SOLUZIONE SCHEDA N.11

ES. 1) a) $\displaystyle\int_E 3\,dx\,dy = \int_{-2}^{2}\left(\int_{1}^{6} 3\,dy\right)dx = 3\cdot\int_{-2}^{2}[y]_1^6\,dx = 3\cdot5\,[x]_{-2}^{2} =$

$= 3\cdot5\cdot4 = 60$

b) $\displaystyle\int_E (5-x)\,dx\,dy = \int_0^3\left(\int_0^5 (5-x)\,dx\right)dy = \int_0^3\left[5x-\frac{x^2}{2}\right]_0^5 dy = \frac{25}{2}\int_0^3 dy = \frac{75}{2}$

c) $\displaystyle\int_E (4-2y)\,dx\,dy = \int_0^1\left(\int_0^1 (4-2y)\,dy\right)dx = \int_0^1\left[4y-y^2\right]_0^1 dx = 3\int_0^1 dx = 3$

d) $\displaystyle\int_E x^2\cdot y\,dx\,dy = \int_0^3\left(\int_1^2 x^2 y\,dy\right)dx = \int_0^3 x^2\left[\frac{y^2}{2}\right]_1^2 dx = \frac{3}{2}\int_0^3 x^2\,dx =$

$\displaystyle = \frac{3}{2}\left[\frac{x^3}{3}\right]_0^3 = \frac{27}{2}$

e) $\displaystyle\int_E (x-3y^2)\,dx\,dy = \int_0^2\left(\int_1^2 (x-3y^2)\,dy\right)dx = \int_0^2\left[xy-y^3\right]_1^2 dx =$

$\displaystyle = \int_0^2 (2x-8-x+1)\,dx = \int_0^2 (x-7)\,dx = \left[\frac{x^2}{2}-7x\right]_0^2 = -12$

f) $\displaystyle\int_E \left(3-\frac{1}{3}x^2\right)dx\,dy = \int_{-2}^{2}\left(\int_{-3}^{3}\left(3-\frac{1}{3}x^2\right)dx\right)dy = \int_{-2}^{2}\left[3x-\frac{x^3}{9}\right]_{-3}^{3} dy =$

$\displaystyle = (9-3-(-9+3))\int_{-2}^{2} dy = 12\,[y]_{-2}^{2} = 48$

g) $\displaystyle\int_E \sqrt{x+y}\,dx\,dy = \int_0^3\left(\int_0^1 (x+y)^{1/2}dx\right)dy = \int_0^3\left[\frac{(x+y)^{3/2}}{3/2}\right]_{x=0}^{x=1} dy =$

$\displaystyle = \frac{2}{3}\int_0^3\left[(1+y)^{3/2}-y^{3/2}\right]dy = \frac{2}{3}\left[\frac{(1+y)^{5/2}}{5/2}-\frac{y^{5/2}}{5/2}\right]_0^3 = \frac{2}{3}\cdot\frac{2}{5}\left[2^5-3^{5/2}-1-0\right]$

$\displaystyle = \frac{4}{15}\left[32-9\sqrt{3}-1\right] = \frac{4}{15}\left(31-9\sqrt{3}\right)$

h) $\displaystyle\int_E 2xy\cos(y^2)\,dx\,dy = \int_1^3\left(\int_0^{\sqrt{\frac{\pi}{2}}} 2xy\cos(y^2)\,dy\right)dx = \int_1^3 x\cdot\left(\int_0^{\sqrt{\frac{\pi}{2}}} 2y\cdot\cos(y^2)\,dy\right)dx =$

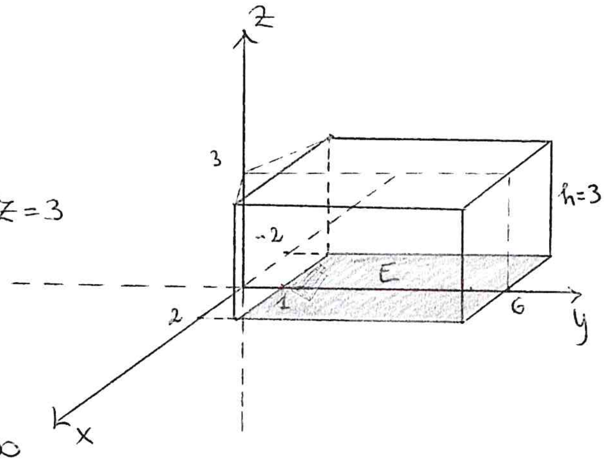
$$= \int_1^3 x \cdot \left[\operatorname{sen}(y^2) \right]_{y=0}^{y=\sqrt{\frac{\pi}{2}}} dx = \int_1^3 x \left[\operatorname{sen}\frac{\pi}{2} - \operatorname{sen} 0 \right] dx =$$

$$= \int_1^3 x \, dx = \left[\frac{x^2}{2} \right]_1^3 = \frac{9}{2} - \frac{1}{2} = 4$$

ES. 2) a) $\int_E 3 \, dx dy \quad f(x,y) = 3 \quad$ grafico $z = 3$

piano orizzontale

L'integrale doppio rappresenta quindi il VOLUME del PARALLELEPIPEDO di base E e altezza 3 $\rightarrow \int_E 3 dxdy = VOL = \text{Area base} \cdot h = (4 \cdot 5) \cdot 3 = 60$

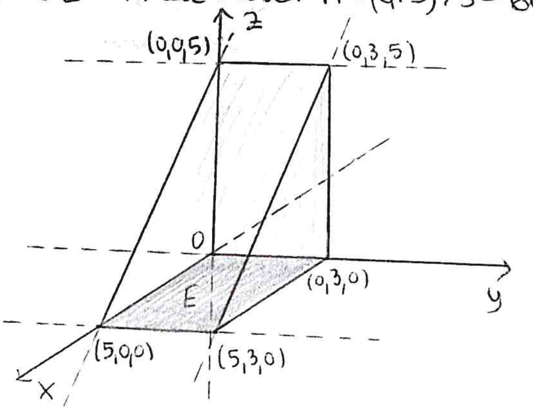


b) $\int_E (5-x) dxdy \quad f(x,y) = 5-x$

grafico $z = 5 - x$ piano inclinato indipendente da y passante per $(0,0,5)$ e $(5,0,0)$

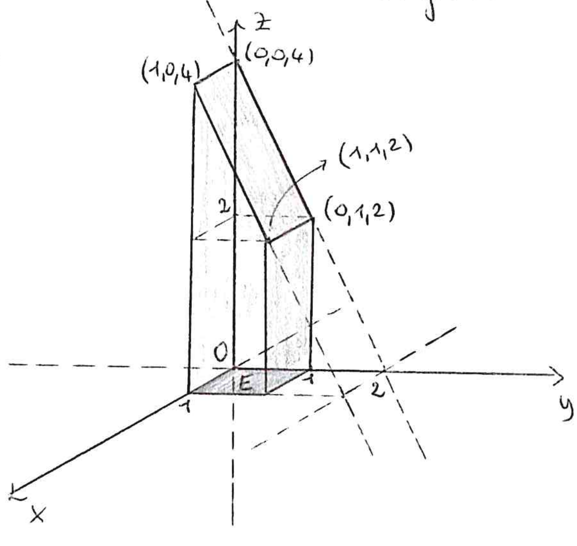
L'integrale doppio rappresenta quindi il volume di un PRISMA a base triangolare di base il triangolo di VERTICI $(0,0,0)$ $(0,0,5)$ $(5,0,0)$ e $h = 3$

$\rightarrow \int_E (5-x) dxdy = A_{base} \cdot h = \frac{25}{2} \cdot 3 = \frac{75}{2}$



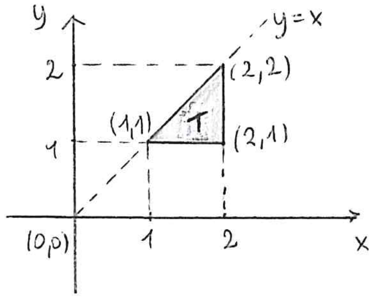
c) $\int_E (4-2y) dxdy \quad f(x,y) = 4 - 2y$

grafico $z = 4 - 2y$ piano inclinato indipendente da x passante per $(0,0,4)$ e $(0,2,0)$



L'integrale doppio rappresenta quindi il volume di un PRIMA a base Trapezoidale di base il TRAPEZIO di VERTICI $(0,0,4)$ $(0,1,2)$ $(0,1,0)$ $(0,0,0)$ e altezza 1 $\rightarrow$ VOLUME $= A_{base} \cdot h = \frac{(B+b) \cdot h}{2} \cdot 1 = \frac{(4+2) \cdot 1}{2} = 3$

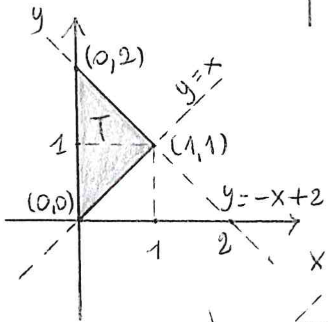
ES. 3) a)



$T_x = \{(x,y)\in\mathbb{R}^2 : 1\le x\le 2,\ 1\le y\le x\}$

$T_y = \{(x,y)\in\mathbb{R}^2 : 1\le y\le 2\ \ y\le x\le 2\}$

b)

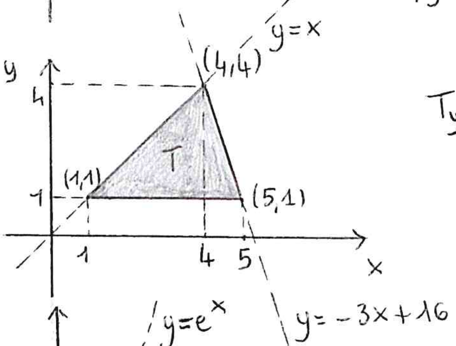


$T_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ x\le y\le -x+2\}$

$T_{1,y} = \{(x,y)\in\mathbb{R}^2 : 0\le y\le 1,\ 0\le x\le y\}$

$T_{2,y} = \{(x,y)\in\mathbb{R}^2 : 1\le y\le 2,\ 0\le x\le 2-y\}$

c)



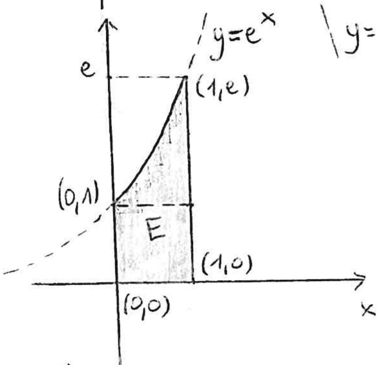
$T_y = \{(x,y)\in\mathbb{R}^2 : 1\le y\le 4,\ y\le x\le -\frac{y}{3}+\frac{16}{3}\}$

$y=-3x+16 \to x = \frac{16}{3}-\frac{y}{3}$

$T_{1,x} = \{(x,y)\in\mathbb{R}^2 : 1\le x\le 4,\ 1\le y\le x\}$

$T_{2,x} = \{(x,y)\in\mathbb{R}^2 : 4\le x\le 5,\ 1\le y\le -3x+16\}$

d)



$E_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ 0\le y\le e^x\}$

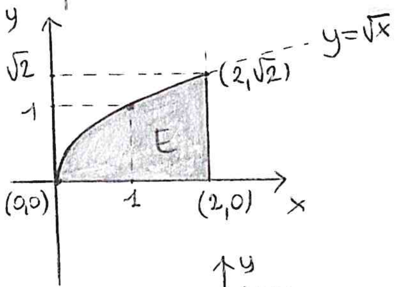
$E_{1,y} = \{(x,y)\in\mathbb{R}^2 : 0\le y\le 1,\ 0\le x\le 1\}$

$E_{2,y} = \{(x,y)\in\mathbb{R}^2 : 1\le y\le e,\ \log y\le x\le 1\}$

$y=e^x \ \updownarrow\ y\ge 1$

$x=\log y$

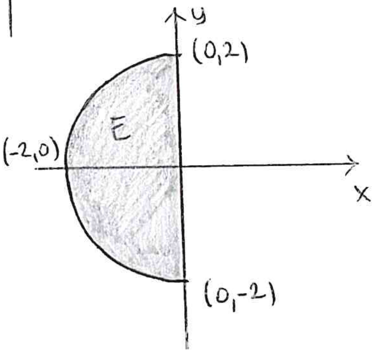
e)



