Assignment 1: Linear Algebra, Differential

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1 Vector Matrix Calculations, pen and paper

1.1 Exercise

let $v = [-2, 3, 4, 1, a]^T$

$$v^T \cdot v = [-2, 3, 4, 1, a] \cdot \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \\ a \end{bmatrix} = (-2 \cdot -2 + 3 \cdot 3 + 4 \cdot 4 + 1 \cdot 1 + a \cdot a) = 30 + a^2$$

$$v \cdot v^T = \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \\ a \end{bmatrix} \cdot [-2, 3, 4, 1, a] =$$

$$= \begin{bmatrix} -2 \cdot -2 & -2 \cdot 3 & -2 \cdot 4 & -2 \cdot 1 & -2 \cdot a \\ 3 \cdot -2 & 3 \cdot 3 & 3 \cdot 4 & 3 \cdot 1 & 3 \cdot a \\ 4 \cdot -2 & 4 \cdot 3 & 4 \cdot 4 & 4 \cdot 1 & 4 \cdot a \\ 1 \cdot -2 & 1 \cdot 3 & 1 \cdot 4 & 1 \cdot 1 & 1 \cdot a \\ a \cdot -2 & a \cdot 3 & a \cdot 4 & a \cdot 1 & a \cdot a \end{bmatrix} = \begin{bmatrix} 4 & -6 & -8 & -2 & -2a \\ -6 & 9 & 12 & 3 & 3a \\ -8 & 12 & 16 & 4 & 4a \\ -2 & 3 & 4 & 1 & a \\ -2a & 3a & 4a & a & a^2 \end{bmatrix}$$

1.2 Exercise

let A and B be

$$A = \begin{bmatrix} 1 & -6 & 1 \\ 2 & -4 & 5 \\ 8 & 6 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 3 \\ -3 & 7 \\ -1 & 1 \end{bmatrix}$$

$$v = B \cdot [x, y]^T = \begin{bmatrix} 2 & 3 \\ -3 & 7 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \cdot x + 3 \cdot y \\ -3 \cdot x + 7 \cdot y \\ -1 \cdot x + 1 \cdot y \end{bmatrix}$$

$$w = A \cdot v = \begin{bmatrix} (2x+3y) + (18x-42y) + (-x+y) \\ (4x+6y) + (12x-28y) + (-5x+5y) \\ (16x+24y) + (-18x+42y) + (-x+y) \end{bmatrix} = \begin{bmatrix} 19x - 38y \\ 11x - 17y \\ -3x + 67y \end{bmatrix}$$

$$C = A \cdot B = \begin{bmatrix} 19 & -38 \\ 11 & -17 \\ -3 & 67 \end{bmatrix}$$

$$z = C \cdot [x, y]^T = \begin{bmatrix} 19 & -38 \\ 11 & -17 \\ -3 & 67 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19x + -38y \\ 11x - 17y \\ -3x + 67y \end{bmatrix}$$

1.3 Exercise

Let A be the matrix:

$$A = \begin{bmatrix} x-3 & -x+3 & -x+5 \\ x-2 & -x+2 & -x+4 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x-3 & -x+3 & -x+5 \\ x-2 & -x+2 & -x+4 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x-3 & -x+3 & -x+5 \\ x-2 & -x+2 & -x+4 \\ -1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} (x^2-6x+9)+(-x^2+5x-6)+(x-5) & (x^2-5x+6)+(-x^2+4x-4)+(x-4) & (-x+3)+(x-2)-1 \\ (-x^2+6x-9)+(x^2-5x+6)+(-x+5) & (-x^+5x-6)+(x^2-4x+4)+(-x+4) & (x-3)+(-x+2)+1 \\ (-x^2+8x-15)+(x^2-7x+12)+(-x+5) & (-x^2+7x-10)+(x^2-6x+8)+(-x+4) & (x-5)+(-x+4)+1 \end{bmatrix}^T =$$

