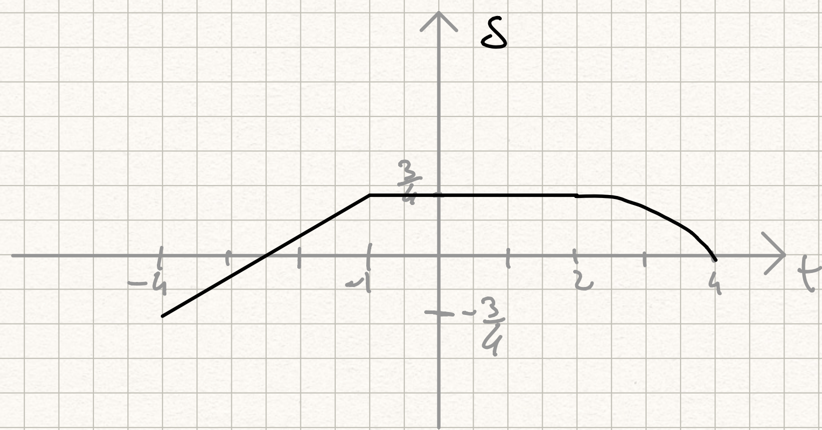
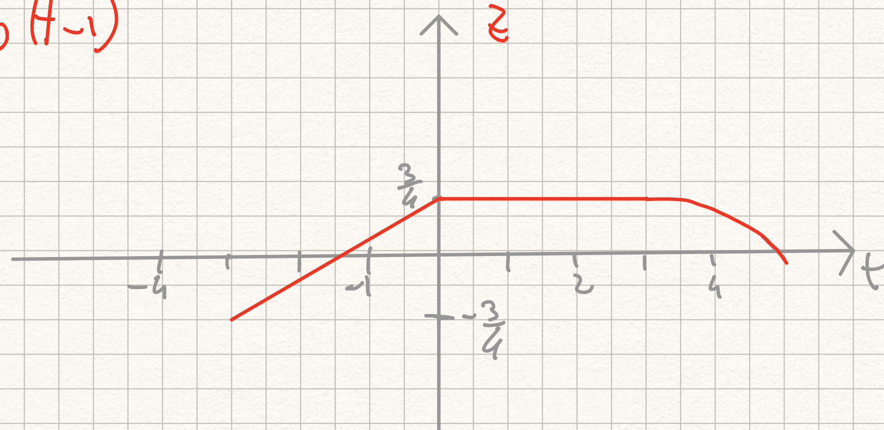
Esercizio 3 Dato $s(t)$ disegnare $q(t) = s(-2t)$ e $z(t) = s(t-1)$

$$s(t) \begin{cases} \frac{5}{4} + \frac{t}{2} & -4 \le t < 1 \\ \frac{3}{4} & -1 \le t \le 2 \\ 1 - \frac{t^2}{16} & 2 < t \le 4 \\ 0 & |t| > 4 \end{cases}$$



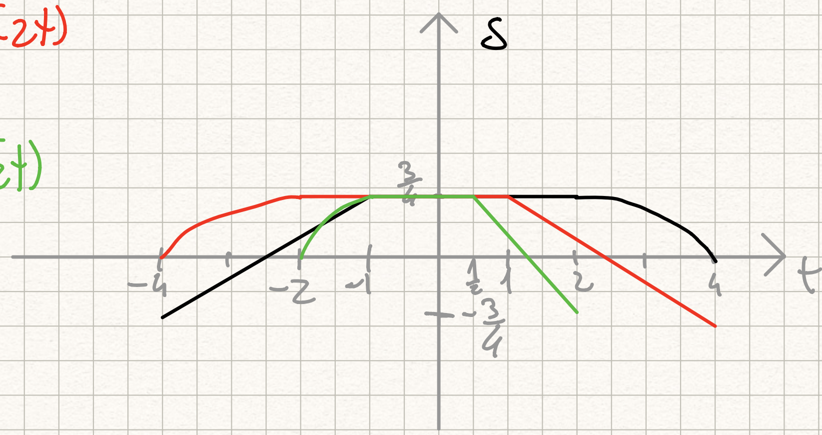
$z(t) = s(t-1)$



$y(t) = s(-2t)$

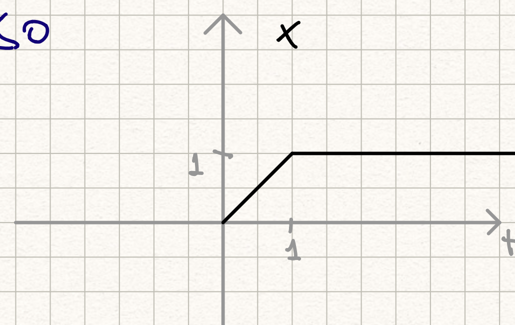
$= -s$

$= -s(2t)$



Esercizio 4

$$x(t) = \begin{cases} 1 & t \geq 1 \\ t & 0 < t < 1 \\ 0 & t \leq 0 \end{cases}$$

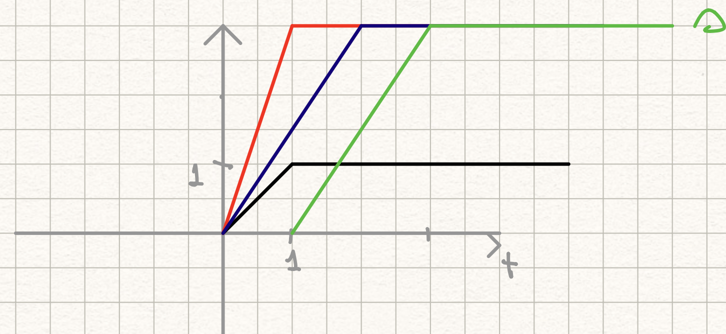


① $s(t) = 3x\left(\frac{t-1}{2}\right)$

$= 3x$

$= 3x\left(\frac{t}{2}\right)$

$= 3x\left(\frac{t-1}{2}\right)$

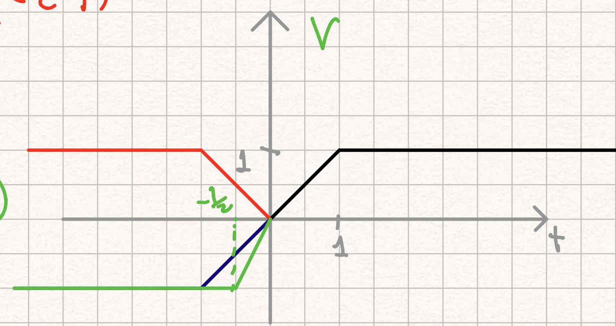


② $v(t) = -x(-2t)$

$= -x$

$= -x(-t)$

$= -x(-2t)$

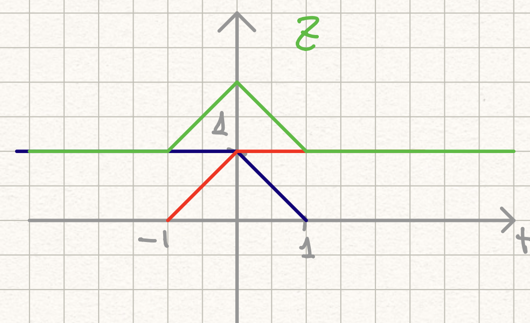


③ $z(t) = x(t+1) + x(1-t)$ $\qquad x(1-t) = -x(t-1)$

$= x(t+1)$

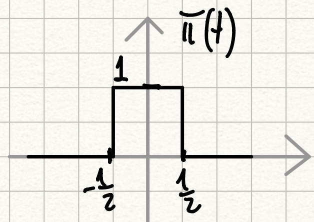
$= x(t+1) - x(t-1)$

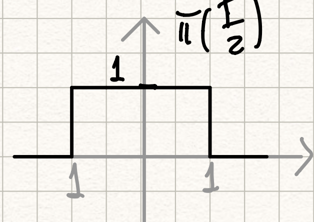
$= x(t+1) - x(t-1)$

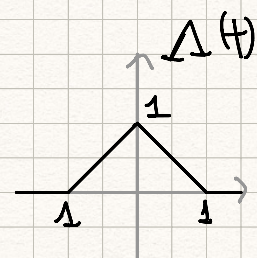


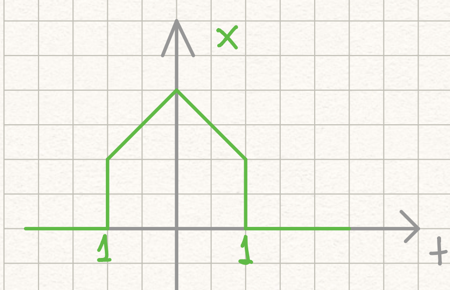
Esercizio 4.1

① $x(t) = \Pi\left(\frac{t}{2}\right) + \Lambda(t)$

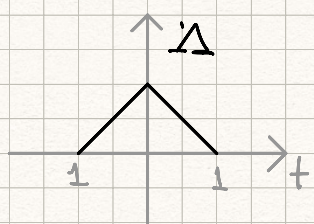


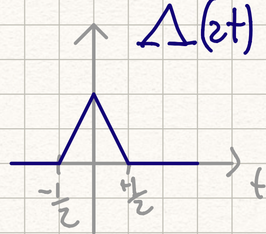


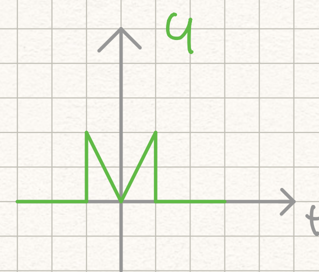




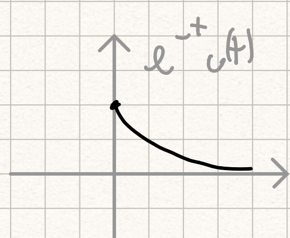
(b) $y(t) = \pi(t) - \Lambda(2t)$

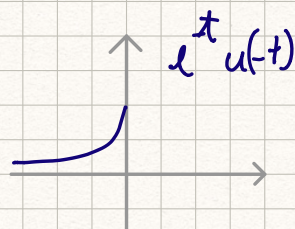


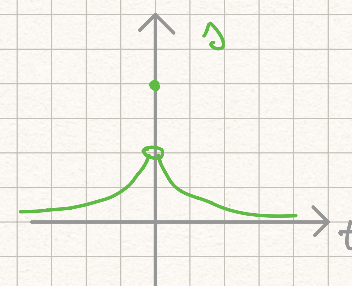




(c) $s(t) = e^{-t}u(t) + e^{+t}u(-t)$







(d) $z(t) = e^{j\pi t} + e^{-j\pi t}$

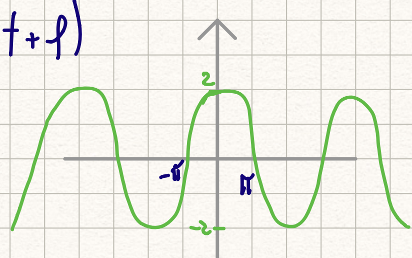
dalla formula di eulero

$e^{j\pi t} + e^{-j\pi t} = 2\cos \pi t$

$z(t) = 2\cos(\pi t)$

$\cos(2\pi f_0 t + \varphi)$

$f_0 = \frac{1}{2}$



Esercizio

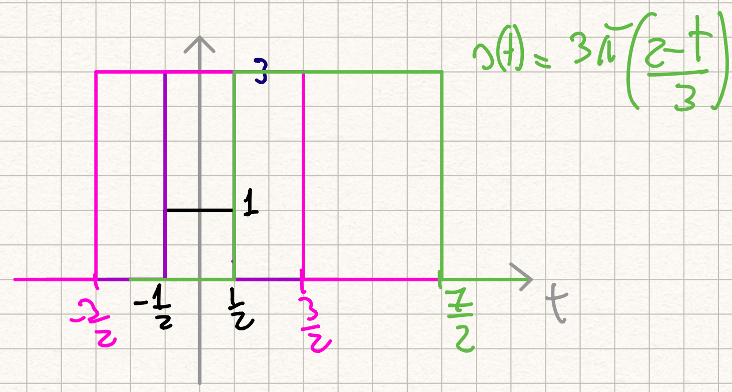
① $3\Pi\left(\frac{2-t}{3}\right)$

$w(t) = 3\Pi$

$y(t) = w\left(\frac{t-2}{3}\right)$

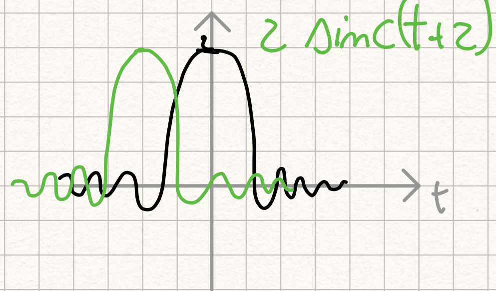
$z(t) = y\left(\frac{t}{3}\right)$

$s(t) = z(t-2)$



② $2\,\mathrm{sinc}\,(t+2)$

$2\sin(t)$

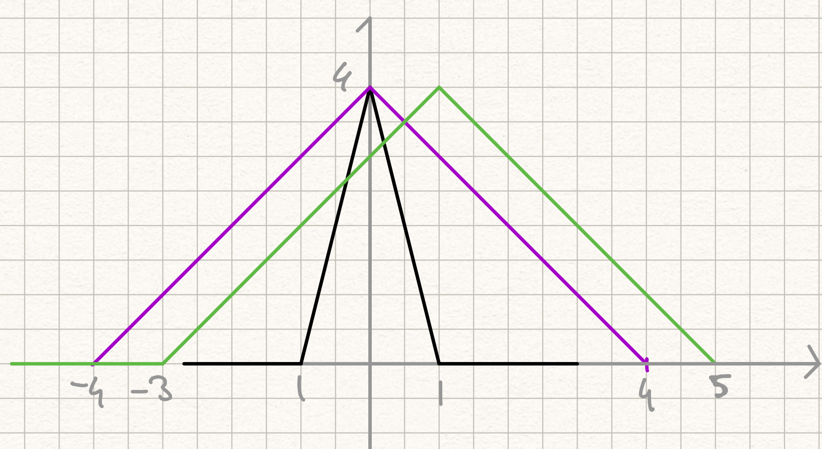


③ $4\,\Lambda\left(\frac{t-1}{4}\right)$

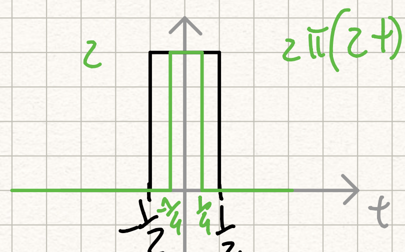
$4\Lambda(t)$

$4\,\Lambda\left(\frac{t}{4}\right)$

$4\,\Lambda\left(\frac{t}{4}-1\right)$

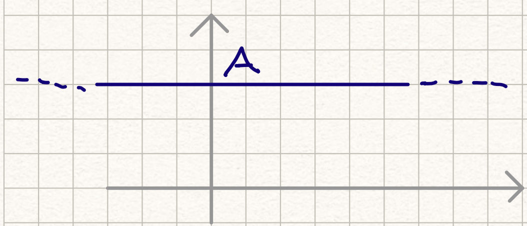


④ $2\Pi(2t)$



Esercizio

1) $x(t) = A$



Durata ∞ $\qquad D = t_2 - T_1$

Area $= 0 \longrightarrow A_x = 0$

$A > 0 \longrightarrow A_x = +\infty$

$A < 0 \longrightarrow A_x = +\infty$

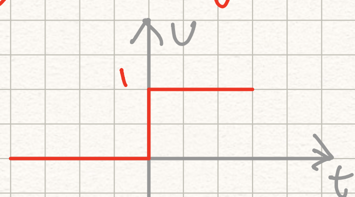
$$\langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt =$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} A\, dt = A$$

$$E_{TOT} = \int_{-\infty}^{+\infty} |x(t)|^2 dt = +\infty$$

$$P_M = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = A^2$$

2) Gradino unitario



Durata $= \infty$

Area $= \infty$

$$x(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

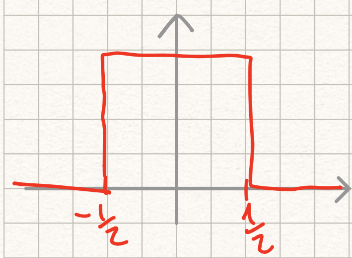
$$\langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} 1 = \frac{1}{2}$$

$$E_{tot} = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \infty$$

$$P_M = \frac{1}{T} \int_{0}^{T/2} |x(t)|^2 dt = \frac{1}{\cancel{T}} \frac{\cancel{T}}{2} = \frac{1}{2}$$

