



Politecnico
di Torino



Progettazione di veicoli
aerospaziali (AA-LZ)

E2. Conceptual Design of hybrid-
electric aircraft

LAB3. HEA
Performance
(Euler integration)

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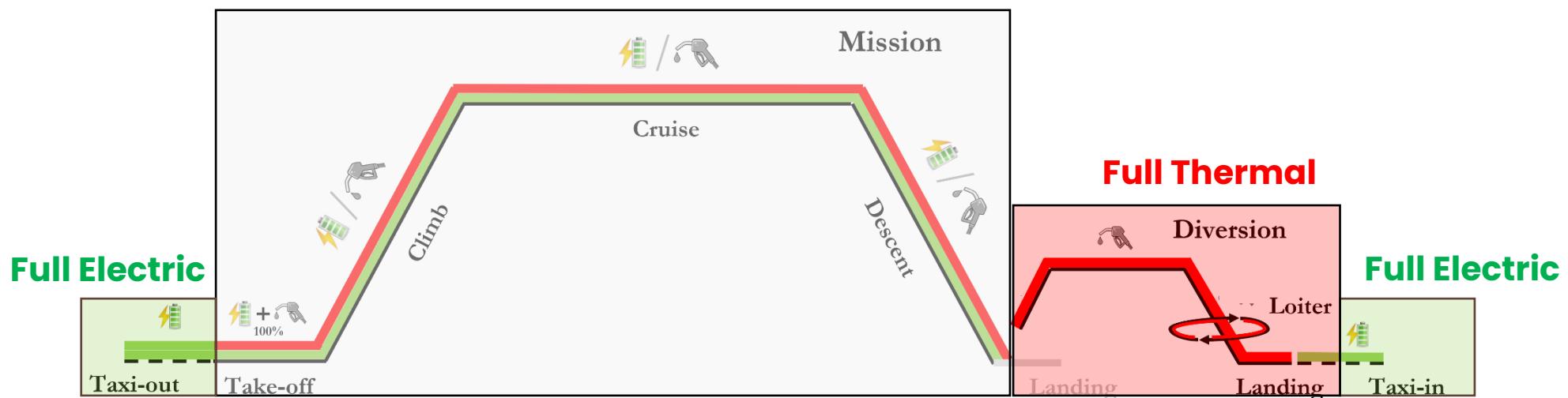
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Analisi di Missione: power management

| | | Thermal Power Fraction | Electric Power Fraction |
|-----------|------------------------|--|---|
| Mission | Taxi-out/in | $\Phi^{\text{ice}}(t) = 0$ | $\Phi^{\text{el}}(t) = 0.07 P_{\text{inst}}^{\text{tot}}$ |
| | Take-off | $\Phi^{\text{ice}}(t) = 1$ | $\Phi^{\text{el}}(t) = 1$ |
| | Climb | $\Phi^{\text{ice}}(t) = \text{const.}$ | $\Phi^{\text{el}}(t) = f(\Phi(t), \Phi_{\text{climb opt}}^{\text{ice}})$ |
| | Cruise | $\Phi^{\text{ice}}(t) = \text{const.}$ | $\Phi^{\text{el}}(t) = f(\Phi(t), \Phi_{\text{cruise opt}}^{\text{ice}})$ |
| | Descent | $\Phi^{\text{ice}}(t) = \text{const.}$ | $\Phi^{\text{el}}(t) = f(\Phi(t), \Phi_{\text{desc opt}}^{\text{ice}})$ |
| Diversion | Climb _{div} | $\Phi^{\text{ice}}(t) = \Phi(t)$ | $\Phi^{\text{el}}(t) = 0$ |
| | Cruise _{div} | $\Phi^{\text{ice}}(t) = \Phi(t)$ | $\Phi^{\text{el}}(t) = 0$ |
| | Descent _{div} | $\Phi^{\text{ice}}(t) = \Phi(t)$ | $\Phi^{\text{el}}(t) = 0$ |

