Politecnico di Torino

MUL²

Progettazione di veicoli aerospaziali (AA-LZ)
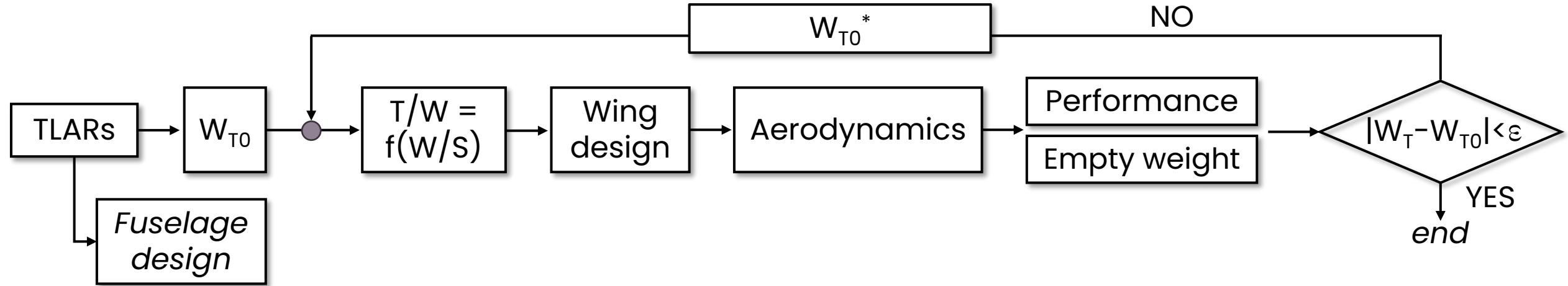
E1. Conceptual Design of subsonic commercial aircraft

9. **Piani di coda**
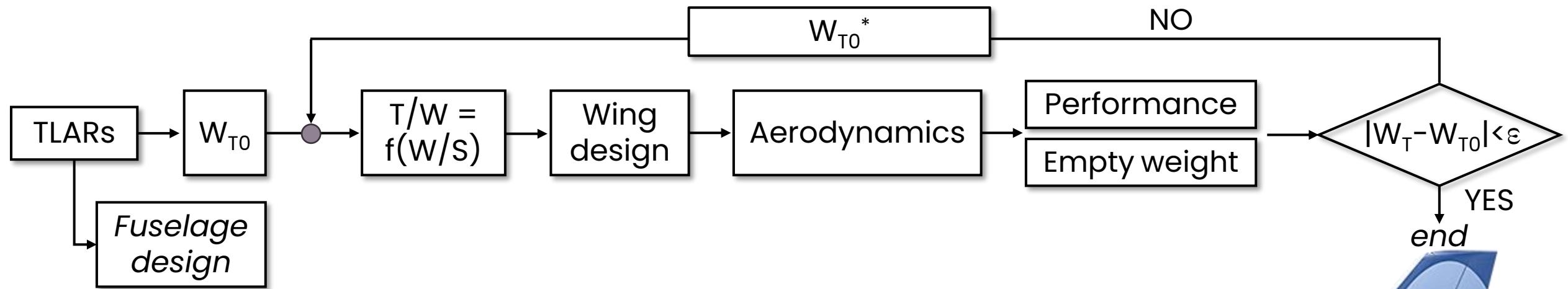
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Recap

Geometry						Weights						Performance						FoMs					
x_1	x_2	x_3	…	…	x_n	w_1	w_2	w_3	…	…	w_n	y_1	y_2	y_3	…	…	y_n	f_1	f_2	f_3	…	…	f_n
x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx
x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx
x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
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x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx	x.xx

Recap


```
                              ┌──────────────┐                              NO
                              │    W_T0*     │─────────────────────────────┐
                              └──────────────┘                             │
                                     │                                     │
                                     ▼                                     ▼
┌────────┐     ┌──────┐     ┌────────────┐   ┌─────────┐   ┌───────────────┐   ┌──────────────┐   ┌─────────────────┐
│ TLARs  │────▶│ W_T0 │──●─▶│  T/W =     │──▶│  Wing   │──▶│  Aerodynamics │──▶│  Performance  │──▶│ |W_T−W_T0|<ε    │
└────────┘     └──────┘     │  f(W/S)    │   │ design  │   │               │   ├──────────────┤   └─────────────────┘
    │                       └────────────┘   └─────────┘   └───────────────┘   │ Empty weight │          │
    │                                                                          └──────────────┘         YES
    ▼                                                                                                     ▼
┌──────────┐                                                                                            end
│ Fuselage │
│  design  │
└──────────┘
```

W_{T0}^{*}

W_{T0}

$T/W = f(W/S)$

$|W_T - W_{T0}| < \varepsilon$

Wing design

Aerodynamics

Performance

Empty weight

TLARs

Fuselage design

NO

YES

end

TLARs → W_{T0} → T/W = f(W/S) → Wing design → Aerodynamics → Performance / Empty weight → $|W_T - W_{T0}| \leq \varepsilon$

$W_{T0}*$

Fuselage design

NO

YES

end

The tail

The **tail** is one the of the lifting surfaces of the aircraft, together with wing. It is composed by two components, **horizontal** tail and **vertical** tail

The **primary function** of the wing is to generate the largest fraction of lift, whereas the tail is supposed to generate a small fraction of the vertical force (positive or negative).

The primary function of the tail are:
1. **Trim** (longitudinal and directional)
2. **Stabilize** (longitudinal and directional)
3. **Control** (longitudinal and directional) via elevator and rudder

Static Stability

Static Stability

Neutral

Stable

Unstable

The static equilibrium is the condition where the sum of all forces (for point mass) and moment (for rigid body) is equal to zero. The equilibrium can be stable, unstable or neutral.

Longitudinal Static Stability

(a) Trimmed (equilibrium) flight

Lift = weight   Thrust = drag   Moments = 0
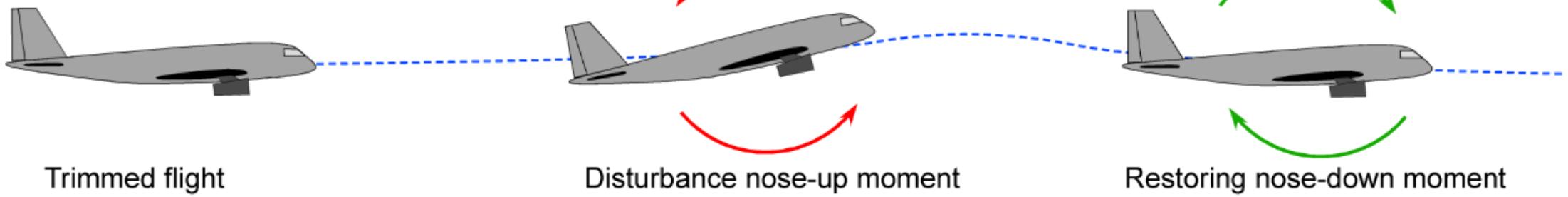
Trimmed flight   Trimmed flight   Trimmed flight

(b) Statically stable

Trimmed flight   Disturbance nose-up moment   Restoring nose-down moment

In case of **static stable** equilibrium the aircraft **returns** to its **trimmed flight condition** when a perturbation (e.g. of the AoA) occurs

Longitudinal Static Stability

(a) Trimmed (equilibrium) flight

Lift = weight   Thrust = drag   Moments = 0

Trimmed flight

Trimmed flight

Trimmed flight

(c) Statically unstable
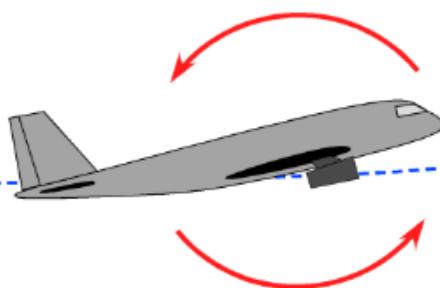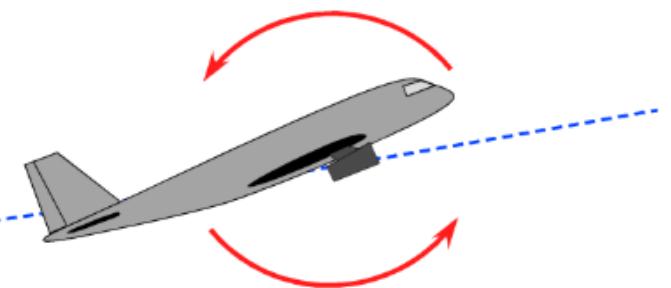
Trimmed flight

Disturbance nose-up moment

Nose-up moment increases

In case of **static unstable** equilibrium the aircraft **deviates** from its **trimmed flight condition** when a perturbation (e.g. of the AoA) occurs

Longitudinal Static Stability

(a) Trimmed (equilibrium) flight

Lift = weight   Thrust = drag   Moments = 0
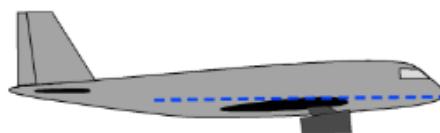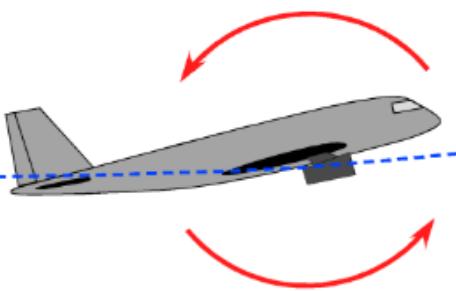
Trimmed flight   Trimmed flight   Trimmed flight
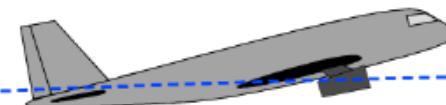
(d) Neutrally stable
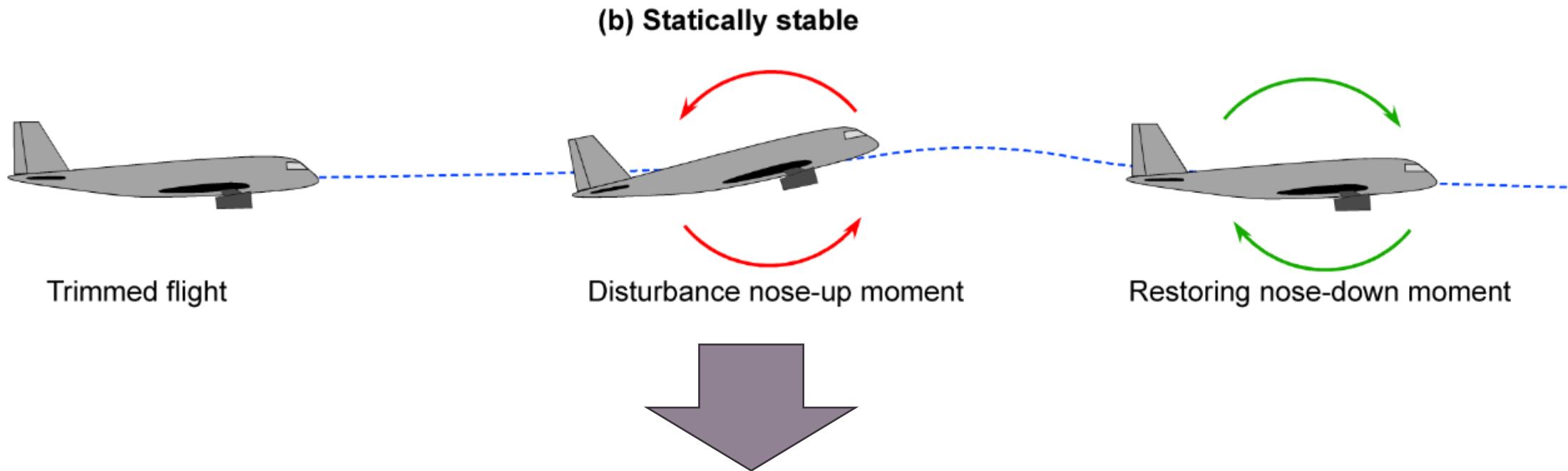
Trimmed flight   Disturbance nose-up moment   No restoring moments

