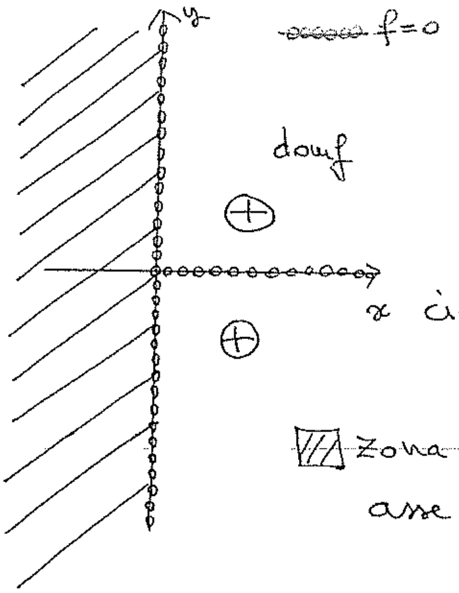
SOLUZIONE SCHEDA N.5

ES.1) a)

$f(x,y)=\sqrt{x}\,\log(1+y^2)$ a) $\mathrm{dom} f = \{(x,y): x \geq 0 \text{ e } \underbrace{1+y^2>0}_{\text{sempre vera } (y^2\geq 0\ \forall y,\ 1+y^2\geq 1)}\} =$

$= \{(x,y): x\geq 0\}$ = Semipiano delle x POSITIVE asse y compreso



$f(x,y)=0 \iff \sqrt{x}=0$ oppure $1+y^2=1$

$\iff x=0$ oppure $y=0$

cioè $f(x,y)=0$ sugli assi (internamente al dominio però)

▨ Zona esclusa dal dom f

asse y compreso nel dom f

$f(x,y)>0 \iff \sqrt{x}\cdot\log(1+y^2)>0 \underset{\substack{\text{essendo}\\ \sqrt{x}\geq 0\ \forall x}}{\iff} \sqrt{x}>0$ e $\log(1+y^2)>0 \iff$

$x>0$ e $1+y^2>1 \iff x>0$ e $y\neq 0$.

Quindi $f(x,y)>0$ in tutti i punti del dominio in cui non è nulla.

b) $\mathrm{dom} f=\{(x,y)\in\mathbb{R}^2: 3y-9x\geq 0,\ 4y\neq 0\}=\{(x,y)\in\mathbb{R}^2: y\geq 3x,\ y\neq 0\}$ SOPRA LA RETTA $y=3x$, asse x escluso

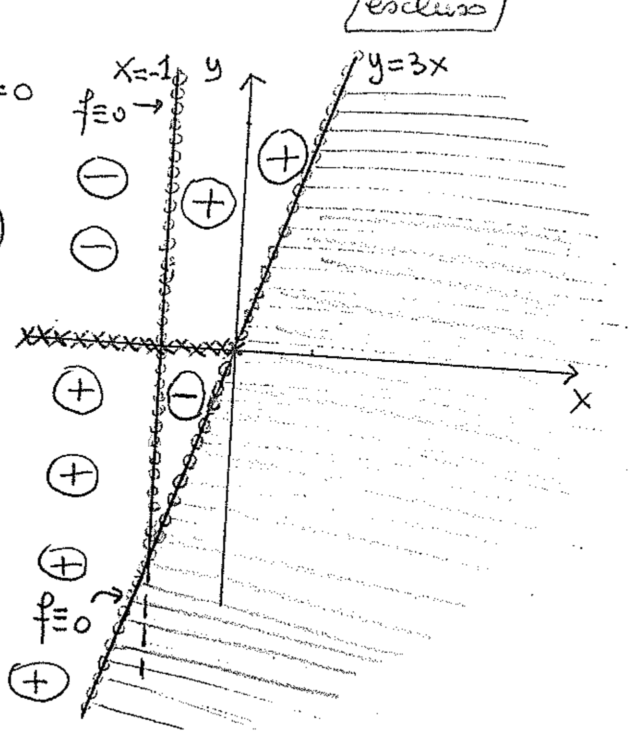
$f(x,y)=0 \iff (2x+2)=0$ oppure $3y-9x=0$

$\iff x=-1$ opp $y=3x$

$f(x,y)>0 \iff \dfrac{2x+2}{4y}\cdot\sqrt{3y-9x}>0$ (essendo $\sqrt{\ }\geq 0$ dove è definita)

$\iff \dfrac{2x+2}{4y}>0$ e $y\neq 3x \iff \left(\begin{cases}2x+2>0\\4y>0\end{cases} \text{ o } \begin{cases}2x+2<0\\4y<0\end{cases}\right)$

e $y\neq 3x \iff \left(\begin{cases}x>-1\\y>0\end{cases} \text{ o } \begin{cases}x<-1\\y<0\end{cases}\right)$ e $y\neq 3x$



