Progettazione di veicoli aerospaziali (AA-LZ)

E1. Conceptual Design of subsonic commercial aircraft

6. Aerodinamica

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Introduction

A brief recap…

The Aircraft Design Process

Multi-Stage

Requirements
Initial Sizing

Conceptual Design

Preliminary Design

Detailed Design

Production
Testing
..Fly!

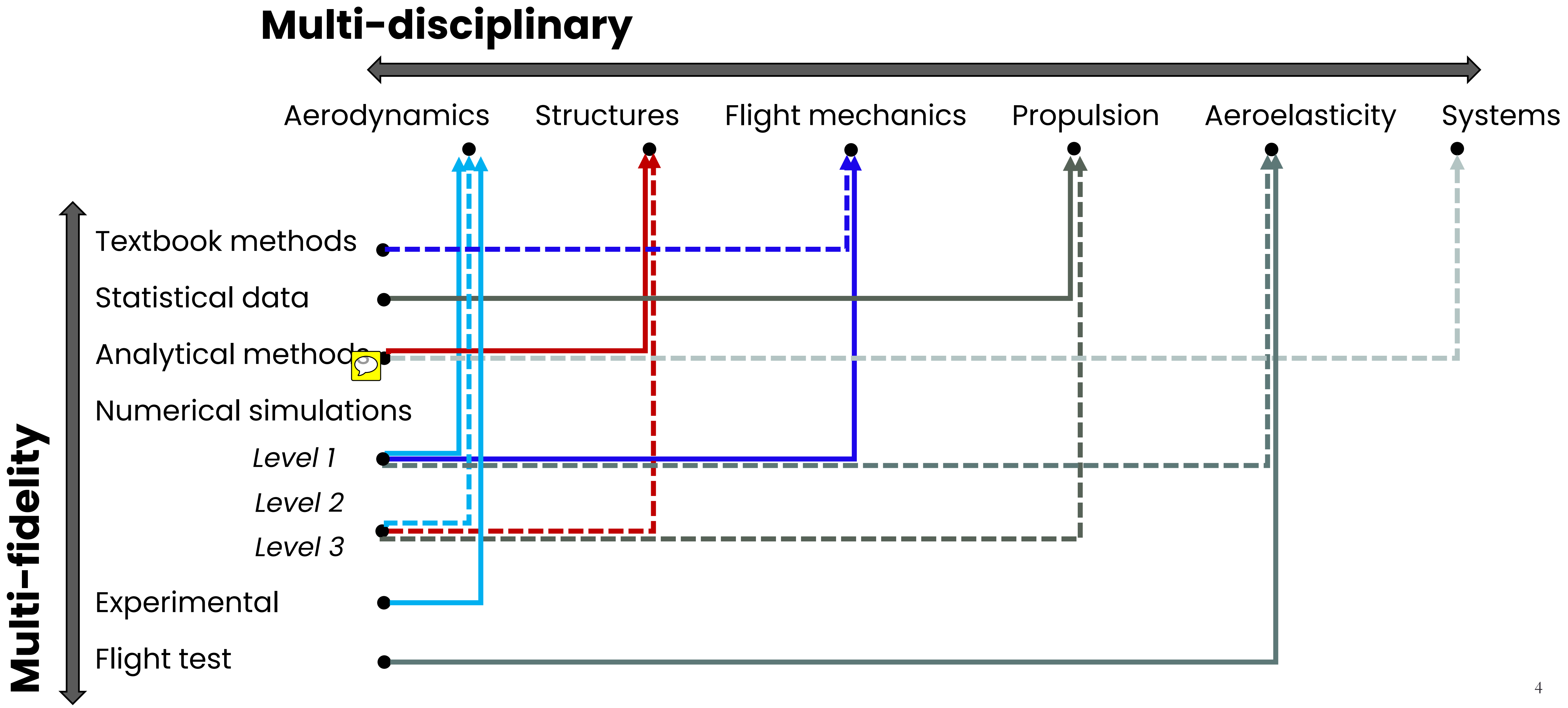
Multi-Disciplinary

- Aerodynamics
- Structures
- Flight mechanics
- Propulsion
- On-board systems
- Weights optimization
- Payload
- Performance
- Control systems

Multi-Fidelity

- Textbook methods
- Statistics/empirical data
- Analytical methods
- Numerical simulations
  - Level 1
  - Level 2
  - Level 3
- Experimental
- Flight test

