Progettazione di veicoli aerospaziali (AA-LZ)

E1. Conceptual Design of subsonic commercial aircraft

4. Matching chart

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A brief recap

Tentative m_{TO}

$$m_{payload} = m_{pax} N_{seats} + m_{baggage} N_{seats} + m_{cargo}$$

$$\frac{m_{fuel}}{m_{TO}} = 1.05\left(1 - \frac{m_{final}}{m_{TO}}\right)$$

$$\frac{m_e}{m_{TO}} = A\, m_{TO}^{C}$$

$$\boldsymbol{m_{TO}} = \frac{m_{crew} + m_{payload}}{1 - \left(\frac{m_{fuel}}{m_{TO}}\right) - \left(\frac{m_{empty}}{m_{TO}}\right)}$$

If $|m_{TO} - \boldsymbol{m_{TO}}| > \text{toll}$

If $|m_{TO} - \boldsymbol{m_{TO}}| < \text{toll}$

$\boldsymbol{m_{TO}}$

Wing Surface and Engine Sizing

In the first step of the aircraft preliminary design phase, the aircraft's most influential parameter, i.e. MTOM, is determined. The second crucial step is the preliminary estimation of wing surface area $\boldsymbol{S_{ref}}$ and engine thrust $\boldsymbol{T}$. In particular, the following parameters shall be estimated:

Wing Loading $\frac{W}{S}$

Thrust-to-Weight ratio $\frac{T}{W}$

Unlike the procedure followed for MTOM estimation, in this case the methodology does not make use of statistic assessment, but it requires a more detailed investigation of aircraft performance requirements and it employs flight mechanics theories.

Set of aircraft performance requirements to be used at this stage

- Landing and take-off
- Climb segments (regulation)
- Cruise speed (from TLARs)

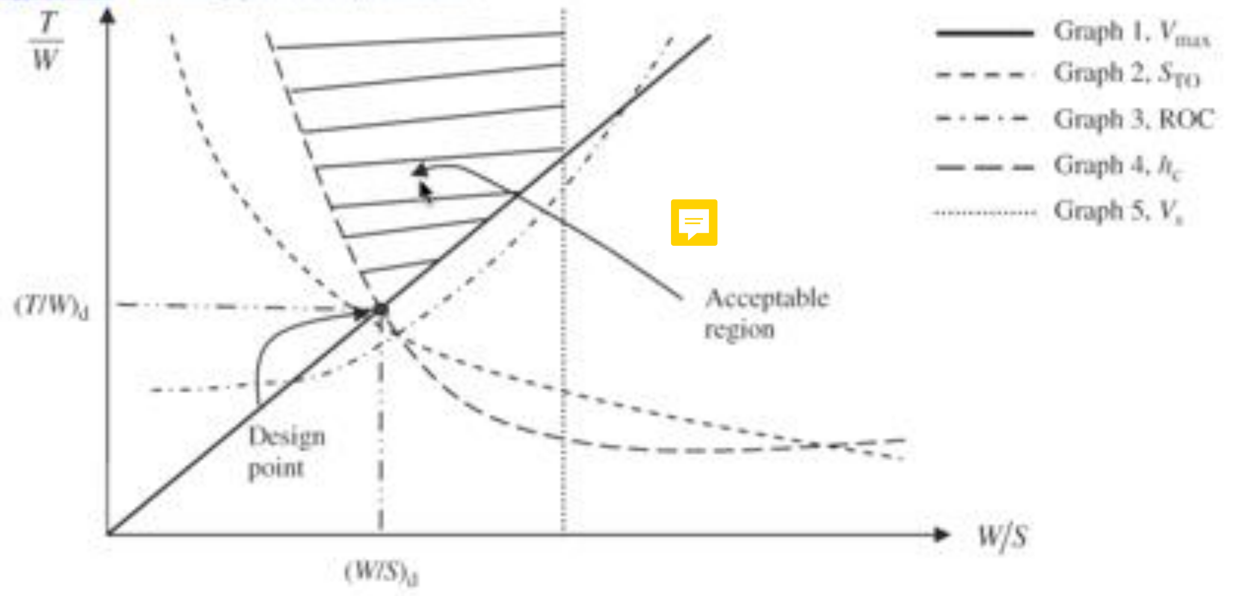
[1] Raymer, Daniel. Aircraft design: a conceptual approach. American Institute of Aeronautics and Astronautics, Inc., 2012.
[2] Sadraey, Mohammad H. Aircraft design: A systems engineering approach. John Wiley & Sons, 2012.

Wing Surface and Engine Sizing

A 4-steps procedure can be followed:

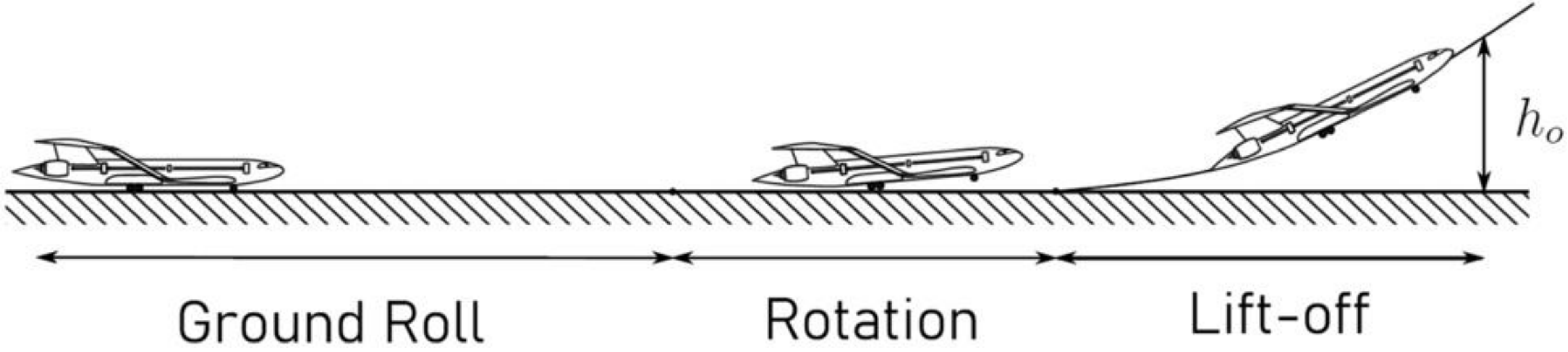
1. Derive equations for each performance requirement, trying to express them as a f$(\frac{W}{S}, \frac{T}{S})$
2. Rewrite equations in the form $\frac{T}{S} = f(Req, W/S)$ and graphically represent the curves in a single plot (aka Matching Chart) with wing loading as horizontal axis and thrust-to-weight ratio as vertical axis.
3. Identify the acceptable region of the diagram
4. Identify the Optimum Design Point inside the acceptable region and evaluate T and S (W is known)

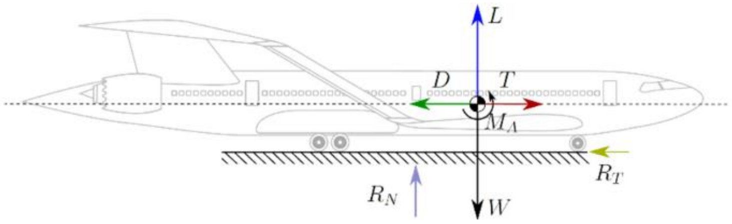
Figure 4.8 Matching plot for a jet aircraft

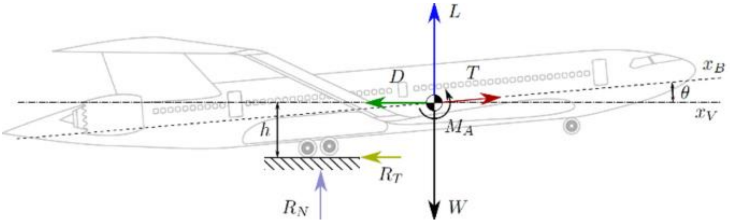


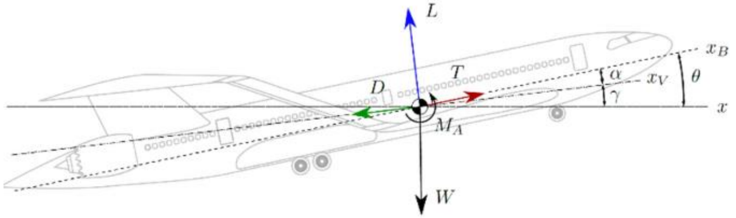
[1] Raymer, Daniel. Aircraft design: a conceptual approach. American Institute of Aeronautics and Astronautics, Inc., 2012.
[2] Sadraey, Mohammad H. Aircraft design: A systems engineering approach. John Wiley & Sons, 2012.

Take-off (3 DOF dynamics)







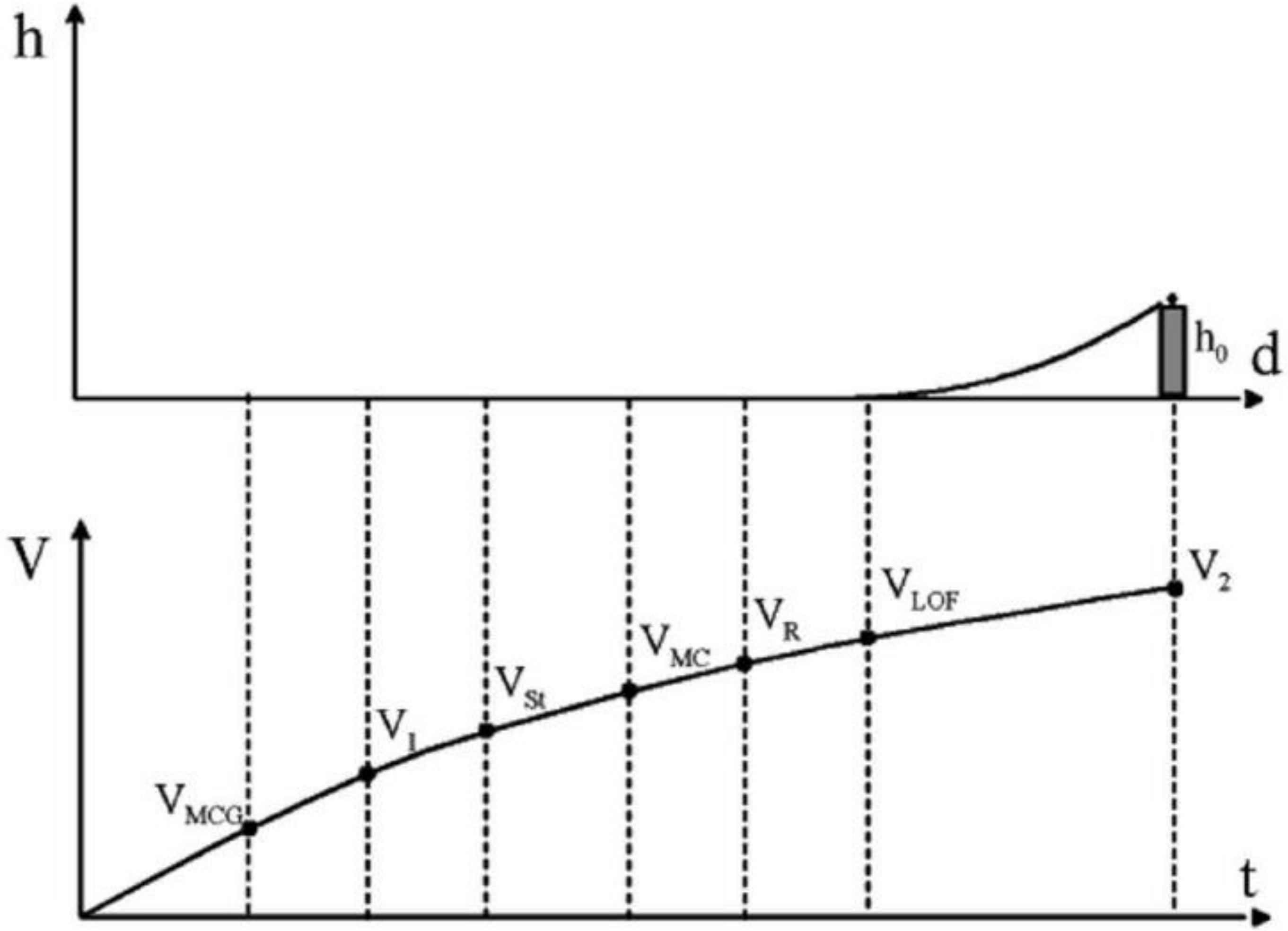


$$
\begin{cases}
\dfrac{W}{g}\dfrac{\mathrm{d}V}{\mathrm{d}t} = T - D - R_{\mathrm{T}} \\
R_{\mathrm{N}} + L = W \\
R_{\mathrm{T}} = \mu R_{\mathrm{N}}
\end{cases}
$$

$$
\begin{cases}
\dfrac{W}{g}\dfrac{\mathrm{d}V}{\mathrm{d}t} = T\cos\theta - D - R_{\mathrm{T}}\,\dfrac{W}{g}\dfrac{\mathrm{d}V}{\mathrm{d}t} = T\cos\theta - D - R_{\mathrm{T}} \\
I_{\mathrm{y}}\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = M_{\mathrm{A}} - R_{\mathrm{N}}\,d - R_{\mathrm{T}}\,h \\
R_{\mathrm{N}} + L + T\sin\theta = W \\
R_{\mathrm{T}} = \mu R_{\mathrm{N}}
\end{cases}
$$

$$
\begin{cases}
\dfrac{W}{g}\dfrac{\mathrm{d}V_x}{\mathrm{d}t} = T\cos\theta - D\cos\gamma - L\sin\gamma \\
\dfrac{W}{g}\dfrac{\mathrm{d}V_z}{\mathrm{d}t} = T\sin\theta - D\sin\gamma + L\cos\gamma - W \\
I_{\mathrm{y}}\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = M_A.
\end{cases}
$$

Take-off (regulations)



[1] Airbus. Getting to grips with aircraft performance

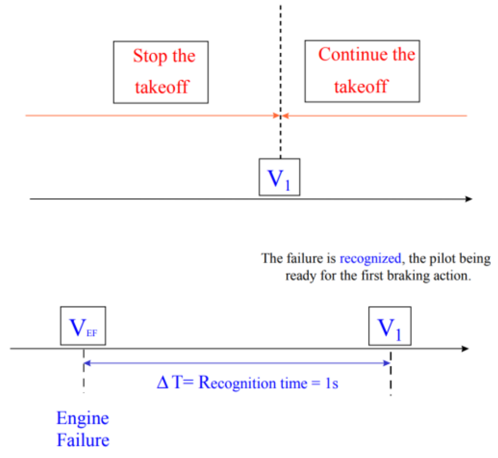
JAR 25.149 Subpart B — *FAR 25.149 Subpart B*

"JAR/FAR 25.149 Minimum control speed
(e) V_{MCG}, the minimum control speed on the ground, is the calibrated airspeed during the take-off run, at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the aeroplane with the use of the primary aerodynamic controls alone (without the use of nose-wheel steering) to enable the take-off to be safely continued using normal piloting skill.