$$= \begin{bmatrix} -2 & 2 & 2 \\ -2 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} -2 & 2 & 2 \\ -2 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x-3 & -x+3 & -x+5 \\ x-2 & -x+2 & -x+4 \\ -1 & 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} (+-2x+6)+(2x-4)-2 & (2x-6)+(-2x+4)+2 & (2x-10)+(-2x+8)+2 \\ (+-2x+6)+(2x-4)-2 & (2x-6)+(-2x+4)+2 & (2x-10)+(-2x+8)+2 \\ (0x-3\cdot 0)+(0x-2\cdot 0)+(0\cdot -1\cdot 0) & (-0x+3\cdot 0)+(-0x+2\cdot 0)+(0\cdot 1) & (-0x+5\cdot 0)+(-0x+4\cdot 0)+(1\cdot 0) \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

1.4 Exercise

Let A be the matrix:

$$A = \begin{bmatrix} 3 & 0 & 4 \\ -2 & 1 & -2 \\ 2 & 0 & 3 \end{bmatrix}$$

and let the vectors $e1$, $e2$ and $e3$ be:

$$e1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A \cdot f1 = e1 = A \cdot [x, y, z]^T = [1, 0, 0]^T = \begin{cases} 3x + 4z = 1 \\ -2x + y - 2z = 0 \\ 2x + 3z = 0 \end{cases} = ... = \begin{cases} x = 3 \\ y = 2 \\ z = -2 \end{cases}$$

$$A \cdot f2 = e2 = A \cdot [x, y, z]^T = [0, 1, 0]^T = \begin{cases} 3x + 4z = 0 \\ -2x + y - 2z = 1 \\ 2x + 3z = 0 \end{cases} = ... = \begin{cases} x = 0 \\ y = 1 \\ z = 0 \end{cases}$$

$$A \cdot f3 = e3 = A \cdot [x, y, z]^T = [0, 0, 1]^T = \begin{cases} 3x + 4z = 0 \\ -2x + y - 2z = 0 \\ 2x + 3z = 1 \end{cases} = ... = \begin{cases} x = -4 \\ y = -2 \\ z = 3 \end{cases}$$

$$f1 = [3, 2, -2]^T \qquad f2 = [0, 1, 0]^T \qquad f3 = [-4, -2, 3]^T$$

Let B be a matrix composed by $f1, f2, f3$ as columns:

$$B = \begin{bmatrix} f1 & f2 & f3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -4 \\ 2 & 1 & -2 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By multiplying the matrix B with matrix A, is possible to retrieve the identity matrix (3x3). The same multiplying $A \cdot B$. This means that B is the inverse matrix of A and A is the inversematrix of B.

1.5 Exercise

Let A and B be respectively:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & ... & a_{1,n} \\ a_{2,1} & a_{2,2} & ... & a_{2,n} \\ ... & ... & ... & ... \\ a_{n,1} & a_{n,2} & ... & a_{n,n} \end{bmatrix} \qquad B = \begin{bmatrix} b_{1,1} & b_{1,2} & ... & b_{1,n} \\ b_{2,1} & b_{2,2} & ... & b_{2,n} \\ ... & ... & ... & ... \\ b_{n,1} & b_{n,2} & ... & b_{n,n} \end{bmatrix}$$

Let the respectively transposed matrixes be:

$$A^T = \begin{bmatrix} a_{1,1} & a_{2,1} & ... & a_{n,1} \\ a_{1,2} & a_{2,2} & ... & a_{n,2} \\ ... & ... & ... & ... \\ a_{1,n} & a_{2,n} & ... & a_{n,n} \end{bmatrix} \qquad B^T = \begin{bmatrix} b_{1,1} & b_{2,1} & ... & b_{n,1} \\ b_{1,2} & b_{2,2} & ... & b_{n,2} \\ ... & ... & ... & ... \\ b_{1,n} & b_{2,n} & ... & b_{n,n} \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} a_{1,1} & a_{1,2} & ... & a_{1,n} \\ a_{2,1} & a_{2,2} & ... & a_{2,n} \\ ... & ... & ... & ... \\ a_{n,1} & a_{n,2} & ... & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} b_{1,1} & b_{1,2} & ... & b_{1,n} \\ b_{2,1} & b_{2,2} & ... & b_{2,n} \\ ... & ... & ... & ... \\ b_{n,1} & b_{n,2} & ... & b_{n,n} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & ... & c_{1,n} \\ c_{2,1} & c_{2,2} & ... & c_{2,n} \\ ... & ... & ... & ... \\ c_{n,1} & c_{n,2} & ... & c_{n,n} \end{bmatrix} = C$$