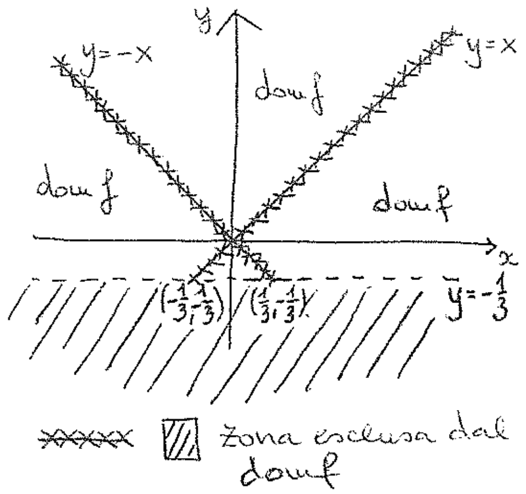
▨ ~~xxxx~~ Zona esclusa dal dom f

$(-1,0)$ e $(0,0)\notin \mathrm{dom} f$

~~ooooo~~ f=0 $\quad y=3x$ è nel dominio a parte $(0,0)$

c) $\mathrm{dom} f = \{(x,y) \in \mathbb{R}^2 : x^2 - y^2 \neq 0,\ 1+3y > 0\} = \{(x,y) \in \mathbb{R}^2 : y \neq \pm x,\ y > -\frac{1}{3}\}$

Semipiano delle $y > -\frac{1}{3}$ escluse le due bisettrici $y=x$ e $y=-x$ (quindi $(0,0) \notin \mathrm{dom} f$). La retta $y = -\frac{1}{3}$ è esclusa dal dominio.



d) $\mathrm{dom} f = \{(x,y) \in \mathbb{R}^2 : 6-|x|-|y| \geq 0,\ |y|-3 > 0\} =$
$= \{(x,y) \in \mathbb{R}^2 : |x|+|y| \leq 6,\ |y| > 3\} \rightarrow y < -3 \cup y > 3$

o - - - $\notin \mathrm{dom} f$
▨ $\notin \mathrm{dom} f$

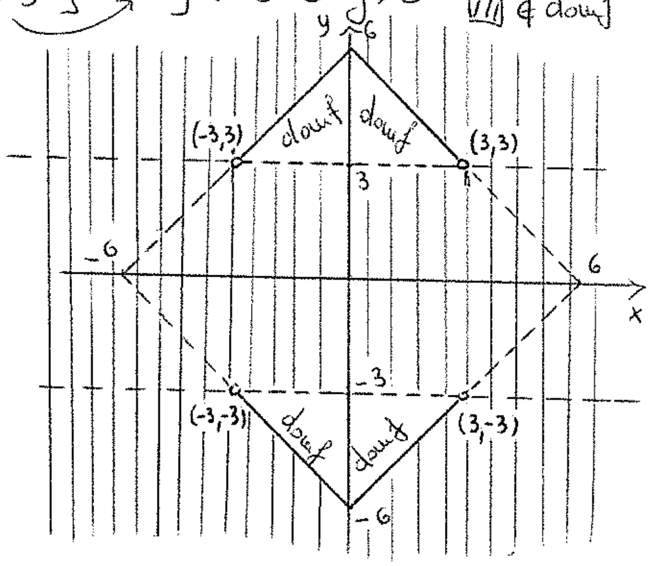
$|x|+|y| \leq 6$ è un ROMBO (è un quadrato ruotato in realtà) di VERTICI $(6,0), (0,6), (-6,0), (0,-6)$: infatti se $x \geq 0, y \geq 0 \rightarrow y \leq 6-x$

se $x \geq 0, y \leq 0 \rightarrow y \geq x-6$

se $x \leq 0, y \geq 0 \rightarrow y \leq x+6$

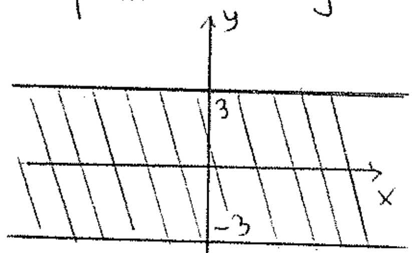
se $x \leq 0, y \leq 0 \rightarrow y \geq -x-6$

Il lati sono compresi



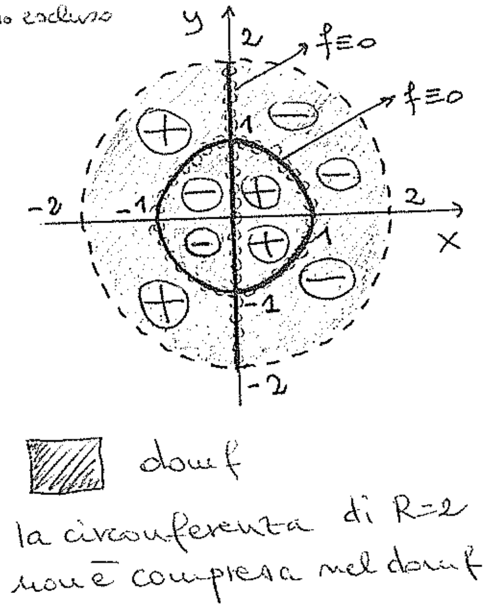
$y < -3$ o $y > 3$ sono tutti i punti del piano esclusi quelli con $-3 \leq y \leq 3$ (che costituiscono una striscia orizzontale)