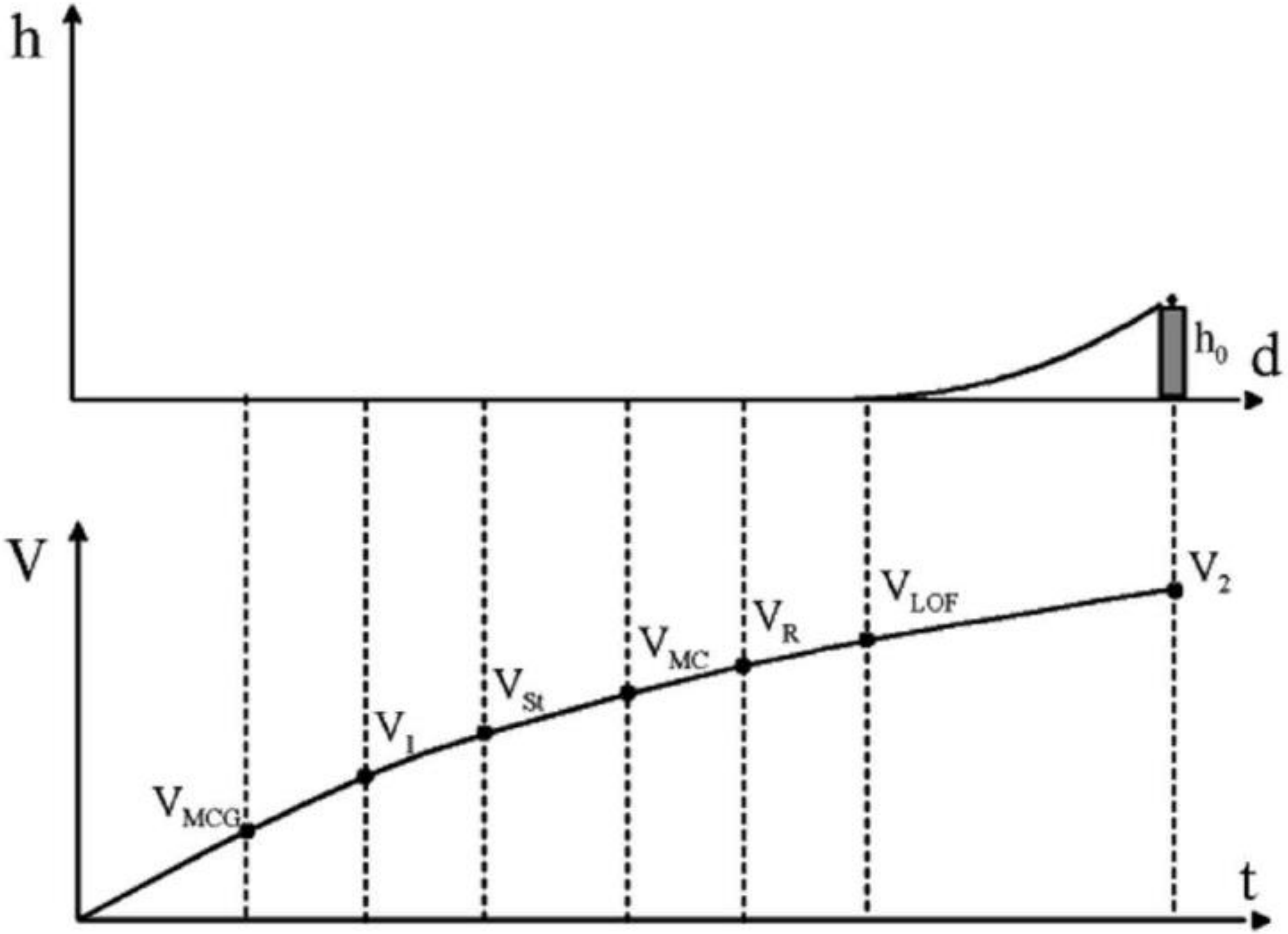
JAR 25.107 Subpart B — *FAR 25.107 Subpart B*

V_1 is the maximum speed at which the crew can decide to reject the takeoff, and is ensured to stop the aircraft within the limits of the runway.

"JAR/FAR 25.107
(a)(2) V_1, in terms of calibrated airspeed, is selected by the applicant; however, V_1 may not be less than V_{EF} plus the speed gained with the critical engine inoperative during the time interval between the instant at which the critical engine is failed, and the instant at which the pilot recognises and reacts to the engine failure, as indicated by the pilot's initiation of the first action (e.g. applying brakes, reducing thrust, deploying speed brakes) to stop the aeroplane during accelerate-stop tests."



Take-off (regulations)



[1] Airbus. Getting to grips with aircraft performance

JAR 25.107 Subpart B	*FAR 25.107 Subpart B*

V_R is the speed at which the pilot initiates the rotation, at the appropriate rate of about 3° per second.

"JAR/FAR 25.107
(e) V_R, in terms of calibrated air speed, […] may not be less than:

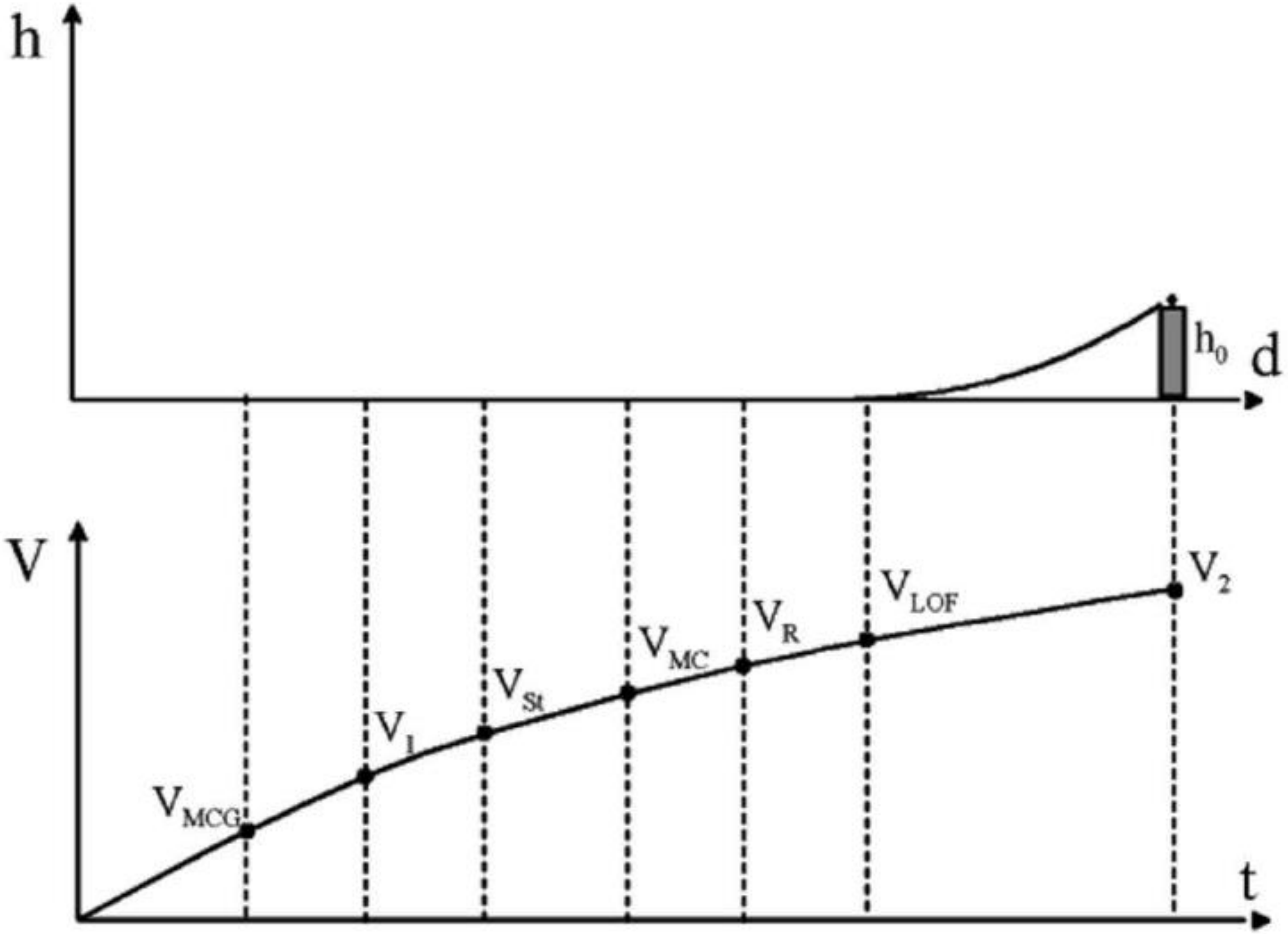
- *V1,*
- *105% of V_{MCA}*
- *The speed that allows reaching V2 before reaching a height of 35 ft above the take-off surface, or*
- *A speed that, if the aeroplane is rotated at its maximum practicable rate, will result in a [satisfactory] V_{LOF}"*

JAR 25.107 Subpart B	*FAR 25.107 Subpart B* *FAR AC 25-7A*

"JAR/FAR 25.107
(f) V_{LOF} is the calibrated airspeed at which the aeroplane first becomes airborne."

Therefore, it is the speed at which the lift overcomes the weight.

Take-off (regulations)



[1] Airbus. Getting to grips with aircraft performance

JAR 25.107 Subpart B	*FAR 25.107 Subpart B*

V_2 is the minimum climb speed that must be reached at a height of 35 feet above the runway surface, in case of an engine failure.

"JAR/FAR 25.107
(b) V_{2min}, in terms of calibrated airspeed, may not be less than:

- ***1.13 V_{SR}*** [1] *(JAR) or **1.2 V_S** (FAR) for turbo-jet powered aeroplanes […]*
- *1.10 times V_{MCA}*

(c) V_2, in terms of calibrated airspeed, must be selected by the applicant to provide at least the gradient of climb required by JAR 25.121(b) but may not be less than:

- *V_{2min}; and*
- *V_R plus the speed increment attained before reaching a height of 35 ft above the take-off surface."*

This speed must be entered by the crew during flight preparation, and is represented by a magenta triangle on the speed scale (see Figure C3).

$V_2 \geq 1.1\ V_{MCA}$

$V_2 \geq 1.13\ V_{s1g}$	(Airbus Fly-By-Wire aircraft) [2]
$V_2 \geq 1.2\ V_s$	(Other Airbus types)

Take-off (regulations)

Balanced field length

The evaluation of the Balanced Field Length (BFL) is crucial for the aircraft performance in the take-off phase. According to FAR, two “main distances” are fundamental for the evaluation of the decision speed (balanced V1) and the BFL: the Take-Off Distance (TOD) and the Accelerate-Stop Distance (ASD).

Take-off (regulations)

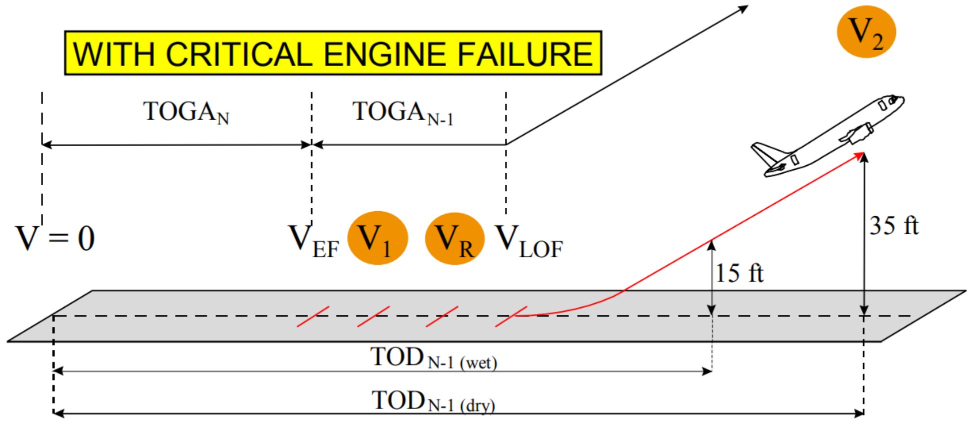
JAR 25.113 Subpart B *FAR 25.113 Subpart B*

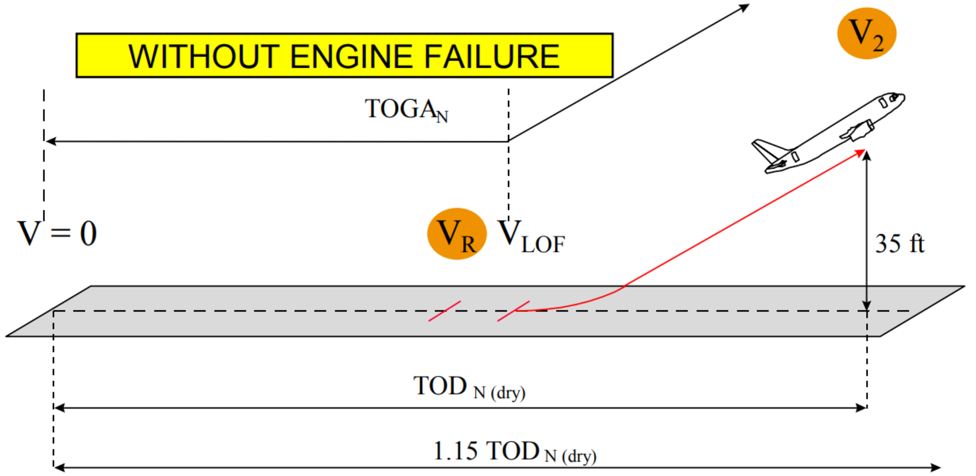
For given operational conditions (temperature, pressure altitude, weight, etc.):

a) The takeoff distance on a **dry** runway is **the greater** of the following values:

- **$TOD_{N\text{-}1\ dry}$** = Distance covered from the brake release to a point at which the aircraft is at 35 feet above the takeoff surface, assuming the failure of the critical engine at V_{EF} and recognized at V_1,
- **1.15 $TOD_{N\ dry}$** = 115% of the distance covered from brake release to a point at which the aircraft is at 35 feet above the takeoff surface, assuming all engines operating.

$$\boxed{\mathbf{TOD_{dry} = \max\ of\ \{TOD_{N\text{-}1\ dry},\ 1.15\ TOD_{N\ dry}\}}}$$





[1] Airbus. Getting to grips with aircraft performance

Take-off (regulations)

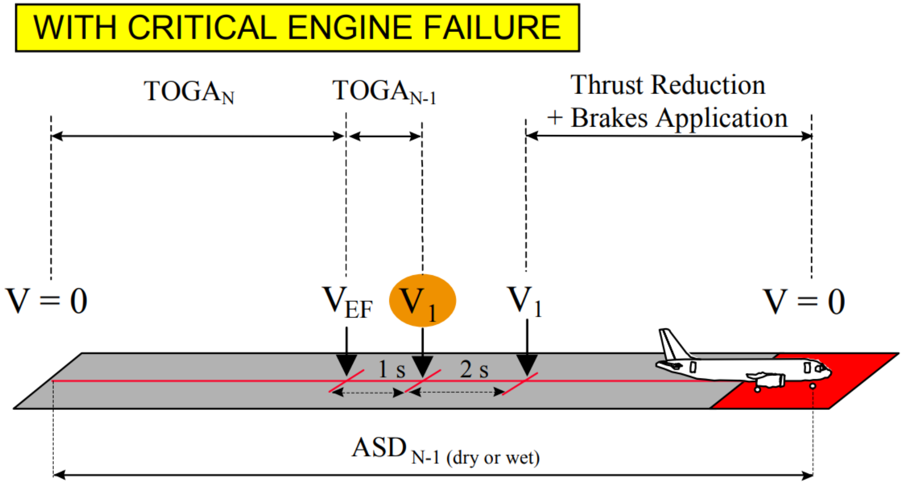
JAR 25.109 Subpart B *FAR 25.109 Subpart B*