In case of **static neutral** equilibrium the aircraft **changes** its **trimmed flight condition** when a perturbation (e.g. of the AoA) occurs

Longitudinal Static Stability

(b) Statically stable

Trimmed flight — Disturbance nose-up moment — Restoring nose-down moment

How this condition can be obtained? What are the main design parameters which affect this aircraft behaviour?
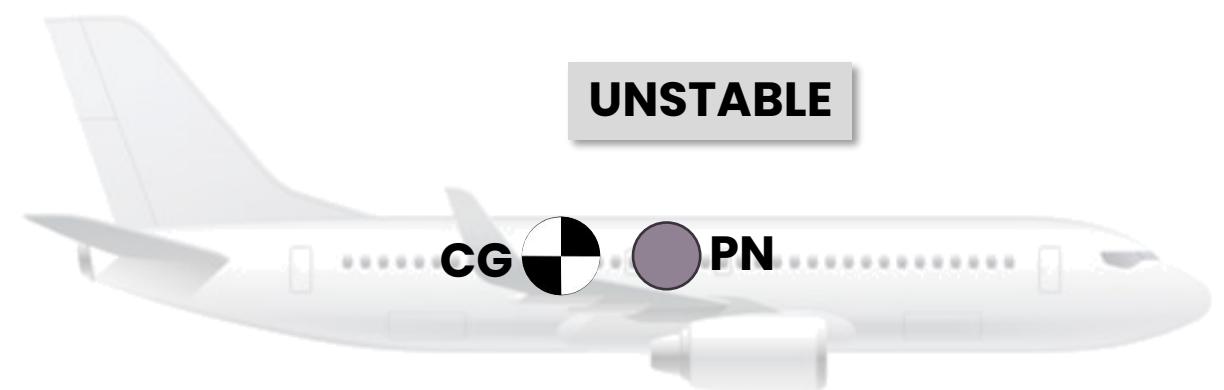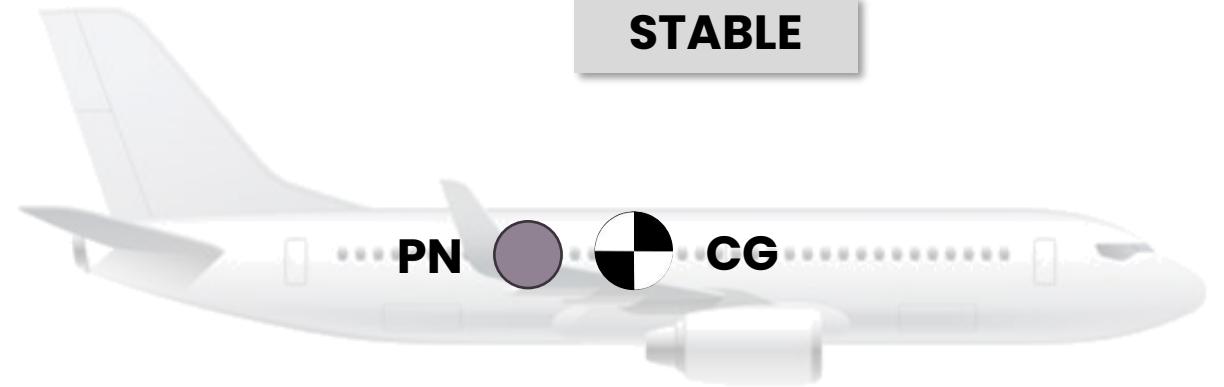
Longitudinal Static Stability

The aircraft **neutral point** is the point w.r.t. the **total moment** generated by the aircraft does **not change**.

Alternatively

The aircraft neutral point is the point where the **lifting forces variations** due to AoA changes are applied.
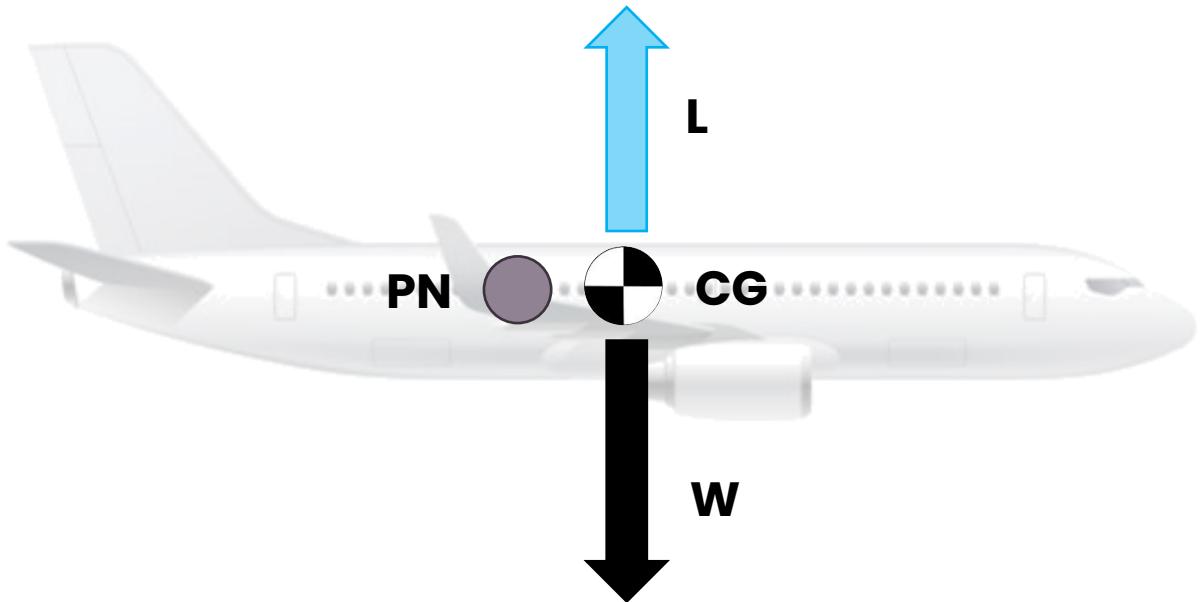
If the neutral point is behind the aircraft center of gravity, the aircraft is statically stable; otherwise, is statically unstable.

STABLE

PN ⬤ ◑ CG

UNSTABLE

CG ◑ ⬤ PN

Longitudinal Static Stability

STABLE

L

PN   CG

W

UNSTABLE

L

CG   PN

W

Longitudinal Static Stability

STABLE

L

PN CG

W

UNSTABLE

L

CG PN

W

AoA perturbation

Longitudinal Static Stability
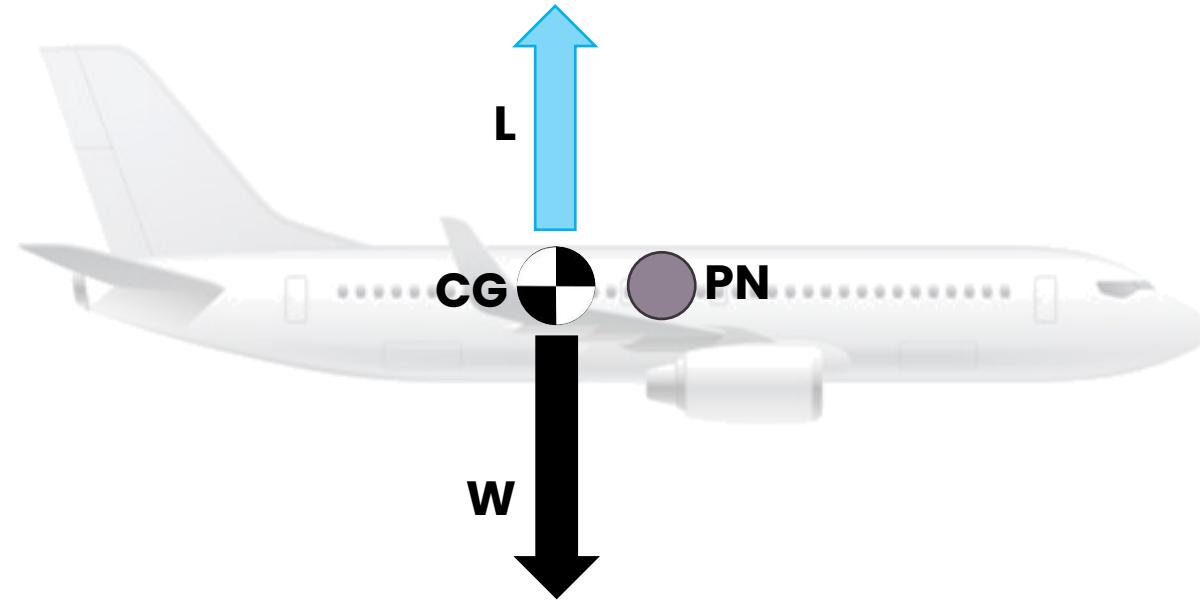
Lift increment

Longitudinal Static Stability
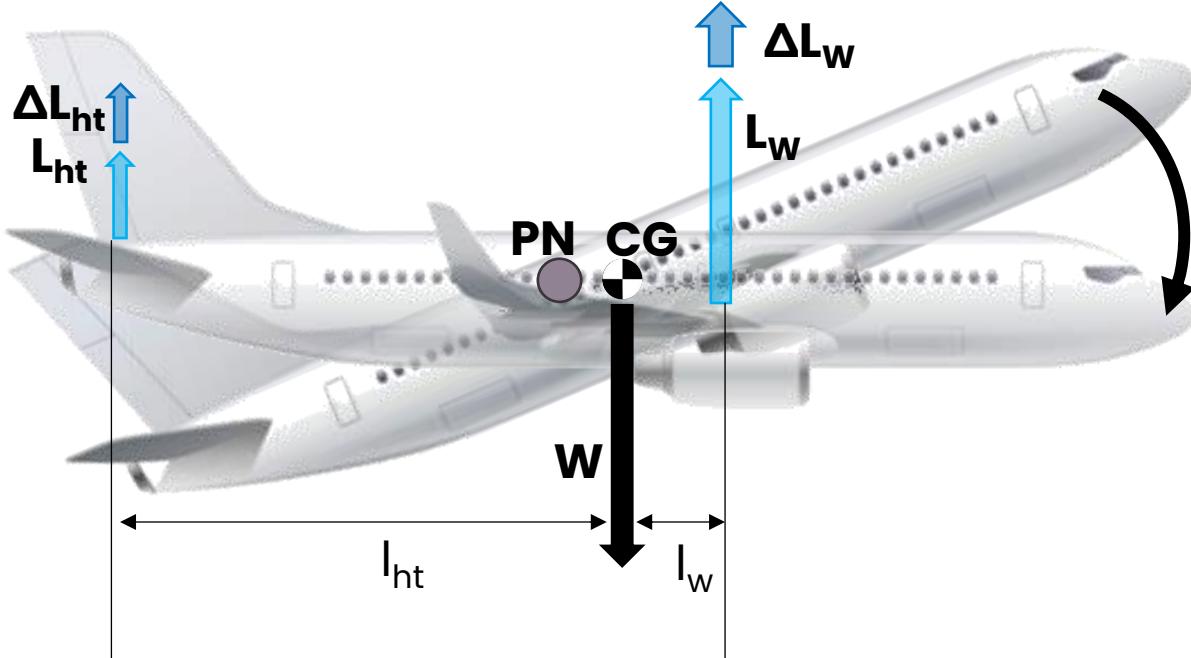
STABLE

UNSTABLE

+ΔL

L

PN

CG

W

L

+ΔL

CG

PN

W

Pitch moment

Longitudinal Static Stability

Euler's second law @ CG

$$\Delta M \leq 0 \; \rightarrow \Delta L_w l_w - \Delta L_{ht} l_{ht} \leq 0$$

$$\frac{\Delta M}{\Delta \alpha} \leq 0 \rightarrow C_{m\alpha} \leq 0$$

Pitch stiffness

Requirements	Stability derivatives	Symbol	Typical value (1/rad)
Static longitudinal stability	Rate of change of pitching moment coefficient with respect to angle of attack	C_{m_α}	−0.3 to −1.5