3) $x(t) = \Pi(t) = \begin{cases} 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0 & \text{altrove} \end{cases}$



Durata $= 1$

$$\text{Area} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pi(t)\, dt = 1$$

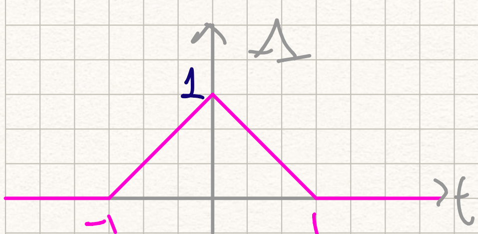
$$\langle x(t) \rangle = \lim_{x \to \infty} \frac{1}{T} \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pi(t)\, dt = 0$$

$$E_{TOT} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pi(t)\, dt = 1\, J.$$

$P_M = 0$

4) Impulso Triangolare

$x(t) = \Lambda(t) = \begin{cases} 1 - |t| & -1 \leq t \leq 1 \\ 0 & \text{altrove} \end{cases}$



Durata $= 2$

$$\text{Area} = \int_{-1}^{1} \Lambda(t) = \int_{-1}^{0} (1+t)\, dt + \int_{0}^{1} (1-t)\, dt$$

$$= 2 \int_{-1}^{0} 1 + t\, dt = 2 \left[1 - \frac{1}{2}\right] = 2 \cdot \frac{1}{2} = 1$$

$$\langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-1}^{1} \Lambda(t)\, dt = \lim_{T \to \infty} \frac{1}{2T} + \frac{1}{2T} = 0$$

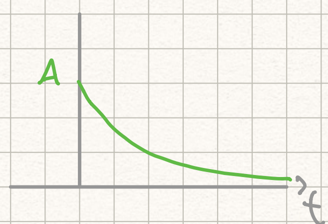
$$E_{TOT} = \int_{-1}^{0} (1+t)^2\,dt + \int_{0}^{1} (1-t)^2\,dt =$$

$$= 2\int_{-1}^{0} t^2+1+2t\,dt = 2\left[\frac{t^3}{3}+t+t^2\right]_{-1}^{0} = 2\left[+\frac{1}{3}\cancel{+1}\cancel{-1}\right) = \frac{2}{3}$$

$P_M = 0$

5) Esponenziale negativo Causale

$$x(t) = A\,e^{-Bt}\,u(t) \qquad A>0 \quad B>0$$



Durata $= \infty$

$$\text{area} = \int_0^{\infty} A e^{-Bt}\,dt$$

$$= A\int_0^{\infty} e^{-Bt} = -\frac{A}{B}\left[e^{-Bt}\right]_0^{\infty}$$

$$= -\frac{A}{B}\left[0-1\right] = \frac{A}{B}$$

$$\langle x(t)\rangle = \lim_{t\to\infty} \frac{1}{T}\int_0^{+\infty} x(t)\,dt = 0$$

in quanto l'area è finita

$$E_{TOT} = \int_0^{\infty} \left|Ae^{-Bt}\right|^2 dt = \frac{A^2}{2B}$$

$$\| \int A^2\, e^{-2Bt}\,dt = \frac{A^2}{2B}$$

$P_M = 0$

6) $x(t) = A\cos(2\pi f_0 t + \varphi)$

Durata $= \infty$

area $= 0$ è finito $\Rightarrow \langle x(t)\rangle = 0$

$$\langle x(t)\rangle = \lim_{t\to\infty} \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) = 0$$

E tot $= +\infty$

$$P_M = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \left[A\cos(2\pi f_0 t + \varphi)\right]^2 dt$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2 \left(\frac{1+\cos(2\pi 2 f_0 t + 2\varphi)}{2}\right) dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{A^2}{2}\,dt + \underbrace{\frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A^2 \frac{\cos(2\pi 2 f_0 t + 2\varphi)}{2}\,dt}_{=0}$$

$$P_M = \frac{A^2}{2}$$

$$T = \frac{1}{2f_0} \quad = \frac{1}{T_0} T_0 \frac{A^2}{2} = \frac{A^2}{2}$$

Esercizio (2bis)

$$|z| = \sqrt{a^2+b^2}$$

$$\angle z = \operatorname{arctg} \frac{b}{a}$$

$$z = |z|\, e^{j\angle z}$$

$z_1 = j$

$z_2 = 1+j$

$z_3 = 1-j$

$z_4 = -\sqrt{3}+j$

$z_5 = 1+\sqrt{3}j$

Soluzioni per gli esercizi contrassegnati da (n) o [n].

(2bis)

$$z_1 = 1\cdot e^{+j\frac{\pi}{2}};\quad z_2 = \sqrt{2}\cdot e^{+j\frac{\pi}{4}};\quad z_3 = \sqrt{2}\cdot e^{-j\frac{\pi}{4}};\quad z_4 = 2\cdot e^{+j\frac{5\pi}{6}};\quad z_5 = 2\cdot e^{+j\frac{\pi}{3}}$$

$$z_6 = 2;\quad z_7 = \sqrt{2}\cdot e^{+j\frac{\pi}{4}} = z_2;\quad z_8 = 1\cdot e^{j\frac{\pi}{2}} = z_1;\quad z_9 = 2\cdot e^{+j\frac{5\pi}{6}} = z_4$$

$z_1 = 1\, e^{j\frac{\pi}{2}}$

$z_2 = \sqrt{2}\, e^{j\frac{\pi}{4}}$

$z_3 = \sqrt{2}\, e^{-j\frac{\pi}{4}}$

$z_4 = 2\, e^{j\frac{5}{6}\pi}$

$z_5 = 2\, e^{j\frac{\pi}{3}}$

$$z_8 = \frac{-\sqrt{3}+j}{1+\sqrt{3}j} = \frac{(-\sqrt{3}+j)(1-\sqrt{3}j)}{(1+\sqrt{3}j)(1-\sqrt{3}j)} = \frac{-\sqrt{3}+3j+j+\sqrt{3}}{4} = \frac{4j}{4} = j$$

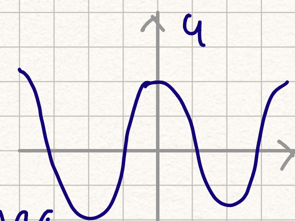
$$= e^{j\frac{\pi}{2}}$$

Esercizio 5

$u(t) = \delta(t+1) + \delta(t) + \delta(t-1)$

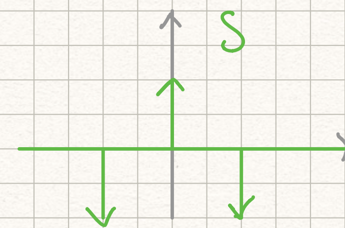
$y(t) = \cos(\pi t) \leftarrow T = 2 \qquad f = \frac{1}{T}$

$s(t) = u(t) \cdot y(t) = ?$



$s(t) = \cos(\pi t) \cdot \delta(t+1) + \cos \pi(t) \delta(t) + \cos(\pi t) \delta(t-1)$

$s(t) = -1\, \delta(t+1) + 1\, \delta(t) - 1\, \delta(t-1)$



Esercizio 8-9

$y(t) = A\, x(t)$ Stazionarietà? Linearità =?

Stazionario se $y_R(t) = y(t-t_0)$

$\downarrow$ $y_R(t) = A\, x_R(t) \leftarrow x(t-t_0)$

$x_R(t) = x(t-t_0)$ $\qquad y_R(t) = A\, x(t-t_0)$

$y(t-t_0) = A x(t-t_0)$

Linearità se $y(t) = T[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$

$x_1(t) \Rightarrow y_1(t) = A x_1(t)$

$x_2(t) \Rightarrow y_2(t) = A x_2(t)$ $\qquad$ OK stazionario

$y(t) = \lambda_1 A x_1(t) + \lambda_2 A x_2(t) = \lambda_1 A x_1(t) + \lambda_2 A x_2(t)$

OK lineare

Esercizio 9 b)

$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau$$

linearità

$$x_1(t) = y_1(t) = \int_{t-T}^{t} x_1(\tau)\, d\tau$$

$$x_2(t) = y_2(t) = \int_{t-T}^{t} x_2(\tau)\, d\tau$$

$$y(t) = \lambda_1 \underbrace{\int_{t-T}^{t} x_1(\tau)\, d\tau}_{y_1(t)} + \lambda_2 \underbrace{\int_{t-T}^{t} x_2(\tau)\, d\tau}_{y_2(t)} = \lambda_1 y_1(t) + \lambda_2 y_2(t)$$

Esercizio 11

① $y(t) = K + x(t)$

② $y(t) = t^2 x(t)$

Stazionarietà, Linearità

① Linearità

$x_1(t) = y_1(t) = K + x_1(t)$

$x_2(t) = y_2(t) = K + x_2(t)$

$$y(t) = K + \left[\lambda_1 x_1(t) + \lambda_2 x_2(t)\right] \neq \lambda_1 (K + x_1(t)) + \lambda_2 (K + x_2(t))$$

① Stazionarietà

$x_R(t) = x(t - t_0) \Rightarrow y_R(t) = K + x(t - t_0)$

$y(t - t_0) = K + x(t - t_0)$

(12) $y(t) = t^2 x(t)$

$x_1(t) = y_1(t) = t^2 x_1(t)$

$x_2(t) = y_2(t) = t^2 x_2(t)$

$y(t) = a_1 \underbrace{\left[t^2 x_1(t) \right]}_{y_1(t)} + a_2 \underbrace{\left[t^2 x_2(t) \right]}_{y_2(t)}$ $\Rightarrow$ è lineare

Stazionarietà

$x_R(t) = x(t-t_0)$ $\underline{y_R(t) = t^2 x(t-t_0)}$

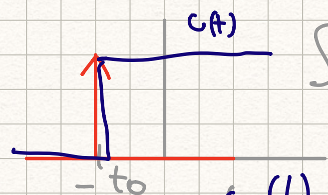
$\underline{y(t-t_0) = (t-t_0)^2 x(t-t_0)}$ non è stazionario

perchè le due equazioni sono diversi

13) Valutare risposta Impulsiva

1) $y(t) = \int_{-\infty}^{t} x(\tau + t_0)\, d\tau$ $h(t) = T[\delta(t)]$

$\hookrightarrow \int_{-\infty}^{t} \delta(\tau + t_0)\, d\tau$



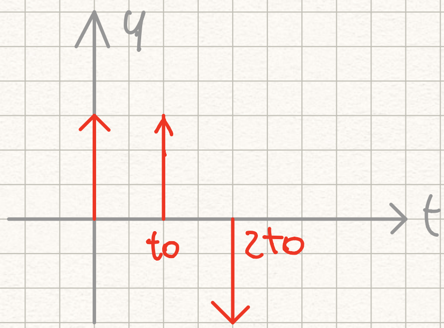
$\int_{-\infty}^{t} h(t)\, dt = u(t)$

$u(t) = \begin{cases} 1 & \text{se } t \geq -t_0 \\ 0 & \text{se } t < -t_0 \end{cases}$

2)

$y(t) = x(t) + x(t-t_0) - x(t-2t_0)$

$\hookrightarrow h(t) = \delta(t) + \delta(t-t_0) - \delta(t-2t_0)$

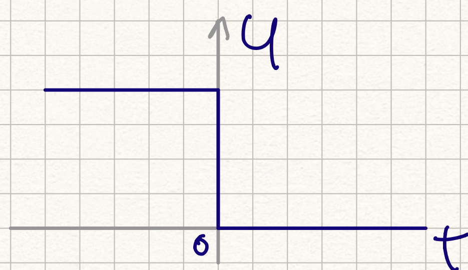


3)

$$y(t) = \int_t^{+\infty} A\, x(\tau)\, d\tau$$

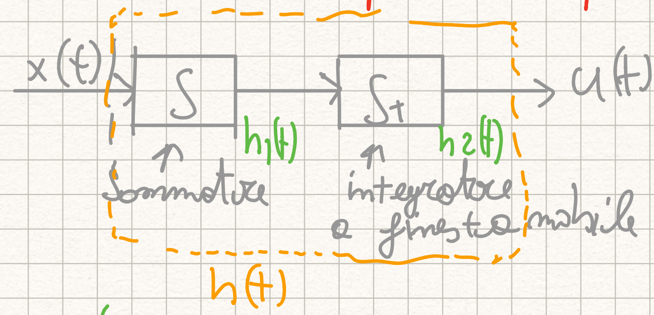
perchè gli estremi di integ sono scambiati

$$= \int_t^{+\infty} A\, \delta(\tau)\, d\tau = A \int_t^{+\infty} \delta\, \tau = u(-t) \Rightarrow h(t) = A\, u(-t)$$



17) Valutare la risposta impulsiva

2)