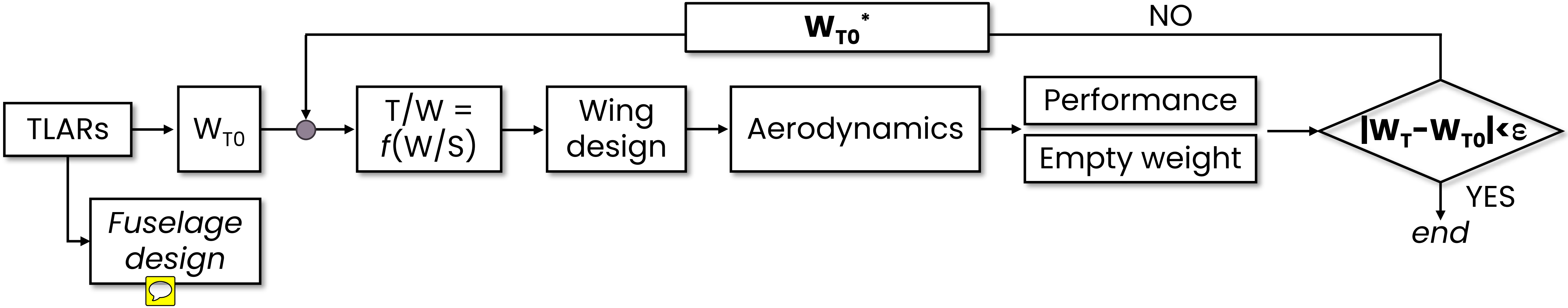
The Aircraft Design Process



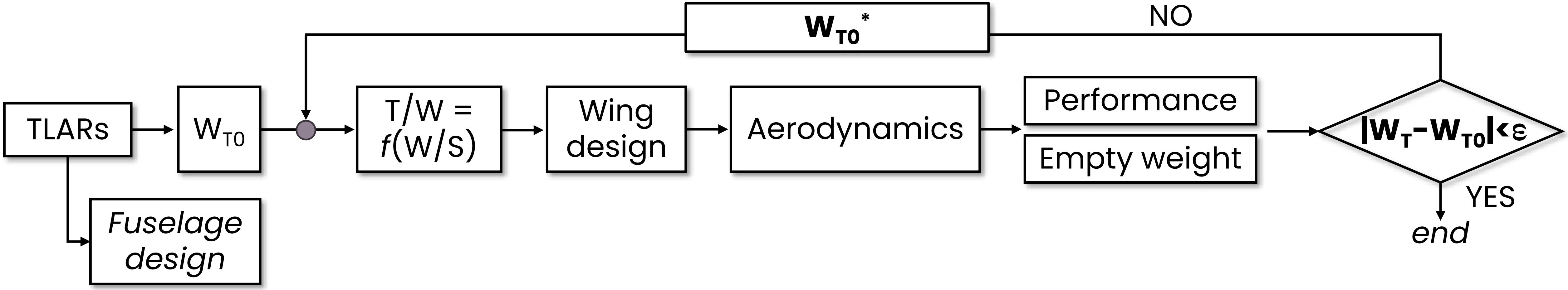
Introduction

The aircraft design process

The Aircraft Design Process

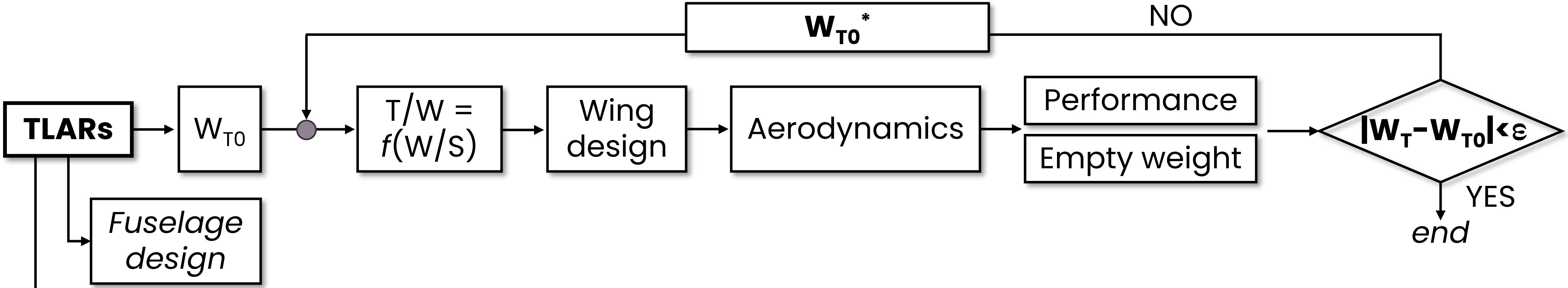


The Aircraft Design Process



Qualche considerazione preliminare su come impostare il vostro codice…

Top Level Aircraft Requirements



Payload

Range

Cruise Mach

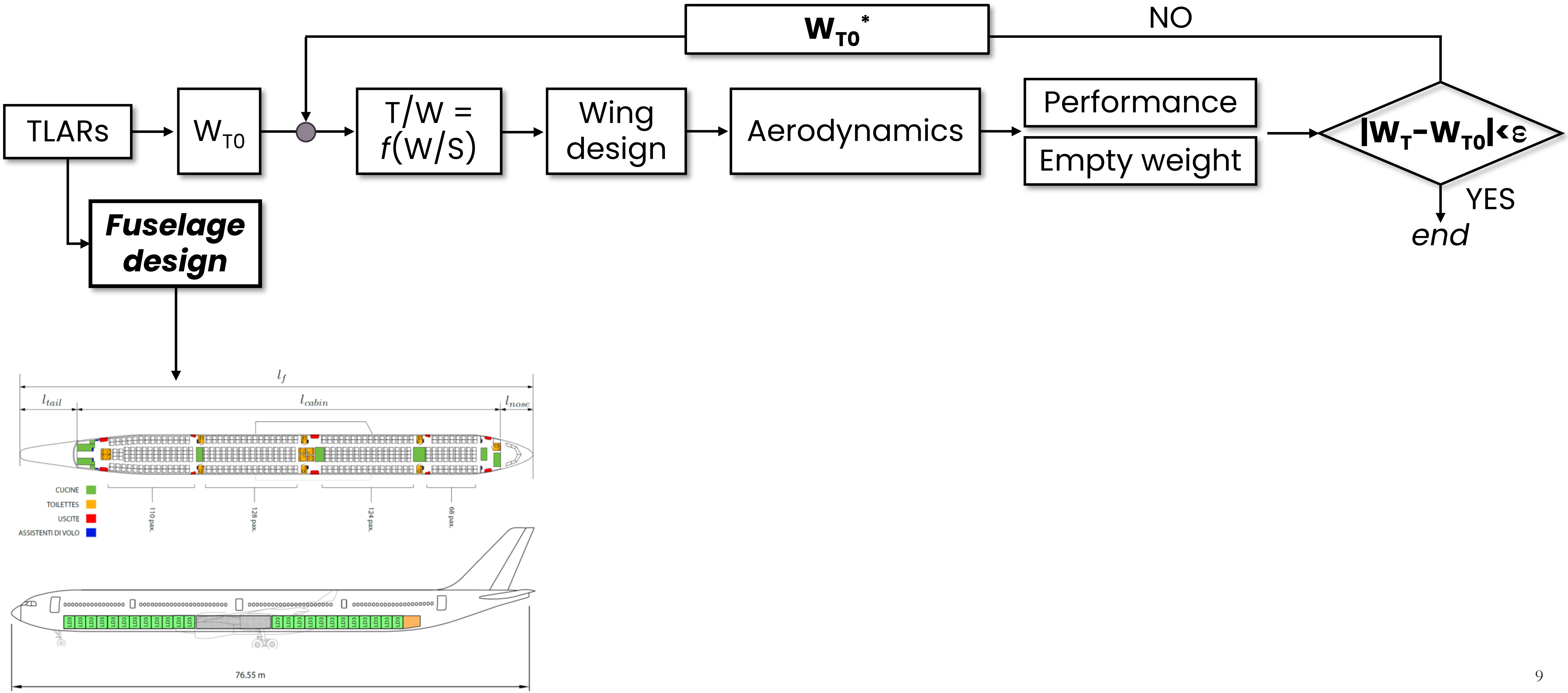
Wingspan

Take-off distance

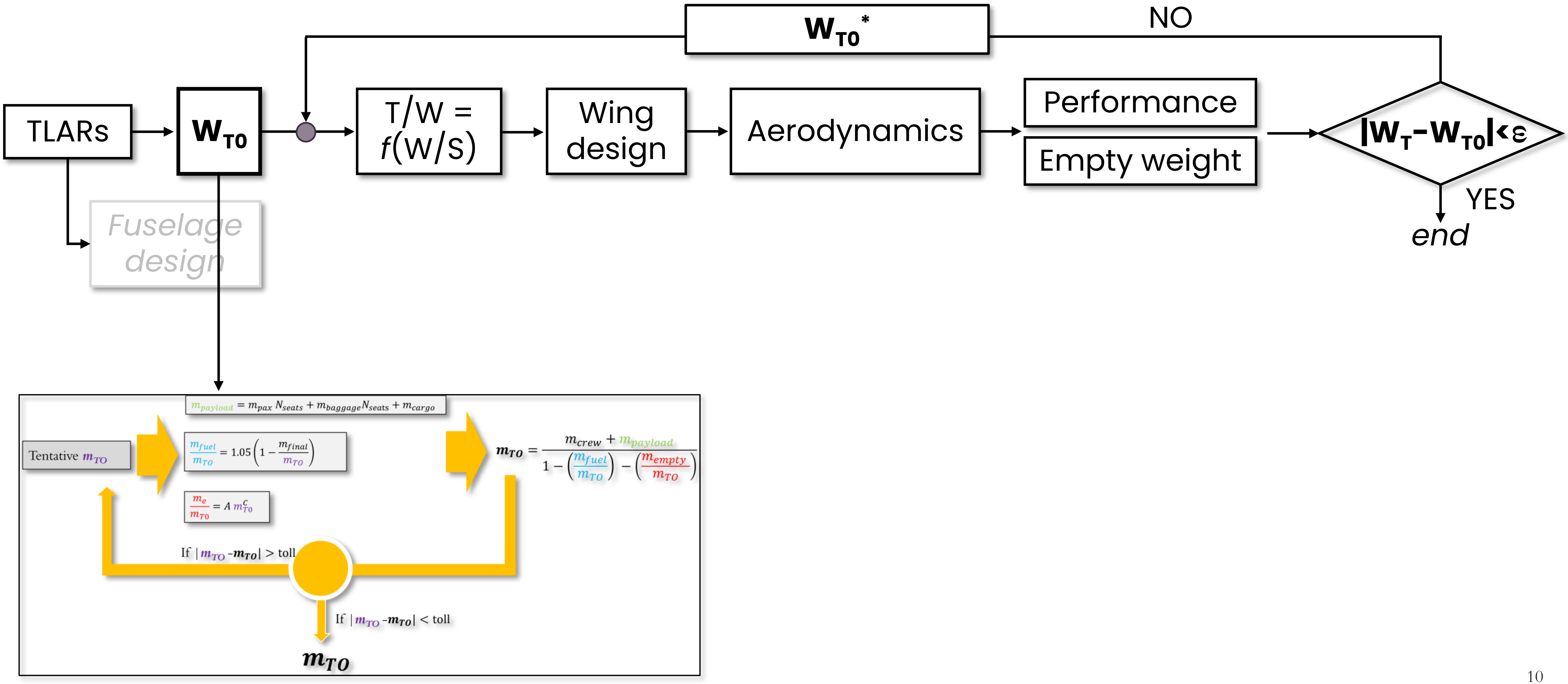
Landing distance

...and others

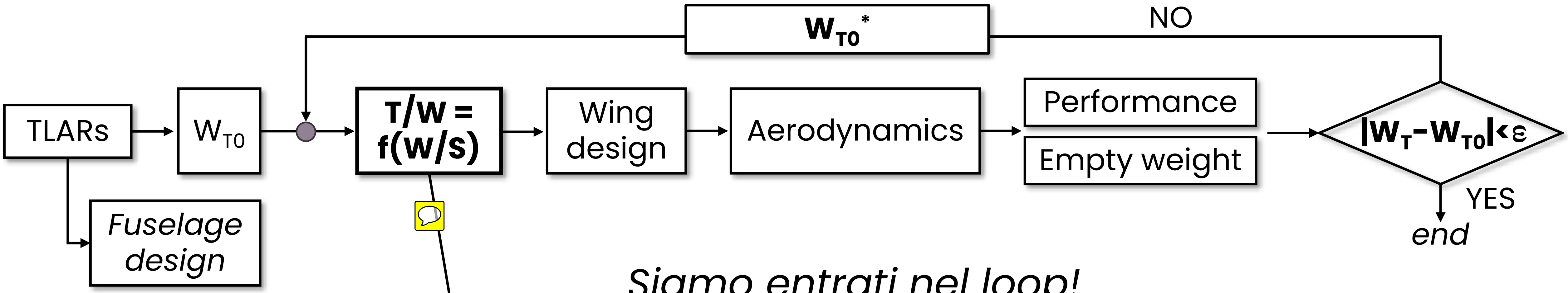
Fuselage Layout



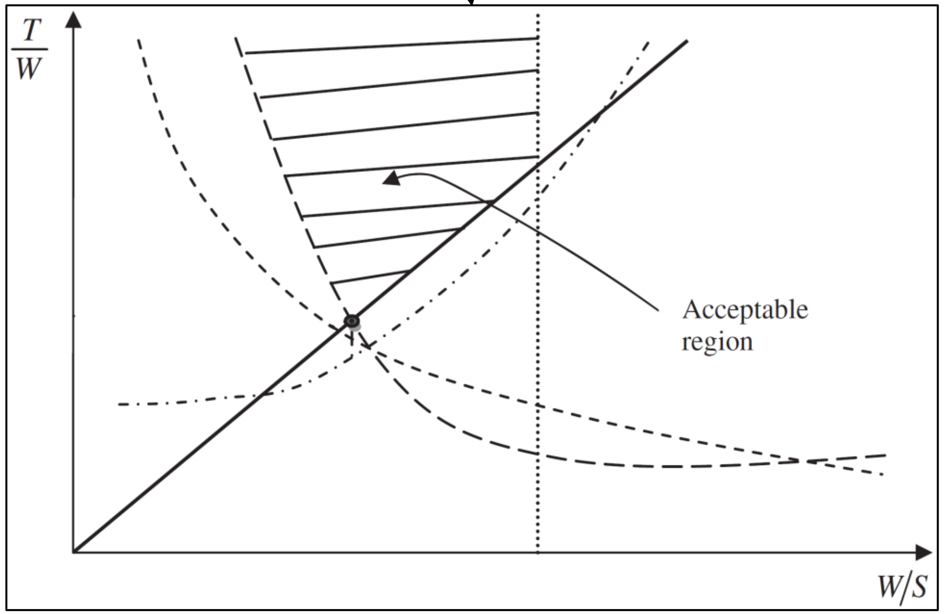
Initial Weight Estimation



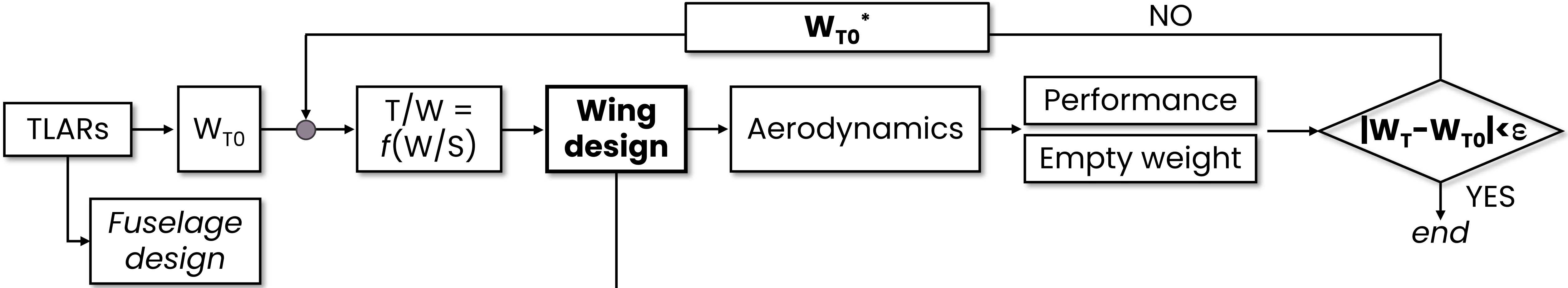
Installed Thrust Calculation



Siamo entrati nel loop!



Aerodynamic Wing Design

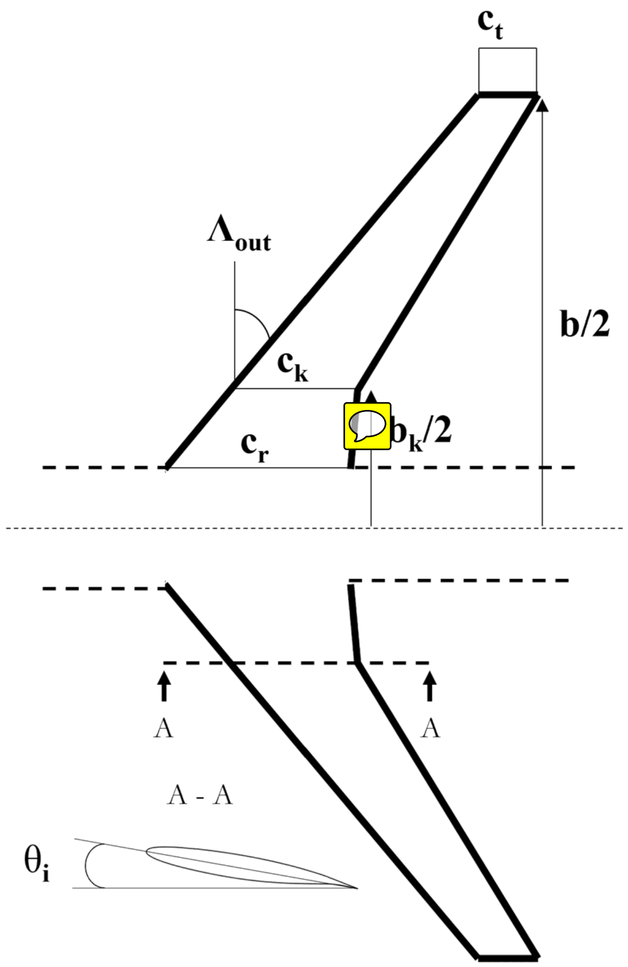


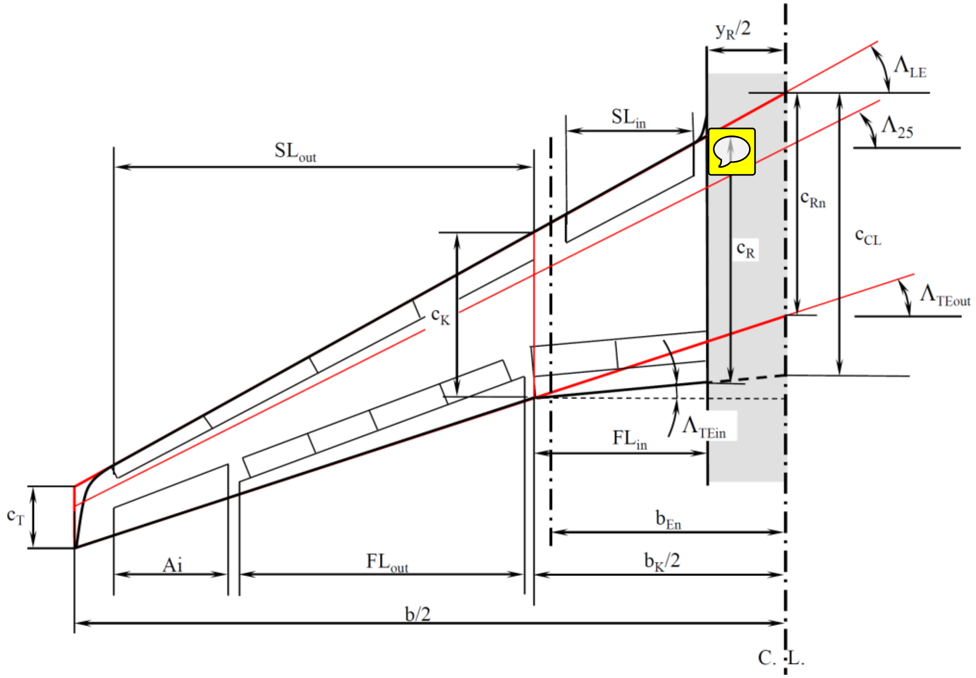
New objective: design aircraft wing planform and assess its aerodynamics!

Wing design: Variables

Design parameters:

- Airfoil
  - **Shape** (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\boldsymbol{\theta(y)}$
- Reference Surface $\boldsymbol{S_{ref}}$
- Aspect Ratio ***AR***
  - Wingspan ***b***
- Taper Ratio $\boldsymbol{\lambda}$
  - Root and tip chords $\boldsymbol{c_t, c_r}$
- Sweep Angle $\boldsymbol{\Lambda}$
- Kink geometry (?)
- *(Dihedral angle Γ)*





Wing design: Airfoil

Design parameters:

- **Airfoil**
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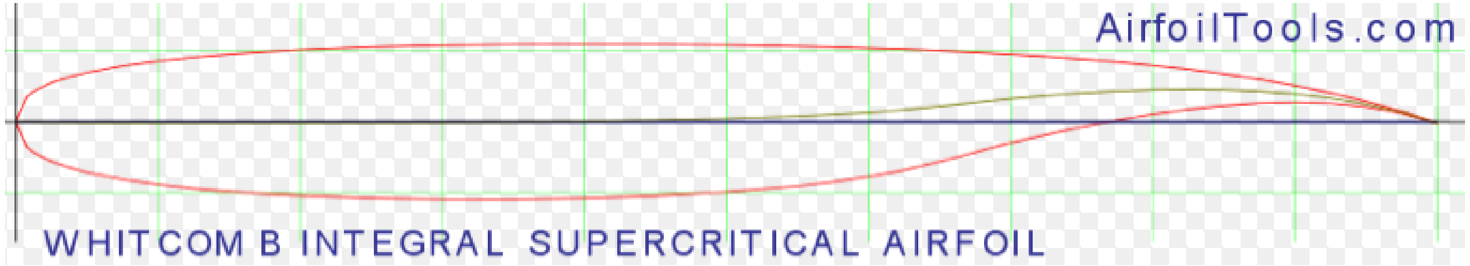
Supercritical airfoils

The basic design approach behind these shapes is to **flatten the upper surface** of the airfoil to reduce flow acceleration and to use a highly cambered aft section to generate the majority of the lift.



There are large databases collecting data and shapes of airfoils (e.g. airfoiltools). You can select a supercritical airfoil (Withcomb, NACA SC20, etc.), and extract its polar curve.

Wing design: Airfoil

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  - **Shape (database)**
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What is transonic?

Critical Mach (M_{crit}) is the Mach number of the freestream at which the **airflow over a point wing first reaches the speed of sound**. In other words, it is the lowest Mach value at which a **supersonic flow** first appears over the aircraft.

It is not a definition that is useful from an engineering perspective.

Drag rise or **drag divergence Mach number** (M_{DD}) is the Mach number at which there is a **rapid and significant increase in aerodynamic drag**, typically due to the formation of **shock waves** associated with local supersonic flow. In practice, M_{DD} is higher than M_{crit} and marks the point where drag increases dramatically due to wave drag.

Represents the operational limit beyond which drag becomes so high that further increases in speed would require a disproportionate expenditure of power.

Wing design: Airfoil

Design parameters:

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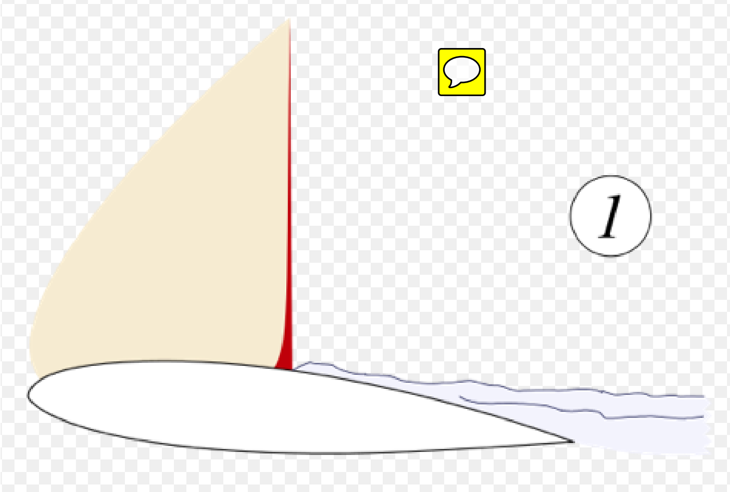
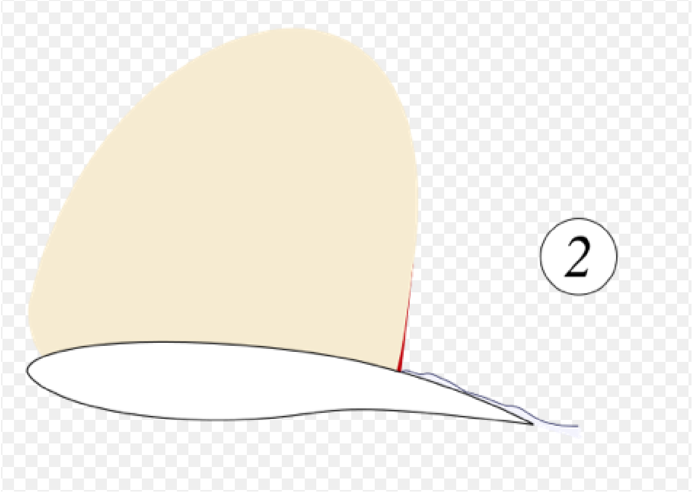


VS



Wing design: Airfoil

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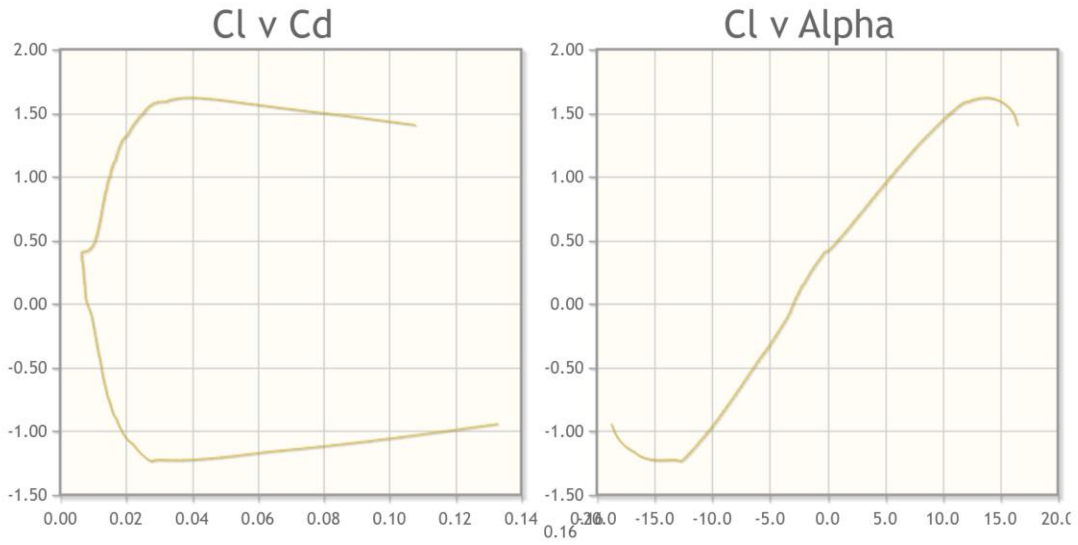

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Effect on transonic performance

Shock wave position: In thicker profiles, the shock wave forms earlier due to increased curvature, accelerating the airflow more significantly.