$E_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 2,\ 0\le y\le \sqrt{x}\}$

$E_y = \{(x,y)\in\mathbb{R}^2 : 0\le y\le \sqrt{2},\ y^2\le x\le 2\}$

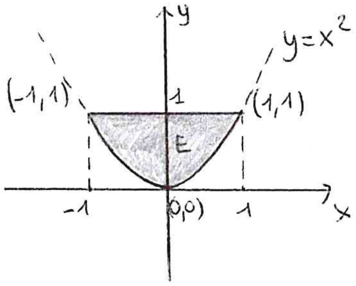
f)



$E_x = \{(x,y)\in\mathbb{R}^2 : -2\le x\le 0,\ -\sqrt{4-x^2}\le y\le \sqrt{4-x^2}\}$

$E_y = \{(x,y)\in\mathbb{R}^2 : -2\le y\le 2,\ -\sqrt{4-y^2}\le x\le 0\}$

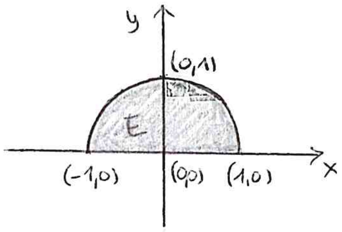
g)



$E_x = \{(x,y)\in\mathbb{R}^2 : -1\le x\le 1,\ x^2\le y\le 1\}$

$E_y = \{(x,y)\in\mathbb{R}^2 : 0\le y\le 1,\ -\sqrt{y}\le x\le \sqrt{y}\}$

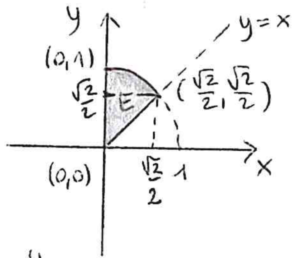
h)



$E_x = \{ (x,y) \in \mathbb{R}^2 : -1 \leq x \leq 1,\ 0 \leq y \leq \sqrt{1-x^2} \}$

$E_y = \{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq 1,\ -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2} \}$

i)

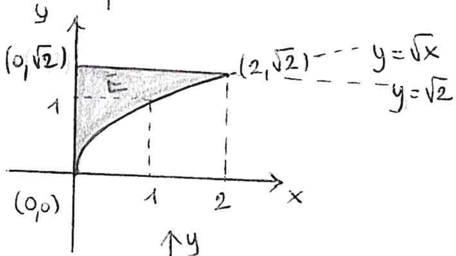


$E_x = \{ (x,y) \in \mathbb{R}^2 : 0 \leq x \leq \frac{\sqrt{2}}{2},\ x \leq y \leq \sqrt{1-x^2} \}$

$E_{1,y} = \{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq \frac{\sqrt{2}}{2},\ 0 \leq x \leq y \}$

$E_{2,y} = \{ (x,y) \in \mathbb{R}^2 : \frac{\sqrt{2}}{2} \leq y \leq 1,\ 0 \leq x \leq \sqrt{1-y^2} \}$

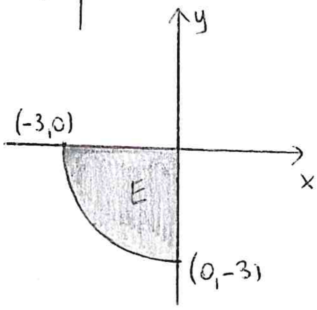
j)



$E_x = \{ (x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2,\ \sqrt{x} \leq y \leq \sqrt{2} \}$

$E_y = \{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq \sqrt{2},\ 0 \leq x \leq y^2 \}$

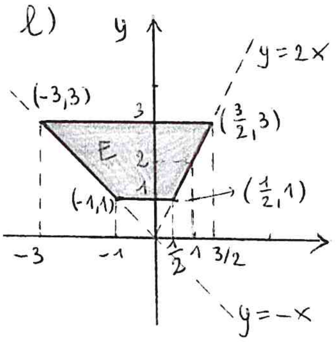
k)



$E_x = \{ (x,y) \in \mathbb{R}^2 : -3 \leq x \leq 0,\ -\sqrt{9-x^2} \leq y \leq 0 \}$

$E_y = \{ (x,y) \in \mathbb{R}^2 : -3 \leq y \leq 0,\ -\sqrt{9-y^2} \leq x \leq 0 \}$

l)



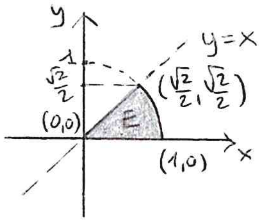
$E_y = \{ (x,y) \in \mathbb{R}^2 : 1 \leq y \leq 3,\ -y \leq x \leq \frac{y}{2} \}$

$E_{1,x} = \{ (x,y) \in \mathbb{R}^2 : -3 \leq x \leq -1,\ -x \leq y \leq 3 \}$

$E_{2,x} = \{ (x,y) \in \mathbb{R}^2 : -1 \leq x \leq \frac{1}{2},\ 1 \leq y \leq 3 \}$

$E_{3,x} = \{ (x,y) \in \mathbb{R}^2 : \frac{1}{2} \leq x \leq \frac{3}{2},\ 2x \leq y \leq 3 \}$

m)

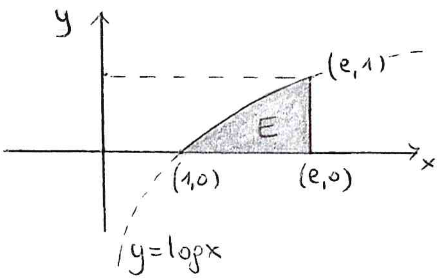


$E_y = \{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq \frac{\sqrt{2}}{2},\ y \leq x \leq \sqrt{1-y^2} \}$

$E_{1,x} = \{ (x,y) \in \mathbb{R}^2 : 0 \leq x \leq \frac{\sqrt{2}}{2},\ 0 \leq y \leq x \}$

$E_{2,x} = \{ (x,y) \in \mathbb{R}^2 : \frac{\sqrt{2}}{2} \leq x \leq 1,\ 0 \leq y \leq \sqrt{1-x^2} \}$

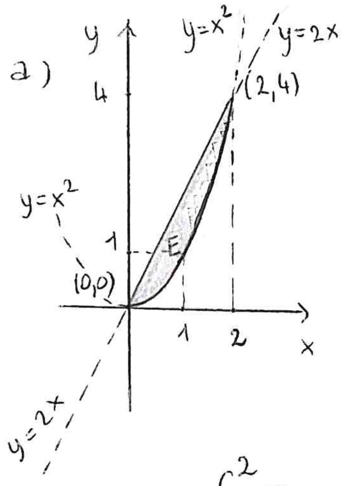
m)



$E_x = \{ (x,y) \in \mathbb{R}^2 : 1 \leq x \leq e,\ 0 \leq y \leq \log x \}$

$E_y = \{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq 1,\ e^y \leq x \leq e \}$

ES.4) a)



NORMALE RISP a x

$E_x=\{(x,y)\in\mathbb{R}^2:\ 0\le x\le 2,\ x^2\le y\le 2x\}$

NORMALE RISP a y

$E_y=\{(x,y)\in\mathbb{R}^2:\ 0\le y\le 4,\ \frac{y}{2}\le x\le\sqrt{y}\}$

$$\int_E (x^2+y^2)\,dx\,dy=\int_0^2\left(\int_{x^2}^{2x}(x^2+y^2)\,dy\right)dx=$$

$$=\int_0^2\left[x^2y+\frac{y^3}{3}\right]_{x^2}^{2x}dx=\int_0^2\left(2x^3+\frac{8}{3}x^3-x^4-\frac{1}{3}x^6\right)dx=$$

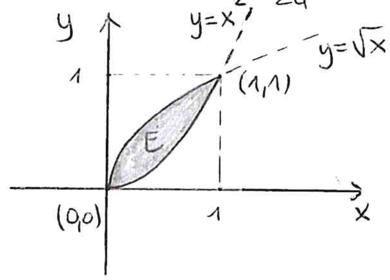
$$=\int_0^2\left(-\frac{1}{3}x^6-x^4+\frac{14}{3}x^3\right)dx=\left[-\frac{1}{21}x^7-\frac{x^5}{5}+\frac{7}{6}x^4\right]_0^2=-\frac{128}{21}-\frac{32}{5}+\frac{\cancel{112}^{\,56}}{\cancel{6}_{\,3}}=\frac{648}{105}=\boxed{\frac{216}{35}}$$

$$\int_E (x^2+y^2)\,dx\,dy=\int_0^4\left(\int_{y/2}^{\sqrt{y}}(x^2+y^2)\,dx\right)dy=\int_0^4\left[\frac{x^3}{3}+y^2x\right]_{x=y/2}^{x=\sqrt{y}}dy=$$

$$=\int_0^4\left(\frac{1}{3}y^{3/2}+y^{5/2}\underbrace{-\frac{1}{24}y^3-\frac{1}{2}y^3}_{-\frac{13}{24}y^3}\right)dy=\left[\frac{1}{3}\frac{y^{5/2}}{5/2}+\frac{y^{7/2}}{7/2}-\frac{13}{96}y^4\right]_0^4=\frac{2}{15}\cdot 2^5+\frac{2}{7}\cdot 2^7-\frac{13}{96}\cdot 4^4=$$

$$=\frac{64}{15}+\frac{256}{7}-\frac{104}{3}=\frac{648}{105}=\boxed{\frac{216}{35}}$$

b)



NORM. risp a x

$E_x=\{(x,y)\in\mathbb{R}^2:\ 0\le x\le 1,\ x^2\le y\le\sqrt{x}\}$

NORM risp a y

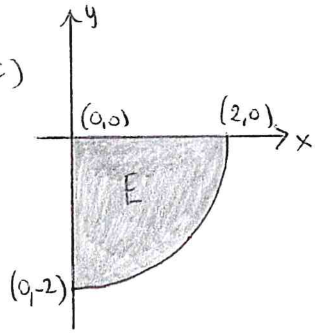
$E_y=\{(x,y)\in\mathbb{R}^2:\ 0\le y\le 1,\ y^2\le x\le\sqrt{y}\}$

$$\int_E (x+y)\,dx\,dy=\int_0^1\left(\int_{x^2}^{\sqrt{x}}(x+y)\,dy\right)dx=\int_0^1\left[xy+\frac{y^2}{2}\right]_{x^2}^{\sqrt{x}}dx=\int_0^1\left[x^{3/2}+\frac{x}{2}-x^3-\frac{1}{2}x^4\right]dx$$

$$=\left[\frac{x^{5/2}}{5/2}+\frac{x^2}{4}-\frac{x^4}{4}-\frac{x^5}{10}\right]_0^1=\frac{2}{5}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{4}}-\frac{1}{10}=\frac{2}{5}-\frac{1}{10}=\boxed{\frac{3}{10}}$$

$\int_E (x+y)\,dx\,dy=\int_0^1\left(\int_{y^2}^{\sqrt{y}}(x+y)\,dx\right)dy$ = esattamente gli stessi passaggi con x e y scambiate.

c)



NORM. risp. a x $E_x=\{(x,y)\in\mathbb{R}^2:\ 0\le x\le 2,\ -\sqrt{4-x^2}\le y\le 0\}$

NORM. risp. a y $E_y=\{(x,y)\in\mathbb{R}^2:\ -2\le y\le 0,\ 0\le x\le\sqrt{4-y^2}\}$

$$\int_E (2x-y)\,dx\,dy=\int_0^2\left(\int_{-\sqrt{4-x^2}}^{0}(2x-y)\,dy\right)dx=\int_0^2\left[2xy-\frac{y^2}{2}\right]_{-\sqrt{4-x^2}}^{0}dx=$$

