrt{3x-\frac{1}{3}x^2}} - \frac{5}{3}x^4 - \frac{\pi-2}{x^2}$$

e) dom f : $6x+1+9x^2>0$ $\quad 9x^2+6x+1>0 \quad (3x+1)^2>0 \ \forall x\neq -\frac{1}{3}$

dom f $=\mathbb{R}\setminus\{-\frac{1}{3}\}$

$$f'(x)= \frac{6+18x}{6x+1+9x^2}$$

retta tg : $f(\frac{1}{3}) = \log(2+1+1)=\log 4 \quad f'(\frac{1}{3})=\frac{12}{4}=3$

eq.^{ne} $y=\log 4 + 3\left(x-\frac{1}{3}\right) \rightarrow y = 3x+(\log 4)-1$

f) dom f $=\mathbb{R}$ $\quad f'(x)= \cos\left(\frac{x^2}{3}\right) - \frac{2}{3}x^2\,\mathrm{sen}\left(\frac{x^2}{3}\right)+\frac{1}{2}e^{\frac{x}{2}}$

g) dom f: $1+\frac{x}{2}>0$ $\quad$ dom f $=]-2,+\infty[$

$$f'(x)= 3\cos x + 3\cdot\log\left(1+\frac{x}{2}\right) + \frac{\frac{3}{2}x}{1+\frac{x}{2}}$$

volendo $=\frac{3x}{2+x}$

$D\left(\frac{3x}{2+x}\right) = \frac{3(2+x)-3x}{(2+x)^2} = \frac{6}{(2+x)^2}$

$$f''(x)= -3\,\mathrm{sen}\,x + \frac{3/2}{1+\frac{x}{2}} + \frac{\frac{3}{2}(1+\frac{x}{2})-\frac{3}{2}x\cdot\frac{1}{2}}{(1+\frac{x}{2})^2}$$

$= \frac{\frac{3}{2}}{(1+\frac{x}{2})^2}$ oppure $\frac{6}{(2+x)^2}$

volendo $\frac{3}{2+x}$

h) dom f $=\mathbb{R}$ $\quad f'(x)=-8x\,e^{-x^2}$

i) dom f: $x^2-1\neq 0 \quad x^2\neq 1$ $\quad$ dom f $=\mathbb{R}\setminus\{-1,1\}$

$$f'(x)= \frac{x^2-1-x(2x)}{(x^2-1)^2} = \frac{-1-x^2}{(x^2-1)^2}$$

16)

Calcolate la retta tangente al grafico delle seguenti funzioni nei punti a fianco indicati

a) $f(x)=3x^2-5x$ $(2,2)$

b) $f(x)=1-x^3$ $(0,1)$

c) $f(x)=\dfrac{x}{1+2x}$ $\left(-\frac{1}{4},-\frac{1}{2}\right)$

d) $f(x)=\dfrac{1}{\sqrt{x+4}}$ $\left(5,\frac{1}{3}\right)$

e) $f(x)=x+\frac{4}{x}$ $(2,4)$

f) $f(x)=x^{5/2}$ $(4,32)$

g) $f(x)=x+\sqrt{x}$ $(1,2)$

h) $f(x)=x^2+2e^x$ $(0,2)$

i) $f(x)=\tan x$ $\left(\frac{\pi}{4},1\right)$

l) $f(x)=e^x\cdot\cos x$ $(0,1)$

m) $f(x)=e^{1-\frac{x^2}{4}}$ $(2,1)$

n) $f(x)=\log(x^2-2x)$ $(-1,\log 3)$

o) $f(x)=\dfrac{x^3}{3}+x^2-3x$ $\left(-4,\frac{20}{3}\right)$

SOL.$^{\text{ne}}$

a) $y=7x-12$ b) $y=1$ c) $y=4x+\frac{1}{2}$ d) $y=-\frac{1}{54}x+\frac{23}{54}$

e) $y=4$ f) $y=20x-48$ g) $y=\frac{3}{2}x+\frac{1}{2}$

h) $y=2x+2$ i) $y=2x+1-\frac{\pi}{2}$ l) $y=x+1$

m) $y=-x+3$ n) $y=-\frac{4}{3}x-\frac{4}{3}+\log 3$ o) $y=5x+\frac{80}{3}$

17) Come esercizio 15 per:

a) $f(x)=\dfrac{1}{(x+3)^2}\cdot\log\left(\frac{1}{2}-x\right)+\sqrt{8-10x-3x^2}+x^6\cdot e^{-3x}-\dfrac{\cos x}{x^2}$

b) $f(x)=\dfrac{1}{x}\cdot\sqrt{(x+2)^2+x}+3e^4+\log(1-2x)-e^{2x}\cdot\sqrt{x}$

c) $f(x)=e^{3x}\cdot\sqrt{7-\frac{1}{7}x^2}+\log(4x^2+1)-\dfrac{\operatorname{sen}(4x)}{3x^2+5x-12}-\log(11-2x)$

d) $f(x)=(x^4-\pi)\cdot\log\left(2x^2-3x+\frac{17}{8}\right)$

e) $f(x)=e^{x-x^2}+\dfrac{5}{x^3}$

f) $f(x)=x\cdot\operatorname{sen}\left(\frac{4}{5}x^2\right)+x^3\cdot\log\left(-\frac{1}{5}x+1\right)$

g) $f(x)=\log(2x+7)+e^{x^2-1}-\dfrac{1}{x^3}$

h) $f(x)=\operatorname{sen}(2x)\cdot\cos(4x)$ (con anche $f'\left(\frac{5}{8}\pi\right)$, $f'\left(\frac{\pi}{3}\right)$)

<u>SOL.</u>ne a) dom f $\begin{cases}(x+3)^2\neq 0\\ \frac{1}{2}-x>0\\ 8-10x-3x^2\geq 0\\ x^2\neq 0\end{cases}$ $\begin{cases}x\neq -3\\ x<\frac{1}{2}\\ -4\leq x\leq\frac{2}{3}\\ x\neq 0\end{cases}$

dom f $=[-4,-3[\,\cup\,]-3,0[\,\cup\,]0,\frac{1}{2}[$

$$f'(x)=-\frac{2}{(x+3)^3}\cdot\log\left(\frac{1}{2}-x\right)+\frac{2}{(x+3)^2}\cdot\frac{1}{\frac{1}{2}-x}+\frac{(-10-6x)}{2\sqrt{8-10x-3x^2}}+$$

$$+6x^5\cdot e^{-3x}+x^6\cdot(-3e^{-3x})-\frac{(-\operatorname{sen}x)\cdot x^2-\cos x\cdot(2x)}{x^4}$$

b) dom f $\begin{cases}x\neq 0\\ (x+2)^2+x\geq 0\\ 1-2x>0\\ x\geq 0\end{cases}$ $\begin{cases}x\neq 0\\ x\leq -4\cup x\geq -1 \quad (x^2+5x+4\geq 0)\\ x<\frac{1}{2}\\ x\geq 0\end{cases}$ dom f $=]0,\frac{1}{2}[$

$$f'(x) = -\frac{1}{x^2}\cdot\sqrt{x^2+5x+4} + \frac{1}{x}\cdot\frac{2x+5}{2\sqrt{x^2+5x+4}} +$$

$$+\frac{-2}{1-2x} - 2e^{2x}\cdot\sqrt{x} - e^{2x}\cdot\frac{1}{2\sqrt{x}}$$

OSS. $3e^4$ è una COSTANTE $\Rightarrow D(3e^4)=0$

c)
$$\text{dom}f \begin{cases} 7-\frac{1}{7}x^2 \geq 0 \\ 4x^2+1>0 \\ 3x^2+5x-12 \neq 0 \\ 11-2x>0 \end{cases} \quad \begin{cases} -7\leq x\leq 7 \\ \forall x \\ x\neq +4/3 \quad x\neq -3 \\ x<\frac{11}{2} \end{cases} \quad \text{dom}f = [-7,-3[\cup]-3,\tfrac{4}{3}[\cup]\tfrac{4}{3},\tfrac{11}{2}[$$

$$f'(x) = 3e^{3x}\cdot\sqrt{7-\frac{1}{7}x^2} + e^{3x}\cdot\frac{-2/7\,x}{\sqrt{7-\frac{1}{7}x^2}} + \frac{8x}{4x^2+1} -$$

$$-\frac{4\cos(4x)\cdot(3x^2+5x-12) - \text{sen}(4x)\cdot(6x+5)}{(3x^2+5x-12)^2} + \frac{2}{11-2x}$$

d) $\text{dom}f:\ 2x^2-3x+\frac{17}{8}>0 \quad \forall x\ (\Delta<0) \quad \text{dom}f=\mathbb{R}$

$$f'(x) = 4x^3\cdot\log\left(2x^2-3x+\frac{17}{8}\right) + (x^4-\pi)\cdot\frac{4x-3}{2x^2-3x+\frac{17}{8}}$$

(oss. π è una costante)

e) $\text{dom}f:\ x^3\neq 0 \to x\neq 0 \quad \text{dom}f=\mathbb{R}\setminus\{0\}$

$$f'(x) = (1-2x)\cdot e^{x-x^2} - \frac{15}{x^4} \qquad \left(D\left(\frac{1}{x^3}\right) = D(x^{-3}) = -3\cdot x^{-4} = -\frac{3}{x^4}\right)$$

f) $\text{dom}f:\ -\frac{1}{5}x+1>0 \quad \text{dom}f =]-\infty,5[$

$$f'(x) = \text{sen}\left(\frac{4}{5}x^2\right) + x\cdot\frac{8}{5}x\cdot\cos\left(\frac{4}{5}x^2\right) + 3x^2\cdot\log\left(-\frac{1}{5}x+1\right) + x^3\cdot\frac{-\frac{1}{5}}{-\frac{1}{5}x+1}$$

g) $\text{dom}f = \begin{cases} 2x+7>0 \\ x^3\neq 0\end{cases} \begin{cases} x>-\frac{7}{2} \\ x\neq 0\end{cases} \quad \text{dom}f =]-\tfrac{7}{2},0[\cup]0,+\infty[$

$$f'(x) = \frac{2}{2x+7} + 2x\cdot e^{x^2-1} + \frac{3}{x^4}$$

h) $\text{dom} f = \mathbb{R}$

$$f'(x) = 2\cos(2x)\cdot\cos(4x) - 4\,\text{sen}(2x)\cdot\text{sen}(4x)$$

$$f'\left(\frac{5}{8}\pi\right) = 2\cos\left(\frac{5}{4}\pi\right)\cdot\cos\left(\frac{5}{2}\pi\right) - 4\,\text{sen}\left(\frac{5}{4}\pi\right)\cdot\text{sen}\left(\frac{5}{2}\pi\right) =$$

$$= 2\cdot\left(-\frac{\sqrt{2}}{2}\right)\cdot 0 - 4\cdot\left(-\frac{\sqrt{2}}{2}\right)\cdot 1 = \boxed{2\sqrt{2}}$$

$$f'\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{2}{3}\pi\right)\cdot\cos\left(\frac{4}{3}\pi\right) - 4\,\text{sen}\left(\frac{2}{3}\pi\right)\cdot\text{sen}\left(\frac{4}{3}\pi\right) =$$

$$= 2\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right) - 4\cdot\frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{2} + 3 = \boxed{\frac{7}{2}}$$

18) Riprendiamo i concetti di PRIMITIVA di una funzione assegnata e di integrale indefinito

Il concetto di primitiva è l'opposto del concetto di derivata: data una funzione $f(x)$ si cerca di risalire ad un'altra funzione $F(x)$ la cui derivata coincida con $f(x)$.

Quindi data una funzione $f(x)$ su un intervallo, si dice PRIMITIVA di $f(x)$ ogni funzione $F(x)$ tale che

$$F'(x) = f(x).$$

Si dimostra che se esiste una primitiva $F(x)$ di una funzione $f(x)$, allora esistono INFINITE PRIMITIVE perché tutte le funzioni $F(x)+c$ lo sono (in quanto la derivata di una costante è nulla).

Si dimostra anche che, per determinare tutte le primitive di una funzione assegnata su un intervallo, è sufficiente trovarne una e poi aggiungere una costante $c \in \mathbb{R}$

$F(x)$ PRIMITIVA $\rightarrow$ $F(x)+c$ tutte le PRIMITIVE $(c \in \mathbb{R})$

Data una funzione $f(x)$ nella variabile x per indicare l'insieme di tutte le PRIMITIVE di $f(x)$ si usa il SIMBOLO di INTEGRALE INDEFINITO ($\int$)

e si scrive

$\int f(x)\,dx$ integrale indefinito di $f(x)$

intendendo $\int f(x)\,dx$ = insieme di tutte le primitive di $f(x)$.

Quindi, individuata una PRIMITIVA $F(x)$ di $f(x)$, si avrà

$$\int f(x)\,dx = \underbrace{F(x) + c}_{\text{infinite primitive al variare di } c \in \mathbb{R}}$$

FORMULE DI INTEGRAZIONE

$f(x)$	$\int f(x)\,dx$
0	c
1	$x + c$
x	$\frac{x^2}{2} + c$
x^α	$\frac{x^{\alpha+1}}{\alpha+1} + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\log\lvert x\rvert + c$
$\operatorname{sen} x$	$-\cos x + c$
$\cos x$	$\operatorname{sen} x + c$
$\frac{1}{\sqrt{x}}$	$2\sqrt{x} + c$

ES.

$\int 2x^5\,dx = \frac{1}{3}x^6 + c$

$\int \frac{1}{4}e^x\,dx = \frac{1}{4}e^x + c$

$\int 3\operatorname{sen} x\,dx = -3\cos x + c$

INTEGRAZIONE DI UNA FUNZIONE COMPOSTA

1) $\int f'(x)\cdot (f(x))^{\alpha} dx = \dfrac{(f(x))^{\alpha+1}}{\alpha+1} + c$

2) $\int \dfrac{f'(x)}{f(x)} dx = \log|f(x)| + c$

3) $\int f'(x)\cdot e^{f(x)} dx = e^{f(x)} + c$

4) $\int f'(x)\cdot \cos f(x)\, dx = \operatorname{sen} f(x) + c$

5) $\int f'(x)\cdot \operatorname{sen} f(x)\, dx = -\cos f(x) + c$

ESEMPI

1) $\int x\cdot\sqrt{2x^2+1}\, dx = \int x\cdot(\underbrace{2x^2+1}_{(f(x))^{\frac{1}{2}}})^{1/2} dx = \frac{1}{4}\int 4x(2x^2+1)^{1/2} dx =$

$f'(x) = 4x$

$= \frac{1}{4}\dfrac{(2x^2+1)^{3/2}}{3/2} + c = \frac{1}{6}(2x^2+1)^{3/2} + c$

(in alternativa cambiate variabile $t = 2x^2+1 \rightarrow dt = 4x\,dx$)

3) $\int e^{3x+1} dx = \frac{1}{3}\int 3\cdot e^{3x+1} dx = \frac{1}{3} e^{3x+1} + c$

$f(x) = 3x+1$
$f'(x) = 3$

(in alternativa $t = 3x+1$
$dt = 3dx$)

4) $\int \operatorname{sen}\left(\frac{x}{4}\right) dx = 4\int \frac{1}{4}\operatorname{sen}\left(\frac{x}{4}\right) dx = -4\cos\left(\frac{x}{4}\right) + c$

$f(x) = \frac{x}{4}$ $f'(x) = \frac{1}{4}$

(oppure $t = \frac{x}{4}$, $dt = \frac{1}{4}dx$)

2) $\int \dfrac{3}{3x+2} dx = \log|3x+2| + c$ (oppure $t = 3x+2$
$dt = 3\,dx$)

$f(x) = 3x+2$ $f'(x) = 3$

19) Dite (motivando la risposta) quali tra le seguenti funzioni costituiscono una primitiva di

$f(x) = \text{sen}(5x)$

$\boxed{1}$ $g(x) = -\frac{1}{5}\cos(5x) - 1$ $\boxed{2}$ $g(x) = \frac{1}{5}\cos(5x)$

$\boxed{3}$ $g(x) = 5\cos(5x)$ $\boxed{4}$ $g(x) = 3 - \frac{1}{5}\cos(5x)$

$f(x) = \frac{1}{2}e^{2x} - \frac{1}{3}\cos\left(\frac{x}{3}\right)$

$\boxed{1}$ $g(x) = e^{2x} + \frac{1}{9}\text{sen}\left(\frac{x}{3}\right)$

$\boxed{2}$ $g(x) = \frac{1}{4}e^{2x} - \text{sen}\left(\frac{x}{3}\right) - 4$

$\boxed{3}$ $g(x) = \frac{1}{2}e^{2x} - \text{sen}\left(\frac{x}{3}\right) + 1$

$f(x) = -\frac{1}{1-x}$ su $[2, 10]$

$\boxed{1}$ $g(x) = -\log(1-x) + 3$

$\boxed{2}$ $g(x) = -\frac{1}{(1-x)^2}$

$\boxed{3}$ $g(x) = \log(1-x) + 4$

<u>SOL.</u>^{ne} Per essere una primitiva dev'essere $g'(x) = f(x)$

$f(x) = \text{sen}(5x)$ 1,4) sì 3) $g(x) = f'(x)$ No 2) $g'(x) = -\text{sen}(5x)$ No

$g'(x) = -\frac{1}{5}\cdot(-\text{sen}(5x)\cdot 5) = \text{sen}(5x)$

$f(x) = \frac{1}{2}e^{2x} - \frac{1}{3}\cos\left(\frac{x}{3}\right)$ 1) $g(x) = f'(x)$ No 2) sì $g'(x) = \frac{1}{4}\cdot 2e^{2x} - \frac{1}{3}\cos\left(\frac{x}{3}\right) = \frac{1}{2}e^{2x} - \frac{1}{3}\cos\left(\frac{x}{3}\right)$