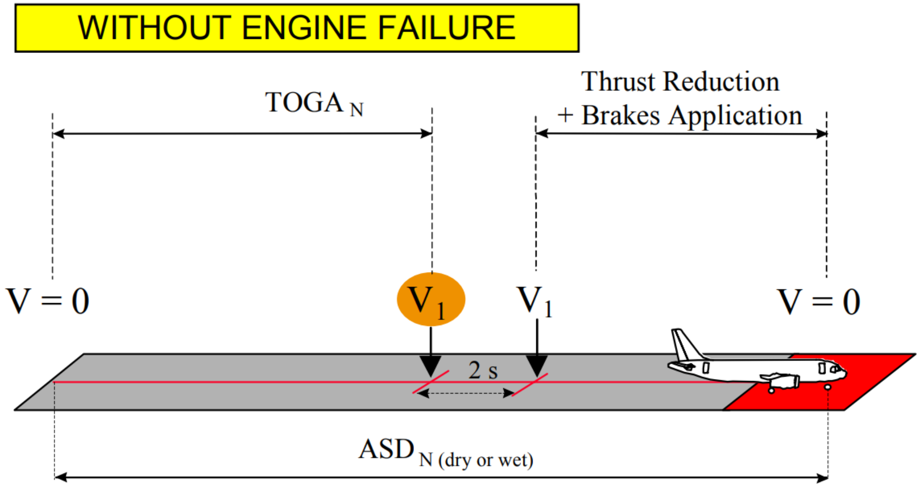
a) The accelerate-stop distance on a **dry runway** is **the greater** of the following values:

- **$ASD_{N-1\ dry}$** = Sum of the distances necessary to:
  - Accelerate the airplane with all engines operating to V_{EF},
  - Accelerate from V_{EF} to V_1[1] assuming the critical engine fails at V_{EF} and the pilot takes the first action to reject the takeoff at V_1
  - Come to a full stop[2,3]
  - Plus a distance equivalent to 2 seconds at constant[4] V_1 speed

- **$ASD_{N\ dry}$** = Sum of the distances necessary to:
  - Accelerate the airplane with all engines operating to V_1 assuming the pilot takes the first action to reject the takeoff at V_1
  - With all engines still operating come to a full stop
  - Plus a distance equivalent to 2 seconds at constant V_1 speed

$$\boxed{ASD_{dry} = \max \text{ of } \{ASD_{N-1\ dry}, ASD_{N\ dry}\}}$$



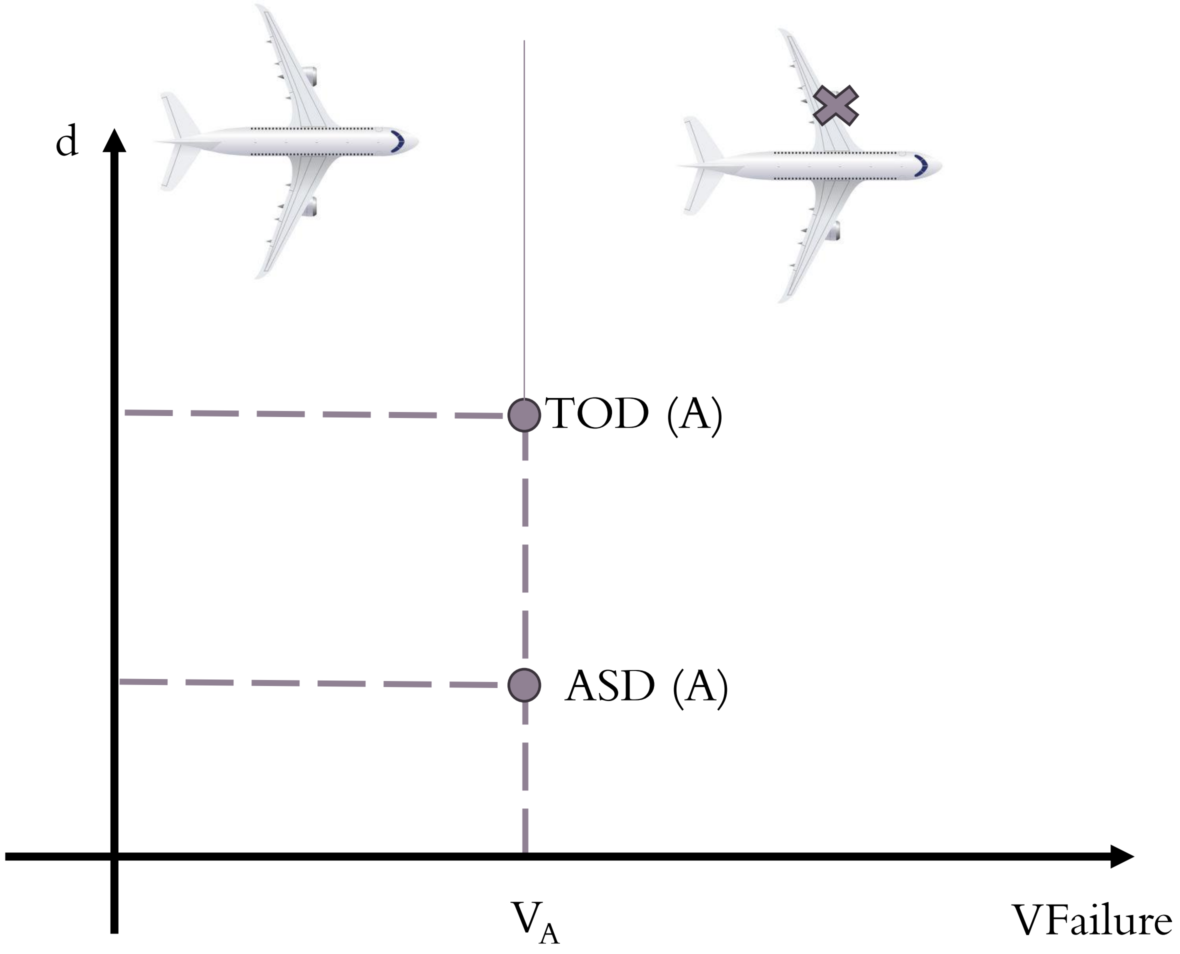


[1] Airbus. Getting to grips with aircraft performance

Take-off (regulations)

Balanced field length

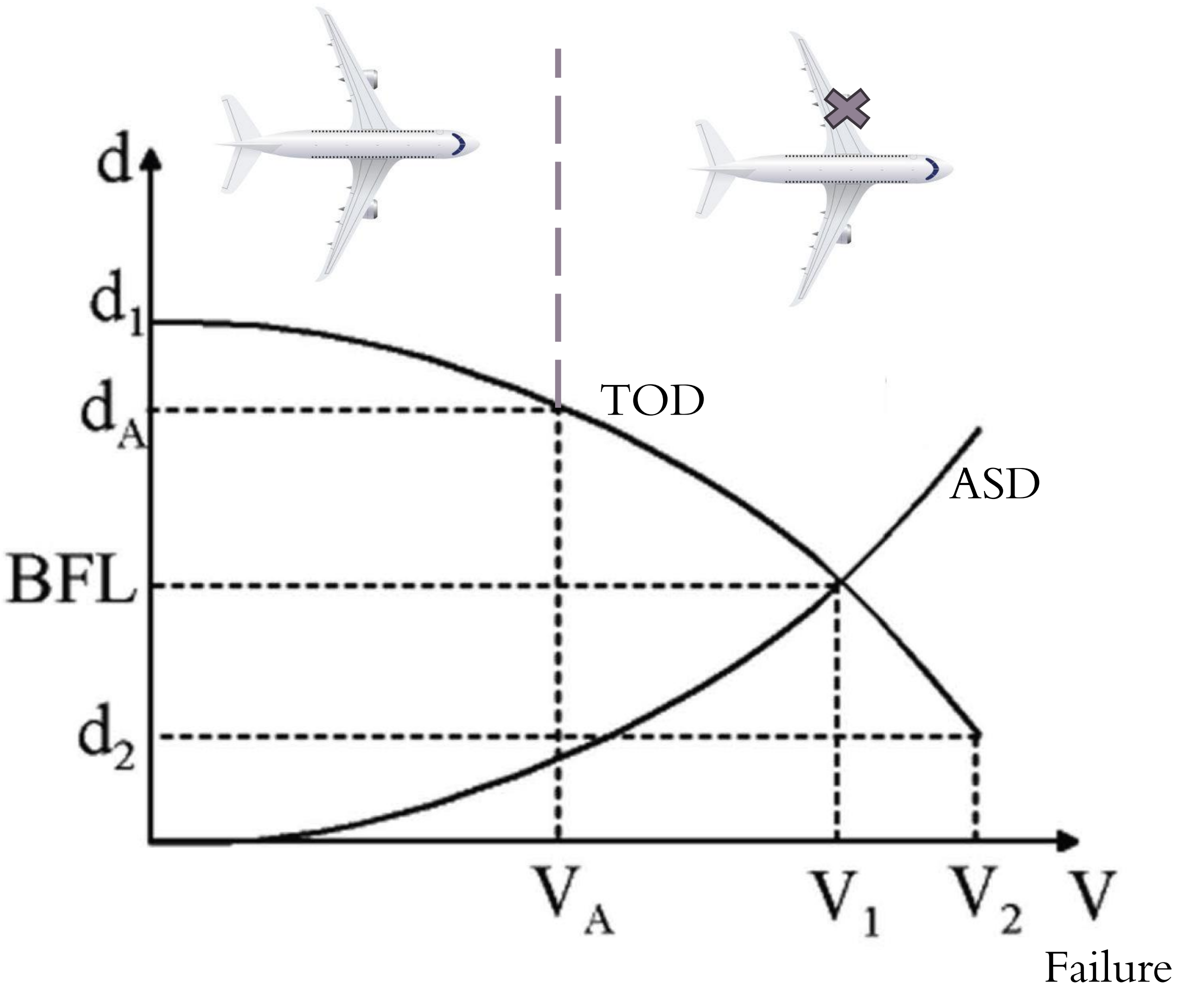
The evaluation of the Balanced Field Length (BFL) is crucial for the aircraft performance in the take-off phase. According to FAR, two "main distances" are fundamental for the evaluation of the decision speed (balanced V1) and the BFL: the Take-Off Distance (TOD) and the Accelerate-Stop Distance (ASD).



Take-off (regulations)

Balanced field length

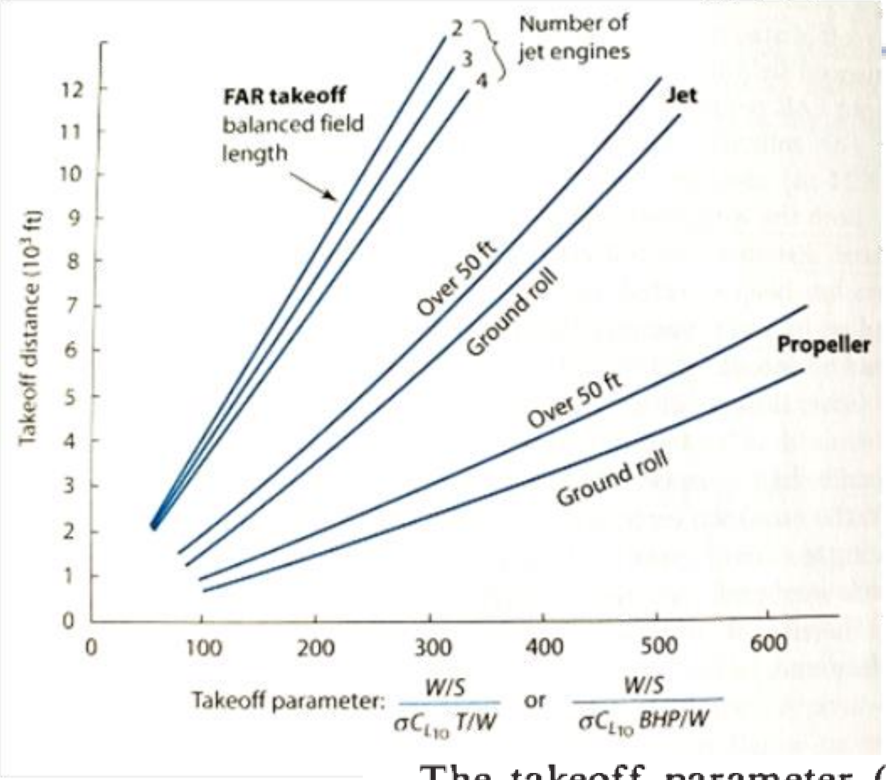
The evaluation of the Balanced Field Length (BFL) is crucial for the aircraft performance in the take-off phase. According to FAR, two “main distances” are fundamental for the evaluation of the decision speed (balanced V1) and the BFL: the Take-Off Distance (TOD) and the Accelerate-Stop Distance (ASD).



Take-off (method 1)

$$\frac{W}{S} = (TOP)\sigma C_{L_{TO}}\left(\frac{T}{W}\right)$$

TOP is the take-off parameter and can be estimated using the plot by Raymer, once the take-off distance is known



$[TOP] = lb/ft^2$

$C_{L_{TO}}$ can be estimated starting from $C_{L_{max}}$. See Roskam to select the proper values, assuming the presence/absence of high lift devices

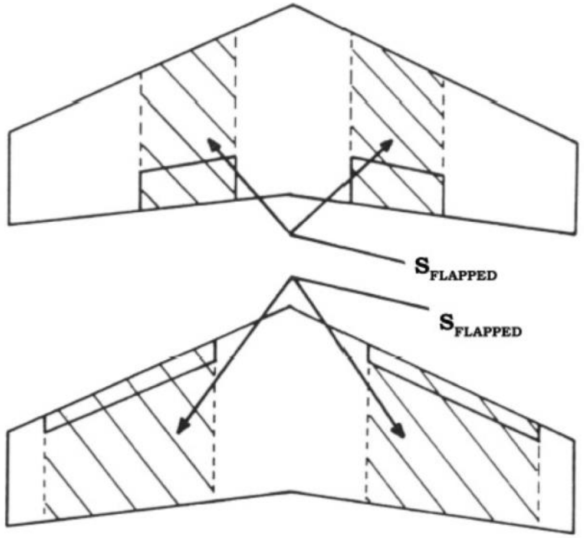
The takeoff parameter (TOP) of Fig. 5.4 is the takeoff wing loading divided by the product of the density ratio (σ), takeoff lift coefficient, and takeoff thrust-to-weight (or horsepower-to-weight) ratio. The density ratio is simply the air density (ρ) at the takeoff altitude divided by the sea level density (0.00238 slugs/cubic ft).