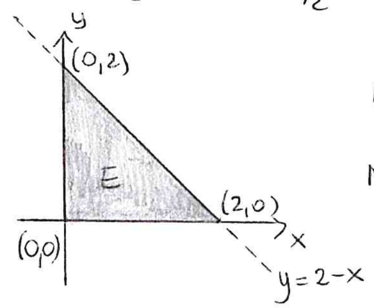
$$= \int_0^2 \left[0 + 2x\sqrt{4-x^2} + \frac{4-x^2}{2}\right] dx = \int_0^2 \left(2x\sqrt{4-x^2} + 2 - \frac{1}{2}x^2\right) dx =$$

$$= \left[-\frac{(4-x^2)^{3/2}}{3/2} + 2x - \frac{x^3}{6}\right]_0^2 = -\frac{2}{3}\cdot 0 + 4 - \frac{8}{6} - \left(-\frac{2}{3}8\right) = 4 - \frac{4}{3} + \frac{16}{3} = \boxed{8}$$

$$\int_E (2x-y)\,dx\,dy = \int_{-2}^{0}\left(\int_0^{\sqrt{4-y^2}} (2x-y)\,dx\right) dy = \int_{-2}^{0} \left[x^2 - xy\right]_0^{\sqrt{4-y^2}} dy = \int_{-2}^{0} \left(4-y^2 - y\sqrt{4-y^2}\right) dy =$$

$$= \left[4y - \frac{y^3}{3} + \frac{1}{2}\frac{(4-y^2)^{3/2}}{3/2}\right]_{-2}^{0} = \frac{1}{3}8 - \left(-8 + \frac{8}{3} + 0\right) = 8/3 + 8 - \frac{8}{3} = \boxed{8}.$$

d)



NORM. risp. a x $E_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 2,\ 0\le y\le 2-x\}$

NORM. risp a y $E_y = \{(x,y)\in\mathbb{R}^2 : 0\le y\le 2,\ 0\le x\le 2-y\}$

$$\int_E (2x+3y)\,dx\,dy = \int_0^2\left(\int_0^{2-x}(2x+3y)\,dy\right)dx =$$

$$= \int_0^2 \left[2xy + 3\frac{y^2}{2}\right]_{y=0}^{y=2-x} dx = \int_0^2 \Big(\underbrace{2x(2-x) + \frac{3}{2}(2-x)^2}_{4x-2x^2+\frac{3}{2}(4+x^2-4x)}\Big) dx = \int_0^2 \left(-\frac{x^2}{2} - 2x + 6\right) dx =$$

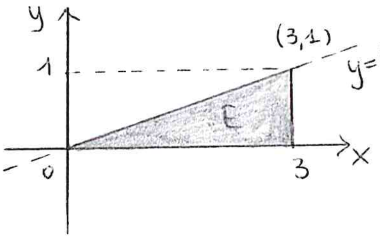
$$= \left[-\frac{x^3}{6} - x^2 + 6x\right]_0^2 = -\frac{8}{6} - 4 + 12 = -\frac{4}{3} + 8 = \boxed{\frac{20}{3}}$$

$$\int_E (2x+3y)\,dx\,dy = \int_0^2\left(\int_0^{2-y}(2x+3y)\,dx\right)dy = \int_0^2 \left[x^2 + 3xy\right]_{x=0}^{x=2-y} dy =$$

$$= \int_0^2 \Big[\underbrace{(2-y)^2 + 3(2-y)y}_{4-4y+y^2+6y-3y^2}\Big] dy = \int_0^2 (-2y^2 + 2y + 4)\,dy = \left[-\frac{2}{3}y^3 + y^2 + 4y\right]_0^2 = -\frac{16}{3} + 4 + 8 = \boxed{\frac{20}{3}}$$

ES. 5) a) $E_y = \{(x,y)\in\mathbb{R}^2 : 0\le y\le 1,\ 3y\le x\le 3\}$

NORM. risp. a y $\qquad x = 3y \to y = \frac{x}{3}$

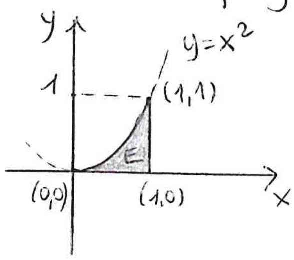


NORM. risp. a x $E_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 3,\ 0\le y\le \frac{x}{3}\}$

$$\int_0^1\left(\int_{3y}^{3} e^{x^2} dx\right) dy = \int_0^3\left(\int_0^{x/3} e^{x^2} dy\right) dx = \int_0^3 e^{x^2}\left[y\right]_0^{x/3} dx = \int_0^3 \frac{x}{3} e^{x^2} dx =$$

$$= \frac{1}{6}\int_0^3 2x e^{x^2} dx = \frac{1}{6}\left[e^{x^2}\right]_0^3 = \frac{1}{6}\left(e^9 - 1\right)$$

b) NORM. risp. a y $E_y=\{(x,y)\in\mathbb{R}^2: 0\le y\le 1,\ \sqrt{y}\le x\le 1\}$



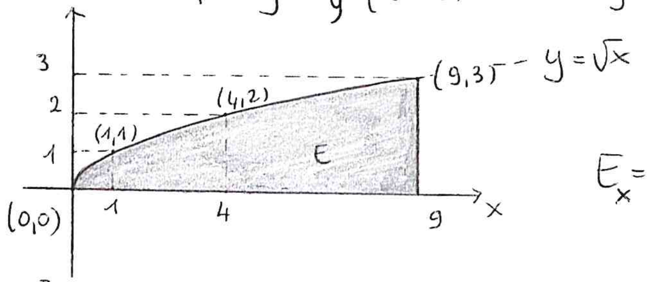
$x\ge 0$ $\quad x=\sqrt{y}\to y=x^2$

NORM. risp. a x $E_x=\{(x,y)\in\mathbb{R}^2: 0\le x\le 1,\ 0\le y\le x^2\}$

$$\int_0^1\left(\int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\right)dy=\int_0^1\left(\int_0^{x^2}\sqrt{x^3+1}\,dy\right)dx=$$

$$=\int_0^1 x^2\sqrt{x^3+1}\,dx=\frac{1}{3}\int_0^1 3x^2(x^3+1)^{1/2}dx=\frac{1}{3}\left[\frac{(x^3+1)^{3/2}}{3/2}\right]_0^1=\frac{2}{9}(2\sqrt{2}-1)$$

c) NORM. risp. a y $E_y=\{(x,y)\in\mathbb{R}^2: 0\le y\le 3,\ y^2\le x\le 9\}$ $\quad x=y^2\to y=\sqrt{x}$, $x\ge 0$



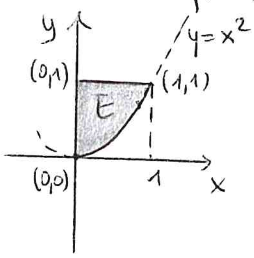
NORM. risp. a x

$E_x=\{(x,y)\in\mathbb{R}^2: 0\le x\le 9,\ 0\le y\le \sqrt{x}\}$

$$\int_0^3\left(\int_{y^2}^9 y\cdot\cos(x^2)\,dx\right)dy=\int_0^9\left(\int_0^{\sqrt{x}} y\cdot\cos(x^2)\,dy\right)dx=\int_0^9\cos(x^2)\left[\frac{y^2}{2}\right]_0^{\sqrt{x}}dx=$$

$$=\int_0^9\frac{1}{2}x\cdot\cos(x^2)\,dx=\frac{1}{4}\int_0^9 2x\cdot\cos(x^2)\,dx=\frac{1}{4}\left[\operatorname{sen}(x^2)\right]_0^9=\frac{1}{4}\operatorname{sen}(81)$$

d) NORM. risp. a x $E_x=\{(x,y)\in\mathbb{R}^2: 0\le x\le 1,\ x^2\le y\le 1\}$ $\quad y=x^2,\ x\ge 0\to x=\sqrt{y}$



NORM. risp. a y $E_y=\{(x,y)\in\mathbb{R}^2: 0\le y\le 1,\ 0\le x\le \sqrt{y}\}$

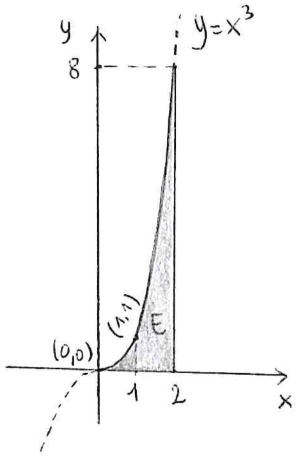
$$\int_0^1\left(\int_{x^2}^1 x^3\operatorname{sen}(y^3)\,dy\right)dx=\int_0^1\left(\int_0 x^3\operatorname{sen}(y^3)\,dx\right)dy=$$

$$=\int_0^1\left(\operatorname{sen}(y^3)\int_0^{\sqrt{y}}x^3dx\right)dy=\int_0^1\operatorname{sen}(y^3)\left[\frac{x^4}{4}\right]_0^{\sqrt{y}}dy=\int_0^1\frac{1}{4}y^2\operatorname{sen}(y^3)\,dy=$$

$$=\frac{1}{12}\int_0^1 3y^2\cdot\operatorname{sen}(y^3)\,dy=\frac{1}{12}\left[-\cos(y^3)\right]_0^1=\frac{1}{12}\left[-\cos 1+\cos 0\right]=\frac{1}{12}(1-\cos 1)$$

e) NORM. risp. a y $E_y=\{(x,y)\in\mathbb{R}^2: 0\le y\le 8,\ \sqrt[3]{y}\le x\le 2\}$ $\quad x\ge 0$ $\ x=\sqrt[3]{y}\to y=x^3$

NORM. risp. a x $E_x=\{(x,y)\in\mathbb{R}^2: 0\le x\le 2,\ 0\le y\le x^3\}$

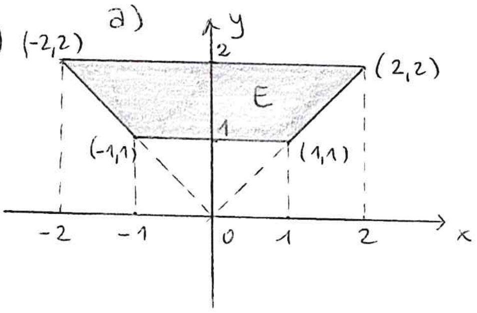


$$\int_0^8 \left(\int_{\sqrt[3]{y}}^2 e^{x^4} dx\right) dy = \int_0^2 \left(\int_0^{x^3} e^{x^4} dy\right) dx =$$

$$= \int_0^2 e^{x^4} [y]_0^{x^3} dx = \int_0^2 x^3 e^{x^4} dx = \frac{1}{4}\int_0^2 (4x^3)\cdot e^{x^4} dx =$$

$$= \frac{1}{4}\left[e^{x^4}\right]_0^2 = \frac{1}{4}(e^{16}-1)$$

ES. 6) a)



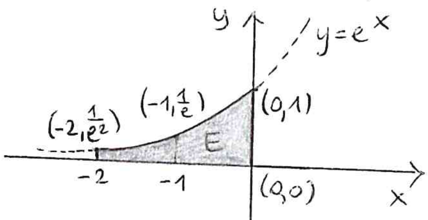
Ex a pag. 20

NORM. risp. a y $E_y = \{(x,y)\in\mathbb{R}^2 : 1\le y\le 2, -y\le x\le y\}$

$$\int_E x^2\cdot y\, dx dy = \int_1^2 \left(\int_{-y}^{y} x^2\cdot y\, dx\right) dy =$$

$$= \int_1^2 y \left[\frac{x^3}{3}\right]_{-y}^{y} dy = \int_1^2 y\left[\frac{y^3}{3} - \left(-\frac{y^3}{3}\right)\right] dy = \int_1^2 \frac{2}{3}y^4 dy = \left[\frac{2}{15}y^5\right]_1^2 = \frac{64}{15} - \frac{2}{15} = \boxed{\frac{62}{15}}$$

b)



NORM. risp. a x $E_x = \{(x,y)\in\mathbb{R}^2 : -2\le x\le 0, 0\le y\le e^x\}$

$$\int_E 2xy\, dx dy = \int_{-2}^0 \left(\int_0^{e^x} 2xy\, dy\right) dx = \int_{-2}^0 x\left[y^2\right]_0^{e^x} dx =$$