The pitch stiffness depends on the relative position wing - centre of gravity – tail

The wing aerodynamic centre is close to the aircraft CG, so, to ensure a proper static margin, a tail it is necessary

Requirements	Stability derivatives	Symbol	Typical value (1/rad)
Static longitudinal stability	Static margin	$h_{np} - h_{cg}$	0.1−0.3

Longitudinal Static Stability

Euler's second law @ CG

$$\Delta M \leq 0 \;\rightarrow\; \Delta L_w l_w - \Delta L_{ht} l_{ht} \leq 0$$
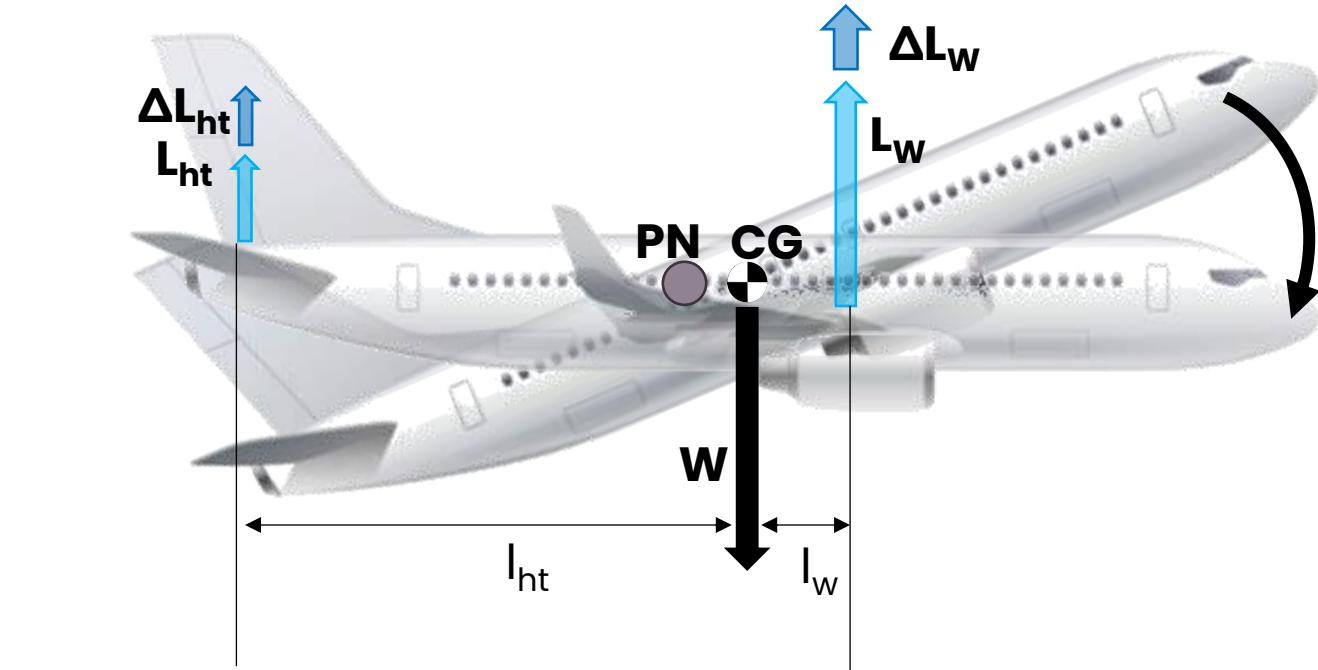
$$\frac{\Delta M}{\Delta \alpha} \leq 0 \rightarrow C_{m\alpha} \leq 0$$

Pitch stiffness

The location and surface of the tail affects the position of aircraft neutral point

A general parameter which is related to the pitch stiffness is the tail volume coefficient V_H

"Big" tail arm allows to reduce tail surface
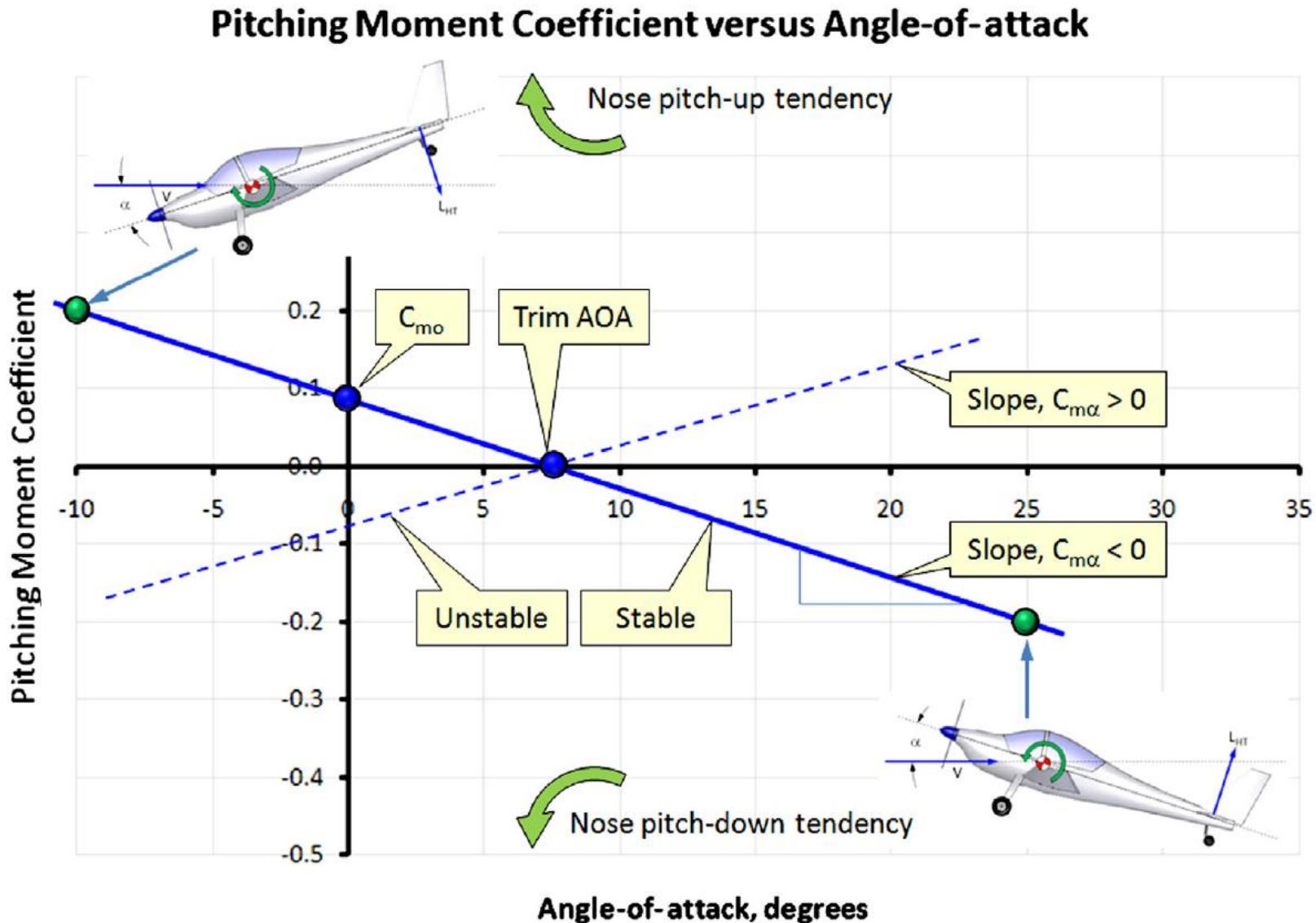
$$V_H = \frac{S_{ht}}{S_w} \frac{l_t}{\bar{c}}$$

$\bar{c}$ is the mean aerodynamic chord of the wing. For a trapezoidal wing, it is calculated according to the following equation

$$\bar{c} = \frac{2}{3} c_r \frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

Longitudinal Static Stability

Pitching Moment Coefficient versus Angle-of-attack
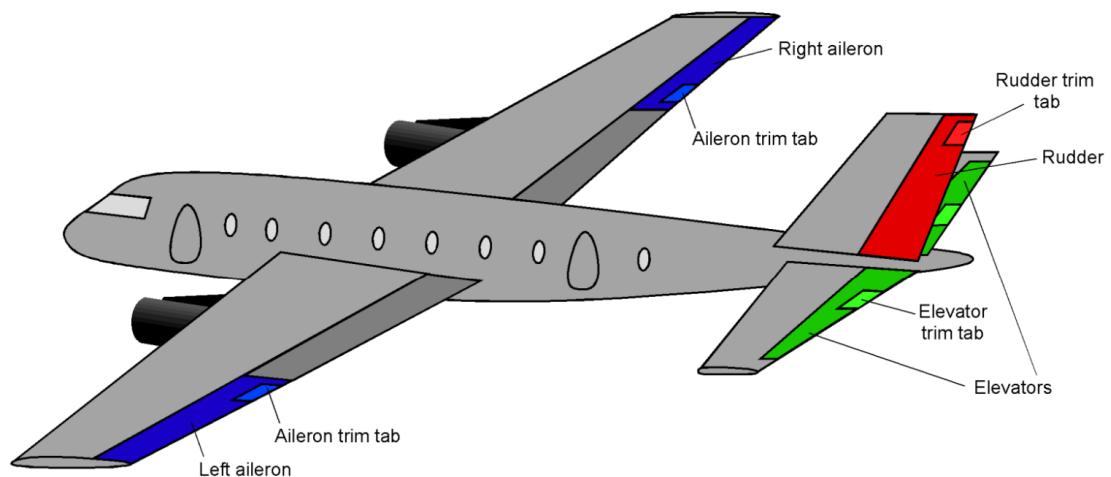
The trim

For an airplane to be in static equilibrium or *trim* at a particular flight condition, the net sum of all the forces and moments acting on the airplane must be zero, i.e., the position and attitude of the aircraft will be in perfect balance about all three flight axes, namely pitch, roll, and yaw.

The use of the flight controls, elevator, ailerons, and rudder, provide the necessary forces and moments to create a trimmed flight condition.