Zona esclusa ▨



Il dom f è costituito dai due triangoli di VERTICI $(3,3)(0,6)(-3,3)$ e $(-3,-3)(0,-6)(3,-3)$ privati del lato su $y=3$ e $y=-3$ rispettivamente. In particolare $(3,3), (-3,3), (-3,-3)$ e $(3,-3) \notin \mathrm{dom} f$.

e) $\text{dom} f = \{ (x,y) \in \mathbb{R}^2 : \frac{4}{3} - \frac{1}{3}(x^2+y^2) > 0 \} = \{ (x,y) : x^2+y^2 < 4 \}$ interno del cerchio $C(0,0)$ $R=2$ bordo escluso



$f(x,y) = 0 \iff x = 0$ opp $\log\left(\frac{4}{3} - \frac{1}{3}(x^2+y^2)\right) = 0$

$\iff x = 0$ opp $\frac{4}{3} - \frac{1}{3}(x^2+y^2) = 1$

$\iff \underset{(\text{asse } y)}{x=0}$ opp $x^2+y^2 = 1$ (circonf $C(0,0)$ $R=1$)

$f(x,y) > 0 \iff x \cdot \log\left(\frac{4}{3} - \frac{1}{3}(x^2+y^2)\right) > 0$

$\iff \left(x > 0 \text{ e } \frac{4}{3} - \frac{1}{3}(x^2+y^2) > 1\right)$ oppure $\left(x < 0 \text{ e } \frac{4}{3} - \frac{1}{3}(x^2+y^2) < 1\right)$

$\iff (x > 0 \text{ e } x^2+y^2 < 1)$ oppure $(x < 0 \text{ e } x^2+y^2 > 1)$

f)

$f(x,y) = (2x-4)\, y \sqrt{25 - x^2}$

$\text{dom} f = \{ (x,y) \in \mathbb{R}^2 : 25 - x^2 \geq 0 \} = \{ (x,y) \in \mathbb{R}^2 : x^2 \leq 25 \} =$

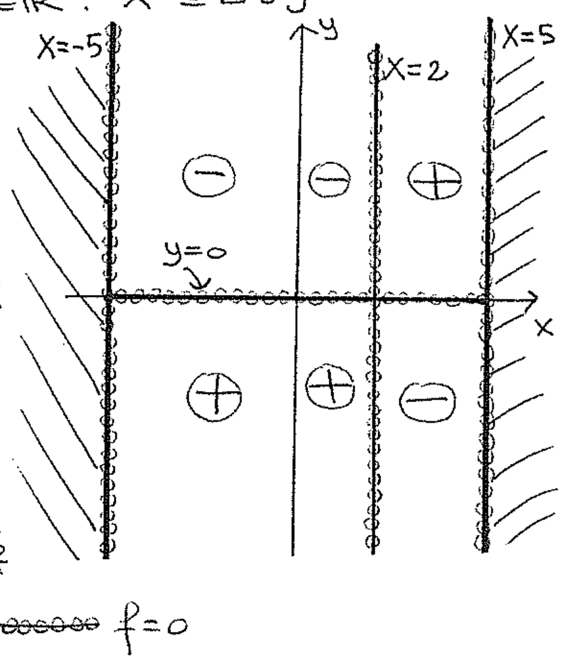
$= \{ (x,y) \in \mathbb{R}^2 : -5 \leq x \leq 5 \}$ STRISCIA di piano tra le rette $x=-5$ e $x=5$, rette COMPRESE

$f(x,y) = 0 \iff (2x-4) \cdot y \cdot \sqrt{25-x^2} = 0$

$\iff x = 2$ opp $y = 0$ opp $x = \pm 5$

$f(x,y) > 0 \iff (2x-4) \cdot y \cdot \underbrace{\sqrt{25-x^2}}_{\geq 0 \text{ sempre nel suo dominio}} > 0$

$\underset{(\text{con } x \neq \pm 5)}{\iff} (2x-4) \cdot y > 0 \iff [(x > 2 \text{ e } y > 0)$ opp. $(x < 2 \text{ e } y < 0)]$ e $x \neq \pm 5$



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g) $\operatorname{dom} f = \{(x,y)\in\mathbb{R}^2 : 9 - 9x^2 - y^2 \geq 0,\ x - y > 0\} =$