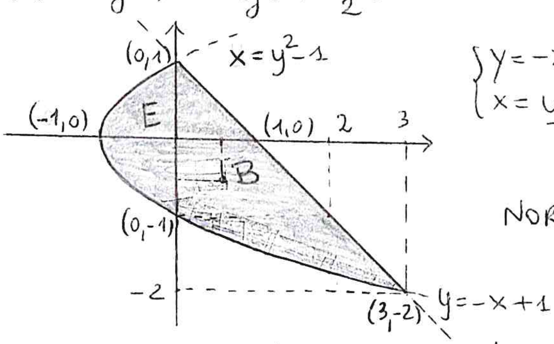
$$= \int_{-2}^0 x e^{2x} dx = \left[\left(\frac{1}{2}x - \frac{1}{4}\right)e^{2x}\right]_{-2}^0 = -\frac{1}{4} - \left(-\frac{5}{4}e^{-4}\right) = \boxed{\frac{5}{4e^4} - \frac{1}{4}}$$

$$\int x e^{2x} dx = \frac{1}{2}x\cdot e^{2x} - \int \frac{1}{2}e^{2x} dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c = \left(\frac{1}{2}x - \frac{1}{4}\right)e^{2x} + c$$

$f(x) = x$ PER PARTI

$f'(x) = 1$ $g'(x) = e^{2x}$ $g(x) = \frac{1}{2}e^{2x}$

c)



$$\begin{cases} y = -x+1 \\ x = y^2-1 \end{cases} \to y = -y^2+2 \to y^2+y-2=0 \quad \begin{matrix} \nearrow y_1 = -2 \\ \searrow y_2 = 1 \end{matrix}$$

$(3,-2)$ $(0,1)$

NORM. risp. a y $E_y = \{(x,y)\in\mathbb{R}^2 : -2\le y\le 1, y^2-1\le x\le 1-y\}$

$$\text{area } E = \int_E 1\, dx dy = \int_{-2}^1 \left(\int_{y^2-1}^{1-y} dx\right) dy = \int_{-2}^1 (\underbrace{1-y-y^2+1}_{-y^2-y+2}) dy = \left[-\frac{y^3}{3} - \frac{y^2}{2} + 2y\right]_{-2}^1 = -\frac{1}{3} - \frac{1}{2} + 2 - \left(\frac{8}{3} - 2 - 4\right) =$$

$$= 8 - 3 - \frac{1}{2} = \frac{9}{2}$$

$$\boxed{\text{area } E = \frac{9}{2}}$$

$$x_B = \frac{1}{\text{area}E}\int_E x\,dx\,dy = \frac{2}{9}\int_{-2}^{1}\left(\int_{y^2-1}^{1-y} x\,dx\right)dy = \frac{2}{9}\int_{-2}^{1}\frac{1}{2}\left[x^2\right]_{y^2-1}^{1-y}dy =$$

$$= \frac{1}{9}\int_{-2}^{1}\underbrace{\left[(1-y)^2-(y^2-1)^2\right]}_{1-2y+y^2-y^4+2y^2-1}dy = \frac{1}{9}\int_{-2}^{1}(-y^4+3y^2-2y)\,dy = \frac{1}{9}\left[-\frac{y^5}{5}+y^3-y^2\right]_{-2}^{1} = \frac{1}{9}\left(-\frac{1}{5}+\cancel{1}-\cancel{1}-\left(\frac{32}{5}-8-4\right)\right) = \frac{1}{9}\left(-\frac{33}{5}+12\right) = \frac{3}{5}$$

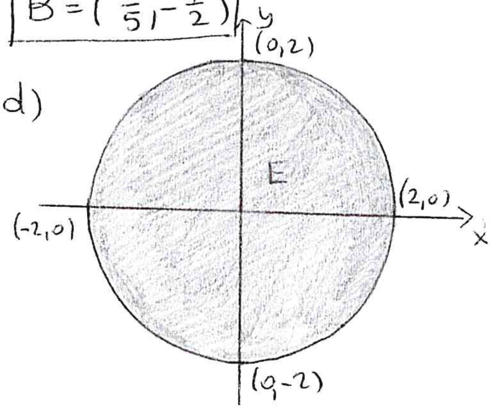
$$y_B = \frac{1}{\text{area}E}\int_E y\,dx\,dy = \frac{2}{9}\int_{-2}^{1}\left(\int_{y^2-1}^{1-y}(y)\,dx\right)dy = \frac{2}{9}\int_{-2}^{1} y\,(1-y-y^2+1)\,dy =$$

$$= \frac{2}{9}\int_{-2}^{1}(-y^3-y^2+2y)\,dy = \frac{2}{9}\left[-\frac{y^4}{4}-\frac{y^3}{3}+y^2\right]_{-2}^{1} = \frac{2}{9}\left(-\frac{1}{4}-\frac{1}{3}+1-\left(-\cancel{4}+\frac{8}{3}+\cancel{4}\right)\right) =$$

$$= \frac{2}{9}\left(-\frac{1}{4}+\underbrace{1-3}_{-2}\right) = \frac{2}{9}\cdot\left(-\frac{9}{4}\right) = -\frac{1}{2}$$

$$\boxed{B = \left(\frac{3}{5}, -\frac{1}{2}\right)}$$

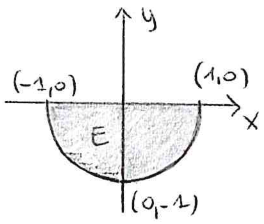
d)



$$\int_E \left(2-\sqrt{x^2+y^2}\right)dx\,dy \underset{\text{COORD. POLARI}}{=} \int_0^{2\pi}\left(\int_0^2 \underbrace{(2-\rho)\rho}_{-\rho^2+2\rho}\,d\rho\right)d\theta =$$

$$= \int_0^{2\pi}\left(\left[-\frac{\rho^3}{3}+\rho^2\right]_0^2\right)d\theta = \left(-\frac{8}{3}+4\right)\cdot\int_0^{2\pi}d\theta = 2\pi\left(\frac{4}{3}\right) = \boxed{\frac{8}{3}\pi}$$

e)

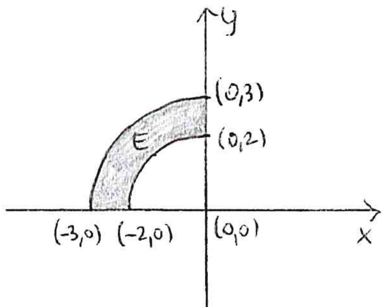


$$\int_E x^2 y\,dx\,dy \underset{\text{COORD. POLARI}}{=} \int_{\pi}^{2\pi}\left(\int_0^1 \rho^3\cos^2\theta\,\text{sen}\theta\,\rho\,d\rho\right)d\theta$$

$$= \int_{\pi}^{2\pi}\text{sen}\theta(\cos\theta)^2\,d\theta\cdot\int_0^1\rho^4\,d\rho = \left[-\frac{(\cos\theta)^3}{3}\right]_{\pi}^{2\pi}\cdot\left[\frac{\rho^5}{5}\right]_0^1 = \left(-\frac{1}{3}-\left(\frac{1}{3}\right)\right)\cdot\frac{1}{5} = \boxed{-\frac{2}{15}}$$

su E $f(x,y) = x^2\cdot y$ è ≤ 0.

f)



$$\int_E y^2\,dx\,dy \underset{\text{COORD. POLARI}}{=} \int_{\frac{\pi}{2}}^{\pi}\left(\int_2^3 \rho^2\,\text{sen}^2\theta\,\rho\,d\rho\right)d\theta =$$

$$= \int_{\pi/2}^{\pi}\text{sen}^2\theta\,d\theta\cdot\int_2^3\rho^3\,d\rho = \left[\frac{1}{2}\theta-\frac{1}{2}\text{sen}\theta\cos\theta\right]_{\frac{\pi}{2}}^{\pi}\cdot\left[\frac{\rho^4}{4}\right]_2^3 =$$

$$= \left(\frac{\pi}{2}-\frac{\pi}{4}\right)\cdot\left(\frac{81}{4}-\frac{16}{4}\right) = \frac{65}{4}\cdot\frac{\pi}{4} = \frac{65}{16}\pi$$

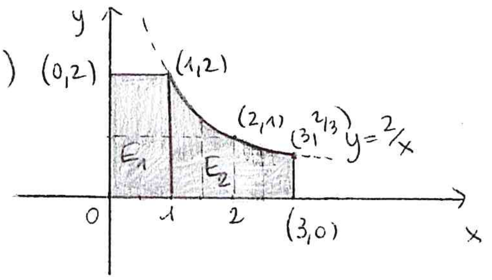
$$\int \text{sen}^2\theta\,d\theta \underset{\text{PER PARTI}}{=} \int \text{sen}\theta\cdot\text{sen}\theta\,d\theta = -\text{sen}\theta\cos\theta + \int\cos^2\theta\,d\theta = -\text{sen}\theta\cos\theta + \int(1-\text{sen}^2\theta)\,d\theta$$

$f(\theta) = \text{sen}\theta$ $f'(\theta) = \cos\theta$

$g'(\theta) = \text{sen}\theta$ $g(\theta) = -\cos\theta$

$$\Rightarrow 2\int\text{sen}^2\theta\,d\theta = \theta - \text{sen}\theta\cos\theta + c$$

$$\Rightarrow \int\text{sen}^2\theta\,d\theta = \frac{1}{2}(\theta - \text{sen}\theta\cos\theta) + c$$

g) 

$0 \le y \le 2$, $0 \le x \le 3$ è un rettangolo

$xy \le 2$ è verificata se $x=0$, se $x>0$

diventa $y \le \frac{2}{x}$ iperbole per $(\frac{1}{2},4)\,(1,2)$

$(\frac{3}{2},\frac{4}{3})\;(2,1)\;(\frac{5}{2},\frac{4}{5})\;(3,\frac{2}{3})$

se $y=2 \to x=1$

$E = E_1 \cup E_2$

$E_{1,x} = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ 0\le y\le 2\}$ rettangolo

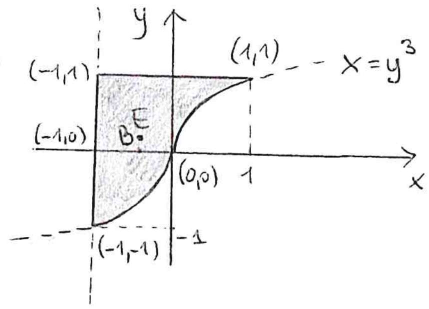
$E_{2,x} = \{(x,y)\in\mathbb{R}^2 : 1\le x\le 3,\ 0\le y\le \frac{2}{x}\}$

$$\int_E \frac{1}{3}x\cdot y\,dxdy = \int_{E_1}\frac{1}{3}x\cdot y\,dxdy + \int_{E_2}\frac{1}{3}x\cdot y\,dxdy = \int_0^1\left(\int_0^2 \frac{1}{3}xy\,dy\right)dx +$$

$$+\int_1^3\left(\int_0^{2/x}\frac{1}{3}xy\,dy\right)dx = \int_0^1 \frac{1}{3}x\left[\frac{y^2}{2}\right]_0^2 dx + \int_1^3 \frac{1}{3}x\left[\frac{y^2}{2}\right]_0^{2/x} dx =$$

$$= \frac{1}{3}\int_0^1 2x\,dx + \frac{1}{3}\int_1^3 x\left[\frac{2}{x^2}\right]dx = \frac{1}{3}[x^2]_0^1 + \frac{2}{3}\int_1^3 \frac{1}{x}\,dx =$$

$$= \frac{1}{3} + \frac{2}{3}[\log x]_1^3 = \boxed{\frac{1}{3}+\frac{2}{3}\log 3}$$

h) 