The longitudinal trim

Euler's first law

$$\sum \boldsymbol{F} = \boldsymbol{0} \rightarrow \begin{array}{c} L_w + L_{ht} = W \\ T = D \end{array}$$
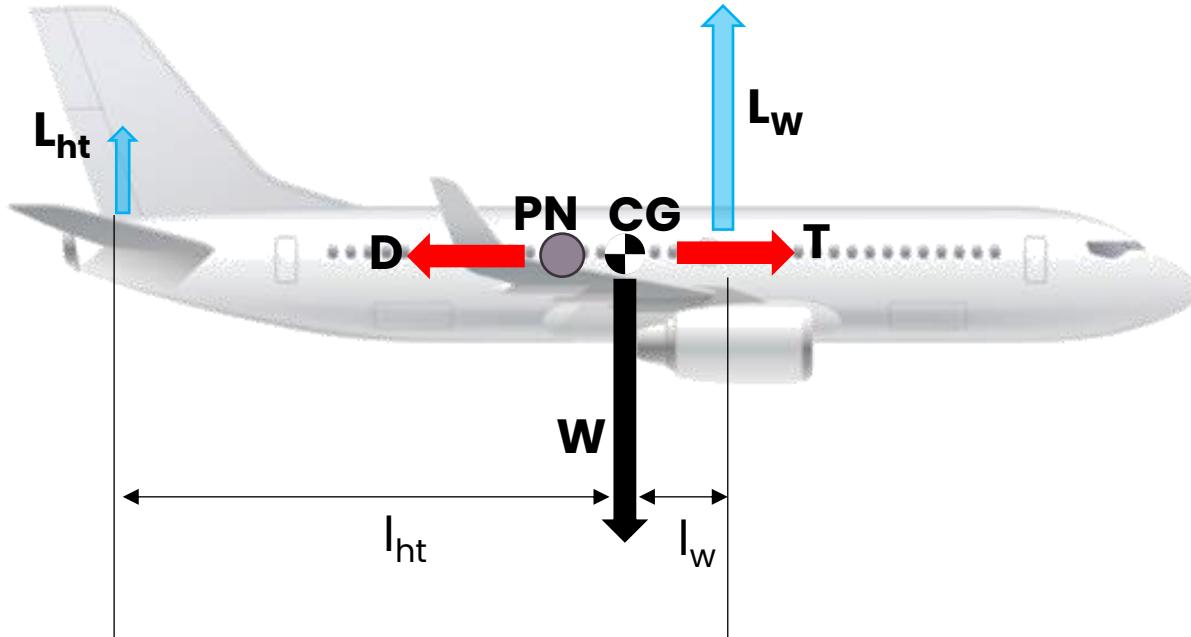
Euler's second law @ c.g.

$$\sum \boldsymbol{M} = \boldsymbol{0} \rightarrow L_w l_w - L_{ht} l_{ht} = 0$$

$$\begin{cases} L_w l_w - L_{ht} l_{ht} = 0 \\ L_w = \dfrac{1}{2}\rho S V^2 CL_w \\ L_{ht} = \dfrac{1}{2}\rho S_{ht} V^2 CL_{ht} \\ l_w = h\bar{c} \end{cases} \longrightarrow \quad h - \dfrac{CL_{ht}}{CL_w}\underbrace{\left|\dfrac{S_{ht}}{S}\dfrac{l_{ht}}{\bar{c}}\right|}_{V_H} = 0$$

The aircraft trim depends on the relative position wing – centre of gravity – tail
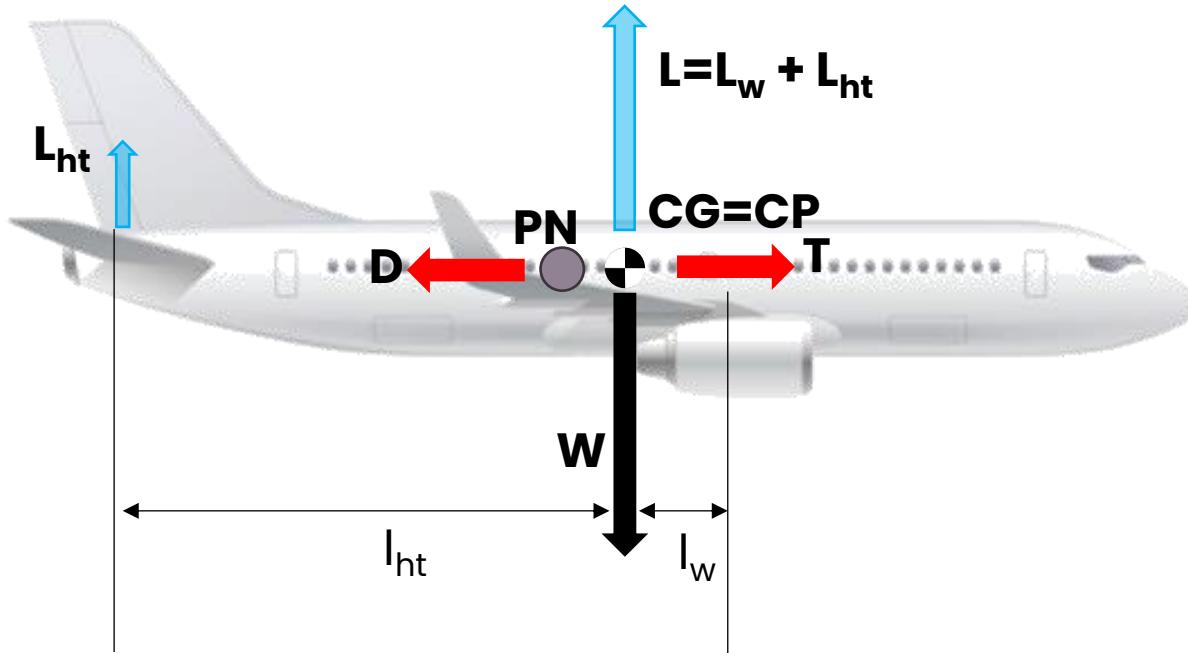
The longitudinal trim

Euler's first law

$$\sum \boldsymbol{F} = \boldsymbol{0} \rightarrow \begin{array}{c} L_w + L_{ht} = W \\ T = D \end{array}$$

Euler's second law @ c.g.

$$\sum \boldsymbol{M} = \boldsymbol{0} \rightarrow L_w l_w - L_{ht} l_{ht} = 0$$
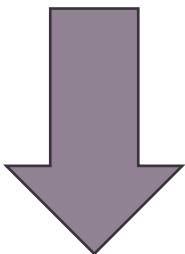

The lifting contributions, in aggregate, of each surface of the aircraft can be assumed to act at a single point: the *center of pressure (CP)*

To trim the aircraft, the CP has to be in the same position of the aircraft CG

The longitudinal trim

The **CG** position generally **changes during flight**. As fuel is burned and the airplane's weight changes, the CG may move forward or aft, depending on the location of the fuel tanks. Therefore, a further trimming by the pilot or flight control system may be required.
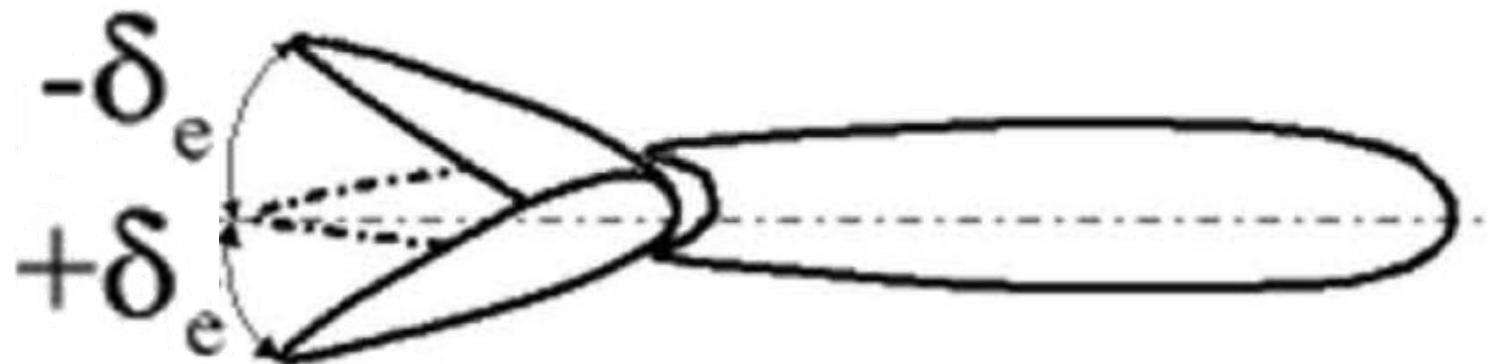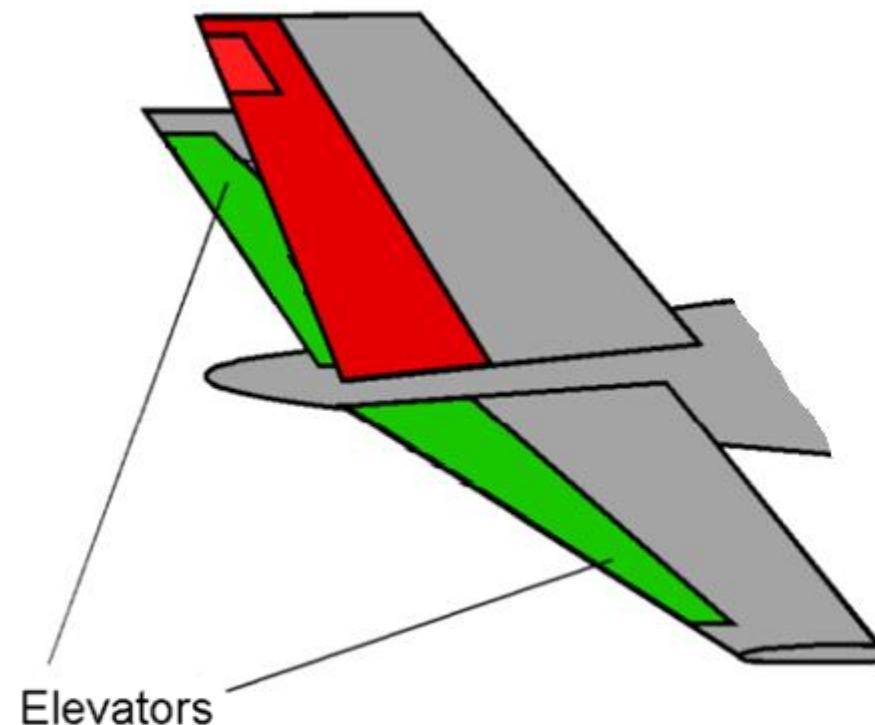
The CP position changes during flight. It depends on trimming condition (aircraft CL)