- **Thin profile:** At transonic speeds, a thinner profile causes lower flow acceleration, reducing shock wave intensity and lowering wave drag.
- **Thick profile:** A thicker profile accelerates the airflow more, generating stronger shock waves and increasing wave drag.

$$M_{DR}(y) = f(\, C_L(y),\; t/c(y) \,)$$

Wing design: Airfoil

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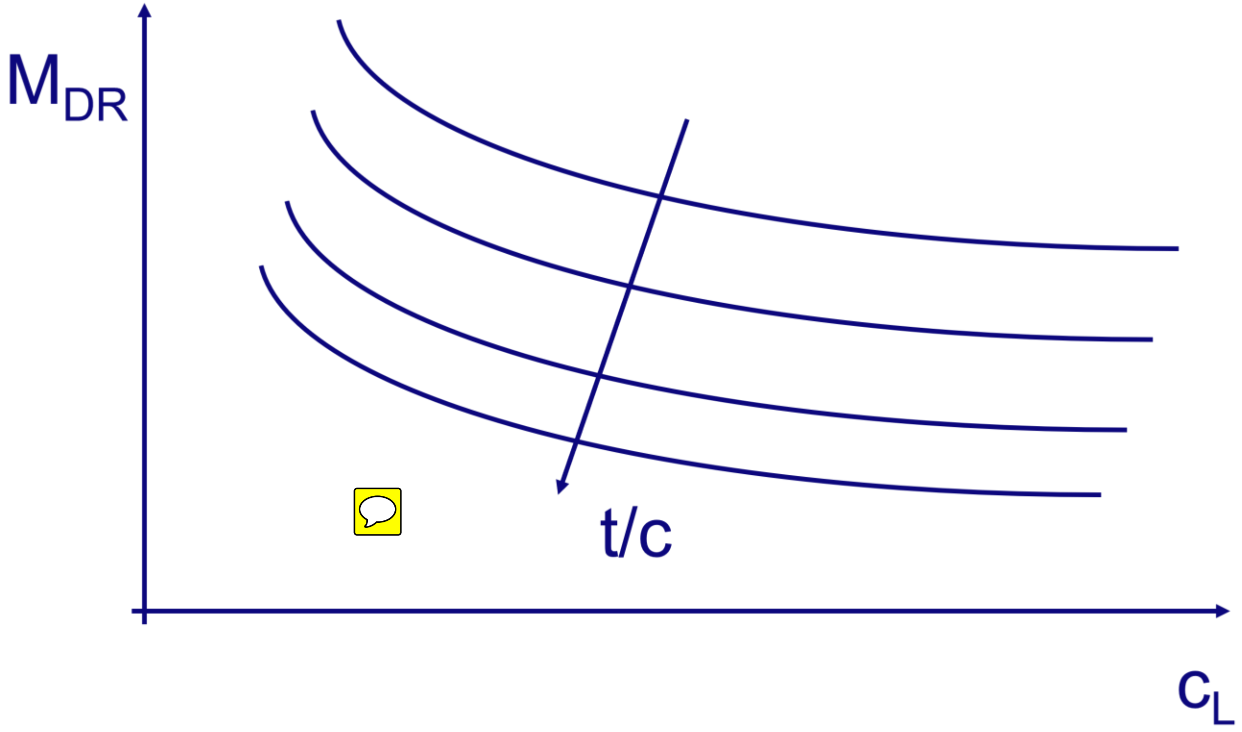
Effect on transonic performance



$$\mathbf{M_{DR}(y)} = f(\,C_L(y),\ t/c(y)\,)$$

Wing design: Airfoil

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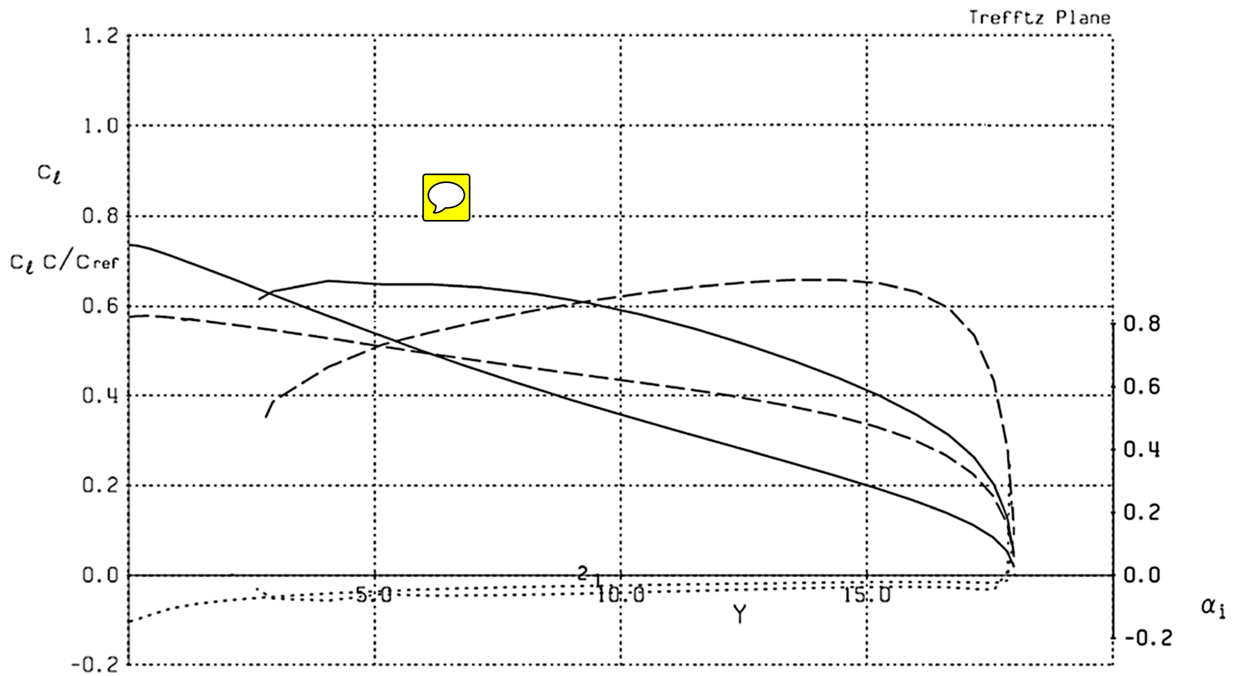
Twist distribution

Relevant effects on

-lift distribution shape (Oswald factor)

-local lift coefficient distribution (wave drag)

-stall location; bending



Wing design: Airfoil

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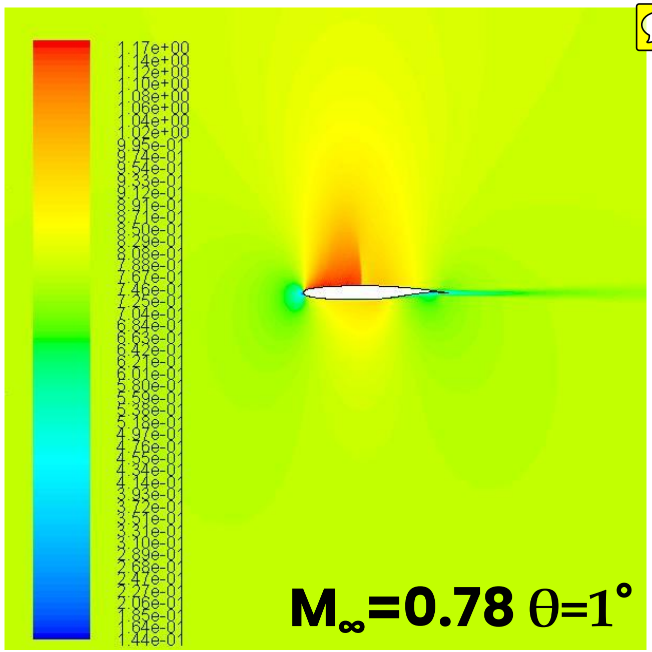

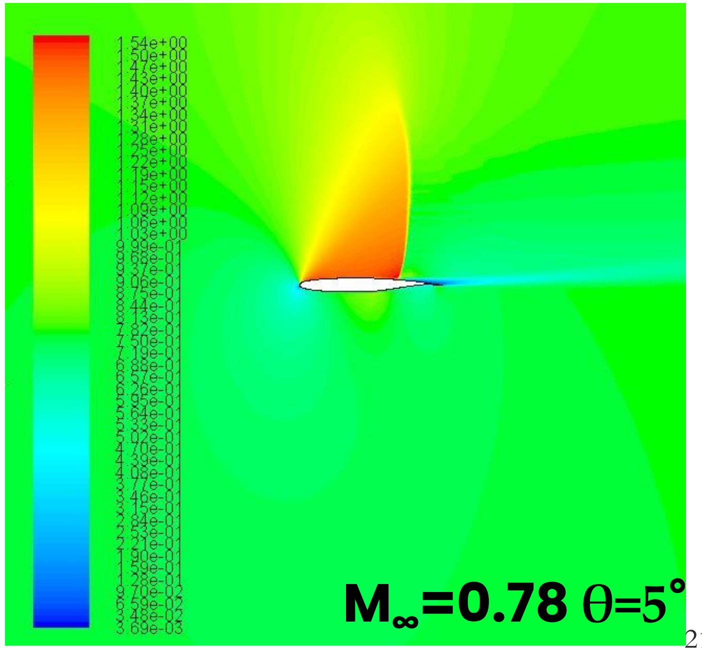

Wing design: Airfoil

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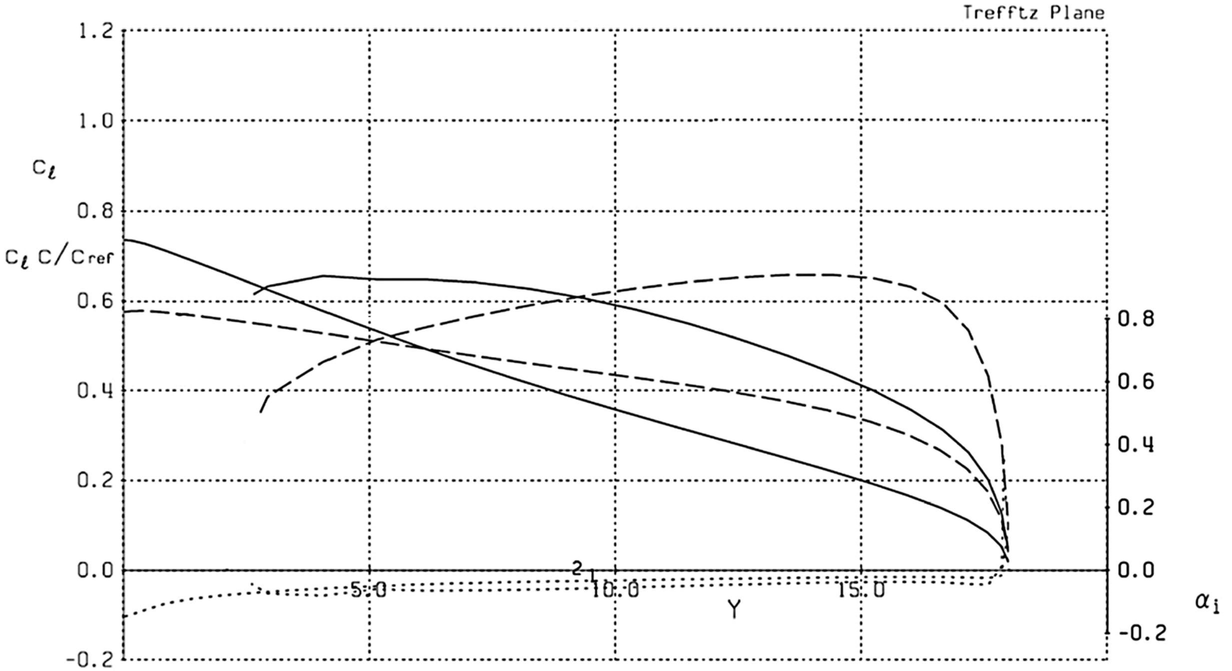

Wing design: Reference Surface

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\theta(y)$
- **Reference Surface S_{ref}**
- Aspect Ratio ***AR***
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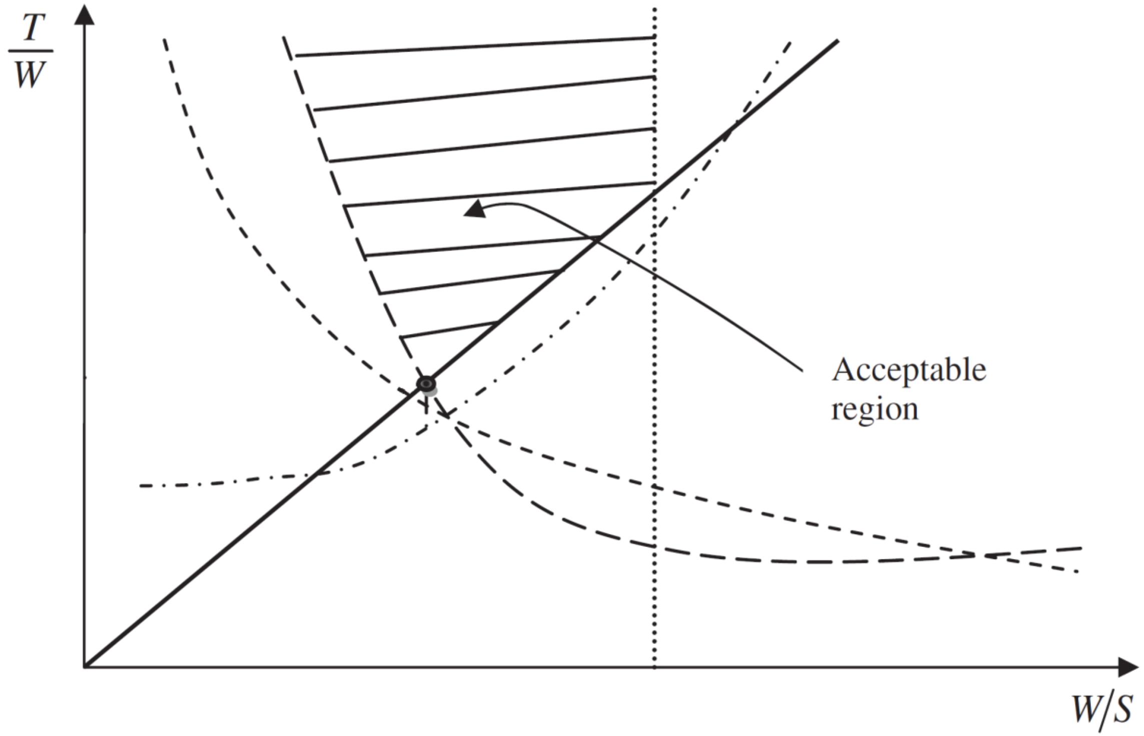
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- Sweep Angle Λ
- Kink geometry (?)
- *(Dihedral angle Γ)*

In this simplified design process, we can extract our reference wing surface $\mathbf{S_{ref}}$ from the design point chosen in the matching chart, and hence from the **design wing loading W/S**, as W_{TO} is known (at each iteration).



Wing design: Reference Surface

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\theta(y)$
- **Reference Surface S_{ref}**
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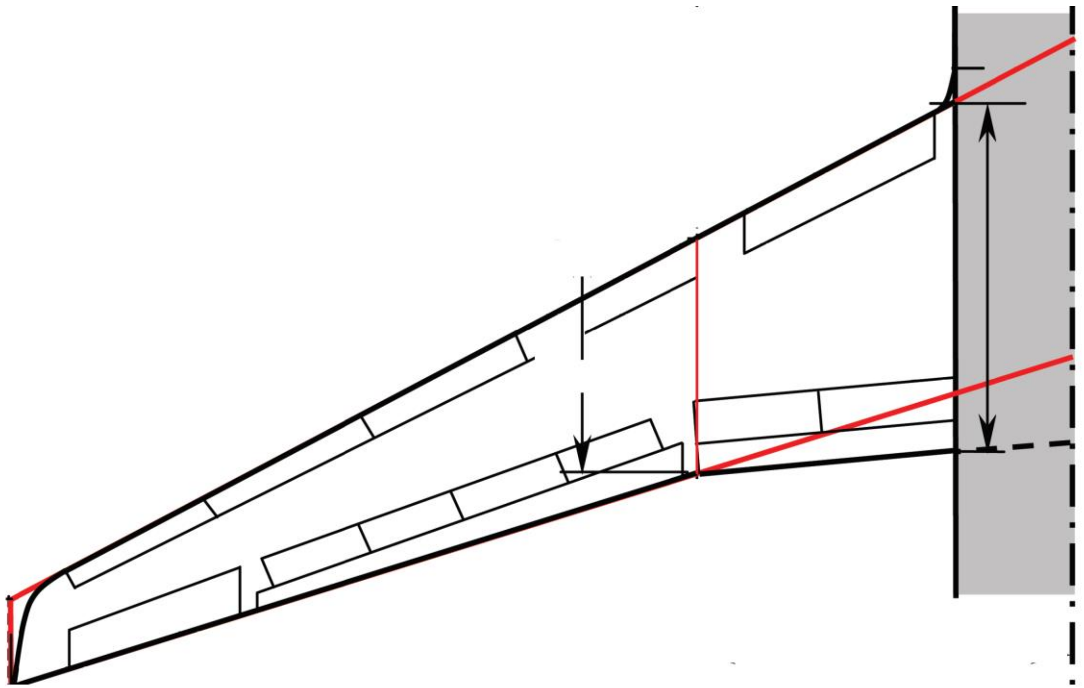

Wing design: Aspect Ratio

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution θ***(y)***
- Reference Surface S_{ref}
- **Aspect Ratio *AR***
  - **Wingspan *b***
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- Sweep Angle Λ
- Kink geometry (?)
- *(Dihedral angle Γ)*

Aspect Ratio *AR* has a relevant impact on wing aerodynamic performance. Particularly, it relevantly affects the **induced drag**, following the relation:

$$C_{Di}{}^{wing} = kC_L{}^2$$

$$k = 1/(\pi\, AR\, e)$$

Let us consider as design point the 25% of the cruise distance. We can evaluate W_{des}, hence we know also the design lift coefficient C_L.

Wing design: Aspect Ratio

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution θ***(y)***
- Reference Surface S_{ref}
- **Aspect Ratio *AR***
  - **Wingspan *b***
- Taper Ratio λ
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- Sweep Angle Λ
- Kink geometry (?)
- *(Dihedral angle Γ)*

We can handle **Aspect Ratio *AR*** has a wing design variable, to perform sensitivity studies with respect to the overall design process

Indeed, **Aspect Ratio *AR*** has a relevant impact also on wing weight, as we will discuss later. **Trade-off solutions** are needed.