Take-off (method 1)

Step 1: Assessment of C_{Lmax} of clean wing (no flap/slat)

- $$C_{L_{max}}^{\text{clean}} = 0.9 \cdot C_{L_{max}}^{2D} \cdot cos(\Lambda_{25})$$

Step 2: Assessment of ΔC_{Lmax} due to flap/slat



Superficie 'flappata'

- $$\underbrace{\Delta C_{L_{max}}^{flap}}_{3D} = 0.92 \cdot \underbrace{\boxed{\Delta C_{l_{max}}^{flap}}}_{2D} \cdot \frac{S_{flapped}}{S} cos(\boxed{\Lambda_{25}})$$

		$C_{L,max}$	$\Delta C_{L,max}$
Clean Airfoil		1,45	-
Plain Flap		2,25	0,80
Single-Slotted Flap		2,60	1,15
Double-Slotted Flap		2,80	1,35
Split Flap		2,40	0,95
Double-Wing (Junkers)		2,25	0,80
Fowler Flap		2,80	1,35
Slat		2,00	0,55
Combinations:			
Plain Flap and Slat		2,45	1,00
Single-Slotted Flap and Slat		2,70	1,25
Double-Slotted Flap and Slat		2,90	1,45
Fowler Flap and Slat		3,00	1,55

Step 3: Assessment of C_{Lmax} of flappend wing

$$C_{Lmax}^{flapped} = C_{Lmax}^{clean} + \Delta C_{Lmax}$$

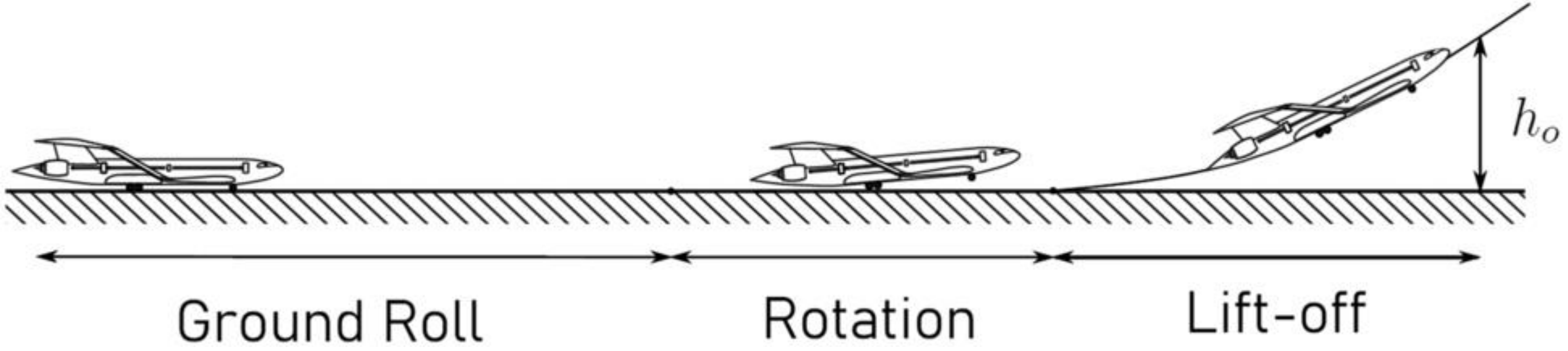
Take-off (method 1)

No.	Aircraft	Type	m_{TO} (kg)	S (m^2)	V_s (knot)	$C_{L_{max}}$
1	Volmer VJ-25 Sunfun	Hang glider/kite	140.5	15.14	13	3.3
2	Manta Fledge III	Sailplane/glider	133	14.95	15	2.4
3	Euro Wing Zephyr II	Microlight	340	15.33	25	2.15
4	Campana AN4	Very light	540	14.31	34	1.97
5	Jurca MJ5 Sirocco	GA two seat	760	10	59	1.32
6	Piper Cherokee	GA single engine	975	15.14	47.3	1.74
7	Cessna 208-L	GA single turboprop	3 629	25.96	61	2.27
8	Short Skyvan 3	Twin turboprop	5 670	35.12	60	2.71
9	Gulfstream II	Business twin jet	29 700	75.2	115	1.8
10	Learjet 25	Business twin jet	6 800	21.5	104	1.77
11	Hawkeye E-2C	Early warning	24 687	65.03	92	2.7
12	DC-9-50	Jet airliner	54 900	86.8	126	2.4
13	Boeing 727-200	Jet airliner	95 000	153.3	117	2.75
14	Airbus 300	Jet airliner	165 000	260	113	3
15	F-14 Tomcat	Fighter	33 720	54.5	110	3.1

Table 4.11 Typical values of maximum lift coefficient and stall speed for different types of aircraft

No.	Aircraft type	$C_{L_{max}}$	V_s (knot)
1	Hang glider/kite	2.5–3.5	10–15
2	Sailplane/glider	1.8–2.5	12–25
3	Microlight	1.8–2.4	20–30
4	Very light	1.6–2.2	30–45
5	GA light	1.6–2.2	40–61
6	Agricultural	1.5–2	45–61
7	Home-built	1.2–1.8	40–70
8	Business jet	1.6–2.6	70–120
9	Jet transport	2.2–3.2	95–130
10	Supersonic fighter	1.8–3.2	100–120

Take-off (method 2)



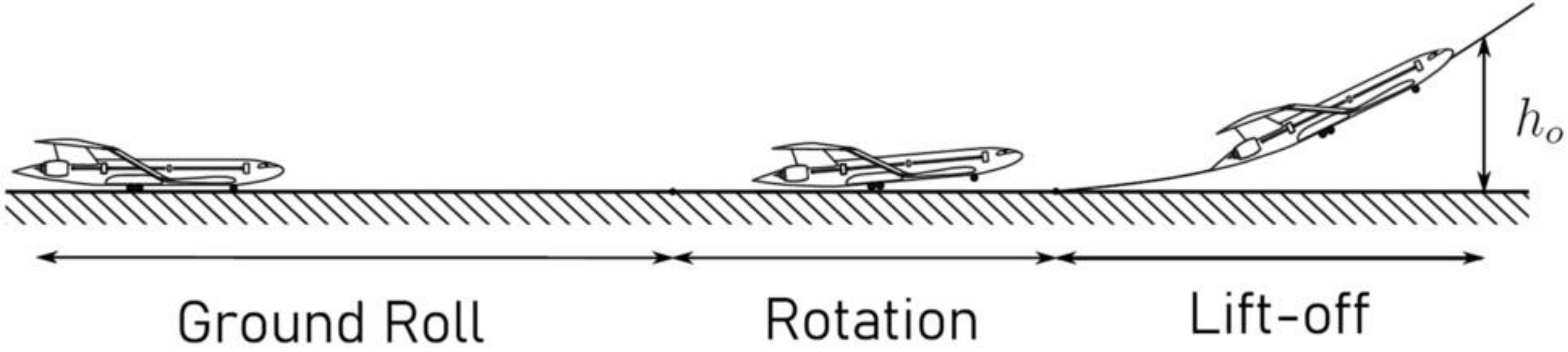
Simplified assumptions:

- Aircraft is a material point
- No wind
- First Newton’s equation of motion is enough
- V is constant during Lift-off
- Aircraft weight is constant
- Thrust is constant
- Acceleration is constant during rotation

Regulations must be considered

1.15 $TOD_{N\ dry}$ = 115% of the distance covered from brake release to a point at which the aircraft is at 35 feet above the takeoff surface, assuming all engines operating.

Take-off (method 2)



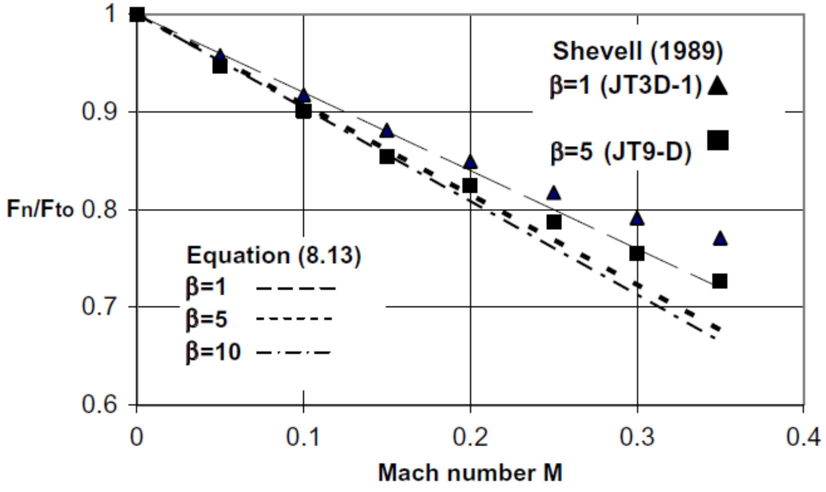


FIGURE 10.3

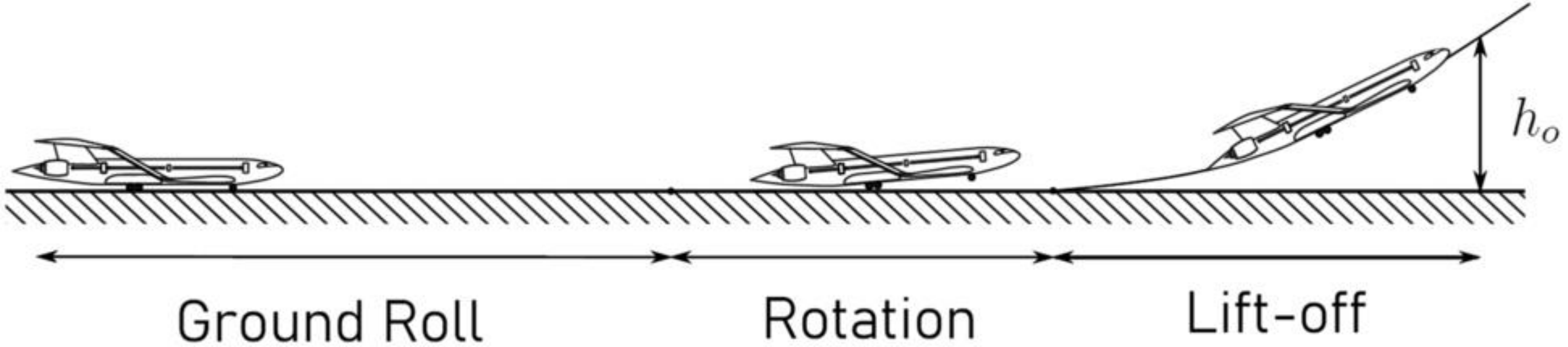
Variation of the ratio of net thrust to static thrust as a function of Mach number for turbofans with bypass ratio β. Data from Shevell (1989).

$$\frac{F_n}{F_g} = 1 - 2M\,\frac{1+\beta}{3+2\beta}$$

0.25

9<BPR<12

Take-off (method 2)



$$x_{GR} = -\frac{k_{VR}^2}{\alpha g \rho C_{L,max}} \frac{W}{S} \log\left(1 - \frac{\alpha}{\frac{T}{W} - \mu}\right)$$

$$x_{RO} = \frac{(k_{VR} + k_{V2})\sqrt{\frac{2\frac{W}{S}}{\rho C_{L,max}}}\Delta \mathrm{T}}{2}$$

$$x_{LO} = \sqrt{2h_0 \frac{k_{V2}^2 \frac{2\frac{W}{S}}{\rho C_{L,max}} - g h_0}{g(n_z - 1)}}$$

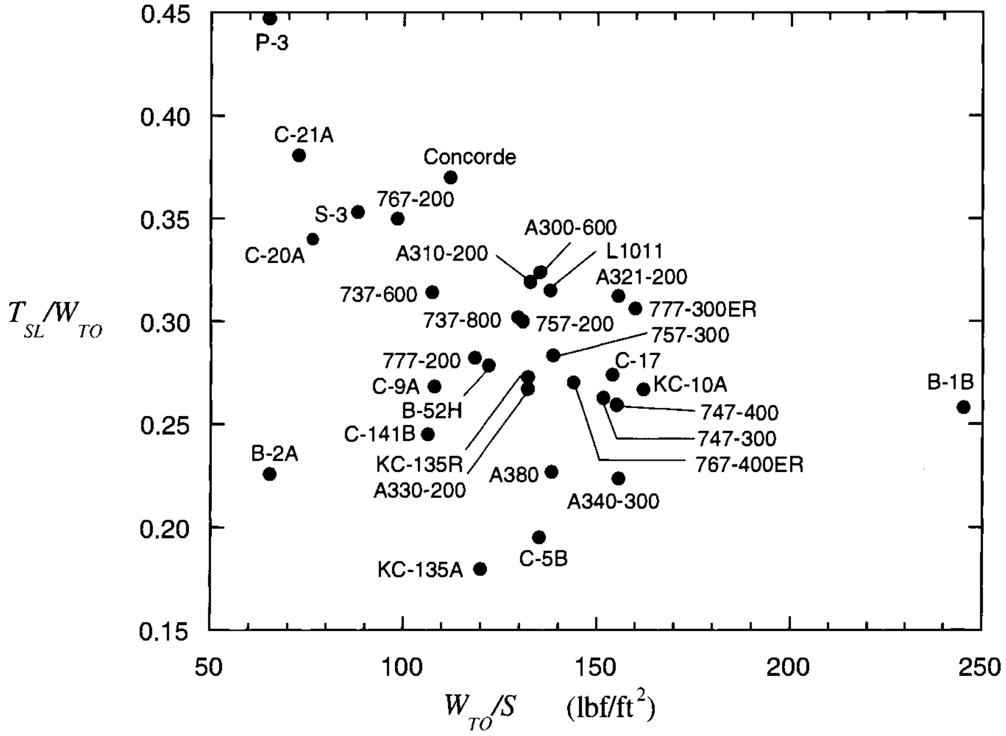
1.15 **TOD$_{\text{N dry}}$** = 115% of the distance covered from brake release to a point at which the aircraft is at 35 feet above the takeoff surface, assuming all engines operating.

$$x_{GR} + x_{RO} + x_{LO} \leq TOD/\sigma$$

$$\boxed{\frac{T}{W}} \leq \mu + \frac{\alpha}{1 - e^{-(TOD/\sigma - x_{RO} - x_{LO})/(\frac{k_{VR}^2}{\alpha g \rho C_{L,max}} * \frac{W}{S})}}$$

$$\frac{F_n}{F_g} = 1 - 2M\frac{1+\beta}{3+2\beta} \longrightarrow \left(\frac{T}{W}\right)_0 = \frac{T/W}{1 - 2M\frac{1+\beta}{3+2\beta}}$$

Wing Surface and Engine Sizing



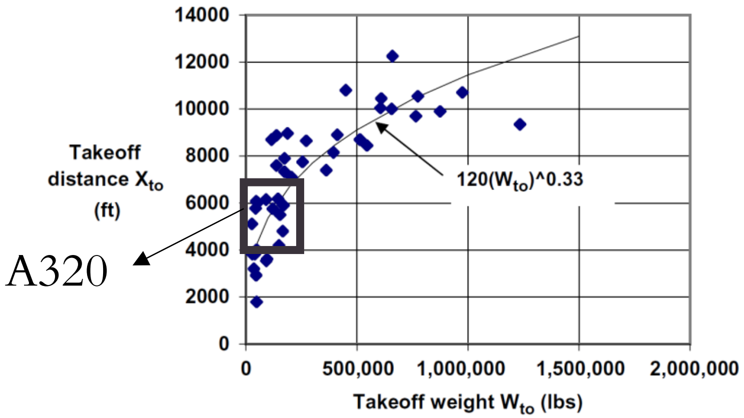


FIGURE 4.13

The nominal values of takeoff distance as a function of takeoff distance taken from the database for 50 commercial airliners.