The longitudinal trim
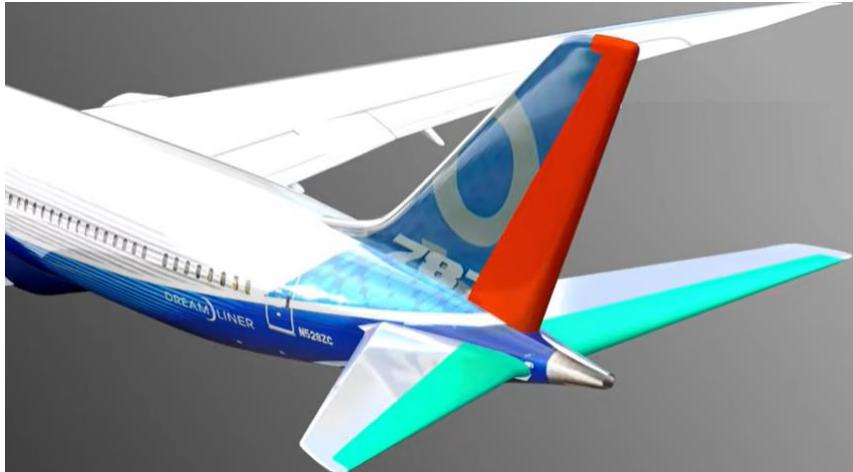
The **CG** position generally **changes during flight**. As fuel is burned and the airplane's weight changes, the CG may move forward or aft, depending on the location of the fuel tanks. Therefore, a further trimming by the pilot or flight control system may be required.
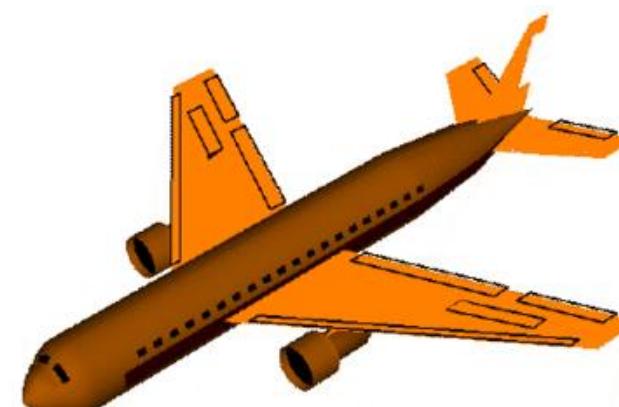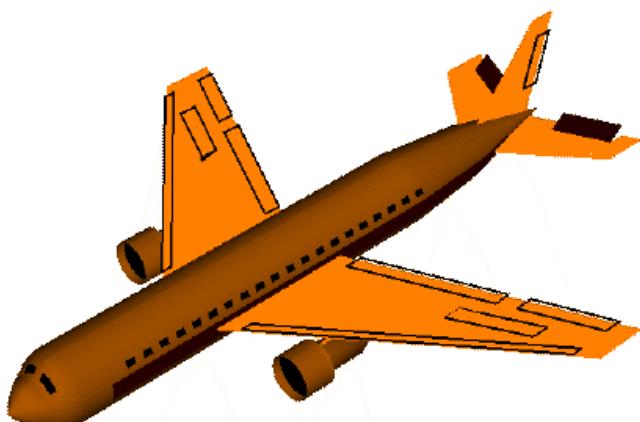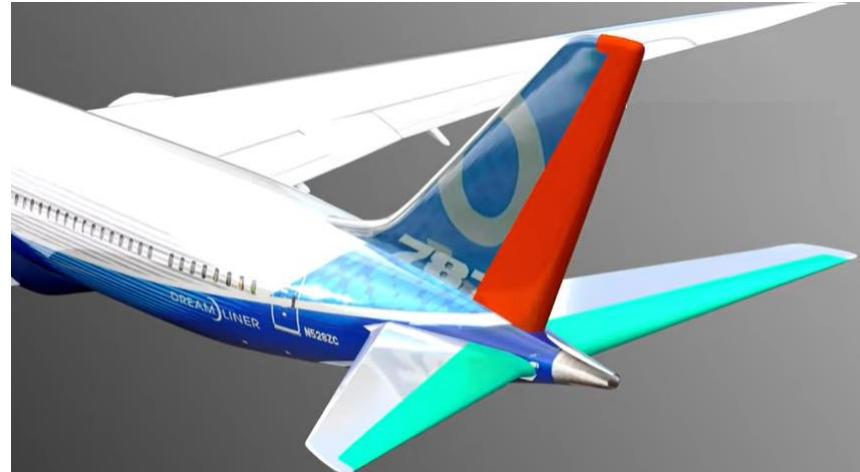

Elevators

Elevator

Rudder

The design of the tail

Horizontal tail

| Select volume coefficient | → | Select tail arm | → | Assess planform area and airfoil | → | Select main geometric parameters | → | Select location and setting angle |

Vertical tail

| Select volume coefficient | → | Select tail arm | → | Assess planform area and airfoil | → | Select main geometric parameters | → | Select location and setting angle |

Identify tail design requirements (trim, stability, control)

Select tail configuration

Check stability

NO

YES

1. longitudinal trim
2. directional trim
3. lateral trim
4. longitudinal stability
5. directional stability
6. lateral stability
7. manufacturability and controllability
8. handling qualities (e.g., passenger comfort)
9. operational requirements (e.g., pilot view)
10. airworthiness (e.g., safety, tail stall, and deep stall)
11. survivability (e.g., spin recovery)
12. size limits (limited height because of hangar space limits)

The design of the tail

Horizontal tail

- Select volume coefficient
- Select tail arm
- Assess planform area and airfoil
- Select main geometric parameters
- Select location and setting angle

Vertical tail

- Select volume coefficient
- Select tail arm
- Assess planform area and airfoil
- Select main geometric parameters
- Select location and setting angle

Identify tail design requirements (trim, stability, control)

Select tail configuration

Check stability

NO

YES

1. longitudinal trim
2. directional trim
3. lateral trim
4. longitudinal stability
5. directional stability
6. lateral stability
7. manufacturability and controllability
8. handling qualities (e.g., passenger comfort)
9. operational requirements (e.g., pilot view)
10. airworthiness (e.g., safety, tail stall, and deep stall)
11. survivability (e.g., spin recovery)
12. size limits (limited height because of hangar space limits)

The design of the tail

Select tail configuration → Select volume coefficient → Select tail arm → Assess planform area and airfoil

Select the tail configuration

Select tail configuration → Select volume coefficient → Select tail arm → Assess planform area and airfoil

conventional

cruciform
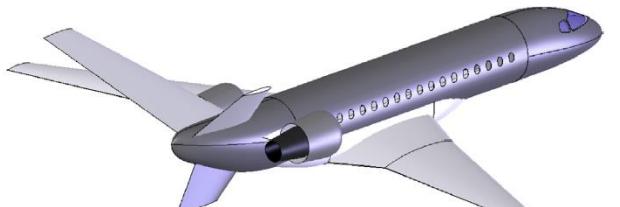
T-tail
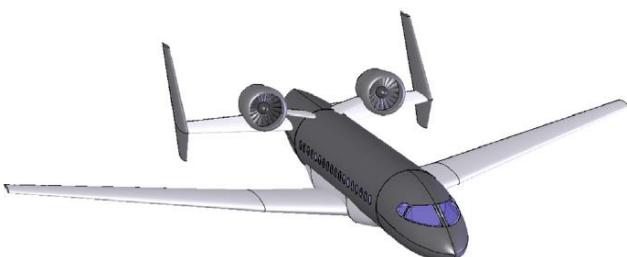
V-tail

V-inverted

Y-tail

Y-tail inverted

H tail

Select the tail configuration

Select tail configuration → Select volume coefficient → Select tail arm → Assess planform area and airfoil

conventional

cruciform

T-tail

V-tail

V-inverted

Y-tail

Y-tail inverted

H tail

Select volume coefficient

Select tail configuration → **Select volume coefficient** → Select tail arm → Assess planform area and airfoil
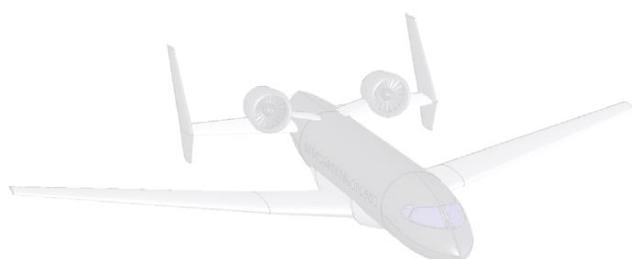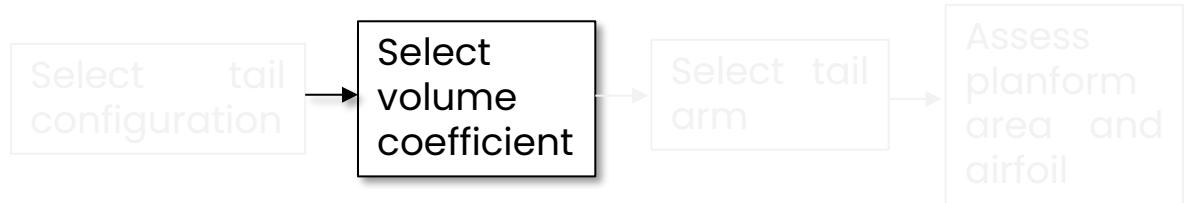
Table 6.4 Typical values for horizontal and vertical tail volume coefficients

No.	Aircraft	Horizontal tail volume coefficient ($\overline{V}_H$)	Vertical tail volume coefficient ($\overline{V}_v$)
1	Glider and motor glider	0.6	0.03
2	Home-built	0.5	0.04
3	GA single prop-driven engine	0.7	0.04
4	GA twin prop-driven engine	0.8	0.07
5	GA with canard	0.6	0.05
6	Agricultural	0.5	0.04
7	Twin turboprop	0.9	0.08
8	Jet trainer	0.7	0.06
9	Fighter aircraft	0.4	0.07
10	Fighter (with canard)	0.1	0.06
11	Bomber/military transport	1	0.08
12	Jet transport	1.1	0.09
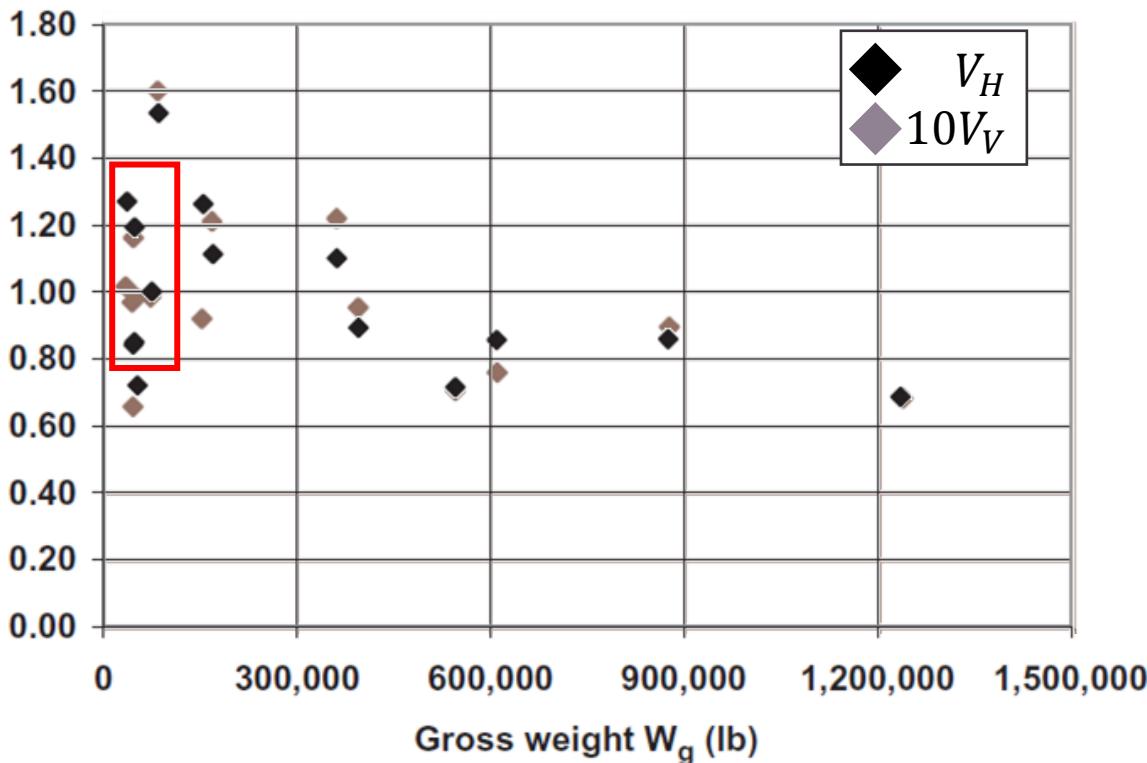
$$V_H = \frac{S_{ht}}{S_w}\frac{l_t}{c}$$