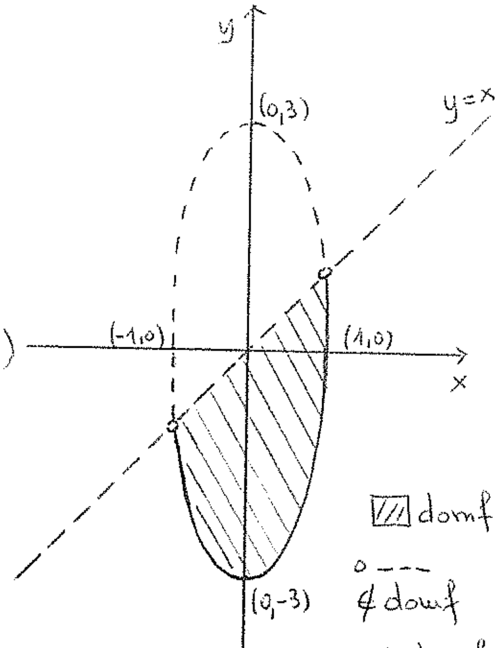
$= \{(x,y)\in\mathbb{R}^2 : x^2 + \frac{y^2}{9} \leq 1,\ y < x\}$

ELLISSE (INTERNO + BORDO) di $C(0,0)$ $a=1$ $b=3$

sotto $y=x$

$y=x$ esclusa

ELLISSE $\cap$ $y=x$: $\left(\frac{3}{10}\sqrt{10}, \frac{3}{10}\sqrt{10}\right)$ e $\left(-\frac{3}{10}\sqrt{10}, -\frac{3}{10}\sqrt{10}\right)$



i punti della bisettrice ∉ dom f

h) $\operatorname{dom} f = \{(x,y)\in\mathbb{R}^2 : |x+y| - 3 \neq 0\} =$

$= \{(x,y)\in\mathbb{R}^2 : |x+y| \neq 3\}$

$|x+y| = 3 \iff x+y=3$ opp $x+y=-3$

$\iff y = -x+3$ opp $y = -x-3$

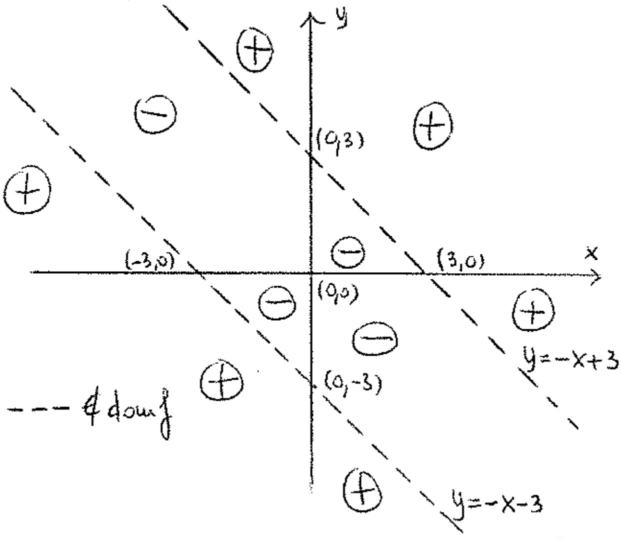
Quindi

$\operatorname{dom} f = \mathbb{R}^2 \setminus \{$le due rette $y=-x+3$, $y=-x-3\}$

$f(x,y)=0 \iff \frac{1}{|x+y|-3} = 0$ mai verificato

(una frazione $\frac{N}{D} = 0 \iff N=0$)

quindi f non si annulla mai



$f(x,y) > 0 \iff |x+y| - 3 > 0 \iff |x+y| > 3 \iff x+y>3$ oppure $x+y<-3$

$\iff y > -x+3$ oppure $y < -x-3$

i) $\operatorname{dom} f = \{(x,y)\in\mathbb{R}^2 : y - x^2 + 2x - 1 \geq 0,\ y - x - 1 \neq 0\} =$

$= \{(x,y)\in\mathbb{R}^2 : y \geq x^2 - 2x + 1 = (x-1)^2,\ y \neq x+1\}$

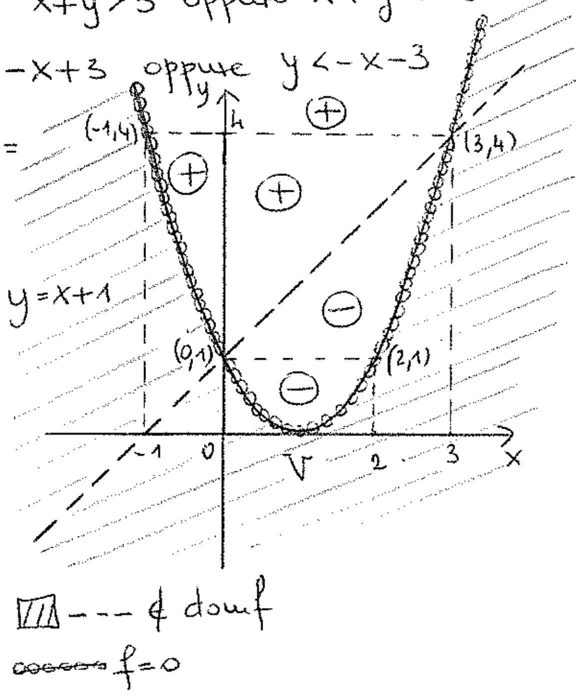
sopra la parabola di $V(1,0)$ verso l'alto (è la parabola $y=x^2$ a ds di 1)