in which $c_{i,j} = a_{i,1}b_{1,j} + a_{i,2}b_{2,j} + ... + a_{i,n}b_{n,j} = \sum_{k=1}^{n} a_{i,k}b_{k,j}$ with $i, j \in [1..n]$.

$$(A \cdot B)^T = \begin{bmatrix} c_{1,1} & c_{2,1} & ... & c_{n,1} \\ c_{1,2} & c_{2,2} & ... & c_{n,2} \\ ... & ... & ... & ... \\ c_{1,n} & c_{2,n} & ... & c_{n,n} \end{bmatrix} = C^T$$

in which $c^T_{i,j} = c_{j,i} = a_{1,i}b_{j,1} + a_{2,i}b_{j,2} + ... + a_{n,i}b_{j,n} = \sum_{k=1}^{n} a_{k,i}b_{j,k}$ with $i, j \in [1..n]$.

$$A^T \cdot B^T = \begin{bmatrix} a_{1,1} & a_{2,1} & ... & a_{n,1} \\ a_{1,2} & a_{2,2} & ... & a_{n,2} \\ ... & ... & ... & ... \\ a_{1,n} & a_{2,n} & ... & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} b_{1,1} & b_{2,1} & ... & b_{n,1} \\ b_{1,2} & b_{2,2} & ... & b_{n,2} \\ ... & ... & ... & ... \\ b_{1,n} & b_{2,n} & ... & b_{n,n} \end{bmatrix} = \begin{bmatrix} d_{1,1} & d_{1,2} & ... & d_{1,n} \\ d_{2,1} & d_{2,2} & ... & d_{2,n} \\ ... & ... & ... & ... \\ d_{n,1} & d_{n,2} & ... & d_{n,n} \end{bmatrix} = D$$

in which $d_{i,j} = a_{i,1}b_{1,j} + a_{2,i}b_{j,2} + ... + a_{n,i}b_{j,n} = \sum_{k=1}^{n} a_{k,i}b_{j,k}$ with $i, j \in [1..n]$.
Comparing C^T and D is easy to see that they are the same matrix:
$d_{i,j} = \sum_{k=1}^{n} a_{k,i}b_{j,k}$ and $c^T_{i,j} = \sum_{k=1}^{n} a_{k,i}b_{j,k}$ with $i, j \in [1..n]$,
therefore $\forall(i, j)$, $d_{i,j} == c^T_{i,j}$

2 Vector Matrix Calculations, Python

2.1 Inner and outer product

Let `a = np.array([1,4,-5,3])` and `b = np.array([3,-1,0,2])`, the inner product is 5 and it is calculated by the function `a@b`. In order to make the handed-in code working, a line has been modified:

```
print(a@b)
a.shape = (4,1) # a matrix with 1 column (a column vector)
b.shape = (4,1) # a matrix with 1 column (a column vector)
print(a)
print(b)
print(b.shape)
print(b.T.shape)
```

```
print(a@b.T) #it was a@b, but the dimensions are not compatible
print(a.T@b)
print(float(a.T@b))
print(a@b.T)
```

the output is shown below:

```
5
[[ 1][ 4][-5][ 3]]
[[ 3][-1][ 0][ 2]]
(4, 1)
(1, 4)
[[  3  -1   0   2][ 12 -4   0   8][-15  5   0 -10][ 9  -3   0   6]]
[[5]]
5.0
[[  3  -1   0   2][ 12 -4   0   8][-15  5   0 -10][ 9  -3   0   6]]
```

