Weak Efficiency and Technical Analysis

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Abstract

This analysis has the aim of testing the weak-form of market efficiency as well as comparing a random trading strategy with technical ones. Initially, tests are conducted to evaluate the weak form of market efficiency, with the aim of determining whether historical price data holds any predictive value. Following this, trading strategies that incorporate technical analysis are assessed. The assets under consideration include EUR/USD, Gold, and the S&P 500. These strategies are evaluated over the period from 2004 to 2024 and are analyzed using key performance metrics, including the Sharpe ratio, the Sortino ratio, and the Probabilistic Sharpe ratio. Additionally, the analysis accounts for trading costs, such as transaction fees in order to provide a more accurate evaluation of strategy performance.

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1 Introduction

Randomness has long been recognized as one of the most controversial and intriguing aspects of both human nature and life in general. Its presence, particularly in the field of finance, introduces a significant layer of complexity to decision-making processes. The pervasive influence of randomness in financial markets has made the task of predicting asset prices and returns exceptionally difficult. Theoretical discussions surrounding randomness in financial markets were first formalized by the Efficient Market Hypothesis (EMH), which posits that markets become increasingly random in response to the introduction of new information. Specifically, the hypothesis suggests that price movements exhibit erratic and unpredictable behavior, making it exceedingly difficult to forecast future returns with accuracy.

The EMH is closely tied to the Random Walk (RW) hypothesis, as the mathematical formulation of the EMH, particularly in its weak form, is represented by the random walk model. This model serves as a discrete approximation of the Geometric Brownian Motion, which is commonly employed to model asset prices in financial markets. According to the random walk hypothesis, future price changes are independent of past price changes, meaning that the path of asset price changes can be considered a random sequence, with no discernible pattern that could provide predictive power for future returns.

The primary aim of this analysis is to empirically test, using both statistical tests and observational techniques, whether major assets in three different markets exhibit randomness, at least in the weak form as described by the EMH, as discussed by Fama [14], [15] and Samuelson [31] The specific assets under consideration are the EUR/USD exchange rate, the S&P 500 index, and Gold. These assets were selected to allow for the exploration of the randomness hypothesis across different types of markets.

The application of statistical tests, alongside empirical methods such as technical analysis, has long been the standard approach for assessing the validity of the EMH. In addition to testing the EMH itself, this study introduces a random trading strategy to assess whether such an approach could yield superior results compared to traditional technical trading strategies. By utilizing a random trading strategy, we aim to better capture the inherent difficulties and uncertainties that characterize financial markets.

A related study by Biondo et al. [3] investigated whether a random trading strategy could outperform traditional technical strategies. Their findings suggested that, on average, random strategies generated particularly compelling results when evaluated within a risk-adjusted framework. The current work aims to extend the analysis conducted by Biondo et al. to provide a broader assessment of the hypothesis of randomness in financial markets. Moreover, the analysis will cover a more recent time period, spanning from 2004 to 2024, allowing for an uptodate examination of how the random walk hypothesis holds up in the context of contemporary financial data.

2 Technical Analysis and EMH

The application of Technical Analysis represents a set of methods utilized for forecasting asset price movements. According to Park and Irwin [28], in the past, particularly during the 1990s, technical analysis was generally effective, with some evidence of profitability. A notable study by Han et al. [16] suggests that the profitability of technical analysis is time-varying, although some evidence of its efficacy remains, especially in the context of the stock market. A significant contribution to the understanding of technical analysis in foreign exchange markets is provided by Menkhoff et al. [26], whose research identifies certain stylized facts. However, it also indicates that there is no clear, consistent evidence regarding either the reasons for its use or its profitability.

In general, the intersection between the Efficient Market Hypothesis and the profit coming from technical analysis, is quite clear. Indeed, to fully capitalize on the benefits of technical analysis, it is imperative to ascertain that our time series, do not demonstrate weak-form efficiency. The notion of weak-form efficiency is articulated within the conceptual framework established by Fama [14], [15]. In this context, our objective is to determine whether the requisite conditions for profiting from technical analysis are present.

The concept of market efficiency is fundamentally interconnected with the random walk model, as delineated in the works of Samuelson [31] and Fama [14], [15]. We can define the RW model as follows:

$$p_t - p_{t-1} = \mu + \epsilon_t = r_t \tag{1}$$

Where p_t is the log price of the asset, μ is the drift parameter, r_t is the log return and ϵ_t the error component. In general, the error component is: $\epsilon_t \sim D(0, \sigma^2)$. The equation (1) is also called RW with drift.

For different specification of ϵ_t in (1) we can have different testable RW-EMH hypothesis, see Campbell et al. [5]. We generally distinguish between three form of RW:

- RW1: the error term ϵ_t is *i.i.d.* and so $\epsilon_t \sim D(0, \sigma^2)$;
- RW2: the error term ϵ_t is *independent* (allows for heteroscedasticity);
- RW3: the error term ϵ_t is *uncorrelated*.

The tests addressing these three forms pertain to weak-form market efficiency. In our analysis, we will focus on specific tests related to the Random Walk Model 3 (RW3) in alignment with our objectives. The RW3 can be evaluated through various statistical tests, whereas the Random Walk Model 2 (RW2) is typically assessed using technical analysis rules. The Random Walk Model 1 (RW1) is generally not taken into consideration due to its limited scope; indeed, it is challenging to empirically identify an RW1, primarily due to the stylized facts observed in financial time series, see Campbell et al. [5]. Consequently, this diminishes interest in conducting tests for it.

The assessment of the weak-form efficiency is considered in order to provide a more rigorous context for evaluating technical analysis. Specifically, an examination of this hypothesis can offer a clearer framework when assessing the effectiveness of technical analysis. As previously noted, testing technical analysis serves as an additional "empirical" method for investigating the presence of RW2. By employing both statistical tests and technical analysis, we can gain a deeper understanding of the potential to exploit historical information contained in prices or returns.

3 Data and Methodology

For this analysis, daily price data sourced from Yahoo Finance was employed. Specifically, the assets selected for the study include the S&P 500, Gold, and EUR/USD. These three assets were deliberately chosen to represent distinct sectors of the market, thereby facilitating a more comprehensive analysis. The aim of examining these diverse markets is to investigate the presence of weak efficiency and the performance of random trading activity across different asset classes and to explore how such trading strategies may perform under varying market conditions. The analysis covers the period from January 2004 to December 2024, providing a broad historical context. This extended time frame ensures the study captures a variety of market dynamics, including periods of high and low volatility, economic recessions, and growth phases, thus enabling a thorough evaluation of the trading strategy's performance.

3.1 Data Description

We begin by presenting a brief description of the data utilized, specifically, a summary of the statistics for the log returns of each asset.

Asset	**Mean (%)**	**Median (%)**	**SD (%)**	**Skewness**	**Kurtosis**
EUR/USD	-0.003465	0.000000	0.7009	0.5859	100.75
Gold	0.03415	0.04689	1.1043	-0.3649	5.42
S&P 500	0.03166	0.07071	1.1909	-0.5323	13.29

Table 1: Summary statistics for daily returns of EUR/USD, Gold, and S&P 500. Data in percentage.

As demonstrated in Table 1, both asymmetry and thick tails are present in the three time series under analysis, confirming the presence of well-established stylized facts in empirical finance, as discussed by Campbell et al. [5]. These stylized facts—specifically the skewness (asymmetry) of returns and the presence of thick tails in the distribution—are commonly observed in financial time series data. Such characteristics are significant because they challenge traditional assumptions underlying many statistical models. For instance, the assumption of normality, often employed in financial modeling and statistical testing, is violated when these features are present. As a result, the presence of asymmetry and thick tails can complicate the accurate modeling of asset returns, potentially leading to unreliable conclusions in statistical analyses and risk assessments.

