[redacted] = [redacted] + [redacted] = [redacted] + [redacted] [redacted] [redacted] [[redacted]] [redacted] $\frac{1}{117}$ [[redacted]]

$$[\text{redacted}] \, \{ [\text{redacted}] \} = \int_{9}^{24} [\text{redacted}]^{\lambda(y-9)} [\text{redacted}] - [\text{redacted}]^{\lambda(y-9)} \Big|_{9}^{24} = [\text{redacted}] - [\text{redacted}]^{\lambda 15} = 0 [\text{redacted}]$$

[redacted] { [redacted] }

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$$[\text{redacted}] - [\text{redacted}] \{ [\text{redacted}] \} - [\text{redacted}] \{ 1 \; [\text{redacted}] \}$$

$$= [\text{redacted}] - ([\text{redacted}])^{10} - ([\text{redacted}])^{9} =$$

$$= [\text{redacted}] - ([\text{redacted}])^{10} - [\text{redacted}] \cdot [\text{redacted}] = [\text{redacted}] - [\text{redacted}] - [\text{redacted}] = [\text{redacted}]$$

PROCESSI STOCASTICI

di 12 (113) Un processo stocastico SSL ha media $\eta_X = 1$ e autocorrelazione $R_X(\tau) = \mathrm{sinc}(B\tau) + 1$. Esso transita in un filtro con risposta impulsiva $h(t) = B\mathrm{sinc}(Bt)$. Verificare se il processo di uscita $Y(t)$ è SSL e, in caso positivo, calcolare η_Y e $R_Y(\tau)$. Calcolare quanto vale la potenza media di $Y(t)$ e quanto quella di $X(t)$.

$X(t)$ SSL con $\eta_X = 1$ $\quad R_X(\tau) = \mathrm{sinc}(B\tau) + 1$

$h(t) = B\,\mathrm{sinc}(Bt)$ $\quad Y(t)$ SSL? $\quad \eta_Y = ?$ $\quad R_Y(\tau) = ?$ $\quad P_Y = ?$ $\quad P_X = ?$

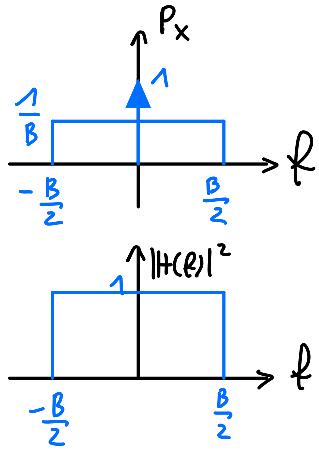
$Y(t)$ SSL (FILTRO NON NE ALTERA STAZIONARIETÀ)

$$P_X = R_X(0) = 1 + 1 = 2$$

$$P_X(f) = \mathcal{F}[R_X(\tau)] = \frac{1}{B} \Pi\left(\frac{f}{B}\right) + \delta(f) \qquad H(f) = \frac{\cancel{B}}{\cancel{B}} \Pi\left(\frac{f}{B}\right)$$

$$\eta_Y = \eta_X \cdot H(0) = 1 \cdot 1 = 1$$

$$P_Y(f) = P_X(f) \cdot |H(f)|^2 = P_X(f)$$



$$R_Y(\tau) = \mathcal{F}[P_Y(f)]^{-1} = \mathrm{sinc}(B\tau) + 1$$

$$P_Y = P_X = 2$$

(114) Dati i due processo SSL $X(t)$ e $Y(t)$ *indipendenti*, con $\eta_X = 1$, $R_X(\tau) = \Pi(\tau) + 1$, $\eta_Y = 2$, $R_Y(\tau) = 2\Pi(\tau) + 4$, calcolare media, autocorrelazione e potenza media di $Z(t) = X(t) + Y(t)$. NOTA: se due processi stocastici sono statisticamente indipendenti, allora qualunque variabile aleatoria (o n-pla di V.A.) estratta dal primo è statisticamente indipendente da qualunque V.A. (o n-pla di V.A.) estratta dal secondo.

$$\eta_Z = E[X(t) + Y(t)] = E[X(t)] + E[Y(t)] = \eta_X + \eta_Y = 1+2 = 3$$

$$R_Z(t_1, t_2) = E\left[\left(X(t_1) + Y(t_1)\right) \cdot \left(X(t_2) + Y(t_2)\right)\right] =$$

$$= E\left[X(t_1)X(t_2) + X(t_1)Y(t_2) + Y(t_1)X(t_2) + Y(t_1)Y(t_2)\right]$$

$$= E[X(t_1)X(t_2)] + E[X(t_1)]E[Y(t_2)] + E[Y(t_1)]E[X(t_2)] + E[Y(t_1)Y(t_2)]$$