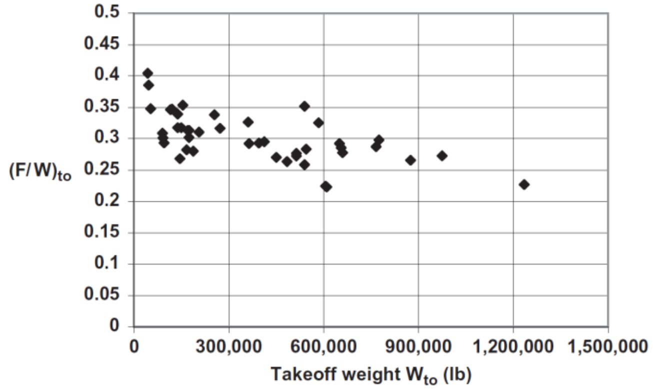
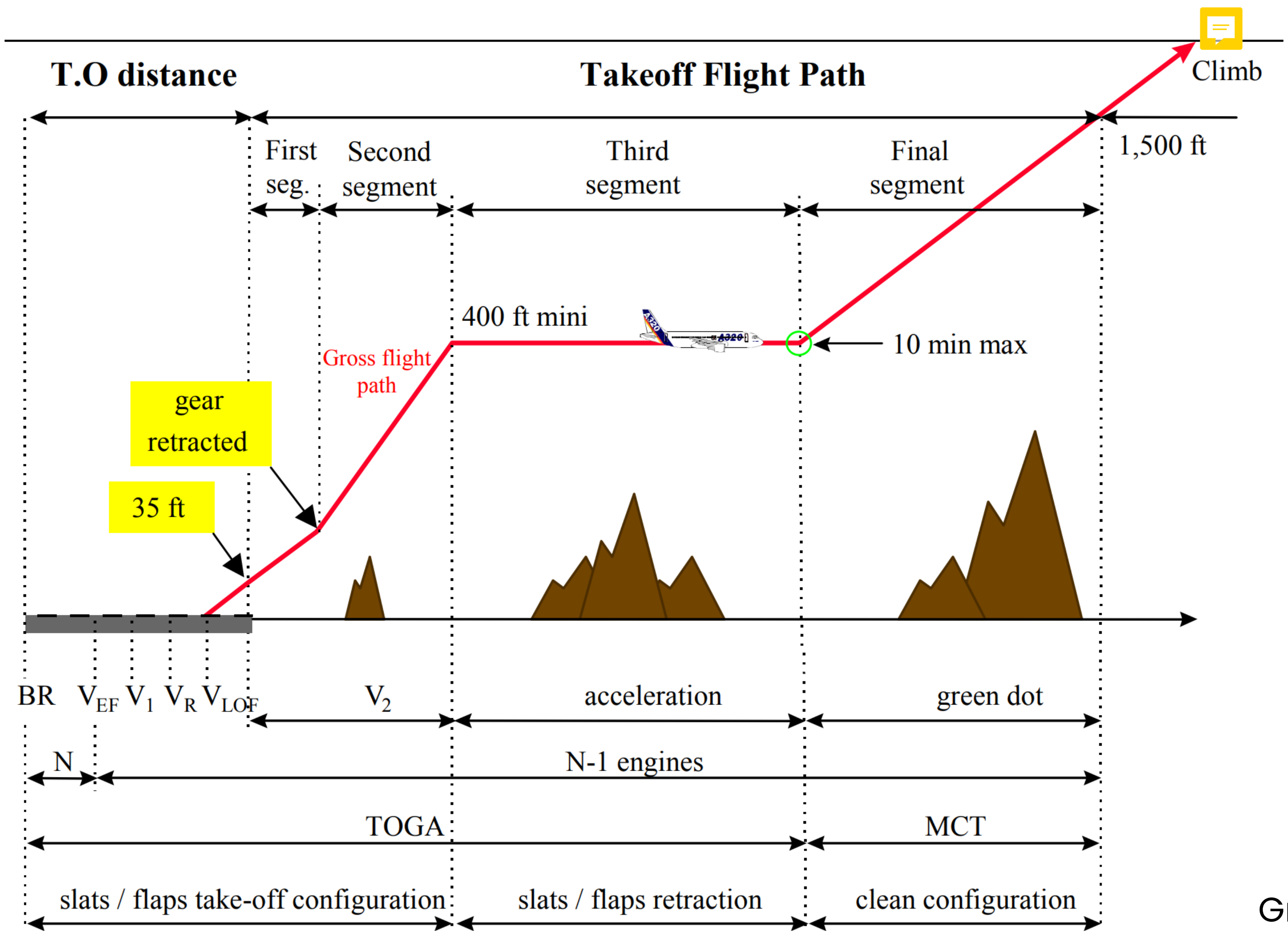


FIGURE 4.17

Takeoff thrust to weight ratio $(F/W)_{to}$ for 43 turbofan-powered airliners is shown as a function of takeoff weight W_{to}.

[1] Mattingly, Aircraft Engine Design
[2] Sforza , Commercial aircraft Design

Climb

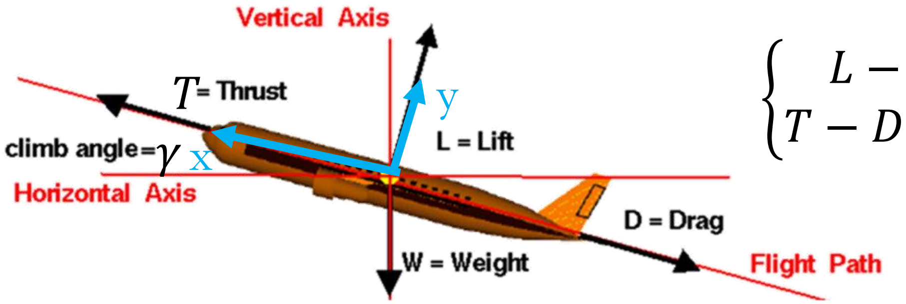


		FIRST SEGMENT	SECOND SEGMENT	FINAL SEGMENT
Minimum climb gradient (N-1) engines	Twin	0.0%	2.4%	1.2%
	Quad	0.5%	3.0%	1.7%
Start when		V_{LOF} reached	Gear fully retracted	En route configuration Achieved
Slats / Flaps Configuration		Takeoff	Takeoff	Clean
Engine rating		TOGA/FLEX	TOGA/FLEX	MCT
Speed reference		V2	V_2	1.25 Vs
Landing gear		Retraction	Retracted	Retracted
Weight reference		Weight at the start of the gear retraction	Weight when the gear is fully retracted	Weight at the end of the acceleration segment
Ground effect		Without	Without	Without

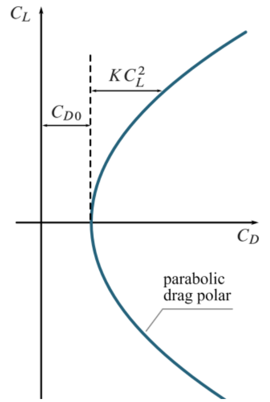
Green dot speed represents the best climb gradient speed

Climb

Thrust must satisfy the steady state climb condition with one engine inoperative, in three different segments. Refer to the previous slide



$$\begin{cases} L - W\cos(\gamma) = 0 \\ T - D - W\sin(\gamma) = 0 \end{cases} \longrightarrow \begin{cases} \frac{1}{2}\rho S V^2 C_L - W\cos(\gamma) = 0 \\ \frac{T}{W} = \frac{\frac{1}{2}\rho S V^2 C_D}{W} + \sin(\gamma) \end{cases}$$



$$\begin{cases} C_L = \frac{2W/S}{\rho V^2}\cos(\gamma) \\ \frac{T}{W} = \frac{\frac{1}{2}\rho S V^2 (C_{D0} + kC_L^2)}{W} + \sin(\gamma) \end{cases}$$

$$\boxed{\begin{gathered} \frac{T}{W} = \frac{\frac{1}{2}\rho V^2 C_{D0}}{W/S} + \frac{1}{2}\rho V^2 k \left(\frac{2W/S}{\rho V^2}\cos(\gamma)\right)^2 + \sin(\gamma) \\ \left(\frac{T}{W}\right)_{engine} = k_{OEI}\frac{T}{W} \qquad k_{OEI} = \begin{cases} 2 \ for\ two\ engines \\ \frac{4}{3} \ for\ four\ engines \end{cases} \end{gathered}}$$

Climb

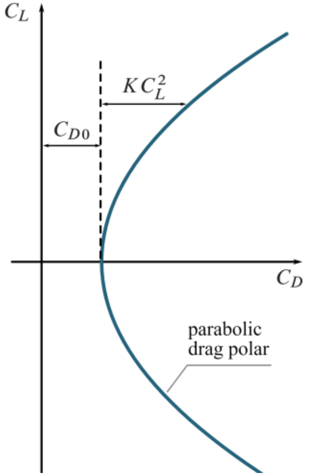
Table 5.8 Typical values of wing aspect ratio

No.	Aircraft type	Aspect ratio
1	Hang glider	4–8
2	Glider (sailplane)	20–40
3	Home-built	4–7
4	General aviation	5–9
5	Jet trainer	4–8
6	Low-subsonic transport	6–9
7	High-subsonic transport	8–12
8	Supersonic fighter	2–4
9	Tactical missile	0.3–1
10	Hypersonic aircraft	1–3

Table 4.12 Typical values of C_{D_o} for different types of aircraft

No.	Aircraft type	C_{D_o}
1	Jet transport	0.015–0.02
2	Turboprop transport	0.018–0.024
3	Twin-engine piston prop	0.022–0.028
4	Small GA with retractable landing gear	0.02–0.03
5	Small GA with fixed landing gear	0.025–0.04
6	Agricultural	0.04–0.07
7	Sailplane/glider	0.012–0.015
8	Supersonic fighter	0.018–0.035
9	Home-built	0.025–0.04
10	Microlight	0.02–0.035

$$\boldsymbol{K} = \frac{1}{\pi e \, AR}, \text{ with } 0.7 < e < 0.95$$



Perkins and Hage (1949) show a curve for estimating the parasite drag area for tricycle landing gear based on takeoff weight, which can be fitted by the following equation:

$$C_{D,LG} = 4.05 \times 10^{-3} \frac{W_{to}^{0.785}}{S} \qquad (9.36)$$

The same equation is also presented by Torenbeek (1982), while Mair and Birdsall (1987) quote data from the Engineering Sciences Data Unit (1987) which gives the same form of the equation, but with different constants, as follows:

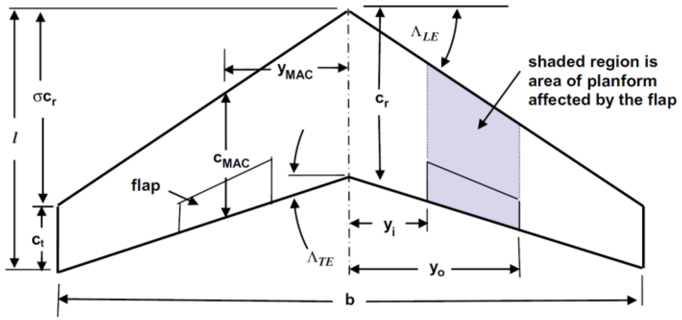
$$C_{D,LG} = 1.79 \times 10^{-3} \frac{W_{to}^{0.785}}{S} \qquad (9.37)$$

They also point out that when the flaps are not extended the landing gear drag is higher:

$$C_{D,LG} = 3.30 \times 10^{-3} \frac{W_{to}^{0.785}}{S} \qquad (9.38)$$

$$C_{D,flap} = 0.9 \left(\frac{c_{flap}}{c}\right)^{1.38} \left(\frac{S_{wflap}}{S}\right) \sin^2 \delta_{flap} \qquad (9.39)$$

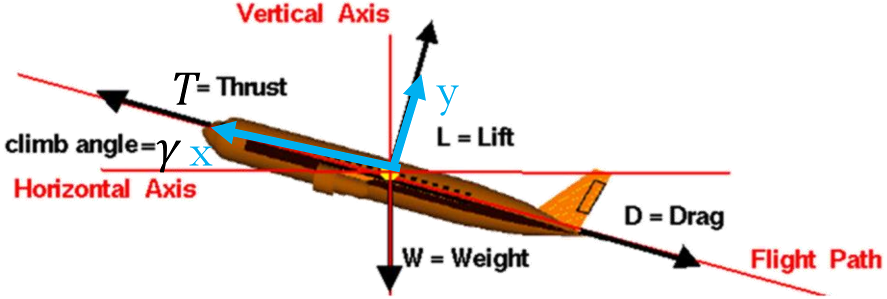
For plain or split flaps the coefficient in Equation (9.39) is increased to 1.7 from 0.9. Here c_{flap}/c represents the flap chord to wing chord ratio and the quantity S_{wflap}/S is the ratio of wing area affected by the trailing edge flap deflection (including both port and starboard wings) to the total wing area, as is shown in Figure 5.2.



S can be evaluated for each W/S, knowing W

Cruise

Thrust must satisfy the steady state climb condition with one engine inoperative, in three different segments. Refer to the previous slide



$$\begin{cases} \dfrac{T}{W} = \dfrac{\frac{1}{2}\rho V^2 C_{D0}}{W/S} + \dfrac{1}{2}\rho V^2 k \left(\dfrac{2W/S}{\rho V^2}\cos(\gamma)\right)^2 + \sin(\gamma) \\ \gamma = 0 \\ \left(\dfrac{T}{W}\right) = \left(\dfrac{T}{W}\right)_{SL} \dfrac{\rho}{\rho_{SL}} \end{cases} \longrightarrow \left(\dfrac{T}{W}\right)_{SL} = \dfrac{\left(\dfrac{T}{W}\right)}{\dfrac{\rho}{\rho_{SL}}}$$

Table 5.8 Typical values of wing aspect ratio

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Table 4.12 Typical values of C_{D_o} for different types of aircraft

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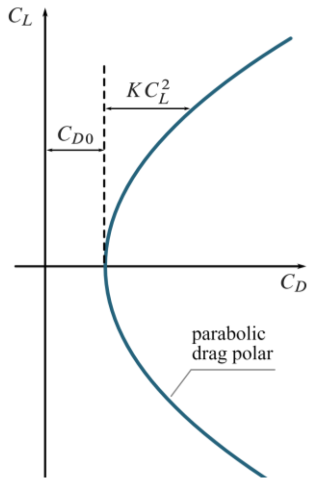
8	Supersonic fighter	0.018–0.035
9	Home-built	0.025–0.04
10	Microlight	0.02–0.035

$\boldsymbol{K} = \dfrac{1}{\pi e \, AR}$, with $0.7 < e < 0.95$



Approach climb

3.3.1. Approach Climb

JAR 25.121 Subpart B *FAR 25.121 Subpart B*

This corresponds to an aircraft's climb capability, assuming that one engine is inoperative. The "approach climb" wording comes from the fact that go-around performance is based on approach configuration, rather than landing configuration. For Airbus fly-by-wire aircraft, the available approach configurations are CONF 2 and 3.