3.2 Methodology

3.2.1 Weak-efficiency tests

Autocorrelation tests As previously defined, the RW3 hypothesis can be tested using an autocorrelation test. It is important to recall that this test is based on the following hypotheses:

$$\mathrm{H}_0 : \rho_1 = \cdots = \rho_p = 0$$
$$\mathrm{H}_1 : \rho_1 \neq \cdots \neq \rho_p \neq 0.$$

Where ρ is the autocorrelation coefficient. In these cases, the tests considered are the Ljung-Box test and the Box-Pierce test. The test statistics are as follows:

$$\mathrm{Q} = T \sum_{k=1}^{p} \hat{\rho}^2 \xrightarrow{d} \chi_p^2$$
$$\mathrm{LB} = T(T+2) \sum_{k=1}^{p} \frac{\hat{\rho}^2}{T-k} \xrightarrow{d} \chi_p^2.$$

In general, these two tests require the specification of the lag order, p, which can be determined using information criteria. Furthermore, if the series r_t is heteroscedastic, the asymptotic distribution of the tests may be incorrect. To address these two problems, Escanciano and Lobato [21] propose an automatic test that is also consistent in the presence of heteroscedasticity. This test is considered in this analysis.

Variance ratio tests Another important method employed to assess the existence of weak-form efficient market hypothesis (EMH) is the variance ratio test. The intuition is that for all three Random Walk (RW), see in Section 2, hypotheses, the variance of RW increments is linear with respect to the time interval. Specifically, if the duration of the interval is doubled, the variance must also double. Consequently, the variance of monthly data should be four times greater than that of weekly data. Specifically, consider the following: let us define the log returns for two periods as: $r_t(2) = r_t + r_{t-1}$, and assume that r_t is covariance stationary. Consequently, the VR test statistic is given by:

$$\mathrm{VR}(2) = \frac{\mathrm{Var}[r_t(2)]}{2\mathrm{Var}[r_t]} = 1 + 2\rho(1).$$

When generalized to q lags, we obtain the following VR statistic:

$$\mathrm{VR}(q) = \frac{\mathrm{Var}[r_t(q)]}{q\mathrm{Var}[r_t]} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho(k).$$

The hypotheses tested are the following:

$$\mathrm{H}_0 : \mathrm{VR}(q) = 1$$
$$\mathrm{H}_1 : \mathrm{VR}(q) \neq 1.$$

Under the null, the distribution of the test statistics is the following: $\frac{\sqrt{Tq}}{\sqrt{2(q-1)}}(\widehat{\mathrm{VR}}(q)-1) \xrightarrow{a} N(0,1)$.

In order to overcome the issues from standard VR tests, the ones defined above, namely the lag selection and the poor asymptotic approximation of the VR sample statistic, we can consider the automatic VR test of Choi [9] with bootstrapped p-values, the method is illustrated by: Kim [18] and Charles et al. [6].

However, the use of the VR test discussed above sometimes may not be sufficient. Specifically, we can consider the test developed by Lo and MacKinlay [22], which adjusts the VR statistic to account for the presence of heteroscedasticity. Another noteworthy VR test is that of Chow and Denning [7], which is a joint test that may offer more informative results compared to the other tests conducted. More specifically the Lo and MacKinlay test statistics are the following:

$$\mathrm{M}_1(q) = \frac{\sqrt{3Tq}}{\sqrt{2(2q-1)(q-1)}}(\widehat{\mathrm{VR}}(q)-1) \xrightarrow{a} N(0,1)$$

$$\mathrm{M}_2(q) = \frac{(\widehat{\mathrm{VR}}(q)-1)}{\sqrt{\phi(q)}} \xrightarrow{a} N(0,1), \quad \phi(q) = \sum_{j=1}^{q}\left[\frac{2(q-j)}{q}\right]^2 \left[\frac{\sum_{t=j+1}^{T}(r_t-\bar{r})^2(r_{t-j}-\bar{r})^2}{[\sum_{t=1}^{T}(r_t-\bar{r})^2]^2}\right].$$

The statistic $\mathrm{M}_2(q)$ accounts for heteroskedasticity. The issues associated with the Lo and MacKinlay test are its poor asymptotic approximation and the fact that it is not a joint test. The poor approximation can be addressed through bootstrapping, as suggested by Kim's method [19]. The issue of joint testing can be tackled using the Chow and Denning test. More specifically, a joint test examines whether $\mathrm{VR}(q) = 1$ for all q. The CD test statistic is as follows:

$$\mathrm{CD} = \sqrt{T} \max_{1\le i\le m} |M_2(q_i)|$$

The CD statistics follows a complex distribution, the studentized maximum modulus [SMM] distribution with m and T degrees of freedom (m is the number of k values). To address the poor asymptotic approximation for both tests, we compute bootstrapped p-values based on Kim's method [19].

3.2.2 Technical Strategies

For this analysis, the strategies were categorized into two primary types: **Momentum** and **Mean-Reverting** strategies. More specifically, all the strategies are based on long and short position, these are:

- **Moving Averages Strategy**: This strategy is based on the crossing of two moving averages, specifically the pairs 20 and 50, as well as 50 and 200, long and short position are considered;

- **Momentum Indicator Strategy**: The momentum indicator was employed, with a long position triggered when the indicator crosses above zero, and a short position when it crosses below zero. Two time lengths were considered: 7 and 14;
- **MACD Indicator Strategy**: The standard MACD strategy, based on the crossing of the signal line and the MACD line, was utilized. The parameters for this strategy are set to 12, 26, and 9;
- **RSI Indicator Strategy**: The standard RSI strategy was applied, with short positions taken when the RSI exceeds 70 and long positions when it falls below 30. Two time lengths were considered: 7 and 14;
- **Random Strategy**: This strategy operates purely on randomness, where buy or sell decisions are made based on a random number generated from a uniform distribution. This approach serves as a baseline to assess the performance of the more systematic strategies.

To clarify, the strategies outlined above are categorized into trend-following and mean-reverting approaches. More specifically, the strategy that employs the momentum indicator may be potentially misleading due to its terminology; however, the approach we consider is based on the principle of momentum/trend-following. The momentum indicator, defined by the formula Momentum $= P - P_n$, where P_n represents the price n periods ago (with n typically set to 7 or 14), results in something similar to the differentiation of the time series. In this process, we attempt to render the series stationary. By considering the crossing of zero as a signal, we anticipate that the series will lose or gain its strength. More generally, once the series is made stationary, we expect it to fluctuate around zero (the center). This fluctuation, combined with the stationary nature of the series, facilitates the interpretation of when the series is losing or gaining strength. The term ”momentum” used here is analogous to its usage in the field of empirical finance, where such strategies typically involve purchasing assets that have exhibited upward momentum over time, while shorting those that have shown a loss of momentum. The concept of ”momentum” itself is derived from the same principles observed in physics. The same consideration can be applied to the use of MACD, see Appel [1] for details.

The RSI strategy, as defined, can be viewed as a mean-reverting strategy. Indeed, while the RSI indicator is inherently a momentum indicator, it is based on the premise that the time series should remain within certain levels—typically 70 and 30. Although the time series may diverge from these levels over extended periods, the fundamental principle behind the strategy is that a reversion is expected when the upper or lower level of the RSI indicator is breached. See Wilder [32] for details.

For moving average strategies, particularly those utilizing crossovers between different moving averages, the underlying rationale is as follows: the moving average of length n represents the general expectations or behavior of certain investors. Consequently, changes in its slope or direction signal shifts in investor expectations and behavior. This is the core concept of moving average strategies, which are inherently trend-following. Moving averages themselves are mathematical functions used to smooth a time series and identify its underlying trend. In fact, in time series analysis, moving averages are essential for determining the trend component when decomposing a time series, as discussed by Hyndman et al. [17].