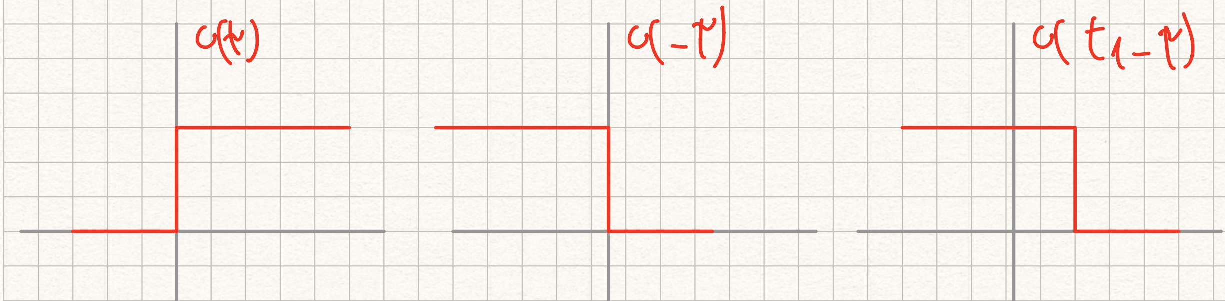


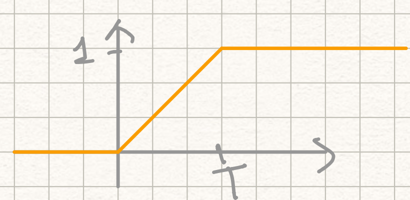
$$h(t) = h_1(t) * h_2(t)$$

$$h_1(t) = u(t)$$

$$h_2(t) = \Pi\left(\frac{t - T/2}{T}\right)$$

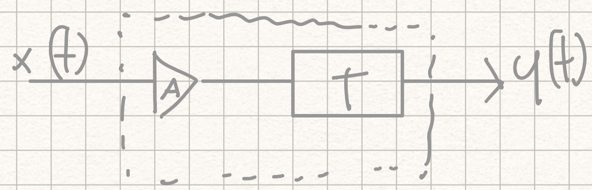
$$\int_{-\infty}^{+\infty} h_2(\tau) \quad h_1(t_1 - \tau)\, dt$$





$$\begin{cases} 0 & t < 0 \\ t & 0 \le t \le T \\ & t > T \rightarrow \int_0^T 1\, d\tau = T \end{cases}$$

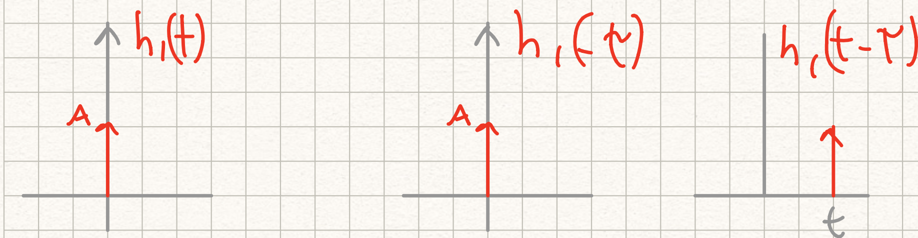
b)

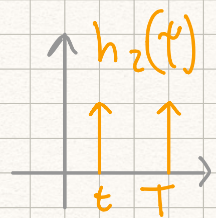


$h_1(t) = A\,\delta(t)$ ← mobile

$h_2(t) = \delta(t-T)$ ← fisso

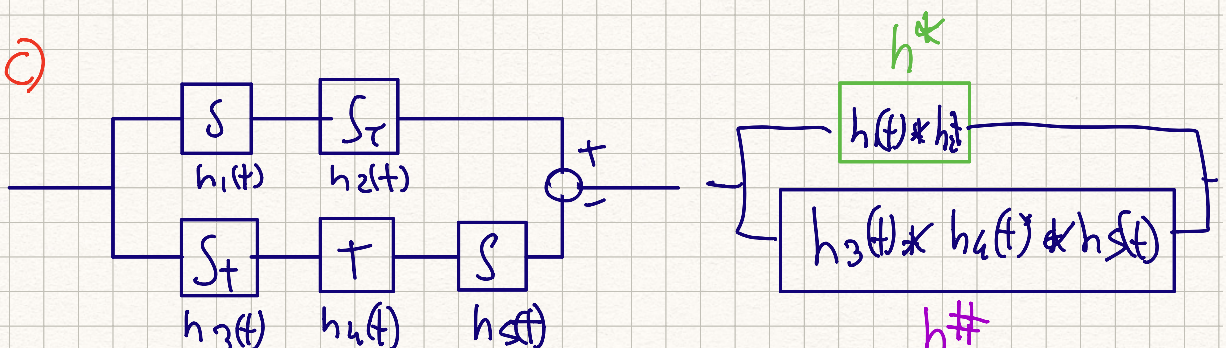
h





$$\Rightarrow \begin{cases} 0 & t<T \\ A & t=T \\ 0 & t>T \end{cases}$$

c)



$$h^* = \begin{cases} 0 & t<0 \\ t & 0 \le t \le T \\ T & t>T \end{cases}$$

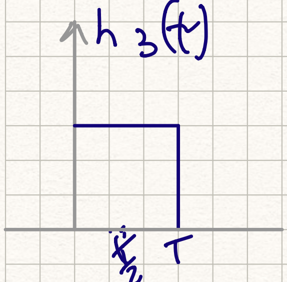
$$h^{\#} = h_3 * (h_4 * h_5) = h' = u(t-T$$

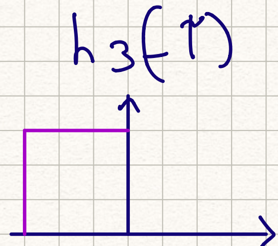
$$h_4 = \delta(t-T)$$
$$h_5 = u(t)$$

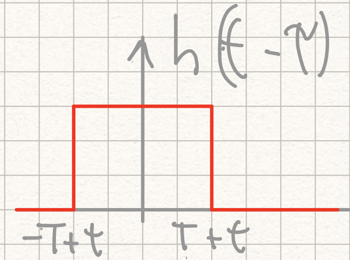
$$h^{\#} = h_3 * h'$$

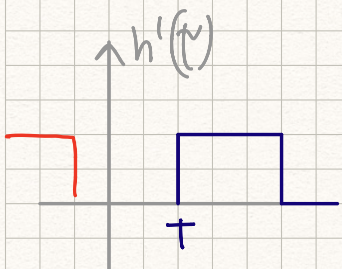
$$h_3(t) = \Pi\left(\frac{t-T/2}{T}\right) \leftarrow \text{mobile}$$

$$h' = u(t-T) \leftarrow \text{fisso}$$

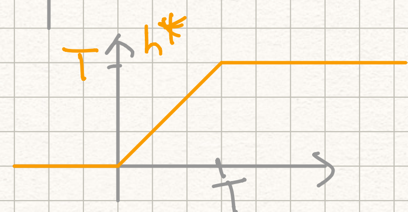




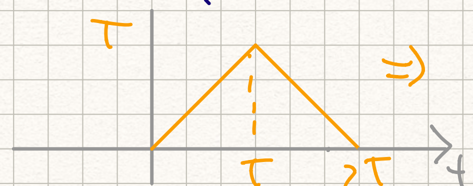




$$\Rightarrow \begin{cases} 0 & t<T \\ \int_T^t 1 = t-T & T<t<2T \\ T & t>2T \end{cases}$$



$$h_{tot} = h^* + (-h^{\#})$$



$$\Rightarrow \Lambda\,\Pi\left(\frac{t-T}{T}\right)$$

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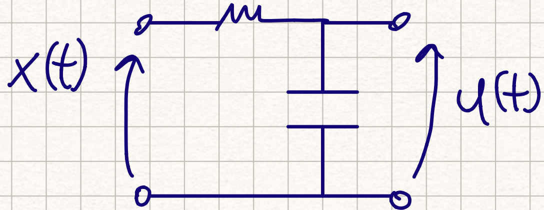
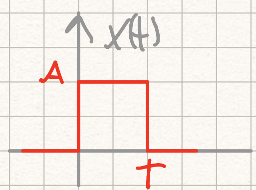
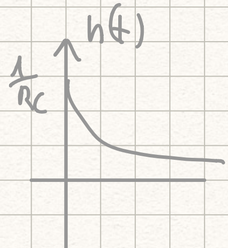


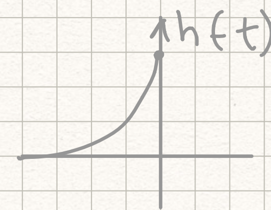
$$x(t) = A\,\Pi\left(\frac{t-\frac{T}{2}}{T}\right)$$

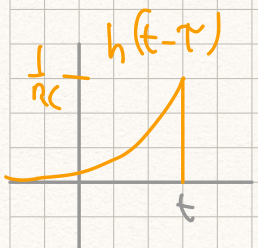
$$h(t) = \frac{1}{RC}\,e^{-t/RC}\,u(t)$$

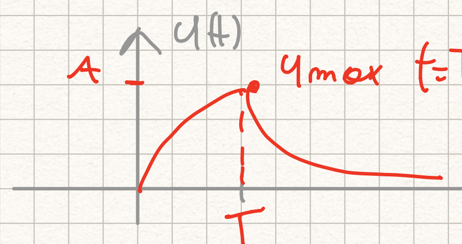
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$











y_{max} $t=T \Rightarrow A\left(1-e^{-T/RC}\right)$

$$y(t)\begin{cases} 0 & t<0 \\ \displaystyle\int_0^t A\frac{1}{RC}e^{-\frac{(t-\tau)}{RC}}d\tau = \frac{A}{RC}\int_0^t e^{-\frac{(t-\tau)}{RC}}d\tau = \frac{A}{\cancel{RC}}\cancel{RC}\left[e^{-\frac{(t-\tau)}{RC}}\right]_0^t & \text{cont} \\ \displaystyle\int_0^T A\frac{1}{RC}e^{-\frac{(t-\tau)}{RC}}d\tau = \frac{A}{\cancel{RC}}\cancel{RC}\left[e^{-\frac{(t-\tau)}{RC}}\right]_0^T = A\left[e^{-\frac{(t-T)}{RC}} - e^{-\frac{t}{RC}}\right] & \end{cases}$$

cont

$$= A\left(1-e^{-t/RC}\right)$$

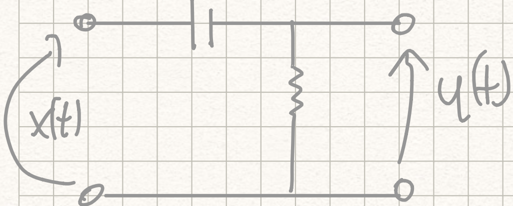
$$A\left[e^{-\frac{(t-T)}{RC}} - e^{-\frac{t}{RC}}\right] = A\,e^{-\frac{T}{RC}}\left[e^{\frac{t}{RC}} - 1\right]$$

$t > T$

$T \ll RC \Rightarrow y_{max} \simeq 0$

$T \gg RC \Rightarrow y_{max} = A$

(22) Ricavare risposta in fase e in Ampiezza del circuito CR



$$h(t) = \delta(t) - \frac{1}{RC} e^{-t/RC} u(t)$$

$$H(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi f t}\, dt$$

$$\int_{-\infty}^{+\infty} \delta(t) - \frac{1}{RC} e^{-t/RC} u(t)\, e^{-j2\pi f t}\, dt =$$

$$= \underbrace{\int_{-\infty}^{+\infty} \delta(t)\, e^{-j2\pi f t}\, dt}_{=1} - \frac{1}{RC}\int_{0}^{\infty} e^{-t/RC} e^{-j2\pi f t}\, dt =$$

tolgo il gradino unitario e metto zero

$$= 1 - \frac{1}{RC}\int_{0}^{+\infty} e^{-\left(\frac{t}{RC} + j2\pi f t\right)}\, dt$$

$$= 1 - \left[\frac{1}{RC}\, \frac{e^{-\left(\frac{t}{RC} + j2\pi f t\right)}}{-\left(\frac{1}{RC} + j2\pi f\right)}\right]_{0}^{\infty} = 1 - \frac{1}{1 + j2\pi RCf}$$

$$= \frac{\cancel{1} + j2\pi RCf - \cancel{1}}{1 + j2\pi RCf}$$

$$A_H(f) = \frac{|2\pi RCf|}{\sqrt{1+(2\pi RCf)^2}}$$

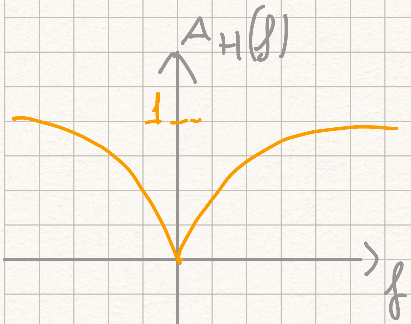
$$\varphi_H(f) = \text{arctg}(num) - \text{arctg}[den] = \text{arctg}\underbrace{\left[\frac{2\pi fRC}{0}\right]}_{\pm\infty} - \text{arctg}\left(\frac{2\pi fRC}{1}\right)$$