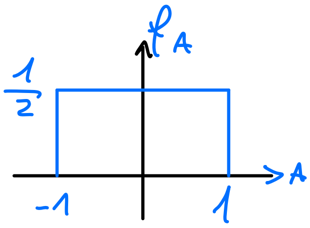
$$= R_X(\tau) + R_Y(\tau) + 2\eta_X\eta_Y$$

$$R_Z(\tau) = \Pi(\tau) + 1 + 2\Pi(\tau) + 4 + 4 = \Pi(\tau) + 2\Pi(\tau) + 9$$

$$P_Z = R_Z(0) = 12$$

ES. 3

$X(t) = 1 + A$ $\quad A \sim \mathcal{U}([-1,1])$



$P_X(f) = ?$ $\quad P_X = ?$

$P_X(f) = \mathcal{F}[R_X(\tau)]$ $\quad \eta_A = 0$ $\quad \sigma_A^2 = \frac{(a-b)^2}{12} = \frac{4}{12} = \frac{1}{3}$

$$R_X(t_1,t_2) = E[(1+A)(1+A)] = E[1 + A + A + A^2] = 1 + 2E[A] + E[A^2]$$
$$= 1 + 2\cdot 0 + \frac{1}{3} = \frac{4}{3}$$

$P_X(f) = \mathcal{F}[R_X(\tau)] = \frac{4}{3}\delta(f)$ $\quad P_X = \int_{-\infty}^{+\infty} P_X(f)\,df = \frac{4}{3}$

ES.

$X(t)$ SSL, $\eta_X = 0$ $\quad R_X(\tau) = A\Lambda\left(\frac{t}{T}\right)$

$X(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow Y(t)$ $\quad Y(t)$ STAZIONARIO? $\quad \eta_Y = ?$ $\quad P_Y(f) = ?$

$R_Y\left(t_1, t_1 + \frac{T}{2}\right) = ?$

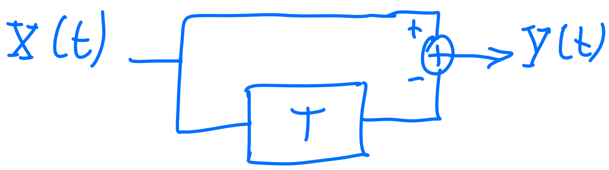
$X(t)$ SSL $\Rightarrow Y$ SSL

$H(f) = j2\pi f$ $\quad \eta_Y = \eta_X \cdot H(0) = 0$

$P_X(f) = \mathcal{F}[R_X(\tau)] = A\cdot T\,\text{sinc}^2(fT)$

$$P_Y(f) = P_X(f)\cdot|H(f)|^2 = AT\,\text{sinc}^2(fT)\cdot|j2\pi f|^2 =$$
$$= AT\frac{\text{sen}^2(\pi f T)}{(\pi f T)^2}\cdot(2\pi f)^2 = \frac{4A}{T}\text{sen}^2(\pi f T) =$$

ES. $X(t)$ S, GAUSSIANO e $\eta_X = 0$ $\qquad R_X(\tau) = \frac{A}{T}\delta(\tau)$



$Y(t)$ S? $\quad \eta_Y$? $\quad P_Y(f) = ?$

η_Y e $R_Y(t_1, t_1 + \frac{T}{2})$

$h(t) = \delta(t) - \delta(t-T)$ $\qquad X(t)$ GAUSSIANO $\Rightarrow X(t)$ SSS $\Rightarrow Y(t)$ SSS

$\eta_Y = \eta_X \cdot H(0) = 0$ $\qquad H(f) = 1 - e^{-j2\pi f T}$

$P_X(f) = \mathcal{F}[R_X(\tau)] = \frac{A}{T}$ $\qquad |H(f)| = 1 + 1 = 2$

$P_Y(f) = P_X(f) \cdot |H(f)|^2 = \frac{A}{T} \cdot 2^2 = 4\frac{A}{T}$

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3) $X(t)$ è un processo stocastico stazionario in senso stretto, caratterizzato dalla funzione di autocorrelazione $R_X(\tau) = \frac{1}{T}\Pi\left(\frac{\tau}{2T}\right)$. Il processo transita attraverso un filtro avente risposta impulsiva $h(t) = \Pi\left(\frac{t - T/2}{T}\right)$, producendo il processo di uscita $Y(t)$: si valuti la potenza media P_Y di $Y(t)$.