3) $g'(x) = e^{2x} - \frac{1}{3}\cos\left(\frac{x}{3}\right)$ No

$f(x) = -\frac{1}{1-x}$ 1) No $g'(x) = \frac{1}{1-x}$ 2) No $g(x) = f'(x)$

3) sì $g'(x) = \frac{1}{1-x}(-1) = -\frac{1}{1-x}$

20) Calcoliamo i seguenti integrali indefiniti:

a) $\int (2e^x + x^3 - x)\,dx$ b) $\int (e^x - \frac{2}{\sqrt{x}})\,dx$

c) $\int (1 + 2\,\mathrm{sen}x - \cos x)\,dx$ d) $\int \frac{1}{x^2}\,dx$

e) $\int (2x^4 - \frac{\sqrt{x}}{x} - 4\,\mathrm{sen}x)\,dx$ f) $\int (1 - x^3 + 15x^4)\,dx$

g) $\int (5\sqrt[4]{x} - 7\sqrt[4]{x^3})\,dx$ h) $\int \frac{10}{x^9}\,dx$

i) $\int (\frac{3}{x^2} - \frac{5}{x^4})\,dx$ j) $\int \cos(4x)\,dx$

k) $\int 3e^{3x}\,dx$ l) $\int e^{5x}\,dx$ m) $\int \sqrt{2x+1}\,dx$

n) $\int 3x\sqrt{1+x^2}\,dx$ o) $\int \frac{2}{3-4x}\,dx$

p) $\int (x^{2/3} + \frac{x^2}{\sqrt{x}})\,dx$ q) $\int (\frac{1}{\sqrt[3]{x}} + 3)\,dx$

r) $\int 2(2x+1)^2\,dx$ s) $\int \frac{1}{(x+1)^2}\,dx$ t) $\int \frac{2x}{\sqrt{1+x^2}}\,dx$

u) $\int (2-x)^6\,dx$ v) $\int \frac{4}{(1+2x)^3}\,dx$ w) $\int e^{x+1}\,dx$

x) $\int (\frac{8}{5}x^5 + \frac{4}{x^{5/2}})\,dx$ y) $\int \frac{3}{2}\,\mathrm{sen}(\frac{x}{6})\,dx$ z) $\int \frac{2}{5}e^{-4x}\,dx$

SOL.^{ue}

OSS. La radice $\sqrt{x} = x^{\frac{1}{2}}$ e sia nelle derivate, sia nel calcolo degli integrali conviene considerare la radice come una potenza.

a) $= 2e^x + \frac{x^4}{4} - \frac{x^2}{2} + c$ b) $= e^x - 4\sqrt{x} + c$

c) $= x - 2\cos x - \operatorname{sen} x + c$ d) $= -\frac{1}{x} + c$

e) $= \int 2x^4 - \frac{1}{\sqrt{x}} - 4\operatorname{sen} x \, dx = \frac{2}{5}x^5 - 2\sqrt{x} + 4\cos x + c$

f) $= x - \frac{x^4}{4} + 3x^5 + c$ g) $= \int (5x^{\frac{1}{4}} - 7x^{3/4})\, dx = 5\frac{x^{5/4}}{5/4} - 7\frac{x^{7/4}}{7/4} + c = 4x^{5/4} - 4x^{7/4} + c$

h) $= \int 10x^{-9} dx = -\frac{5}{4x^8} + c$

i) $= \int (3x^{-2} - 5x^{-4})\, dx = -\frac{3}{x} + \frac{5}{3x^3} + c$ j) $= \frac{1}{4}\operatorname{sen}(4x) + c$

k) $= e^{3x} + c$ l) $= \frac{1}{5}e^{5x} + c$

m) $= \int (2x+1)^{\frac{1}{2}} dx = \frac{1}{2}\int \underbrace{2(2x+1)^{\frac{1}{2}}}_{f'(x)(f(x))^\alpha} dx = \frac{1}{2}\frac{(2x+1)^{3/2}}{\frac{3}{2}} + c = \frac{1}{3}(2x+1)^{3/2} + c$

oppure cambiate variabile $t = 2x+1$ $dt = 2dx$ ....

n) $= \frac{3}{2}\frac{(1+x^2)^{3/2}}{3/2} + c = (1+x^2)^{3/2} + c$

o) $= -\frac{1}{2}\int \frac{-4}{3-4x} dx = -\frac{1}{2}\log|3-4x| + c$ (oppure cambiamento di variabile $t = 3-4x$)

p) $= \int (x^{2/3} + x^{2-\frac{1}{2}})\, dx = \int (x^{2/3} + x^{3/2})\, dx = \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + c$

q) $= \int x^{-\frac{1}{3}} + 3 \, dx = \frac{3}{2}x^{2/3} + 3x + c$ r) $= \frac{1}{3}(2x+1)^3 + c$

s) $= -\frac{1}{(x+1)} + c$ t) $= 2\sqrt{1+x^2} + c$ u) $= -\frac{1}{7}(2-x)^7 + c$

v) $= -\frac{1}{(1+2x)^2} + c$ w) $= e^{x+1} + c$ x) $= \frac{4}{15}x^6 - \frac{8}{3x^{3/2}} + c$

y) $= -9\cos\left(\frac{x}{6}\right) + c$ z) $= -\frac{1}{10}e^{-4x} + c$

INTEGRAZIONE PER PARTI

$$\int f'(x)\cdot g(x)\,dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\,dx$$

SI UTILIZZA NEI SEGUENTI CASI

$P_m(x)$ = polinomio di grado m

→ $\int \underset{g}{P_m(x)} \cdot \underset{f}{e^x}\,dx$

integrando l'esponenziale rimane uguale, mentre derivando il polinomio si abbassa di grado

ES. $\int (3x-x^2)\,e^x\,dx =$

$g(x) = 3x-x^2$ $\quad f'(x) = e^x$

$g'(x) = 3-2x$ $\quad f(x) = e^x$

$= e^x(3x-x^2) - \int e^x\cdot(3-2x)\,dx =$

$g(x) = 3-2x$ $\quad f'(x) = e^x$

$g'(x) = -2$ $\quad f(x) = e^x$

$= e^x(3x-x^2) - \left[e^x(3-2x) - \int e^x(-2)\,dx\right] =$

$= e^x(3x-x^2) - \left[e^x(3-2x) + 2e^x\right] + c =$

$= e^x(3x-x^2-3+2x-2) + c =$

$= e^x(-x^2+5x-5) + c$

Verifica

$D\left[(-x^2+5x-5)\,e^x\right] = (-2x+5)\,e^x + (-x^2+5x-5)\,e^x =$

$= e^x(-2x+\cancel{5}-x^2+5x-\cancel{5}) =$

$= e^x(-x^2+3x)$ ok

$$\rightarrow \int \underset{\downarrow\, g}{P_m(x)} \cdot \begin{matrix} \text{sen}\, x \\ \text{sen}(\alpha x) \\ \cos x \\ \cos(\alpha x) \end{matrix}\, dx$$

$\hookrightarrow f'$

es. $\int (2x+3)\,\text{sen}\, x\, dx =$

$g(x) = 2x+3 \quad g'(x) = 2$

$f'(x) = \text{sen}\, x$

$f(x) = -\cos x$

$= -\cos x \cdot (2x+3) - \int -\cos x\,(2)\, dx =$

$= -\cos x\,(2x+3) - \int -2\cos x\, dx =$

$= -\cos x\,(2x+3) + 2\int \cos x\, dx =$

$= -\cos x\,(2x+3) + 2\,\text{sen}\, x + c$

Verifica

$-2\cancel{\cos x} + (2x+3)\,\text{sen}\, x + 2\cancel{\cos x} =$

$= (2x+3)\,\text{sen}\, x \quad$ ok

$$\rightarrow \int \underset{f'}{\ldots}\ \underset{\downarrow\, g}{\begin{matrix}\log x \\ \log(\alpha x)\end{matrix}}\, dx$$

oss: In generale può funzionare con e^x, e^{2x}, sen x, sen(3x) ... log x, log(2x) una non quando c'è una composizione $\rightarrow e^{x^2}$, sen(x^3-3), $\log(x^2)$

si prende come g il log per derivarlo e trasformarlo in $\frac{1}{x}$ (più semplice).

es. $\int \log x\, dx =$

$g(x) = \log(x) \quad g'(x) = \frac{1}{x}$

$f'(x) = 1 \quad f(x) = x$

$= x \cdot \log(x) - \int x \cdot \frac{1}{x}\, dx =$

dom: $x > 0$

$I \subset\,]0, +\infty[$

$= x\log x - x + c =$

$= x(\log x - 1) + c$

Verifica

$1\,(\log x - 1) + x\left(\frac{1}{x}\right) = \log x - 1 + 1 = \log x \quad$ ok

$\rightarrow \int \underset{\substack{\operatorname{sen}(\alpha x)\\ \operatorname{sen}^2(\alpha x)}}{\operatorname{sen}(\alpha x)} \cdot \underset{\operatorname{sen}(\beta x)}{\cos(\beta x)}\, dx$ → è indifferente quale prendere come f' o come g → si ripete il procedimento 2 volte e si porta al 1° membro. La 2ª volta non invertire nell'attribuire f' e g se no si torna indietro.

$\rightarrow \int \underset{\substack{\cos(\alpha x)\\ \downarrow \\ g}}{\operatorname{sen}(\alpha x)} \cdot \underset{\substack{\downarrow \\ f'}}{e^{\beta x}}\, dx$

es. $\int \operatorname{sen}^2 x\, dx = \int \operatorname{sen} x \cdot \operatorname{sen} x\, dx =$

$f'(x) = \operatorname{sen} x \qquad f(x) = -\cos x$

$g(x) = \operatorname{sen} x \qquad g'(x) = \cos x$

$= -\cos x \cdot \operatorname{sen} x - \int -\cos x \cdot \cos x\, dx =$

$= -\cos x \cdot \operatorname{sen} x + \int \cos^2 x\, dx =$

$= -\cos x \cdot \operatorname{sen} x + \int (1 - \operatorname{sen}^2 x)\, dx =$

$= -\operatorname{sen} x \cos x + \int dx - \int \operatorname{sen}^2 x\, dx \Rightarrow$

$\int \operatorname{sen}^2 x\, dx = -\operatorname{sen} x \cos x + x - \int \operatorname{sen}^2 x\, dx \Rightarrow$

$2\int \operatorname{sen}^2 x\, dx = -\operatorname{sen} x \cos x + x + c$

$\int \operatorname{sen}^2 x\, dx = \dfrac{-\operatorname{sen} x \cos x + x}{2} + c$

Verifica

$\frac{1}{2}\left[1 + (-\cos x \cdot \cos x + \operatorname{sen}^2 x)\right] =$

$= \frac{1}{2}(1 - \cos^2 x + \operatorname{sen}^2 x) =$

$= \dfrac{\operatorname{sen}^2 x + \operatorname{sen}^2 x}{2} = \operatorname{sen}^2 x$

$$\int \underset{g}{\cos x} \cdot \underset{f'(x)}{e^x}\, dx = \qquad g(x)=\cos x \quad g'(x)=-\operatorname{sen}x$$

$$f'(x)=e^x \quad f(x)=e^x$$

$$I \subset \mathbb{R}$$

$$= e^x \cdot \cos x - \overbrace{\int e^x \cdot (-\operatorname{sen}x)\, dx}^{+\int e^x \operatorname{sen}x\, dx} =$$

$$g(x)=\operatorname{sen}x \quad g'(x)=\cos x \quad f'(x)=e^x \quad f(x)=e^x$$

$$= e^x \cdot \cos x + \left[\operatorname{sen}x\, e^x - \int \cos x \cdot e^x dx\right] =$$

$$= e^x \cos x + \operatorname{sen}x\, e^x - \int \cos x\, e^x\, dx$$

porto al 1° membro

$$2\int \cos x\, e^x = (\cos x + \operatorname{sen}x)\, e^x + c$$

$$\int \cos x\, e^x = \frac{1}{2}(\operatorname{sen}x + \cos x)\, e^x + c$$

Verifica

$$\frac{1}{2}\left[e^x(\operatorname{sen}x + \cos x) + e^x(\cos x - \operatorname{sen}x)\right] =$$

$$= \frac{1}{2} e^x \left(\cancel{\operatorname{sen}x} + \cos x + \cos x - \cancel{\operatorname{sen}x}\right) = \frac{e^x \cdot 2\cos x}{2} =$$

$$= e^x \cos x \quad \text{ok.}$$

20/2) Calcoliamo i seguenti integrali indefiniti:

i) $\int \frac{x^2+x+1}{x}\,dx$ ii) $\int (x^3-1)^2\,dx$ iii) $\int \frac{x^5+x^4+1}{x^2}\,dx$

iv) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$ v) $\int x^2\cdot\sqrt{x^3+1}\,dx$ vi) $\int x\cdot\cos x\,dx$

vii) $\int \frac{x}{\sqrt{x^2+1}}\,dx$ viii) $\int x\cdot e^{2x}\,dx$ ix) $\int x\cdot\operatorname{sen}(4x)\,dx$

x) $\int e^{2x+1}\,dx$ xi) $\int \operatorname{sen}^2 x\cdot\cos x\,dx$

Sol.ne i) $\int (x+1+\frac{1}{x})\,dx = \frac{x^2}{2}+x+\log|x|+c$ vale $\forall$ intervallo $I\subset]-\infty,0[$, $I\subset]0,+\infty[$

$\left(\log|x| = \begin{cases}\log x & x>0\\ \log(-x) & x<0\end{cases}\right)$

ii) $\int (x^6-2x^3+1)\,dx = \frac{x^7}{7}-\frac{1}{2}x^4+x+c$ $\forall I\subset\mathbb{R}$

iii) $\int (x^3+x^2+\frac{1}{x^2})\,dx = \frac{1}{4}x^4+\frac{1}{3}x^3-\frac{1}{x}+c$ $\forall I\subset]-\infty,0[$, $I\subset]0,+\infty[$

iv) $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx \underset{t=\sqrt{x}}{=} \int \frac{\operatorname{sen}t}{\cancel{t}}\,2\cancel{t}\,dt = \int 2\operatorname{sen}t\,dt = -2\cos t+c$

$t=\sqrt{x}$
$dt=\frac{1}{2\sqrt{x}}\,dx$
$dx = 2t\,dt$

$= -2\cos\sqrt{x}+c$ $\forall I\subset]0,+\infty[$

oppure $= 2\int \frac{1}{2\sqrt{x}}\cdot\operatorname{sen}\sqrt{x}\,dx$ e poi $\int f'(x)\cdot\operatorname{sen}(f(x))\,dx = -\cos(f(x))+c$ con $f(x)=\sqrt{x}$

v) $\frac{1}{3}\int (3x^2)(x^3+1)^{1/2}\,dx = \frac{1}{3}\frac{(x^3+1)^{3/2}}{3/2}+c = \frac{2}{9}(x^3+1)^{3/2}+c$ $\forall I\subset[-1,+\infty[$

$f'(x)=3x^2$ $f(x)=x^3+1$

$\int f'(x)\cdot(f(x))^\alpha\,dx = \frac{1}{\alpha+1}\cdot(f(x))^{\alpha+1}+c$

oppure $t=x^3+1$

vi) PER PARTI $\int x\cdot\cos x\,dx = x\operatorname{sen}x - \int \operatorname{sen}x\,dx = x\operatorname{sen}x+\cos x+c$ $\forall I\subset\mathbb{R}$

$f(x)=x$ $f'(x)=1$
$g'(x)=\cos x$ $g(x)=\operatorname{sen}x$

vii) $\frac{1}{2}\int 2x\cdot(x^2+1)^{-\frac{1}{2}}dx = \frac{1}{2}\frac{(x^2+1)^{\frac{1}{2}}}{1/2}+c = \sqrt{x^2+1}+c \quad \forall I\subset\mathbb{R}$

$f'=2x \quad f(x)=x^2+1$

oppure $t=x^2+1$

$\frac{1}{4}\int 2e^{2x}dx$ oppure $t=2x$

viii) PER PARTI $\int x\cdot e^{2x}dx = \frac{1}{2}xe^{2x}-\int\frac{1}{2}e^{2x}dx = \frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+c$

$f(x)=x \quad f'=1$

$g'(x)=e^{2x} \quad g(x)=\frac{1}{2}e^{2x}$

$= \frac{1}{2}(x-\frac{1}{2})e^{2x}+c \quad \forall I\subset\mathbb{R}$

ix) PER PARTI $\int x\cdot \text{sen}(4x)\,dx = -\frac{1}{4}x\cos(4x)+\frac{1}{4}\int\cos(4x)\,dx =$

$\hookrightarrow \frac{1}{16}\int 4\cos(4x)\,dx$ oppure $t=4x$

$\forall I\subset\mathbb{R}$

$f(x)=x \quad f'=1$

$g'=\text{sen}(4x) \quad g(x)=-\frac{1}{4}\cos(4x)$

$= -\frac{1}{4}x\cos(4x)+\frac{1}{16}\text{sen}(4x)+c$

x) $\int e^{2x+1}dx = \frac{1}{2}e^{2x+1}+c \quad \forall I\subset\mathbb{R} \quad (\frac{1}{2}\int 2e^{2x+1}$ oppure $t=2x+1)$

xi) $\int \cos x\,(\text{sen}x)^2dx = \frac{1}{3}(\text{sen}x)^3+c$ oppure $t=\text{sen}x$