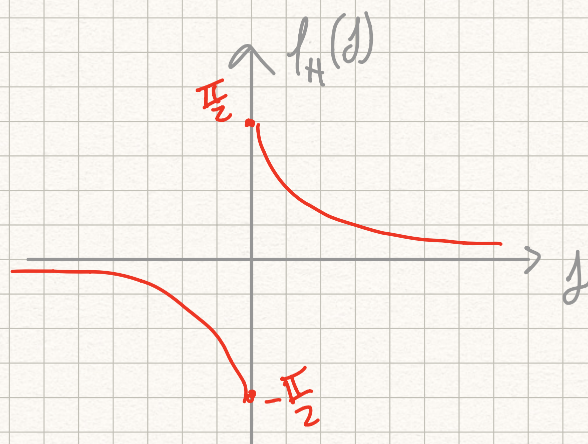
$$\varphi_H(0^+) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$\varphi_H(0^-) = -\frac{\pi}{2} - 0 = -\frac{\pi}{2}$$

$$\varphi_H(\infty) = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

$$\varphi_H(-\infty) = -\frac{\pi}{2} + \frac{\pi}{2} = 0$$





(26) Calcolare il coefficiente di Fourier

1) $x(t) = \text{sen}(2\pi f_0 t)$ $\qquad X_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-j2\pi k f_0 t} dt$

$$X = \frac{1}{T_0}\int_{T_0} \text{sen}\, 2\pi f_0 t\; e^{-j2\pi k f_0 t} dt$$

$$\boxed{e^{-j\pi f} = \cos(\pi f) - \text{sen}(\pi f)}$$

$$= \frac{1}{T_0}\int_{T_0} \text{sen}(2\pi f_0 t)\left[\cancel{\cos(2\pi k f_0 t)} - j\,\text{sen}\, 2\pi k f_0 t\right] dt$$

si elimina in quanto il segnale da calcolare è dispari e invece coseno è pari

$$= \frac{1}{T_0}\int_{T_0} -j\,\text{sen}(2\pi f_0 t)\,\text{sen}(2\pi k f_0 t)\, dt =$$

$$\boxed{\text{sen}(\alpha)\,\text{sen}(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)}$$

$$= \frac{-j}{T_0}\left[\int_{T_0}\frac{1}{2}\cos 2\pi(k-1)f_0 t\, dt - \int \frac{1}{2}\cos 2\pi(k+1) f_0 t\, dt\right]$$

$$= \frac{-j}{2T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(2\pi(k-1)f_0 t)\, dt + \frac{j}{2T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(2\pi(k+1)f_0 t)\, dt$$

$$= -\frac{j}{2\cancel{T_0}}\left[\frac{\text{sen}\, 2\pi(k-1)f_0 t}{2\pi(k-1)\cancel{f_0}}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} + \frac{j}{2\cancel{T_0}}\left[\frac{\text{sen}\, 2\pi(k+1)f_0 t}{2\pi(k+1)\cancel{f_0}}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}}$$

$$= -\frac{j}{\cancel{2}}\,\cancel{2}\left[\frac{\text{sen}(\pi(k-1)\cancel{f_0}(\frac{\cancel{T_0}}{2}))}{2\pi(k-1)}\right] + \frac{j}{\cancel{2}}\,\cancel{2}\left[\frac{\text{sen}\,\pi(k+1)}{2\pi(k+1)}\right]$$

$$= -\frac{j}{2}\,\frac{\text{sen}\,\pi(k-1)}{\pi(k-1)} + \frac{j}{2}\,\frac{\text{sen}(\pi(k+1))}{\pi(k+1)} =$$

$$= -\frac{j}{2}\,\text{sinc}(k-1) + \frac{j}{2}\,\text{sinc}(k+1)$$

$$X_k = \begin{cases} -\frac{j}{2} & k=1 \Rightarrow \frac{1}{2} e^{-j\frac{\pi}{2}} \\ \frac{j}{2} & k=-1 \Rightarrow \frac{1}{2} e^{+j\frac{\pi}{2}} \\ 0 & k \neq \pm 1 \end{cases}$$

2) $x(t) = \cos(2\pi f_0 t)$

cos

$$X_k = \frac{1}{T_0}\int_{T_0} (\cos 2\pi f_0 t)\left[\cos(2\pi k f_0 t) - j\cancel{\sin(2\pi k f_0 t)}\right] dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{1}{2}\left[\cos(2\pi f_0 t - 2\pi k f_0 t) + \cos(2\pi f_0 t + 2\pi k f_0 t)\right] dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{1}{2}\left[\cos 2\pi(1-k) f_0 t + \cos 2\pi(k+1) f_0 t\right]$$

$$= \frac{1}{2\cancel{T_0}}\left\{\left[\frac{\sin(2\pi(1-k) f_0 t)}{2\pi(1-k)\cancel{f_0}}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} + \left[\frac{\sin(2\pi(k+1) f_0 t)}{2\pi(k+1) f}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}}\right\}$$

$$= \frac{1}{2}\left[\frac{\sin\left(\pi(1-k)\cancel{f_0}\frac{\cancel{T_0}}{2}\right)}{2\pi(1-k)} - \frac{\sin \pi(1-k) f_0 \left(-\frac{T_0}{2}\right)}{2\pi(1-k)} + \sin\left(\cancel{2}\pi(k+1)\cancel{f_0}\left(\frac{\cancel{T_0}}{2}\right)\right) + \right.$$

$$\left. - \frac{\sin\left(\cancel{2}\pi(k+1)\cancel{f_0}\left(\frac{\cancel{T_0}}{2}\right)\right)}{2\pi(1-k)}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{2}\sin\left(\frac{\pi(1-k)}{\cancel{2}}\right)}{\cancel{2}\pi(1-k)} + \frac{\cancel{2}\sin\left(\frac{\pi(k+1)}{\cancel{2}}\right)}{\cancel{2}\pi(k+1)}\right]$$

$$= \frac{1}{2}\frac{\operatorname{sen}(\pi(1-k))}{\pi(1-k)} + \frac{1}{2}\frac{\operatorname{sen}(\pi(k+1))}{\pi(k+1)}$$

$$= \frac{1}{2}\operatorname{sinc}(k+1) + \frac{1}{2}\operatorname{sinc}(k-1)$$

$$X_k = \begin{cases} \frac{1}{2} & \text{se } k = \pm 1 \\ 0 & \text{altrove} \end{cases}$$

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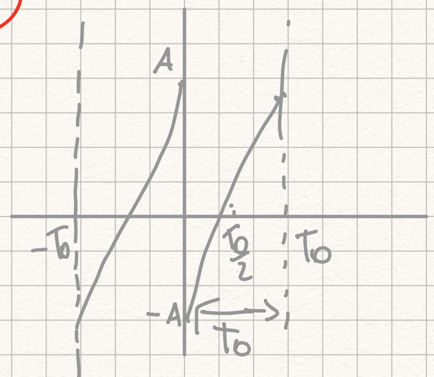


$$\boxed{\begin{array}{l} X_k = A_k e^{j\varphi_k} \quad \text{N.B} \\ X_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j2\pi k f_0 t}\, dt \end{array}}$$

$$x(t) = \frac{2A}{T_0}\left(t - \frac{T_0}{2}\right)$$

$$X_k = \frac{1}{T_0}\int \frac{2A}{T_0}\left(t-\frac{T_0}{2}\right)\left[-j\operatorname{sen}(2\pi k f_0 t)\right] dt$$

$$= \frac{1}{T_0}\frac{2A}{T_0}\left[\int_{T_0} -jt\operatorname{sen}(2\pi k f_0 t)\, dt + j\frac{T_0}{2}\int_{T_0} \operatorname{sen} 2\pi k f_0 t\, dt\right]$$

$$= \frac{2A}{T_0^2}\left[-j\int_{T_0} t\operatorname{sen} 2\pi k f_0 t\, dt + j\frac{T_0}{2}\int_{T_0} \operatorname{sen} 2\pi k f_0 t\, dt\right]$$

Calcolo integrale di $\int t \ \operatorname{sen}(2\pi K f_0 t)\,dt$

$$-t\,\frac{\cos(2\pi K f_0 t)}{2\pi K f_0} - \int \frac{\cos 2\pi K t}{2\pi K f_0}\,dt$$

$$= \frac{2A}{T_0^2}\left[+J\left\{\left[\frac{t\cos(2\pi K f_0 t)}{2\pi K f_0}\right]_0^{T_0} - \frac{J T_0}{2}\int_{T_0} \frac{\cos(2\pi K f_0 t)}{2\pi K f_0}\,dt\right\}\right.$$

$$= \frac{2A}{T_0^{\,2}}\left[J\left\{\frac{T_0\cos(2\pi K)}{2\pi K f_0} - \left[\frac{\operatorname{sen}(2\pi K f_0 t_0)}{(2\pi K f_0)^2}\right]_0^{t_0}\right\}\right.$$

$$+\frac{J T_0}{2}\left[\frac{\cos(2\pi K f_0 t)}{2\pi K f_0}\right]_0^{t_0} =$$

$$= J2A\,\frac{\cos(2\pi K)}{2\pi K} - J2A\,\frac{\operatorname{sen} 2\pi K}{(2\pi K)^2} - JA\left[\frac{\cos(2\pi K)-1}{2\pi K}\right]$$

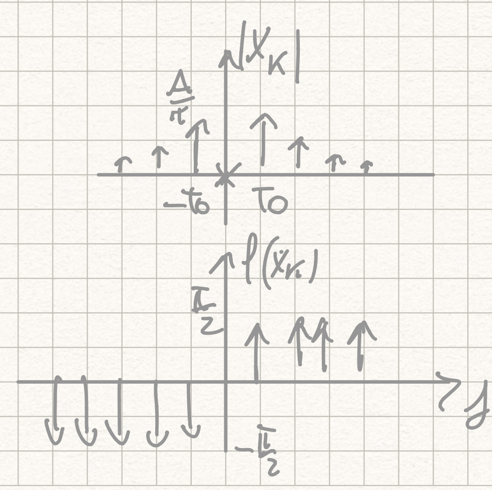
$$= J2A\,\frac{\cos(2\pi K)}{2\pi K} - J2A\,\frac{\operatorname{sen}(2\pi K)}{(2\pi K)^2} - JA\,\frac{\cos(2\pi K)-1}{2\pi K}$$

$$= JA\,\frac{2(2\pi K)\cos(2\pi K) - 2\operatorname{sen}(2\pi K) - 2\operatorname{sen}^2(\pi K)}{(2\pi K)^2}$$

NB

$$\cos(x) - 1 = 2\operatorname{sen}^2\left(\frac{x}{2}\right)$$

$$X_K = \frac{JA}{\pi K}$$



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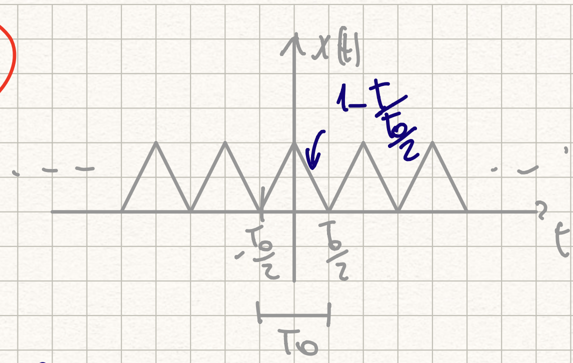


$$x(t) = \sum_{m=-\infty}^{\infty} \Lambda\left(\frac{t - m T_0}{T_0/2}\right)$$

$$\bar{X}_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j2\pi k f_0 t}\, dt$$

$$X_k = \frac{1}{T_0}\, 2 \int_0^{\frac{T_0}{2}} \left(1 - \frac{t}{T_0/2}\right) \cos(2\pi k f_0 t)\, dt$$

perchè pari e simmetrico

$$= \frac{2}{T_0}\int_0^{\frac{T_0}{2}} \cos(2\pi k f_0 t)\, dt - \frac{2}{T_0}\int_0^{\frac{T_0}{2}} t \cos(2\pi k f_0 t)\, dt]$$

$$= \frac{2}{T_0}\left[\frac{\operatorname{sen}(2\pi k f_0 t)}{2\pi k f_0}\right]_0^{\frac{T_0}{2}} - \frac{2}{T_0}\left\{\left[t\,\frac{\operatorname{sen}(2\pi k f_0 t)}{2\pi k f_0}\right]_0^{\frac{T_0}{2}} - \int_0^{\frac{T_0}{2}} \frac{\operatorname{sen}(2\pi k f_0 t)}{2\pi k f_0}\, dt\right]$$

$$= \frac{2}{T_0}\left[\frac{\operatorname{sen}(2\pi k f_0 \frac{T_0}{2})}{2\pi k f_0}\right) - \frac{2}{T_0}\left[\frac{T_0}{2}\,\frac{\operatorname{sen}(2\pi k f_0 \frac{T_0}{2})}{2\pi k f_0} - \frac{\cos(2\pi k f_0 \frac{T_0}{2})}{2\pi k f}\right]\right]$$

$$= \frac{2}{T_0}\,\frac{\operatorname{sen}\pi k}{2\pi k f_0} - \frac{\cancel{4}^{2}}{\cancel{T_0}^{2}}\,\frac{T_0}{2}\,\frac{\operatorname{sen}\pi k}{2\pi k f_0} + \frac{4}{T_0^2}\,\frac{\cos\pi k}{(2\pi k f_0)^{2}}{}^{-1}$$

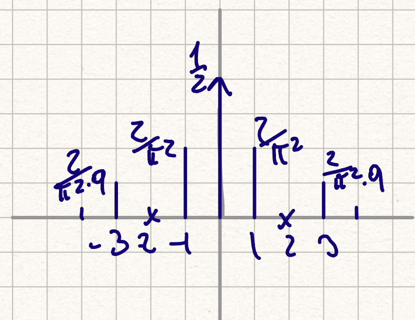
$$\cancel{\text{sen}\,\frac{\pi k}{\pi k}} - \cancel{\frac{\text{sen}\,\pi k}{\pi k}} + \frac{\cancel{T_0}}{\cancel{T_0^2}}\,\frac{\cos \pi k - 1}{(2\pi k f_0)^2}$$

$$= \frac{\cos(\pi k) - 1}{(\pi k)^2} \Rightarrow \text{Per prop di prima} \quad \frac{1\,\cancel{2}\,\text{sen}\left(\frac{\pi}{2} k\right)^2}{2\cdot 4 \left(\frac{\pi k}{2}\right)^2} = \frac{1}{2}\,\text{sinc}(k)$$

$$\Uparrow$$

$$2\,\frac{\sin^2\left(\frac{\pi}{2} k\right)}{(\pi k)^2}$$

$$X_k : \begin{cases} \frac{1}{2} & k = 0 \\ 0 & k \text{ pari} \\ \frac{2}{\pi^2 k^2} & k \text{ dispari} \end{cases}$$



Esercizio 34

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$x(t) = \Pi(t)$ $\qquad Y(f) = ? \qquad \mathcal{F}^{-1}[Y(f)] = ?$

$h(t) = \Pi(t)$

$y(t) = x(t) * h(t)$

$Y(f) = X(f)\, H(f)$

$X(f) = \mathrm{sinc}(f)$

$H(f) = \mathrm{sinc}(f)$

$Y(f) = \mathrm{sinc}(f) \cdot \mathrm{sinc}(f) = \mathrm{sinc}^2(f)$

$y(t) = \mathcal{F}^{-1}[\mathrm{sinc}^2(f)] = \Lambda(t)$

Esercizio 44 Calcolare $\mathcal{F}[x(t) = e^{-|t|}]$ e disegnare il grafico

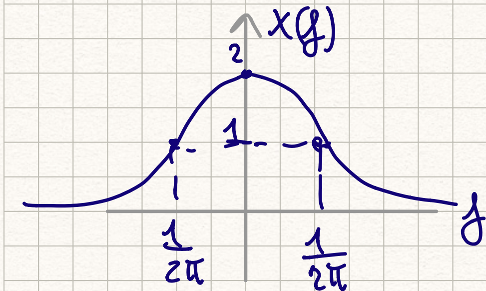
Sup $x(t) = s(t) + s(-t)$ $\qquad e^{-t}u(t) \longleftrightarrow \dfrac{1}{1+j2\pi f}$