3.3.1.1. Aircraft Configuration

- One engine inoperative
- TOGA thrust
- Gear retracted
- Slats and flaps in approach configuration (CONF 2 or 3 in most cases)
- $1.23\ V_{S1g} \leq V \leq 1.41\ V_{S1g}$ and check that $V \geq VMCL$

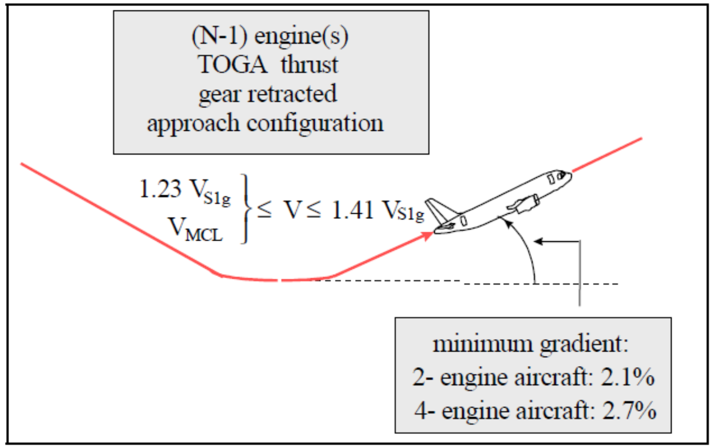
$$\frac{T}{W} = \frac{\frac{1}{2}\rho V^2 C_{D0}}{W/S} + \frac{1}{2}\rho V^2 k\left(\frac{2W/S}{\rho V^2}\cos(\gamma)\right)^2 + \sin(\gamma)$$

$$\left(\frac{T}{W}\right)_{engine} = k_{OEI}\frac{T}{W}$$

3.3.1.2. Requirements

The minimum gradients to be demonstrated:

		Approach Climb
Minimum climb gradient one engine out	Twin	2.1%
	Quad	2.7%



Landing climb

3.3.2. Landing Climb

JAR 25.119 Subpart B	*FAR 25.119 Subpart B*

The objective of this constraint is to ensure aircraft climb capability in case of a missed approach with all engines operating. The “Landing climb” wording comes from the fact that go-around performance is based on landing configuration. For Airbus FBW, the available landing configurations are CONF 3 and FULL.

3.3.2.1. Configuration

- N engines
- Thrust available 8 seconds after initiation of thrust control movement from minimum flight idle to TOGA thrust
- Gear extended
- Slats and flaps in landing configuration (CONF 3 or FULL)
- $1.13\ V_{S1g} \leq V \leq 1.23\ V_{S1g}$ and check that $V \geq V_{MCL}$.

$$\frac{T}{W} = \frac{\frac{1}{2}\rho V^2 C_{D0}}{W/S} + \frac{1}{2}\rho V^2 k \left(\frac{2W/S}{\rho V^2}\cos(\gamma)\right)^2 + \sin(\gamma)$$

$$\left(\frac{T}{W}\right)_{engine} = k_{OEI}\frac{T}{W}$$

3.3.2.2. Requirements

The minimum gradient to be demonstrated is **3.2%** for all aircraft types.

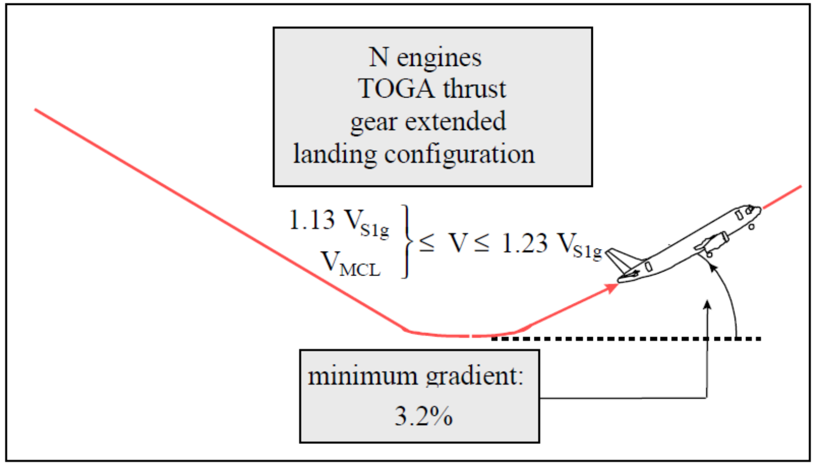
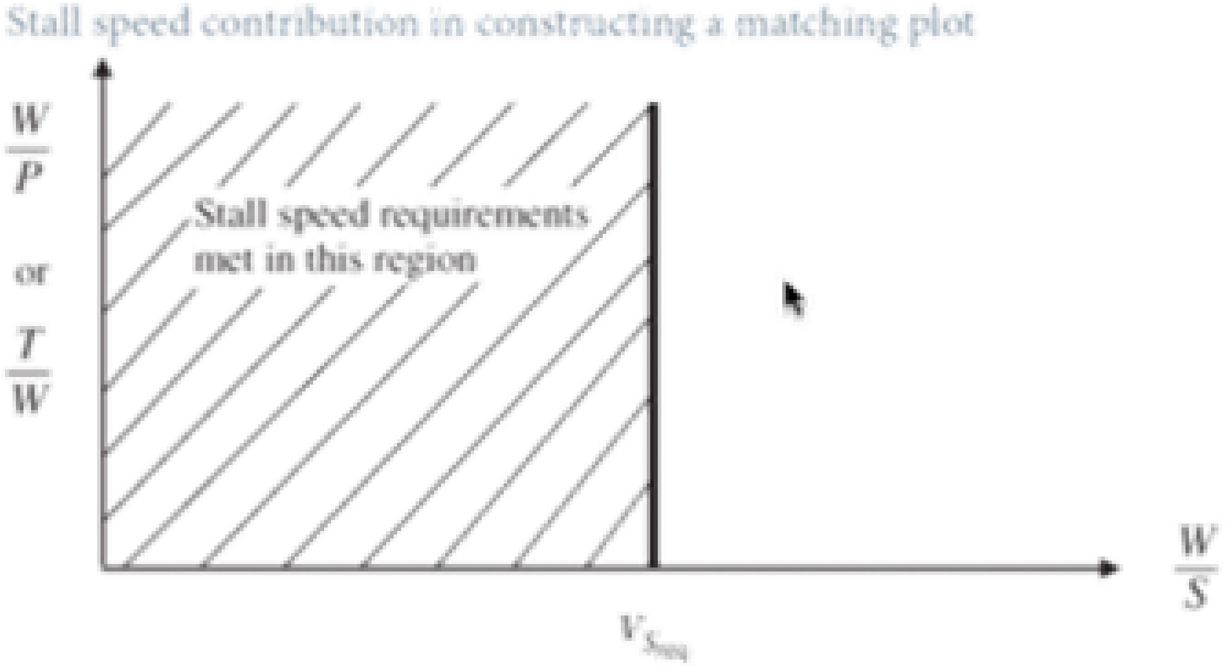


Figure E8: Minimum Air Climb Gradients - Landing Climb

Stall Speed Equation (Vs)

Fixed-wing aircraft have a **minimum airspeed** to be airborne, thus a limit to the minimum allowable speed exists and it is referred to as **stall speed**. To guarantee longitudinal trim at any flight speed ($L = W$), when aircraft speed lowers down, being closer to stall, the aircraft lift coefficient must be increased, becoming closer to $\mathbf{C_{Lmax}}$.



$$\mathbf{L \geq W} \qquad \boldsymbol{L = \frac{1}{2}\rho V_s^2 S C_{Lmax}} \qquad \boldsymbol{\frac{W}{S} \leq \frac{1}{2}\rho V_s^2 C_{Lmax}}$$

[1] Ackert, Shannon. "Aircraft payload-range analysis for financiers." Aircraft Monitor: San Francisco, CA, USA (2013).

Stall Speed Equation (Vs)

V_s is stall speed.

A suggestion might be to set this requirement starting from the required approach speed, duly considering safety factors accounting for rearward gust or wind shear.

$$\frac{W}{S} \leq \frac{1}{2}\rho V_s^2 C_{Lmax}$$

is the air density evaluated in **approach conditions**

is the maximum lift coefficient. Very difficult to estimate without knowing the wing geometry.

Usually:
$0.8 \leq C_{Lmax} \leq 1.2$ for no-flap wing
$1.5 \leq C_{Lmax} \leq 3.5$ for flap wing

Stall Speed Equation (V_s)

is the air density evaluated in **approach conditions**

$$\frac{W}{S} \leq \frac{1}{2}\rho V_s^2 C_{Lmax}$$

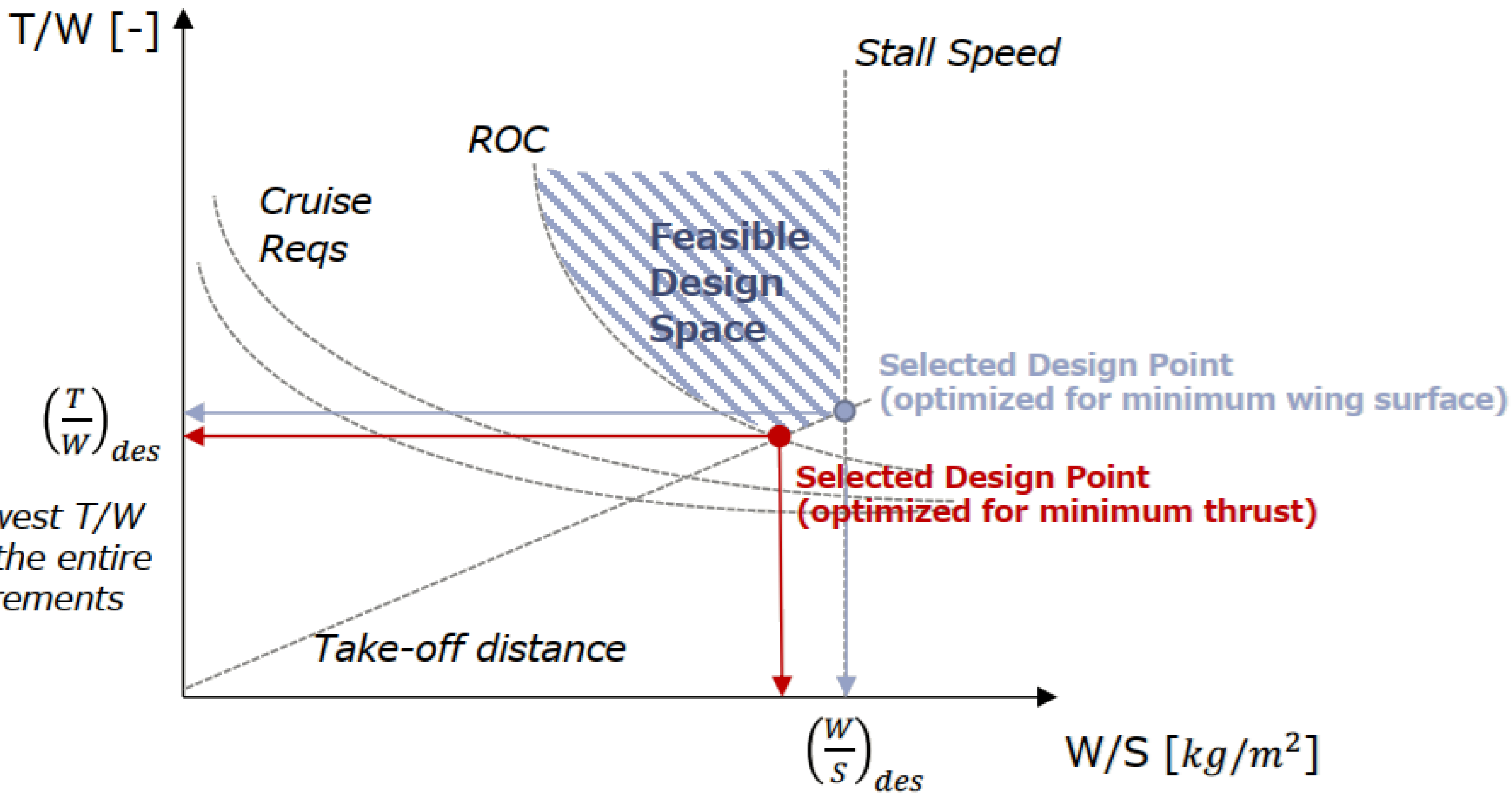
is the maximum lift coefficient. Very difficult to estimate without knowing the wing geometry.

Usually:
$0.8 \leq C_{Lmax} \leq 1.2$ for no-flap wing
$1.5 \leq C_{Lmax} \leq 3.5$ for flap wing

No.	Aircraft	Type	m_{TO} (kg)	S (m^2)	V_s (knot)	$C_{l_{max}}$
1	Volmer VJ-25 Sunfun	Hang glider/kite	140.5	15.14	13	3.3
2	Manta Fledge III	Sailplane/glider	133	14.95	15	2.4
3	Euro Wing Zephyr II	Microlight	340	15.33	25	2.15
4	Campana AN4	Very light	540	14.31	34	1.97
5	Jurca MJ5 Sirocco	GA two seat	760	10	59	1.32
6	Piper Cherokee	GA single engine	975	15.14	47.3	1.74
7	Cessna 208-L	GA single turboprop	3 629	25.96	61	2.27
8	Short Skyvan 3	Twin turboprop	5 670	35.12	60	2.71
9	Gulfstream II	Business twin jet	29 700	75.2	115	1.8
10	Learjet 25	Business twin jet	6 800	21.5	104	1.77
11	Hawkeye E-2C	Early warning	24 687	65.03	92	2.7
12	DC-9-50	Jet airliner	54 900	86.8	126	2.4
13	Boeing 727-200	Jet airliner	95 000	153.3	117	2.75
14	Airbus 300	Jet airliner	165 000	260	113	3
15	F-14 Tomcat	Fighter	33 720	54.5	110	3.1

Table 4.11 Typical values of maximum lift coefficient and stall speed for different types of aircraft

Wing Surface and Engine Sizing



Select the lowest T/W able to meet the entire set of requirements

Select the highest W/S to design the smallest wing surface able to meet the requirements

$$T_{des} = \left(\frac{T}{W}\right)_{des} m_{TO} g$$

$$S_{des} = \frac{m_{TO}}{\left(\frac{W}{S}\right)_{des}}$$

Task 4.1

- **Build the Matching Chart for your case study and define Wing Surface and Engine Thrust**

Task 4.1

Back-up slides

A brief recap

$$m_{TO} = \frac{m_{crew} + m_{payload}}{1 - \left(\frac{m_{fuel}}{m_{TO}}\right) - \left(\frac{m_{empty}}{m_{TO}}\right)} \rightarrow m_{TO} = m_{crew} + m_{payload} + \left(\frac{m_{fuel}}{m_{TO}}\right) m_{TO} + \left(\frac{m_{empty}}{m_{TO}}\right) m_{TO}$$

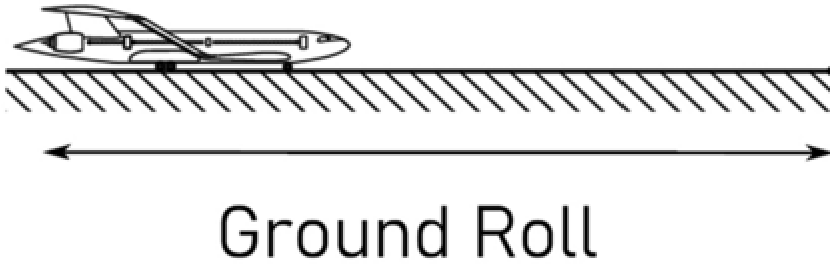
A brief recap

$$m_{TO} = \frac{m_{crew} + m_{payload}}{1 - \left(\frac{m_{fuel}}{m_{TO}}\right) - \left(\frac{m_{empty}}{m_{TO}}\right)} \rightarrow m_{TO} = m_{crew} + m_{payload} + \left(\frac{m_{fuel}}{m_{TO}}\right) m_{TO} + \left(\frac{m_{empty}}{m_{TO}}\right) m_{TO}$$

$$\begin{cases} (m_{TO})_0 = m_{TO,0} \\ (m_{TO})_{k+1} = m_{crew} + m_{payload} + \left(\frac{m_{fuel}}{m_{TO}}\right)(m_{TO})_k + \left(\frac{m_{empty}}{m_{TO}}\right)(m_{TO})_k = m_0 + \lambda (m_{TO})_k \end{cases}$$

$$\begin{aligned} (m_{TO})_1 &= m_0 + \lambda (m_{TO})_0 \\ (m_{TO})_2 &= m_0 + \lambda (m_{TO})_1 = m_0 + \lambda m_0 + \lambda (m_{TO})_0 \\ (m_{TO})_3 &= m_0 + \lambda (m_{TO})_2 = m_0 + \lambda m_0 + \lambda^2 m_0 + \lambda^3 (m_{TO})_0 \\ &\vdots \\ (m_{TO})_{k+1} &= \sum_{i=0}^{k} \lambda^i m_0 + \lambda^{k+1} (m_{TO})_0 \end{aligned}$$