The last strategy can be considered a benchmark. Specifically, the strategy based on randomness is consistent with results obtained in the context of exchange rate forecasting. When seeking a model that effectively captures the dynamics of nominal exchange rates, the Random Walk model has been shown to yield more accurate forecasts compared to structural models, such as Purchasing Power Parity (PPP), or other macroeconomic models, such as the monetary model. See Meese and Rogoff [25], Rossi [30], Engel et al. [13], Cheung et al. [8], as well as the challenges associated with using macroeconomic data in forecasting, as discussed by Christoffersen et al. [10].

Moreover, the Random strategy, is based on the work of Biondo et al. [3], who explore the use of random trading, where the decision to take a long or short position is made randomly.

It is important to note that no optimization process is considered for the technical strategies discussed here. This is due to the fact that optimization can be time-consuming and requires a degree of discretion in choosing the appropriate approach. Specifically, when performing optimization, several factors must be considered, such as the type of optimization (exhaustive or genetic), the number of parameters involved, and the scheme (in-sample, out-of-sample, rolling, anchored, or static).

3.2.3 Performance Evaluation

None of the strategies employed involve any in-sample or out-of-sample splits, and no optimization is performed. Instead, standard parameterizations of these indicators/oscillators are used, including those for Moving Averages, Momentum, RSI, and MACD. The settings for these indicators are those originally established upon their publication. This approach ensures that the analysis remains unbiased by using the standard parameters.

The metrics involved in the analysis of the strategies are the ones generally used for assessing any trading strategy. These are:

- Sharpe Ratio;
- Probabilistic Sharpe Ratio;
- Sortino Ratio.

These metrics incorporate the main components when evaluating trading strategies: risk and return. It was also considered the known Probabilistic Sharpe Ratio developed by Bailey and de Prado [12]. In particular, this version of the Sharpe Ratio enable us to better evaluate the strategies. The PSR takes into account the skewness, kurtosis and the optimal length of the track record. By doing so, it accounts for non-normality, as well as adjusting for the estimation errors. The PSR is defined as follows:

$$\widehat{PSR}(SR^*) = \mathbb{P}[SR \leq SR^*] = \int_{-\infty}^{\widehat{SR}} z(SR \mid SR^*, \hat{\sigma}_{\widehat{SR}})\, dSR.$$

Where in the above equation $\hat{\sigma}_{\widehat{SR}} = \sqrt{\frac{1-\hat{\gamma}_3\widehat{SR}+\frac{\hat{\gamma}_4-1}{4}\widehat{SR}^2}{N-1}}$. The PSR is then rewritten simply as follows:

$$\widehat{PSR}(SR^*) = Z\left[\frac{(\widehat{SR} - SR^*)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma}_3\widehat{SR} + \frac{\hat{\gamma}_4-1}{4}\widehat{SR}^2}}\right].$$

Where Z is the cdf of a standard normal distribution, with mean SR^*, and standard deviation equal to $\hat{\sigma}_{\widehat{SR}}$. On the other hand, $\hat{\gamma}_3$ and $\hat{\gamma}_4$ are the estimates of skewness and kurtosis respectively. The PSR gives you the probability of actually being superior with respect to a SR benchmark denoted as SR^*. This probability should be compared with the confidence level considered, for instance 95%.

However, PSR does not focus on the presence of autocorrelation within the strategy returns, indeed, as denoted by Lo [23], when autocorrelation is present, the time aggregation factor (naively $\sqrt{T}$) might be ineffective as well as the inference on the SR can be misleading. However, even if might be useful to adjust the SR for the autocorrelation present in the returns, we avoid it since it requires an appropriate modeling of the time series, an example could be the AR(1) model.

Following the calculation of the metrics involved in the analysis, a bootstrapping procedure was also employed. Specifically, the returns of the strategies were bootstrapped to obtain a distribution of the risk and return metrics considered. More precisely, a block bootstrapping method was applied to account for the potential autocorrelation present within the strategy returns.

4 Empirical Analysis

This section presents the empirical results, specifically those derived from the Random Walk Hypothesis and the outcomes of the technical strategies. The results of the strategies, as well as the tests, consider the time period outlined in Section 3. More specifically, the tests account for both the entire sample and the rolling window approach, specifically they consider only the daily returns. In contrast, the strategies do not involve any optimization process and therefore consider the entire sample.

4.1 Test for weak efficiency

Autocorrelation Test To evaluate the weak form of the Efficient Market Hypothesis (EMH) and examine potential informational patterns within the historical data, we employed the Automatic Portmanteau test, as proposed by Escanciano and Lobato [21]. This test is specifically designed to assess the presence of autocorrelation in financial return series. Under the null hypothesis of the test, it is assumed that there is no autocorrelation in the return series, more specifically the null is that the autocorrelation coefficients are jointly zero, which aligns with the premise of the weak form of EMH, where past price movements are believed to contain no predictive power for future prices. For the purpose of this analysis, we set the maximum lag to 10, which allows for a reasonable examination of autocorrelation over a moderate time horizon.

The test was conducted using two approaches: the first considered the entire sample period, while the second utilized a rolling window technique. The rolling window approach offers several advantages, as it allows for the detection of temporal variations in the autocorrelation structure. This method is particularly valuable in the context of financial markets, where the absence or presence of autocorrelation may not be constant over time. By applying this rolling window approach, we are able to identify periods in which the null hypothesis may be rejected, signaling potential deviations from the efficient market hypothesis. Such periods could indicate the presence of informational patterns, where historical data might provide predictive signals, suggesting that technical analysis could be beneficial in those specific windows. Thus, this dual approach provides a more nuanced understanding of the market efficiency dynamics over time.

The following Table provides the results over the entire sample.

Asset	P-value Auto Q
EUR/USD	0.05639429
Gold	0.8491317
S&P 500	0.0004386

Table 2: Auto Q Portmanteau test for daily returns. Entire sample considered.

As presented in Table 2, the results reveal that for EUR/USD, we fail to reject the null hypothesis of no autocorrelation at both the 1% and 5% significance levels. In contrast, for Gold, there is stronger evidence in favor of the null hypothesis, indicating the absence of autocorrelation. However, for the S&P 500, we reject the null hypothesis of no autocorrelation, suggesting the presence of autocorrelation in the return series. Based on these findings, we conclude that, at least over the entire sample period, only the S&P 500 demonstrates a strong violation of weak form market efficiency, as evidenced by the significant, at different level, autocorrelation in its returns. This result implies that historical price movements in the S&P 500 may contain predictable patterns, challenging the weak form of the Efficient Market Hypothesis for this particular asset. Moving on to the rolling window analysis, the results can be observed in the following figures. Specifically, a window of 504 observations (equivalent to 2 years) was used, with the window advancing by one observation at a time.

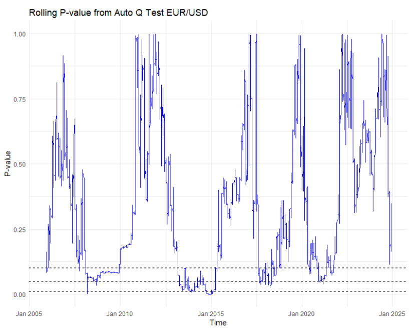

Figure 1: Rolling P-value from Auto Q Test EUR/USD.