$E_y = \{(x,y)\in\mathbb{R}^2 : -1\le y\le 1,\ -1\le x\le y^3\}$

NORM rispa y

oppure NORM risp. a x $\quad E_x = \{(x,y)\in\mathbb{R}^2 : -1\le x\le 1,\ \sqrt[3]{x}\le y\le 1\}$

$$\text{area } E = \int_{-1}^1\left(\int_{-1}^{y^3}dx\right)dy = \int_{-1}^1 (y^3+1)\,dy = \left[\frac{y^4}{4}+y\right]_{-1}^1 = \frac{1}{4}+1-\left(\frac{1}{4}-1\right) = 2$$

$$x_B = \frac{1}{2}\int_{-1}^1\left(\int_{-1}^{y^3}x\,dx\right)dy = \frac{1}{2}\int_{-1}^1\left[\frac{x^2}{2}\right]_{-1}^{y^3}dy = \frac{1}{4}\int_{-1}^1 (y^6-1)\,dy = \frac{1}{4}\left[\frac{y^7}{7}-y\right]_{-1}^1 =$$

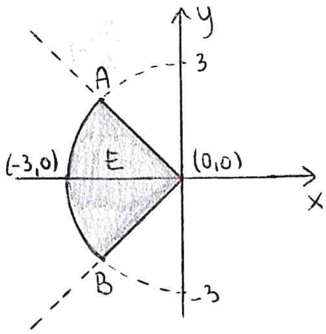
$$= \frac{1}{4}\left[\frac{1}{7}-1-\left(-\frac{1}{7}+1\right)\right] = \frac{1}{4}\left(-\frac{12}{7}\right) = -\frac{3}{7}$$

$$y_B = \frac{1}{2}\int_{-1}^1\left(\int_{-1}^{y^3}y\,dx\right)dy = \frac{1}{2}\int_{-1}^1 y(y^3+1)\,dy =$$

$$= \frac{1}{2}\int_{-1}^1 (y^4+y)\,dy = \frac{1}{2}\left[\frac{y^5}{5}+\frac{y^2}{2}\right]_{-1}^1 = \frac{1}{2}\left[\frac{1}{5}+\frac{1}{2}-\left(-\frac{1}{5}+\frac{1}{2}\right)\right] = \frac{1}{5}$$

$B = (-\frac{3}{7}, \frac{1}{5})$

i)



$A=\left(-3\frac{\sqrt{2}}{2},3\frac{\sqrt{2}}{2}\right)$

$B=\left(-3\frac{\sqrt{2}}{2},-3\frac{\sqrt{2}}{2}\right)$

$$\int_E x^2\,dx\,dy \underset{\text{COORD POLARI}}{=} \int_{\frac{3}{4}\pi}^{\frac{5}{4}\pi}\left(\int_0^3 \rho^2\cos^2\theta\cdot\rho\,d\rho\right)d\theta =$$

$E_{\rho,\theta}$: $0\le\rho\le3$, $\frac{3}{4}\pi\le\theta\le\frac{5}{4}\pi$

$$=\int_{\frac{3}{4}\pi}^{\frac{5}{4}\pi}\cos^2\theta\,d\theta\cdot\int_0^3\rho^3\,d\rho=\left[\frac{1}{2}\theta+\frac{1}{2}\operatorname{sen}\theta\cos\theta\right]_{\frac{3}{4}\pi}^{\frac{5}{4}\pi}\cdot\left[\frac{\rho^4}{4}\right]_0^3=$$

$$=\left[\frac{\pi}{4}+\frac{1}{2}\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}+\frac{1}{2}\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\right]\cdot\frac{81}{4}=\left(\frac{\pi}{4}+\frac{1}{2}\right)\cdot\frac{81}{4}$$

$\frac{1}{2}\left[\frac{5}{4}\pi-\frac{3}{4}\pi\right]$

$$=\boxed{\frac{81}{16}\pi+\frac{81}{8}}$$

$$\int\cos^2\theta\,d\theta \underset{\text{PER PARTI}}{=} \operatorname{sen}\theta\cos\theta+\int\operatorname{sen}^2\theta\,d\theta=$$

$f(\theta)=\cos\theta$ $\quad f'(\theta)=-\operatorname{sen}\theta$

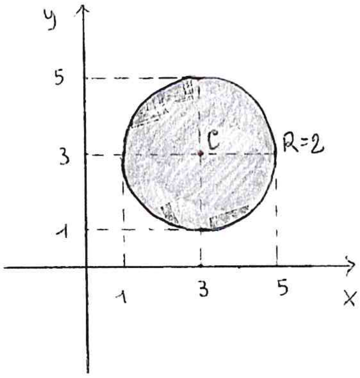
$g'(\theta)=\cos\theta$ $\quad g(\theta)=\operatorname{sen}\theta$

$$=\operatorname{sen}\theta\cos\theta+\theta-\int\cos^2\theta\,d\theta$$

$$2\int\cos^2\theta\,d\theta=\theta+\operatorname{sen}\theta\cos\theta+c$$

$$\int\cos^2\theta=\frac{1}{2}(\theta+\operatorname{sen}\theta\cos\theta)+c$$

j)



E è il CERCHIO di C(3,3) e R=2

Con il CAMB. di VARIABILE a COORDINATE POLARI CENTRATE in (3,3) $\begin{cases}x=3+\rho\cos\theta\\ y=3+\rho\operatorname{sen}\theta\end{cases}$ diventa

$E_{\rho,\theta}$: $0\le\rho\le2$, $0\le\theta\le2\pi$

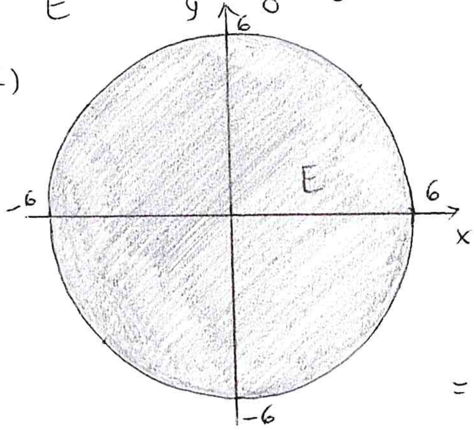
1° modo

$$\int_E 2\,dx\,dy=2\int_E dx\,dy=2\text{ area }E=2(\pi\cdot2^2)=\boxed{8\pi}$$ oppure

2° modo (COORD POL)

$$2\int_E dx\,dy=2\int_0^{2\pi}\left(\int_0^2\rho\,d\rho\right)d\theta=2\cdot\int_0^{2\pi}d\theta\cdot\left[\frac{\rho^2}{2}\right]_0^2=2\cdot2\pi\cdot2=\boxed{8\pi}$$

k)



E = cerchio C(0,0) R=6

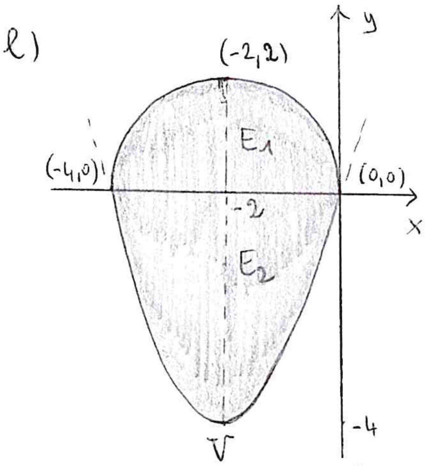
$$\int_E\sqrt{36-x^2-y^2}\,dx\,dy \underset{\text{COORD. POLARI}}{=} \int_0^{2\pi}\int_0^6\sqrt{36-\rho^2}\,\rho\,d\rho=$$

$$=2\pi\cdot\left(-\frac{1}{2}\right)\int_0^6(-2\rho)(36-\rho^2)^{1/2}d\rho=$$

$$=-\pi\left[\frac{(36-\rho^2)^{3/2}}{3/2}\right]_0^6=-\frac{2}{3}\pi\left[0-36^{3/2}\right]=$$

$$=-\frac{2}{3}\pi(-6^3)=\frac{2}{3}\pi\cdot\overset{72}{216}=\boxed{144\pi}$$

l)



E : sopra la parabola e sotto la semicirconf.

$y = x^2 + 4x$ parabola verso l'alto

$V(-2,-4)$ $\cap$ asse x $x=0, x=-4$

$y = \sqrt{-x^2-4x} \to y^2 = -x^2-4x \quad x^2+y^2+4x=0$

$-x^2-4x \geq 0 \quad +x^2+4x \leq 0 \quad x \in [-4,0]$

$y \geq 0$

$(x+2)^2 + y^2 = 4$ circonf $C(-2,0)$ e $R=2$

$y = \sqrt{-x^2-4x} = \sqrt{4-(x+2)^2}$ è la metà superiore della circonferenza

$E = E_1 \cup E_2 \quad E_1 = \{(x,y) \in \mathbb{R}^2 : (x+2)^2+y^2 \leq 4, \ y \geq 0\}$

$E_{2,x} = \{(x,y) \in \mathbb{R}^2 : -4 \leq x \leq 0, \ x^2+4x \leq y \leq 0\}$

$\boxed{\text{area } E} = \text{area } E_1 + \text{area } E_2 = \frac{1}{2}\pi \cdot 4 + \int_{-4}^{0} \left(\int_{x^2+4x}^{0} dy \right) dx = 2\pi + \int_{-4}^{0} (-x^2-4x)\, dx =$

$= 2\pi + \left[-\frac{x^3}{3} - 2x^2 \right]_{-4}^{0} = 2\pi + \left[0 - \left(\frac{64}{3} - 32 \right) \right] = \boxed{2\pi + \frac{32}{3}}$

$\underline{\int_E x \, dx dy} = \int_{E_1} x\, dxdy + \int_{E_2} x\, dx dy = \int_0^2 \int_0^\pi (-2+\rho\cos\theta)\rho \, d\rho d\theta + \int_{-4}^{0} \left(\int_{x^2+4x}^{0} x\, dy \right) dx =$

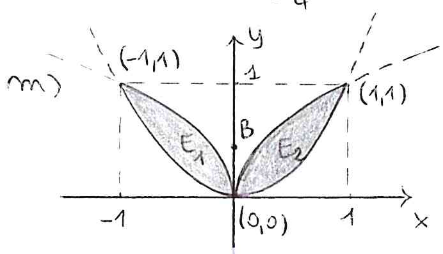
COORD POLARI centrate in $(-2,0)$ $\begin{cases} x = -2+\rho\cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$

$= -\pi \int_0^2 2\rho \, d\rho + \int_0^2 \rho^2 d\rho \cdot \underbrace{\int_0^\pi \cos\theta \, d\theta}_{= [\operatorname{sen}\theta]_0^\pi = 0} + \int_{-4}^{0} \underbrace{x(-x^2-4x)}_{-x^3-4x^2} dx = -\pi \left[\rho^2 \right]_0^2 + 0 +$

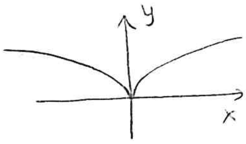
$+ \left[-\frac{x^4}{4} - \frac{4}{3}x^3 \right]_{-4}^{0} = -4\pi + \left[0 - \left(-64 + \frac{256}{3} \right) \right] = \boxed{-4\pi - \frac{64}{3}} \to x_B = \frac{-4\pi - \frac{64}{3}}{2\pi + \frac{32}{3}} = -2$

$64 - \frac{256}{3}$

m)



$y = \sqrt{|x|} \to \text{dom} = \mathbb{R} \qquad y = \sqrt{|x|} = \begin{cases} \sqrt{x} & x \geq 0 \\ \sqrt{-x} & x \leq 0 \end{cases}$



$E = E_1 \cup E_2 \quad E_1 = E \cap x \leq 0 \quad E_2 = E \cap x \geq 0$

E è simmetrico rispetto all'asse y, il baricentro ha sicuramente $x_B = 0$ e $0 < y_B < 1$. $\boxed{\text{area } E} = 2\int_0^1 \left(\int_{x^2}^{\sqrt{x}} dy \right) dx = 2\int_0^1 (\sqrt{x} - x^2)\, dx =$

$= 2\left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^1 = 2\left(\frac{2}{3} - \frac{1}{3} \right) = \boxed{\frac{2}{3}}$ $\qquad \int_E x\, dxdy = 0$ per simmetria da cui $\boxed{x_B = 0}$:

infatti $f(x,y) = x$ assume valori di segno opposto su E_1 e E_2 $\to \int_{E_1} x\, dxdy = -\int_{E_2} x\, dxdy$