$s(t) = e^{-t}u(t)$ $\qquad x(-t) \longleftrightarrow X(-f)$

$$X(f) = \mathcal{F}[s(t) + s(-t)] = \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f}$$

$$= \frac{1 - \cancel{j2\pi f} + 1 + \cancel{j2\pi f}}{(1+j2\pi f)(1-j2\pi f)} = \frac{2}{1+(2\pi f)^2}$$

Disegno lo spettro $X(f)$



Esercizio 45 Calcolare spettro $x(t) = \mathrm{sinc}(t)$

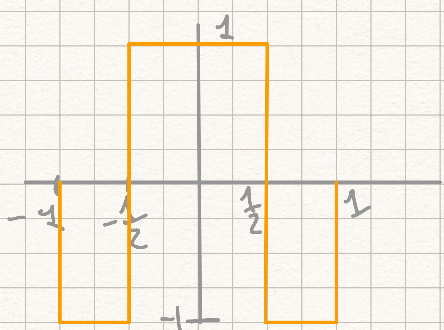
$E_x = ?$

$$X(t) \longleftrightarrow x(-f)$$

$X(f) = \sqcap(-f)$ ma è pari $\Rightarrow \sqcap(f)$

$$E_x = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \, dt = \left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right] = 1 \, J$$

Esercizio 45 bis T.d.F di $x(t)$



$x(t) \; s_1(t) + s_2(t)$

$s_1(t) = \sqcap(t)$

$s_2(t) = \sqcap\left(\frac{t}{2}\right)$

$x(t) = 2\,s_1(t) - s_2(t)$

$X(f) = 2S_1(f) - S_2(f) \Rightarrow 2\,\text{sinc}(f) - 2\,\text{sinc}\,2f$

Esercizio 46

$s(t) = e^{-\frac{t}{T}} u(t)$

$w(t) = e^{-t} u(t) \Rightarrow w(t) = s(t \cdot T)$

$S(f) = \dfrac{1}{1+j2\pi f T}$ $\qquad \Downarrow\; W(f) = \dfrac{1}{T} S\left(\dfrac{f}{T}\right)$

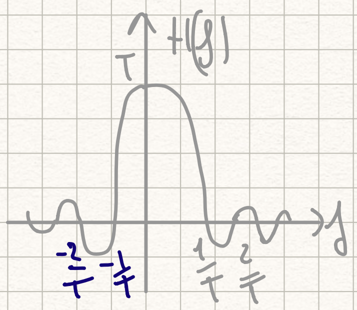
$W(f) = \dfrac{1}{\cancel{T}} \dfrac{1}{1+j2\pi f \cancel{T} \cdot \frac{1}{\cancel{T}}} \cancel{T} = \dfrac{1}{1+j2\pi f}$

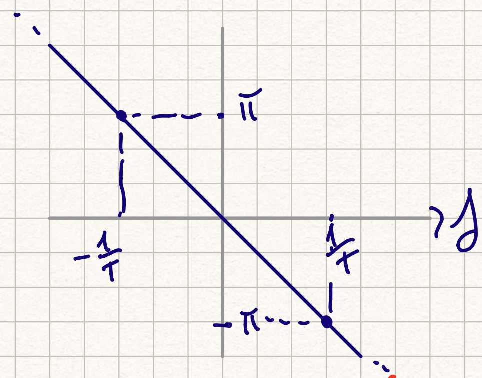
Esercizio 49 Calcolare

$h(t) = \Pi\left(\dfrac{t - \frac{T}{2}}{T}\right)$

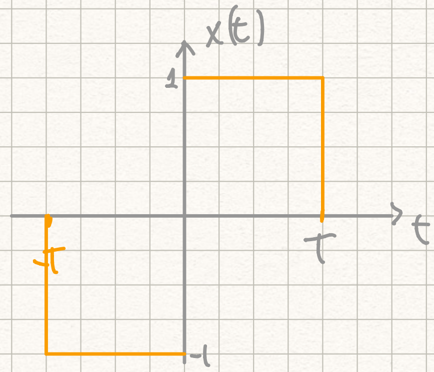
$H(f) = ?$ Disegnare grafico di modulo e fase di $H(f)$

$H(f) = \underbrace{|T|\,\text{sinc}(fT)}_{A_H(f)}\; \underbrace{e^{-j2\pi f \frac{T}{2}}}_{e^{-j\pi f T}}$ $\qquad T = \dfrac{1}{f}$





Esercizio 51 g.d.F A_x φ_x di $X(f)$



$s_1(t) = \Pi\left(\frac{t}{T} - \frac{T}{2}\right)$

$s_2(t)\ \Pi\left(\frac{t}{T} + \frac{T}{2}\right)$

$$T\,\text{sinc}(Tf)\,e^{-j2\pi f\frac{T}{2}} - T\,\text{sinc}\,Tf\,e^{-j2\pi f\left(-\frac{T}{2}\right)}$$

$$= jT\,\text{sinc}(Tf)\left[\frac{e^{-j\pi fT} - e^{j\pi fT}}{2j}\right]$$ eulero

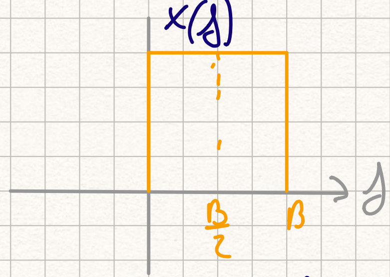
$$= -2jT\,\text{sinc}(fT)\,(\text{sen}\,\pi fT)$$

$$= -2jT\,\frac{\sin(\pi fT)}{\pi fT}\,\text{sen}(\pi fT)$$

$$A_x = 2T\,\frac{\text{sen}^2(\pi fT)}{(\pi fT)}$$

$$\varphi_x = -j = \begin{cases} -\frac{\pi}{2} & f > 0 \\ \frac{\pi}{2} & f < 0 \end{cases}$$

5a Data $X(f)$ Calcolare $x(t)$



$$X(t)\, e^{-j2\pi f t} \longleftrightarrow X(f+f_0)$$

$$X(f) = \Pi\left(\frac{f-\frac{B}{2}}{B}\right)$$

$x(t) = ?$

$$x(t) = B \operatorname{sinc}(Bt)\, e^{+j2\pi t \frac{B}{2}} \Rightarrow B \operatorname{sinc}(Bt)\, e^{j\pi B t}$$

5b

$$x(t) = \Pi\left(\frac{t}{T}\right) \cos(2\pi f_0 t)$$

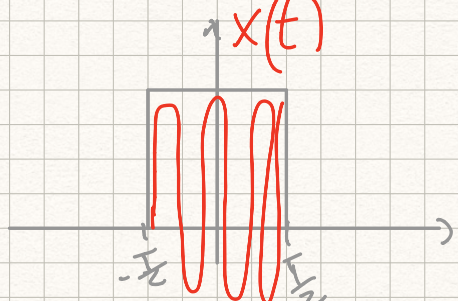
$f_0 = \frac{20}{T} \Rightarrow T_0 = \frac{T}{20}$

$$x(t)\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2} X(f-f_0) + \frac{1}{2} X(f+f_0),$$

Trovare $\mathcal{F}[x(t)]$?

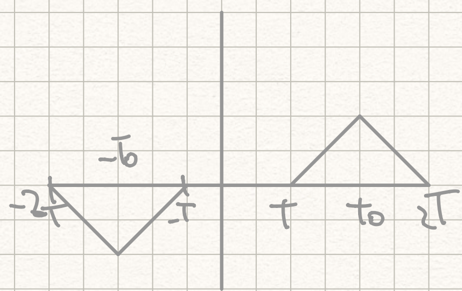
$$X(f) = \frac{T}{2} \operatorname{sinc}(T(f-f_0)) + \frac{T}{2} \operatorname{sinc}(T(f+f_0))$$

$$= \frac{T}{2} \operatorname{sinc}(T(f-20)) - \frac{T}{2} \operatorname{sinc}(T(f+20))$$



Esercizio

$$x(t) = \Lambda\left(\frac{t-t_0}{T}\right) - \Lambda\left(\frac{t+t_0}{T}\right)$$



$$X(f) = T\,\mathrm{sinc}^2(fT)\,e^{-j2\pi f t_0} - T\,\mathrm{sinc}^2(fT)\,e^{j2\pi f t_0}$$

$$= -T\,\mathrm{sinc}^2(fT)\left[e^{-j2\pi f t_0} - e^{j2\pi f t_0}\right)$$

aggiungendo

$$= -2jT\,\mathrm{sinc}^2 fT\left[\frac{e^{j2\pi f t_0} - e^{-j2\pi f t_0}}{2j}\right] \rightarrow \text{eulero}$$

$$= -2jT\,\mathrm{sinc}^2(fT)\;\mathrm{sen}(2\pi f t_0)$$

$$A_x = 2T\,\mathrm{sinc}^2(fT)\;\mathrm{sen}(2\pi f t_0)$$

$$\varphi_x = -j \cdot \begin{cases} -\frac{\pi}{2} & f > 0 \\ \frac{\pi}{2} & f < 0 \end{cases}$$

Esercizio 67

$X(f)$?

$x(t) = t \cdot \Pi\left(t - \frac{1}{2}\right)$

$-j2\pi t \, x(t) \xleftrightarrow{\mathcal{F}} \frac{dX(f)}{df}$

$= \frac{1}{-j2\pi}\left(-j2\pi t \underbrace{\Pi\left(t-\frac{1}{2}\right)}_{s(t)}\right)$

$S(f) = \text{sinc}(f) e^{-j\pi f}$

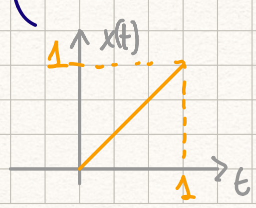
$$\frac{1}{-j2\pi} \frac{d\,\text{sinc}(f)\, e^{-j\pi f}}{df}.$$

sinc la posso scrivere come $\frac{\sin(\pi f)}{\pi f} e^{-j\pi f}$

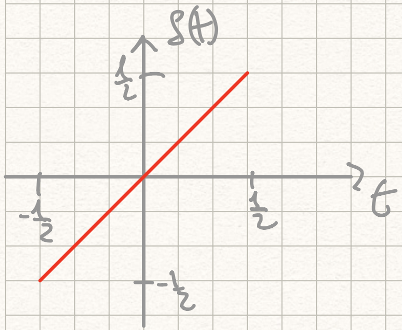
Faccio la derivata $= \dfrac{\pi f \, \pi \cos(\pi f) - \pi \sin(\pi f) e^{-j\pi f} \; \pi \overbrace{\frac{\sin(\pi f)}{\pi f}}^{\text{sinc}} e^{-j\pi f}}{(\pi f)^2}$

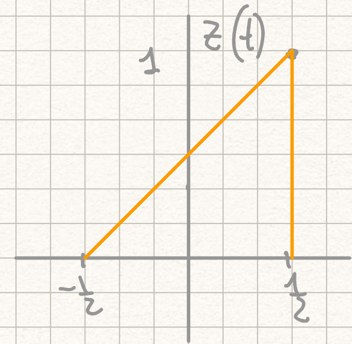
$$X(f) = \frac{1}{-j2\cancel{\pi}}\left[\frac{\cancel{\pi}\pi f\cos(\pi f) - \pi\sin(\pi f) - j\cancel{\pi}\,\text{sinc}(f)}{(\pi f)^2}\right] e^{-j\pi f}$$

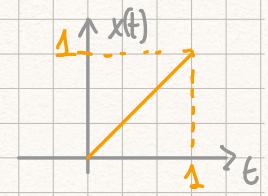
$$= \left\{\frac{1}{2}\,\text{sinc}(f) + \frac{1}{2j\pi f}\left[\text{sinc}(f) - \cos(\pi f)\right]\right\} e^{-j\pi f}$$



Esercizio 68







Risolvo z(t)

$$z(t) = x\left(t + \frac{1}{2}\right)$$

$$Z(f) = X(f)\, e^{+j\pi} \Rightarrow \left\{ \frac{1}{2}\,\mathrm{sinc}(f) + \frac{1}{2j\pi f}\left(\mathrm{sinc}(f) - \cos(\pi f)\right) \right\} \cancel{e^{-j\pi f}}\, \cancel{e^{j\pi f}}$$

Risolvo s(t)

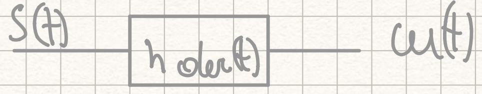
$$s(t) = z(t) - \frac{1}{2}\,\Pi(t)$$

$$S(f) = \cancel{\frac{1}{2}\,\mathrm{sinc}(f)} + \frac{1}{2j\pi f}\left[\mathrm{sinc}(f) - \cos(\pi f)\right] - \cancel{\frac{1}{2}\,\mathrm{sinc}(f)}$$

$$= \frac{1}{2j\pi f}\left[\mathrm{sinc}(f) - \cos(\pi f)\right]$$

Esercizio 6p bis ??