esclusa la retta $y=x+1$

$f(x,y) = 0 \iff y = x^2 - 2x + 1$ e $y \neq x+1$

$f=0$ in tutti i punti della parabola esclusi $(0,1)$ e $(3,4)$ che ∉ dom f

$f(x,y) > 0 \iff y \neq x^2 - 2x + 1$ e $y > x+1$



j) $\text{dom} f = \{(x,y) \in \mathbb{R}^2 : x - 3y \neq 0\} =$

$= \{(x,y) \in \mathbb{R}^2 : y \neq \frac{1}{3}x\}$

tutto $\mathbb{R}^2$ tranne la retta $y = \frac{1}{3}x$

$f(x,y) = 0 \iff \dfrac{4x-2y}{x-3y} = 0 \iff 4x - 2y = 0$ e $x - 3y \neq 0$

$\iff y = 2x$ ← retta su cui f vale 0
escluso $(0,0)$ che $\notin \text{dom} f$

$f(x,y) > 0 \iff \dfrac{4x-2y}{x-3y} > 0 \iff$

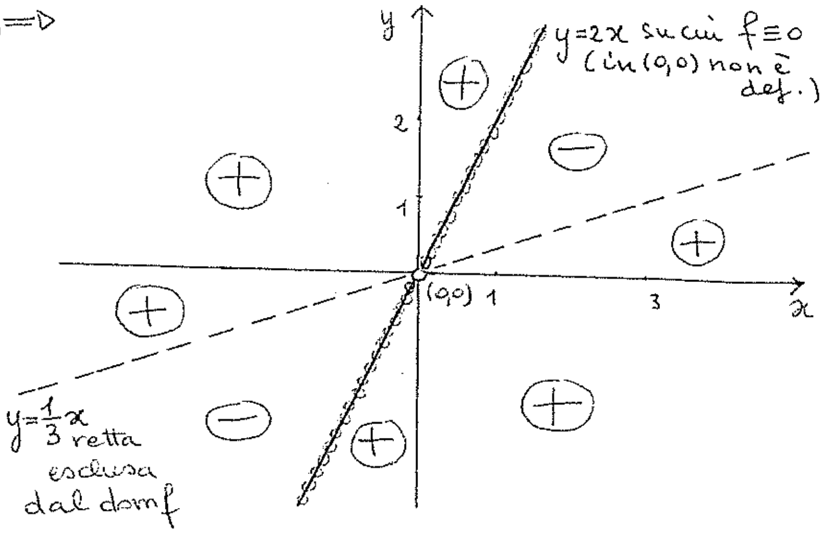
$(4x - 2y > 0$ e $x - 3y > 0)$

oppure

$(4x - 2y < 0$ e $x - 3y < 0)$

$\iff (y < 2x$ e $y < \frac{1}{3}x)$ oppure

$(y > 2x$ e $y > \frac{1}{3}x)$



k) $\text{dom} f = \{(x,y) \in \mathbb{R}^2 : 4 - y \geq 0$ e $16 - x^2 - y^2 - 6x > 0\} =$

$= \{(x,y) \in \mathbb{R}^2 : y \leq 4$ e $(x+3)^2 + y^2 < 25\}$

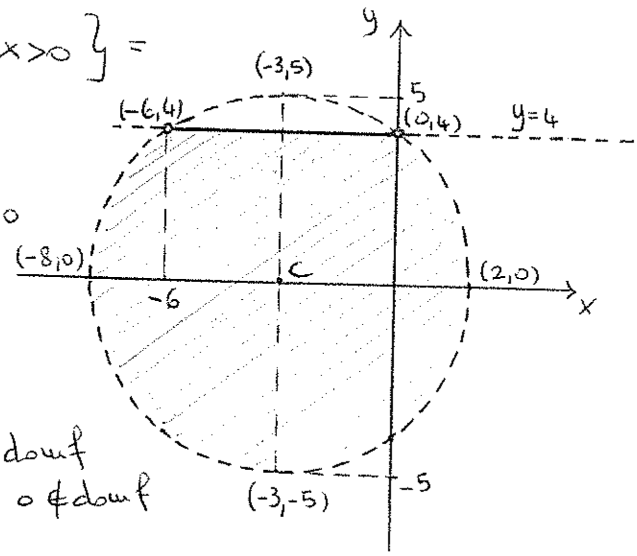
semipiano
retta $y = 4$ compresa

INTERNO del CERCHIO
di $C(-3,0)$ e $R = 5$
(bordo escluso)