Take-off (method 2)



$$\begin{cases} \frac{W}{g}\frac{\mathrm{d}V}{\mathrm{d}t} = T - D - R_{\mathrm{T}} \\ R_{\mathrm{N}} + L = W \\ R_{\mathrm{T}} = \mu R_{\mathrm{N}} \end{cases}$$

$$L = \frac{1}{2}\rho S V_R^2 C_L = W$$

$$T - \frac{1}{2}\rho V^2 S C_D - \mu(W - L) = \frac{W}{g}\frac{dV}{dt}$$

$$T - \frac{1}{2}\rho V^2 S C_D - \mu\left(W - \frac{1}{2}1\rho V^2 S C_L\right) = \frac{W}{g}\frac{dV}{dt}$$

$$T - \mu W - \frac{1}{2}\rho S(C_D - \mu C_L)V^2 = \frac{W}{g}\frac{dV}{dx}\frac{dx}{dt}$$

$$\left(\frac{T}{W} - \mu\right) - \frac{\frac{1}{2}\rho(C_D - \mu C_L)V^2}{W/S} = \frac{1}{g}\frac{dV}{dx}V$$

$$\underbrace{\left(\frac{T}{W} - \mu\right)}_{c_2} - \underbrace{\frac{\cancel{\rho}(C_D - \mu C_L)}{\frac{1}{2}\cancel{\rho}V_R^2 C_L}}_{c_1}(V^2/2) = \frac{1}{g}\frac{d\left(\frac{V^2}{2}\right)}{dx}$$

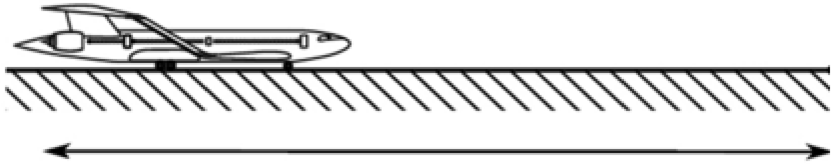
$$c_2 - c_1\frac{V^2}{2} = \frac{1}{g}\frac{d\left(\frac{V^2}{2}\right)}{dx}$$

$$0.015 < \mu < 0.02$$

$$\frac{C_D - \mu C_L}{C_L} = \frac{1 - \mu(L/D)}{L/D}$$

$$12 < \frac{L}{D} < 16$$

Take-off (method 2)



Ground Roll

$$\begin{cases} \frac{W}{g}\frac{\mathrm{d}V}{\mathrm{d}t} = T - D - R_{\mathrm{T}} \\ R_{\mathrm{N}} + L = W \\ R_{\mathrm{T}} = \mu R_{\mathrm{N}} \end{cases}$$

$$\frac{c_1}{c_2} = \frac{\dfrac{1-\mu(L/D)}{\left(\frac{1}{2}V_R^2\right)L/D}}{\dfrac{T}{W}-\mu} = \frac{\alpha}{\left(\frac{1}{2}V_R^2\right)\left(\frac{T}{W}-\mu\right)}$$

$$V_R = k_{VR}\sqrt{\frac{2\frac{W}{S}}{\rho C_{L,max}}}$$

$$c_2 - c_1\frac{V^2}{2} = \frac{1}{g}\frac{d\left(\frac{V^2}{2}\right)}{dx}$$

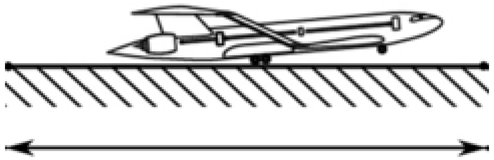
$$g\int_0^{x_{GR}} dx = \int_0^{V_R}\frac{d(V^2/2)}{c_2 - c_1\frac{V^2}{2}}$$

$$gx_{GR} = -\frac{1}{c_1}\left(\log\left(c_2 - c_1\frac{V_R^2}{2}\right) - \log c_2\right)$$

$$gx_{GR} = -\frac{1}{c_1}\left(\log\left(1 - \frac{c_1}{c_2}\frac{V_R^2}{2}\right)\right)$$

$$x_{GR} = -\frac{k_{VR}^2}{\alpha g\rho C_{L,max}}\frac{W}{S}\log\left(1 - \frac{\alpha}{\frac{T}{W}-\mu}\right)$$

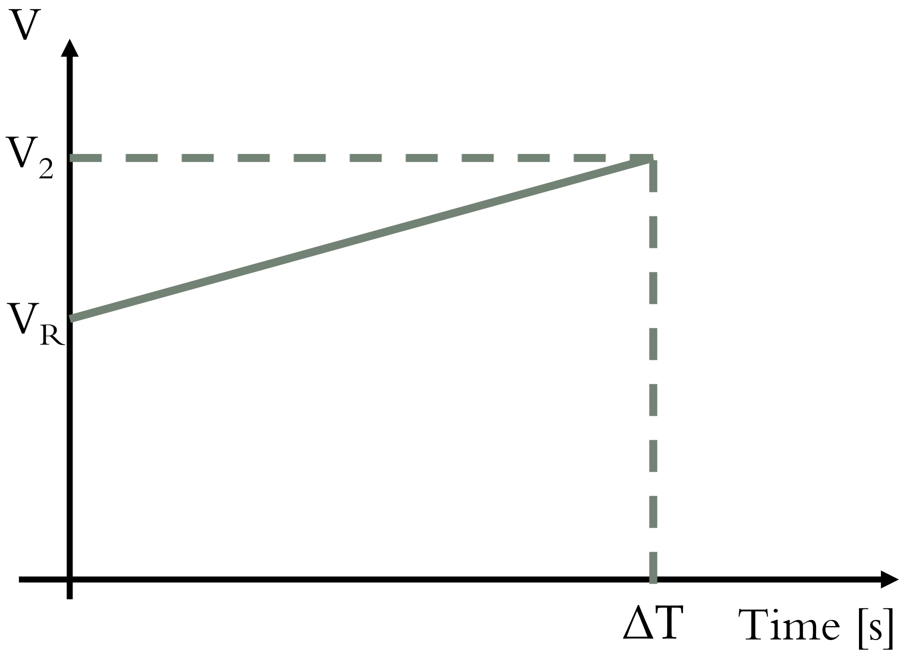
Take-off (method 2)



Rotation

Simplified assumptions:

- Acceleration is constant during rotation
- Aircraft accelerate from V_R to V_2
- This phase lasts 3s

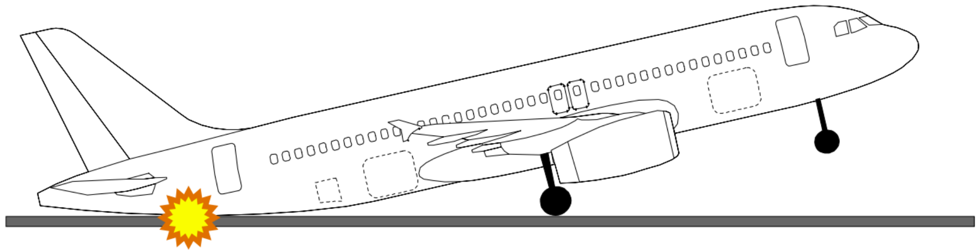


$$x_{RO} = \frac{(V_R + V_2)\Delta T}{2} = k_{VR}$$

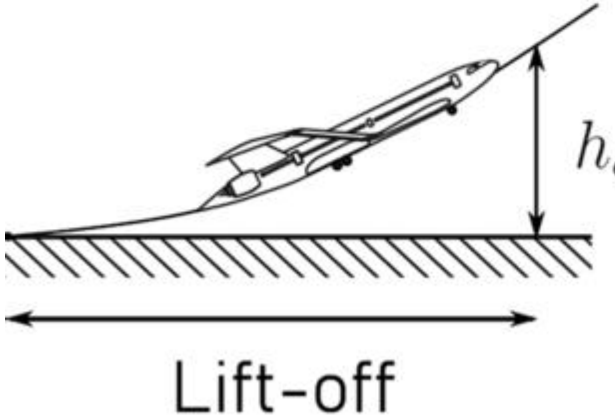
$$k_{VR} = 1.1$$

$$x_{RO} = \frac{(k_{VR} + k_{V2})\sqrt{\dfrac{2\dfrac{W}{S}}{\rho C_{L,max}}}\Delta T}{2} = k_{VR}$$

$$k_{V2} = 1.2$$



Take-off (method 2)



Simplified assumptions:

- The lift-off phase is approximated by a trajectory with constant radius
- Aircraft speed is equal to V_2

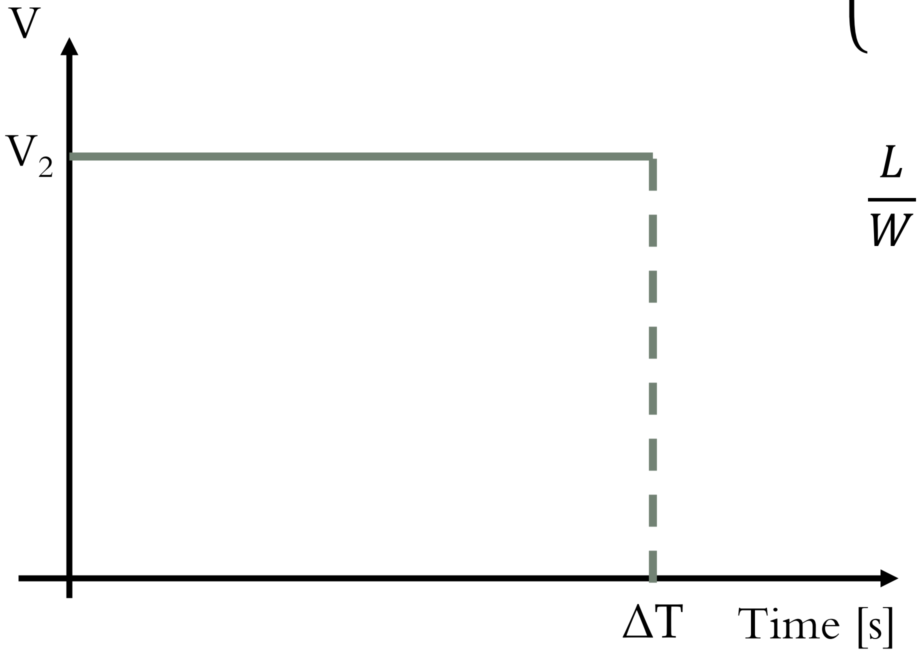
Regulation:

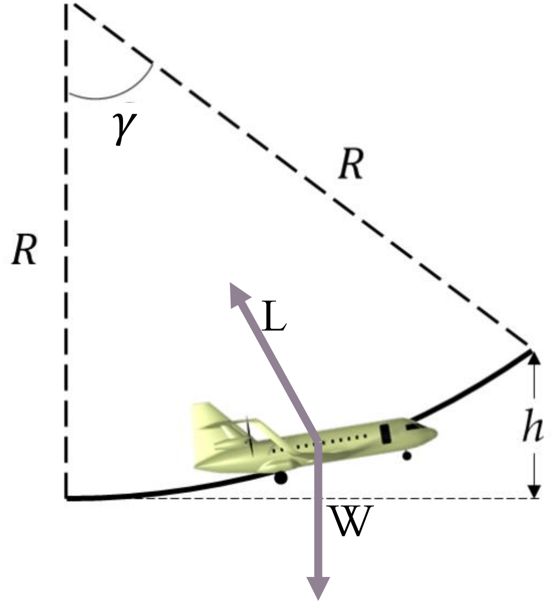
- This phase ends when $h > h_0 = 35ft$

$$\begin{cases} L - W\cos(\gamma) = \dfrac{W}{g}\dfrac{V_2^2}{R} \\ R(1-\cos(\gamma)) = h_0 \\ x_{LO} = R\sin(\gamma) \end{cases}$$

$$\frac{L}{W} = n_z = 1 - \frac{h_0}{R} + \frac{V_2^2}{gR}$$

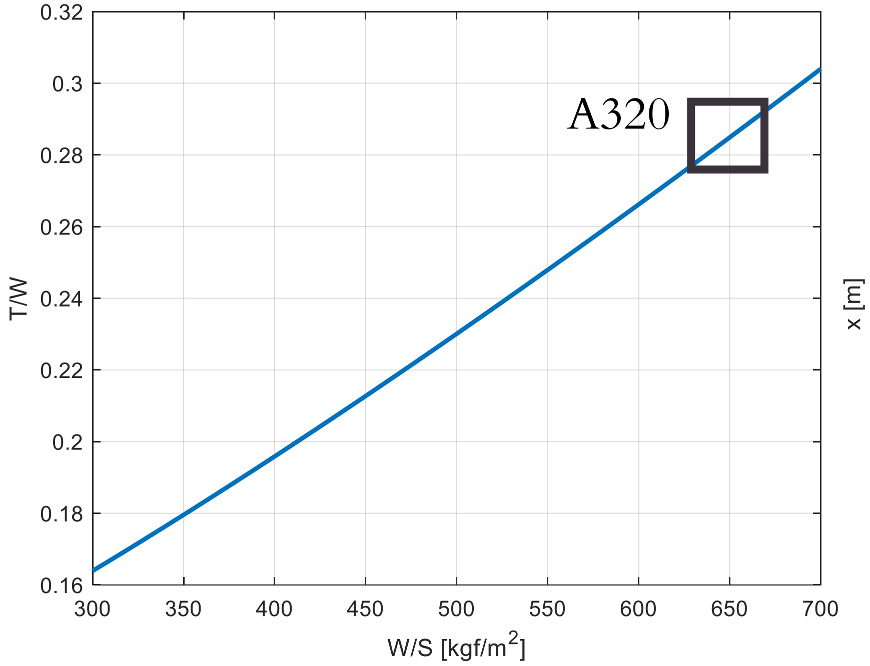
$$R = \frac{\dfrac{k_{V2}^2 \dfrac{2\frac{W}{S}}{\rho C_{L,max}}}{g} - h_0}{n_z - 1} \longrightarrow x_{LO} = R\sqrt{1-\left(1-\frac{h_0}{R}\right)^2} \sim \sqrt{2h_0R}$$