$f'=\cos x \quad f(x)=\text{sen}x$

20/3) Calcoliamo i seguenti integrali indefiniti:

i) $\int(\cos x)^4\,\text{sen}x\,dx$ ii) $\int\frac{x}{(1+x^2)^2}dx$

iii) $\int\frac{\log x}{x}dx$ iv) $\int(1+x+x^2)^3(1+2x)\,dx$

v) $\int x\sqrt[3]{1+x^2}\,dx$ vi) $\int xe^{x^2}dx$

vii) $\int(2-x)^6dx$ viii) $\int e^{\text{sen}x}\cdot\cos x\,dx$

ix) $\int\frac{e^x+1}{e^x}dx$ x) $\int x^2\cdot\text{sen}(3x)\,dx$

xi) $\int(x^3+\log x)^3(3x^2+\frac{1}{x})\,dx$ xii) $\int(\text{sen}x+1)\cos x\,dx$

xiii) $\int\tan x\,dx$ xiv) $\int x^2\cdot e^x\,dx$

xv) $\int \frac{4}{(1+2x)^3}\,dx$ xvi) $\int e^{2x}\cdot \operatorname{sen}x\,dx$

xvii) $\int \frac{(\log x)^2}{x}\,dx$ xviii) $\int \frac{1+4x}{\sqrt{1+x+2x^2}}\,dx$

xix) $\int x^4 \log x\,dx$ xx) $\int e^x \sqrt{1+e^x}\,dx$.

xxi) $\int x^2 \cos(3x)\,dx$ xxii) $\int \sqrt[5]{3-5x}\,dx$

xxiii) $\int (t^3-1)e^t\,dt$ xxiv) $\int \frac{e^x}{e^x+1}\,dx$

xxv) $\int x^2 \log x\,dx$ xxvi) $\int (\operatorname{sen}x)^2\,dx$

xxvii) $\int x\cdot \operatorname{sen}x\,dx$ xxviii) $\int e^{3x}\cdot \operatorname{sen}(2x)\,dx$

xxix) $\int (\operatorname{sen}x)^3\,dx$ xxx) $\int e^{-x}\cdot \cos(2x)\,dx$.

xxxi) $\int (2x-1)\cdot \log x\,dx$ xxxii) $\int x\cdot e^{-x}\,dx$

xxxiii) $\int \sqrt{x}\cdot \log x\,dx$

xxxiv) $\int (\cos x)^2\,dx$ xxxv) $\int \operatorname{sen}x \cos x\,dx$

xxxvi) $\int (1-x)^3\,dx$ xxxvii) $\int (2-x)^{5/2}\,dx$

xxxviii) $\int \operatorname{sen}x \cos^2 x\,dx$ xxxix) $\int (3-2x)^4\,dx$

xL) $\int \operatorname{sen}^2 x \cos x\,dx$

Sol.^{ne} i) $I \subset \mathbb{R}$ $\int (\cos x)^4 \cdot \operatorname{sen} x \, dx = -\int \underbrace{(-\operatorname{sen} x)}_{f'(x)} \cdot \underbrace{(\cos x)^4}_{(f(x))^\alpha} dx =$

$= -\dfrac{(\cos x)^5}{5} + c = -\frac{1}{5}(\cos x)^5 + c$ oppure

$\int (\cos x)^4 \cdot \operatorname{sen} x \, dx = \int t^4 \cdot \operatorname{sen} x \cdot \dfrac{dt}{-\operatorname{sen} x} = -\int t^4 dt =$

$t = \cos x$

$dt = -\operatorname{sen} x \, dx$

$dx = \dfrac{dt}{-\operatorname{sen} x}$

$= -\dfrac{t^5}{5} + c = -\dfrac{(\cos x)^5}{5} + c$

ii) $I \subset \mathbb{R}$ $\int \dfrac{x}{(1+x^2)^2} dx = \frac{1}{2} \int \underbrace{2x(1+x^2)^{-2}}_{f'(x)\cdot(f(x))^\alpha} dx = \frac{1}{2} \dfrac{(1+x^2)^{-1}}{-1} + c =$

$= -\dfrac{1}{2(1+x^2)} + c$

oppure $\int \dfrac{x}{(1+x^2)^2} dx = \int \dfrac{x}{t^2} \dfrac{dt}{2x} = \frac{1}{2} \int \frac{1}{t^2} dt = -\frac{1}{2t} + c =$

$t = 1+x^2$

$dt = 2x dx$

$dx = \dfrac{dt}{2x}$

$= -\dfrac{1}{2(1+x^2)} + c$

iii) $I \subset]0,+\infty[$ $\int \dfrac{\log x}{x} dx = \int \underbrace{\frac{1}{x}}_{f'(x)} \cdot \underbrace{\log x}_{f(x)} dx = \dfrac{(\log x)^2}{2} + c$

oppure $\int \dfrac{\log x}{x} d = \int \dfrac{t}{x} \cdot x dt = \int t \, dt = \dfrac{t^2}{2} + c =$

$t = \log x$

$dt = \frac{1}{x} dx$

$dx = x \, dt$

$= \frac{1}{2}(\log x)^2 + c$

iv) $I \subset \mathbb{R}$ $\displaystyle\int \underbrace{(1+x+x^2)^3}_{(f(x))^3} \cdot \underbrace{(1+2x)}_{f'(x)}\,dx = \frac{(1+x+x^2)^4}{4} + c$

oppure $\displaystyle\int (1+x+x^2)^3 \cdot (1+2x)\,dx \underset{\downarrow}{=} \int t^3 \cdot (1+2x) \cdot \frac{dt}{1+2x} =$

$t = 1+x+x^2$
$dt = (1+2x)\,dx$
$dx = \dfrac{dt}{1+2x}$

$= \displaystyle\int t^3\,dt = \frac{1}{4}t^4 + c$

$= \frac{1}{4}(1+x+x^2)^4 + c$.

v) $I \subset \mathbb{R}$ (nessuna condizione) $\displaystyle\int x\sqrt[3]{1+x^2}\,dx = \frac{1}{2}\int \underbrace{2x\,(1+x^2)^{1/3}}_{f'(x)\cdot(f(x))^\alpha}\,dx = \frac{1}{2}\,\frac{(1+x^2)^{4/3}}{4/3} + c =$

$= \dfrac{3}{8}\underbrace{\sqrt[3]{(1+x^2)^4}}_{=(1+x^2)\sqrt[3]{1+x^2}} + c$

oppure $\displaystyle\int x\sqrt[3]{1+x^2}\,dx \underset{\downarrow}{=} \int x\sqrt[3]{t}\,\frac{dt}{2x} = \frac{1}{2}\int \sqrt[3]{t}\,dt = \frac{1}{2}\,\frac{t^{4/3}}{4/3} + c$

$t = 1+x^2$
$dt = 2x\,dx$ $\qquad = \dfrac{3}{8}(1+x^2)^{4/3} + c$
$dx = \dfrac{dt}{2x}$

vi) $I \subset \mathbb{R}$ $\displaystyle\int x e^{x^2}\,dx = \frac{1}{2}\int \underbrace{2x\,e^{x^2}}_{f'(x)\cdot e^{f(x)}}\,dx = \frac{1}{2}e^{x^2} + c$

oppure $\displaystyle\int x e^{x^2}\,dx \underset{\downarrow}{=} \int x\,e^t\,\frac{dt}{2x} = \frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t + c =$

$t = x^2$ $\qquad = \frac{1}{2}e^{x^2} + c$
$dt = 2x\,dx$
$dx = \dfrac{dt}{2x}$

vii) $\displaystyle\int (2-x)^6\,dx = -\int \underbrace{-1\cdot(2-x)^6}_{f'(x)\cdot(f(x))^\alpha}\,dx = -\frac{(2-x)^7}{7} + c =$

$I \subset \mathbb{R}$ $\qquad = -\frac{1}{7}(2-x)^7 + c$

oppure $\displaystyle\int (2-x)^6\,dx \underset{\downarrow}{=} \int t^6(-dt) = -\int t^6\,dt = -\frac{t^7}{7} + c$

$t = 2-x$ $\qquad = -\dfrac{(2-x)^7}{7} + c$
$dt = -dx$
$dx = -dt$

viii) $I \subset \mathbb{R}$ $\displaystyle\int \underbrace{e^{\operatorname{sen}x}}_{e^{f(x)}} \cdot \underbrace{\cos x}_{f'(x)}\, dx = e^{\operatorname{sen}x} + c$

oppure $\displaystyle\int e^{\operatorname{sen}x} \cdot \cos x \, dx = \int e^t \, dt = e^t + c = e^{\operatorname{sen}x} + c$

$t = \operatorname{sen}x$
$dt = \cos x \, dx$

ix) $I \subset \mathbb{R}$ $\displaystyle\int \frac{e^x + 1}{e^x} \, dx = \int 1 + \frac{1}{e^x} \, dx = \int 1 + e^{-x} \, dx =$

$e^x \neq 0 \ \forall x$ $\quad = x - e^{-x} + c$

x) $I \subset \mathbb{R}$ $\displaystyle\int x^2 \cdot \operatorname{sen}(3x) \, dx = -\frac{1}{3}x^2 \cos(3x) + \int \frac{1}{3}\cos(3x) \cdot 2x \, dx =$ PER PARTI

$g(x) = x^2$ $\quad f'(x) = \operatorname{sen}(3x)$
$g'(x) = 2x$ $\quad f(x) = -\frac{1}{3}\cos(3x)$

$= -\frac{1}{3}x^2 \cdot \cos(3x) + \frac{2}{3}\displaystyle\int x \cdot \cos(3x) \, dx = -\frac{1}{3}x^2 \cdot \cos(3x)$ PER PARTI

$g(x) = x$ $\quad f'(x) = \cos(3x)$
$g'(x) = 1$ $\quad f(x) = \frac{1}{3}\operatorname{sen}(3x)$

$+ \frac{2}{3}\left[\frac{1}{3} x \operatorname{sen}(3x) - \displaystyle\int \frac{1}{3}\operatorname{sen}(3x) \, dx \right] = -\frac{1}{3}x^2 \cos(3x)$

$+ \frac{2}{9} x \operatorname{sen}(3x) + \frac{2}{9} \cdot \frac{1}{3} \displaystyle\int -3 \operatorname{sen}(3x) \, dx =$

$= -\frac{1}{3}x^2 \cdot \cos(3x) + \frac{2}{9} x \operatorname{sen}(3x) + \frac{2}{27}\cos(3x) + c$

$= \left(\frac{2}{27} - \frac{1}{3}x^2 \right) \cos(3x) + \frac{2}{9} x \operatorname{sen}(3x) + c.$

xi) $I \subset]0, +\infty[$ $\displaystyle\int \underbrace{(x^3 + \log x)^3}_{(f(x))^3} \cdot \underbrace{\left(3x^2 + \frac{1}{x}\right)}_{f'(x)} dx = \frac{1}{4}(x^3 + \log x)^4 + c$

oppure

$\displaystyle\int (x^3 + \log x)^3 \left(3x^2 + \frac{1}{x}\right) dx = \int t^3 \, dt = \frac{1}{4}t^4 + c = \frac{1}{4}(x^3 + \log x)^4 + c$

$t = x^3 + \log x$
$dt = \left(3x^2 + \frac{1}{x}\right) dx$

xii) $I \subset \mathbb{R}$ $\int \underset{f(x)}{(\operatorname{sen}x+1)} \underset{f'(x)}{\cos x}\, dx = \frac{1}{2}(\operatorname{sen}x+1)^2+c$

oppure $\int (\operatorname{sen}x+1)\cos x\, dx \underset{\downarrow}{=} \int t\, dt = \frac{t^2}{2}+c = \frac{1}{2}(\operatorname{sen}x+1)^2+c$

$t = \operatorname{sen}x+1$

$dt = \cos x\, dx$

xiii) $I \subset \operatorname{dom}\tan x$ $\int \tan x\, dx = \int \frac{\operatorname{sen}x}{\cos x}dx = -\int \frac{-\operatorname{sen}x}{\cos x}dx =$

I intervallo $\subset \mathbb{R}\setminus\{\frac{\pi}{2}+k\pi\}$

$= -\int \frac{f'(x)}{f(x)}dx = -\log \underbrace{|\cos x|}_{\text{il segno di } \cos x \text{ dipende poi dall'intervallo}} + c$

con $f(x) = \cos x$

oppure $\int \frac{\operatorname{sen}x}{\cos x}dx \underset{\downarrow}{=} \int \frac{1}{t}(-dt) = -\int \frac{1}{t}dt = -\log|t|+c$
$= -\log|\cos x|+c$

$t = \cos x$

$dt = -\operatorname{sen}x\, dx$

$-dt = \operatorname{sen}x\, dx$

xiv) $I \subset \mathbb{R}$ $\int x^2 e^x dx = x^2e^x - \int 2x e^x = x^2e^x - \left[2xe^x - \int 2e^x dx\right] =$

PER PARTI: $g(x) = x^2$, $f'(x) = e^x$, $g'(x) = 2x$, $f(x) = e^x$

PER PARTI: $g(x) = 2x$, $f'(x) = e^x$, $g'(x) = 2$, $f(x) = e^x$

$= x^2 \cdot e^x - 2x \cdot e^x + \int 2e^x dx = (x^2-2x+2)e^x + c$

xv) $I \subset]-\infty, -\frac{1}{2}[$ o $I \subset]-\frac{1}{2}, +\infty[$ $\int \frac{4}{(1+2x)^3}dx =$

$= 2\int \underset{f'(x)}{2} \cdot \underset{(f(x))^\alpha}{(1+2x)^{-3}} dx = 2\,\frac{(1+2x)^{-2}}{-2} = -\frac{1}{(1+2x)^2}+c$

oppure $\int \frac{4}{(1+2x)^3}dx \underset{\downarrow}{=} \int \frac{4}{t^3}\frac{1}{2}dt = 2\int \frac{1}{t^3}dt = -\frac{1}{t^2}+c =$

$t = 1+2x$

$dt = 2dx$

$dx = \frac{1}{2}dt$

$= -\frac{1}{(1+2x)^2}+c$

xvi) $I \subset \mathbb{R}$ $\int e^{2x} \cdot \operatorname{sen}x\, dx = -\cos x \cdot e^{2x} + 2\int \cos x \cdot e^{2x} dx =$

PER PARTI: $g(x) = e^{2x}$, $f'(x) = \operatorname{sen}x$, $g'(x) = 2e^{2x}$, $f(x) = -\cos x$

PER PARTI: $f'(x) = \cos x$, $g(x) = e^{2x}$, $f(x) = \operatorname{sen}x$, $g'(x) = 2e^{2x}$

$= -\cos x \cdot e^{2x} + 2\left[\operatorname{sen} x \cdot e^{2x} - \int 2 \operatorname{sen} x\, e^{2x} dx \right] =$

$= (2\operatorname{sen} x - \cos x)\, e^{2x} - 4 \int \operatorname{sen} x\, e^{2x} dx \quad \Rightarrow$

$5 \int \operatorname{sen} x \cdot e^{2x} dx = (2 \operatorname{sen} x - \cos x) \cdot e^{2x} + c$

$\int \operatorname{sen} x\, e^{2x} dx = \frac{1}{5} (2 \operatorname{sen} x - \cos x)\, e^{2x} + c$

xvii) $I \subset]0, +\infty[$ $\quad \int \frac{(\log x)^2}{x} dx = \int \underbrace{\frac{1}{x} (\log x)^2}_{f'(x) \cdot (f(x))^2} dx = \frac{1}{3} (\log x)^3 + c$

(oppure $t = \log x$)

xviii) $I \subset \mathbb{R}$ $\quad \int \frac{1+4x}{\sqrt{1+x+2x^2}} dx = \int \underbrace{(1+4x)}_{f'(x)} \cdot \underbrace{(1+x+2x^2)^{-\frac{1}{2}}}_{(f(x))^\alpha} dx =$

$2x^2 + x + 1 > 0 \ \forall x$

$\Delta = 1 - 8 < 0$ $\qquad = 2 \sqrt{1+x+2x^2} + c$

(oppure $t = 1 + x + 2x^2$)

xix) $I \subset]0, +\infty[$ $\quad \int x^4 \cdot \log x\, dx \underset{\text{PER PARTI}}{=} \frac{1}{5} x^5 \cdot \log x - \int \frac{1}{5} x^5 \cdot \frac{1}{x} dx =$

$f'(x) = x^4 \quad g(x) = \log x$

$f(x) = \frac{x^5}{5} \quad g'(x) = \frac{1}{x}$

$= \frac{1}{5} x^5 \cdot \log x - \frac{1}{5} \int x^4 dx = \frac{1}{5} x^5 \left(\log x - \frac{1}{5} \right) + c$

xx) $I \subset \mathbb{R}$ $\quad \int e^x \sqrt{1+e^x} dx = \int \underbrace{e^x}_{f'(x)} \cdot \underbrace{(1+e^x)^{1/2}}_{(f(x))^\alpha} dx =$

$1 + e^x \geq 0 \ \forall x$

$= \frac{(1+e^x)^{3/2}}{3/2} + c = \frac{2}{3} (1+e^x)^{3/2} + c$

(oppure $t = 1 + e^x$)

xxi) $I \subset \mathbb{R}$ $\quad \int x^2 \cos(3x)\, dx \underset{\text{PER PARTI}}{=} \frac{1}{3} x^2 \operatorname{sen}(3x) - \int \frac{2}{3} x \operatorname{sen}(3x) dx =$

$g(x) = x^2 \quad f'(x) = \cos(3x)$

$g'(x) = 2x \quad f(x) = \frac{1}{3} \operatorname{sen}(3x)$

$= \frac{1}{3} x^2 \operatorname{sen}(3x) - \frac{2}{3} \int x \operatorname{sen}(3x) dx$

$g(x) = x \quad f'(x) = \operatorname{sen}(3x)$

$g'(x) = 1 \quad f(x) = -\frac{1}{3} \cos(3x)$

$\underset{\text{PER PARTI}}{=} \frac{1}{3} x^2 \operatorname{sen}(3x) - \frac{2}{3} \left[-\frac{1}{3} x \cos(3x) + \int \frac{1}{3} \cos(3x)\, dx \right] =$

$$= \frac{1}{3}x^2 \operatorname{sen}(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{9}\int \cos(3x)\,dx =$$

$$= \left(\frac{1}{3}x^2 - \frac{2}{27}\right)\operatorname{sen}(3x) + \frac{2}{9}x\cos(3x) + c$$

xxii) $I \subset \mathbb{R}$ (nessuna cond.)