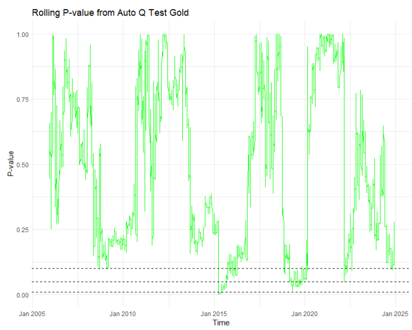

Figure 2: Rolling P-value from Auto Q Test Gold.

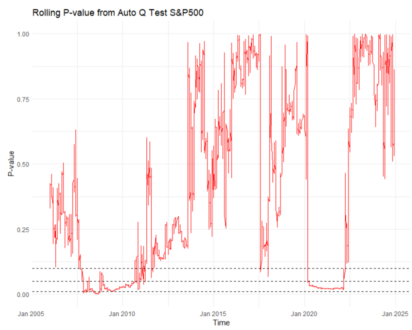

Figure 3: Rolling P-value from Auto Q Test S&P 500.

As illustrated in Figures 1, 2, and 3, based on the rolling window analysis, we observe occasional instances of autocorrelation. However, in recent years, there has been no strong or consistent presence of autocorrelation within the returns. Notably, for EUR/USD and the S&P 500, autocorrelation appears more frequently and persists for longer periods compared to Gold, where such patterns are less pronounced. However, it is important to note that the periods during which autocorrelation is observed coincide with times of crisis. Specifically, for the S&P 500 and EUR/USD, the low values observed are likely attributable to market crashes, however, the Q test used is robust to heteroscedasticity and so the results should be reliable.

From these results, it might be inferred that the presence of autocorrelation windows is more likely to occur in the S&P 500 than in the other two assets. This suggests that, at certain points in time, the returns of the S&P 500 may exhibit patterns that could potentially be exploited. The occurrence of such autocorrelation windows implies the possibility of generating profitable trading strategies, as technical analysis techniques are specifically designed to exploit such informational patterns in asset prices. Therefore, the identification of these windows of autocorrelation opens up the potential for further exploration of trading strategies that capitalize on historical price movements. However, as previously noted, the rejection of the null hypothesis tends to coincide with market crashes. Specifically, during such downturns, the leverage effect becomes evident, resulting in consecutive returns of the same sign. This phenomenon can therefore influence the outcomes of statistical tests. Consequently, it is crucial to consider the broader economic context to more accurately assess these events.

On the other hand, even if there is any presence of autocorrelation within the data, this results might be generally difficult to be exploited in reality due to the costs. Indeed, in order to better capture this consideration, let us observe directly the acf and pacf over the entire sample.

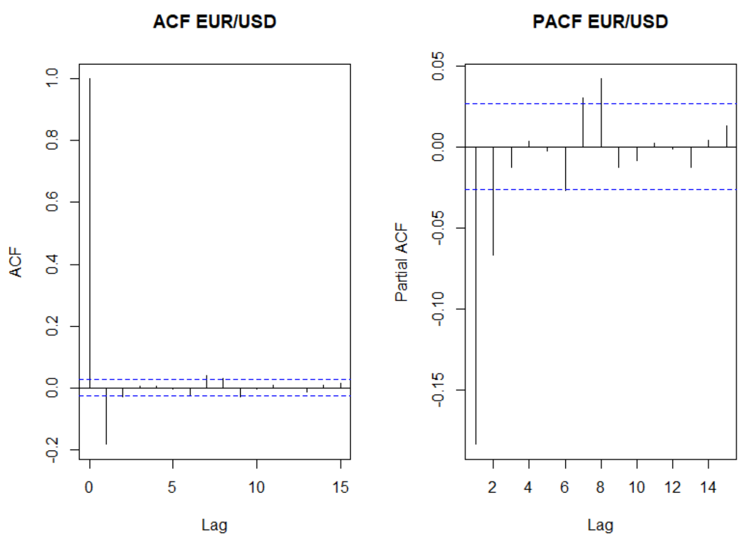

Figure 4: ACF and PACF for daily returns of EUR/USD.

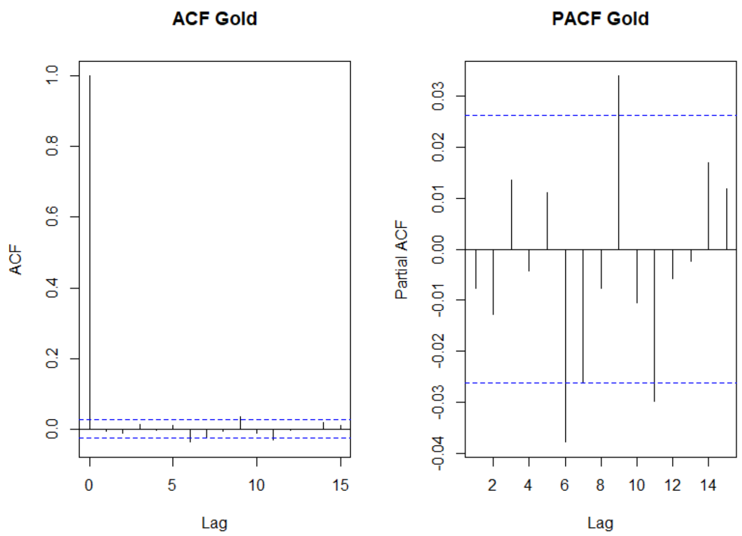

Figure 5: ACF and PACF for for daily returns of Gold.

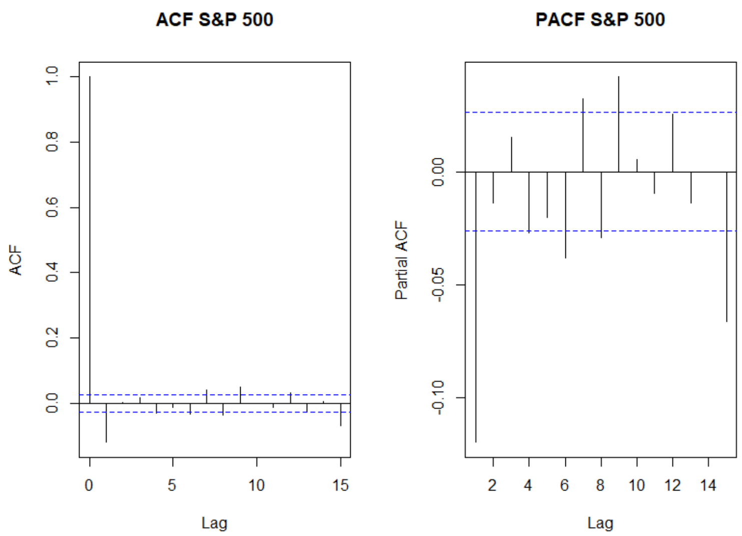

Figure 6: ACF and PACF for for daily returns of S&P 500.

From Figures 4, 5, and 6, it is observed that significant lags are present. This observation is particularly notable for EUR/USD and the S&P 500, which is consistent with the results from the test in Table 2. However, it is also important to note that, despite the daily time scale, the autocorrelations are relatively small, with a significance level of approximately 0.02. Given their modest magnitude, it is likely that this pattern has not been "eliminated" due to its intrinsic weakness. As a result, the low magnitude of these autocorrelations, especially at this time scale, suggests that they may not be sustainable when accounting for costs. Since the magnitude of the autocorrelations is related to the intensity and to the likelihood of observing such an event, their small size implies that they are unlikely to be effective for exploitation.

Variance Ratio Test After having investigated the autocorrelation test we can observe the results from the variance ratio tests. Again as already mentioned previously to overcome the issues from standard VR tests, namely the lag selection and the poor asymptotic approximation of the VR sample statistic, we consider the automatic VR test of Choi [9] with bootstrapped p-values, the method is illustrated by: Kim [18] and Charles et al. [6]. The wild bootstrap with Mammen distribution is considered. A number of 10,000 bootstrap replication was considered.