$\begin{cases} y = 4 \\ (x+3)^2 + y^2 = 25 \end{cases} \to (x+3)^2 = 9 \to x + 3 = \pm 3$

$x = 0$
$x = -6$

$\cap$ in $(0,4)$ e $(-6,4)$



Il bordo della circonferenza è escluso dal dominio -

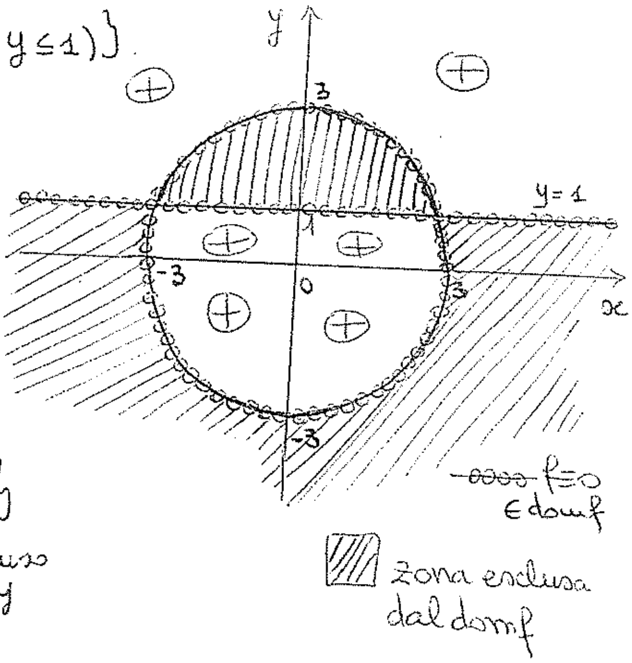
I punti della retta $y=4$ appartengono al dominio per

$x \in]-6, 0[$, $(-6,4)$ e $(0,4)$ esclusi -

l) $\text{dom} f = \{(x,y) \in \mathbb{R}^2 : (x^2+y^2-9)(y-1) \geq 0\} =$

$= \{(x,y) : (x^2+y^2-9 \geq 0 \ \underline{e} \ y-1 \geq 0) \ \underline{o} \ (x^2+y^2-9 \leq 0 \ \underline{e} \ y-1 \leq 0)\}$

$= \{(x,y) : (x^2+y^2 \geq 9 \ \underline{e} \ y \geq 1) \ \underline{o} \ (x^2+y^2 \leq 9 \ \underline{e} \ y \leq 1)\}$



$f(x,y) = 0 \iff x^2+y^2 = 9 \ \underline{o} \ y = 1$

$f(x,y) > 0 \quad \forall (x,y) \in \text{dom} f$ esclusi i p.ti in cui vale 0

m) $\text{dom} f = \left\{(x,y) \in \mathbb{R}^2 : \dfrac{|x+2y|-1}{x} \geq 0, \ x \neq 0\right\}$ ↳ escluso asse y

$\begin{cases} |x+2y|-1 \geq 0 \\ x > 0 \end{cases}$ oppure $\begin{cases} |x+2y|-1 \leq 0 \\ x < 0 \end{cases}$

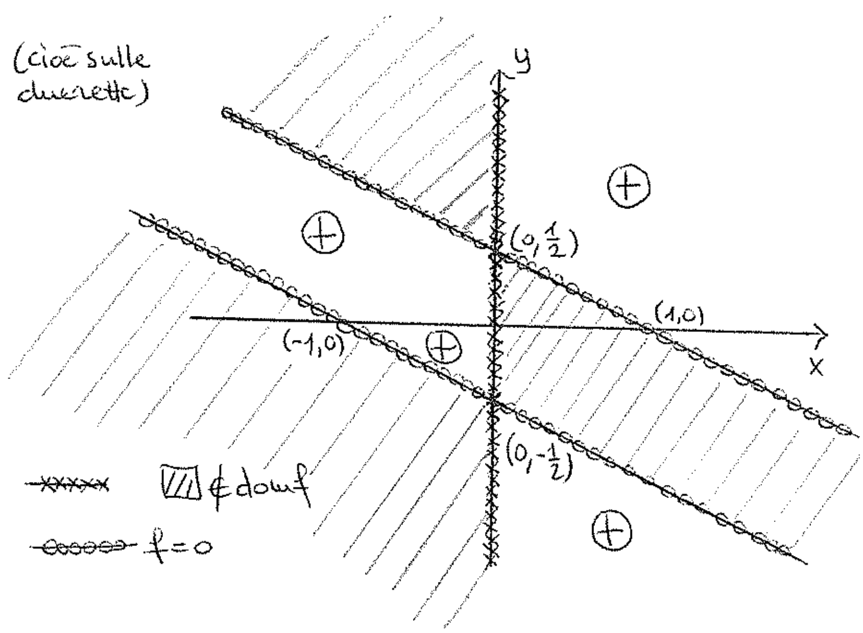
$|x+2y| \geq 1 \iff x+2y \geq 1 \ \text{o} \ x+2y \leq -1 \qquad |x+2y| \leq 1 \iff -1 \leq x+2y \leq 1$