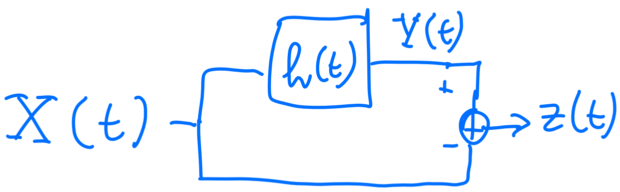
$H(f) = T \operatorname{sinc}(fT)\, e^{-j\pi f T}$ $\qquad P_X(f) = \mathcal{F}[R_X(\tau)] = 2 \operatorname{sinc}(2fT)$

$P_Y(f) = P_X(f) \cdot |H(f)|^2 = 2 \operatorname{sinc}(2fT) \cdot T^2 \operatorname{sinc}^2(fT)$

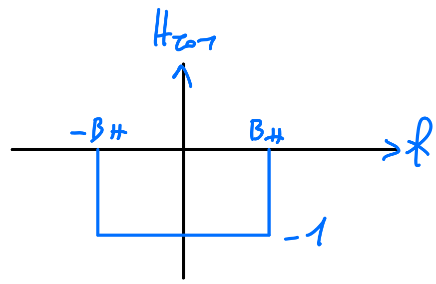
$R_Y(\tau) = R_X(\tau) * h(\tau) * h(-\tau) = \frac{1}{\cancel{T}}\Pi\left(\frac{\tau}{2T}\right) * \cancel{T}\Lambda\left(\frac{\tau}{T}\right) =$

costruzione, disegn...

3) Il processo $X(t)$ è un rumore bianco, gaussiano e a media nulla, con densità spettrale di potenza $P_X(f) = K$. Il processo $X(t)$ transita in un filtro con risposta impulsiva $h(t) = \delta(t) - 2B_H \mathrm{sinc}(2B_H t)$, producendo in uscita il processo $Y(t)$. Determinare la probabilità che ad un dato istante t_1, il valore di $Y(t_1)$ superi quello di $X(t_1)$, ovvero, detto $Z(t) = Y(t) - X(t)$, determinare $P\{Z(t_1) \geq 0\}$.



$$Z(t) = Y(t) - X(t)$$



$$R_X(\tau) = \mathcal{F}^{-1}[P_X(f)] = K\,\delta(\tau)$$

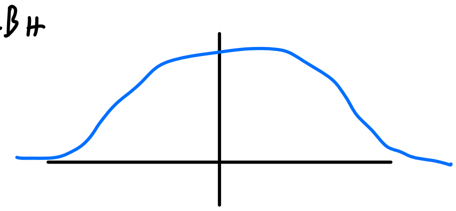
$$h_{tot}(t) = \cancel{\delta(t)} - 2B_H \mathrm{sinc}(2B_H t) - \cancel{\delta(t)}$$

$$= -2B_H \mathrm{sinc}(2B_H t) \longleftrightarrow H_{tot}(f) = -\Pi\left(\frac{f}{2B_H}\right)$$

$$P_Z(f) = P_X(f) \cdot |H_{tot}(f)|^2 = K\,\Pi\left(\frac{f}{2B_H}\right) \qquad P_Z = K \cdot 2B_H$$

$$\eta_Z = \eta_X \cdot H(0) = 0 \qquad \sigma_Z^2 = P_Z - \eta_Z^2 = K 2B_H$$

$$f_Z(z_1; t_1) = \frac{1}{\sqrt{2\pi\sigma_Z^2}} \exp\left\{-\frac{z_1^2}{2\sigma_Z^2}\right\}$$



$$P\{Z(t_1) \geq 0\} = \int_0^{+\infty} f_Z(z_1; t_1)\, dz = \frac{1}{2}$$

ES.

$X(t)$ HA $\eta_X = \sqrt{3}$ $\quad R_X(\tau) = 3 + N_0\,\delta(\tau)$

$$X(t) \longrightarrow \boxed{\int_T} \longrightarrow Y(t)$$

η_Y? $\quad P_Y(f)$? $\quad$ Y S? $\quad P_Y$?

$$h(t) = \Pi\left(\frac{t - T/2}{T}\right) \qquad H(f) = T\,\mathrm{sinc}(fT)\,e^{-j\pi f T}$$

$$\eta_Y = \eta_X \cdot H(0) = \sqrt{3} \cdot T \qquad P_X(f) = 3\,\delta(f) + N_0$$

$$P_Y(f) = P_X(f) \cdot |H(f)|^2 = \left[3\,\delta(f) + N_0\right] \cdot T^2 \mathrm{sinc}^2(fT) =$$

$$= 3T^2\delta(f) + N_0 T^2 \mathrm{sinc}^2(fT)$$

$$R_Y(\tau) = \mathcal{F}^{-1}\left[P_Y(f)\right] = 3T^2 + N_0 T\,\Lambda\left(\frac{\tau}{T}\right) \qquad P_Y = R_Y(0) = 3T^2 + N_0 T$$