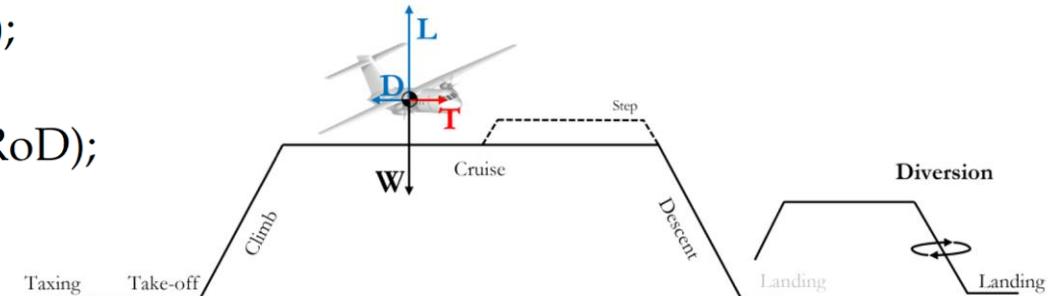
Hybrid power supply





Analisi di Missione: programmi di volo

- Taxi-out: ground manoeuvring with constant power supply for 240 s;
- Take-off: full-power supply for 45 s;
- Climb: constant indicated air speed (IAS) and rate of climb (RoC);
- Cruise: constant speed and altitude;
- Descent: constant indicated air speed (IAS) and rate of descent (RoD);
- Loiter: 30 min of level flight at maximum L/D;
- Approach: constant RoD;
- Landing: neglected;
- Taxi-in: ground manoeuvring with constant power supply for 240 s.

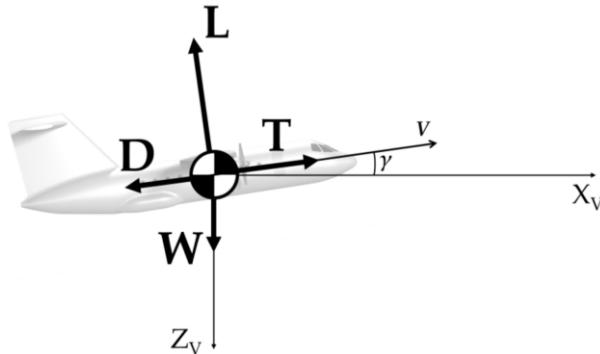


| Mission | | | Diversion | |
|----------------|--------------|-------------------|--------------|-------------------|
| Climb | IAS = 170 kt | RoC = 900 ft/min | IAS = 150 kt | RoC = 600 ft/min |
| Cruise | Mach = 0.4 | h = 6100 m | Mach = 0.27 | h = 3050 m |
| Descent | IAS = 220 kt | RoD = -1100ft/min | IAS = 150 kt | RoD = -1100ft/min |



Simulazione missione

- 1. Velivolo punto materiale**
- 2. Volo nel piano verticale**
- 3. Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

+condizioni iniziali

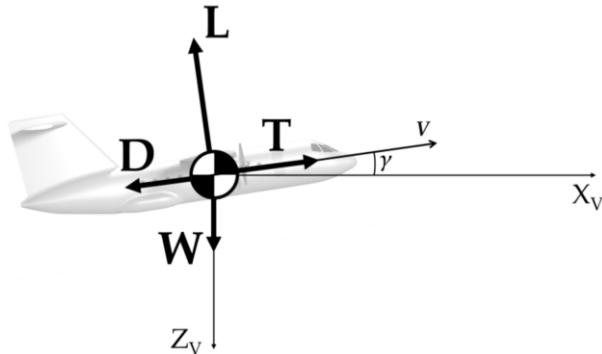
$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \end{array} \right.$$

$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$



Simulazione missione

- Velivolo punto materiale**
- Volo nel piano verticale**
- Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right. + \text{condizioni iniziali}$$

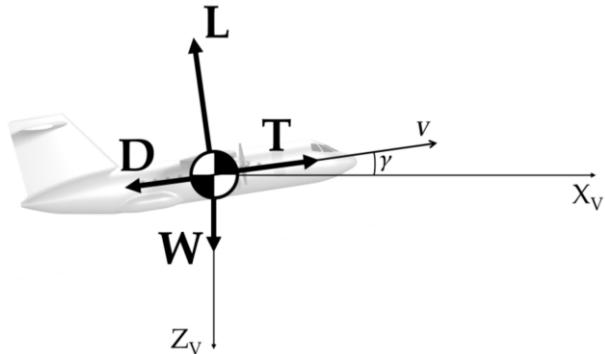
$$\left\{ \begin{array}{l} P_{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P_{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \\ \Phi^{ice}(t) = \frac{P_{ice}(t)}{P_{ice,inst}} \quad \Phi^{el}(t) = \frac{P_{emot}(t)}{P_{emot,inst}} \end{array} \right.$$