Asset	P-value Auto VR Test
EUR/USD	0.0173
Gold	0.8
S&P 500	0.013

Table 3: Auto VR Test for daily returns. Entire sample considered.

Based on the results presented in Table 3, we draw the same conclusion as from the autocorrelation test. Specifically, we fail to reject the null hypothesis of a random walk for Gold. Similarly, we do not reject the null hypothesis for EUR/USD at the 1% significance level, and the same conclusion applies to the S&P 500.

We can also examine the results using a rolling window approach, as previously performed, with a window size of 504 observations, and applying the same methodology utilized for the autocorrelation test. We are still considering the same test as in Table 3.

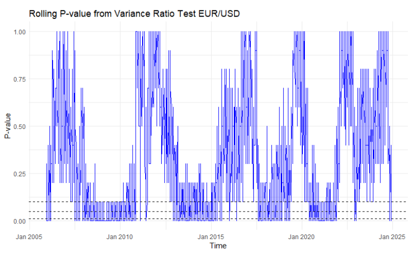

Figure 7: Rolling p-value from Variance Ratio Test EUR.

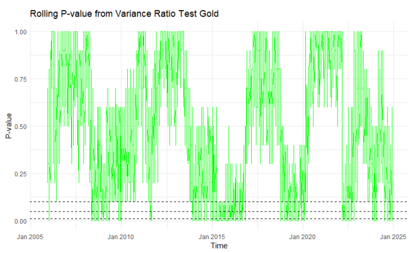

Figure 8: Rolling p-value from Variance Ratio Test Gold.

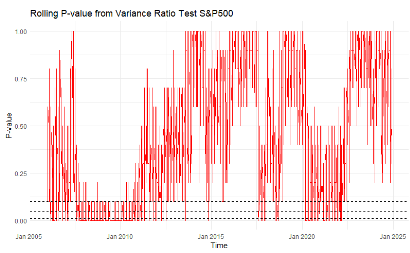

Figure 9: Rolling p-value from Variance Ratio Test S&P 500.

From Figures 7, 8, and 9, a similar pattern emerges as observed in the autocorrelation tests. In general, there appear to be periods of "weak inefficiency" that may potentially be exploitable, for example, through technical analysis.

The use of the VR test discussed above may not be sufficient. We thus present both the heteroscedasticity-adjusted VR test by Lo and MacKinlay and the Chow and Denning test over the entire sample, considering lag values of 2, 4, 6, 8, and 10 for the VR test. To address the poor asymptotic approximation for both tests, we compute bootstrapped p-values based on Kim's method [19], using the Mammen distribution. The following tables present the results for the entire sample, with 10,000 bootstrap replications.

Asset	$q = 2$	$q = 4$	$q = 6$	$q = 8$	$q = 10$
EUR/USD	0.0171	0.0078	0.0112	0.0133	0.0256
Gold	0.806	0.661	0.778	0.538	0.4465
S&P 500	0	0.0102	0.01074	0.0231	0.0328

Table 4: LM VR test, p-values for different lags, daily returns considered. Entire sample considered.

Asset	P-value CD Test
EUR/USD	0.0245
Gold	0.725
S&P 500	0.0032

Table 5: CD Test up to lag 10 for daily returns. Entire sample considered.

As observed in Tables 4 and 5, the results are consistent with those presented in Table 3. Specifically, the evidence of weak-efficiency is most pronounced for Gold, while there is weaker evidence for EUR/USD and even less for the S&P 500.

After examining the presence of weak form efficiency in line with the RW3 hypothesis, we can conclude that for assets such as the S&P 500 and EUR/USD, there may be potential to derive meaningful results through the application of technical analysis. In general, these assets present opportunities to achieve better outcomes compared to a Buy-and-Hold strategy.

In the literature, when testing for market efficiency, Oh et al. [27] observe that the FX market in Europe and North America was efficient at least until the early 2000s. However, in the context of Uncovered Interest Parity (UIP), Czech et al. [11] present contrasting results, indicating periods of inefficiency in certain historical contexts.

It is important to note that for the FX market, a key method for testing market efficiency involves assessing the equilibrium relation defined by the UIP, a topic that has been extensively studied in the literature.

On the other hand, the strong rejection of the null hypothesis of no autocorrelation in stock indexes, at least for daily data, is well-supported, as demonstrated by Campbell et al. [5], and for the European Markets by Borges [4], in which some markets tends to be not efficient. Furthermore, with respect to commodities, Kristoufek et al. [20] indicate that some efficiency tends to persist, especially within metals.

The results obtained are generally consistent with those observed in the literature, even though the studies refer to different time periods. However, the inconsistency in addressing the question of weak-form efficiency is also evident in our analysis using the rolling window approach, where periods of inefficiency are observed.

It is worth noting that another potential approach for testing the presence of weak-efficiency is to examine long-range dependence. More specifically, Lo [24] defined the concept of long-range dependence as the existence of non-trivial dependencies within a time series that may not be fully captured by conventional statistical methods.

However, such methods sometimes can be problematic and generally lack robust statistical testing frameworks. As highlighted by Pernagallo [29], reliance on Hurst exponent methodologies can be misleading. For this reason, the popular statistical tests employed in this analysis were chosen to circumvent these potential issues.

After testing for the presence of RWH, specifically RW3, we note that technical analysis strategies can be employed to test for RW2, as outlined by Campbell et al. [5]. In particular, the application of technical analysis provides an empirical approach to testing the RW2 hypothesis.

It is important to clarify that the tests conducted assume a linear dependence between the returns and their lags, as seen in the autocorrelation tests. The same holds for the variance ratio tests, which remain dependent on the autocorrelation coefficients. Specifically, the VR statistics is the following:

$$\mathrm{VR}(q) = \frac{\mathrm{Var}[r_t(q)]}{q\mathrm{Var}[r_t]} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho(k).$$

As observed, the statistics are dependent on the autocorrelation coefficients, $\rho(k)$, meaning that we are operating within a ”linear” dependence framework. An alternative approach could involve testing for other forms of dependence, such as that investigated by Mutual Information, which is typically considered ”non-linear.” However, this method is not included in the present analysis. Instead, through technical analysis, we are empirically testing for potential dependencies between past and future data in a discretionary manner. Indeed, the way in which we process information through the tools of technical analysis and the price time series does not always align with standard methods for measuring statistical dependence. Therefore, technical analysis could serve as a potential method for assessing the effectiveness of historical data in forecasting future prices or returns.

4.2 EUR/USD Trading Results

We begin by presenting the results for the EUR/USD asset. Specifically, the results are provided both with and without accounting for transaction costs. The following table presents the annualized Sharpe and Sortino ratios for the strategies considered. A zero risk-free return is assumed.

Asset	SR	PSR	SorR	SR (fees)	PSR(fees)	SorR (fees)
MAs (20,50)	0.09	65.57%	0.12	0.085	64.74%	0.11
MAs (50,200)	0.06	59.85%	0.08	0.054	59.80%	0.07
RSI (7)	0.26	88.77%	0.39	0.24	87.27%	0.36
RSI (14)	0.06	59.92%	0.08	0.05	59.30%	0.07
Mom (7)	-0.64	0.05%	-0.79	-0.68	0.025%	-0.83
Mom (14)	-0.25	12.04%	-0.33	-0.28	9.45%	-0.37
MACD (12,26,9)	-0.42	2.42%	-0.56	-0.44	2.08%	-0.58
Random Strategy	-0.11	29.09%	-0.16	-0.21	15.85%	-0.30
Buy-and-Hold	-0.08	35.74%	-0.11	/	/	/

Table 6: Sharpe, PSR and Sortino Ratio on an annual basis. The PSR considers a benchmark SR^* of zero. The fees considered are at 2% based on the return.