$$0.8 \leq V_H \leq 1.35$$
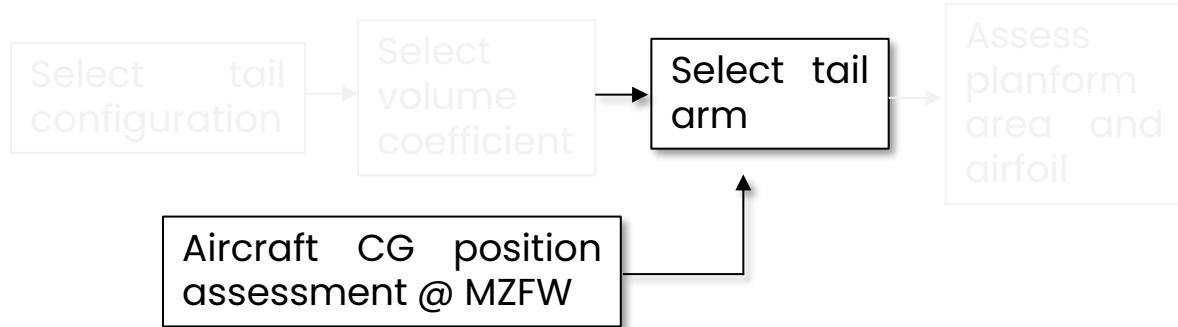
$$V_V = \frac{S_v}{S_w}\frac{l_v}{b}$$

$$0.08 \leq V_V \leq 0.14$$

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW
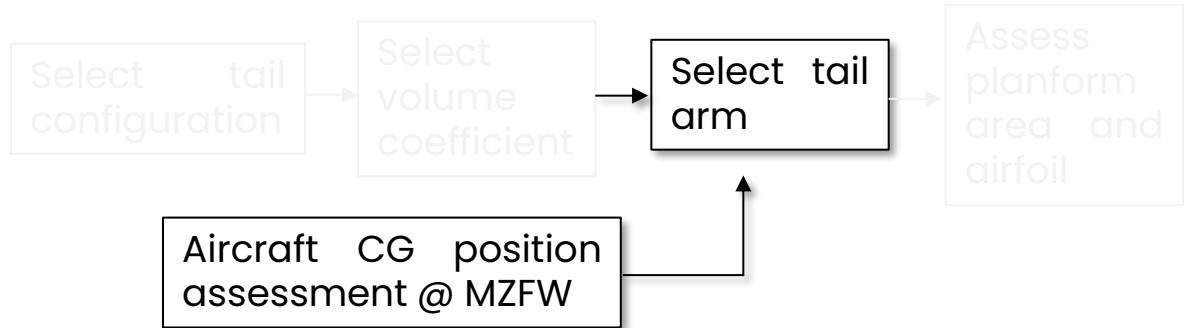
The aircraft CG $(x_{CG,a})$ is calculated by estimation of CG $(x_{CG,i})$ position of each i-th aircraft component
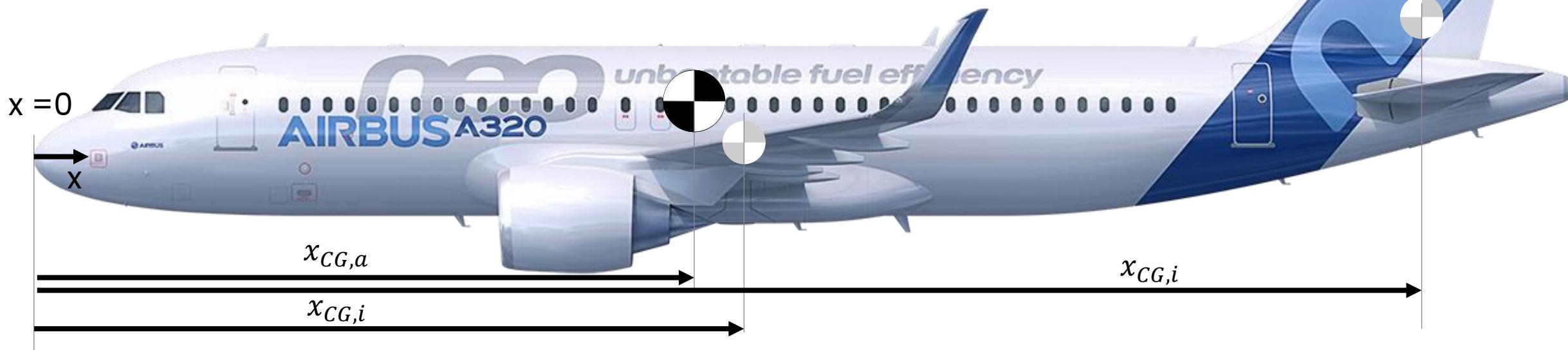
$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW

$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$

x = 0

x

$x_{CG,a}$

$x_{CG,i}$

$x_{CG,i}$

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW
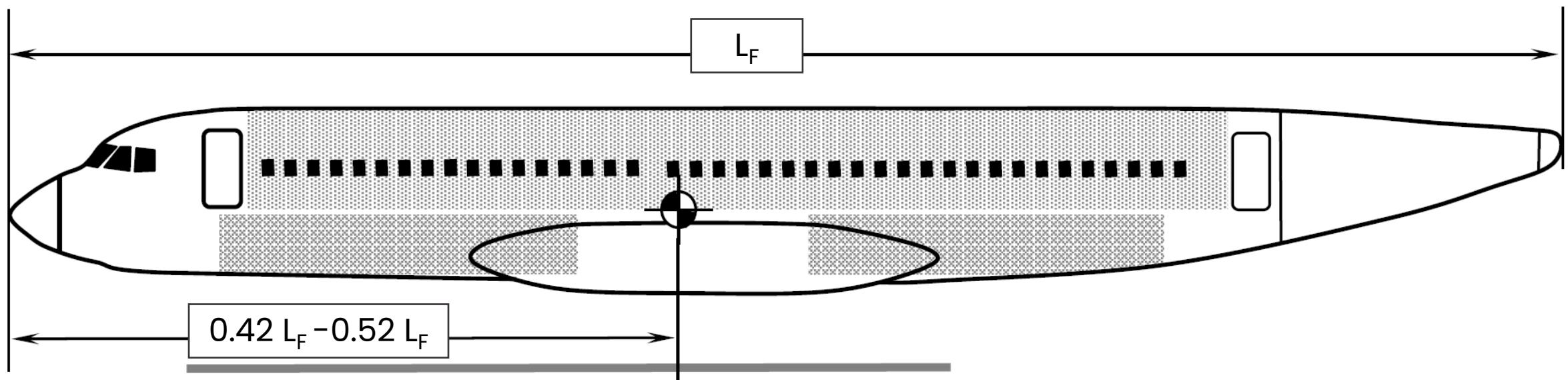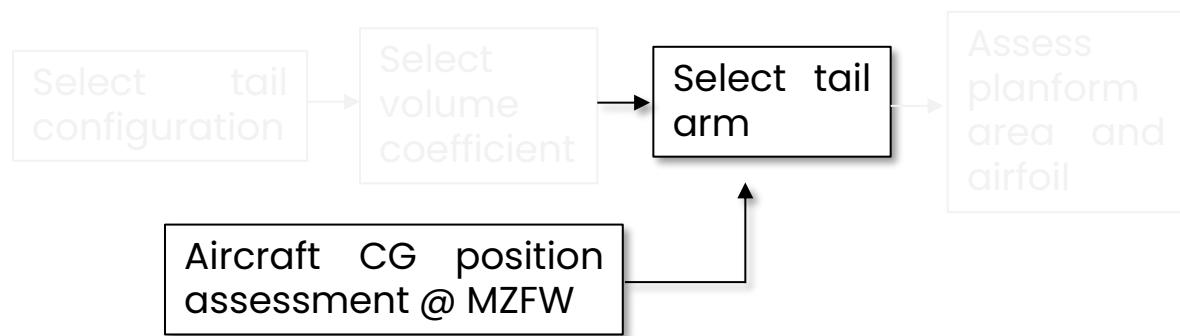
$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$

Fuselage (structure, payload, operatings and landing gear)

L_F

$0.42\ L_F - 0.52\ L_F$

Select tail arm

Select tail configuration → Select volume coefficient → Select tail arm → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW
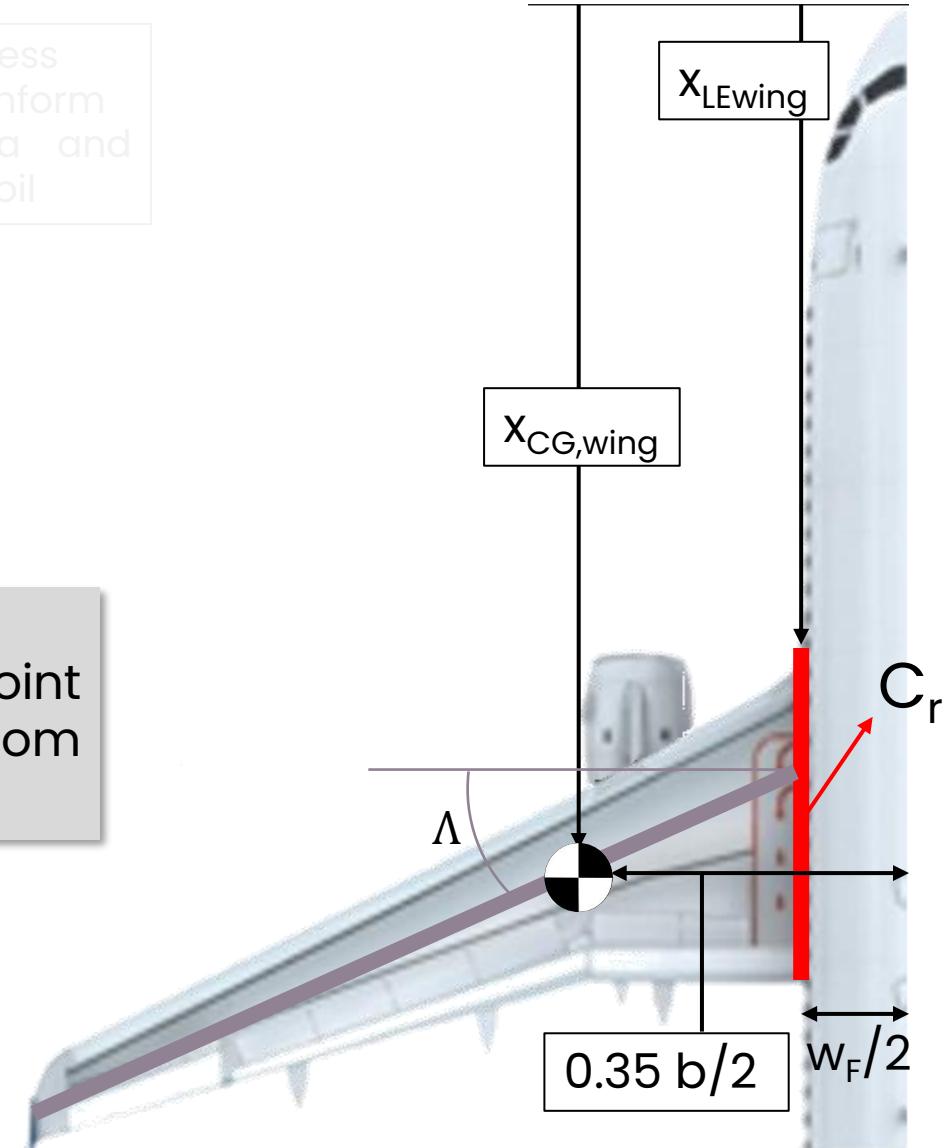
$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$ **Wing**