The cross product made by the vector `t = np.array([2,0,-1])` and itself, and the other cross and inner products are in the following piece of code:

```
t = np.array([2,0,-1])
t.shape = (3,1)
np.cross(t.T, t.T) #cross(t,t) == [0,0,0]
a = np.array([-2,6,1])
a.shape = (3,1)
b = np.cross(t.T, a.T) #cross(t,a) == [6,0,12]
c = np.cross(a.T, t.T) #cross(a,t) == [-6,0,-12]
t.T@b.T #inner(t,b) == 0
a.T@b.T #inner(a,b) == 0
```

2.2 3D vector

Let the matrix m be:

$$m = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

a function that takes in input a 3D vector and calculates that matrix is

```
def get_t_matrix(vec):
  m = np.zeros((vec.size, vec.size))
```

```
    m[0,1] = -vec[2]
    m[0,2] = vec[1]
    m[1,0] = vec[2]
    m[1,2] = -vec[0]
    m[2,0] = -vec[1]
    m[2,1] = vec[0]
    return m
```

Then, it is possible to make a matrix vector product as shown here below:

```
m = get_t_matrix(t)
print(m.dot(t)) #[[0.][0.][0.]]
print(m.dot(a)) #[[6.][0.][12.]]
```

Let the vector v be $v = [x, y, z]^T$, the vector $t = [t1, t2, t3]^T$ and assuming that the operation $v \to t$ x v is linear, then:

$$f \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t1 \\ t2 \\ t3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t1x + t1y + t1z \\ t2x + t2y + t2z \\ t3x + t3y + t3z \end{bmatrix}$$

As is it possible to see in the code below, the matrix m^T is equal to $-m$, the transpose of m^2 is exactly the same matrix as m^2, but this is not true with m^3.

```
print(-m==m.T) #True

m2 = m.dot(m)
print(m2==m2.T) #True

m3 = m.dot(m.dot(m))
print(m3==m3.T) #False
```

3 Derivatives

3.1 Functions of one variable

Let the function $f(x)$ be $e^{-\frac{x^2}{2}}$ its derivative $f'(x)$ is $-e^{-\frac{x^2}{2}}x$, and it is calculated in this way:

1) Using the chain rule:

$$\frac{\partial}{\partial x} e^{-\frac{x^2}{2}} = \frac{\partial e^u}{\partial u}\frac{\partial u}{\partial x}$$

in which,

$$\frac{\partial}{\partial u}(e^u) = e^u \qquad u = -\frac{x^2}{2}$$

so, is possible to re-write it as:

$$e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}\left(-\frac{x^2}{2}\right)\right)$$

2) Factor out constants:

$$-\frac{1}{2}\left(\frac{\partial}{\partial x}(x^2)\right) e^{-\frac{x^2}{2}}$$

3) Use the power rule:

$$-\frac{1}{2}\, e^{-\frac{x^2}{2}}\, 2x$$

4) Simplify the expression:

$$f'(x) = -e^{-\frac{x^2}{2}} x$$

The second derivative $f"(x)$ is equal to $e^{-\frac{x^2}{2}}(x^2-1)$ and can be computed in this way:

1) Factor out constants:

$$-\left(\frac{\partial}{\partial x}\left(e^{-\frac{x^2}{2}} x\right)\right)$$

2) Leibniz rule:

$$-x\left(\frac{\partial}{\partial x}\left(e^{-\frac{x^2}{2}}\right)\right) + e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}(x)\right)$$

3) Chain rule:

$$-e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}(x)\right) - e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}\left(-\frac{x^2}{2}\right)\right) x$$

4) Simplifying:

$$-e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}(x)\right) + \frac{1}{2}e^{-\frac{x^2}{2}} x\left(\frac{\partial}{\partial x}(x^2)\right)$$