From Table 6, it can be observed that the moving average and RSI strategies generate positive Sharpe ratios, although their PSRs are not particularly high, none of them are larger than the conventional confidence interval, generally at 90% to 99%. However, these strategies produce a higher Sortino ratio compared to the Sharpe ratio, indicating a lower downside risk in their respective time series.

In contrast, the other technical strategies perform only marginally better than a random strategy, all of which yield negative risk-return metrics. Like the other strategies, this approach is based on long and short positions.

When transaction costs are accounted for, the results remain largely unchanged. This is primarily because, for the technical strategies, fees are incurred only when there is a change in position or, more generally, when a new order is placed. This fee structure can be likened to a bid-ask spread, as the daily return is reduced by 2%.

Additionally, it is observed that most of all the strategies underperform, generating negative Sharpe or Sortino, in the context of this asset. This outcome is further evidenced by the negative Sharpe ratio of the Buy-and-Hold strategy. The poor performance might be attributed to the fact that, unlike commodities or stocks, currencies generally do not exhibit a clear trend. Moreover, the technical difficulties associated with forecasting methods, particularly over short horizons, as discussed in Section 3, complicate trading activities in this market. Further supporting this observation are the results of tests for weak-form market efficiency conducted previously.

4.2.1 Bootstrap EUR/USD Results

We now proceed with bootstrapping the time series of the strategies to obtain more robust statistics. The bootstrap method is generally useful in this context. We apply a block bootstrap approach to account for the autocorrelation present in the returns of the strategies. The following Tables present the results for the previous metrics, excluding the Sharpe ratio (SR), along with the empirical 95% quantiles computed for the distributions of these metrics. The SR is omitted for computational efficiency, as the PSR provides a clearer interpretation of the SR.

More specifically, a total of 10,000 bootstrap replications were performed, using a block length of 3, which is reasonable considering the potential for up to 3 lags of memory, especially in trend-following strategies. Generally, the block length should be determined based on the sample pacf or acf. In our case, to avoid introducing any bias, this point was not considered.

Asset	**PSR**	**SorR**	**PSR (fees)**	**SorR (fees)**
MAs (20,50)	65.92%	0.12	64.56%	0.11
	(10.47%, 98.5%)	(-0.37, 0.66)	(9.53%, 98.14)	(-0.39, 0.66)
MAs (50,200)	60.53%	0.07	59.56%	0.07
	(7.49%, 97.57%)	(-0.43, 0.61)	(7.37%, 97.56%)	(-0.43, 0.61)
RSI (7)	88.99%	0.38	87.27%	0.35
	(27.36%, 99.92%)	(-0.18, 1.02)	(24.52%, 99.90%)	(-0.19, 1.012)
RSI (14)	59.97%	0.07	59.42%	0.07
	(7.17%, 97.03%)	(-0.44, 0.62)	(7.29%, 97.65%)	(-0.44, 0.63)
Mom (7)	0.05%	-0.83	0.025%	-0.81
	(0.000053%, 14.4%)	(-1.26, -0.31)	(0.00002%, 10.41%)	(-1.26, -0.30)

Table 7: Median PSR and Sortino Ratio on an annual basis for strategies up to Momentum (7). The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

Asset	PSR	SorR	PSR (fees)	SorR (fees)
Mom (14)	12.53%	-0.34	9.39%	-0.34
	(0.07%, 77.03%)	(-0.85, 0.23)	(0.04%, 72,37%)	(-0.85, 0.22)
MACD	2.29%	-0.56	2.43%	-0.57
	(0.0022%, 44.9%)	(-1.08, -0.02)	(0.0025%, 46.9%)	(-1.09, -0.03)
Random	28.88%	-0.17	15.45%	-0.31
	(0.62%, 91.31%)	(-0.73, 0.44)	(0.12%, 81.27%)	(-0.87, 0.26)
BH	35.08%	-0.11	/	/
	(1.85%, 91.18%)	(-0.61, 0.42)	/	/

Table 8: PSR and Sortino Ratio on an annual basis for strategies from Momentum (14) onwards. The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

From Tables 7 and 8, it can be observed that only the Moving Averages and RSI strategies yield statistically significant results when evaluating the PSR metrics. However, these results are based on the 97.5% quantile, indicating that only 250 out of 10,000 replications achieve such outcomes. Based on these findings, it can be concluded that, although the random strategy proves ineffective, the performance of the various strategies is generally comparable. In fact, none of the strategies exhibit notably high Sharpe or Sortino ratios. One possible explanation for this could be the inherent complexity of the EUR/USD asset, which may be more intricate than other assets, as previously noted.

In conclusion, the fact that only a few strategies yielded favorable results, with positive Sharpe and Sortino ratios, was insufficient, as the PSR was not sufficiently high to demonstrate a clear ability of the strategies to outperform a zero-skill benchmark. Consequently, although the random strategy did not prove to be effective, it is noteworthy that only marginal differences were observed between them. This suggests a potential limitation of these types of technical analysis methods when applied to the EUR/USD asset.

4.3 S&P 500 Trading Results

Having reviewed the results for the EUR/USD, we now turn to the results for the S&P 500, again considering the same metrics previously used.

Asset	**SR**	**PSR**	**SorR**	**SR (fees)**	**PSR(fees)**	**SorR (fees)**
MAs (20,50)	0.017	53.18%	0.024	0.085	52.13%	0.016
MAs (50,200)	0.33	92.85%	0.456	0.32	92.63%	0.451
RSI (7)	0.27	89.15%	0.43	0.24	87.28%	0.39
RSI (14)	-0.12	28.93%	-0.18	-0.13	28.31%	-0.19
Mom (7)	-0.15	23.90%	-0.21	-0.19	19.13%	-0.26
Mom (14)	-0.04	42.60%	-0.06	-0.06	37.99%	-0.09
MACD (12,26,9)	-0.04	42.35%	-0.06	-0.05	39.94%	-0.08
Random Strategy	0.0047	50.86%	0.006	-0.1	32.37%	-0.14
Buy-and-Hold	0.42	97.23%	0.58	/	/	/

Table 9: Sharpe, PSR and Sortino Ratio on an annual basis. The PSR considers a benchmark SR^* of zero. The fees considered are at 2% based on the return.

From Table 9, we observe that the same strategies that performed well with the EUR/USD asset also yield favorable results in this case, particularly the moving average strategies and the 7-period RSI. Apart from the MA(50,200) strategy, these results are not comparable to the Buy-and-Hold strategy. In general, only the MA(50,200) strategy produces results similar to Buy-and-Hold, likely because it mimics a Buy-and-Hold approach. Given the strong trend of the S&P 500 index over the past 20 years, this strategy has performed well, maybe due to the inherent trend in the data. So that, the MA(50,200) went well, mainly due to its long trades.

The performance of the other technical strategies, however, was subpar. The random strategy also underperformed, although its results were not significantly worse than those of the other technical strategies.

When accounting for fees, the situation remains similar to what was observed for the EUR/USD asset. Overall, we find that, although the tests conducted previously show evidence of autocorrelation, the moving autocorrelation test was a valuable inclusion. This test reveals that only in certain windows, past information does have significance, a finding that aligns with the results obtained in Table 9.

4.3.1 Bootstrap S&P 500 Results

As in the previous analysis, we employ the bootstrap method to obtain more robust risk and return metrics. The procedure follows the same approach as before, with a block bootstrap of length 3 being applied. The results are presented in the following tables.