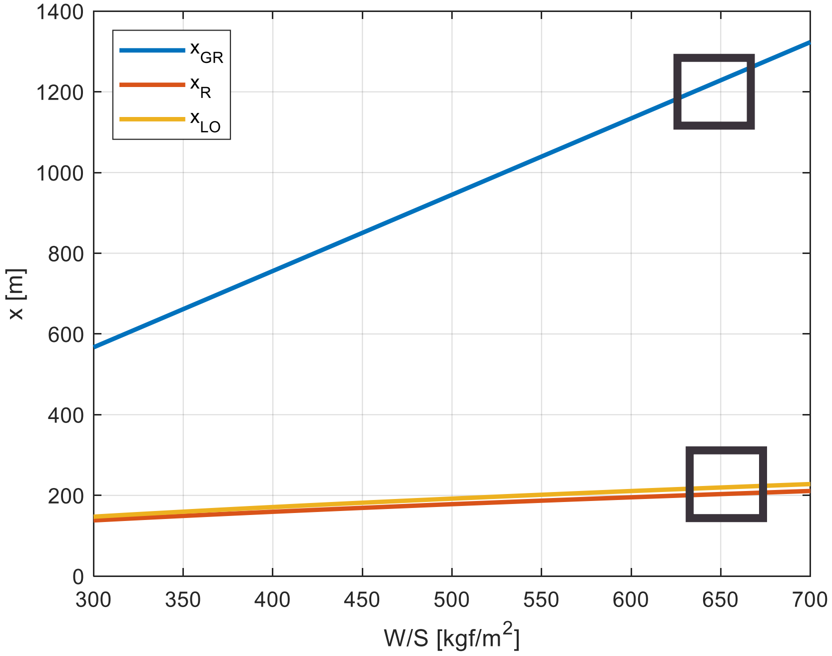


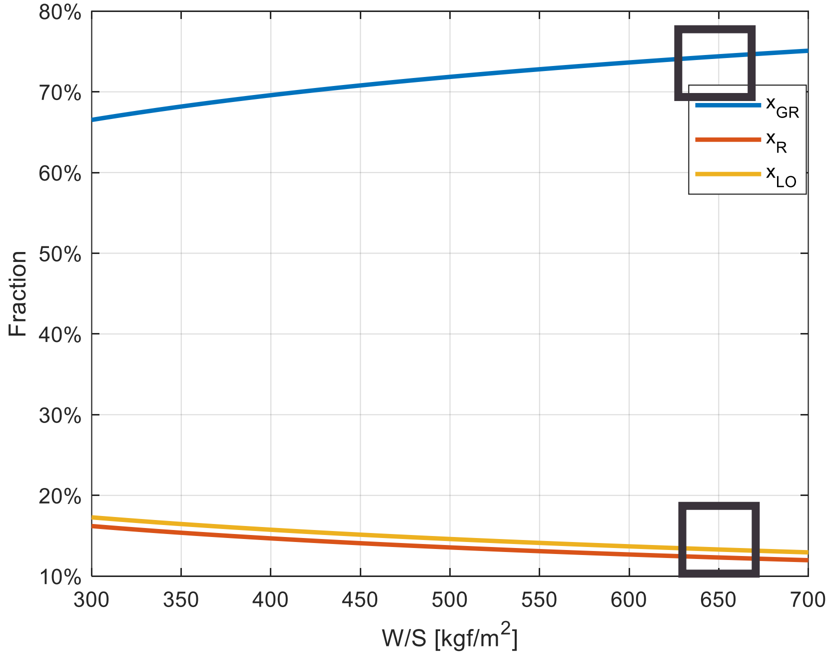
Take-off (calculation)

$L/D = 12, TOD = 2000$ m,
$BPR = 12, C_{L,max}=3$



$L/D = 12, BPR = 12, T/W = 0.3, C_{L,max}=3$





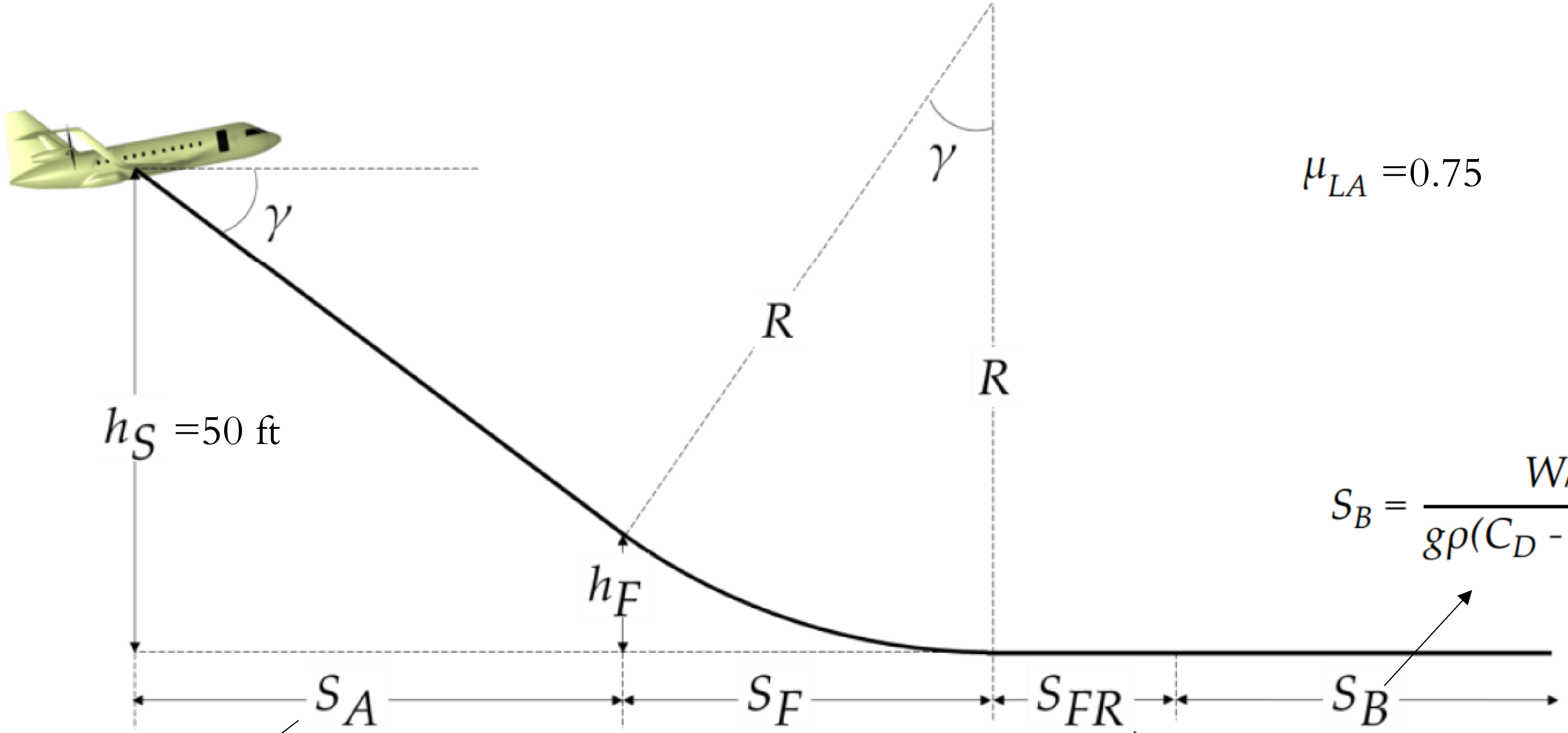
Landing

FAR 121.195 prescribes that the aircraft weight must be selected to guarantee that actual landing distance (*ALD*) must be lower than the 60% of the landing distance available (*LDA*). To compute the actual landing distance, the landing trajectory has been divided into 4 segments: approach, flare, free rolling and brake distance (as shown in Figure B-5). The main assumptions of each phase are detailed in Table B-3.

Table B-3. Main assumption for the calculation of ALD.

Segment	Assumption
Approach	The aircraft flies along a straight line with slope (γ) of 3 deg. The aircraft speed (V_A) is $1.3V_S$
Flare	The aircraft flies along a circular trajectory. The aircraft flare speed is the mean value between V_A and V_{TD} and the vertical load factor (n_Z) is 1.15
Free rolling	The aircraft moves along the runway with constant speed ($V_{TD} = 1.05\ V_S$) for 1.5s (t_{TD}).
Brake	The aircraft brakes are activated. No thrust reverser has been included, so aircraft thrust has been considered equal to zero.

Landing



μ_{LA} =0.75

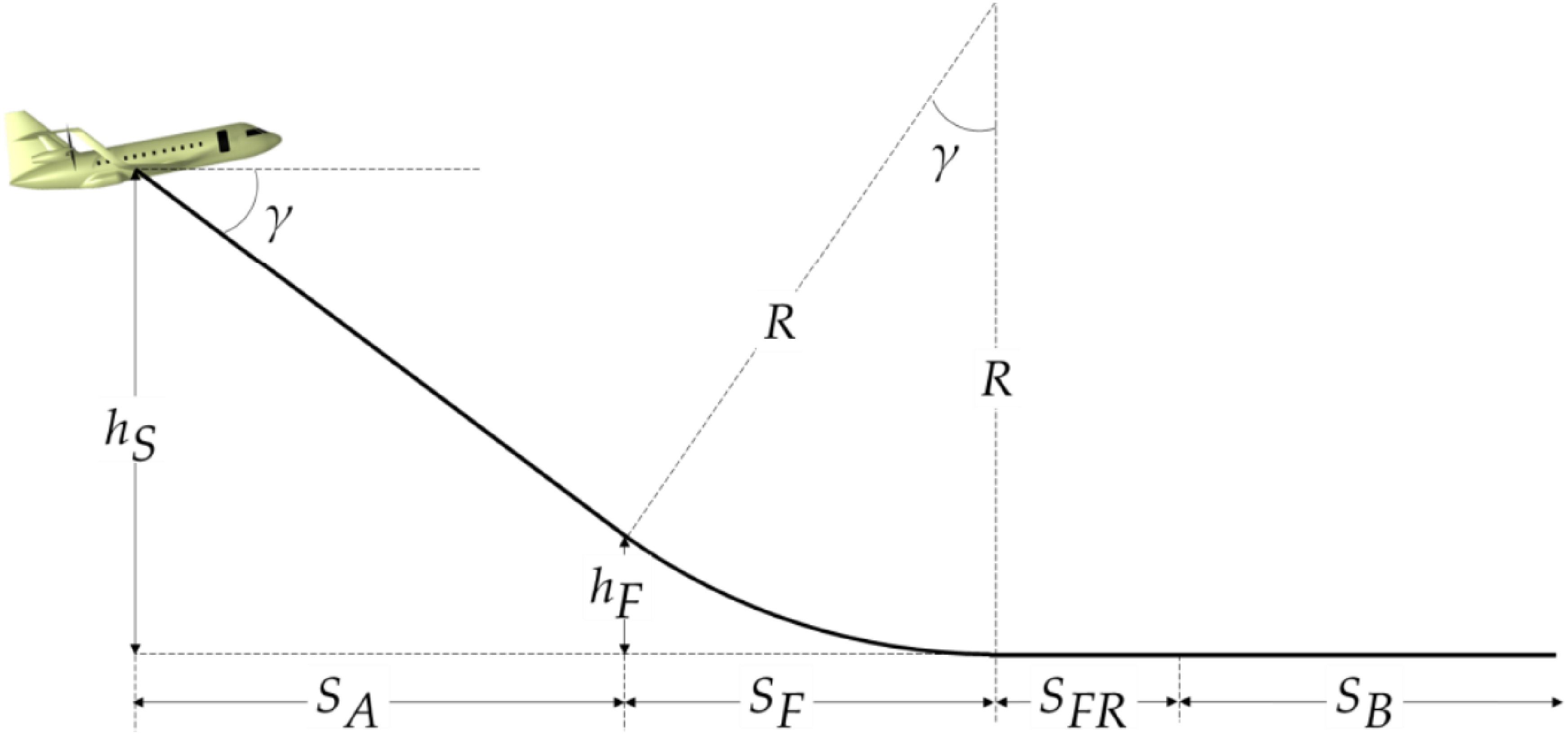
$$S_B = \frac{W/S}{g\rho(C_D - \mu_{LA} C_L)} \log\left(1 + k_{VTD}^2 \frac{C_D - \mu_{LA} C_L}{\mu_{LA} C_{L,max}}\right)$$

$$S_A = \frac{h_S - h_F}{tan(\gamma)}$$

$$\begin{cases} R(1 - cos(\gamma)) = h_F \\ R\,sin(\gamma) = S_F \\ V_F^2 = gR(n_Z - 1) \end{cases} \longrightarrow \begin{cases} S_F = \dfrac{V_F^2\, sin(\gamma)}{g(n_Z - 1)} \\ h_F = S_F \dfrac{1 - cos(\gamma)}{sin(\gamma)} \end{cases}$$

$$S_{FR} = V_{FR} t_{FR} = k_{VFR} \sqrt{2 \frac{W/S}{\rho\, C_{L,max}}}\ t_{FR}$$

Landing



$$ALD = S_A + S_F + S_{FR} + S_B = \frac{h_S}{tan(\gamma)} + \frac{(k_{VTD} + k_{VA})^2}{4g(n_Z - 1)} \frac{1 - cos(\gamma)}{sin(\gamma)} 2 \frac{W/S}{\rho\ C_{L,max}} + \frac{k_{VTD} + k_{VA}}{2} \sqrt{2 \frac{W/S}{\rho\ C_{L,max}}}\ t_{FR} + \frac{W/S}{g\rho(C_D - \mu_{LA} C_L)} log\left(1 + k_{VTD}^2 \frac{C_D - \mu_{LA} C_L}{\mu_{LA} C_{L,max}}\right)$$

$$ALD \leq 0.6\ LDA$$

Landing

$$ALD = S_A + S_F + S_{FR} + S_B = \frac{h_S}{tan(\gamma)} + \frac{(k_{VTD} + k_{VA})^2}{4g(n_Z - 1)} \frac{1 - cos(\gamma)}{sin(\gamma)} 2 \frac{W/S}{\rho\ C_{L,max}} + \frac{k_{VTD} + k_{VA}}{2} \sqrt{2 \frac{W/S}{\rho\ C_{L,max}}}\ t_{FR} + \frac{W/S}{g\rho(C_D - \mu_{LA} C_L)} log\left(1 + k_{VTD}^2 \frac{C_D - \mu_{LA} C_L}{\mu_{LA} C_{L,max}}\right)$$

$$ALD \leq 0.6\ LDA$$

$$\begin{cases} \dfrac{W}{S} \leq \left(\dfrac{-\dfrac{k_{VTD} + k_{VA}}{2} \sqrt{\dfrac{2}{\rho C_{L,max}}}\ t_{FR} + \sqrt{\Delta}}{2\xi} \right)^2 \\ \Delta = \dfrac{(k_{VTD} + k_{VA})^2}{2} \dfrac{1}{\rho C_{L,max}}\ t_{FR}^2 - 4\xi \left(\dfrac{h_S}{tan(\gamma)} - 0.6 LDA \right) \\ \xi = \left(\dfrac{(k_{VTD} + k_{VA})^2}{4g(n_Z - 1)}\ \dfrac{1 - cos(\gamma)}{sin(\gamma)}\ \dfrac{2}{\rho\ CL_{max}} + \dfrac{1}{\rho g(C_D - \mu_{LA} C_L)} log\left(1 + k_{VTD}^2 \dfrac{C_D - \mu_{LA} C_L}{\mu_{LA} C_{L,max}} \right)\right) \end{cases}$$