Assumption

x_{CGwing} (of half wing) is located to the x-coordinate of the point which belongs to the sweep angle line, and its distance from aircraft center line is the 35% of the half wingspan
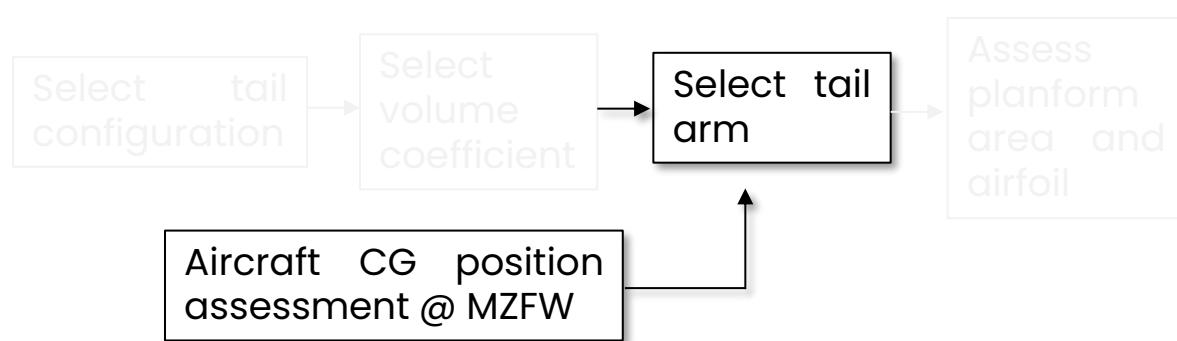
$$x_{CG,wing} = x_{LEwing} + 0.25 c_r + \left(0.35 \frac{b}{2} - \frac{w_F}{2} \right) \tan(\Lambda)$$

$$0.42 L_F \le x_{LEwing} \le 0.52 L_F$$

x_{LEwing}

$x_{CG,wing}$

C_r

Λ

0.35 b/2 w_F/2

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW
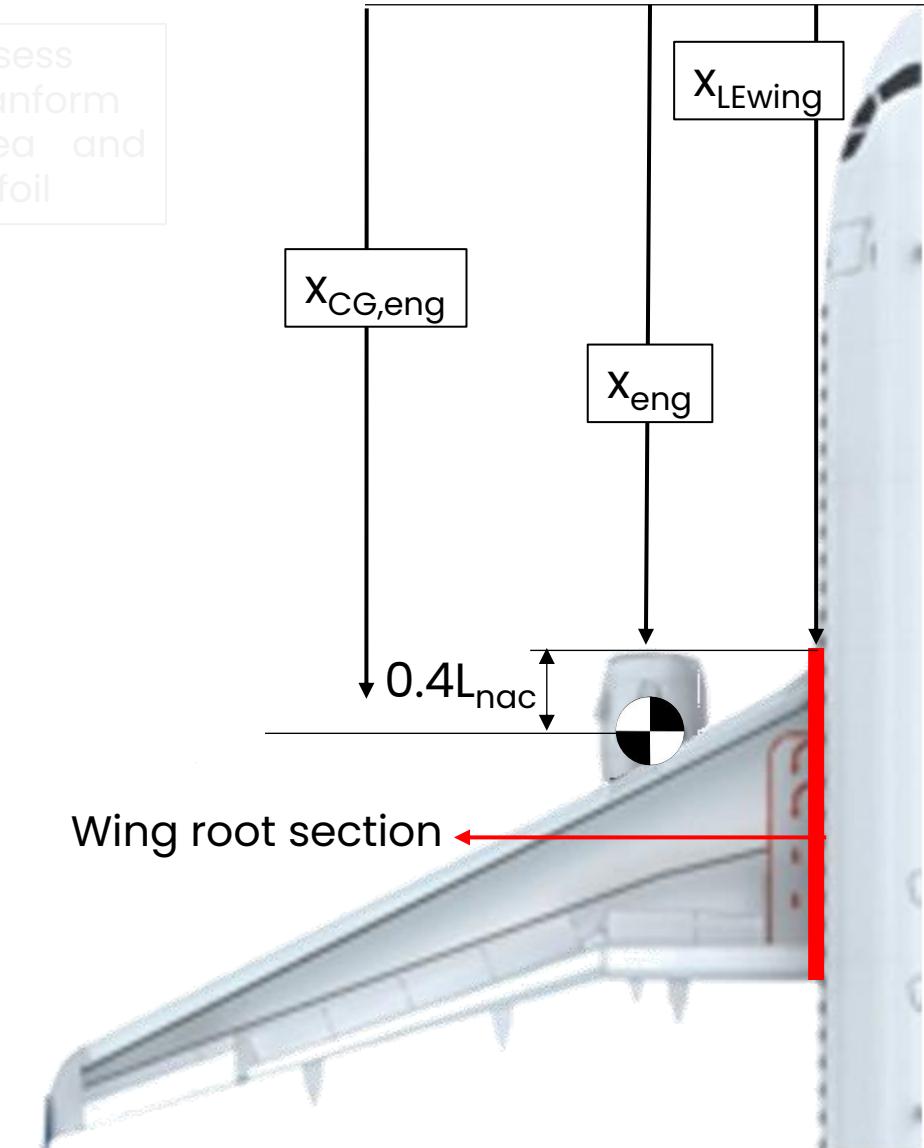
$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$

Propulsion (engine and nacelle)
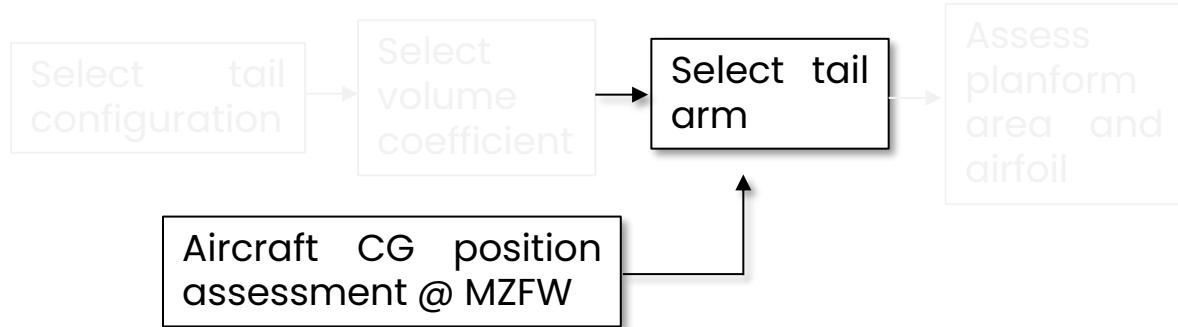
Assumption
x_{eng} is equal to the x coordinate of LE of wing root section

$$x_{CG,eng} = x_{LEwing} + 0.4 L_{nac}$$
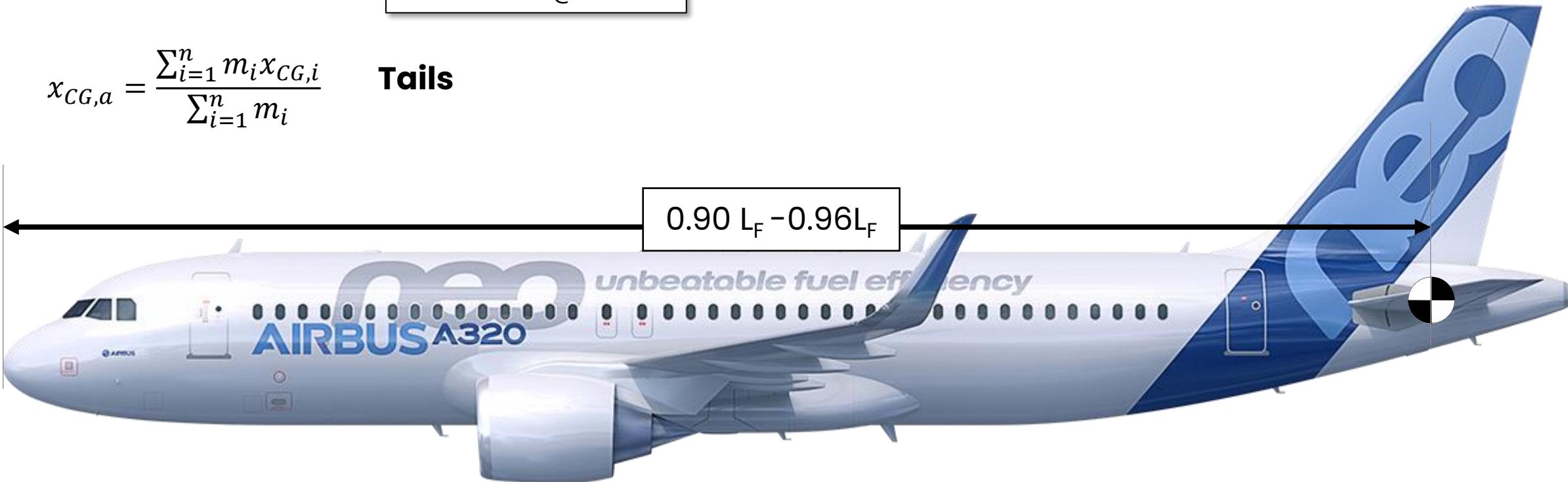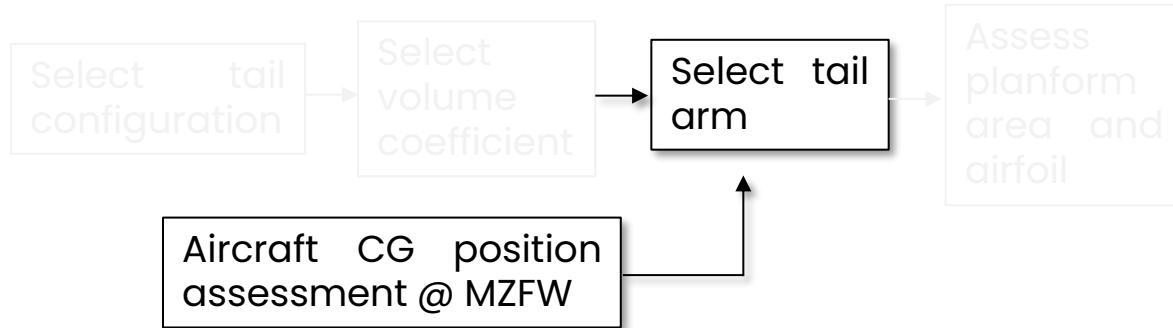
x_{LEwing}