Asset	PSR	SorR	PSR (fees)	SorR (fees)
MAs (20,50)	53.39%	0.026	51.42%	0.028
	(4.32%, 96.97%)	(-0.51, 0.56)	(4.29%, 96.56%)	(-0.53, 0.58)
MAs (50,200)	92.73%	0.45	92.62%	0.45
	(36.08%, 99.93%)	(-0.09, 1.03)	(35.81%, 99.94%)	(-0.11, 1.03)
RSI (7)	89.13%	0.41	87.06%	0.37
	(21.87%, 99.95%)	(-0.27, 1.13)	(18.03%, 99.93%)	(-0.30, 1.10)
RSI (14)	28.36%	-0.18	27.90%	-0.19
	(0.58%, 91.39%)	(-0.81, 0.47)	(0.66%, 91.57%)	(-0.83, 0.45)
Mom (7)	23.68%	-0.21	23.71%	-0.26
	(0.59%, 85.47%)	(-0.75, 0.35)	(0.52%, 86.26%)	(-0.77, 0.34)

Table 10: Median PSR and Sortino Ratio on an annual basis for strategies up to Momentum (7). The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

Asset	PSR	SorR	PSR (fees)	SorR (fees)
Mom (14)	41.99%	-0.051	42.5%	-0.06
	(2.075%, 94.86%)	(-0.61, 0.52)	(2.02%, 94.64%)	(-0.62, 0.52)
MACD	42.35%	-0.06	39.19%	-0.08
	(2.29%, 94.64%)	(-0.62, 0.53)	(1.83%, 93.26%)	(-0.65, 0.50)
Random	50.52%	-0.0012	33.18%	-0.15
	(2.65%, 97.15%)	(-0.61, 0.63)	(0.78%, 92.27%)	(-0.75, 0.49)
BH	97.28%	0.57	/	/
	(55.86%, 99.90%)	(0.031, 1.13)	/	/

Table 11: PSR and Sortino Ratio on an annual basis for strategies from Momentum (14) onwards. The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

From Tables 10 and 11, it is evident that the moving averages strategy (50,200) and the RSI 7 strategy emerge as relatively effective alternatives for trading. However, despite the promising performance of these strategies, the Buy-and-Hold strategy continues to demonstrate superior success when compared to the others under evaluation.

A noteworthy observation from the results is that, while the outcomes appear more favorable relative to the EUR/USD asset, there are instances in which we are unable to reject the null hypothesis of no skill in managing, even when considering the top 250 results. This finding reinforces the conclusions presented in the tests previously conducted, where it was emphasized that autocorrelation is observed in only a limited number of periods. Such a result implies that the market offers only a narrow window of opportunity to leverage past return information during specific time intervals, which further suggests that the potential for exploiting historical price movements is constrained. This observation points to a more complex market dynamic, where consistent strategies may struggle to achieve sustained out-performance over time.

4.4 Gold Trading Results

To conclude, the results for the Gold are presented, more specifically, the same risk and returns metrics were considered as for EUR/USD and S&P 500.

Asset	**SR**	**PSR**	**SorR**	**SR (fees)**	**PSR(fees)**	**SorR (fees)**
MAs (20,50)	-0.019	46.44%	-0.026	-0.021	46.04%	-0.029
MAs (50,200)	0.21	82.72%	0.288	0.20	82.67%	0.28
RSI (7)	-0.31	7.52%	-0.44	-0.33	6.10%	-0.47
RSI (14)	-0.48	1.28%	-0.68	-0.49	1.23%	-0.69
Momentum (7)	-0.084	35.06%	-0.11	-0.12	27.64%	-0.18
Momentum (14)	-0.14	25.14%	-0.20	-0.17	21.19%	-0.24
MACD (12,26,9)	0.15	75.60%	0.21	0.14	74.24%	0.20
Random Strategy	0.21	83.59%	0.30	0.1	68.03%	0.14
Buy-and-Hold	0.5	98.74%	0.68	/	/	/

Table 12: Sharpe, PSR and Sortino Ratio on an annual basis. The PSR considers a benchmark SR^* of zero. The fees considered are at 2% based on the return.

From Table 12, a similar situation to that observed with the S&P 500 emerges. Specifically, the Buy-and-Hold strategy yields the best results, followed by the moving average strategy (50,200), the MACD, and the random trading strategy, all of which also perform reasonably well. However, none of these strategies produce outstanding results when considering the PSR, as none, aside from Buy-and-Hold, achieve at least 90%.

These results partially confirm the findings presented in the tests, where the presence of autocorrelation, observed across different windows, suggests limited prospects for outperforming the Buy-and-Hold strategy. To further understand the results presented in Table 12, we apply the bootstrap method, as was done with the other assets. This approach is useful for assessing the uncertainty within the estimations.

4.4.1 Bootstrap Gold Results

The bootstrap procedure considered is the same as the one considered for the other assets, again only PSR and Sortino ratio are considered.

Asset	**PSR**	**SorR**	**PSR (fees)**	**SorR (fees)**
MAs (20,50)	46.67%	-0.031	44.85%	-0.028
	(1.81%, 96.20%)	(-0.62, 0.57)	(1.85%, 96.77%)	(-0.62, 0.56)
MAs (50,200)	82.57%	0.28	83.07%	0.28
	(15.42%, 99.77%)	(-0.3, 0.89)	(15.64%, 99.81%)	(-0.33, 0.89)
RSI (7)	7.39%	-0.46	5.82%	-0.48
	(0.034%, 67.65%)	(-1.03, 0.14)	(0.02%, 65.65%)	(-1.06, 0.12)
RSI (14)	1.29%	-0.71	1.30%	-0.71
	(0.0017%, 38.66%)	(-1.31, -0.11)	(0.0015%, 37.33%)	(-1.31, -0.097)
Mom (7)	35.89%	-0.12	27.73%	-0.18
	(0.91%, 94.87%)	(-0.70, 0.48)	(0.48%, 91.31%)	(-0.77, 0.44)

Table 13: Median PSR and Sortino Ratio on an annual basis for strategies up to Momentum (7). The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

Asset	**PSR**	**SorR**	**PSR (fees)**	**SorR (fees)**
Mom (14)	24.53%	-0.20	20.85%	-0.24
	(0.40%, 90.23%)	(-0.79, 0.39)	(0.29%, 87.43%)	(-0.83, 0.34)
MACD	75.77%	0.21	74.45%	0.20
	(11.30%, 99.48%)	(-0.36, 0.80)	(10.19%, 99.54%)	(-0.39, 0.80)
Random	83.33%	0.30	68.59%	0.14
	(16.75%, 99.83%)	(-0.30, 0.91)	(6.72%, 99.21%)	(-0.43, 0.77)
BH	97.28%	0.57	/	/
	(55.86%, 99.90%)	(0.031, 1.13)	/	/

Table 14: PSR and Sortino Ratio on an annual basis for strategies from Momentum (14) onwards. The PSR considers a benchmark SR^* of zero. The fees considered are 2% based on the return. In parenthesis are the 2.5% and 97.5% quantiles.

From Tables 13 and 14, it can be observed that none of the strategies, aside from MACD, moving average strategy (50,200) and random, generally yield favorable results, as previously indicated in Table 12. Furthermore, the results reinforce similar observations as earlier, where only a few strategies generate a good PSR value, despite being ranked among the top 250 outcomes out of 10,000 bootstrap replications.

To conclude we observe that, the difficulties in beating the BH were higher in the Gold asset, on the other hand this aspect was generally already clear when observing the efficiency tests, in which when performing the test on the entire sample, no sign of autocorrelation were found, on the other hand few windows of autocorrelation instead were found even if less frequently with respect to the other two assets.