$$P_{fly} = TV$$



Simulazione missione

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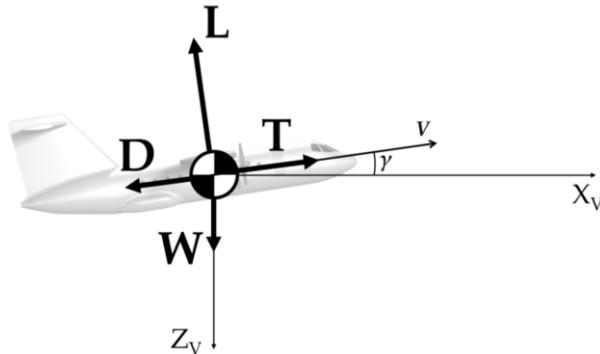
$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \\ \Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P_{emot}(t)}{P_{inst}^{emot}} \end{array} \right.$$

$$W(t + dt) = W(t) + \dot{W}(t) dt$$



Simulazione missione

- 1. Velivolo punto materiale**
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- 3. Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

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$$\Phi^{ice}(t) = \frac{P_{ice}(t)}{P_{ice,inst}} \quad \Phi^{el}(t) = \frac{P_{emot}(t)}{P_{emot,inst}}$$

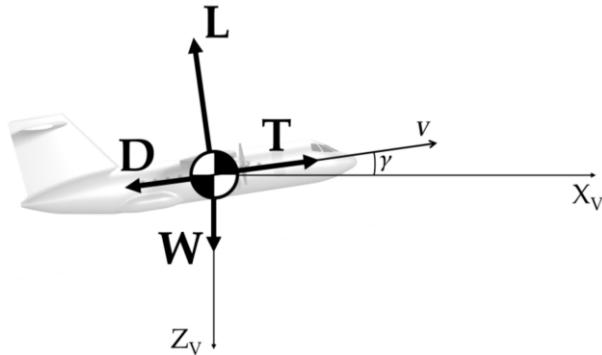
Se assegniamo un valore noto a Φ^{ice}

$$W(t + dt) = W(t) + \boxed{\dot{W}(t)} dt$$



Simulazione missione

1. **Velivolo punto materiale**
2. **Volo nel piano verticale**
3. **Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right. + \text{condizioni iniziali}$$
$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \end{array} \right.$$
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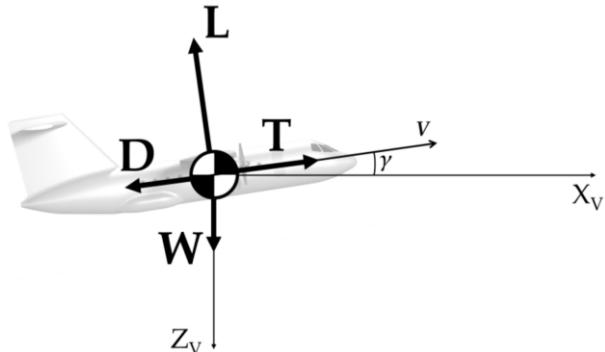
Se assegniamo un valore noto a Φ^{el}

$$W(t + dt) = W(t) + \boxed{\dot{W}(t)} dt$$



Simulazione missione

- 1. Velivolo punto materiale**
- 2. Volo nel piano verticale**
- 3. Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

+condizioni iniziali
 $W(t_0) = W_0$

$$W(t_1) = W(t_0) + \dot{W}(t_0) dt$$

$$W(t_2) = W(t_1) + \dot{W}(t_1) dt$$

... ...

$$W(t_{k+1}) = W(t_k) + \dot{W}(t_k) dt$$

$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P^{nec} \end{array} \right.$$

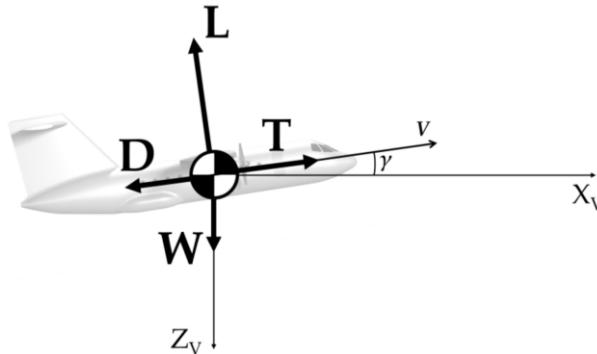
$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$

$$\dot{W}(t_k) = -k_C P^{ice}(t_k)$$



Simulazione missione - Crociera

- 1. Velivolo punto materiale**
- 2. Volo nel piano verticale**
- 3. Volo quasi-stazionario**