$s(t) = t \cdot \Pi(t)$ $\qquad w(t) \qquad H(f) = j2\pi f$

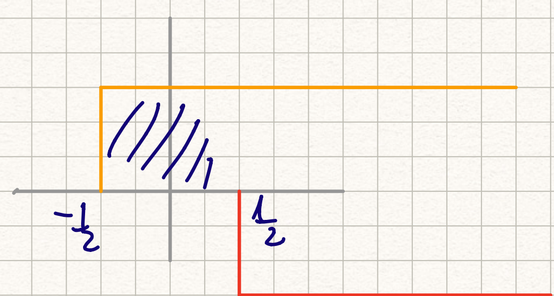


$$W(f) = S(f) \cdot j2\pi f$$

$$= \frac{1}{\cancel{j2\pi} f}\left[\text{sinc}(f) - \cos(\pi f)\right] \cdot j\cancel{2\pi} f \Rightarrow W(f) = \text{sinc}(f) - \cos(\pi f)$$

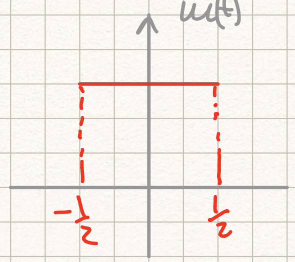
$s(t) = t \cdot \Pi(t)$ $\qquad w(t) = \frac{ds(t)}{dt}$

$$\Pi(t) = \underbrace{u\left(t+\frac{1}{2}\right)} - u\left(t-\frac{1}{2}\right)$$



$$w(t) = \frac{d\Pi(t)}{dt} = \frac{d}{dt}\left\{u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)\right\} = \delta\left(t+\frac{1}{2}\right) - \delta\left(t-\frac{1}{2}\right)$$

$$\frac{d\, t\,\Pi(t)}{dt} = \Pi(t) + \overset{-\frac{1}{2}}{t}\,\delta\left(t+\frac{1}{2}\right) - \overset{\frac{1}{2}}{t}\,\delta\left(t-\frac{1}{2}\right)$$



Esercizio 69

$S(f) = ?$ $\qquad s(t) = \mathrm{sinc}^2(t) = x(t) \cdot x(t)$

$x(t) = \mathrm{sinc}(t)$

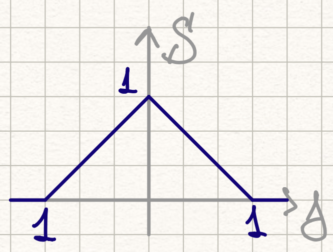
N.B

$$\boxed{\begin{array}{l} x(t) \cdot y(t) \xleftrightarrow{\mathcal{F}} X(f) * Y(f) \\ x(t) * y(t) \xleftrightarrow{\mathcal{F}} X(f) \cdot Y(f) \end{array}}$$

$S(f) = X(f) * X(f)$

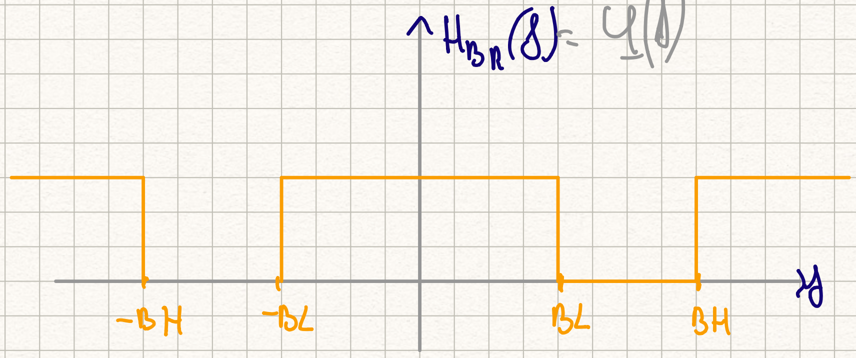
$X(f) = \Pi(f)$

$S(f) = \Pi(f) * \Pi(f) = \Lambda(f)$



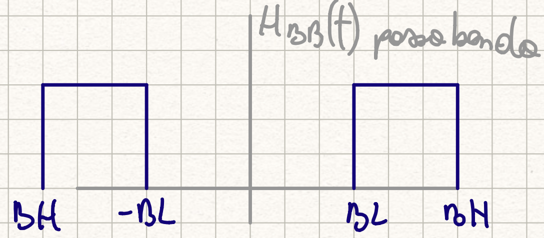
Esercizio 74

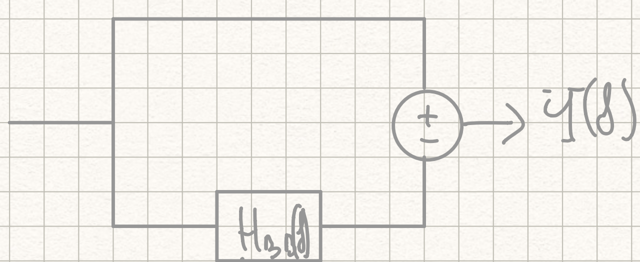
Schema a blocchi ?



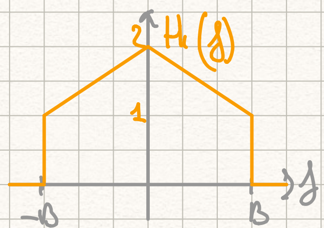
$$f_0 = \frac{B_L + B_H}{2}$$

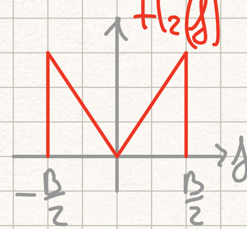
$$B = B_H - B_L$$

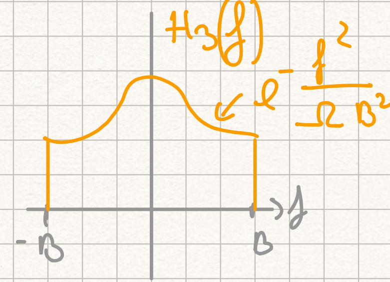




Esercizio 76



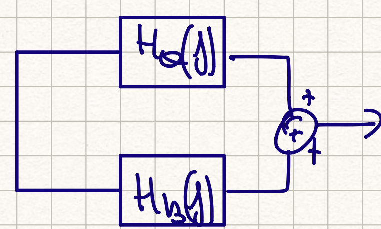




$H_1(f)$

$$H_a(f) = \Pi\left(\frac{f}{2B}\right)$$

$$H_b(f) = \Lambda\left(\frac{f}{B}\right)$$



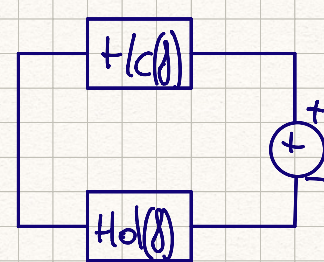
$$h_a(t) = 2B \operatorname{sinc}(Bt)$$

$$h_b(t) = B \operatorname{sinc}^2(Bt)$$

$H2(f)$

$$H_c(f) = \Pi\left(\frac{f}{B}\right)$$

$$H_d(f) = \Lambda\left(\frac{2f}{B}\right)$$



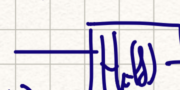
$$h_c(t) = B \operatorname{sinc}(Bt)$$

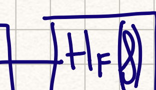
$$h_d(t) = \frac{B}{2} \operatorname{sinc}^2\left(\frac{B}{2} t\right)$$

$H_3(f)$

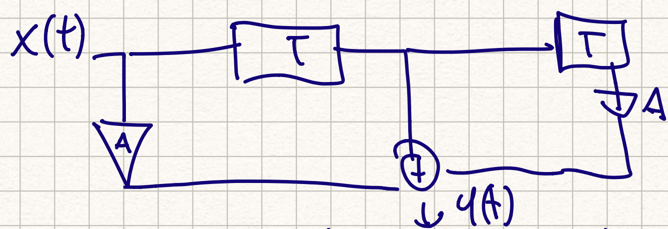
$$H_E(f) = e^{-\frac{f^2}{2B^2}} \qquad h_e(t) = \sqrt{2\pi}\ t\ e^{-2\pi^2 t^2 T^2}$$

$$H_F(f) = \Pi\left(\frac{f}{2B}\right) \qquad h_f(t) = 2B \operatorname{sinc}(2Bt)$$





Esercizio 83



$$h(t) = A\,\delta(t) + \delta(t-T) + A\,\delta(t-2T)$$

$\Updownarrow \mathcal{F}$

$$H(f) = A + e^{-j2\pi f t} + A\,e^{-j2\pi f 2t}$$

$$\Rightarrow e^{-j2\pi f t}\left(A\,e^{j2\pi f t} + 1 + A\,e^{-j2\pi f t}\right)$$

$$\Rightarrow e^{-j2\pi f t}\left(1 + 2A\cos(2\pi f t)\right)$$

$$A_H = 1 + 2A\cos(2\pi f t)$$

$$\varphi_H = -2\pi f t$$

85

N.B $x(t)$ reale $\rightarrow X(f) = X^*(-f)$

$$x(t) = e^{j(2\pi f t + \varphi)}$$

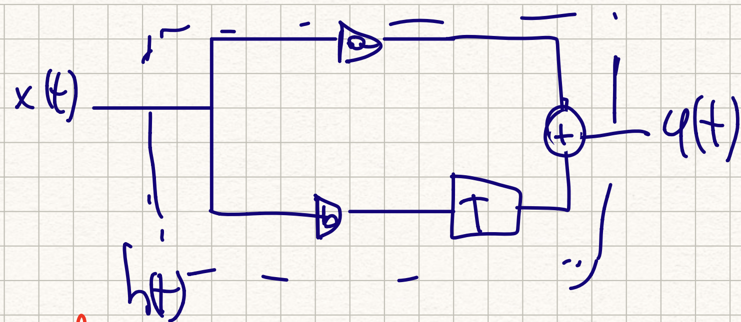
$$\Rightarrow \underbrace{e^{j2\pi f T}}_{\substack{\uparrow \\ \text{traslazione in frequenza}}} + e^{j\varphi} \Rightarrow X(f) = e^{j\varphi}\,\delta(f-f_0)$$

traslazione in frequenza $\Rightarrow \delta(f-f_0)$

$X(f)$ Hermitiana $\Rightarrow \varphi = k\pi$

Antihermitiana $X(f)$ puramente immaginario $\Rightarrow \varphi = \pm\frac{\pi}{2} + k\pi$

82



Risposta impulsiva

$$h(t) = a\,\delta(t) + b\,\delta(t-T)$$

N.B. $e^{-j\varphi} = \cos(\varphi) - j\,\mathrm{sen}(\varphi)$

$$\Updownarrow \mathcal{F}$$

$$H(f) = a + b\,e^{-j2\pi f T} = a + b\cos(2\pi f T) - jb\,\mathrm{sen}(2\pi f T)$$

89

$x_0(t)$ e $s(t) \Rightarrow X_0(f)$ e $S(f)$ coincidono?

$$f_0 X_0(k f_0) = f_0 S(k f_0)$$

$$x_0(t) = \Lambda\left(\frac{t}{\frac{3T_0}{4}}\right) \qquad s(t) = \frac{2}{3}\Pi\left(\frac{t}{T_0}\right) + \frac{1}{3}\Lambda\left(\frac{t}{T_0/4}\right)$$

$$X_0(f) = \frac{3T_0}{4}\,\mathrm{sinc}\left(\frac{3}{4}T_0 f\right)$$

$$S(f) = \frac{2}{3}T_0\,\mathrm{sinc}(f T_0) + \frac{1}{3}\frac{T_0}{4}\,\mathrm{sinc}^2\left(\frac{T_0}{4}f\right)$$

$$f_0 X_0(k f_0) = \cancel{f_0}\frac{3}{4}\cancel{T_0}\,\mathrm{sinc}\left(\frac{3}{4}\cancel{T_0}k\cancel{f_0}\right) = \frac{3}{4}\,\mathrm{sinc}\left(\frac{3}{4}k\right)$$

$$f_0 S(kf_0) = \cancel{f_0}\frac{2}{3} \cancel{T_0} \operatorname{sinc}(\cancel{f_0} k \cancel{T_0}) + \cancel{f_0}\frac{1}{3}\frac{\cancel{T_0}}{4} \operatorname{sinc}^2\left(\frac{\cancel{T_0}}{4} k \cancel{f_0}\right)$$

$$= \frac{2}{3} \operatorname{sinc}(k) + \frac{1}{12} \operatorname{sinc}^2\left(\frac{k}{4}\right)$$

$$f_0 X_0(kf_0) = \begin{cases} \frac{3}{4} & k=0 \\ \frac{3}{4} \frac{\sin^2\left(\frac{3}{4}\pi k\right)}{\left(\frac{3}{4}k\pi\right)^2} & k \neq 0 \end{cases}$$

$$f_0 S_0(kf_0) = \begin{cases} \frac{2}{3} + \frac{1}{12} = \frac{3}{4} & k=0 \\ \frac{1}{12} \frac{\sin^2\left(\frac{\pi}{4} k\right)}{\left(\frac{\pi}{4} k\right)^2} \end{cases}$$

non considero

$\frac{2}{3}\operatorname{sinc}(k)$ vale zero per i naturali cioè 1,2,3 ecc.

Trovo una similitudine tra $f_0 X_0(kf_0)$ e $f_0 S_0(kf_0)$

Dalla $f_0 X_0(kf_0)$ ho:

$$\frac{\cancel{3}^{\,1}}{4} \frac{\operatorname{sinc}^2\left(\frac{3}{4}\pi k\right)}{\cancel{9}_3\left(\frac{1}{4}\pi\right)^2} = \frac{1}{12} \frac{\operatorname{sinc}^2\left(\frac{3}{4}\pi k\right)}{\left(\frac{\pi k}{4}\right)^2}$$

$\frac{3}{4}\pi = \pi - \frac{\pi}{4}$ e quindi hanno lo stesso valore.

9i

X_K ? di $x(t) = \sum_{m=-\infty}^{\infty} x_0(t - m T_0)$ $\qquad$ $Y_K = X_K (-1)^K$

$y_K \longrightarrow y(t) = \sum_{m=-\infty}^{\infty} x_0\left(t - \frac{T_0}{2} - m T_0\right)$ $\qquad$ $y_0 = x_0\left(t - \frac{T_0}{2}\right)$

$Y_K = f_0 Y_0 (K f_0) = \underbrace{f_0 X_0 (K f_0)}_{X_K} e^{-j 2\pi K f_0 \frac{T_0}{2}} \Rightarrow y_K = X_K e^{-j\pi K}$

$\Rightarrow y_K = X_K (-1)^K$

9u

$X_K = X(-K) = ?$

$x(t)$ reale e pari

$\hookrightarrow x(t) = x(-t)$

$x(t) = \sum_{m=-\infty}^{\infty} x_0(t - m T_0) = x(-t) = \sum_{m=-\infty}^{\infty} x_0(-t - m T_0)$

$= \sum_{m=-\infty}^{\infty} x_0(-(t - m T_0))$??

post $m = -m$

$X_K = f_0 X_0 (K f_0) = f_0 X_0 (-K f_0) = X(-K)$

9j

$x(t) = \sum_{m=-\infty}^{\infty} x_0(t - m T_0) \longrightarrow z(t) = x\left(\frac{t}{T}\right) = \sum_{m=-\infty}^{\infty} x_0\left(\frac{t}{T} - m T_0\right)$

X_K e Z_K $\qquad$ $T_0' = T_0 \cdot T$ $\qquad$ $f_0' = \frac{f_0}{T}$

$K f_0$ $\qquad$ $f_0 X_0(K f_0)$

$x(t)$ $\qquad$ $x\left(\frac{t}{T}\right)$

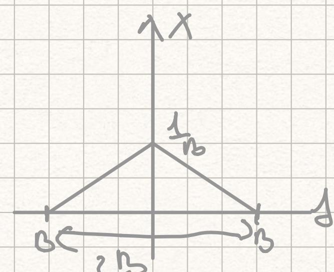
$z_k = X_k \quad k f_0' = k \frac{f_0}{T}$