$\to \int_{E_1} x\, dxdy + \int_{E_2} x\, dxdy = 0 \to \int_E x\, dxdy = 0.$

Con il calcolo $\int_E x\,dx\,dy = \int_{E_1} x\,dx\,dy + \int_{E_2} x\,dx\,dy =$

$$= \int_{-1}^{0}\left(\int_{x^2}^{\sqrt{-x}} x\,dy\right)dx + \int_0^1\left(\int_{x^2}^{\sqrt{x}} x\,dy\right)dx = \int_{-1}^0 (\underbrace{x\cdot\sqrt{-x}}_{-(-x)^{3/2}} - x^3)\,dx + \int_0^1 (\underbrace{x\sqrt{x}}_{x^{3/2}} - x^3)\,dx =$$

$$= \left[\frac{(-x)^{5/2}}{5/2}\right]_{-1}^0 - \left[\frac{x^4}{4}\right]_{-1}^0 + \left[\frac{x^{5/2}}{5/2}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 = 0 - \frac{2}{5} - \left[0-\frac14\right] + \frac25 - \frac14 = \boxed{0}$$

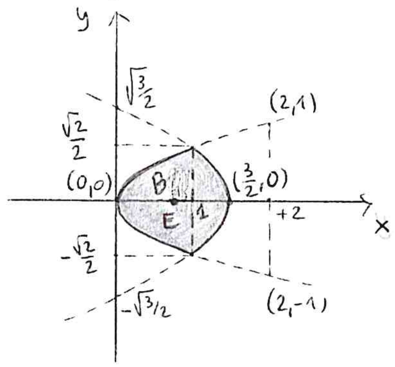
$$\boxed{y_B} = \frac{1}{\text{area}\,E}\int_E y\,dx\,dy = \frac{2}{\text{area}\,E}\cdot\int_{E_2} y\,dx\,dy = 3\int_0^1\left(\int_{x^2}^{\sqrt{x}} y\,dy\right)dx = 3\int_0^1 \left[\frac{y^2}{2}\right]_{x^2}^{\sqrt{x}} dx =$$

per simmetria $\int_{E_1} y\,dx\,dy = \int_{E_2} y\,dx\,dy$ perchè $f(x,y)=y$ assume gli stessi valori su E_1 e E_2

$B(0, \frac{9}{20})$

$$= \frac32 \int_0^1 (x-x^4)\,dx = \frac32\left[\frac{x^2}{2}-\frac{x^5}{5}\right]_0^1 = \frac32\left[\frac12-\frac15\right] = \frac32\cdot\frac{3}{10} = \boxed{\frac{9}{20}}$$

n)



$x=2y^2$ parabola di asse x $V(0,0)$ per $(2,1)$ e $(2,-1)$

$3-2x=2y^2 \to 2x = 3-2y^2 \quad x=-y^2+\frac32$ parabola di asse x $V(0,\frac32)$ per $(0,\pm\sqrt{\frac32})$

$$\begin{cases} x=2y^2 \\ x=-y^2+\frac32\end{cases} \to 2y^2=-y^2=\frac32 \quad 3y^2=\frac32 \quad y^2=\frac12 \quad \boxed{\begin{matrix} y=\pm\frac{\sqrt2}{2} \\ x=1\end{matrix}}$$

Condizioni : $x \geq 2y^2$, $x \leq -y^2+\frac32$ $\qquad E_y=\left\{(x,y)\in\mathbb{R}^2 : 2y^2 \leq x \leq -y^2+\frac32,\ -\frac{\sqrt2}{2} \leq y \leq \frac{\sqrt2}{2}\right\}$

NORM. risp. a y

$$\boxed{\text{area}\,E} = \int_{-\frac{\sqrt2}{2}}^{\sqrt2/2}\left(\int_{2y^2}^{-y^2+3/2} dx\right)dy = \int_{-\frac{\sqrt2}{2}}^{\frac{\sqrt2}{2}} (\underbrace{-y^2+\frac32-2y^2}_{-3y^2+\frac32})\,dy = \left[-y^3+\frac32 y\right]_{-\frac{\sqrt2}{2}}^{\sqrt2/2} = -\frac12\frac{\sqrt2}{2}+\frac32\frac{\sqrt2}{2} -$$

$$-\left(\frac12\frac{\sqrt2}{2}-\frac32\frac{\sqrt2}{2}\right) = -\frac{\sqrt2}{4}+\frac34\sqrt2-\frac{\sqrt2}{4}+\frac34\sqrt2 = \boxed{\sqrt2}$$

oppure per SIMMETRIA (E è SIMMETRICO rispetto all'asse x) area $E = 2\int_0^{\sqrt2/2}\left(\int_{2y^2}^{-y^2+3/2} dx\right)dy$

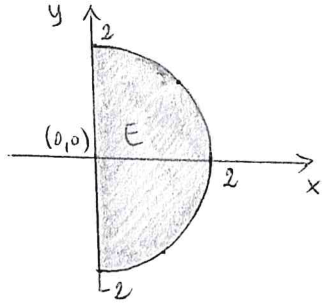
$$\boxed{x_B} = \frac{1}{\sqrt2}\cdot 2\int_0^{\sqrt2/2}\left(\int_{2y^2}^{-y^2+3/2} x\,dx\right)dy =$$

$$= \sqrt2\int_0^{\sqrt2/2}\left[\frac{x^2}{2}\right]_{2y^2}^{-y^2+\frac32} dy = \frac{\sqrt2}{2}\int_0^{\sqrt2/2}\left(y^4-3y^2+\frac94-4y^4\right)dy = \frac{\sqrt2}{2}\left[-3\frac{y^5}{5}-y^3+\frac94 y\right]_0^{\frac{\sqrt2}{2}} =$$

$$= \frac{\sqrt2}{2}\left[-\frac35\cdot\frac{\cancel{4}\sqrt2}{\cancel{32}_{8}} - \frac{2\sqrt2}{8}+\frac98\sqrt2\right] = \frac{\sqrt2}{2}\left[-\frac{3}{40}-\frac28+\frac98\right]\cdot\sqrt2 = \left(\frac{-3-10+45}{40}\right) = \frac{32}{40} = \boxed{\frac45}$$

Per simmetria $\boxed{y_B=0}$ $\qquad B=(\frac45;0)$

o)



$x=\sqrt{4-y^2}$ è la metà destra della circonferenza di $C(0,0)$ $R=2$

$$\int_E e^{\sqrt{x^2+y^2}}\,dxdy \underset{\text{COORD POLARI}}{=} \int_{-\pi/2}^{\pi/2}\left(\int_0^2 e^{\rho}\cdot\rho\, d\rho\right)d\theta =$$

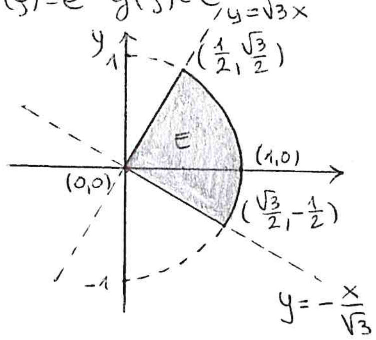
$$= \pi\cdot\left[(\rho-1)e^{\rho}\right]_0^2 = \boxed{\pi\left[e^2+1\right]}$$

$$\int \rho e^{\rho}\,d\rho \underset{\text{PER PARTI}}{=} \rho e^{\rho}-\int e^{\rho}\,d\rho = (\rho-1)e^{\rho}+c$$

$f(\rho)=\rho$ $f'(\rho)=1$

$g'(\rho)=e^{\rho}$ $g(\rho)=e^{\rho}$

p)



$y=-\frac{x}{\sqrt{3}}$ è la retta per l'origine di coeff. angolare $m=-\frac{1}{\sqrt{3}}=\tan\left(-\frac{\pi}{6}\right)$

$y=\sqrt{3}x$ è la retta per l'origine di coeff. angolare $m=\sqrt{3}=\tan\left(\frac{\pi}{3}\right)$

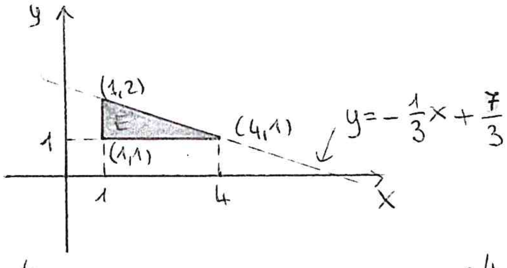
$$\int_E \frac{x}{1+x^2+y^2}\,dxdy = \int_{-\pi/6}^{\pi/3}\left(\int_0^1 \frac{\rho\cos\theta}{1+\rho^2}\rho\,d\rho\right)d\theta = \int_{-\pi/6}^{\pi/3}\cos\theta\,d\theta\cdot\int_0^1\frac{\rho^2}{1+\rho^2}\,d\rho =$$

$$= \left[\operatorname{sen}\theta\right]_{-\pi/6}^{\pi/3}\cdot\left[\rho-\arctan\rho\right]_0^1 = \left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\left(1-\arctan 1\right) = \boxed{\left(\frac{\sqrt{3}+1}{2}\right)\cdot\left(1-\frac{\pi}{4}\right)}$$

$\arctan 0 = 0$

$$\int\frac{\rho^2}{1+\rho^2}\,d\rho = \int\frac{1+\rho^2-1}{1+\rho^2}\,d\rho = \int\left(1-\frac{1}{1+\rho^2}\right)d\rho = \rho-\arctan\rho+c$$

q)



NORM. risp. a x

$$E_x=\left\{(x,y)\in\mathbb{R}^2:\ 1\le x\le 4,\ 1\le y\le -\tfrac{1}{3}x+\tfrac{7}{3}\right\}$$

$$\int_E xy\,dxdy = \int_1^4\left(\int_1^{-\frac{1}{3}x+\frac{7}{3}} xy\,dy\right)dx = \int_1^4 x\left[\frac{y^2}{2}\right]_1^{-\frac{1}{3}x+\frac{7}{3}}dx =$$

$$= \int_1^4\frac{x}{2}\Big[\underbrace{\left(-\tfrac{1}{3}x+\tfrac{7}{3}\right)^2}_{\frac{1}{9}x^2-\frac{14}{9}x+\frac{49}{9}}-1\Big]dx = \frac{1}{18}\int_1^4\left(x^3-14x^2+40x\right)dx = \frac{1}{18}\left[\frac{x^4}{4}-\frac{14}{3}x^3+20x^2\right]_1^4 =$$

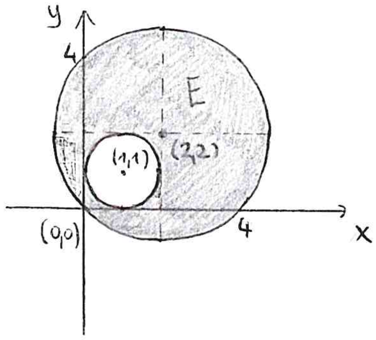
$$= \frac{1}{18}\left[64-\frac{896}{3}+320-\frac{1}{4}+\frac{14}{3}-20\right] = \frac{1}{18}\left(364-\frac{1}{4}-\frac{882}{3}\right) = \frac{1}{\cancel{18}_2}\cdot\frac{\cancel{837}\,93}{12} = \frac{93}{24} = \boxed{\frac{31}{8}}$$

Il calcolo con $E_y=\left\{(x,y)\in\mathbb{R}^2:\ 1\le y\le 2,\ 1\le x\le -3y+7\right\}$ NORM. risp. a y risulta più semplice: $\int_E xy\,dxdy = \int_1^2\left(\int_1^{-3y+7} xy\,dx\right)dy = \int_1^2 y\left[\frac{x^2}{2}\right]_1^{-3y+7}dy =$

$$= \frac{1}{2}\int_1^2 \underbrace{y\,(9y^2-42y+49-1)}_{9y^3-42y^2+48y}\,dy = \frac{1}{2}\left[9\frac{y^4}{4}-14y^3+24y^2\right]_1^2 =$$

$$= \frac{1}{2}\left[36-112+96-\frac{9}{4}+14-24\right] = \frac{1}{2}\left(10-\frac{9}{4}\right) = \frac{1}{2}\,\frac{31}{4} = \boxed{\frac{31}{8}}$$

c)