$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right. + \text{condizioni iniziali}$$

$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \\ \Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P_{emot}(t)}{P_{inst}^{emot}} \end{array} \right.$$

Crociera

$$\gamma = 0$$

$$\dot{W}(t_k) = -k_C P_{ice}(t_k)$$

Variazione peso

$$\dot{x}(t_k) = V_x(t_k)$$

Traiettoria

$$\dot{z}(t_k) = V_z(t_k)$$

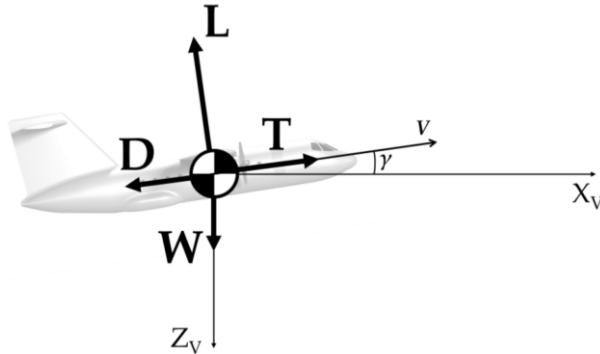
$$\dot{E}_b(t_k) = P_b(t_k)$$

Variazione di carica delle batterie



Simulazione missione - Crociera

- 1. Velivolo punto materiale**
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$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

+condizioni iniziali

$$\left\{ \begin{array}{l} P^{fly} = DV + VW \sin \gamma \\ P_{ice} + P_{emot} = \frac{P^{fly}}{(\eta_{gear} \eta_{prop})} = P_{nec} \end{array} \right.$$

$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P_{emot}(t)}{P_{inst}^{emot}}$$

Crociera

$$\gamma = 0$$

$$\dot{W}(t_k) = -k_C \textcolor{red}{P_{ice}}(t_k)$$

$$\textcolor{red}{P_{ice}} = f(\Phi^{ice})$$

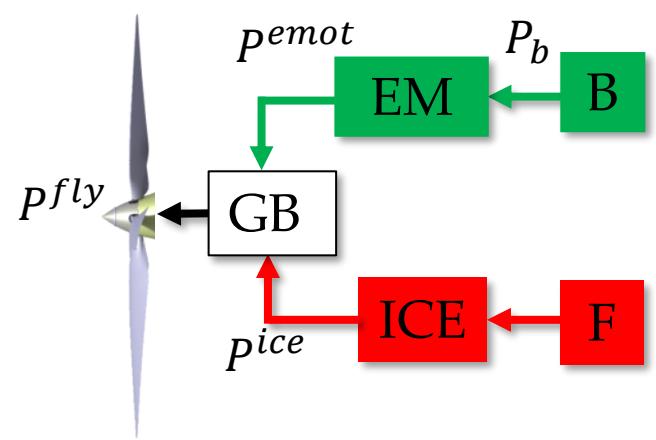
$$\textcolor{green}{P_b} = \textcolor{green}{P_{emot}} / (\eta_{em})$$

$$\dot{E}_b(t_k) = \textcolor{green}{P_b}(t_k)$$

$$\eta_{em} = 0,95$$

$$\eta_{gear} = 0,98$$

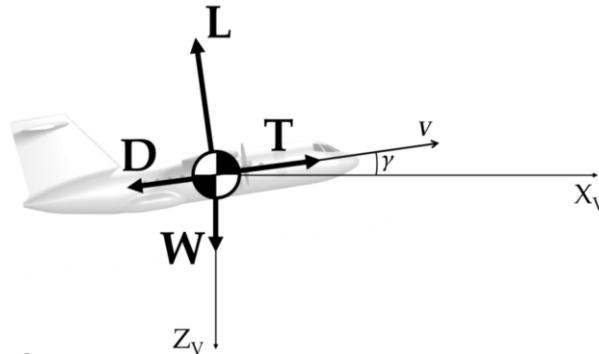
$$\eta_{prop} = 0,85$$





Simulazione missione - Salita

- 1. Velivolo punto materiale**
- 2. Volo nel piano verticale**
- 3. Volo quasi-stazionario**