$$\int \sqrt[5]{3-5x}\,dx = \int (3-5x)^{1/5}dx = -\frac{1}{5}\int \underbrace{-5}_{f'(x)}\underbrace{(3-5x)^{1/5}}_{(f(x))^{1/5}}dx$$

$$= -\frac{1}{5}\cdot\frac{(3-5x)^{6/5}}{6/5} + c = -\frac{1}{6}(3-5x)^{6/5} + c$$

(oppure $t = 3-5x$)

$$\sqrt[5]{(3-5x)^6} = (3-5x)\sqrt[5]{3-5x}$$

xxiii) $I \subset \mathbb{R}$ $\int (t^3-1)e^t\,dt =$ PER PARTI

$g(t) = t^3-1$ $f'(t) = e^t$

$g'(t) = 3t^2$ $f(t) = e^t$

$$= (t^3-1)e^t - \int 3t^2 e^t\,dx \underset{\text{PER PARTI}}{=} (t^3-1)e^t - \left[3t^2\cdot e^t - \int 6te^t\,dt\right] =$$

$g(t) = 3t^2$ $f' = f = e^t$

$g'(t) = 6t$

$$= (t^3-3t^2-1)e^t + 6\int te^t\,dt \underset{\text{PER PARTI}}{=} (t^3-3t^2-1)e^t + 6\left[te^t - \int e^t dt\right] =$$

$g(t) = t$ $f' = f = e^t$

$g'(t) = 1$

$$= (t^3-3t^2+6t-7)e^t + c$$

xxiv) $I \subset \mathbb{R}$ $\int \frac{e^x}{e^x+1}\,dx = \int \frac{f'(x)}{f(x)}\,dx$ con $f(x) = 1+e^x$ $= \log|e^x+1| + c =$

$e^x+1 \neq 0 \ \forall x \in \mathbb{R}$

$$= \log(1+e^x) + c \qquad (1+e^x > 0 \ \forall x)$$

(oppure $t = e^x+1$)

xxv) $I \subset]0,+\infty[$ $\int x^2\cdot\log x\,dx \underset{\text{PER PARTI}}{=} \frac{1}{3}x^3\cdot\log x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx =$

$f'(x) = x^2$ $g(x) = \log x$

$f(x) = \frac{x^3}{3}$ $g'(x) = \frac{1}{x}$

$$= \frac{1}{3}x^3\cdot\log x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3\cdot\log x - \frac{x^3}{9} + c =$$

$$= \frac{1}{3}x^3\left(\log x - \frac{1}{3}\right) + c$$

xxvi) svolto a pag. 68 (PER PARTI)

xxvii) $I \subset \mathbb{R}$ $\int x \cdot \operatorname{sen}x\,dx \underset{\text{PER PARTI}}{=} -x\cos x + \int \cos x\,dx =$

$g(x)=x$ $f'(x)=\operatorname{sen}x$
$g'(x)=1$ $f(x)=-\cos x$

$= -x\cos x + \operatorname{sen}x + c$

xxviii) $I \subset \mathbb{R}$ $\int e^{3x}\cdot \operatorname{sen}(2x)\,dx \underset{\text{PER PARTI}}{=} \frac{1}{3}\operatorname{sen}(2x)\,e^{3x} - \int \frac{1}{3}e^{3x}\cdot 2\cos(2x)\,dx =$

$f'(x)=e^{3x}$ $g(x)=\operatorname{sen}(2x)$
$f(x)=\frac{e^{3x}}{3}$ $g'(x)=2\cos(2x)$

$= \frac{1}{3}\operatorname{sen}(2x)e^{3x} - \frac{2}{3}\int e^{3x}\cos(2x)\,dx \underset{\text{PER PARTI}}{=} \frac{1}{3}\operatorname{sen}(2x)e^{3x} -$

$f'(x)=e^{3x}$ $g(x)=\cos(2x)$
$f(x)=\frac{e^{3x}}{3}$ $g'(x)=-2\operatorname{sen}(2x)$

$-\frac{2}{3}\left[\frac{1}{3}e^{3x}\cos(2x) + \int \frac{1}{3}e^{3x}\,2\operatorname{sen}(2x)\,dx\right] = \frac{1}{3}\operatorname{sen}(2x)e^{3x}$

$-\frac{2}{9}\cos(2x)e^{3x} - \frac{4}{9}\int e^{3x}\cdot\operatorname{sen}(2x)\,dx \quad \Rightarrow$

$\left(1+\frac{4}{9}\right)\int e^{3x}\operatorname{sen}(2x)\,dx = \frac{1}{3}e^{3x}\left(\operatorname{sen}(2x) - \frac{2}{3}\cos(2x)\right) + c$

$\frac{13}{9}$ $\int e^{3x}\cdot\operatorname{sen}(2x)\,dx = \frac{3}{13}e^{3x}\left(\operatorname{sen}(2x)-\frac{2}{3}\cos(2x)\right) + c$

xxix) $I \subset \mathbb{R}$ $\int (\operatorname{sen}x)^3dx = \int \operatorname{sen}x\,(\operatorname{sen}x)^2dx = \int \operatorname{sen}x\,(1-\cos^2x)\,dx =$

$= \int \operatorname{sen}x\,dx + \int(-\operatorname{sen}x)\cos^2x\,dx = -\cos x + \frac{\cos^3x}{3} + c$

xxx) $I \subset \mathbb{R}$ $\int e^{-x}\cdot\cos(2x)\,dx \underset{\text{PER PARTI}}{=} -e^{-x}\cos(2x) - \int e^{-x}\,2\operatorname{sen}(2x)\,dx =$

$f'(x)=e^{-x}$ $g(x)=\cos(2x)$
$f(x)=-e^{-x}$ $g'(x)=-2\operatorname{sen}(2x)$

$= -e^{-x}\cos(2x) - 2\int e^{-x}\operatorname{sen}(2x)\,dx \underset{\text{PER PARTI}}{=} -e^{-x}\cos(2x) - 2\left[-\operatorname{sen}(2x)e^{-x} +\right.$

$f'(x)=e^{-x}$ $g(x)=\operatorname{sen}(2x)$
$f(x)=-e^{-x}$ $g'(x)=2\cos(2x)$

$\left. + \int e^{-x}\cdot 2\cdot\cos(2x)\,dx\right] = -e^{-x}\cos(2x) + 2e^{-x}\operatorname{sen}(2x) - 4\int e^{-x}\cos(2x)\,dx$

$\Rightarrow \quad 5\int e^{-x}\cos(2x)\,dx = e^{-x}\left(2\operatorname{sen}(2x) - \cos(2x)\right) + c$

$\int e^{-x}\cos(2x)\,dx = \frac{1}{5}e^{-x}\left(2\operatorname{sen}(2x) - \cos(2x)\right) + c$

XXXi) $I \subset]0,+\infty[$ $\int (2x-1)\cdot \log x = (x^2-x)\cdot \log x - \int \frac{x^2-x}{x} dx =$ PER PARTI

$f'(x)=2x-1$ $g(x)=\log x$

$f(x)=x^2-x$ $g'(x)=\frac{1}{x}$

$= (x^2-x)\log x - \int (x-1)\, dx = (x^2-x)\log x - \frac{x^2}{2} + x + c$

XXXii) $I \subset \mathbb{R}$ $\int x\cdot e^{-x} dx = -xe^{-x} + \int e^{-x} dx = -(x+1)e^{-x} + c$ PER PARTI

$g(x)=x$ $f'(x)=e^{-x}$

$g'(x)=1$ $f(x)=-e^{-x}$

XXXiii) $I \subset]0,+\infty[$ $\int \sqrt{x}\cdot \log x \, dx = \frac{2}{3}x^{3/2}\cdot \log x - \int \frac{2}{3}\sqrt{x}\, dx =$ PER PARTI

$f'(x)=\sqrt{x}$ $g(x)=\log x$

$f(x)=\frac{2}{3}x^{3/2}$ $g'(x)=\frac{1}{x}$

$= \frac{2}{3}x^{3/2}\cdot \log x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + c = \frac{2}{3}x^{3/2}\left(\log x - \frac{2}{3}\right) + c$

XXXiv) $\int \cos^2 x\, dx = \frac{1}{2}(x + \operatorname{sen} x \cos x) + c$ (PER PARTI)

XXXv) $\int \operatorname{sen} x \cos x \, dx = \frac{1}{2}\operatorname{sen}^2 x + c$ $\left(\int f(x)\cdot f'(x)\, dx = \frac{(f(x))^2}{2} + c \right)$ oppure $t = \operatorname{sen} x$

XXXvi) $\int (1-x)^3 dx = -\frac{(1-x)^4}{4} + c$ $\left(\int (f(x))^3\cdot f'(x)\, dx = \frac{(f(x))^4}{4} + c \right)$ oppure $t=1-x$

$f(x)=1-x$ $f'(x)=-1$

XXXvii) $\int (2-x)^{5/2} dx = -\frac{(2-x)^{7/2}}{7/2} + c$ $\left(\int (f(x))^{5/2}\cdot f'(x)\, dx = \frac{(f(x))^{7/2}}{7/2} + c \right)$ oppure $t=2-x$

XXXviii) $\int \operatorname{sen} x\cdot (\cos x)^2 dx = -\frac{(\cos x)^3}{3} + c$ $\left(\int f'(x)\cdot (f(x))^2 dx = \frac{(f(x))^3}{3} + c \right)$ oppure $t=\cos x$

XXXix) $\int (3-2x)^4 dx = -\frac{1}{2}\,\frac{(3-2x)^5}{5} + c$ $\left(\int f'(x)\cdot (f(x))^4 dx = \frac{(f(x))^5}{5} + c \right)$ oppure $t=3-2x$

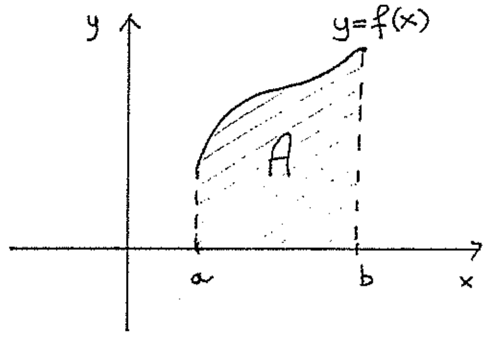
XL) $\int \operatorname{sen}^2 x\cdot \cos x\, dx = \frac{(\operatorname{sen} x)^3}{3} + c$ $\left(\int (f(x))^2\cdot f'(x)\, dx = \frac{(f(x))^3}{3} + c \right)$ oppure $\operatorname{sen} x = t$

21) Riprendiamo il concetto di integrale definito,
il suo significato geometrico e il Teorema Fondamentale del calcolo integrale che consente di calcolarlo.
Applichiamo i risultati al calcolo dell'area.

L'integrale definito di una funzione $f(x)$ su un intervallo $[a,b]$ è un numero reale positivo, nullo o negativo che si indica con
$$\int_a^b f(x)\,dx.$$

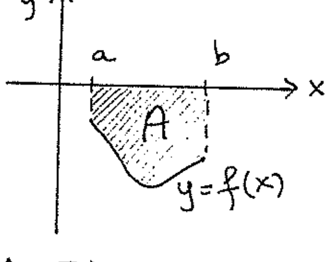
Tale valore viene matematicamente costruito, utilizzando l'area di rettangolini che stanno al di sotto e al di sopra del grafico della funzione, in modo tale che

$$\text{se } f \geq 0 \text{ su } [a,b] \Rightarrow \int_a^b f(x)\,dx = \text{area } A$$ dove

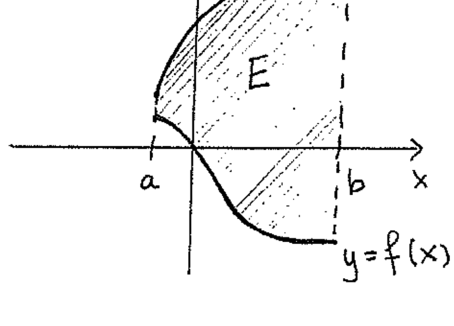


A è la regione di piano compresa tra il grafico di f e l'asse x.

Se invece $f \leq 0$ su $[a,b]$, allora
$$\int_a^b f(x)\,dx = -\text{area } A$$

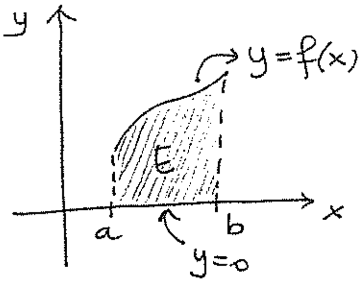


Si dimostra la formula dell'AREA
$$\text{area } E = \int_a^b \left(g(x) - f(x)\right) dx$$



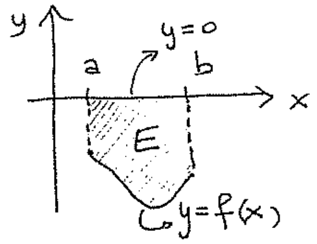
integrale della differenza tra la funzione che delimita sopra l'insieme e quella che lo delimita da sotto.

Nel caso



$$\text{area } E = \int_a^b (f(x)-0))dx = \int_a^b f(x)\,dx$$

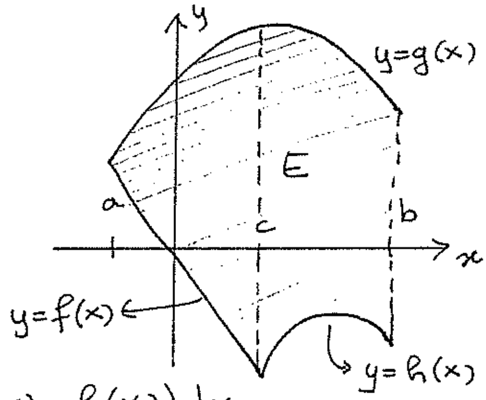
mentre nel caso



$$\text{area } E = \int_a^b (0-f(x))dx = -\int_a^b f(x)dx$$

come già detto prima.

Infine se fosse



il calcolo va spezzato

$$\text{area } E = \int_a^c (g(x)-f(x))dx + \int_c^b (g(x)-h(x))\,dx$$

Per calcolare un integrale definito $\int_a^b f(x)\,dx$ si usa il Teorema Fondamentale del Calcolo Integrale che afferma che

$$\int_a^b f(x)dx = F(b)-F(a)$$

dove $F(x)$ è una qualunque primitiva di $f(x)$.

Trovata dunque una qualunque primitiva di $f(x)$ basta calcolare la differenza tra il valore nell'estremo superiore dell'integrale e quello nell'estremo inferiore.

ES. $$\int_1^4 3x^2\,dx = \left[x^3\right]_{x=1}^{x=4} = 4^3-1^3 = 63 \qquad \left(\begin{array}{l} f(x)=3x^2 \\ F(x)=x^3 \end{array}\right)$$

22) Calcoliamo i seguenti integrali definiti

a) $\int_0^3 (x-1)\,dx$ b) $\int_0^1 (4x^5+3x^2+1)\,dx$ c) $\int_1^9 \left(\frac{2t^2+t^2\sqrt{t}-1}{t^2}\right) dt$

d) $\int_0^4 \sqrt{x}\,dx$ e) $\int_1^2 \frac{3}{t^4}\,dt$ f) $\int_{-1}^0 (2x-e^x)\,dx$ g_1) $\int_1^9 \frac{1}{\sqrt{x}}\,dx$

g_2) $\int_1^4 \frac{1}{x^{3/2}}\,dx$ h) $\int_{\pi/4}^{\pi/3} 2\,\text{sen}\,t\,dt$ i) $\int_{-1}^3 (2x^3-\frac{1}{9}x+\frac{1}{6})\,dx$

j) $\int_{\log 3}^{\log 6} 8e^x\,dx$ k) $\int_{2\pi}^{8\pi} \frac{3}{2}\cos\left(\frac{x}{6}\right)\,dx$ l) $\int_{-2}^0 (x-x^2)\,dx$

m) $\int_{1/2}^2 \frac{1}{(1+x)^2}\,dx$ n) $\int_1^2 \frac{1}{(2-3x)^2}\,dx$ o) $\int_0^4 5x^{3/2}\,dx$

p) $\int_0^2 (x-1)^{25}\,dx$ q) $\int_0^7 \sqrt{4+3x}\,dx$ r) $\int_0^1 x^2(1+2x^3)^5\,dx$

s) $\int_0^{\pi/2} 9\,\text{sen}\left(\frac{x}{3}\right)\,dx$ t) $\int_0^1 \cos(\pi t)\,dt$

u) $\int_0^1 \frac{1}{12}e^{6x}\,dx$ v) $\int_1^2 \frac{2}{x^6}\,dx$ w) $\int_0^2 2x\sqrt{1+x^2}\,dx$

x) $\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}}\,dx$ y) $\int_0^{\pi/4} (\text{sen}(2x))\,dx$