$x_{CG,eng}$

x_{eng}

$0.4L_{nac}$

Wing root section

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW

$$x_{CG,a} = \frac{\sum_{i=1}^{n} m_i x_{CG,i}}{\sum_{i=1}^{n} m_i}$$ **Tails**

$0.90\ L_F - 0.96 L_F$

Select tail arm

Select tail configuration → Select volume coefficient → **Select tail arm** → Assess planform area and airfoil

Aircraft CG position assessment @ MZFW
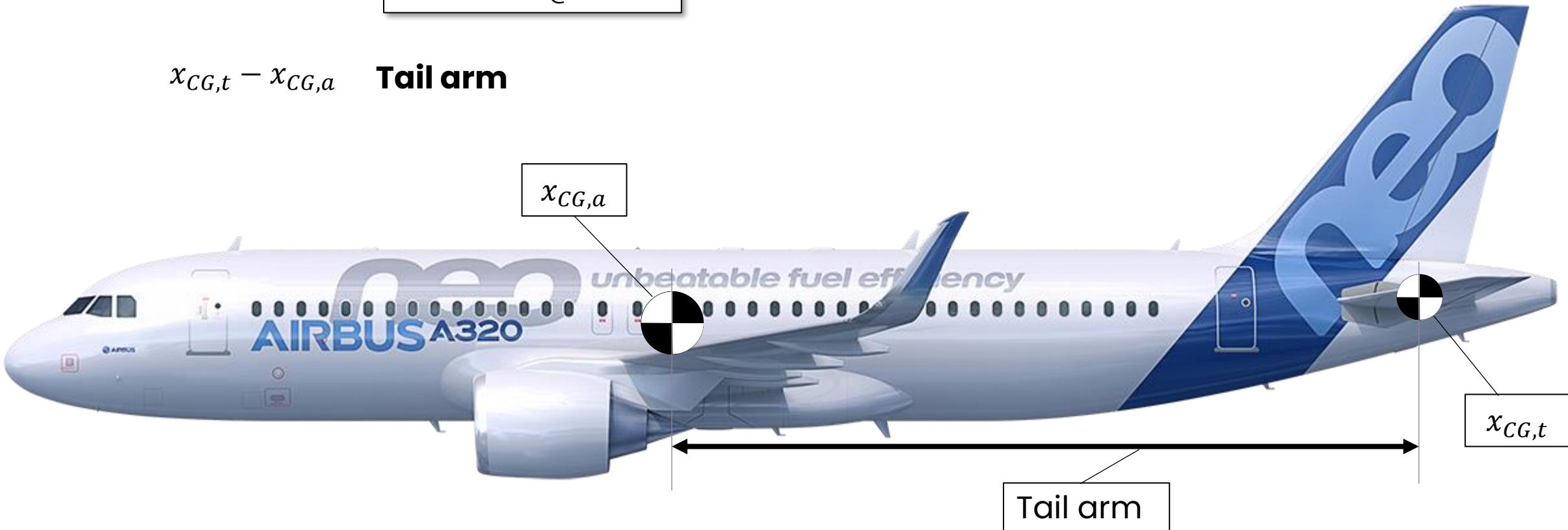
$x_{CG,t} - x_{CG,a}$ **Tail arm**

$x_{CG,a}$

$x_{CG,t}$

Tail arm

Assess planform area and airfoil

Identify tail design requirements (trim, stability, control)

Select tail configuration

Select volume coefficient

Select tail arm → Assess planform area and airfoil

- Planform area of tail can be now calculated by using volume coefficient and tail arm

- The airfoil section must be able to create sometimes a positive and sometimes a negative lift. For this reason, a symmetric airfoil section is a suitable candidate for a horizontal tail.

- To ensure the symmetricity of the aircraft about the xz plane, the vertical airfoil section must be symmetric.

Tail design at glance

Steps for your homework:
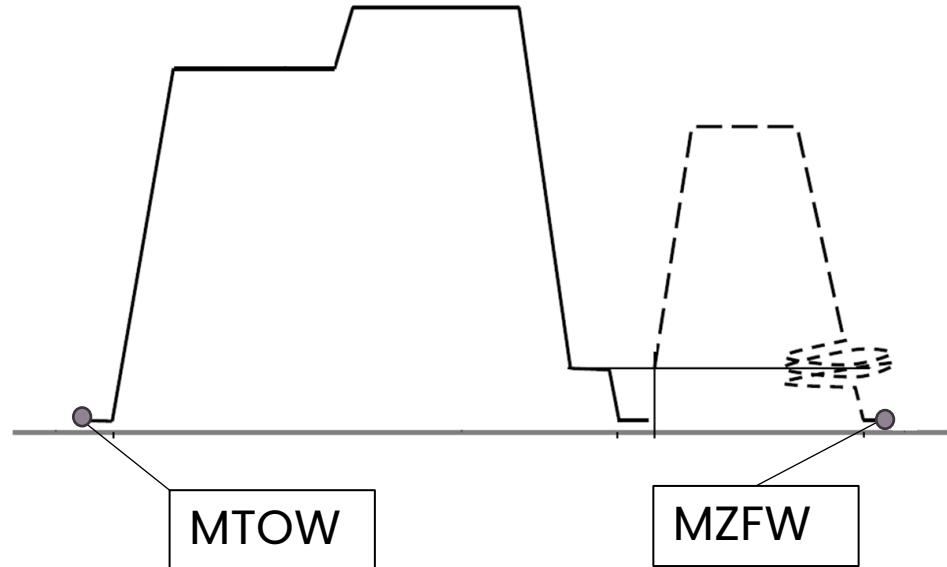
1. Select the tail volume (slide 31)

2. Assess aircraft CG @ MZFW and tail arm (slides 32-38)

3. Assess planform area of tail by using tail volume coefficient and tail arm and select the airfoil (slide 39)

4. Re-do step n.2 @ MTOW and calculate in this condition the tail volume

MTOW

MZFW

DESIGN

$$0.8 \leq V_H \ @ \ MZFW \leq 1.35$$
$$0.08 \leq V_V \ @ \ MZFW \leq 0.14$$
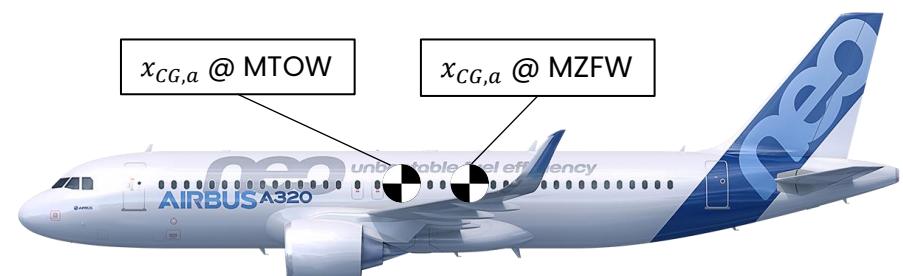
CHECK

$$0.8 \leq V_H \ @ \ MTOW \leq 1.35$$
$$0.08 \leq V_V \ @ \ MTOW \leq 0.14$$

$x_{CG,a}$ @ MTOW

$x_{CG,a}$ @ MZFW

Task 8.1
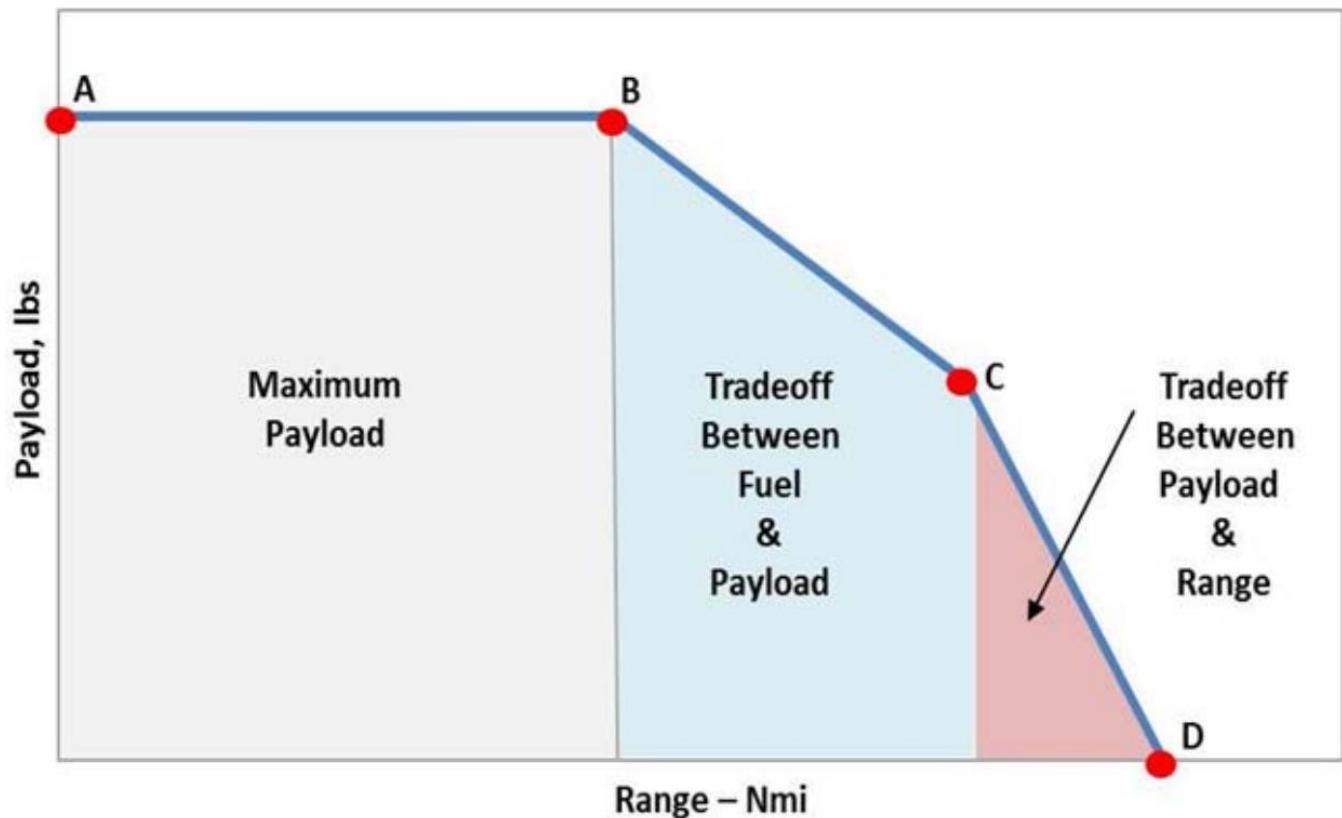
Task 8.1.a: Design the tail of your selected configuration and check the tail volume at MZFW

Re-assessment of payload-range

Now you can calculate the payload-range diagram of your selected configuration

Task 8.1

Task 8.1.b: estimate the payload-range diagram of your selected configuration