$\begin{cases} y \geq -\frac{1}{2}x + \frac{1}{2} \ \cup \ y \leq -\frac{1}{2}x - \frac{1}{2} \\ x > 0 \end{cases}$ oppure $\begin{cases} -\frac{1}{2}x - \frac{1}{2} \leq y \leq -\frac{1}{2}x + \frac{1}{2} \\ x < 0 \end{cases}$

$f(x,y) = 0 \iff |x+2y|-1 = 0$

$\iff y = -\frac{1}{2}x + \frac{1}{2} \ \cup \ y = -\frac{1}{2}x - \frac{1}{2}$ (cioè sulle due rette)

esclusi i punti $(0, \frac{1}{2})$ e $(0, -\frac{1}{2})$ che $\notin \text{dom} f$



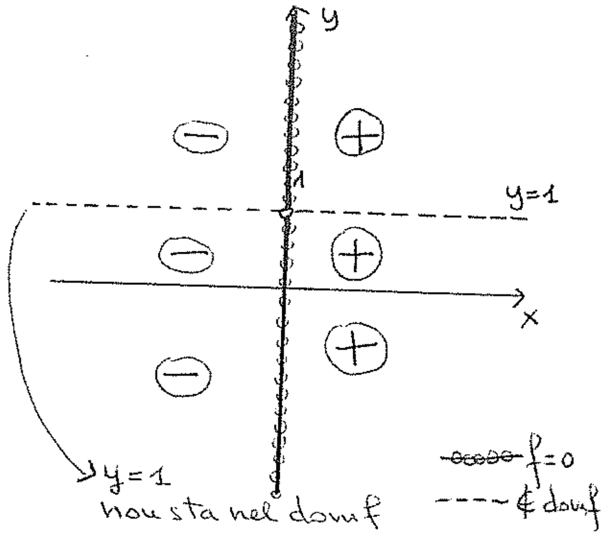
$f(x,y) > 0 \quad \forall (x,y) \in \text{dom} f$ in cui f non è nulla (perché $\sqrt{\ldots} \geq 0$ per ogni punto in cui è definita).

2) i) $\mathrm{dom} f = \{(x,y)\in\mathbb{R}^2 : (y-1)^2 \neq 0\} = \{(x,y) : y-1\neq 0\} = \{(x,y)\in\mathbb{R}^2 : y\neq 1\}$
$= \mathbb{R}^2 \setminus$ la retta $y=1$

$f(x,y)=0 \iff \frac{x}{(y-1)^2}\cdot \underbrace{e^{3y}}_{e^{3y}\neq 0\ \forall y} = 0 \iff \underbrace{x=0}_{\text{asse } y}$

$f(x,y)>0 \iff \frac{x}{(y-1)^2}e^{3y}>0 \iff x>0$

$(y-1)^2>0$ sempre nel dom f
$e^{3y}>0$ sempre



ii) $\mathrm{dom} f = \mathbb{R}^2$

$f(x,y)=0 \iff |y|\,x(2x-4)e^{y^2-x}=0 \iff$

$|y|=0$ o $x=0$ $\quad 2x-4=0$ o $\underbrace{e^{y^2-x}=0}_{\text{mai}}$

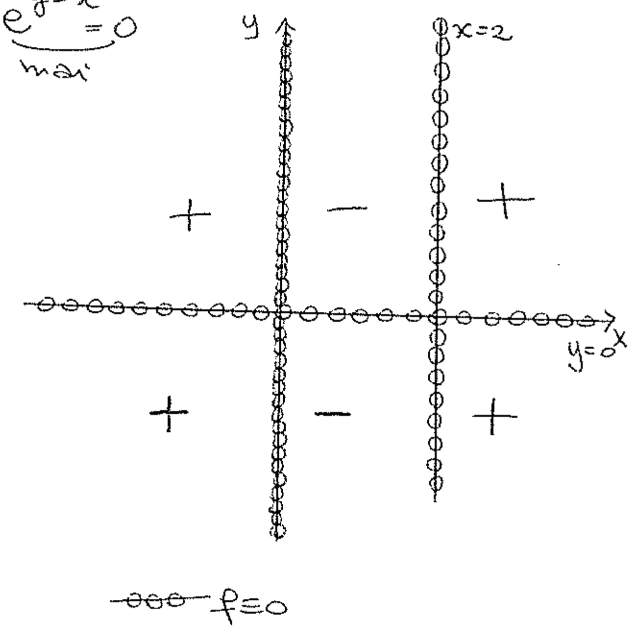
$\iff \underbrace{y=0}_{\text{asse } x}$ o $\underbrace{x=0}_{\text{asse } y}$ o $x=2$