22 bis) a) $\int_1^3 6x\sqrt{3x^2-2}\,dx$ b) $\int_2^6 \frac{1}{\sqrt{3x-2}}\,dx$ c) $\int_0^1 \frac{4}{(1+4x)^2}\,dx$

d) $\int_{\pi/6}^{\frac{7}{24}\pi} \cos(4x)\,dx$ e) $\int_{-1}^3 (2x-x^3+\frac{1}{3})\,dx$ f) $\int_{-2}^1 (2x-x^3+\frac{4}{3}x^2)\,dx$

g) $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2 x}{\cos^2 x}\,dx$ h) $\displaystyle\int_{-1}^{2} |x-x^2|\,dx$ i) $\displaystyle\int_1^{e} \frac{\log x}{x}\,dx$

j) $\displaystyle\int_0^{\pi/2} e^{\cos x}\cdot \operatorname{sen} x\,dx$ k) $\displaystyle\int_1^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ l) $\displaystyle\int_1^{2} \frac{1}{3x+1}\,dx$

m) $\displaystyle\int_{-2}^{-1} \frac{1}{3x+1}\,dx$ n) $\displaystyle\int_0^{\pi/2} x\cdot\cos(2x)\,dx$ o) $\displaystyle\int_0^{1} (x^2+1)e^{-x}\,dx$

p) $\displaystyle\int_1^{4} \log\sqrt{x}\,dx$ q) $\displaystyle\int_0^{\frac{\pi^2}{4}} \operatorname{sen}\sqrt{x}\,dx$ r) $\displaystyle\int_0^{1} x^3 e^{x^2}\,dx$

s) $\displaystyle\int_0^{\pi} (\cos x)^2\,dx$ t) $\displaystyle\int_0^{\pi/2} (\cos x)^3\,dx$

SOL.^{ne}

22) a) $\left[\frac{x^2}{2}-x\right]_{x=0}^{x=3}=\frac{3}{2}$ b) $\left[\frac{2}{3}x^6+x^3+x\right]_{x=0}^{x=1}=\frac{8}{3}$

c) $\left[2t+\frac{2}{3}t^{3/2}+\frac{1}{t}\right]_{t=1}^{t=9}=33-\frac{5}{9}=\frac{292}{9}$

d) $\left[\frac{2}{3}x^{3/2}\right]_{x=0}^{x=4}=\frac{16}{3}$ e) $\left[-\frac{1}{t^3}\right]_{t=1}^{t=2}=\frac{7}{8}$ f) $\left[x^2-e^x\right]_{x=-1}^{x=0} = -2+\frac{1}{e}$

g_1) $\left[2\sqrt{x}\right]_{x=1}^{x=9}=6-2=4$

g_2) $\left[-\frac{2}{\sqrt{x}}\right]_1^4=1$ h) $\left[-2\cos t\right]_{t=\pi/4}^{t=\pi/3}=\sqrt{2}-1$

i) $\left[\frac{1}{2}x^4-\frac{1}{18}x^2+\frac{1}{6}x\right]_{-1}^{3}=\frac{81}{2}-\frac{1}{2}+\frac{1}{2}-\left(\frac{1}{2}-\frac{1}{18}-\frac{1}{6}\right)=\frac{362}{9}$

j) $\left[8e^x\right]_{x=\log 3}^{x=\log 6}=8(6-3)=24$ K) $\left[9\,\mathrm{sen}\frac{x}{6}\right]_{x=2\pi}^{x=8\pi}=9\left(\mathrm{sen}\frac{4}{3}\pi-\mathrm{sen}\frac{\pi}{3}\right) = 9(-\sqrt{3})=-9\sqrt{3}$

l) $=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{0}=0-\left(2+\frac{8}{3}\right)=-\frac{14}{3}$ m) $\left[-\frac{1}{(1+x)}\right]_{\frac{1}{2}}^{2}=\frac{1}{3}$

n) $\left[\frac{1}{3(2-3x)}\right]_{x=1}^{x=2}=\frac{1}{4}$ o) $\left[2x^{5/2}\right]_{x=0}^{x=4}=2\cdot 4^{5/2}-0=64$ $\quad 4^{5/2}=(\sqrt{4})^5=2^5=32$

p) $\left[\frac{(x-1)^{26}}{26}\right]_{x=0}^{x=2}=0$ q) $\left[\frac{2}{9}(4+3x)^{3/2}\right]_{x=0}^{x=7}=\frac{234}{9}$

r) $\left[\frac{(1+2x^3)^6}{36}\right]_{x=0}^{x=1}=\frac{728}{36}=\frac{182}{9}$

s) $\left[-27\cos\left(\frac{x}{3}\right)\right]_{x=0}^{x=\pi/2}=27\left(1-\frac{\sqrt{3}}{2}\right)$ t) $\left[\frac{1}{\pi}\mathrm{sen}(\pi t)\right]_{t=0}^{t=1}=0$

u) $\left[\frac{1}{72}e^{6x}\right]_{x=0}^{x=1}=\frac{1}{72}(e^6-1)$ v) $\left[-\frac{2}{5x^5}\right]_{x=1}^{x=2}=-\frac{1}{80}+\frac{2}{5}=\frac{31}{80}$

w) $\left[\frac{2}{3}(1+x^2)^{3/2}\right]_{x=0}^{x=2}=\frac{2}{3}\left[5^{3/2}-1^{3/2}\right]=\frac{2}{3}(5\sqrt{5}-1)$

x) $\left[\frac{1}{2}\,\frac{(1+2x)^{1/3}}{1/3}\right]_{x=0}^{x=13} = \frac{3}{2}\left[(1+2x)^{1/3}\right]_{x=0}^{x=13} = \frac{3}{2}\left(\sqrt[3]{27}-\sqrt[3]{1}\right) = 3$

y) $\left[-\frac{\cos(2x)}{2}\right]_{x=0}^{x=\pi/4} = \frac{1}{2}$

22bis) a) $\left[\frac{2}{3}(3x^2-2)^{3/2}\right]_{x=1}^{x=3} = \frac{2}{3}\left(25^{3/2}-1^{3/2}\right) = \frac{2}{3}(5^3-1) = \frac{248}{3}$

b) $\left[\frac{2}{3}\sqrt{3x-2}\right]_{x=2}^{x=6} = \frac{4}{3}$

c) $\left[-\frac{1}{(1+4x)}\right]_{x=0}^{x=1} = -\frac{1}{5}+1 = \frac{4}{5}$

d) $\left[\frac{1}{4}\,\mathrm{sen}(4x)\right]_{x=\pi/6}^{x=\frac{7}{24}\pi} = \frac{1}{4}\left(\mathrm{sen}\left(\frac{7}{6}\pi\right)-\mathrm{sen}\left(\frac{2}{3}\pi\right)\right) = \frac{1}{4}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right) = -\frac{1+\sqrt{3}}{8}$

e) $\left[x^2-\frac{x^4}{4}+\frac{1}{3}x\right]_{-1}^{3} = 9-\frac{81}{4}+1-\left(1-\frac{1}{4}-\frac{1}{3}\right) = -\frac{32}{3}$

f) $\left[x^2-\frac{x^4}{4}+\frac{4}{9}x^3\right]_{-2}^{1} = 1-\frac{1}{4}+\frac{4}{9}-\left(4-4-\frac{32}{9}\right) = \frac{19}{4}$

g) $\left[\tan x + x\right]_0^{\pi/4} = \tan\frac{\pi}{4}+\frac{\pi}{4}-\underbrace{\tan 0}_{=0}-0 = 1+\frac{\pi}{4}$

h) $\int_{-1}^{2}|x-x^2|\,dx = \int_{-1}^{0}(x^2-x)\,dx+\int_{0}^{1}(x-x^2)\,dx+\int_{1}^{2}(x^2-x)\,dx =$

$x-x^2\geq 0 \iff 0\leq x\leq 1 \rightarrow |x-x^2| = \begin{cases} x-x^2 & 0\leq x\leq 1 \\ x^2-x & x<0 \cup x>1 \end{cases}$

$x(1-x)\geq 0$

$= \left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{0}^{1}+\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{1}^{2} = -\left(-\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left[\frac{8}{3}-2-\left(\frac{1}{3}-\frac{1}{2}\right)\right]$

$= \cancel{\frac{1}{3}}+\frac{1}{2}+\frac{1}{2}-\cancel{\frac{1}{3}}+\frac{7}{3}-2+\frac{1}{2} = \frac{7}{3}-\frac{1}{2} = \frac{11}{6}$

i) $\left[\frac{1}{2}(\log x)^2\right]_1^e = \frac{1}{2}$ $\quad(\log e = 1, \log 1 = 0)$

j) $\left[-e^{\cos x}\right]_0^{\pi/2} = -e^{\cos\frac{\pi}{2}} + e^{\cos 0} = -e^0 + e^1 = e - 1$

k) $\left[2e^{\sqrt{x}}\right]_1^4 = 2e^{\sqrt{4}} - 2e = 2e^2 - 2e$

$\downarrow \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int \underbrace{\frac{1}{2\sqrt{x}}e^{\sqrt{x}}}_{f'(x)e^{f(x)}}dx = 2e^{\sqrt{x}} + c$ $\quad$ oppure $t = \sqrt{x}$

l) $\left[\frac{1}{3}\log(3x+1)\right]_1^2 = \frac{1}{3}(\log 7 - \log 4) = \frac{1}{3}\log\frac{7}{4}$ $\quad$ su $[1,2]$ $\log|3x+1| = \log(3x+1)$

m) $\left[\frac{1}{3}\log|3x+1|\right]_{-2}^{-1} = \left[\frac{1}{3}\log(-3x-1)\right]_{-2}^{-1} = \frac{1}{3}(\log 2 - \log 5) = \frac{1}{3}\log\frac{2}{5}$
(che è <0)

n) $\int x\cos(2x)\,dx \underset{\text{PER PARTI}}{=} \frac{1}{2}x\,\mathrm{sen}(2x) - \int\frac{1}{2}\mathrm{sen}(2x)\,dx =$

$f(x) = x \to f' = 1$ $\quad = \frac{1}{2}x\,\mathrm{sen}(2x) + \frac{1}{4}\cos(2x) + c$

$g'(x) = \cos(2x)$

$g(x) = \frac{1}{2}\mathrm{sen}(2x)$ $\quad \left[\frac{1}{2}x\,\mathrm{sen}(2x) + \frac{1}{4}\cos(2x)\right]_0^{\pi/2} = \frac{1}{2}\frac{\pi}{2}\overset{0}{\mathrm{sen}\,\pi} + \frac{1}{4}\cos\pi$

$-0 - \frac{1}{4}\cos 0 = -\frac{1}{2}$

o) $\int(x^2+1)\cdot e^{-x}dx = -(x^2+1)e^{-x} + \int 2xe^{-x}dx \underset{\text{PER PARTI}}{=} -(x^2+1)e^{-x} - 2xe^{-x} +$

$f(x) = x^2+1$ $f' = 2x$ PER PARTI $\quad f(x) = 2x$ $f' = 2$

$g'(x) = e^{-x}$ $g(x) = -e^{-x}$ $\quad g'(x) = e^{-x}$ $g(x) = -e^{-x}$

$+\int 2e^{-x}dx = -(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x} + c = -(x^2+2x+3)e^{-x} + c$

$\left[-(x^2+2x+3)e^{-x}\right]_0^1 = -6e^{-1} - (-3e^0) = 3 - \frac{6}{e}$

p) $\int\log\sqrt{x}\,dx \underset{\substack{t=\sqrt{x}\\ dt=\frac{1}{2\sqrt{x}}dx}}{=} \int 2t\log t\,dt \underset{\text{PER PARTI}}{=} t^2\cdot\log t - \int t^2\cdot\frac{1}{t}dt = t^2\cdot\log t - \int t\,dt$

$f'(t) = 2t$ $\quad f(t) = t^2$ $\quad g(t) = \log t$ $\quad g'(t) = \frac{1}{t}$ $\quad = t^2\log t - \frac{t^2}{2} + c$

$= \left[x\log\sqrt{x} - \frac{1}{2}x\right]_1^4 = 4\log 2 - 2 - 1\cdot\overset{0}{\log 1} + \frac{1}{2} = 4\log 2 - \frac{3}{2}$

q) $\int \operatorname{sen}\sqrt{x}\,dx = \int 2t \operatorname{sen} t\,dt \underset{\text{PER PARTI}}{=} -2t\cos t + \int 2\cos t =$

$t=\sqrt{x}$ $f(t)=2t$

$f'=2$

$g'(t)=\operatorname{sen} t$ $\qquad = -2t\cos t + 2\operatorname{sen} t + c$

$g(t)=-\cos t$

$$\left[-2\sqrt{x}\cos\sqrt{x} + 2\operatorname{sen}\sqrt{x}\right]_0^{\pi^2/4} = -2\frac{\pi}{2}\underbrace{\cos\frac{\pi}{2}}_{0} + 2\underbrace{\operatorname{sen}\frac{\pi}{2}}_{1} - 0 - 0 = 2$$

r) $\int x^3 e^{x^2}dx = \frac{1}{2}\int x^2\cdot(2xe^{x^2})\,dx \underset{\text{PER PARTI}}{=} \frac{1}{2}\left(x^2e^{x^2} - \int 2xe^{x^2}dx\right) =$

$f(x)=x^2$ $f'=2x$

$g'(x)=2x\,e^{x^2}$ $\qquad = \frac{1}{2}(x^2-1)e^{x^2}+c$

$g(x)=e^{x^2}$

oppure $t=x^2$ e poi PER PARTI

$$\left[\frac{1}{2}(x^2-1)e^{x^2}\right]_0^1 = 0-\left(-\frac{1}{2}\right)e^0 = 1/2$$

s) $\int(\cos x)^2dx = \int \cos x\cdot\cos x\,dx \underset{\text{PER PARTI}}{=} \operatorname{sen}x\cos x + \int \operatorname{sen}^2 x\,dx =$

$f(x)=\cos x$ $\qquad = \operatorname{sen}x\cos x + \int(1-\cos^2 x)\,dx =$

$f'=-\operatorname{sen}x$

$g'=\cos x$ $\qquad = \operatorname{sen}x\cos x + x - \int\cos^2 x\,dx$

$g=\operatorname{sen}x$

$\to 2\int(\cos x)^2dx = x+\operatorname{sen}x\cos x \to \int(\cos x)^2dx = \frac{1}{2}(x+\operatorname{sen}x\cos x)+c$

$$\left[\frac{1}{2}(x+\operatorname{sen}x\cos x)\right]_0^{\pi} = \frac{1}{2}(\pi+0-0-0) = \frac{\pi}{2}$$

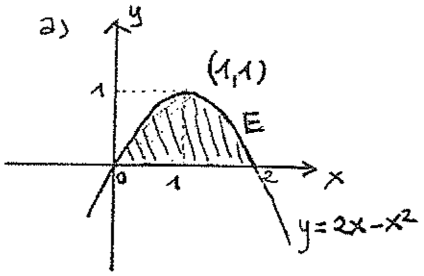
t) $\int(\cos x)^3dx = \int\cos x\cdot\cos^2 x\,dx = \int\cos x(1-\operatorname{sen}^2 x)\,dx = \int(\cos x - \cos x$

$= \int(\cos x - \operatorname{sen}^2 x\cos x)\,dx = \operatorname{sen}x - \frac{(\operatorname{sen}x)^3}{3}+c$

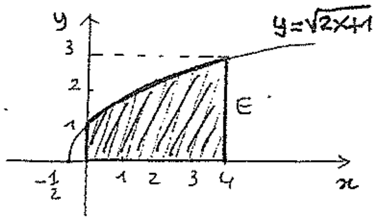
$$\left[\operatorname{sen}x - \frac{(\operatorname{sen}x)^3}{3}\right]_0^{\pi/2} = 1-\frac{1}{3}-0+0 = \frac{2}{3}$$

23) Calcoliamo l'area degli insiemi E disegnati in figura

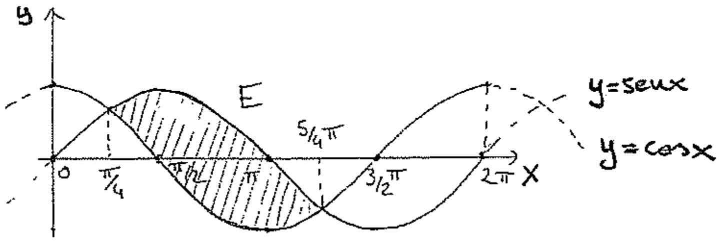
a)



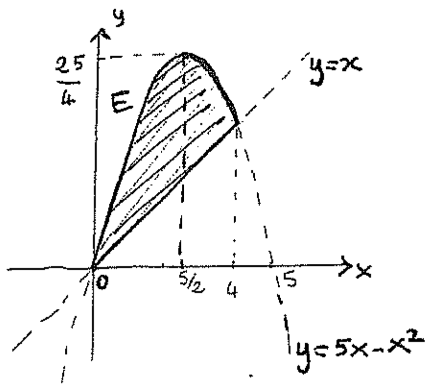
b)



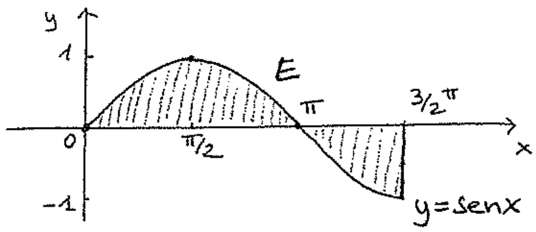
c)



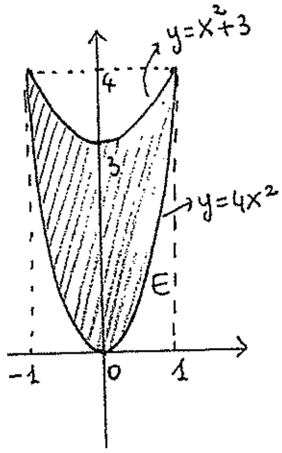
d)



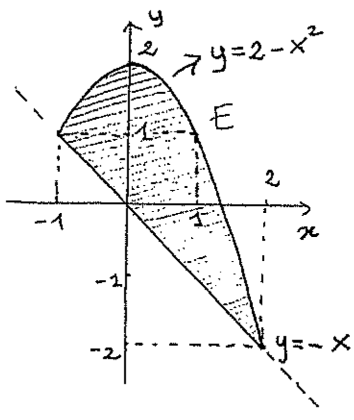
e)