ES. PARZIALE

$X(t)$ SSL $\quad$ E INDIP. DA $Y(t)$ $\quad R_X(\tau) = \mathrm{sinc}(2f_0\tau)$

$$Y(t) = a \cdot \cos(2\pi f_0 t + \Phi) \qquad \Phi \sim \mathcal{U}([0, 2\pi])$$

$$R_Y(t_1, t_2) = E\left[a\cos(2\pi f_0 t_1 + \varphi) \cdot a\cos(2\pi f_0 t_2 + \varphi)\right]$$

$$= \frac{a^2}{2}\cos\left(2\pi f_0 (t_2 - t_1)\right)$$

$$\eta_Y = E\left[a\cos(2\pi f_0 t_1 + \Phi)\right] = \frac{1}{2\pi}\int_0^{2\pi} a\cos(2\pi f_0 t_1 + \varphi)\,d\varphi = 0$$

$$Z(t) = X(t) \cdot Y(t) \qquad \eta_Z = \eta_X \cdot \eta_Y = 0$$

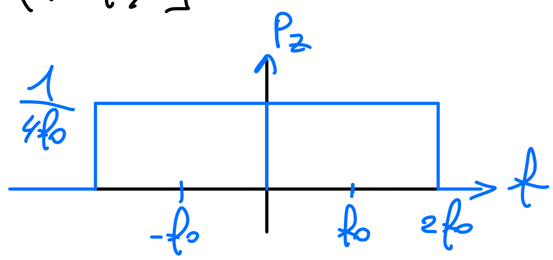
$$R_Z(t_1,t_2) = E\left[\left(X(t_1)\cdot Y(t_1)\right)\left(X(t_2)\cdot Y(t_2)\right)\right] =$$

$$\overset{|}{=} E\left[X(t_1)\,X(t_2)\right] E\left[Y(t_1)\,Y(t_2)\right] = R_X(\tau)\cdot R_Y(\tau)$$

$$P_Z(f) = \mathcal{F}\left[R_Z(\tau)\right] = \mathcal{F}\left[R_X(\tau)\right] * \mathcal{F}\left[R_Y(\tau)\right]$$

$$\overset{|}{=} \frac{1}{2f_0}\,\Pi\left(\frac{f}{2f_0}\right) * \frac{1}{2}\left[\delta\left(f+f_0\right) + \delta\left(f-f_0\right)\right]$$

$$= \frac{1}{4f_0}\,\Pi\left(\frac{f+f_0}{2f_0}\right) + \frac{1}{4f_0}\,\Pi\left(\frac{f-f_0}{2f_0}\right)$$

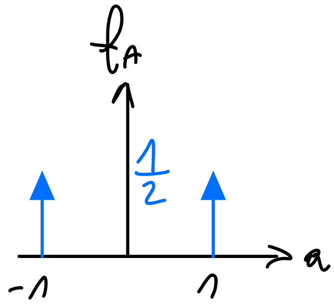


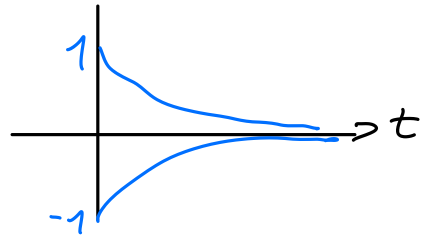
$$\overset{|}{=} \frac{1}{4f_0}\,\Pi\left(\frac{f}{4f_0}\right)$$

$$P_Z = \cancel{4f_0}\cdot\frac{1}{\cancel{4f_0}} = 1$$

ES.

$$X(t) = A\, e^{-\frac{t}{\tau}}\, u(t)$$





$$\eta_X = E\left[A\, e^{-\frac{t}{\tau}} u(t)\right] = E[A] \cdot e^{-\frac{t}{\tau}} u(t) = \left(-1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}\right) e^{-\frac{t}{\tau}} u(t) = 0$$

$$R_X(t_1, t_2) = E\left[A\, e^{-\frac{t_1}{\tau}} u(t_1) \cdot A\, e^{-\frac{t_2}{\tau}} u(t_2)\right] = \underbrace{E\left[A^2\right]}_{=1} \cdot e^{-\frac{(t_2+t_1)}{\tau}} u(t_1)\, u(t_2)$$

$$= \begin{cases} e^{-\frac{(t_2+t_1)}{\tau}} & \text{PER } t_1 > 0 \text{ e } t_2 > 0 \\ 0 & \end{cases}$$

DIPENDE DAGLI ISTANTI, NON DALLA DIFFERENZA

$\Downarrow$

X È NON STAZ.