$f(x,y)>0 \iff |y|(2x^2-4x)\cdot e^{y^2-x}>0$

$\iff \underset{(\text{con } y\neq 0)}{(2x^2-4x)>0}$ $\quad (e^{y^2-x}>0$ sempre$)$
$|y|\geq 0$ sempre

$\iff (x<0 \text{ e } y\neq 0)$ o $(x>2 \text{ e } y\neq 0)$

$2x^2-4x>0 \iff x<0$ o $x>2$ (valori esterni)



iii) $\mathrm{dom} f = \mathbb{R}^2$ $\quad f(x,y)=0 \iff 6x^2y - 2y^2 = 0$

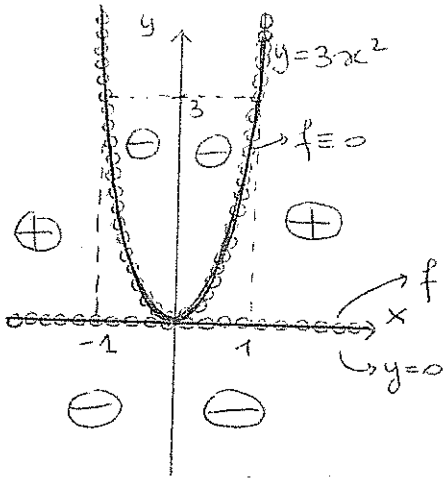
$\iff 2y(3x^2-y)=0 \iff y=0$ o $y=3x^2$

$y=0 \to$ asse x

$y=3x^2 \to$ parabola di $V(0,0)$ verso l'alto apertura 3

$x=\pm 1$ $y=3$

$x=\pm 2$ $y=12$



$f(x,y)>0 \iff 2y(3x^2-y)>0 \iff$

$\begin{cases} y>0 \\ y<3x^2 \end{cases}$ o $\begin{cases} y<0 \\ y>3x^2 \end{cases}$ mai verificate contemporaneamente

$\iff \begin{cases} y>0 \\ y<3x^2 \end{cases}$

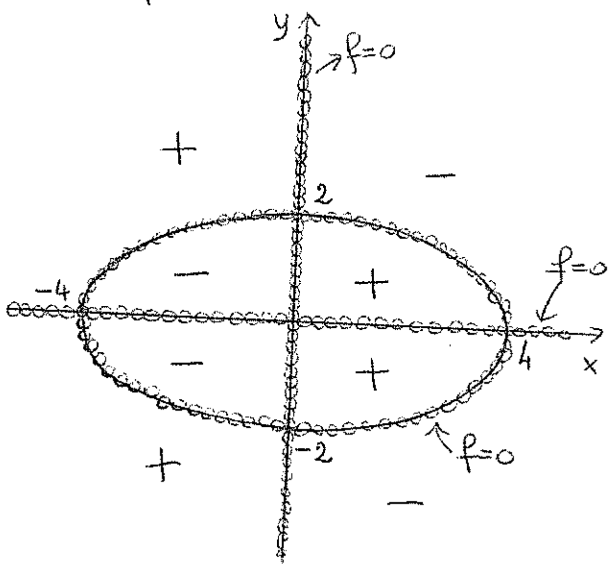
iv) $\mathrm{dom} f = \mathbb{R}^2$ $\quad f(x,y)=0 \iff x\cdot y^2\cdot\left(1-\frac{x^2}{16}-\frac{y^2}{4}\right)=0 \iff$

$x=0$ o $y=0$ o $\frac{x^2}{16}+\frac{y^2}{4}=1$

$x=0 \to$ asse y

$y=0 \to$ asse x

$\frac{x^2}{16}+\frac{y^2}{4}=1 \to$ ellisse $C(0,0)$ $a=4$ $b=2$



$f(x,y)>0 \iff x\cdot y^2\left(1-\frac{x^2}{16}-\frac{y^2}{4}\right)>0$

$\underset{y^2\geq 0\ \forall y}{\iff}$ $x\cdot\left(1-\frac{x^2}{16}-\frac{y^2}{4}\right)>0$ e $y\neq 0$

$\iff \left(\begin{cases} x>0 \\ \frac{x^2}{16}+\frac{y^2}{4}<1 \end{cases}\right.$ o $\left.\begin{cases} x<0 \\ \frac{x^2}{16}+\frac{y^2}{4}>1 \end{cases}\right)$ e $y\neq 0$