Typical values can be between 7-12.
Known AR and S_{ref}, it is easy to extract wingspan, as:

$$\mathbf{AR = b^2 / S_{ref}}$$

Wing design: Aspect Ratio

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution θ***(y)***
- Reference Surface S_{ref}
- **Aspect Ratio *AR***
  - **Wingspan *b***
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Indeed, **Aspect Ratio *AR*** has a relevant impact also on wing weight, as we will discuss later. **Trade-off solutions** are needed.

Rember the airport apron **constraints** from TLARS!

$$\mathbf{AR} = \boxed{\mathbf{b}}^2 / \mathbf{S_{ref}}$$

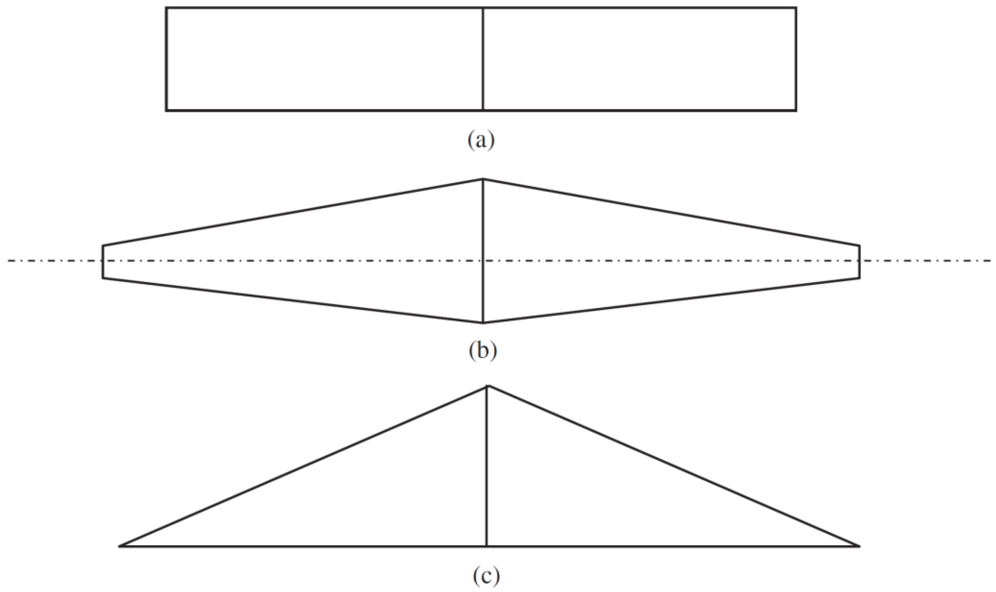
Wing design: Taper Ratio

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\theta(y)$
- Reference Surface S_{ref}
- Aspect Ratio ***AR***
  - Wingspan ***b***
- **Taper Ratio λ**
  - **Root and tip chords c_t , c_r**
- Sweep Angle Λ
- Kink geometry (?)
- *(Dihedral angle Γ)*

The **taper ratio (λ)** of a wing is the ratio between the length of the chord at the wingtip (c_{tip}) and the length of the chord at the root (c_{root}).

A wing with a low taper ratio will have a narrower wingtip compared to the root, while a wing with a high taper ratio will have a more uniform profile along its length.



Wings with various taper ratios: (a) Rectangle ($\lambda = 1$); (b) Trapezoid $0 < \lambda < 1$ (straight tapered); and (c) Triangle (delta) $\lambda = 0$

Wing design: Taper Ratio

Design parameters:

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  - Shape (database)
  - Thickness-chord ratio ***t/c***
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- Aspect Ratio ***AR***
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- **Taper Ratio** λ
  - **Root and tip chords** c_t, c_r
- Sweep Angle Λ
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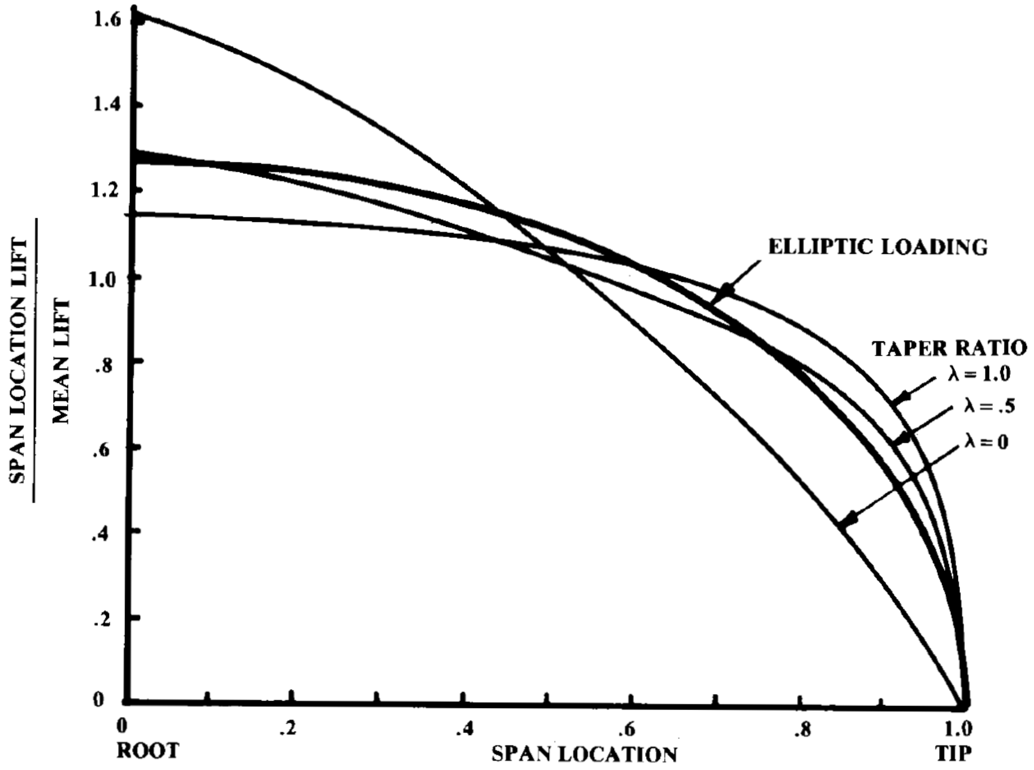
- *(Dihedral angle* Γ*)*

A possible theoretical expression of Oswald factor:

$$e_{th} = \frac{1}{1 + f(\lambda) \cdot AR} \cdot \cos \Lambda$$



Wing design: Taper Ratio

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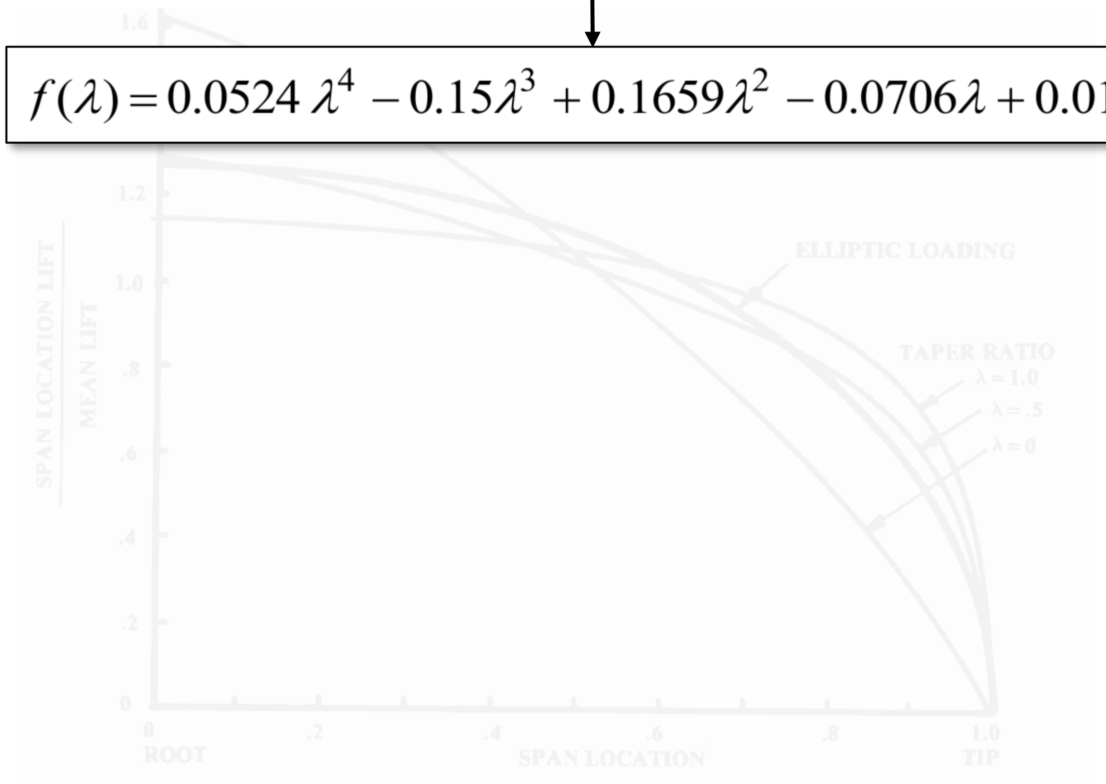
$$e_{th} = \frac{1}{1 + \boxed{f(\lambda)} \cdot AR} \cdot \cos\Lambda$$

$$f(\lambda) = 0.0524\,\lambda^4 - 0.15\lambda^3 + 0.1659\lambda^2 - 0.0706\lambda + 0.0119$$



Wing design: Sweep Angle

Design parameters:

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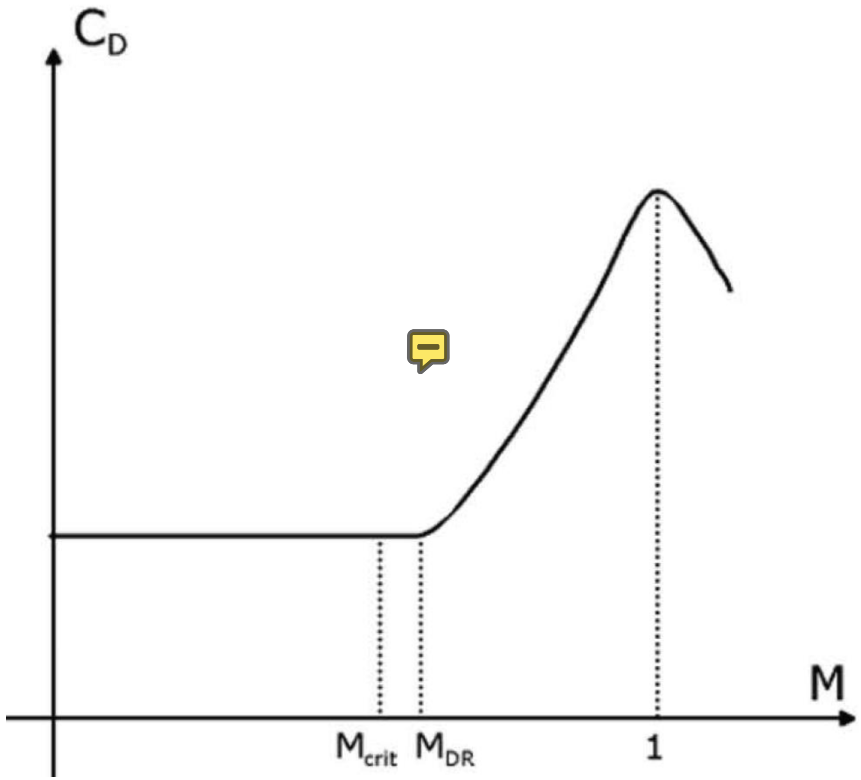
  - Wingspan ***b***
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- **Sweep Angle Λ**
- Kink geometry (?)
- *(Dihedral angle Γ)*

Sweep angle Λ_{25} has a relevant impact on wing aerodynamic performance. Particularly, it relevantly affects the **wave drag**



Wing design: Sweep Angle

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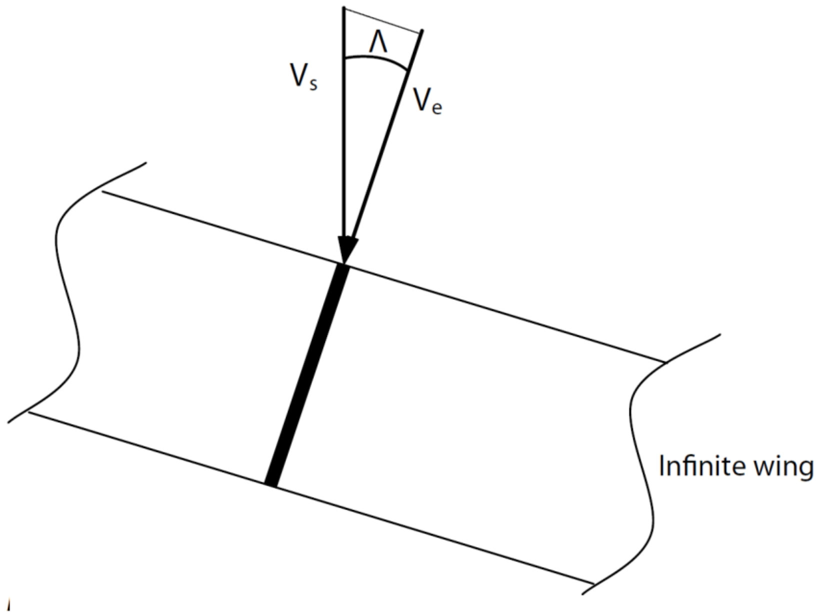
In theory, the formation of the shock wave on a swept wing is not determined by the freestream velocity, but by **its component orthogonal to the leading edge**. Thanks to the sweep, we can fly at Mach numbers higher than the drag divergence Mach number of individual airfoils without encountering significant increases in drag.



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$$M_{dd} = \frac{k_A}{\cos\Lambda} - \frac{(t/c)}{\cos^2\Lambda} - \frac{k_C C_L}{\cos^3\Lambda} \quad \text{with} \quad \begin{array}{l} k_A = 0.90 \\ k_C = 1/10 \end{array}$$

$$M_{crit} = M_{dd} - \left(\frac{0.1}{80}\right)^{1/3}$$

$$C_{Dwave} = 20(M - M_{crit})^4 \text{ for } M > M_{crit}$$

Let us consider as design point the 25% of the cruise distance. We can evaluate W_{des}, hence we know also the design lift coefficient C_L.

Wing design: Sweep Angle

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We can handle **Sweep angle Λ_{25}** has a wing design variable, to perform sensitivity studies with respect to the overall design process

Indeed, **Sweep angle Λ_{25}** has a relevant impact also on wing weight, as we will discuss later. **Trade-off solutions** are needed.

Typical values can be between 15°-35°.

Wing design: Sweep Angle

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  - Twist distribution θ*(y)*
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Typical values can be between 15°-35°.

$$\Delta C_{L_{max_{3D}}}^{flap} = 0.92 \cdot \Delta C_{l_{max_{2D}}}^{flap} \cdot \frac{S_{flapped}}{S} cos(\boxed{\Lambda_{25}})$$

Rember the impact of **Sweep angle Λ_{25}** on C_{LMax}!

Wing design: Kink

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\theta(y)$
- Reference Surface S_{ref}
- Aspect Ratio ***AR***
  - Wingspan ***b***
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- Sweep Angle Λ
- **Kink geometry *(?)***
- *(Dihedral angle Γ)*

It is sometimes advantageous to employ a cranked wing planform, that is, one with **discontinuous changes in sweepback angle of the leading and/or trailing edges**.

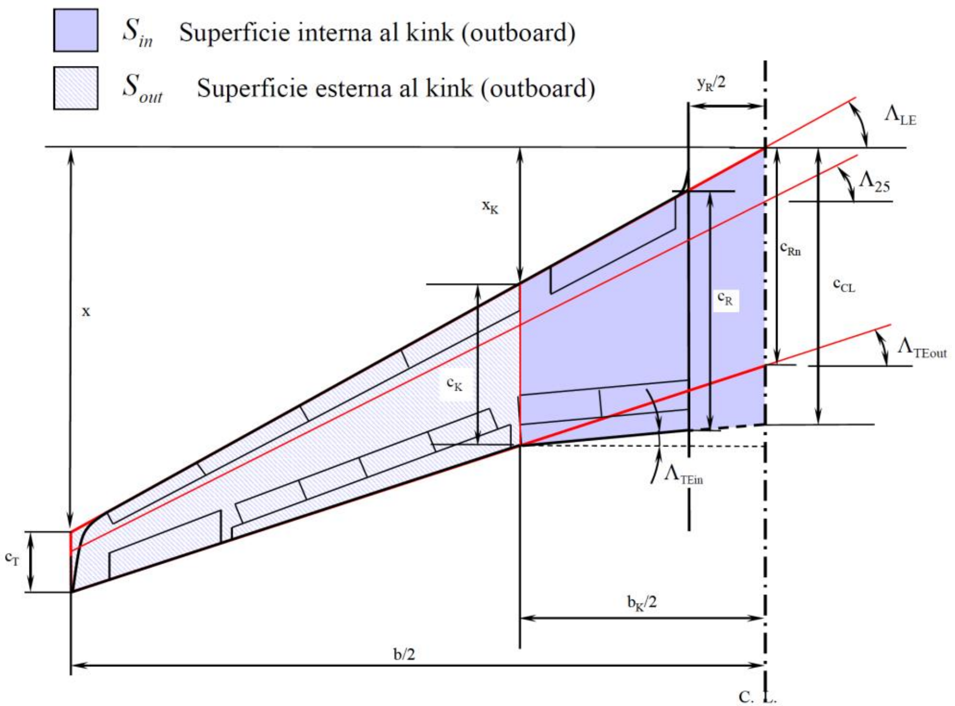
Some commercial airliners have essentially zero sweepback along the trailing edge in the central portion of the wing.