Esercizio 99

$x(t) = \text{sinc}^2 Bt$ $\qquad f_c \geq B_x$

$f_c = 3B$

$X(f) = \frac{1}{B} \Lambda\left(\frac{f}{B}\right)$

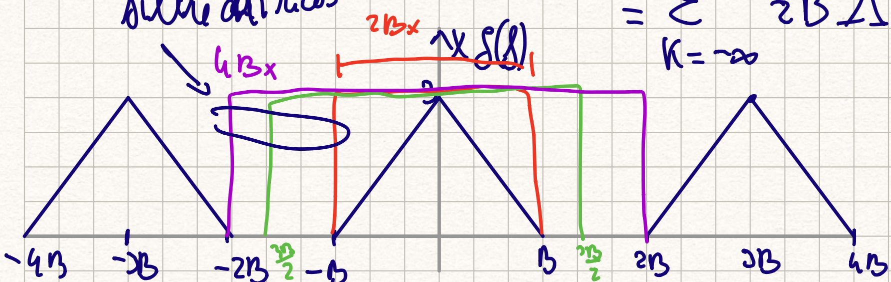


OK verificato Nyquist

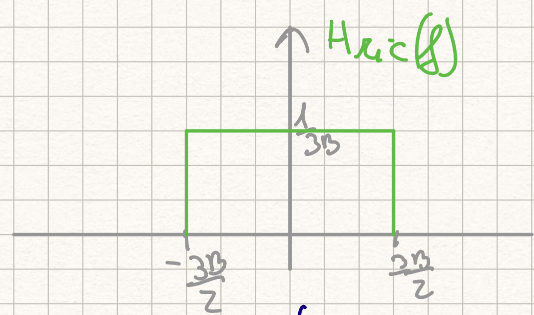
$$X_\delta(f) = \sum_{-\infty}^{\infty} f_0 X(f - k f_0) = \sum_{k=-\infty}^{k} 3\cancel{B} \frac{1}{\cancel{B}} \Lambda\left(\frac{f - k3B}{B}\right)$$

$$= \sum_{k=-\infty}^{k} 2B \Lambda\left(\frac{f - k3B}{B}\right)$$

filtri di ricostruzione consentiti



$H_{ric}(f) = T_c \, \Pi\left(\frac{f}{f_c}\right) = \frac{1}{3B} \Pi\left(\frac{f}{3B}\right)$



$X_{ric}(f) = X_\delta(f) \cdot H_{ric}(f) = \frac{1}{B} \Lambda\left(\frac{f}{B}\right)$

$$2B \leq B_{H_{ric}} \leq 4B$$

$$H_{ric}(f) = T_c \, \Pi\left(\frac{f}{B_H}\right)$$

101 bis

$$x(t) = \sin(2\pi f_0 t)$$

$$T_c = \frac{1}{f_0}$$

$$x_\delta(f) = \sum_{k=-\infty}^{+\infty} \sin\left(2\pi \cancel{f_0} n \frac{1}{\cancel{f_0}}\right) \delta\left(t - \frac{n}{f_0}\right) = 0$$

$$\sin(2\pi n) = 0$$

$f = 2 f_0 > f_0 \Rightarrow$ non è rispettato Nyquist.

$$X_\delta(f) = \frac{1}{2j} \delta(f - f_0) - \frac{1}{2j} \delta(f + f_0)$$

$$\hookrightarrow X_\delta(f) = \sum_{k=-\infty}^{\infty} \frac{1}{2j} \left[f_0 \, \delta(f - f_0 - k f_0) - f_0 \, \delta(f + f_0 - k f_0) \right]$$

$$= \sum_{k=-\infty}^{\infty} \frac{f_0}{2j} \left[\delta\left(f - (k+1) f_0\right) - \delta\left(f - (k-1) f_0\right) \right]$$

Spettro del segnale nullo in quanto le δ di dirac si elidono

103 $\quad B = 10\ KHz$

$f_c = 44100\ Hz$ $\qquad\qquad H_{eq-ric}(f) = ?$

$S(t) = \Pi\left(100 f_c t - \frac{1}{2}\right)$

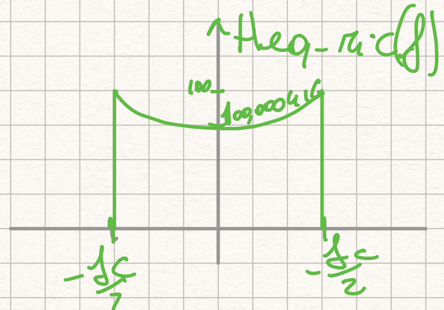
→ risolvo

$$S(f) = \frac{1}{100 f_c}\ \text{sinc}\left(\frac{f}{100 f_c}\right) e^{-J\pi f}$$

$$H_{eq-ric}(f) = \frac{H_{ric}(f)}{|S(f)|} \qquad \boxed{H_{ric} = T_c\ \Pi\left(\frac{f}{f_c}\right)}$$

$$= 100 \cancel{f_c} \frac{\cancel{T_c}}{\text{sinc}\left(\frac{f}{100 f_c}\right)} \Pi\left(\frac{f}{f_c}\right) = \frac{100}{\text{sinc}\left(\frac{f}{100 f_c}\right)} \Pi\left(\frac{f}{f_c}\right)$$

$$H_{eq-ric}(f) = \begin{cases} \dfrac{100}{\text{sinc}\left(\frac{f}{100 f_c}\right)} & |f| \le \dfrac{f_c}{2} \\ 0 & \text{altrove} \end{cases}$$



$H_{eq\,ric}(0) = 100 \Rightarrow \min$ valore minimo

$H_{eq,ric}\left(\frac{f_c}{2}\right) = 100{,}000416$ valore massimo

104

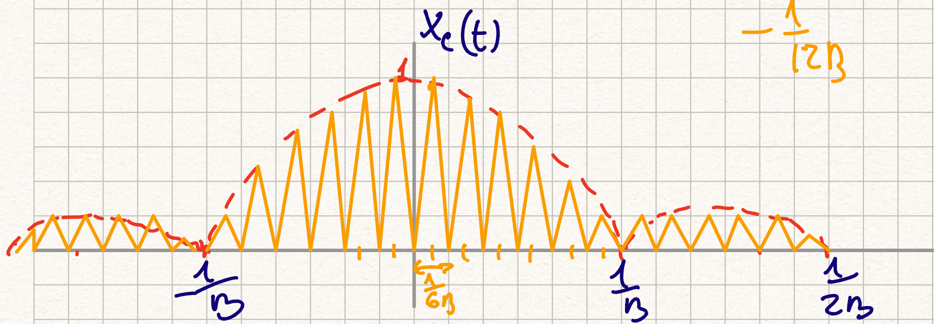
$x(t) = \sin^2(Bt) \Rightarrow X(f) = \frac{1}{B}\Lambda\left(\frac{f}{B}\right)$ $\quad B_x = 2B < f_c$

$f_c = 3B$

$$s(t) = \Lambda\left(\frac{t - \frac{T_c}{4}}{\frac{T_c}{4}}\right) \Rightarrow s(t) = \Lambda\left(\frac{t - \frac{1}{12B}}{\frac{1}{12B}}\right)$$

$-\frac{1}{12B}$ $\quad \frac{1}{12B}$

$\frac{\sin \pi K}{\pi K}$



$X_c(f) = X(f) \cdot S(f) =$

$$= \left[\sum_{-\infty}^{\infty} \overbrace{3B}^{f_c} \frac{1}{B}\Lambda\left(\frac{f - K\overbrace{3B}^{f_c}}{B}\right)\right] \frac{1}{\underbrace{12B}_{4f_c}} \operatorname{sinc}^2\left(\frac{f}{\underbrace{12B}_{4f_c}}\right) e^{-j2\pi f \frac{1}{12B}}$$

$$H_{eq-ric}(f) = \frac{H_{ric}(f)}{|S(f)|} = 4\cancel{f_c}\,\frac{\cancel{f_c}}{\operatorname{sinc}\left(\frac{f}{4f_c}\right)}\,\Pi\left(\frac{f}{f_c}\right)$$

$$\Rightarrow \begin{cases} \dfrac{4}{\operatorname{sinc}^2\left(\frac{f}{4f_c}\right)} & |f| \le \frac{f_c}{2} \\ 0 & |f| \ge \frac{f_c}{2} \end{cases}$$

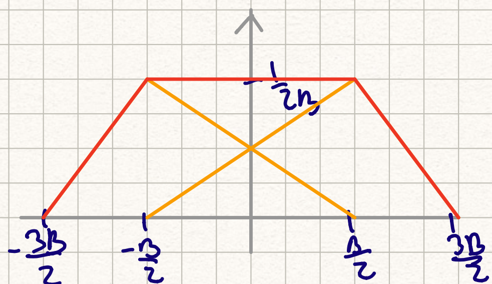
Esercizio esame 22/1/04 ②

$x(t) = \text{sinc}^2(Bt)\cos(\pi B t)$

$f_c = 40\,\text{KHz}$

$B = 20\,\text{KHz}$

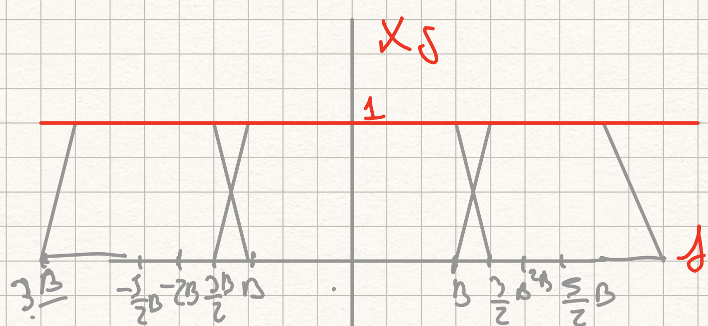
$$X(f) = \frac{1}{2B}\Lambda\left(\frac{f-\frac{B}{2}}{B}\right) + \frac{1}{2B}\Lambda\left(\frac{f+\frac{B}{2}}{B}\right)$$



$B_X = \frac{3}{2}B \cdot 2 = 60\,\text{KHz} > f_c$ ALIASING

$x_{ric}(t) \neq x(t)$

$$X_s(f) = \sum_{-\infty}^{\infty} f_c\, X(f-kf_c) = 2B\, X(f-k2B)$$



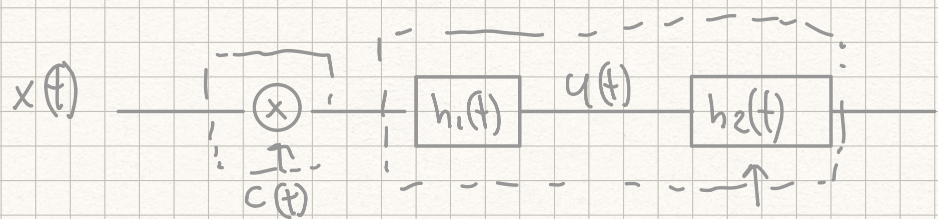
$$X_{ric}(f) = X_s(f)\cdot H_{ric}(f) = 1\,\frac{1}{2B}\,\Pi\left(\frac{f}{2B}\right) = \underset{\substack{\uparrow \\ x_{ric}(t)}}{\text{sinc}(2Bt)}$$

(2/2/07) Esercizio 2

$$h_1(t) = \Lambda\ 2 f_c t$$

f_c

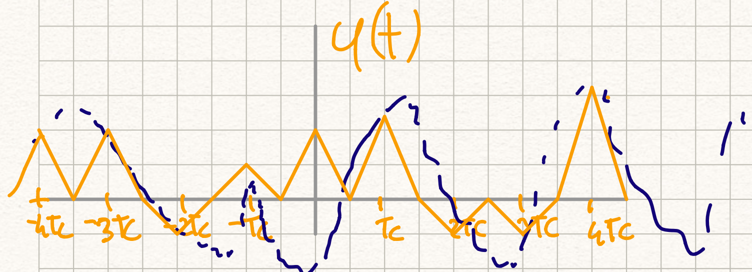
$$y(t) = x(t) * h_1(t)$$



$$x_s(t) = \left[\sum_{-\infty}^{\infty} x(nT_c)\,\delta(t - nT_c)\right]$$

$$y(t) = \Big(\sum_{-\infty}^{\infty} x(nT_c)\,\underbrace{\delta(t - nT_c) * \Lambda\left(\frac{2}{T_c}t\right)}_{s(t) * \delta(t-t_0) = s(t-t_0)}$$

$$\Rightarrow y(t) = \sum_{-\infty}^{\infty} x(nT_c)\,\Lambda\left(\frac{2}{T_c}(t - nT_c)\right)$$



$$X_s(f) = \sum_{-\infty}^{+\infty} f_c \, X(f - k f_c)$$

$$H_1(f) = \frac{T_c}{2} \operatorname{sinc}^2\left(\frac{f T_c}{2}\right) = \frac{1}{2 f_c} \operatorname{sinc}^2\left(\frac{f}{2 f_c}\right)$$

$$Y(f) = \sum_{k=-\infty}^{\infty} f_c \, X(f - k f_0) \, H_1(f)$$

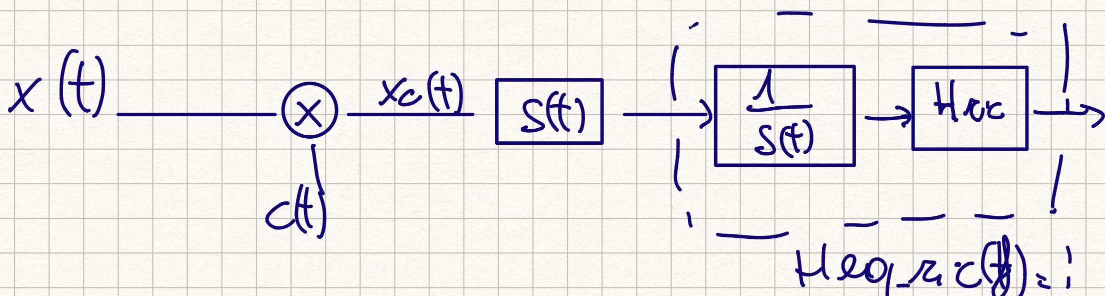
$$H_2(f) = \frac{H_{ric}(f)}{H_1(f)} = \frac{\cancel{1/f_c}\, T_c \, \Pi\left(\frac{f}{f_c}\right)}{\operatorname{sinc}^2\left(\frac{f}{2 f_c}\right)} \Rightarrow$$

$$H_2(f) = \begin{cases} \dfrac{2}{\operatorname{sinc}^2\left(\frac{f}{2 f_c}\right)} & |f| \leq \dfrac{f_c}{2} \\ 0 & |f| > \dfrac{f_c}{2} \end{cases}$$