E è il CERCHIO di $C(2,2)$ $R=2\sqrt{2}$ (passa per $(0,0)$)

privato del cerchio di $C(1,1)$ $R=1$

$E = E_1 - E_2$ $\quad E_1: (x-2)^2+(y-2)^2 \le 8$

$\qquad\qquad\quad E_2: (x-1)^2+(y-1)^2 \le 1$

$$\int_E x^2\,dx\,dy = \int_{E_1} x^2\,dx\,dy - \int_{E_2} x^2\,dx\,dy = 48\pi - \frac{5}{4}\pi = \boxed{\frac{187}{4}\pi}$$

$$\int_{E_1} x^2\,dx\,dy \underset{\substack{\text{CAMB} \\ \text{a var POLARI}}}{=} \int_0^{2\pi}\int_0^{2\sqrt{2}} (2+\rho\cos\vartheta)^2\rho\,d\rho\,d\vartheta = \int_0^{2\pi}\int_0^{2\sqrt{2}} (4\rho+\rho^3\cos^2\vartheta+4\rho^2\cos\vartheta)\,d\rho\,d\vartheta =$$

$$\begin{cases} x = 2+\rho\cos\vartheta \\ y = 2+\rho\,\text{sen}\,\vartheta \end{cases}$$

$$= 2\pi\left[2\rho^2\right]_0^{2\sqrt{2}} + \int_0^{2\pi}\cos^2\vartheta\,d\vartheta \cdot \int_0^{2\sqrt{2}}\rho^3\,d\rho + 4\underbrace{\int_0^{2\pi}\cos\vartheta\,d\vartheta}_{0} \cdot \int_0^{2\sqrt{2}}\rho^2\,d\rho =$$

$$= 32\pi + \left[\frac{1}{2}\vartheta + \frac{1}{2}\,\text{sen}\,\vartheta\cos\vartheta\right]_0^{2\pi}\left[\frac{\rho^4}{4}\right]_0^{2\sqrt{2}} = 32\pi + \pi\,\frac{64}{4} = 48\pi$$

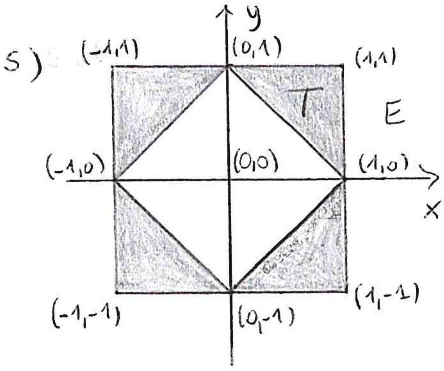
$$\int_{E_2} x^2\,dx\,dy \underset{\substack{\text{CAMB} \\ \text{a VAR. POLARI}}}{=} \int_0^{2\pi}\int_0^{1} (1+\rho\cos\vartheta)^2\rho\,d\rho\,d\vartheta = \int_0^{2\pi}\int_0^{1} (\rho+\rho^3\cos^2\vartheta+2\rho^2\cos\vartheta)\,d\rho\,d\vartheta =$$

$$\begin{cases} x = 1+\rho\cos\vartheta \\ y = 1+\rho\,\text{sen}\,\vartheta \end{cases}$$

$$= 2\pi\left[\frac{\rho^2}{2}\right]_0^1 + \int_0^{2\pi}\cos^2\vartheta\,d\vartheta \cdot \int_0^1\rho^3\,d\rho + 2\underbrace{\int_0^{2\pi}\cos\vartheta\,d\vartheta}_{0} \cdot \int_0^1\rho^2\,d\rho =$$

$$= \pi + \pi\cdot\frac{1}{4} = \frac{5}{4}\pi$$

s)



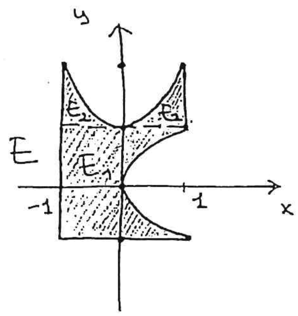
E è unione di 4 triangoli, su ciascuno dei quali per simmetria la funzione $f(x,y)=x^2$ assume gli stessi valori.

Detto $T_x = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ 1-x\le y\le 1\}$

risulta $\int_E x^2\,dx\,dy = 4\int_T x^2\,dx\,dy = 4\int_0^1\left(\int_{1-x}^1 x^2\,dy\right)dx = 4\int_0^1 x^2(1-(1-x))\,dx =$

$= 4\int_0^1 x^3\,dx = \left[x^4\right]_0^1 = \boxed{1}$

t) L'insieme E non è normale né rispetto a x, né rispetto a y. Tuttavia possiamo scrivere $E=E_1\cup E_2$



con $E_{1,y} = \{(x,y) : -1\le y\le 1,\ -1\le x\le y^2\}$

$E_{2,x} = \{(x,y) : -1\le x\le 1,\ 1\le y\le 1+x^2\}$.

E_1 è normale rispetto a y, mentre E_2 è normale rispetto a x. Allora possiamo scrivere

$$\int_E x^2y\,dx\,dy = \int_{E_1} x^2y\,dx\,dy + \int_{E_2} x^2y\,dx\,dy = \int_{-1}^1\left(\int_{-1}^{y^2} x^2y\,dx\right)dy + \int_{-1}^1\left(\int_1^{1+x^2} x^2y\,dy\right)dx =$$

$$= \int_{-1}^1 y\left[\frac{x^3}{3}\right]_{-1}^{y^2} dy + \int_{-1}^1 x^2\left[\frac{y^2}{2}\right]_1^{(1+x^2)} dx = \frac{1}{3}\int_{-1}^1 (y^7+y)\,dy + \frac{1}{2}\int_{-1}^1 (x^6+2x^4)\,dx =$$

$$= \frac{1}{3}\left[\frac{y^8}{8}+\frac{y^2}{2}\right]_{-1}^1 + \frac{1}{2}\left[\frac{x^7}{7}+2\frac{x^5}{5}\right]_{-1}^1 = 0 + \left(\frac{1}{7}+\frac{2}{5}\right) = \boxed{\frac{19}{35}}$$

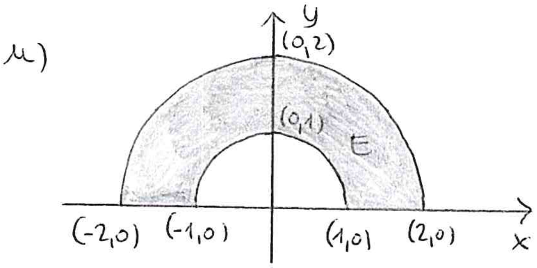
Abbiamo ottenuto che $\int_{E_1} x^2y\,dx\,dy = 0$.

Infatti E_1 è simmetrico rispetto all'asse x e la funzione $f(x,y)=x^2y$ assume valori di segno opposto sulla parte di E_1 con $y\ge 0$ e su quella con $y\le 0$. Allora $\int_{E_1\cap\{y\le 0\}} x^2y\,dx\,dy = -\int_{E_1\cap\{y\ge 0\}} x^2y\,dx\,dy$ per cui $\int_{E_1} x^2y\,dx\,dy = 0$.

Per simmetria si poteva semplificare anche il calcolo dell' $\int_{E_2}$: $f(x,y)=x^2y$ assume gli stessi valori su $E_2\cap\{x\ge 0\}$ e su $E_2\cap\{x\le 0\}$ $\Rightarrow$

$$\int_{E_2} x^2y\,dx\,dy = 2\int_{E_2\cap\{x\ge 0\}} x^2y\,dx\,dy.$$

u)



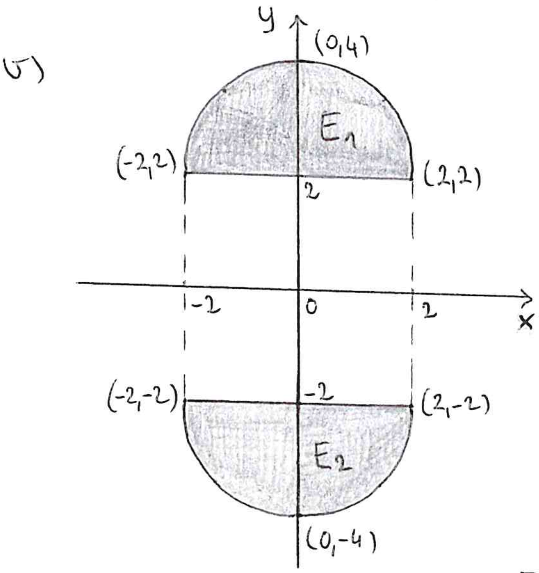
$$\int_E (3x+4y^2)\,dx\,dy \underset{\text{COORD. POLARI}}{=}$$

$$=\int_0^\pi \left(\int_1^2 (3\rho\cos\theta+4\rho^2 \operatorname{sen}^2\theta)\rho\,d\rho\right)d\theta =$$

$$=\underbrace{\int_0^\pi \cos\theta\,d\theta}_{=0}\cdot\int_1^2 3\rho^2\,d\rho + \int_0^\pi \operatorname{sen}^2\theta\,d\theta\cdot\int_1^2 4\rho^3\,d\rho =$$

$$=\left[\frac12\theta-\frac12\operatorname{sen}\theta\cos\theta\right]_0^\pi\cdot\left[\rho^4\right]_1^2 = \frac{\pi}{2}\cdot(16-1)=\boxed{\frac{15}{2}\pi}$$

v)



$y=2+\sqrt{4-x^2}$ è la metà superiore della circonferenza di $C(0,2)$ e $R=2$

Se $y\geq 0$ la condizione significa

$$2\leq y\leq 2+\sqrt{4-x^2} \longrightarrow E_1$$

Se $y\leq 0$ la condizione diventa

$$2\leq -y\leq 2+\sqrt{4-x^2} \quad \text{ossia}$$

$$-2-\sqrt{4-x^2}\leq y\leq -2 \longrightarrow E_2$$

$E=E_1\cup E_2$

$y=-2-\sqrt{4-x^2}$ è la metà inferiore della circonferenza di $C(0,-2)$ e $R=2$

$$\int_E y^2\,dx\,dy \underset{\substack{\text{PER}\\ \text{SIMMETRIA}}}{=} 4\int_{E_1\cap x\geq 0} y^2\,dx\,dy \underset{\substack{\text{COORD}\\ \text{POLARI}}}{=} 4\int_0^{\pi/2}\left(\int_0^2 \underbrace{(2+\rho\operatorname{sen}\theta)^2}_{(4+\rho^2\operatorname{sen}^2\theta+4\rho\operatorname{sen}\theta)}\rho\,d\rho\right)d\theta =$$

con centro $(0,2)$
$$\begin{cases} x=\rho\cos\theta \\ y=2+\rho\operatorname{sen}\theta \end{cases}$$

$$=4\left[\frac{\pi}{2}\cdot\int_0^2 4\rho\,d\rho+\int_0^{\pi/2}\operatorname{sen}^2\theta\,d\theta\cdot\int_0^2\rho^3\,d\rho+\int_0^{\pi/2}\operatorname{sen}\theta\,d\theta\cdot\int_0^2 4\rho^2\,d\rho\right]=$$

$$=4\left[\frac{\pi}{2}\left[2\rho^2\right]_0^2+\left[\frac12\theta-\frac12\operatorname{sen}\theta\cos\theta\right]_0^{\pi/2}\cdot\left[\frac{\rho^4}{4}\right]_0^2+\left[-\cos\theta\right]_0^{\pi/2}\cdot\left[4\frac{\rho^3}{3}\right]_0^2\right]=$$

$$=4\left[4\pi+\frac{\pi}{4}\cdot 4+1\cdot\frac{32}{3}\right]=\boxed{20\pi+\frac{128}{3}}$$

area $E = \pi R^2 = \boxed{4\pi}$

Il Baricentro di E è chiaramente in $\boxed{(0,0)}$.