Salita

$$\gamma = \arctan(V_z/V_x)$$

$$V_z = R\omega C$$

$$\rho = f(h)$$

$$V = f(IAS)$$

$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

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$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$

$$\dot{W}(t_k) = -k_C P_{ice}(t_k)$$

Variazione peso

$$\dot{x}(t_k) = V_x(t_k)$$

Traiettoria

$$\dot{z}(t_k) = V_z(t_k)$$

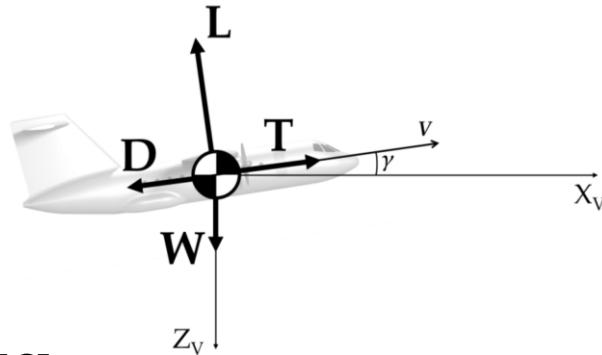
$$\dot{E}_b(t_k) = P_b(t_k)$$

Variazione di carica delle batterie



Simulazione missione - Salita

- 1. Velivolo punto materiale**
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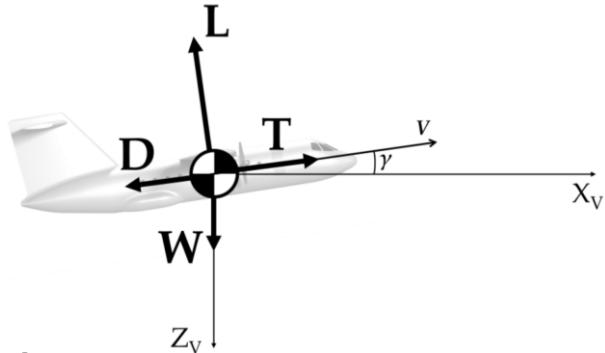
$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$





Simulazione missione - Salita

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- 2. Volo nel piano verticale**
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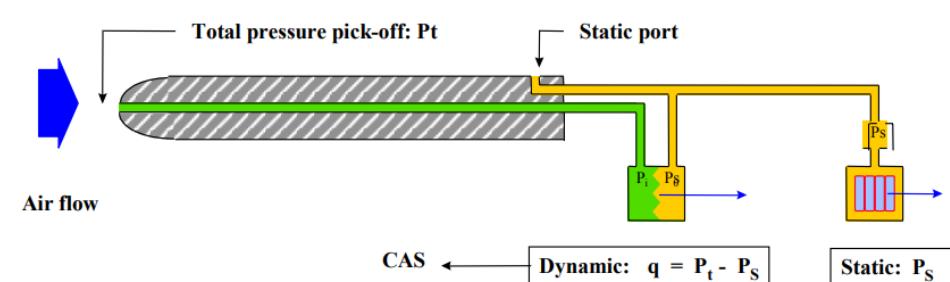
$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} \dot{V} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

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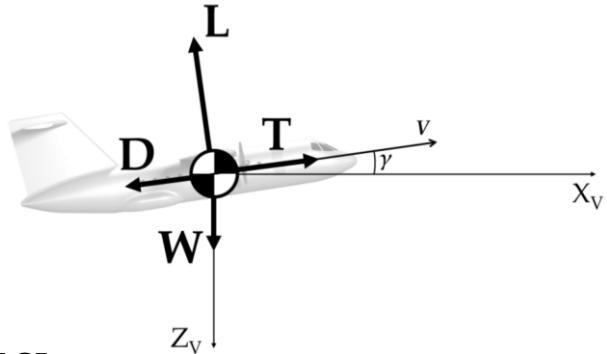
La velocità dell'aria calibrata (CAS) si ottiene dalla differenza tra la pressione totale (P_t) e la pressione statica (P_s). Questa differenza è chiamata pressione dinamica (q).