5) Leibniz rule:

$$-e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}(x)\right)+2x\frac{1}{2}e^{-\frac{x^2}{2}}x$$

6) Simplifying:

$$f"(x)=-e^{-\frac{x^2}{2}}\left(\frac{\partial}{\partial x}(x)\right)+x^2e^{-\frac{x^2}{2}}=-e^{-\frac{x^2}{2}}1+x^2e^{-\frac{x^2}{2}}=e^{-\frac{x^2}{2}}(x^2-1)$$

The three functions are plotted here below:

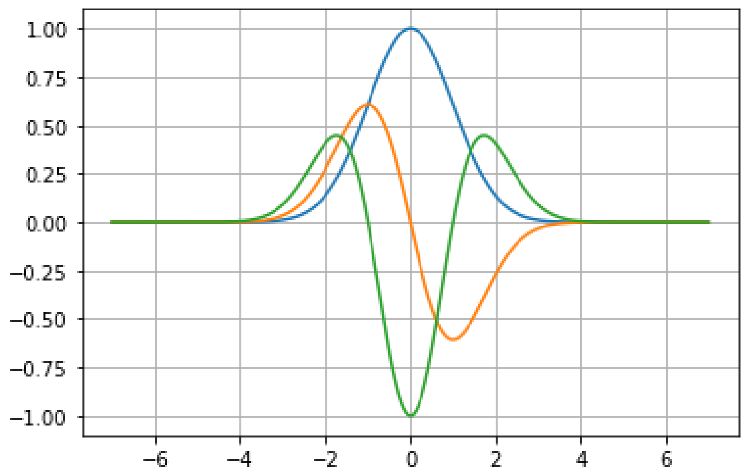

Figure 1: $f(x)$ (blue), $f'(x)$ (orange) and $f"(x)$ (green) with $x \in [-7,7]$

Let $g(t)$ be the function $\frac{cos(t)}{sin(t)}$, its derivative $g'(t)$ is $-csc^2(t)$ and it can be computed as shown here below:
1) As $\frac{cos(t)}{sin(t)}$ can be re-written as $cot(t)$, it is possible then to use the chain rule:

$$\frac{\partial}{\partial t}\left(cot(t)\right) = \frac{\partial cot(u)}{\partial u}\frac{\partial u}{\partial t}$$

assuming that

$$\frac{\partial}{\partial u}(cot(u)) = -csc^2(u) \qquad u = t$$

so, is possible to re-write it as:

$$-csc^2(t)\left(\frac{\partial}{\partial t}(t)\right)$$

2) Since the derivative of t is equal to 1, the solution is:

$$g'(t) = -csc^2(t)$$

3.2 Functions of several variables

Let the function $f(x,t)$ be:

$$f(x,t) = \frac{1}{\sqrt{2\pi t}}e^{-\frac{x^2}{2t}}$$

The first partial derivative of $f(x,t)$ w.r.t. x is equal to $-\frac{e^{-\frac{x^2}{2t}}\cdot x}{2\sqrt{2\pi}\sqrt{t}}$ and it is calculated in this way:

$$\frac{\partial f}{\partial x}(x,t) = \frac{1}{\sqrt{2\pi}\sqrt{t}}\cdot\left(\frac{\partial}{\partial x}\left(e^{-\frac{x^2}{2t}}\right)\right) =$$

using the chain rule and simplifying:

$$= \frac{1}{\sqrt{2\pi}\sqrt{t}}\cdot e^{-\frac{x^2}{2t}}\cdot\left(\frac{\partial}{\partial x}\left(-\frac{x^2}{2t}\right)\right) = \frac{e^{-\frac{x^2}{2t}}}{\sqrt{2\pi}\sqrt{t}}\cdot\left(-\frac{\frac{\partial}{\partial x}(x^2)}{2t}\right) = -\frac{e^{-\frac{x^2}{2t}}\cdot\left(\frac{\partial}{\partial x}(x^2)\right)}{2\sqrt{2\pi}\sqrt{t}} =$$

using the Leibniz rule and simplifying:

$$= -\frac{e^{-\frac{x^2}{2t}}}{2\sqrt{2\pi}\sqrt{t}}\cdot 2x = -\frac{e^{-\frac{x^2}{2t}}\cdot x}{2\sqrt{2\pi}\sqrt{t}}$$