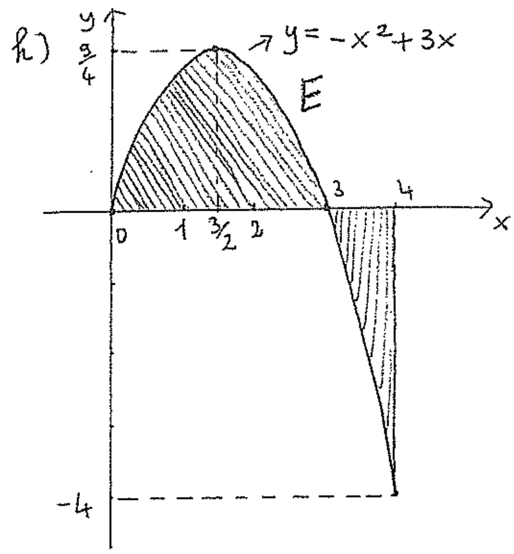
f)



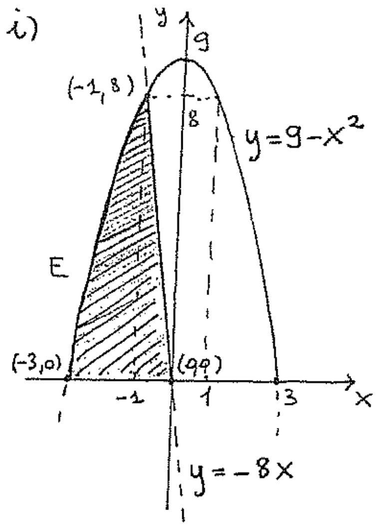
g)



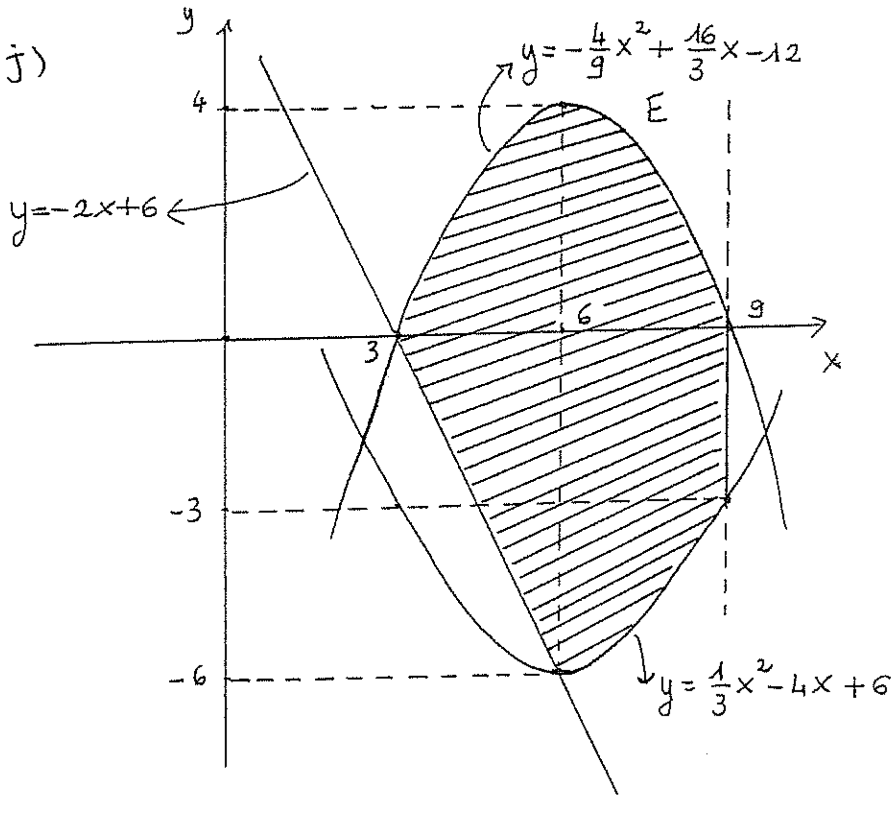
h)


y
$\frac{9}{4}$
$y=-x^2+3x$
E
0
1
$3/2$
2
3
4
x
-4

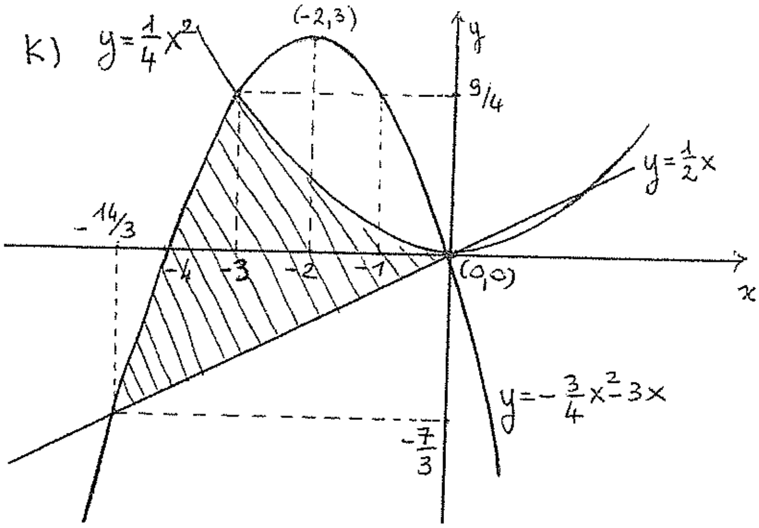
i)


y
9
(-1,8)
8
$y=9-x^2$
E
(-3,0)
(0,0)
-1
1
3
x
$y=-8x$

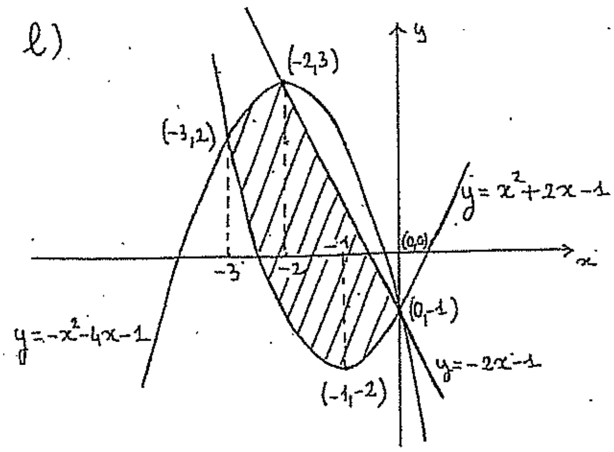
j)


y
$y=-\frac{4}{9}x^2+\frac{16}{3}x-12$
4
E
$y=-2x+6$
6
9
3
x
-3
-6
$y=\frac{1}{3}x^2-4x+6$

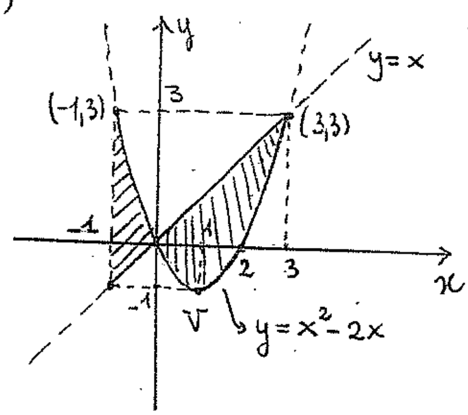
k)



l)



m)



SOL.^{ue} 23) a) $\text{area } E=\int_0^2 (2x-x^2)\,dx=\left[x^2-\frac{x^3}{3}\right]_0^2=4-\frac{8}{3}=\boxed{\frac{4}{3}}\,m^2$

b) $\text{area } E=\int_0^4 \sqrt{2x+1}\,dx=\left[\frac{1}{2}\,\frac{(2x+1)^{3/2}}{3/2}\right]_0^4=\left[\frac{1}{3}(2x+1)^{3/2}\right]_0^4=\frac{1}{3}\,3^3-\frac{1}{3}=$

$=3^2-\frac{1}{3}=9-\frac{1}{3}=\boxed{\frac{26}{3}}\,m^2\simeq 8{,}66\,m^2$

c) $\int_{\frac{\pi}{4}}^{5/4\pi}(\operatorname{sen}x-\cos x)\,dx=\left[-\cos x-\operatorname{sen}x\right]_{\pi/4}^{5/4\pi}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)=2\sqrt{2}$

$\simeq 2{,}83\ m^2$

d) $\text{area } E=\int_0^4 (5x-x^2-x)\,dx=\int_0^4(-x^2+4x)\,dx=\left[-\frac{x^3}{3}+2x^2\right]_0^4=-\frac{64}{3}+32$

$=\boxed{\frac{32}{3}}\simeq 10.66\,m^2$

e) $\text{area } E = \int_0^{\pi} (\operatorname{sen} x - 0)\, dx + \int_{\pi}^{\frac{3}{2}\pi} (0 - \operatorname{sen} x)\, dx =$

$= \int_0^{\pi} \operatorname{sen} x\, dx - \int_{\pi}^{\frac{3}{2}\pi} \operatorname{sen} x\, dx = \left[-\cos x\right]_0^{\pi} - \left[-\cos x\right]_{\pi}^{3/2\pi} = 1 + 1 - (0 - 1) = 3\, m^2$

f) $\text{area } E = \int_{-1}^{1} (x^2 + 3 - 4x^2)\, dx = \int_{-1}^{1} (-3x^2 + 3)\, dx = \left[-x^3 + 3x\right]_{-1}^{1} = 4\, m^2$

g) $\text{area } E = \int_{-1}^{2} \left[(2 - x^2) - (-x)\right] dx = \int_{-1}^{2} (-x^2 + x + 2)\, dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2} =$

$= \frac{9}{2} = 4{,}5\ m^2$

h) $\text{area } E = \int_0^3 (-x^2 + 3x)\, dx + \int_3^4 (0 - (-x^2 + 3x))\, dx =$

$= \int_0^3 (-x^2 + 3x)\, dx - \int_3^4 (-x^2 + 3x)\, dx =$

$= \left[-\frac{x^3}{3} + 3\frac{x^2}{2}\right]_0^3 - \left[-\frac{x^3}{3} + 3\frac{x^2}{2}\right]_3^4 = \frac{19}{3}\ m^2 \simeq 6{,}3\ m^2$

i) $\text{area } E = \int_{-3}^{-1} (9 - x^2)\, dx + \int_{-1}^{0} -8x\, dx = \left[9x - \frac{x^3}{3}\right]_{-3}^{-1} + \left[-4x^2\right]_{-1}^{0} =$

$= \left(-9 + \frac{1}{3} - (-27 + 9)\right) + (0 - (-4)) = \frac{40}{3}\ m^2 \simeq 13{,}3\ m^2$

j) $\text{area } E = \int_3^6 \left(-\frac{4}{9}x^2 + \frac{16}{3}x - 12\right) - (-2x + 6)\, dx + \int_6^9 \left(-\frac{4}{9}x^2 + \frac{16}{3}x - 12\right) -$

$- \left(\frac{1}{3}x^2 - 4x + 6\right) dx = \int_3^6 \left(-\frac{4}{9}x^2 + \frac{22}{3}x - 18\right) dx + \int_6^9 \left(-\frac{7}{9}x^2 + \frac{28}{3}x - 18\right) dx =$

$= \left[-\frac{4}{27}x^3 + \frac{11}{3}x^2 - 18x\right]_3^6 + \left[-\frac{7}{27}x^3 + \frac{14}{3}x^2 - 18x\right]_6^9 = 17 + 23 = 40$

K) $\text{area } E = \int_{-14/3}^{-3} \left(-\frac{3}{4}x^2 - 3x\right) - \left(\frac{1}{2}x\right) dx + \int_{-3}^{0} \left(\frac{1}{4}x^2 - \frac{1}{2}x\right) dx =$

$= \left[-\frac{1}{4}x^3 - \frac{7}{4}x^2\right]_{-\frac{14}{3}}^{-3} + \left[\frac{1}{12}x^3 - \frac{1}{4}x^2\right]_{-3}^{0} = \frac{100}{27} + \frac{9}{2} = \frac{443}{54}$

l) $\text{area } E = \displaystyle\int_{-3}^{-2} (-x^2-4x-1)-(x^2+2x-1)\,dx$

$$+\int_{-2}^{0}(-2x-1)-(x^2+2x-1)\,dx = \int_{-3}^{-2}(-2x^2-6x)\,dx+\int_{-2}^{0}(-x^2-4x)\,dx =$$

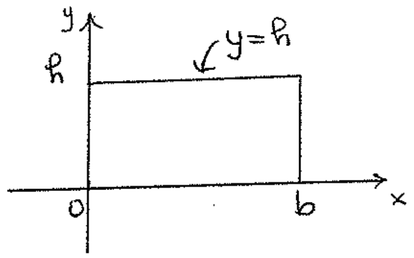
$$=\left[-\frac{2}{3}x^3-3x^2\right]_{-3}^{-2}+\left[-\frac{x^3}{3}-2x^2\right]_{-2}^{0} = \frac{7}{3}+\frac{16}{3}=\boxed{\frac{23}{3}}$$

m) $\text{Area} = \displaystyle\int_{-1}^{0}\underbrace{(x^2-2x)-x}_{x^2-3x}\,dx+\int_{0}^{3}x-(x^2-2x)\,dx = \left[\frac{x^3}{3}-3\frac{x^2}{2}\right]_{-1}^{0}+\left[3\frac{x^2}{2}-\frac{x^3}{3}\right]_{0}^{3}=$

$$=\left[0-\left(-\frac{1}{3}-\frac{3}{2}\right)\right]+\left[\frac{27}{2}-9\right]=\frac{11}{6}+\frac{9}{2}=\boxed{\frac{19}{3}}$$

OSSERVAZIONE (IMPORTANTE). Tutte le formule per l'area di una figura elementare (es. rettangolo, triangolo, trapezio ecc.) si possono ridimostrare mediante gli integrali e la formula dell'area. Integrali e aree sono strettamente connessi tra loro.

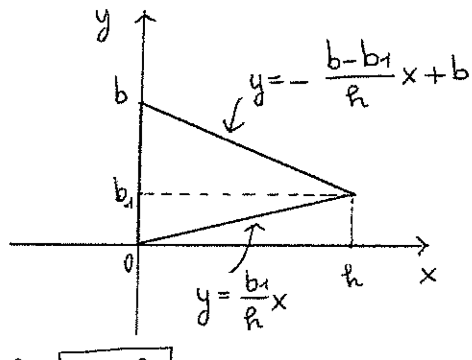
ES. Area di un rettangolo $\boxed{A=b\cdot h}$
b = base
h = altezza



Con gli integrali:

$$A=\int_0^b h\,dx = h\int_0^b dx = h\cdot[x]_{x=0}^{x=b} =$$
$$= h\,[b-0]=\boxed{b\cdot h}$$

ES. Area di un triangolo $A=\frac{1}{2}b\cdot h$
b = base
h = altezza



Con gli integrali

$$A=\int_0^h\left[-\frac{b-b_1}{h}x+b-\frac{b_1}{h}x\right]dx =$$

$$=\int_0^h\left[-\frac{b}{h}x+b\right]dx=\left[-\frac{b}{h}\cdot\frac{x^2}{2}+bx\right]_0^h=-\frac{1}{2}b\cdot h+b\cdot h=\boxed{\frac{1}{2}b\cdot h}$$

24) BARICENTRO DI UNA FIGURA PIANA

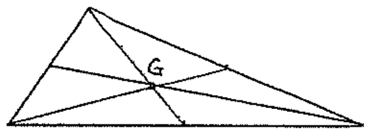
Si consideri una lamina di densità uniforme che occupi una regione del piano. Il BARICENTRO della lamina (o CENTRO DI MASSA) è il punto P su cui la lamina resta in equilibrio orizzontale.

Se la figura è simmetrica rispetto alla linea I, allora il baricentro si trova su I.

Ad esempio è chiaro che il baricentro di un rettangolo è il suo centro perchè si trova sulle due mediane (cioè sulle due rette congiungenti il punto centrale di due lati opposti 

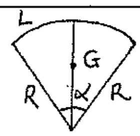
).

In un triangolo si dimostra che il baricentro è il punto di incontro delle 3 mediane (una mediana è il segmento congiungente un vertice con il punto medio del lato opposto).

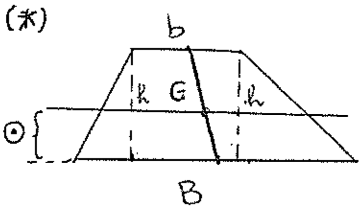


Il baricentro di una figura piana dipende dai MOMENTI e dall'AREA della figura. Entrambi si calcolano utilizzando gli integrali, quindi non stupitevi se nella trattazione teorica degli argomenti continuerete ad incontrare INTEGRALI.