29/10/03 Campionamento

$x(t) = \text{sinc}(t) \qquad T_c = 1$

$s(t) = \Pi\left(10t - \frac{1}{2}\right) \qquad H_{eq-ric}(f) = ?$



$$s(t) = \Pi\left(\frac{t - \frac{1}{20}}{\frac{1}{10}}\right)$$

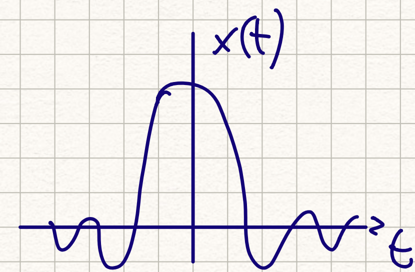
$$x_c(t) = \left[\sum_{n=-\infty}^{\infty} x(nT_c)\,\delta(t - nT_c)\right] * s(t) =$$

$$= \sum_{n=-\infty}^{\infty} x(nT_c)\, s(t - nT_c)$$

Prop. campionatrice δ

$$\boxed{\delta(t - nT_c) * c(t) = s(t - nT_c)}$$

$$x(nT_c) = \begin{cases} 1 & n = 0 \\ 0 & \text{altrove} \end{cases}$$

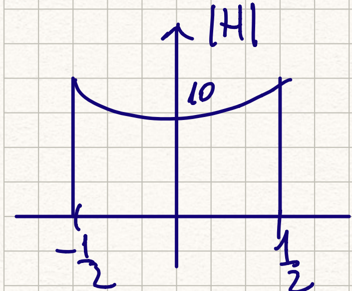


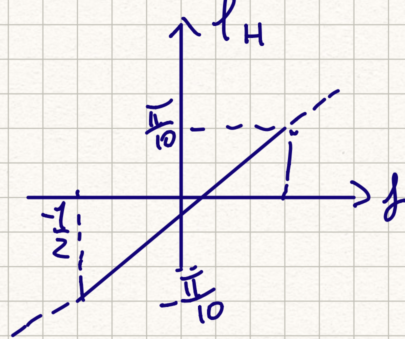
$x_c(t) = s(t)$

$f_c \geq B_x \rightarrow X(f) = \Pi(f) \Leftarrow B_x = 1 = \frac{1}{T_c} = 1$ Nyquist OK

$$H_{eq-rc}(f) = \frac{T_c \, \Pi\left(\frac{f}{f_c}\right)}{S(f)} = \frac{10 \cdot 1}{\mathrm{sinc}\left(\frac{f}{10}\right)} e^{+j2\pi \frac{f}{10}} \Pi\left(\frac{f}{f_0}\right)$$

$$S(f) = \frac{1}{10} \mathrm{sinc}\left(\frac{f}{10}\right) e^{-j2\pi f \frac{1}{20}} \qquad = \begin{cases} \frac{10}{\mathrm{sinc}\left(\frac{f}{10}\right)} e^{j2\pi \frac{f}{20}} & |f| < \frac{1}{2} \\ \emptyset & |f| > \frac{1}{2} \end{cases}$$





$2\pi f \frac{1}{20} = \frac{\pi f}{10}$

2

22|01|04

$$x(t) = \cos(10\pi t)\cos(8\pi t) \Rightarrow x(t) = \overset{\text{modulante}}{m}(t) \cdot \overset{\text{portante}}{p}(t)$$

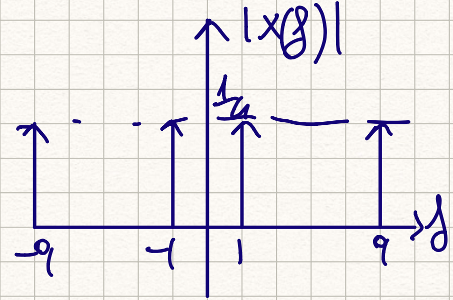
$$h(t) = \mathrm{sinc}^2(2t)$$

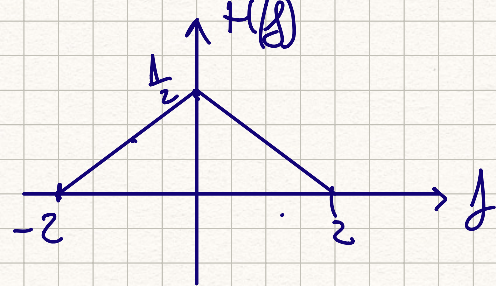
$$Y(f) = X(f) \cdot H(f) \longleftrightarrow y(t)$$

$$x(t) = \cos(2\pi \underset{\substack{\uparrow \\ f_0}}{5} t)\cos(2\pi \underset{\substack{\uparrow \\ f_0}}{4} t)$$

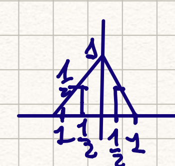
$$M(f) = \frac{1}{2}\delta(f-5) + \frac{1}{2}\delta(f+5) \Rightarrow X(f) = \frac{1}{2}M(f-4) + \frac{1}{2}M(f+4)$$

$$X(f) = \frac{1}{4}\delta(f-9) + \frac{1}{4}\delta(f-1) + \frac{1}{4}\delta(f+1) + \frac{1}{4}\delta(f+9)$$





$$H(f) = \frac{1}{2}\Lambda\left(\frac{f}{2}\right)$$



$$Y(f) = X(f) \cdot H(f) \Rightarrow \begin{cases} f \pm 1 \Rightarrow \frac{1}{2}\Lambda\left(\frac{f}{2}\right) = \frac{1}{4} \\ \end{cases}$$

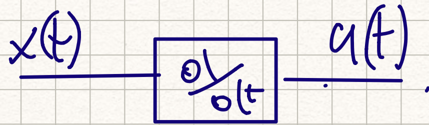
$$Y(f) = \frac{1}{16}\delta(f-1) + \frac{1}{16}\delta(f+1)$$

$$\updownarrow \mathcal{F}$$

$$y(t) = \frac{1}{8}\cos(2\pi\cdot 1\cdot t)$$

24/02/04

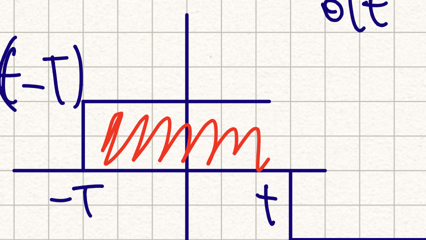
$$x(t) = \Pi\left(\frac{t}{2\pi}\right)$$



$$\frac{d\,u(t)}{dt} = \delta t$$

$$x(t) = u(t+T) - u(t-T)$$

$$y(t) = \frac{dx(t)}{dt} = \delta(t+T) - \delta(t-T)$$



$$X(f) = 2T\,\mathrm{sinc}(2Tf) = \cancel{2T}\,\frac{\sin 2\pi f t}{\cancel{2}\pi f \cancel{T}}$$

$$Y(f) = j2\cancel{\pi f}\cdot\frac{\mathrm{sen}(2\pi f t)}{\pi f} \qquad \frac{dx(t)}{dt} \longleftrightarrow j2\pi f X(f)$$

$$= 2j\,\mathrm{sen}(2\pi f t) \Rightarrow \cancel{2j}\left(\frac{e^{j2\pi f t} - e^{-j2\pi f t}}{\cancel{2j}}\right) =$$

$$= e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}$$

$$x(t) = m(t)\cdot\cos(2\pi f_0 t + \varphi)$$

7/7/06 $\quad x(t) = ? \rightarrow$

N.B. $X(f) = \frac{1}{2} M(f - f_0)\, e^{+j\varphi} + \frac{1}{2} M(f + f_0)\, e^{-j\varphi}$

$$X(f) = \frac{T}{2}\left[\mathrm{sinc}\underbrace{\left(fT - \frac{1}{2}\right)}_{=\left(fT - T\frac{1}{2T}\right) \Rightarrow f_0 = \frac{1}{2T}} + \mathrm{sinc}\left(fT + \frac{1}{2}\right) \right]$$

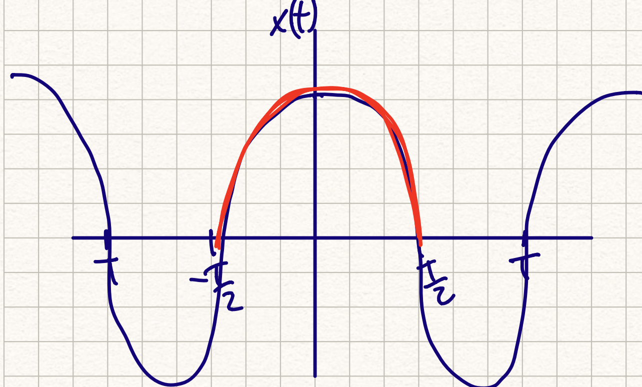
divido per $T \rightarrow \left(f - \overset{f_0}{\frac{1}{2T}}\right)$

$$M(f) = T\, \mathrm{sinc}(fT)$$

$$\Updownarrow$$

$$m(t) = \frac{\cancel{T}}{\cancel{T}}\, \Pi\left(\frac{t}{T}\right)$$

$$x(t) = \Pi\left(\frac{t}{T}\right) \cos\left(2\pi \overset{f_0}{\frac{1}{2T}} \cdot t\right)$$



24/01/05

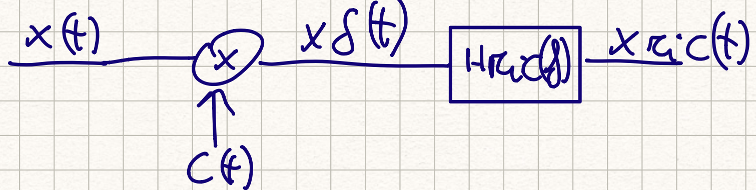
$$x(t) = B\,\mathrm{sinc}\left(2Bt - \frac{1}{2}\right) + B\,\mathrm{sinc}\left(2Bt + \frac{1}{2}\right)$$

$f_c = 4B$

$x_{ric}(t) \overset{?}{=} x(t)$

div per 2B

$$\left(B - \frac{1}{2\cdot 2B}\right) \quad f_0 = \frac{1}{4B}$$



$$2B(t - t_0)$$

$$\cancel{2B}\cdot\frac{1}{\cancel{4B}} = \frac{1}{2} \qquad T$$

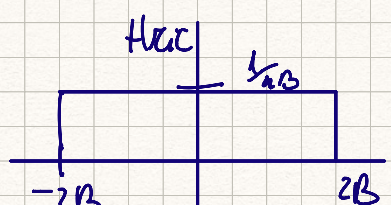
$$X(f) = \frac{\cancel{B}}{2\cancel{B}}\,\Pi\left(\frac{f}{2B}\right) e^{-j2\pi\frac{1}{4B}} + \frac{B}{2B}\,\Pi\left(\frac{f}{2B}\right) e^{+j2\pi\frac{1}{4B}}$$

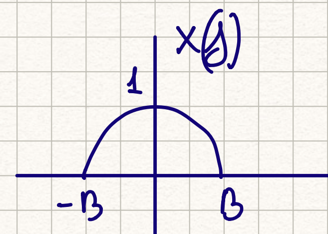
$\hookrightarrow f \geq B_x \Rightarrow 4B \geq 2B$ ok Nyquist.

$$= \frac{1}{2}\,\Pi\left(\frac{f}{2B}\right) e^{-j\pi\frac{1}{2B}} + \frac{1}{2}\,\Pi\left(\frac{f}{2B}\right) e^{+j\pi\frac{1}{2B}}$$

$$= \Pi\left(\frac{f}{2B}\right)\cos\left(\pi f \frac{1}{2B}\right)$$

$$H_{ric}(f) = T_c\,\Pi\left(\frac{f}{f_0}\right) \Rightarrow \frac{1}{4B}\,\Pi\left(\frac{f}{4B}\right)$$





4/2/08 $x(t) = A \cdot \mathrm{sinc}(3Bt)$

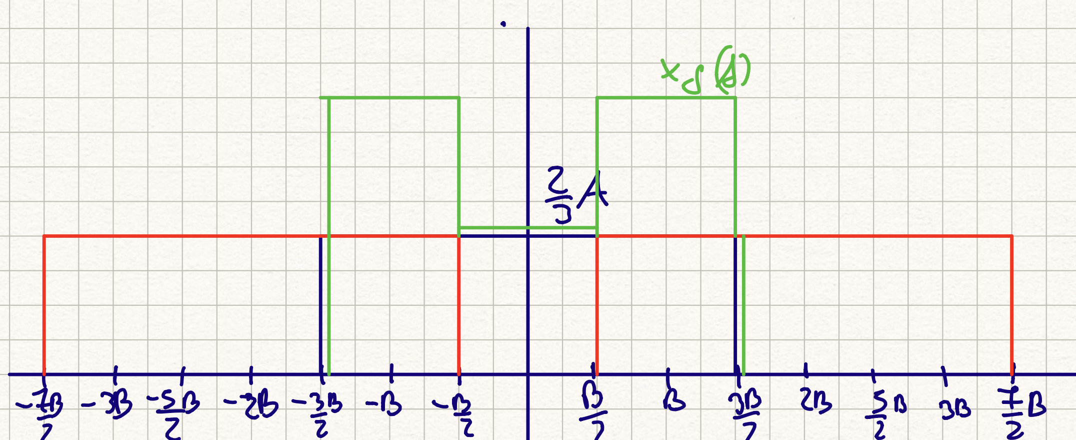
$f_C = 2B$

$$X(f) = \frac{A}{3B}\,\Pi\left(\frac{f}{3B}\right)$$

$f_C \geq B_X \Rightarrow 2B \geq 3B$ No Nyquist

$$X_\delta(f) = \sum_{-\infty}^{\infty} f_C \cdot X(f - k f_C) = \sum_{-\infty}^{\infty} 2B\,\frac{A}{3B}\,\Pi\left(\frac{f}{3B} - k2B\right)$$

$$= \sum_{-\infty}^{\infty} \frac{2}{3} A\, \Pi\left(\frac{f}{3B} - k2B\right)$$



$$X_{ric}(f) = X_\delta(f)\, T_C \cdot \Pi\left(\frac{f}{f_C}\right) = \frac{\cancel{4}^{\,2}}{3} A\left[1 - \frac{1}{2}\Pi\left(\frac{f}{B}\right)\right]\frac{1}{2B}\,\Pi\left(\frac{f}{2B}\right)$$

$$= \frac{2A}{3B}\left[\Pi\left(\frac{f}{2B}\right) - \frac{1}{2\pi}\left(\frac{f}{B}\right)\right] = X_{ric}(f)$$

$$x_{ric}(t) = \frac{2A}{3B}\left[2B\,\mathrm{sinc}(2Bf) - \frac{1}{2\pi}B\,\mathrm{sinc}(fB)\right]$$

$$x_{ric}(t) = \frac{4A}{3}\,\mathrm{sinc}(2Bf) - \frac{A}{3\pi}\sin(fB)$$