5 Conclusions

The analysis highlighted some of the complexity of assessing weak-form efficiency. Specifically, we observed that, based on the autocorrelation tests and variance ratio tests, when the entire sample is considered, we reject the null hypothesis for the S&P 500 and, to a lesser extent, for EUR/USD, but fail to reject it for Gold. However, when applying a rolling window approach, we identify periods in which both autocorrelation and rejection of the null hypothesis according to the variance ratio tests are present, suggesting potential opportunities to exploit past information within the time series. Nonetheless, it is important to note that some of these periods of null rejection coincide with market crises, which may influence the results. Therefore, caution is necessary when interpreting results based on the rolling window approach.

In contrast, when testing for RW2 through technical analysis, we find that only for EUR/USD do certain strategies outperform the Buy-and-Hold approach. For both the S&P 500 and Gold, no strategies performed better than Buy-and-Hold.

When comparing the results from statistical tests and technical analysis, we observe occasional discrepancies. For assets such as EUR/USD, where there is limited evidence of weak efficiency, certain technical strategies have outperformed the Buy-and-Hold strategy. However, for the S&P 500, despite stronger evidence of weak efficiency from the statistical tests, none of the technical strategies have surpassed the Buy-and-Hold approach. The discrepancy with the Buy-and-Hold strategy is notable, as it is considered a passive approach that does not rely on modeling past data to extrapolate future information or make predictions, or, more generally, this approach does not rely on complex assumptions. It is important to acknowledge that the statistical tests used in this analysis may have inherent limitations and potential flaws, and therefore, caution is advised when interpreting these results.

Moreover, the use of a random approach to trading, as highlighted by Biondo et al. [3], represents an intriguing foundation for further study. This approach serves as an interesting starting point for examining trading strategies from a new perspective. The results obtained in this analysis suggest that random trading, when compared to traditional technical analysis strategies, can serve as a viable alternative "benchmark." In some cases, the performance of random strategies is either superior to or not significantly different from that of technical analysis, which opens up the possibility for a more comprehensive evaluation of trading strategies. This comparison allows for an interesting contrast between technical strategies, which rely on historical price data and patterns, and strategies that are entirely based on randomness, thereby shedding light on the potential benefits of incorporating randomness into trading decisions.

From the results obtained in Section 4, it becomes evident that utilizing technical information derived from past data in the same time series may not always be effective or reliable. In fact, relying solely on historical data for technical analysis can often lead to suboptimal decision-making, especially in markets that are inherently volatile and complex. It is also important to note that the consideration of random alternatives is a common practice in the assessment of risk and return metrics. Hypothesis testing, already incorporate this assumption, indeed, it helps us to determine whether the observed results significantly deviate from what would be expected by chance.

Ultimately, the findings of this analysis contribute to the broader conversation about the role of randomness in financial decision-making.

References

[1] Appel, G. (2005). Technical analysis: power tools for active investors. FT Press.

[2] Bailey, D. H., & Lopez de Prado, M. (2012). The Sharpe ratio efficient frontier. *Journal of Risk*, *15*(2), 13.

[3] Biondo, A. E., Pluchino, A., Rapisarda, A., & Helbing, D. (2013). Are random trading strategies more successful than technical ones?. PloS one, 8(7), e68344.

[4] Borges, M. R. (2010). Efficient market hypothesis in European stock markets. The European Journal of Finance, 16(7), 711-726.

[5] Campbell, J. Y., Lo, A. W., MacKinlay, A. C., & Whitelaw, R. F. (1998). The econometrics of financial markets. *Macroeconomic Dynamics*, *2*(4), 559-562.

[6] Charles, A., Darné, O., & Kim, J. H. (2011). Small sample properties of alternative tests for martingale difference hypothesis. Economics Letters, 110(2), 151-154.

[7] Chow, K. V., & Denning, K. C. (1993). A simple multiple variance ratio test. *Journal of econometrics*, *58*(3), 385-401.

[8] Cheung, Y. W., Chinn, M. D., Pascual, A. G., & Zhang, Y. (2019). *Exchange rate prediction redux: New models, new data, new currencies. Journal of International Money and Finance*, *95*, 332-362.

[9] Choi, I. (1999). Testing the random walk hypothesis for real exchange rates. Journal of Applied Econometrics, 14(3), 293-308.

[10] Christoffersen, P., Ghysels, E., & Swanson, N. R. (2002). *Let's get "real" about using economic data. Journal of Empirical Finance*, *9*(3), 343-360.

[11] Czech, K. A., & Waszkowski, A. (2012). Foreign exchange market efficiency: Empirical results for the USD/EUR market. e-Finanse: Financial Internet Quarterly, 8(3), 1-9.

[12] Bailey, D. H., & Lopez de Prado, M. (2012). The Sharpe ratio efficient frontier. *Journal of Risk*, *15*(2), 13.

[13] Engel, C., Lee, D., Liu, C., Liu, C., & Wu, S. P. Y. (2019). *The uncovered interest parity puzzle, exchange rate forecasting, and Taylor rules. Journal of International Money and Finance*, *95*, 317-331.

[14] Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. The Journal of Finance, 25(2), 383–.

[15] Fama, E. F. (1965). The behavior of stock-market prices. The journal of Business, 38(1), 34-105.

[16] Han, Y., Liu, Y., Zhou, G., & Zhu, Y. (2024). Technical analysis in the stock market: A review. Handbook of Investment Analysis, Portfolio Management, and Financial Derivatives: In 4 Volumes, 1893-1928.

[17] Hyndman, R.J., & Athanasopoulos, G. (2021) *Forecasting: principles and practice*, 3rd edition, OTexts: Melbourne, Australia. OTexts.com/fpp3.

[18] Kim, J. H. (2009). Automatic variance ratio test under conditional heteroskedasticity. Finance Research Letters, 6(3), 179-185.

[19] Kim, J. H. (2006). Wild bootstrapping variance ratio tests. Economics letters, 92(1), 38-43.

[20] Kristoufek, L., & Vosvrda, M. (2014). Commodity futures and market efficiency. Energy Economics, 42, 50-57.

[21] Escanciano, J. C., & Lobato, I. N. (2009). An automatic portmanteau test for serial correlation. Journal of Econometrics, 151(2), 140-149.

[22] Lo, A. W., & MacKinlay, A. C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test. The review of financial studies, 1(1), 41-66.

[23] Lo, A. W. (2002). The statistics of Sharpe ratios. *Financial analysts journal*, *58*(4), 36-52.

[24] Lo, A. W. (1991). Long-term memory in stock market prices. Econometrica: Journal of the Econometric Society, 1279-1313.

[25] Meese, R. A., & Rogoff, K. (1983). *Empirical exchange rate models of the seventies: Do they fit out of sample? Journal of international economics*, *14*(1-2), 3-24.

[26] Menkhoff, L., & Taylor, M. P. (2007). The obstinate passion of foreign exchange professionals: technical analysis. Journal of Economic Literature, 45(4), 936-972.

[27] Oh, G., Kim, S., & Eom, C. (2007). Market efficiency in foreign exchange markets. Physica A: Statistical Mechanics and its Applications, 382(1), 209-212.

[28] Park, C. H., & Irwin, S. H. (2004). The profitability of technical analysis: A review.

[29] Pernagallo, G. (2025). Random walks, Hurst exponent, and market efficiency. Quality & Quantity, 1-23.

[30] Rossi, B. (2013). *Exchange rate predictability. Journal of economic literature*, *51*(4), 1063-1119.

[31] Samuelson, Paul A. 1965a. Proof That Properly Anticipated Prices Fluctuate Randomly. *Industrial Management Review, 6(2): 41-49.* 20.

[32] Wilder, J. W. (1978). New concepts in technical trading systems. Greensboro, NC.