$\boxed{\text{TABELLA}}$ con PERIMETRO, AREA e BARICENTRO DELLE FIGURE PIANE ELEMENTARI

	PERIMETRO	AREA	BARICENTRO G
RETTANGOLO	$2b+2h$	$A=b\cdot h$	CENTRO
QUADRATO	4ℓ	ℓ^2	CENTRO
PARALLELOGRAMMA	$2b+2\ell_{obliquo}$	$b\cdot h$	CENTRO
TRIANGOLO	$\ell_1+\ell_2+b$	$A=\frac{1}{2}b\cdot h$	VERTICI $A(x_A,y_A)$ $B(x_B,y_B)$ $C(x_C,y_C)$ $G\left(\frac{1}{3}(x_A+x_B+x_C), \frac{1}{3}(y_A+y_B+y_C)\right)$
ROMBO	4ℓ	$A=\frac{D\cdot d}{2}$ o $A=b\cdot h$ D = diagonale maggiore d = diagonale minore	CENTRO
TRAPEZIO	$B+b+\ell_{obliquo1}+\ell_{obliquo2}$ B = base maggiore b = base minore	$A=\frac{(B+b)\cdot h}{2}$	G si trova sulla MEDIANA delle due BASI e a $\frac{h}{3}\frac{(2b+B)}{(B+b)}$ dalla base MAGGIORE (*)
CERCHIO	$2\pi R$	$A=\pi R^2$	CENTRO
SEMICERCHIO	$\pi R+2R$	$A=\frac{\pi R^2}{2}$	 CENTRO $C(x_c,y_c)$ $G\left(x_c, y_c+\frac{4}{3}\frac{R}{\pi}\right)$ (**)
CORONA CIRCOLARE	$2\pi R+2\pi r$ $r<R$	$A=\pi R^2-\pi r^2$	CENTRO
SETTORE CIRCOLARE	$2R+\alpha_{rad}R$ α_{rad} = misura dell'angolo in radianti	$A=\frac{1}{2}\alpha_{rad}\cdot R^2$ o $A=\frac{1}{2}L\cdot R$	 CENTRO (x_c,y_c) $G\left(x_c,y_c+\frac{2}{3}\frac{R\cdot sen(\alpha/2)}{\left(\frac{\alpha_{rad}}{2}\right)}\right)$ (**)

(*)



La mediana delle due basi è il segmento congiungente i punti medi delle due basi. Poi G si trova $\odot\ \frac{h}{3}\cdot\frac{(2b+B)}{(B+b)}$ al di sopra della base maggiore

(**) E' chiaro che se il semicerchio o il settore circolare sono posizionati in modo diverso, allora il baricentro G rimane nello stesso punto rispetto all'insieme, ma le sue coordinate cartesiane saranno diverse. Ad es. se si considera il semicerchio  le coordinate di G saranno $G\left(x_c+\frac{4}{3}\frac{R}{\pi}, y_c\right)$

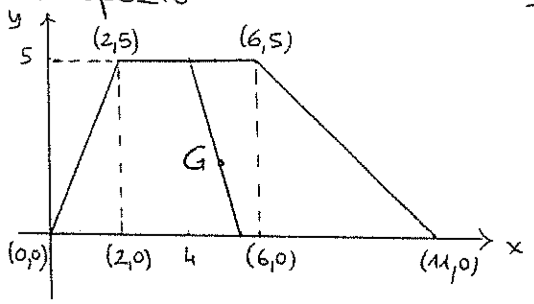
OSSERVAZIONI IMPORTANTI

- La collocazione del baricentro rispetto all'insieme NON DIPENDE dal SISTEMA DI RIFERIMENTO, ma le COORDINATE del baricentro sì, perchè in generale se collochiamo una figura geometrica nel piano in un'altra posizione le coordinate dei suoi punti cambiano.
  La formula assegnata per il baricentro del triangolo funziona con qualunque sistema di riferimento perchè dipende dalle coordinate. Non c'è nessun problema anche quando il baricentro coincide con il centro della figura. Invece con trapezi, semicerchi e settori circolari le coordinate del baricentro dipendono da come è posizionata la figura.
- Per calcolare il baricentro di una figura COMPOSTA di forme geometriche semplici è possibile calcolare il baricentro di ogni figura e poi sommare i baricentri pesandoli con le aree: 2 figure E_1, E_2 baricentri G_1, G_2
  $E_1 \cup E_2 = E$ E_1 e E_2 non sovrapposte

$\Rightarrow$ Baricentro di E:

$$G = \left(\frac{\text{area } E_1 \cdot x_{G_1} + \text{area } E_2 \cdot x_{G_2}}{\text{area } E}, \frac{\text{area } E_1 \cdot y_{G_1} + \text{area } E_2 \cdot y_{G_2}}{\text{area } E} \right)$$

ESEMPIO - Calcoliamo in due modi diversi il baricentro del trapezio



1° modo:

$B = 11 \quad b = 4 \quad h = 5$

La mediana congiunge i punti $(\frac{11}{2}, 0)$ e $(4,5)$:

eq.ne $y = -\frac{10}{3}x + \frac{55}{3}$ da cui $x = \frac{11}{2} - \frac{3}{10}y$

$$y_G = \frac{h}{3} \frac{(2b+B)}{(b+B)} = \frac{19}{9}$$

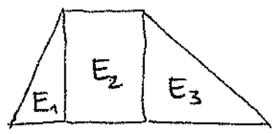
$$\Rightarrow x_G = \frac{11}{2} - \frac{3}{10} \cdot \frac{19}{9} = \frac{73}{15}$$

$$\boxed{G = \left(\frac{73}{15}, \frac{19}{9}\right)}$$

OSS. Nel calcolo di y_G si è tenuto conto che la base B è su $y=0$.

2° modo

Scomponiamo il trapezio in due triangoli e 1 rettangolo



$G_1 = (\frac{4}{3}, \frac{5}{3}) \quad \text{Area } E_1 = 5$

$G_2 = (4, \frac{5}{2}) \quad \text{Area } E_2 = 20$

$G_3 = (\frac{23}{3}, \frac{5}{3}) \quad \text{Area } E_3 = \frac{25}{2}$

$A_{\text{Trapezio}} = \frac{75}{2}$

$$x_G = \frac{1}{\frac{75}{2}} \cdot \left(5 \cdot \frac{4}{3} + 20 \cdot 4 + \frac{25}{2} \cdot \frac{23}{3}\right) = \frac{2}{75} \cdot \left(\frac{20}{3} + 80 + \frac{575}{6}\right) =$$

$$= \frac{\cancel{2}}{\cancel{75}_{15}} \cdot \frac{\cancel{1095}^{219}}{\cancel{6}_3} = \frac{\cancel{219}^{73}}{15 \cdot \cancel{3}} = \frac{73}{15}$$

$$y_G = \frac{1}{\frac{75}{2}} \cdot \left(5 \cdot \frac{5}{3} + 20 \cdot \frac{5}{2} + \frac{25}{2} \cdot \frac{5}{3}\right) = \frac{2}{75}\left(\frac{25}{3} + 50 + \frac{125}{6}\right) =$$

$$= \frac{\cancel{2}}{\cancel{75}_3} \cdot \frac{\cancel{475}^{19}}{\cancel{6}_3} = \frac{19}{9} \rightarrow \boxed{G = \left(\frac{73}{15}, \frac{19}{9}\right)}$$

ESERCIZI VARI (dai compiti degli anni precedenti)

1a) Se $f(x)=\frac{1}{(x+4)^2}\,\log(-x)+\sqrt{5-14x-3x^2}+x^5\cdot e^{-2x}-\frac{\operatorname{sen} x}{x}$, allora

domf = $[-5,-4[\ \cup\ ]-4,0[$

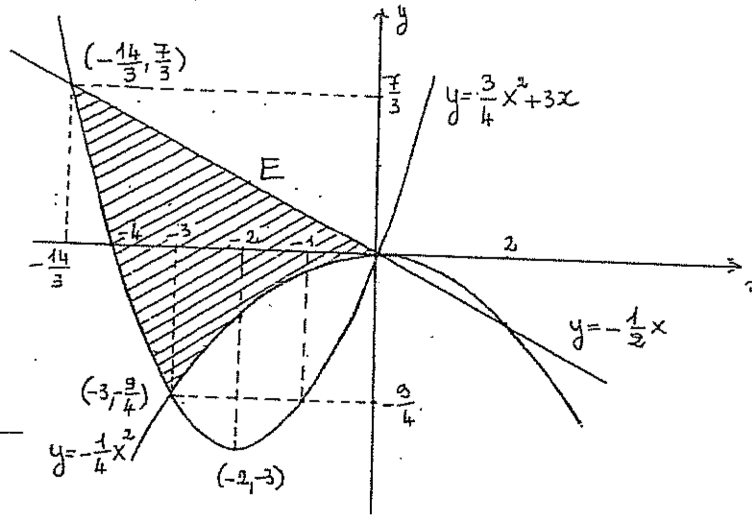
$f'(x)=-\frac{2}{(x+4)^3}\cdot\log(-x)+\frac{1}{(x+4)^2}\cdot\frac{1}{x}-\frac{7+3x}{\sqrt{5-14x-3x^2}}+5x^4e^{-2x}-2x^5e^{-2x}-\frac{x\cdot\cos x-\operatorname{sen}x}{x^2}$

1b) Completate

$$\int \overbrace{\frac{3}{(x+3)^4}}^{3\cdot(x+3)^{-4}}-5x^7-\frac{1}{4}\cos\left(\frac{x}{2}\right)-\overbrace{\frac{x^3}{\sqrt{x}}}^{x^{5/2}}-3e^{x/3}\,dx=-\frac{1}{(x+3)^3}-\frac{5}{8}x^8-\frac{1}{2}\operatorname{sen}\left(\frac{x}{2}\right)-\frac{2}{7}x^{7/2}-9e^{x/3}+c$$

1c) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.

$$\int_{-\frac{14}{3}}^{-3}\left(-\frac{1}{2}x-\left(\frac{3}{4}x^2+3x\right)\right)dx+\int_{-3}^{0}\left(-\frac{1}{2}x-\left(-\frac{1}{4}x^2\right)\right)dx$$



Risposta: ... $\frac{443}{54}$

1a) Se $f(x)=\frac{1}{(x+3)^2}\,\log\left(\frac{1}{2}-x\right)+\sqrt{8-10x-3x^2}+x^6\cdot e^{-3x}-\frac{\cos x}{x^2}$, allora

domf = $[-4,-3[\ \cup\ ]-3,0[\ \cup\ ]0,\frac{1}{2}[$

$f'(x)=-\frac{2}{(x+3)^3}\cdot\log\left(\frac{1}{2}-x\right)+\frac{1}{(x+3)^2}\cdot\frac{-1}{\frac{1}{2}-x}+\frac{-10-6x}{2\sqrt{8-10x-3x^2}}+6x^5e^{-3x}-3x^6e^{-3x}-\frac{(-\operatorname{sen}x)x^2-\cos x\cdot(2x)}{x^4}$

$=\frac{\operatorname{sen}x}{x^2}+\frac{2\cos x}{x^3}$

1b) Completate

$$\int \overbrace{\frac{7}{(x+4)^5}}^{7\cdot(x+4)^{-5}}-3x^8-\frac{1}{9}\cos\left(\frac{x}{3}\right)+\overbrace{\frac{x^5}{\sqrt{x}}}^{x^{9/2}}-2e^{x/2}\,dx=-\frac{7}{4(x+4)^4}-\frac{x^9}{3}-\frac{1}{3}\operatorname{sen}\left(\frac{x}{3}\right)+\frac{2}{11}x^{\frac{11}{2}}-4e^{x/2}+c$$

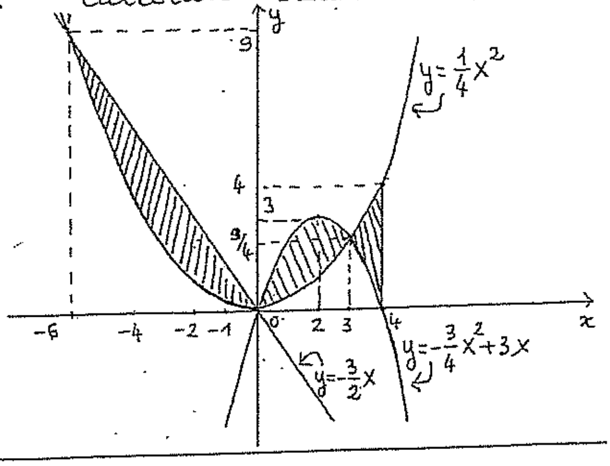
$x^4 \neq 0,\ -\frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{4} \geq 0,\ 2x+5>0 \Rightarrow x \neq 0,\ -3 \leq x \leq 1,\ x > -\frac{5}{2}$

1a) Se $f(x) = \frac{1}{x^4}\sqrt{-\frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{4}} + \frac{\cos x}{2\pi^2} + \log(2x+5)$, allora

domf = ... $]-\frac{5}{2}, 0[\ \cup\ ]0,1]$

$f'(x) = -\frac{4}{x^5}\cdot\sqrt{-\frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{4}} + \frac{1}{x^4}\cdot\frac{\left(-\frac{1}{2} - \frac{1}{2}x\right)}{2\sqrt{-\frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{4}}} - \frac{1}{2\pi^2}\cdot \text{sen}\,x + \frac{2}{2x+5}$

1b) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\int_{-6}^{0}\left[-\frac{3}{2}x - \frac{1}{4}x^2\right]dx + \int_{0}^{3}\left[-\frac{3}{4}x^2+3x-\left(\frac{1}{4}x^2\right)\right]dx + \int_{3}^{4}\left[\frac{1}{4}x^2 - \left(-\frac{3}{4}x^2+3x\right)\right]dx$$

Risposta: ... $\boxed{\frac{46}{3}}$

$x^3 \neq 0 \quad \left(\frac{7}{2}x-1\right)^2 + 6x \geq 0$

$2-x>0$

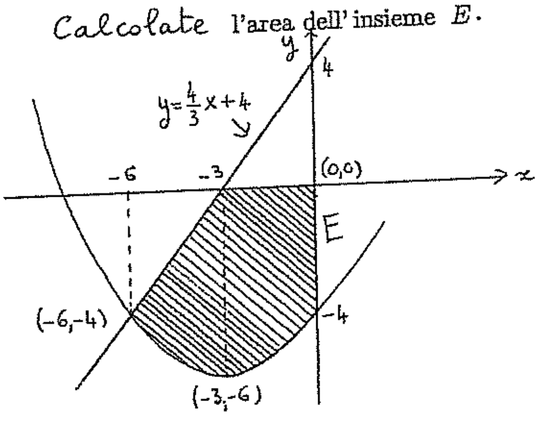
$4x+1>0$

1a) Se $f(x) = \frac{1}{x^3}\sqrt{\left(\frac{7}{2}x-1\right)^2 + 6x} + \frac{\text{sen}\,x}{4\,e^2} + \log(2-x) + \log(4x+1)$, allora

domf = ... $]-\frac{1}{4}, 0[\ \cup\ ]0,2[$

$f'(x) = \ldots -\frac{3}{x^4}\cdot\sqrt{\left(\frac{7}{2}x-1\right)^2+6x} + \frac{1}{x^3}\,\frac{\frac{49}{2}x-7}{2\sqrt{\left(\frac{7}{2}x-1\right)^2+6x}} + \frac{1}{4e^2}\cos x - \frac{1}{2-x} + \frac{4}{4x+1}$

1b) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\int_{-6}^{-3}\left[\frac{4}{3}x+4-\left(\frac{2}{9}(x+3)^2-6\right)\right]dx + \int_{-3}^{0} -\frac{2}{9}(x+3)^2+6\,dx$$

Risposta: ... $\boxed{26}$

1c) Completate

$$\int \left[\underbrace{\frac{4}{(x+1)^5}}_{4\cdot(x+1)^{-5}} - 3x^3 - e^{x/3} - \underbrace{\frac{\sqrt{x}}{x^3}}_{x^{-5/2}} - 2\cos(2x) + 3\right]dx = -\frac{1}{(x+1)^4} - \frac{3}{4}x^4 - 3e^{x/3} + \frac{2}{3x^{3/2}}$$

$- \text{sen}(2x) + 3x + c$

ANALISI 2 -100-

1c) Completate

$$\int [\frac{1}{6}\,\mathrm{sen}(6x) - 2e^{\frac{1}{4}x} + \overbrace{\frac{4}{(x+2)^4}}^{4\cdot(x+2)^{-4}} - \frac{1}{5}x^3 + \overbrace{\frac{\sqrt{x}}{x^3}}^{x^{-5/2}}]\,dx = -\frac{1}{36}\cos(6x) - 8e^{\frac{1}{4}x} - \frac{4}{3(x+2)^3} - \frac{1}{20}x^4 - \frac{2}{3x^{3/2}} + c$$

1c) Completate

$$\int [\frac{1}{5}\cos(5x) - 4e^{\frac{1}{2}x} + \overbrace{\frac{8}{(x+4)^4}}^{8\cdot(x+4)^{-4}} - \frac{1}{6}x^3 + \overbrace{\frac{\sqrt{x}}{x^2}}^{x^{-3/2}}]\,dx = \frac{1}{25}\,\mathrm{sen}(5x) - 8e^{\frac{1}{2}x} - \frac{8}{3(x+4)^3} - \frac{1}{24}x^4 - \frac{2}{\sqrt{x}} + c$$

$$\mathrm{dom}f \begin{cases} 5-x-\frac{2}{5}x^2 \geq 0 \\ 3x^2-4x+\frac{4}{3} > 0 \\ x^4-1 \neq 0 \end{cases} \qquad \begin{cases} x \in [-5, 5/2] \\ \forall x \neq 2/3 \\ x \neq \pm 1 \end{cases}$$

1a) Se $f(x) = e^{\frac{x}{2}}\sqrt{5 - x - \frac{2}{5}x^2} + \log(3x^2 - 4x + \frac{4}{3}) + \frac{\cos x}{x^4 - 1}$, allora

domf = $[-5,-1[\,\cup\,]-1,\frac{2}{3}[\,\cup\,]\frac{2}{3},1[\,\cup\,]1,\frac{5}{2}]$

$f'(x) = \ldots \frac{1}{2}e^{\frac{x}{2}}\sqrt{5-x-\frac{2}{5}x^2} + e^{x/2}\cdot\frac{(-1-\frac{4}{5}x)}{2\sqrt{5-x-\frac{2}{5}x^2}} + \frac{6x-4}{3x^2-4x+4/3} + \frac{-(x^4-1)\,\mathrm{sen}x - 4x^3\cos x}{(x^4-1)^2}$

1b) Completate

$$\int \overbrace{\frac{x\sqrt{x}}{4}}^{x^{3/2}} + e^{\frac{x}{5}} - \frac{1}{2}\,\mathrm{sen}(4x) + \overbrace{\frac{3}{8x^5}}^{\frac{3}{8}x^{-5}} - \overbrace{\frac{2}{(x-3)^4}}^{2(x-3)^{-4}}\,dx = \frac{1}{4}\frac{x^{5/2}}{5/2} + 5e^{x/5} + \frac{1}{8}\cos(4x) + \frac{3}{8}\frac{x^{-4}}{-4} - 2\frac{(x-3)^{-3}}{-3} + c =$$

$$\boxed{\frac{1}{10}x^{5/2} + 5e^{x/5} + \frac{1}{8}\cos(4x) - \frac{3}{32x^4} + \frac{2}{3(x-3)^3} + c}$$

TUTOR ?