The first partial derivative of $f(x,t)$ w.r.t. t is equal to $\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{2\cdot\sqrt{2\pi}\cdot t^{5/2}}$ and it is calculated in this way:

$$\frac{\partial f}{\partial t}(x,t) = \frac{\frac{\partial}{\partial t}\left(\frac{e^{-\frac{x^2}{2t}}}{\sqrt{t}}\right)}{\sqrt{2\pi}} =$$

using the Leibniz rule, the chain rule and simplifying:

$$= \frac{-\frac{e^{-\frac{x^2}{2t}}\cdot x^2\cdot\left(\frac{\partial}{\partial t}\left(\frac{1}{t}\right)\right)}{2\sqrt{t}} + e^{-\frac{x^2}{2t}}\cdot\left(\frac{\partial}{\partial t}\left(t^{-1/2}\right)\right)}{\sqrt{2\pi}} =$$

using the Leibniz rule two more times, after simplifying:

$$= \frac{-\frac{e^{-\frac{x^2}{2t}}}{2t^{3/2}} + \frac{e^{-\frac{x^2}{2t}}\cdot x^2}{2t^{5/2}}}{\sqrt{2\pi}} = \frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{2\cdot\sqrt{2\pi}\cdot t^{5/2}}$$

The second partial derivative of $f(x,t)$ w.r.t. x is equal to $\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{\sqrt{2\pi}\cdot t^{5/2}}$ and it is calculated in this way:

$$\frac{\partial^2 f}{\partial x^2}(x,t) = \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{2\pi}\sqrt{t}}\cdot\left(\frac{\partial}{\partial x}\left(e^{-\frac{x^2}{2t}}\right)\right)\right) =$$

after chain rule, power rule and product rule, simplifying:

$$= -\frac{-\frac{e^{-\frac{x^2}{2t}}\cdot x^2}{t} + e^{-\frac{x^2}{2t}}\cdot\left(\frac{\partial}{\partial x}(x)\right)}{\sqrt{2\pi}\cdot t^{3/2}} = -\frac{-\frac{e^{-\frac{x^2}{2t}}\cdot x^2}{t} + e^{-\frac{x^2}{2t}}\cdot 1}{\sqrt{2\pi}\cdot t^{3/2}} = \frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{\sqrt{2\pi}\cdot t^{3/2}}$$

As is possible to see, $\frac{\partial f}{\partial t}(x,t)$ is equal to $\frac{1}{2}\cdot\frac{\partial^2 f}{\partial x^2}(x,t)$:

$$\frac{\partial f}{\partial t}(x,t) = \frac{1}{2}\cdot\frac{\partial^2 f}{\partial x^2}(x,t)$$

$$\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{2\cdot\sqrt{2\pi}\cdot t^{5/2}} = \frac{1}{2}\cdot\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{\sqrt{2\pi}\cdot t^{5/2}}$$

$$\frac{1}{2}\cdot\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{\sqrt{2\pi}\cdot t^{5/2}} = \frac{1}{2}\cdot\frac{e^{-\frac{x^2}{2t}}\cdot(-t+x^2)}{\sqrt{2\pi}\cdot t^{5/2}}$$

The three functions are plotted here below, with $t \in \{1, 2, 4\}$:

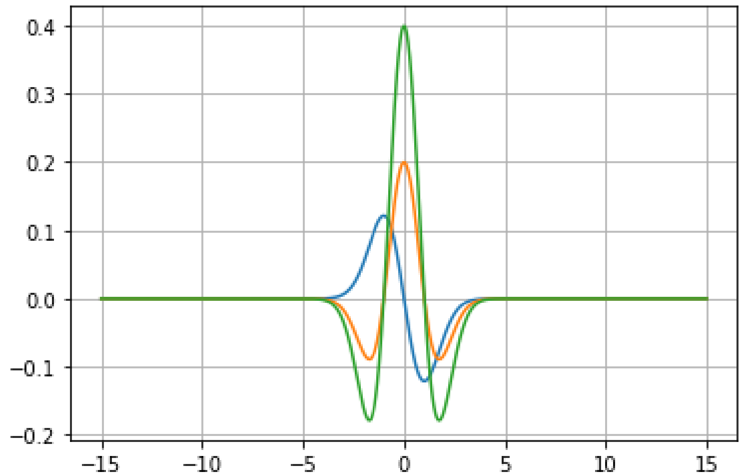

Figure 2: $\frac{\partial f}{\partial x}(x,t)$ (blue), $\frac{\partial f}{\partial t}(x,t)$ (orange) and $\frac{\partial^2 f}{\partial x^2}(x,t)$ (green) with $x \in [-15, 15]$ and $t = 1$

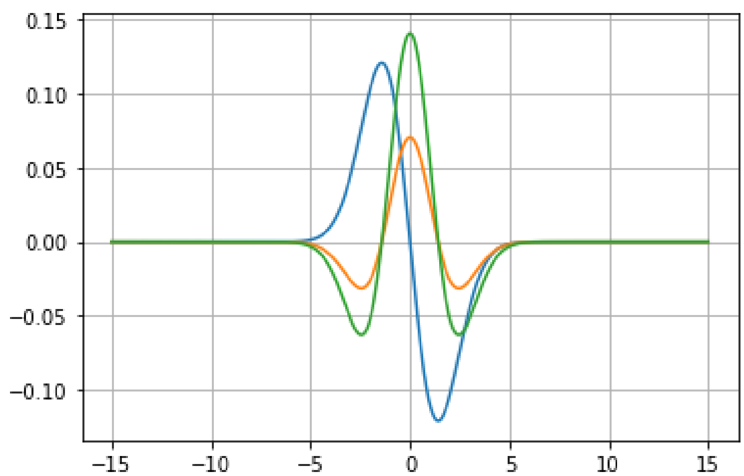

Figure 3: $\frac{\partial f}{\partial x}(x,t)$ (blue), $\frac{\partial f}{\partial t}(x,t)$ (orange) and $\frac{\partial^2 f}{\partial x^2}(x,t)$ (green) with $x \in [-15, 15]$ and $t = 2$

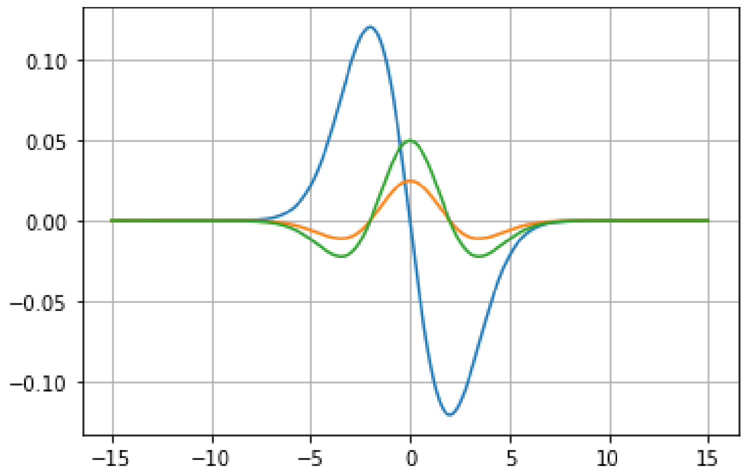

Figure 4: $\frac{\partial f}{\partial x}(x,t)$ (blue), $\frac{\partial f}{\partial t}(x,t)$ (orange) and $\frac{\partial^2 f}{\partial x^2}(x,t)$ (green) with $x \in [-15, 15]$ and $t = 4$