Wing design: Kink

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution $\theta(y)$
- Reference Surface S_{ref}
- Aspect Ratio ***AR***
  - Wingspan ***b***
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- Sweep Angle Λ
- **Kink geometry *(?)***
- *(Dihedral angle Γ)*

This design feature allows for better **stowage of landing gear**, and a stronger wing root structure. In addition to these intuitively obvious attributes, the longer chord length in the vicinity of the fuselage junction provides for improved aerodynamic flow characteristics over the wing. It enables also more effective structural integration and/or a simplified trailing edge high-lift system installation.

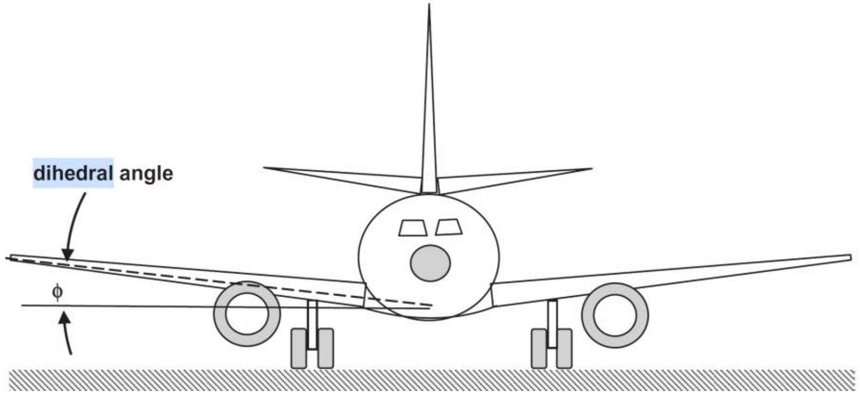


Wing design: Dihedral Angle

Design parameters:

- Airfoil
  - Shape (database)
  - Thickness-chord ratio ***t/c***
  - Twist distribution θ***(y)***
- Reference Surface S_{ref}
- Aspect Ratio ***AR***
  - Wingspan ***b***
- Taper Ratio λ
  - Root and tip chords c_t, c_r
- Sweep Angle Λ
- Kink geometry (?)
- ***(Dihedral angle*** Γ***)***

The planform views discussed thus far cannot illustrate another common wing characteristic, **dihedral angle**. This is the angle that a wing makes with the horizontal plane. It is typically a small angle, on the order of 3–8º. This angle has an influence on the stability of an aircraft in roll.



Parametric Wing Design

Design Variables:

- Reference Surface $\boldsymbol{S_{ref}}$
- Cruise Mach $\boldsymbol{M}$
- Thickness-chord ratio $\boldsymbol{t/c}$
- Aspect Ratio $\boldsymbol{AR}$
- Taper Ratio λ
- Sweep Angle Λ

Parametric Wing Design

Design Variables:

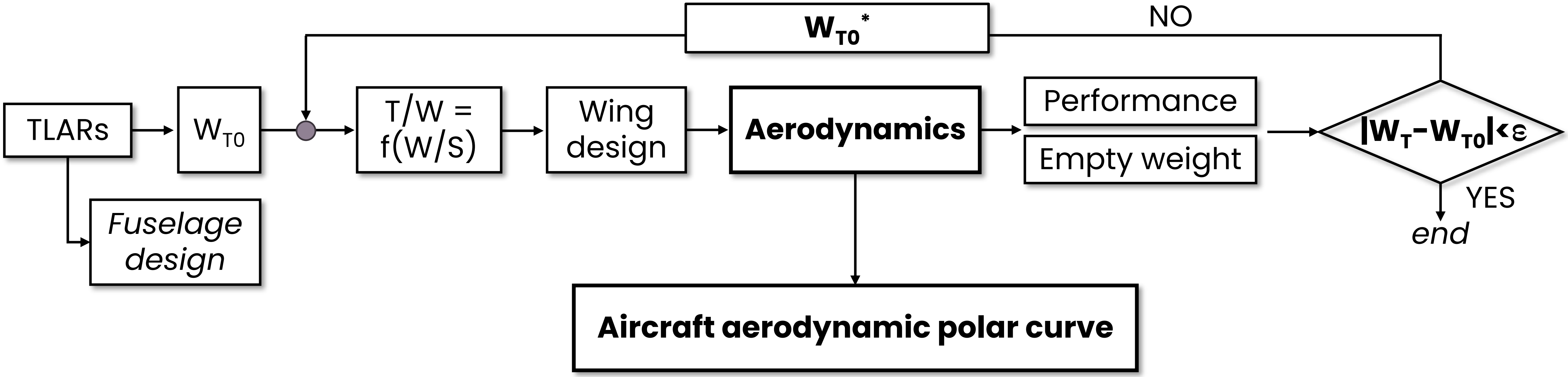
- Reference Surface $\boldsymbol{S_{ref}}$ → $\mathbf{(W/S)_{vect}} = [(W/S)_1 \ (W/S)_2 \ \dots \ (W/S)_k]$
- Cruise Mach $\boldsymbol{M}$ → $\mathbf{M_{vect}} = [M_1 \ M_2 \ \dots \ M_k]$
- Thickness-chord ratio $\boldsymbol{t/c}$ → $\mathbf{(t/c)_{vect}} = [(t/c)_1 \ (t/c)_2 \ \dots \ (t/c)_k]$
- Aspect Ratio $\boldsymbol{AR}$ → $\mathbf{AR_{vect}} = [AR_1 \ AR_2 \ \dots \ AR_k]$
- Taper Ratio $\boldsymbol{\lambda}$ → $\boldsymbol{\lambda}_{\mathbf{vect}} = [\lambda_1 \ \lambda_2 \ \dots \ \lambda_k]$
- Sweep Angle $\boldsymbol{\Lambda}$ → $\boldsymbol{\Lambda}_{\mathbf{vect}} = [\Lambda_1 \ \Lambda_2 \ \dots \ \Lambda_k]$

Parametric Wing Design

Typical values:

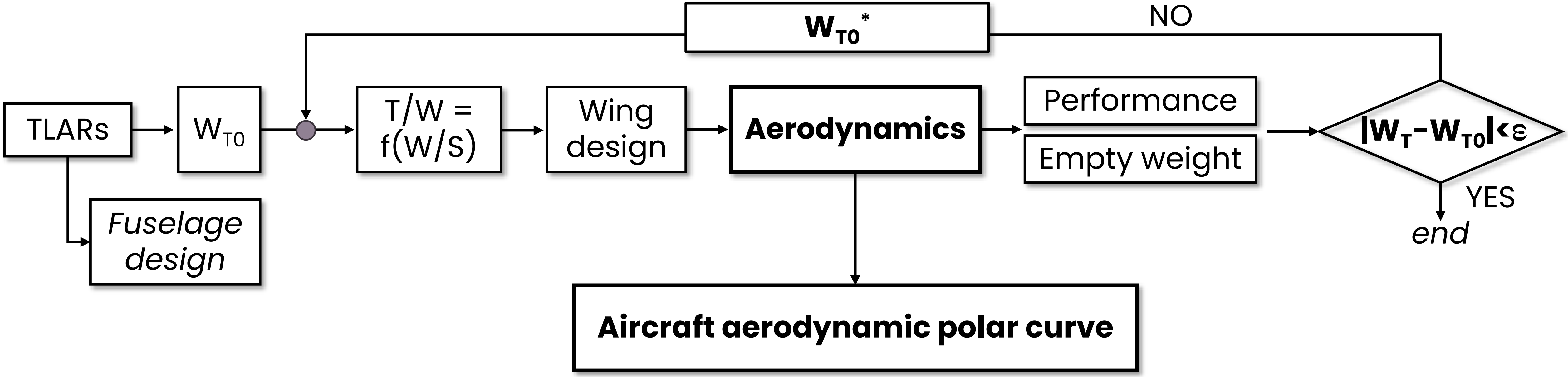
- Reference Surface $\boldsymbol{S_{ref}}$ — 550-750 kg/m^2
- Cruise Mach $\boldsymbol{M}$ — 0.75-0.82
- Thickness-chord ratio $\boldsymbol{t/c}$ — 0.10-0.15
- Aspect Ratio $\boldsymbol{AR}$ — 7-12
- Taper Ratio $\boldsymbol{\lambda}$ — 0.20-0.40
- Sweep Angle $\boldsymbol{\Lambda}$ — 15°-35°

Aircraft Polar Curve



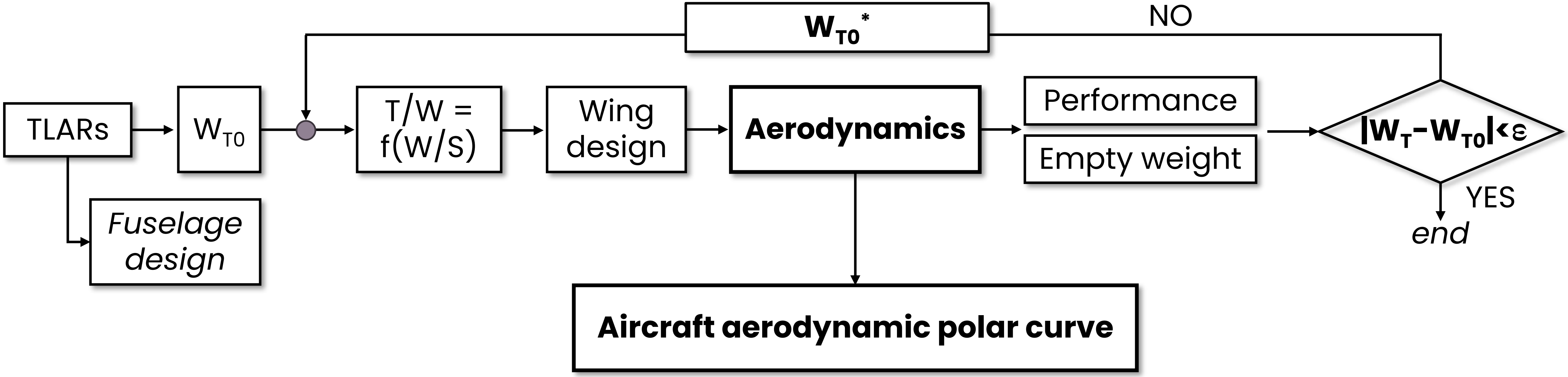
$$C_{Dtot} = C_{D0} + C_{Di} + C_{Dw} + C_{Dtrim} + C_{Dinter} + \ldots$$

Aircraft Polar Curve



$$C_{Dtot}=C_{D0}+C_{Di}+C_{Dw}+\cancel{C_{Dtrim}}+\cancel{C_{Dinter}}+\dots$$

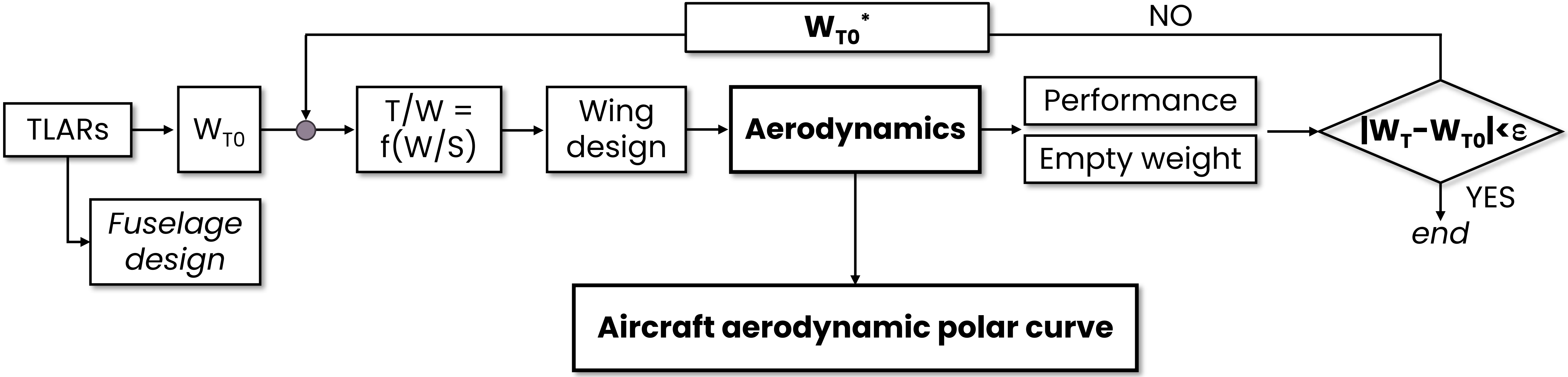
Aircraft Polar Curve



$$C_{Dtot} = C_{D0} + \boxed{C_{Di} + C_{DW}}$$

'mainly' wing contribution

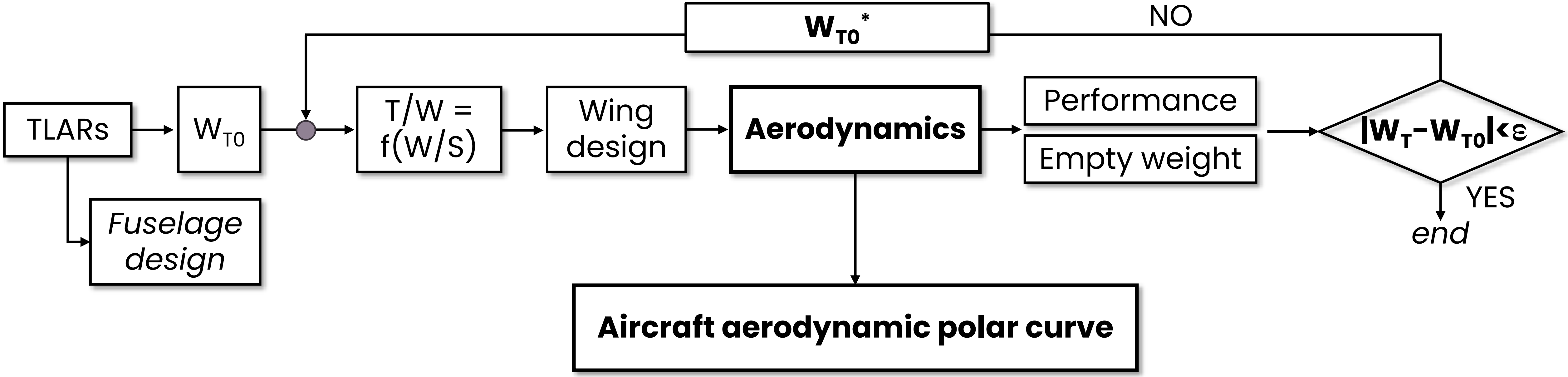
Aircraft Polar Curve



$$C_{Dtot} = \boxed{C_{D0}} + C_{Di} + C_{Dw}$$

whole aircraft

Aircraft Polar Curve

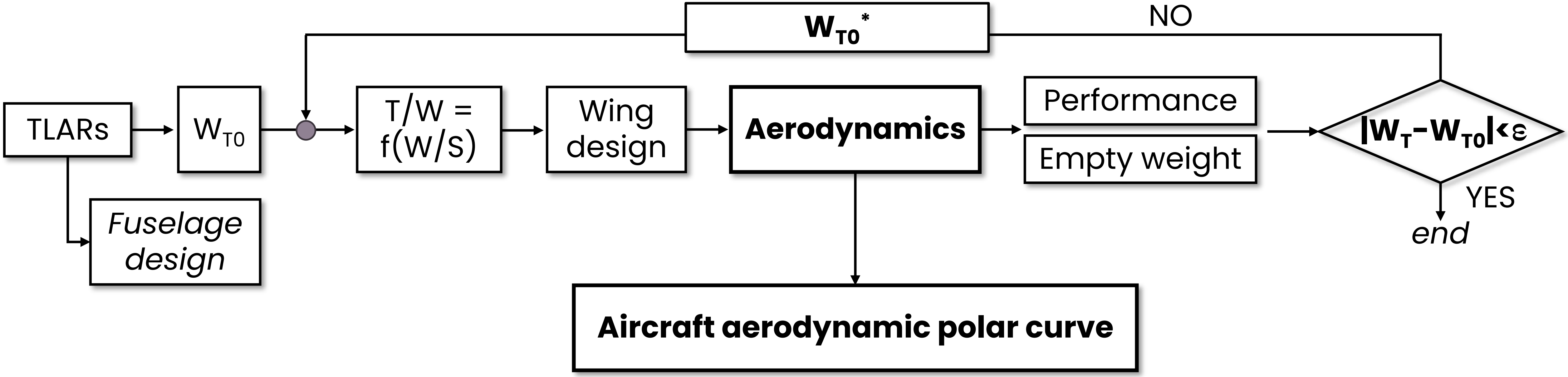


$$C_{Dtot} = \boxed{C_{D0}} + C_{Di} + C_{Dw}$$

whole aircraft

$$C_{D0} = C_{D0}^{wing} + C_{D0}^{fus} + C_{D0}^{tail} + C_{D0}^{nac} + \ldots$$

Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot FF_i \cdot Q_i \cdot S_{Wi}}{S}$$

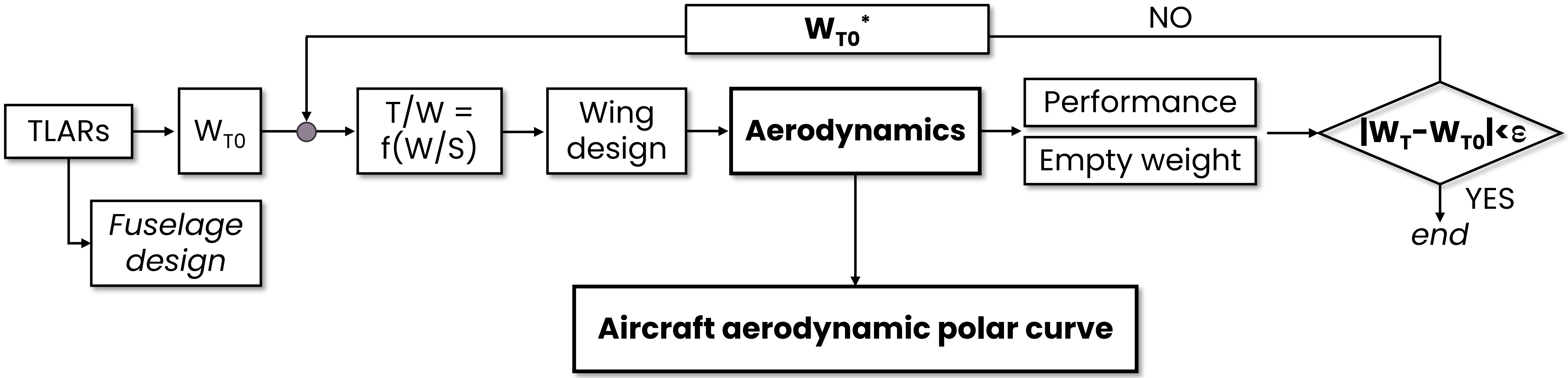
c_{Fi} : coefficiente di attrito equivalente

FF_i : fattore di forma

Q_i : fattore di interferenza

S_{wi} : superficie bagnata

Aircraft Polar Curve

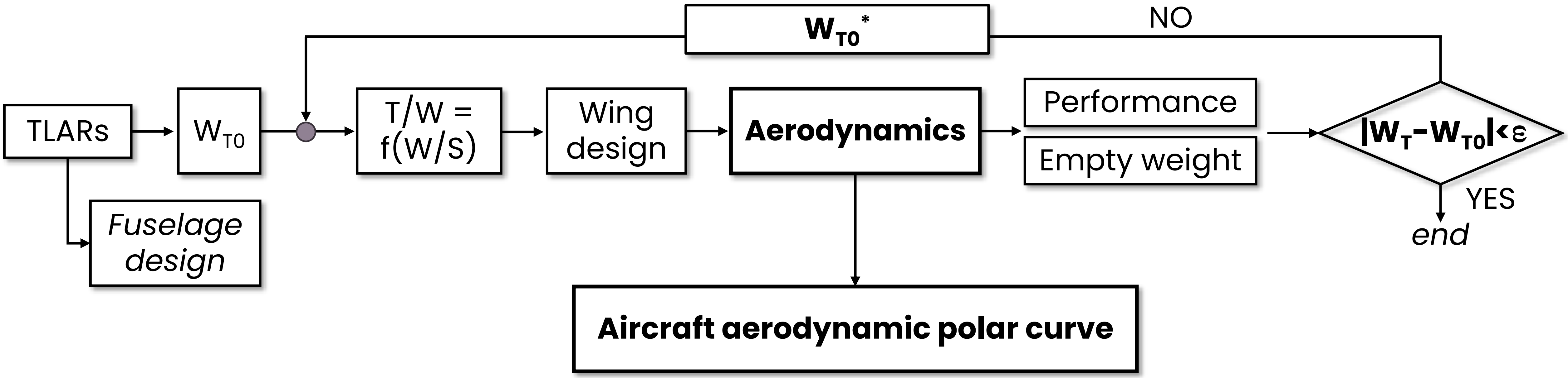


$$C_{D_0} = \frac{\sum_i \boxed{c_{Fi}} \cdot FF_i \cdot Q_i \cdot S_{Wi}}{S}$$

c_{Fi} : coefficiente di attrito equivalente

The **equivalent skin friction coefficient** (c_F) is derived on the analogy with a flat plate at zero incidence. It mainly depends on the Reynolds number (Re), Mach number (M), and surface roughness. The Reynolds number provides an indication of whether the flow is **laminar or turbulent**. In this case, we will assume turbulent flow along the entire wing.

Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i \boxed{c_{Fi}} \cdot FF_i \cdot Q_i \cdot S_{Wi}}{S}$$

c_{Fi} : coefficiente di attrito equivalente

Reynolds number (Re)

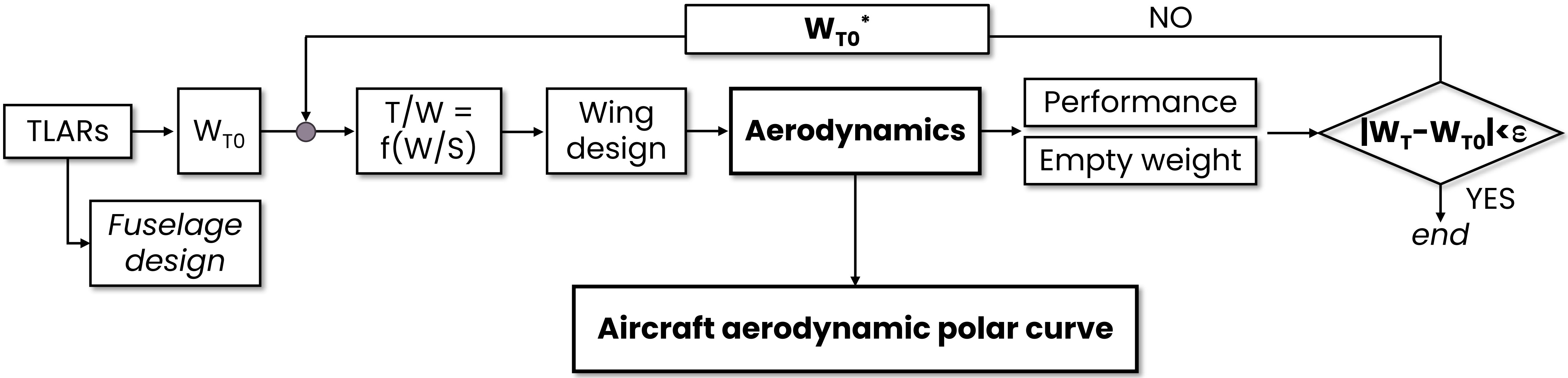
$$\Re e = \frac{V \cdot L}{\nu}$$

V : è la velocità asintotica del flusso

L : la lunghezza caratteristica del corpo

ν : la viscosità cinematica ($\nu = \mu/\rho\ [m]^2\ [s]^{-1}$)

Aircraft Polar Curve

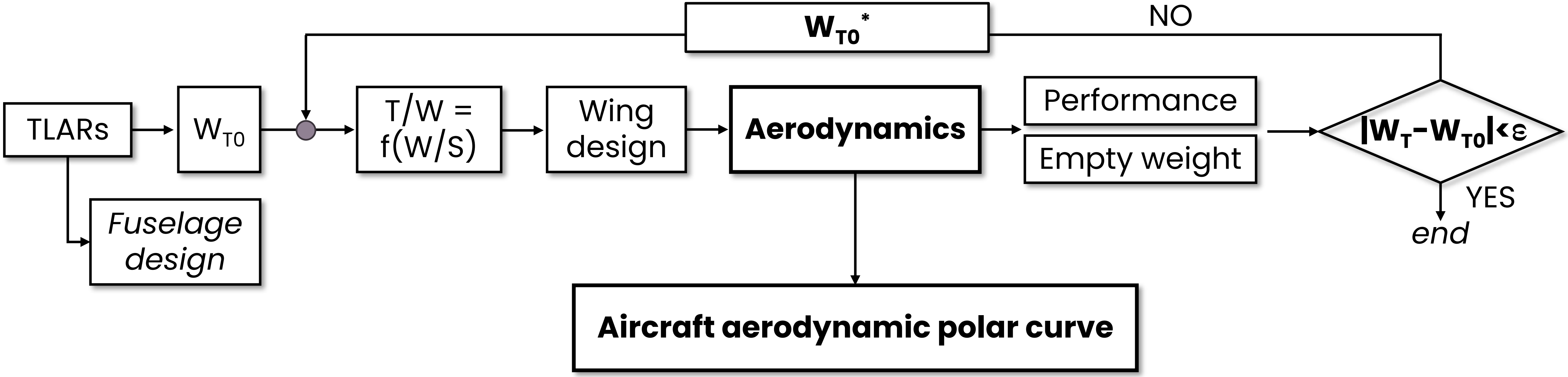


$$C_{D_0} = \frac{\sum_i \boxed{c_{Fi}} \cdot FF_i \cdot Q_i \cdot S_{Wi}}{S}$$

$$c_{F_{TURB}} = \frac{0.455}{\left[Log_{10}(Re)\right]^{2.58}\left[1+0.144 \cdot M^2\right]^{0.65}}$$

c_{Fi} : coefficiente di attrito equivalente

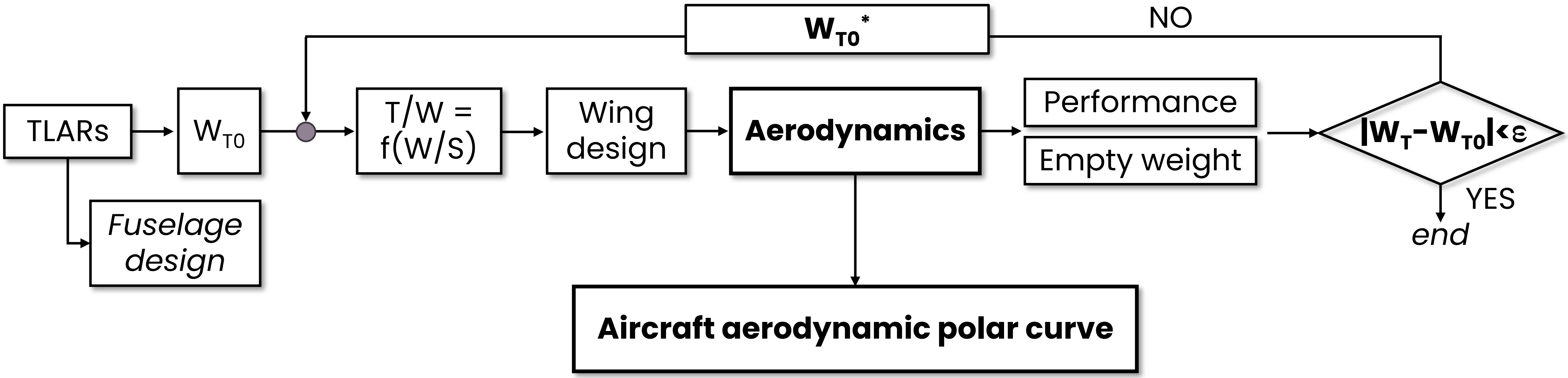
Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot \boxed{FF_i} \cdot Q_i \cdot S_{Wi}}{S}$$

The **form factor** accounts for pressure drag due to boundary layer and viscous separation in the estimation of subsonic parasitic drag. The formulas used are considered valid up to the drag divergence Mach number and vary depending on the component being analyzed

Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot \boxed{FF_i} \cdot Q_i \cdot S_{Wi}}{S}$$

Ali :

$$FF_i = \left[1 + \frac{0.6}{(x/c)_m} \cdot \left(\frac{t}{c}\right) + 100 \cdot \left(\frac{t}{c}\right)^4\right] \cdot \left[1.34 \cdot M^{0.18} \cdot (\cos \Lambda_m)^{0.28}\right]$$

$(x/c)_m \sim 0.5$

Fusoliera :

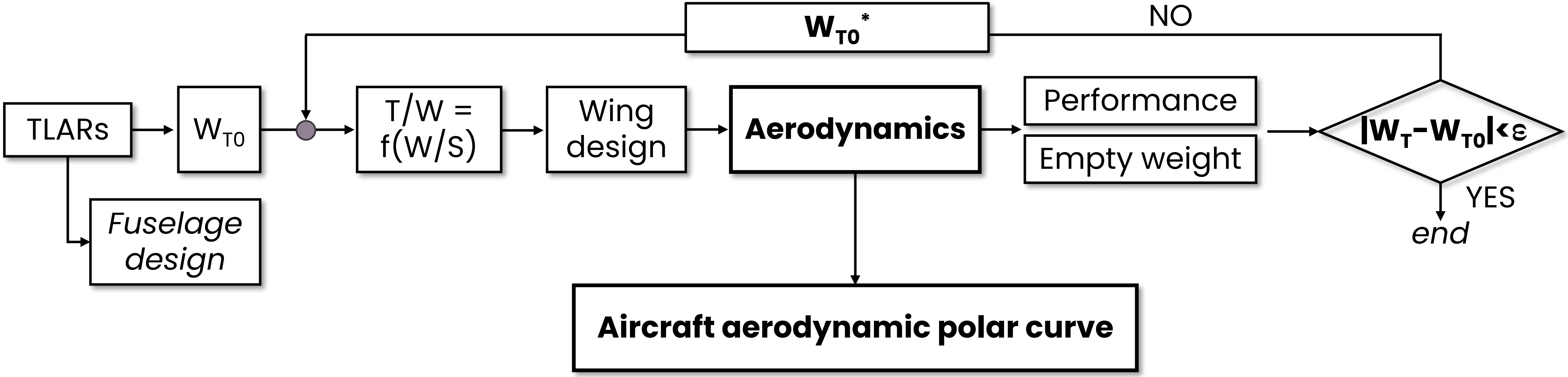
$$FF_i = \left(1 + \frac{60}{f^3} + \frac{f}{400}\right)$$

$$f = \frac{l}{d} = \frac{l}{\sqrt{(4/\pi)A_{max}}}$$

Lunghezza

Area massima della sezione

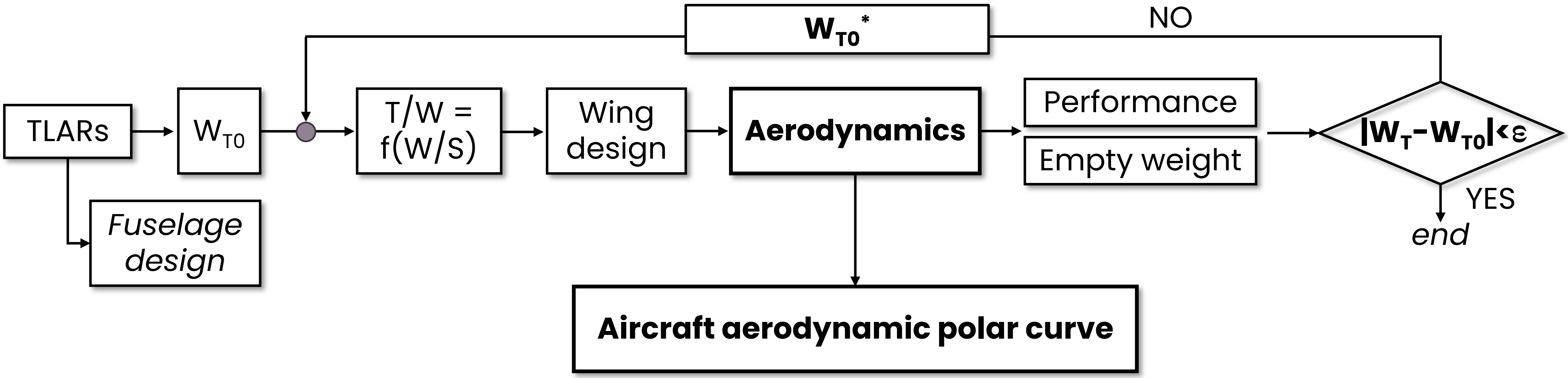
Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot FF_i \cdot \boxed{Q_i} \cdot S_{Wi}}{S}$$

Parasitic drag is increased by the mutual interference between the aircraft components. To account for this physical phenomenon in drag estimation, the **interference factor Q** is introduced.