-101- ANALISI 2

$x_{1,2} = \frac{-7 \pm \sqrt{49+32}}{4} = \frac{-7 \pm 9}{4}$

1a) Se $f(x) = \sqrt{4-7x-2x^2} + x^4 \cdot \log(3x^2 - 2x + \frac{1}{3}) + \frac{e^{3x}}{x} - \cos x$, allora

domf = $[-4,0[\,\cup\,]0,\frac{1}{3}[\,\cup\,]\frac{1}{3},\frac{1}{2}]$

$$\begin{cases} 4-7x-2x^2 \geq 0 \\ 3x^2-2x+1/3 > 0 \\ x \neq 0 \end{cases} \quad \begin{cases} 2x^2+7x-4 \leq 0 \\ 3(x-\frac{1}{3})^2 > 0 \\ x \neq 0 \end{cases} \quad \begin{cases} -4 \leq x \leq \frac{1}{2} \\ x \neq 1/3 \\ x \neq 0 \end{cases}$$

$$f'(x) = \frac{-7-4x}{2\sqrt{4-7x-2x^2}} + 4x^3 \cdot \log(3x^2-2x+\tfrac{1}{3}) + x^4 \cdot \frac{6x-2}{3x^2-2x+1/3} + \frac{3xe^{3x}-e^{3x}}{x^2} + \operatorname{sen} x$$

1b) Completate

$\frac{1}{4}x^{-\frac{1}{2}}$ $\quad \frac{7}{3}x^{-5}$

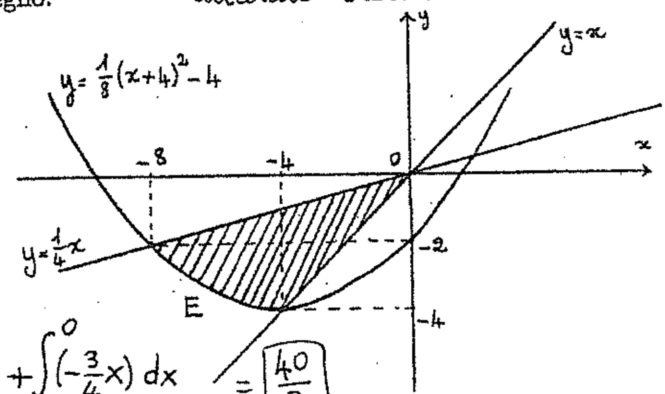
$$\int \frac{1}{4\sqrt{x}} - \frac{7}{3x^5} - e^{\frac{x}{4}} + \cos(5x) - \frac{3}{(x-3)^2}\, dx = \frac{1}{2}\sqrt{x} + \frac{7}{12x^4} - 4e^{\frac{x}{4}} + \frac{1}{5}\operatorname{sen}(5x) + \frac{3}{(x-3)} + c$$

$\forall I \subset]0,3[$

$\forall I \subset]3,+\infty[$

dom $\begin{cases} x > 0 \\ x^5 \neq 0 \\ (x-3)^2 \neq 0 \end{cases} \quad \begin{cases} x > 0 \\ x \neq 0 \\ x \neq 3 \end{cases}$ dom$=]0,3[\,\cup\,]3,+\infty[$

1c) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\text{area}E = \int_{-8}^{-4} \left[\frac{1}{4}x - \left(\frac{1}{8}(x+4)^2 - 4\right)\right] dx + \int_{-4}^{0} \left[\frac{1}{4}x - x\right] dx =$$

Risposta: $= \int_{-8}^{-4} \left(-\frac{1}{8}x^2 - \frac{3}{4}x + 2\right) dx + \int_{-4}^{0} \left(-\frac{3}{4}x\right) dx = \boxed{\frac{40}{3}}$

1b) Completate

dom $\begin{cases} x \geq 0 \\ x^4 \neq 0 \end{cases} \begin{cases} x \geq 0 \\ x \neq 0 \end{cases}$

$$\int \frac{x^3\sqrt{x}}{6} - \frac{5}{4x^4} + e^{\frac{x}{6}} - \cos(5x)\, dx = \frac{1}{27}x^{9/2} + \frac{5}{12x^3} + 6e^{\frac{x}{6}} - \frac{1}{5}\operatorname{sen}(5x) + c$$

$\frac{x^{7/2}}{6} \quad -\frac{5}{4}x^{-4}$

$-102-$ ANALISI 2

$\text{dom} f \begin{cases} 3-8x^2+10x \geq 0 \\ 8x+1+16x^2>0 \end{cases} \begin{cases} 8x^2-10x-3 \leq 0 \\ (4x+1)^2>0 \end{cases} \begin{cases} x \in [-\frac{1}{4}, \frac{3}{2}] \\ x \neq -\frac{1}{4} \end{cases}$

1a) Se $f(x) = x^5 \sqrt{3-8x^2+10x} + \log(8x+1+16x^2) + e^{\cos(x/2)}$, allora

dom f $=]-\frac{1}{4}, \frac{3}{2}]$

$$f'(x) = 5x^4 \sqrt{3-8x^2+10x} + x^5 \cdot \frac{(-16x+10)}{2\sqrt{3-8x^2+10x}} + \frac{8+32x}{8x+1+16x^2} - \frac{1}{2} \operatorname{sen}\frac{x}{2} \cdot e^{\cos\frac{x}{2}}$$

$\text{dom} f \begin{cases} 32-2x^2 \geq 0 \\ x^2>0 \\ 2x^2+x-6 \neq 0 \end{cases} \begin{cases} x^2 \leq 16 \to [-4,4] \\ x \neq 0 \\ x \neq -2, \frac{3}{2} \end{cases}$

1a) Se $f(x) = e^{2x} \sqrt{32-2x^2} + \log(x^2) + \dfrac{2x-1}{2x^2+x-6}$, allora

dom f $= [-4,-2[\cup]-2,0[\cup]0,\frac{3}{2}[\cup]\frac{3}{2},4]$

$$f'(x) = 2e^{2x} \sqrt{32-2x^2} - \frac{e^{2x}(2x)}{\sqrt{32-2x^2}} + \frac{2}{x} + \frac{\overbrace{2(2x^2+x-6)-(2x-1)\cdot(4x+1)}^{-4x^2+4x-11}}{(2x^2+x-6)^2}$$

1b) Completate

$$\int \frac{1}{3}\cos(\frac{x}{6}) - 10\,e^{5x} + \frac{3}{2x^6} + \overbrace{\frac{x}{4\sqrt{x}}}^{\frac{1}{4}\sqrt{x} = \frac{1}{4}x^{1/2}}\, dx = 2 \operatorname{sen}\left(\frac{x}{6}\right) - 2e^{5x} + \frac{3}{2}\frac{x^{-5}}{(-5)} + \frac{1}{4}\frac{x^{3/2}}{3/2} + c$$

$\text{dom} f \begin{cases} x^6 \neq 0 \\ x \geq 0 \\ \sqrt{x} \neq 0 \end{cases} \begin{cases} x \neq 0 \\ x \geq 0 \\ x>0 \end{cases}$

$$= 2\operatorname{sen}\left(\frac{x}{6}\right) - 2e^{5x} - \frac{3}{10x^5} + \frac{1}{6}x^{3/2} + c$$

dom f $=]0,+\infty[$ $\quad \forall I \subset]0,+\infty[$

1b) Completate

$$\int [2\cos(4x) - 3e^{\frac{x}{2}} + \frac{3}{(x+2)^5} - \frac{\sqrt{x}}{x}]\,dx = \frac{1}{2}\operatorname{sen}(4x) - 6e^{\frac{x}{2}} + 3\frac{(x+2)^{-4}}{-4} - 2\sqrt{x} + c$$

dom: $x \neq -2$, $x \neq 0$, $x \geq 0$ $\quad$ dom $=]0,+\infty[$ $\quad \forall I \subset]0,+\infty[$

$-\frac{1}{\sqrt{x}}$

$$= \frac{1}{2}\operatorname{sen}(4x) - 6e^{\frac{x}{2}} - \frac{3}{4(x+2)^4} - 2\sqrt{x} + c$$

-103- ANALISI 2

$$\text{dom} f \begin{cases} x^2 \neq 0 \\ (\frac{5}{2}x-1)^2+8x \geq 0 \\ 1-x>0 \\ 3x+1>0 \end{cases} \quad \begin{cases} x\neq 0 \\ \forall x \\ x<1 \\ x>-\frac{1}{3} \end{cases}$$

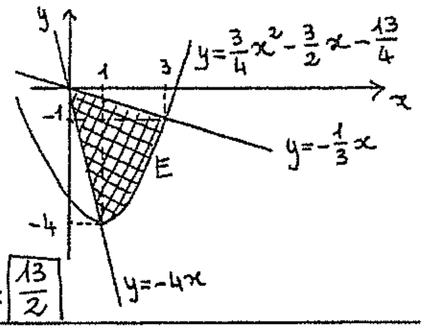
$$\frac{25}{4}x^2+3x+1 \geq 0 \qquad \Delta<0$$

1a) Se $f(x) = \dfrac{1}{x^2}\sqrt{(\frac{5}{2}x-1)^2+8x} + 3e^2 + \log(1-x) + \log(3x+1)$, allora

domf = $]-\frac{1}{3},0[\,\cup\,]0,1[$

$f'(x) = -\dfrac{2}{x^3}\cdot\sqrt{\frac{25}{4}x^2+3x+1} + \dfrac{1}{x^2}\cdot\dfrac{\frac{25}{2}x+3}{2\sqrt{\frac{25}{4}x^2+3x+1}} - \dfrac{1}{1-x} + \dfrac{3}{3x+1}$

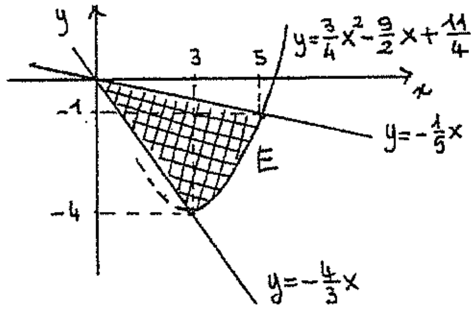
1c) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\text{area } E = \int_0^1 \left[-\frac{1}{3}x-(-4x)\right]dx + \int_1^3 \left[-\frac{1}{3}x-\left(\frac{3}{4}x^2-\frac{3}{2}x-\frac{13}{4}\right)\right]dx$$

Risposta: $\text{area } E = \int_0^1 \frac{11}{3}x\,dx + \int_1^3 \left(-\frac{3}{4}x^2+\frac{7}{6}x+\frac{13}{4}\right)dx = \boxed{\frac{13}{2}}$

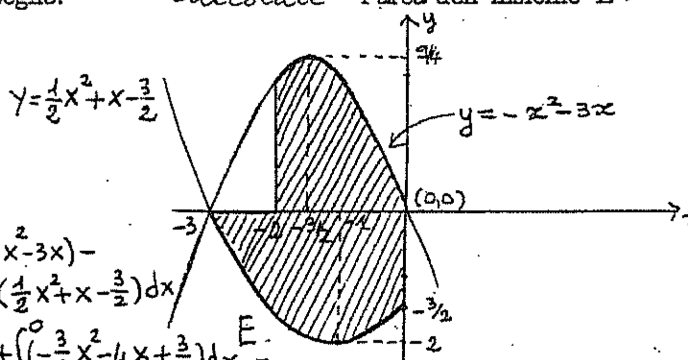
1c) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\text{area } E = \int_0^3 \left[-\frac{1}{5}x-\left(-\frac{4}{3}x\right)\right]dx + \int_3^5 \left[-\frac{1}{5}x-\left(\frac{3}{4}x^2-\frac{9}{2}x+\frac{11}{4}\right)\right]dx$$
$$= \int_0^3 \frac{17}{15}x\,dx + \int_3^5 \left(-\frac{3}{4}x^2+\frac{43}{10}x-\frac{11}{4}\right)dx$$

Risposta: ... $\boxed{\frac{19}{2}}$

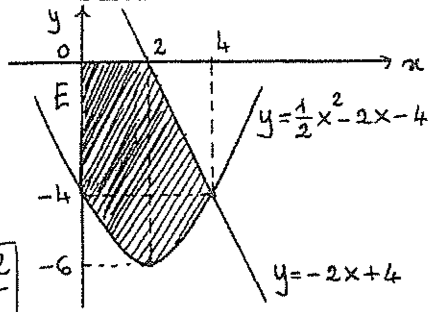
1c) Considerate l'insieme E del disegno. Calcolate l'area dell'insieme E.



$$\text{area } E = \int_{-3}^{-2} \left[0-\left(\frac{1}{2}x^2+x-\frac{3}{2}\right)\right]dx + \int_{-2}^{0} (-x^2-3x) - \left(\frac{1}{2}x^2+x-\frac{3}{2}\right)dx$$

Risposta: $\ldots = \int_{-3}^{-2}\left(-\frac{1}{2}x^2-x+\frac{3}{2}\right)dx + \int_{-2}^{0}\left(-\frac{3}{2}x^2-4x+\frac{3}{2}\right)dx = \boxed{\frac{47}{6}}$

1c) Considerate l'insieme E del disegno. l'area dell' insieme E



$$\text{area } E = \int_0^2 \left[-\left(\tfrac{1}{2}x^2-2x-4\right)\right]dx + \int_2^4 \left[(-2x+4)\right.$$
$$+\int_2^4 \left[(-2x+4)-\left(\tfrac{1}{2}x^2-2x-4\right)\right]dx$$

Risposta: ... $\int_0^2 \left(-\frac{1}{2}x^2+2x+4\right)dx + \int_2^4 \left(-\frac{1}{2}x^2+8\right)dx = \boxed{\frac{52}{3}}$

$$\to \text{domf} = \begin{cases} 5-6x^2+13x \geq 0 \\ 6x+1+9x^2 > 0 \end{cases} \begin{cases} 6x^2-13x-5 \leq 0 \\ (3x+1)^2 > 0 \end{cases} \begin{cases} x \in [-\frac{1}{3}, \frac{5}{2}] \\ x \neq -\frac{1}{3} \end{cases}$$

1a) Se $f(x) = x^4\sqrt{5-6x^2+13x} + \log(6x+1+9x^2) + e^{\operatorname{sen}(x/2)}$, allora

domf = ... $\left]-\frac{1}{3}, \frac{5}{2}\right]$

$f'(x) = \ldots\; 4x^3\cdot\sqrt{5-6x^2+13x} + \dfrac{x^4\cdot(-12x+13)}{2\sqrt{5-6x^2+13x}} + \dfrac{6+18x}{6x+1+9x^2} + \dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)\cdot e^{\operatorname{sen}\left(\frac{x}{2}\right)}$

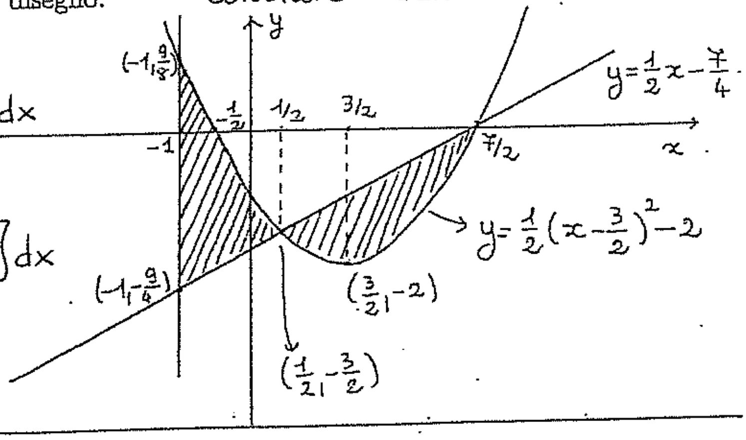
1b) Completate

dom $\begin{cases} x \geq 0 \\ x^5 \neq 0 \end{cases} \begin{cases} x \geq 0 \\ x \neq 0 \end{cases}$ $\to$ dom $=]0,+\infty[$; la risposta vale $\forall I \subset]0,+\infty[$

$$\int \frac{x^2\sqrt{x}}{5} - \frac{6}{5x^5} + e^{\frac{x}{5}} - \cos(6x)\,dx = \ldots \frac{2}{35}x^{7/2} + \frac{3}{10x^4} + 5e^{\frac{x}{5}} - \frac{1}{6}\operatorname{sen}(6x) + C$$

$x^2\cdot x^{\frac{1}{2}} = x^{5/2}$ $\qquad -\frac{6}{5}x^{-5}$

1b) Considerate l'insieme E del disegno. Calcolate l'area dell' insieme E.



$$\int_{-1}^{1/2} \left[\tfrac{1}{2}\left(x-\tfrac{3}{2}\right)^2-2-\left(\tfrac{1}{2}x-\tfrac{7}{4}\right)\right]dx$$
$$+\int_{1/2}^{7/2} \left[\tfrac{1}{2}x-\tfrac{7}{4}-\left(\tfrac{1}{2}\left(x-\tfrac{3}{2}\right)^2-2\right)\right]dx$$

Risposta: ... $\frac{9}{2}$