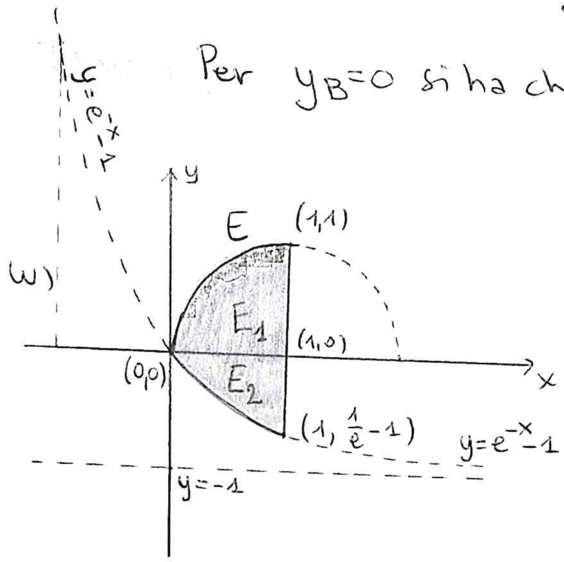
Calcoliamo ad es. $x_B = \frac{1}{4\pi}\int_E x\,dx\,dy$; per simmetria si ha

$$\int_{E_1} x\,dx\,dy = 0 \quad e \quad \int_{E_2} x\,dx\,dy = 0 \quad \Rightarrow \quad \int_E x\,dx\,dy = 0 .$$

Calcoliamo ad es. $\displaystyle\int_{E_1} x\,dx\,dy \underset{\substack{\text{COORD}\\ \text{POLARI}}}{=} \int_0^{\pi}\int_0^{2} (\rho\cos\vartheta)\rho\, d\rho\, d\vartheta = \underbrace{\int_0^{\pi}\cos\vartheta\, d\vartheta}_{=0} \cdot \int_0^{2}\rho^2 d\rho = 0$

$$\begin{cases} x = \rho\cos\vartheta \\ y = 2 + \rho\,\text{sen}\vartheta \end{cases}$$

Per $y_B = 0$ si ha che $\displaystyle\int_{E_2} y\,dx\,dy = -\int_{E_1} y\,dx\,dy$.



$E = E_1 \cup E_2$

$$y = \sqrt{2x - x^2} \quad \to \quad y^2 = 2x - x^2$$
$$y \geq 0 \qquad x^2 + y^2 - 2x = 0$$
$$(x-1)^2 + y^2 = 1$$

quindi $y = \sqrt{2x - x^2}$ è la metà superiore della circonferenza di $C(1,0)$ e $R=1$

area $E_1 = \frac{\pi}{4}$ $\qquad$ area $\displaystyle E_2 = \int_0^1 \left(\int_{e^{-x}-1}^{0} dy\right) dx = \int_0^1 (-e^{-x} + 1)\,dx = \left[e^{-x} + x\right]_0^1 = \frac{1}{e} + 1 - 1 = \frac{1}{e}$

$$E_{2,x} = \left\{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\ e^{-x} - 1 \leq y \leq 0\right\}$$

area E = area E_1 + area E_2 = $\boxed{\frac{\pi}{4} + \frac{1}{e}}$

$$\int_E x\,dx\,dy = \int_{E_1} x\,dx\,dy + \int_{E_2} x\,dx\,dy \underset{\substack{\text{COORD}\\ \text{POLARI}}}{=} \int_{\pi/2}^{\pi}\left(\int_0^1 (1 + \rho\cos\vartheta)\rho\, d\rho\right) d\vartheta + \int_0^1 \left(\int_{e^{-x}-1}^{0} x\, dy\right) dx =$$

$$\begin{cases} x = 1 + \rho\cos\vartheta \\ y = \rho\,\text{sen}\vartheta \end{cases}$$

$$= \frac{\pi}{2}\int_0^1 \rho\, d\rho + \int_{\pi/2}^{\pi}\cos\vartheta\, d\vartheta \cdot \int_0^1 \rho^2 d\rho + \int_0^1 (x - x e^{-x})\,dx = \frac{\pi}{2}\left[\frac{\rho^2}{2}\right]_0^1 + \left[\text{sen}\vartheta\right]_{\pi/2}^{\pi}\left[\frac{\rho^3}{3}\right]_0^1$$

$$+ \left[\frac{x^2}{2} + (x+1)e^{-x}\right]_0^1 = \frac{\pi}{4} - \frac{1}{3} + \frac{1}{2} + \frac{2}{e} - 1 = \boxed{\frac{\pi}{4} + \frac{2}{e} - \frac{5}{6}}$$

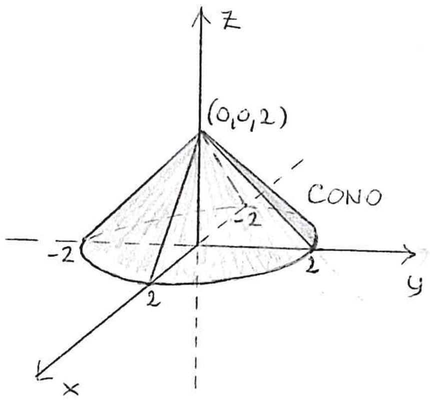
$$\int -x e^{-x}\,dx = \int x(-e^{-x})\,dx \underset{\substack{\text{PER}\\ \text{PARTI}}}{=} x e^{-x} - \int e^{-x} dx = x e^{-x} + e^{-x} + c = (x+1)e^{-x}$$

$f = x \quad f' = 1$

$g' = -e^{-x} \quad g = e^{-x}$

7) d) $\int_E (2-\sqrt{x^2+y^2})\,dx\,dy$ $\quad f(x,y) = 2-\sqrt{x^2+y^2}$ $\quad E: x^2+y^2 \leq 4$

grafico $z = 2-\sqrt{x^2+y^2}$ che è un CONO CIRCOLARE di $V(0,0,2)$, verso il basso, $\hat{ap} = 45°$, $\cap z=0$ su $x^2+y^2=4$ $\quad R=2$

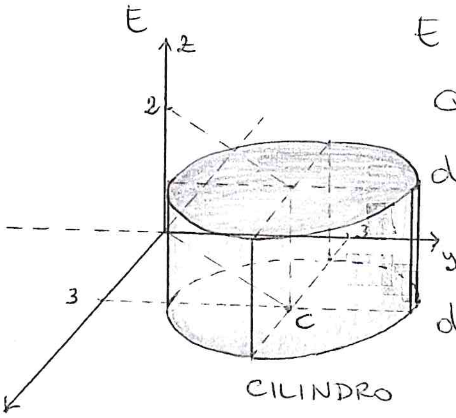


Quindi l'integrale rappresenta il volume del cono (di base $x^2+y^2 \leq 4$ e $h=2$) e vale $VOL = \frac{1}{3}\underbrace{\pi \cdot 4}_{A_{base}} \cdot \underbrace{2}_{h} = \frac{8}{3}\pi$

$\hookrightarrow$ Cono $= \{(x,y,z) \in \mathbb{R}^3 : \underbrace{x^2+y^2 \leq 4}_{(x,y)\in E},\ 0 \leq z \leq 2-\sqrt{x^2+y^2}\}$

j) $\int_E 2\,dx\,dy$ $\quad f(x,y)=2$ grafico $z=2$ piano orizzontale

$E: (x-3)^2+(y-3)^2 \leq 4$



Quindi l'integrale rappresenta il volume del CILINDRO

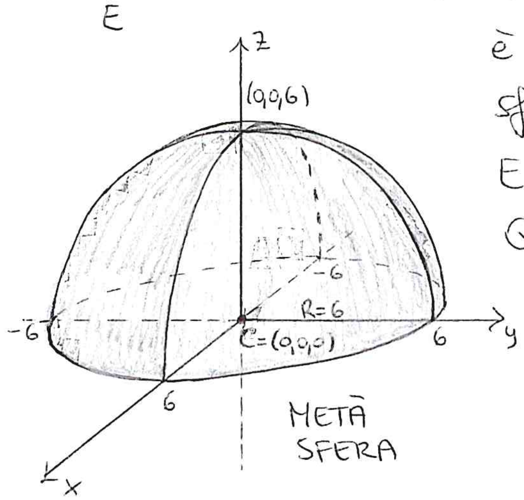
CILINDRO $= \{(x,y,z) \in \mathbb{R}^3 : \underbrace{(x-3)^2+(y-3)^2 \leq 4}_{(x,y)\in E},\ 0 \leq z \leq 2\}$

di base il cerchio $C(3,3)$ e $R=2$ e $h=2$.

$VOL = A_{base} \cdot h = 4\pi \cdot 2 = 8\pi$

k) $\int_E \sqrt{36-x^2-y^2}\,dx\,dy$ $\quad f(x,y) = \sqrt{36-x^2-y^2}$ $\quad$ grafico $z = \sqrt{36-x^2-y^2}$ è la metà superiore della superficie sferica di $C(0,0,0)$ e $R=6$

$E: x^2+y^2 \leq 36$



Quindi l'integrale rappresenta il volume di metà SFERA

metà SFERA $= \{(x,y,z) \in \mathbb{R}^3 : \underbrace{x^2+y^2 \leq 36}_{(x,y)\in E},\ 0 \leq z \leq \sqrt{36-x^2-y^2}\}$

$VOL = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi \cdot 6^3 = \frac{2}{\cancel{3}}\pi \cdot 2 \overset{72}{\cancel{\times 6}} = 144\pi$

es. 6) a) $E_{1,x}=\{(x,y)\in\mathbb{R}^2 : -2\le x\le -1 , -x\le y\le 2\}$

$E_{2,x}=\{(x,y)\in\mathbb{R}^2 : -1\le x\le 1 , 1\le y\le 2\}$

$E_{3,x}=\{(x,y)\in\mathbb{R}^2 : 1\le x\le 2 , x\le y\le 2\}$

c) $E_{1,x}=\{(x,y)\in\mathbb{R}^2 : -1\le x\le 0, -\sqrt{x+1}\le y\le \sqrt{x+1}\}$

$E_{2,x}=\{(x,y)\in\mathbb{R}^2 : 0\le x\le 3, -\sqrt{x+1}\le y\le -x+1\}$

g) $E_{1,y}=\{(x,y)\in\mathbb{R}^2 : 0\le y\le \frac{2}{3} , 0\le x\le 3\}$

$E_{2,y}=\{(x,y)\in\mathbb{R}^2 : \frac{2}{3}\le y\le 2 , 0\le x\le \frac{2}{y}\}$

l) $E_{1,x}=\{(x,y)\in\mathbb{R}^2 : -4\le x\le 0, 0\le y\le \sqrt{-x^2-4x}\}$

$E_{1,y}=\{(x,y)\in\mathbb{R}^2 : 0\le y\le 2 , -2-\sqrt{4-y^2}\le x\le -2+\sqrt{4-y^2}\}$

$E_{2,y}=\{(x,y)\in\mathbb{R}^2 : -4\le y\le 0, -2-\sqrt{y+4}\le x\le -2+\sqrt{y+4}\}$

parabola $y=x^2+4x=(x+2)^2-4 \rightarrow (x+2)^2=y+4$ $\quad x+2=\pm\sqrt{y+4}$

$x=-2\pm\sqrt{y+4}$ $\quad$ + metà siu $\quad$ − metà ds

circonf $C(-2,0)$ $R=2$ eq.^{ne} $(x+2)^2+y^2=4$ $\quad (x+2)^2=4-y^2$

$x+2=\pm\sqrt{4-y^2}$

$x=-2\pm\sqrt{4-y^2}$

− metà di sinistra

+ metà di destra

m) $E_{1,x}=\{(x,y)\in\mathbb{R}^2 : 0\le x\le 1, -\sqrt{\frac{x}{2}}\le y\le \sqrt{\frac{x}{2}}\}$ $\quad x=2y^2$

$y^2=\frac{x}{2}$ $\quad y=\pm\sqrt{\frac{x}{2}}$

− metà INF + metà SUP

$E_{2,x}=\{(x,y)\in\mathbb{R}^2 : 1\le x\le \frac{3}{2} , -\sqrt{\frac{3}{2}-x}\le y\le \sqrt{\frac{3}{2}-x}\}$

$2y^2=3-2x$ $\quad y^2=\frac{3}{2}-x$ $\quad y=\pm\sqrt{\frac{3}{2}-x}$ $\quad$ − metà INF

+ metà SUP