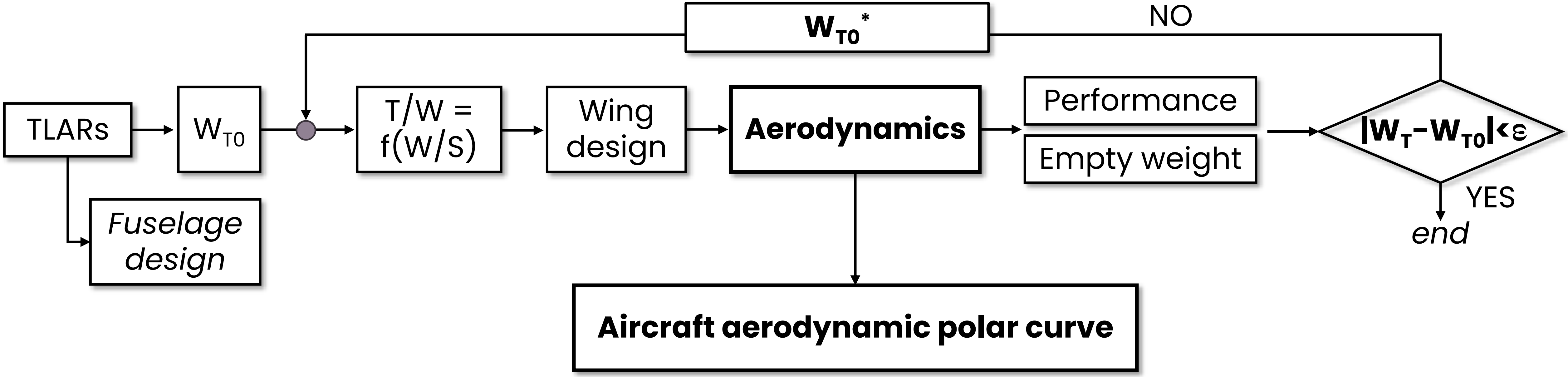
Aircraft Polar Curve



$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot FF_i \cdot \boxed{Q_i} \cdot S_{Wi}}{S}$$

- 1.0 : per la **fusoliera**
- 1.0 : per **ali** alte o basse ben raccordate
  - 1.1–1.4 : per ali basse con raccordi poco curati
- 1.05 : per **code** di tipo **convenzionale**
- 1.3 : per **gondole** montate in fusoliera o sulle ali
  - 1.0 : se sono montati a più di un diametro

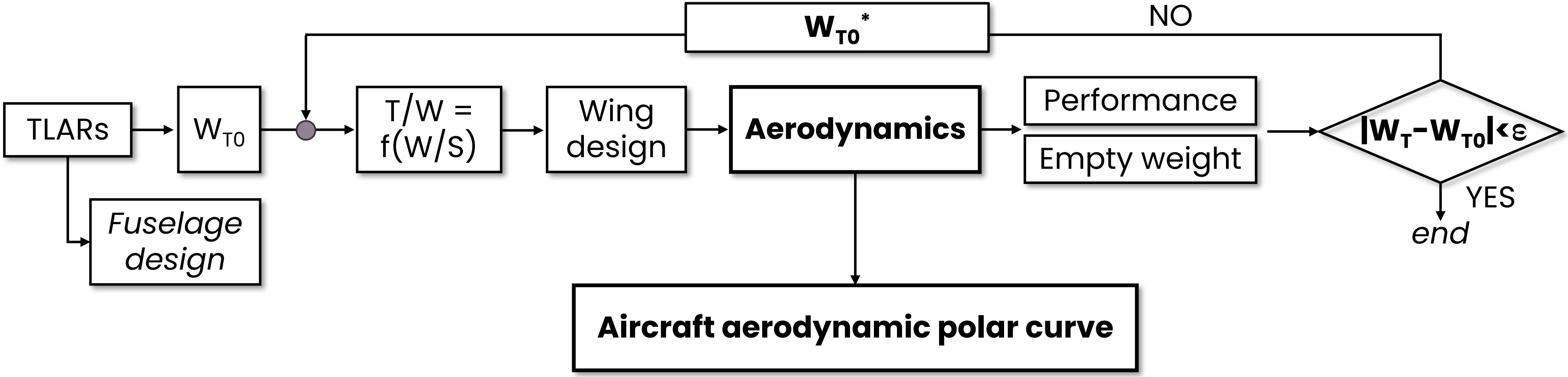
Aircraft Polar Curve



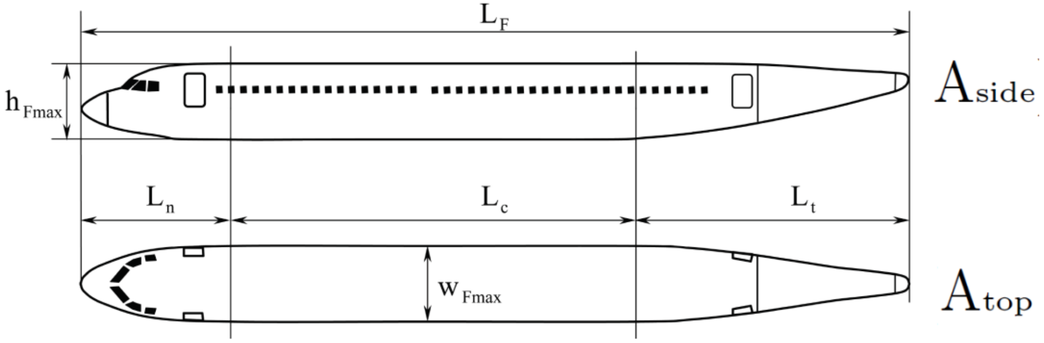
$$C_{D_0} = \frac{\sum_i c_{Fi} \cdot FF_i \cdot Q_i \cdot \boxed{S_{Wi}}}{S}$$

Wetted surfaces of fuselage, wings, tail, nacelles, etc.

Aircraft Polar Curve



S_w Fuselage

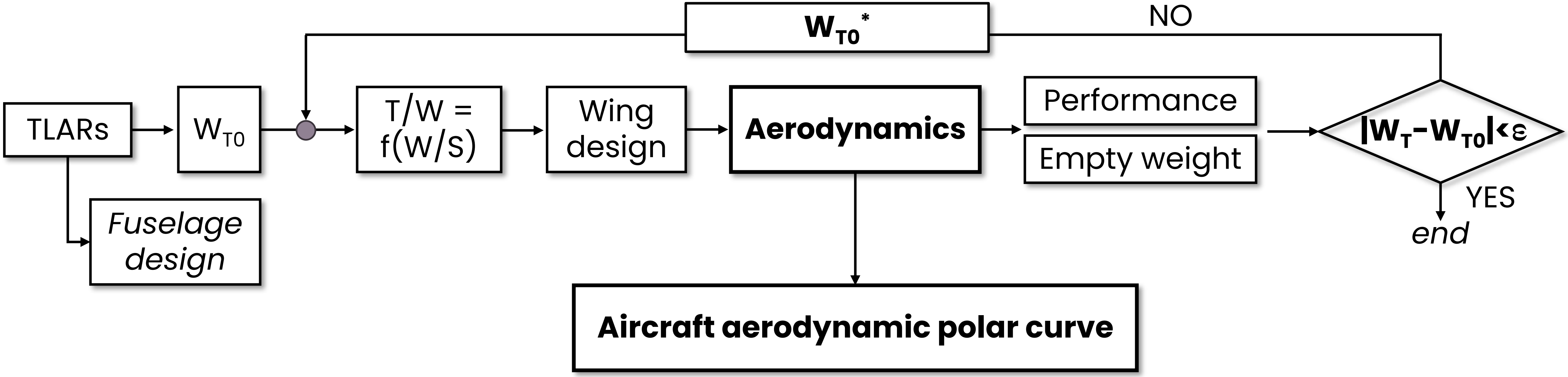


$$S_{wet\_fus} = 3.4 \cdot \frac{A_{top} + A_{side}}{2}$$

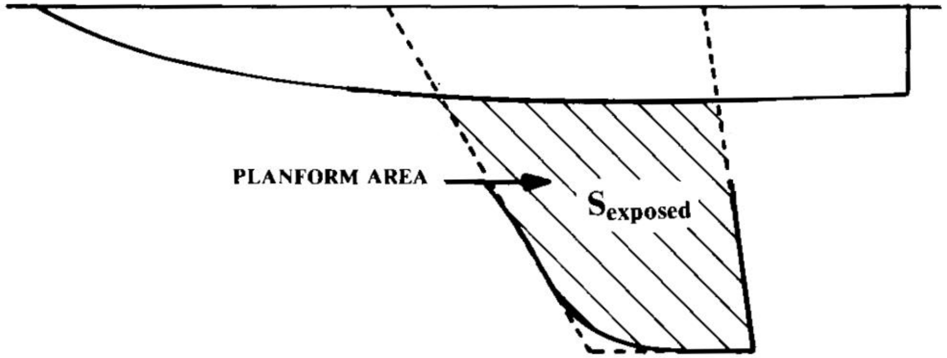
or

$$S_{wet}^{cyl} = k\pi\, d^{cyl} l^{cyl}$$

Aircraft Polar Curve

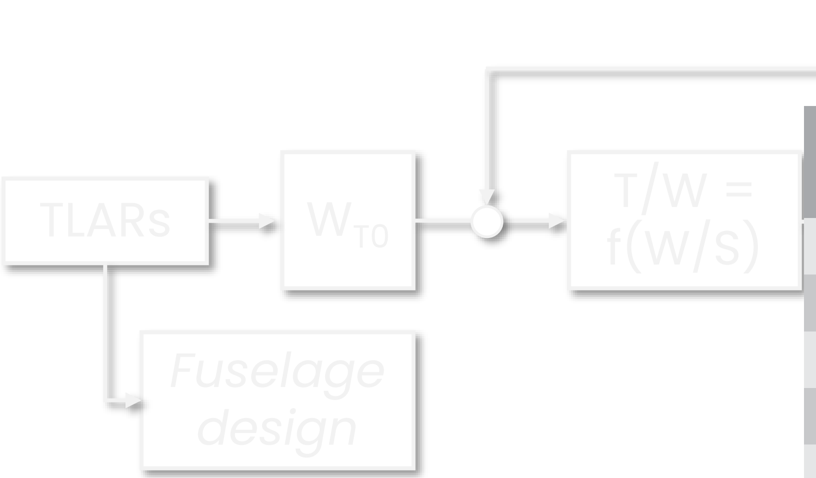


S_w Wing



$$S_{wet}^{Wing} = S_{exposed}\,[1.997 + 0.52\,(t/c)]$$

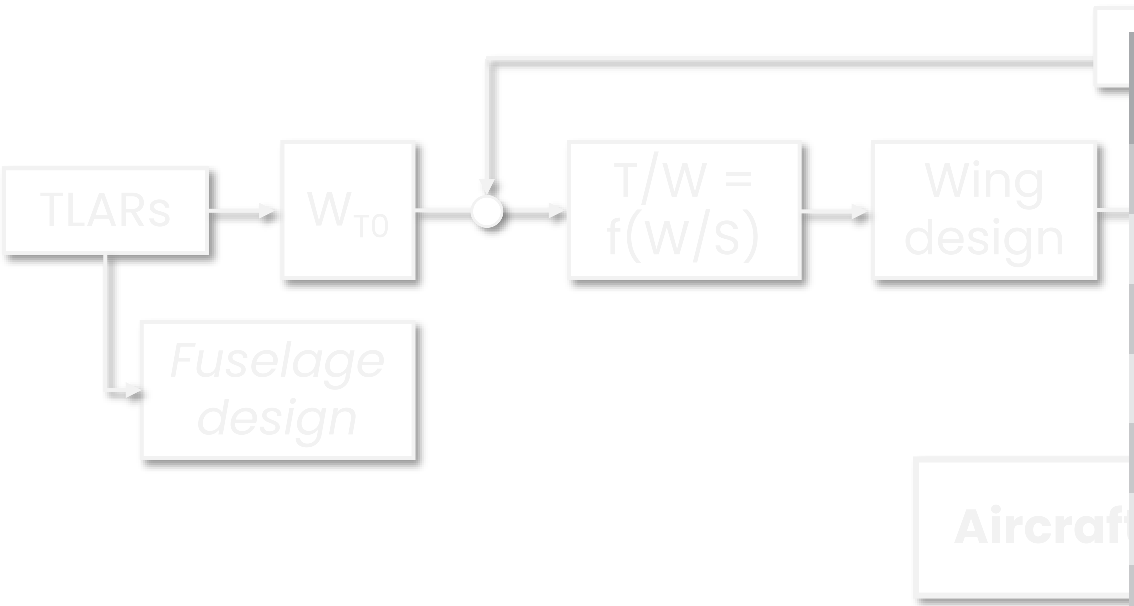
Aircraft Polar Curve



No.	Aircraft	m_{TO} (kg)	Tail type	$(t/C)_{max}$ (%)	S_h/S	AR_h	λ_h	Λ_h (deg)	i_h (deg) +	i_h (deg) –
1	Wright Flyer	420	Moving	Low	0.16	5.7	1	0	–	–
2	Cessna 177	1 100	Fixed	10.5	0.2	4	1	0	–	–
3	Cessna Citation I	5 375	Fixed	9	0.26	5.2	0.5	–	–	–
4	Beech Starship	6 759	Fixed	–	0.22	10.2	0.5	33	–	–
5	Fokker F-27	19 773	Fixed	14	0.23	6	0.4	0	–	–
6	Boeing 737-100	50 300	Adjustable	10.5	0.32	4.16	0.38	30	–	–
7	Boeing 707-320	151 320	Adjustable	11.6	0.216	3.37	0.42	35	0.5	14
8	Boeing 747-100	333 390	Adjustable	9	0.267	3.6	0.26	37	1	12
9	DC-8-10	141 000	Adjustable	8.75	0.203	4.04	0.33	35	2	10
10	Airbus 300B	165 000	Adjustable	–	0.27	4.13	0.5	32.5	3	12
11	Lockheed C-130 Hercules	70 305	Fixed	12	0.313	5.2	0.36	7.5	–	–
12	Lockheed L-1011	211 000	Adjustable	8	0.37	4	0.33	35	0	14
13	Lockheed C-5A	381 000	Adjustable	10	0.156	4.9	0.36	24.5	4	12
14	Eurofighter 2000	21 000	Movable	–	0.048	3.4	0.34	45	–	–
15	F-15 Eagle	36 741	Movable	–	0.183	2.3	0.36	48	–	–

Tailplanes
(horizontal)

Aircraft Polar Curve



No.	Aircraft	Type	$(t/C)_{max}$ (%)	S_v/S	AR_v	Λ_v (deg)
1	Wright Flyer	First aircraft in history	Low	0.045	6.3	0
2	Cessna 177	GA single-prop engine	7.5	0.107	1.41	35
3	C-130 Hercules	Large turboprop cargo	15	0.18	1.84	18.8
4	DC-9/10	Large jet transport	11	0.19	0.95	43.5
5	Cessna Citation I	Business jet	10	0.191	1.58	33
6	Fokker F-27	Turboprop transport	15	0.203	1.55	33
7	Boeing 737-100	Large jet transport	12	0.27	1.88	35
8	Beechjet 400A	Business jet transport	12	0.263	1	55
9	DC-8-10	Large jet transport	9.85	0.122	1.91	35
10	Airbus 300B	Large jet transport	12.5	0.204	1.623	40
11	C-17A	Heavy jet cargo	9	0.195	1.36	36
12	Eurofighter 2000	Fighter	7	0.096	1.3	45
13	F-15 Eagle	Fighter	7	0.346[a]	1.3	35

Tailplanes
(vertical)

Aircraft Polar Curve

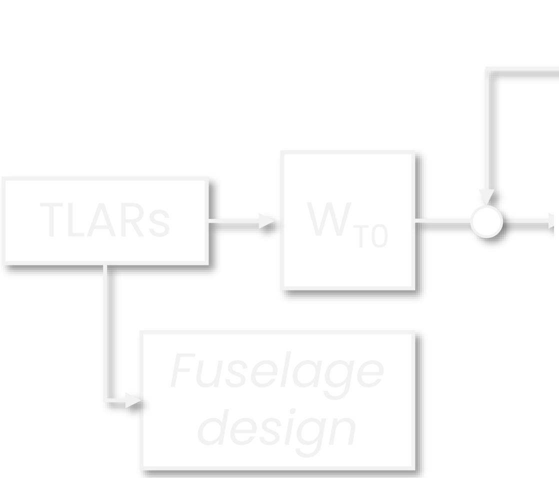


Table 6.1 Horizontal Tail Properties for a Range of Airliners[a]