Simulazione missione - Salita



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3. **Volo quasi-stazionario**



Salita

$$\gamma = \arctan(V_z/V_x)$$

$$V_z = R\dot{\gamma}C$$

$$\rho = f(h)$$

$$V = f(IAS)$$

$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

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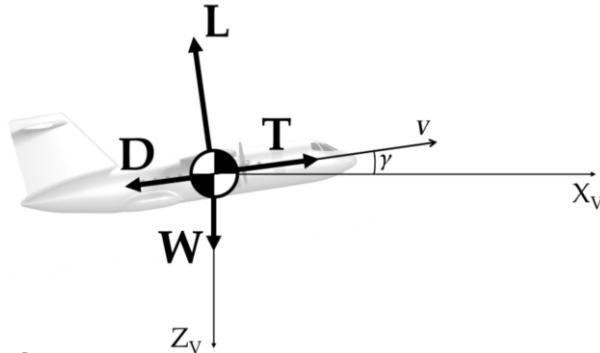


La velocità dell'aria indicata (IAS) è la velocità indicata dall'**indicatore di velocità** dell'aria. Tuttavia, a seconda dell'angolo d'attacco dell'aeromobile, della configurazione dei flap, della vicinanza al suolo (effetto suolo o meno), della direzione del vento e di altri **parametri** vengono introdotti alcuni **errori** di misura, principalmente sulla **pressione statica**. Ciò comporta una piccola differenza tra i valori **CAS** e **IAS**.



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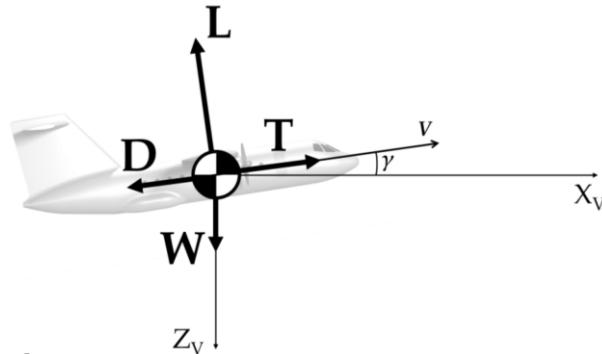


Successivamente, si **corregge** la IAS per gli effetti della **compressibilità** dell'aria



Simulazione missione - Salita

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$$V_z = R\dot{\theta}C$$

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$$V = f(IAS)$$

$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} \dot{V} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

+condizioni iniziali

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$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$



$\gamma_{air} = 1.4$ Ideal diatomic gas value

$p_t = p + \frac{1}{2} \rho_0 IAS^2$ [Pa] Total pressure read by the IAS anemometer

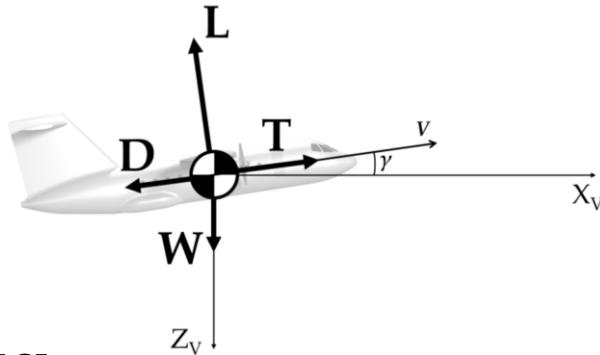
$$EAS = \sqrt{2 \frac{a^2}{\gamma_{air}-1} \left(\frac{\rho}{\rho_0} \right) \left(\left(\frac{p_t}{p} \right)^{\frac{\gamma_{air}-1}{\gamma_{air}}} - 1 \right)}$$

[m/s]



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Salita

$$\gamma = \arctan(V_z/V_x)$$

$$V_z = R\dot{\gamma}C$$

$$\rho = f(h)$$

$$V = f(IAS)$$

$$\left\{ \begin{array}{l} \frac{W}{g} \dot{V} = T - D - W \sin \gamma \\ \frac{W}{g} V \dot{\gamma} = L - W \cos \gamma \\ V_x = V \cos \gamma \\ V_z = -V \sin \gamma \\ \dot{W} = -k_C P_{ice} \end{array} \right.$$

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$$\Phi^{ice}(t) = \frac{P^{ice}(t)}{P_{inst}^{ice}} \quad \Phi^{el}(t) = \frac{P^{emot}(t)}{P_{inst}^{emot}}$$



Infine, si corregge l'EAS per tenere conto della variazione della densità dell'aria

$$TAS = EAS \sqrt{\frac{\rho_0}{\rho}}$$



End