Aircraft	Gross Weight W_g (lb)	Wing Area S (sq.ft.)	S_h/S	Horizontal Tail Span b_h (ft)	A_h	Taper λ_h	Sweep Λ_h (deg)
Dash 8 Q100	36,300	585	0.28	26.8	4.43	0.80	6.5
XAC MA60	48,050	810	0.22	31.0	5.49	0.44	20.0
ATR72-500	48,500	657	0.20	24.2	4.37	0.56	8.0
ERJ145LR	46,275	551	0.24	24.8	4.68	0.59	20.0
CRJ200LR	53,000	587	0.19	20.5	3.72	0.50	34.5
CRJ700ER	75,250	739	0.24	28.0	4.36	0.40	33.0
E175	85,517	783	0.32	32.8	4.29	0.50	35.0
B737-700ER	154,500	1341	0.33	47.1	5.04	0.33	36.0
A320-200	169,800	1320	0.26	40.9	4.84	0.31	32.5
A310-300	361,600	2360	0.30	53.2	3.99	0.44	37.0
B767-200ER	395,000	3050	0.28	61.1	4.42	0.30	36.0
B777-200	545,000	4605	0.25	70.6	4.40	0.33	39.5
A340-300	609,580	3890	0.20	63.5	5.12	0.40	32.5
B747-400	875,000	5650	0.26	72.8	3.57	0.24	42.5
A380-800	1,234,600	9104	0.24	99.7	4.45	0.37	37.5

[a] *Manufacturers: Airbus (A320, A310, A340, A380), Avions de Transport Regional (ATR72), Boeing (B737, B747, B767, B777), Bombardier (CRJ200, CRJ700, Dash8), Embraer (ERJ145, E175), Xian Aircraft Industry Co. (XAC MA60).*

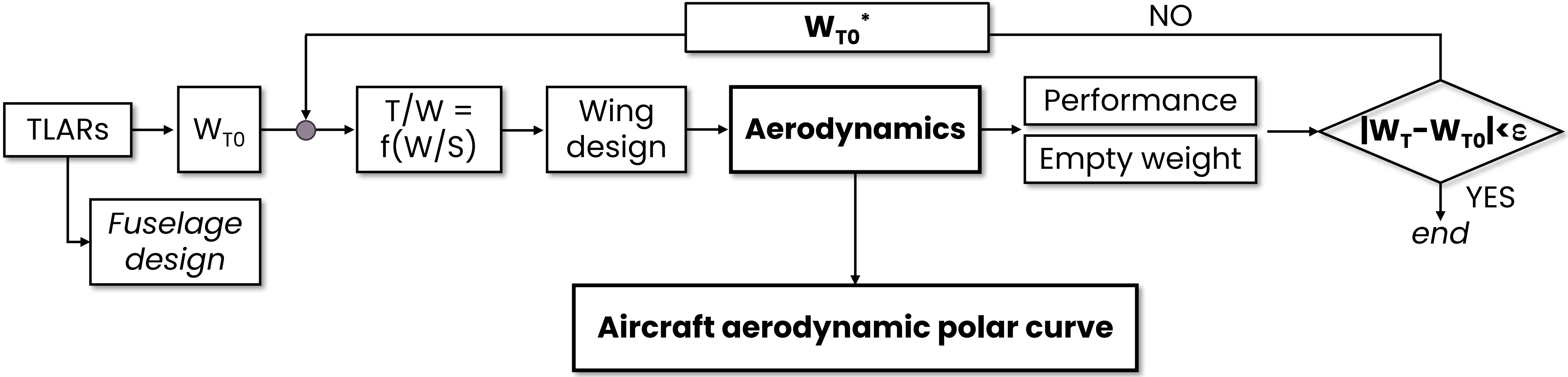
Table 6.2 Vertical Tail Properties for a Range of Airliners[a]

Aircraft	Gross Weight W_g (lb)	Wing Area S (sq.ft.)	S_v/S	Vertical Tail Height b_v	A_v	Taper λ_v	Sweep Λ_v (deg)
Dash 8 Q100	36,300	585	0.28	14.0	1.20	0.67	32.0
XAC MA60	48,050	810	0.18	15.2	1.57	0.40	27.0
ATR72-500	48,500	657	0.26	15.8	1.48	0.50	38.5
ERJ145LR	46,275	551	0.18	11.4	1.29	0.64	36.0
CRJ200LR	53,000	587	0.19	11.4	1.16	0.69	44.5
CRJ700ER	75,250	739	0.17	12.0	1.16	0.71	40.0
E175	85,517	783	0.29	20.5	1.84	0.30	40.0
B737-700ER	154,500	1341	0.22	26.8	2.45	0.20	37.0
A320-200	169,800	1320	0.24	23.8	1.77	0.30	39.0
A310-300	361,600	2360	0.26	32.3	1.70	0.34	44.0
B767-200ER	395,000	3050	0.21	33.5	1.77	0.33	45.0
B777-200	545,000	4605	0.18	40.7	2.03	0.25	45.0
A340-300	609,580	3890	0.16	33.6	1.81	0.34	44.0
B747-400	875,000	5650	0.19	38.3	1.39	0.31	49.5
A380-800	1,234,600	9104	0.17	52.6	1.76	0.37	44.0

[a] *Manufacturers: Airbus (A320, A310, A340, A380), Avions de Transport Regional (ATR72), Boeing (B737, B747, B767, B777), Bombardier (CRJ200, CRJ700, Dash8), Embraer (ERJ145, E175), Xian Aircraft Industry Co. (XAC MA60).*

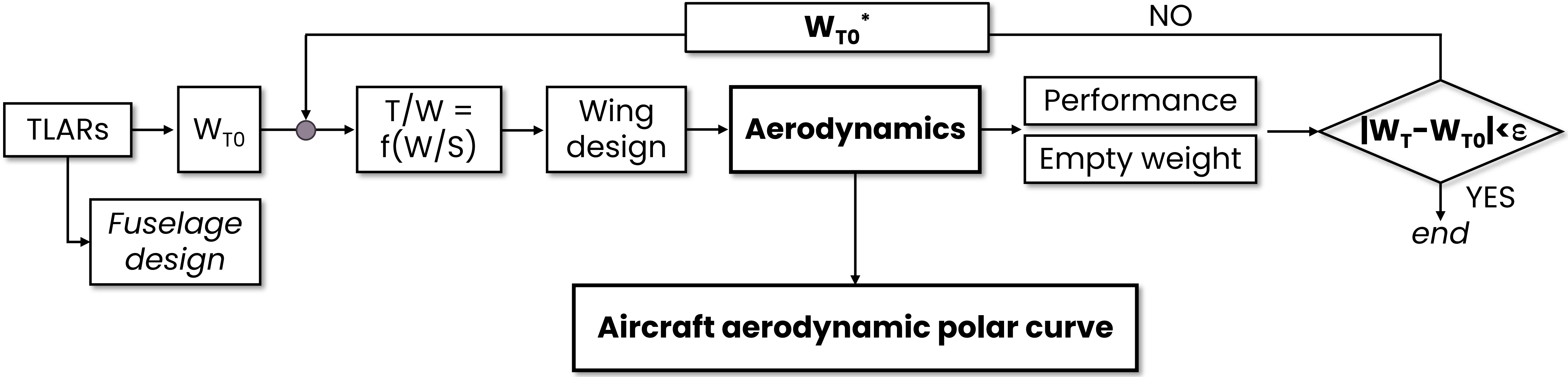
Tailplanes

Aircraft Polar Curve



S_w Tailplanes → *You can repeat the same procedure done for the main wing*

Aircraft Polar Curve



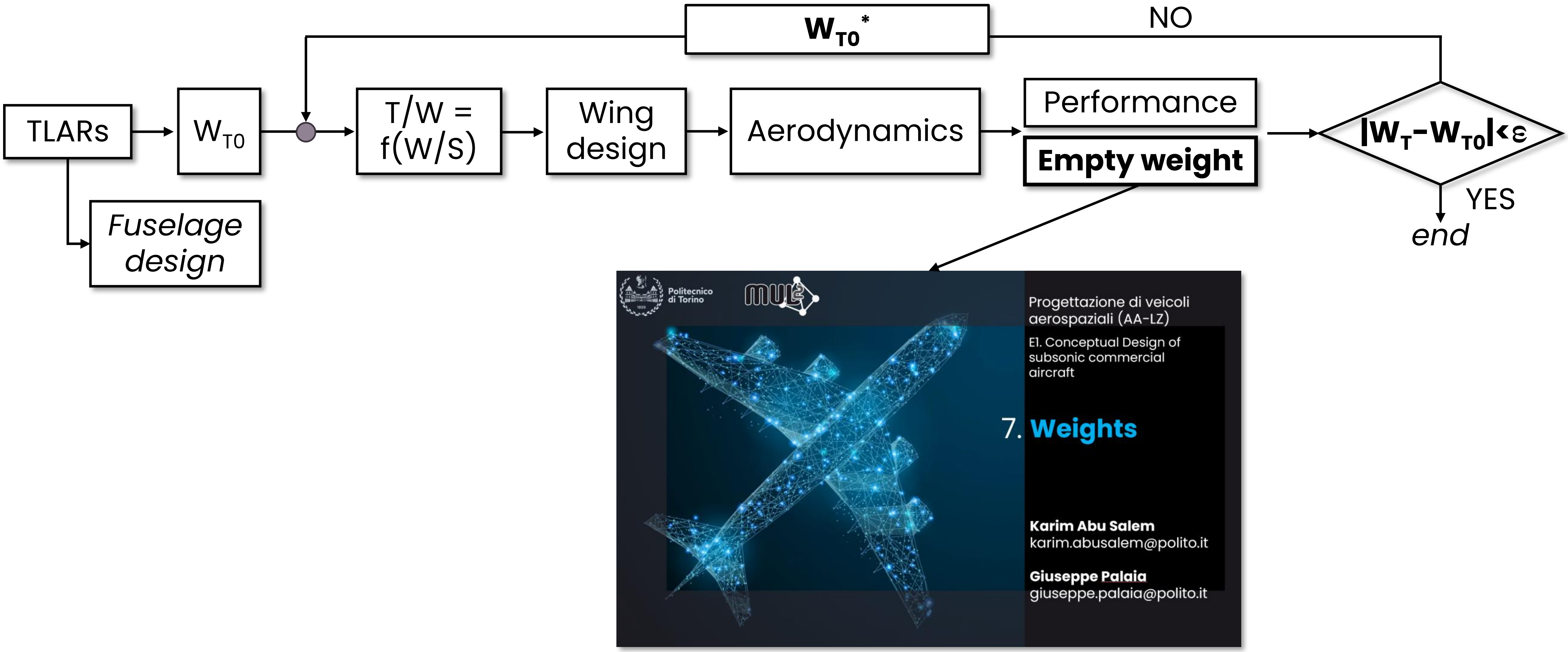
S_w Nacelle

$$D_{nac} = 0.04\sqrt{T_{eng}}$$

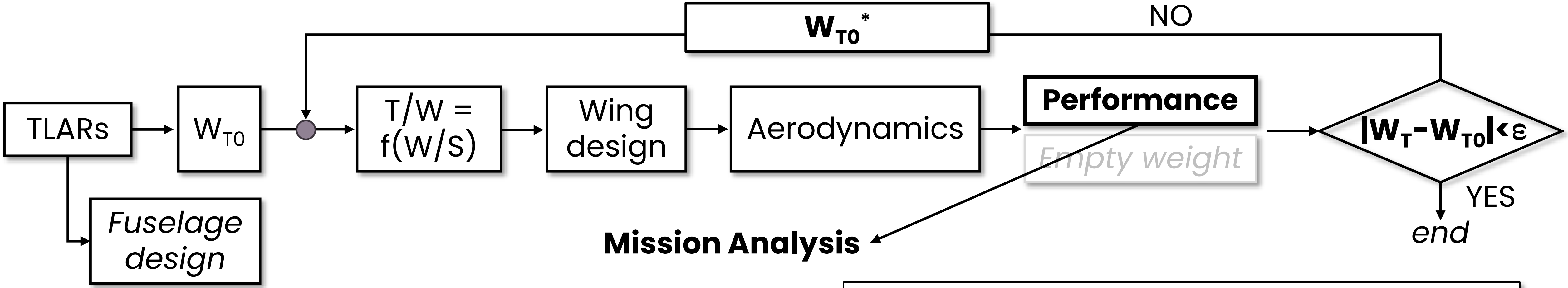
$$L_{nac} = 0.07\sqrt{T_{eng}}$$

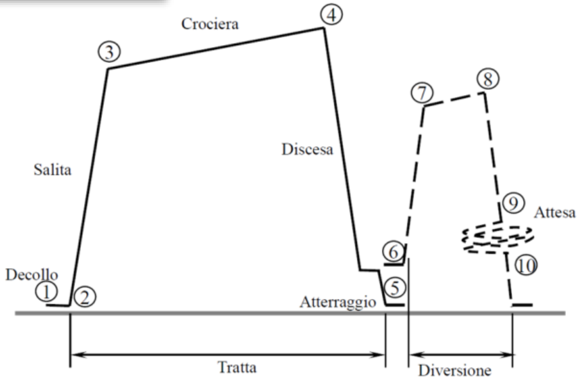
$$S_{wet\ nac} = \pi\, D_{nac}\, L_{nac}$$

Weights Update



Performance Assessment





$$\left(\frac{W_2^{(i)}}{W_{TO}^{(i)}}\right)=0.970 \qquad \left(\frac{W_7^{(i)}}{W_6^{(i)}}\right)\cong 1$$

$$\left(\frac{W_3^{(i)}}{W_2^{(i)}}\right)=0.985 \qquad \left(\frac{W_8^{(i)}}{W_7^{(i)}}\right)=?$$

$$\left(\frac{W_4^{(i)}}{W_3^{(i)}}\right)=? \qquad \left(\frac{W_9^{(i)}}{W_8^{(i)}}\right)\cong 1$$

$$\left(\frac{W_5^{(i)}}{W_4^{(i)}}\right)=0.995 \qquad \left(\frac{W_{10}^{(i)}}{W_9^{(i)}}\right)=?$$

$$W_{TO}^{(i)} \longrightarrow \boxed{\begin{aligned}
&\left(\frac{W_F^{(i)}}{W_{TO}^{(i)}}\right)=\left(\frac{W_{F-MIS}^{(i)}}{W_{TO}^{(i)}}\right)+\left(\frac{W_{F-RIS}^{(i)}}{W_{TO}^{(i)}}\right)\\
&\left(\frac{W_{F-MIS}^{(i)}}{W_{TO}^{(i)}}\right)=1-\left(\frac{W_2^{(i)}}{W_{TO}^{(i)}}\right)\left(\frac{W_3^{(i)}}{W_2^{(i)}}\right)\left(\frac{W_4^{(i)}}{W_3^{(i)}}\right)\left(\frac{W_5^{(i)}}{W_4^{(i)}}\right)\\
&\left(\frac{W_{F-RIS}^{(i)}}{W_{TO}^{(i)}}\right)=0.05\frac{W_{F-MIS}^{(i)}}{W_{TO}^{(i)}}+\left(1-\frac{W_{F-MIS}^{(i)}}{W_{TO}^{(i)}}\right)\left[1-\left(\frac{W_7^{(i)}}{W_6^{(i)}}\right)\left(\frac{W_8^{(i)}}{W_7^{(i)}}\right)\left(\frac{W_9^{(i)}}{W_8^{(i)}}\right)\left(\frac{W_{10}^{(i)}}{W_9^{(i)}}\right)\right]
\end{aligned}} \longrightarrow \left(\frac{W_F^{(i)}}{W_{TO}^{(i)}}\right)$$

$$\frac{dx}{dW}=-\frac{V}{c\cdot T} \xrightarrow{T=D \quad L=W} \frac{dx}{dW}=-\frac{V\cdot L}{c\cdot D\cdot L}=-\frac{V\cdot E}{c\cdot W}$$

$$\boxed{\frac{dW}{W}=-\frac{c\cdot dx}{V\cdot E}}$$

Ipotesi:

$V =$ Costante Assetto Costante $\Longrightarrow$ $E =$ Costante

$$\boxed{\frac{W_{i+1}}{W_i}=e^{-\frac{c\cdot\Delta x_i}{V\cdot E}}} \qquad \boxed{\frac{W_{i+1}}{W_i}=e^{-\frac{C\,\Delta t}{E}}}$$

Autonomia chilometrica Autonomia oraria

Configurations Matrix

Summary of the set of configurations designed

Main configuration features should be inserted in the configuration matrix, e.g.:

Geo: S_{ref}, b, c_r, c_t, Λ, λ, AR, t/c, W/S, $S_{wet\ wing}$, Vol_{tank}, *and others*

Perfo: E_{cruise}, M, m_{fuel}, C_{D0}, C_{Dw}, C_{Di}, C_{dtot}, e, C_{Ltrim}, C_{Lmax}, T/W, T_{max}, *and others*

Weights: MTOW, W_{eo}, W_{wing}, *and others*

Geometry						**Weights**						**Performance**						**FoMs**					
x_1	x_2	x_3	…	…	x_n	w_1	w_2	w_3	…	…	w_n	y_1	y_2	y_3	…	…	y_n	f_1	f_2	f_3	…	…	f_n
x.xx	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*
x.xx	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*
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x.xx	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*	*x.xx*

Task 2.1

Provisional Task: Start to build the design convergence loop and test the sub-parts of the code

The task will be assigned after the lesson focusing on weights breakdown assessment

Extra content

Back-up content for class open discussion

Title