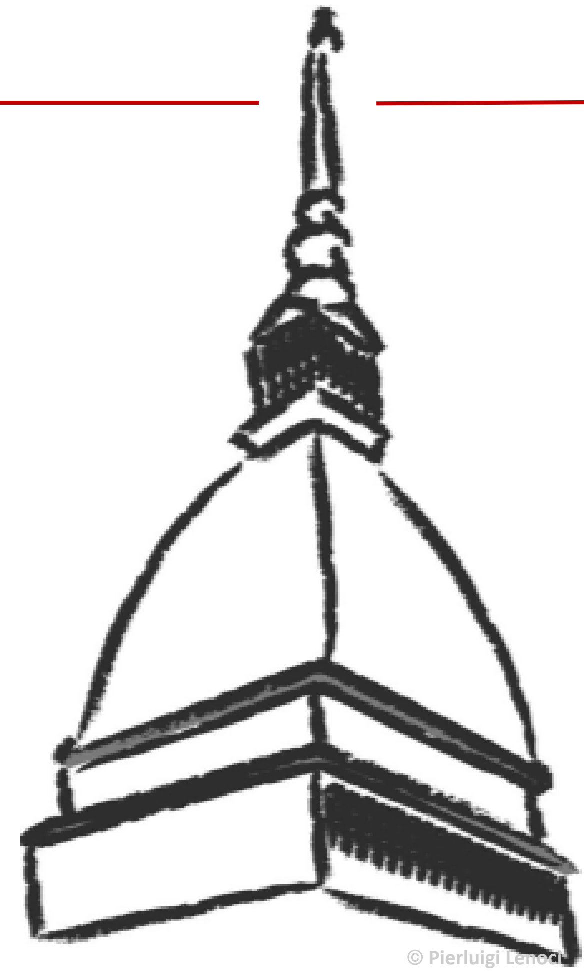
# Daniele Botto
# Costruzione di Motori per Aeromobili - Machine Design
# Fatigue - Chapter 2






## Chapters

# Index of contents

## Basic properties, data and representations

# 1. Basic ideas on fatigue data & Wöhler curve (1/10)

The Stress-Life method is the classical method for fatigue analysis of metals and has its origins in the work of Wöhler from about 1850.

A stress amplitude $\sigma_a$ is produced at a suitable location on specimen, and the number $N$ of cycles to failure is found.

Wöhler called a "safe stress level" the one below which failure does not occur.

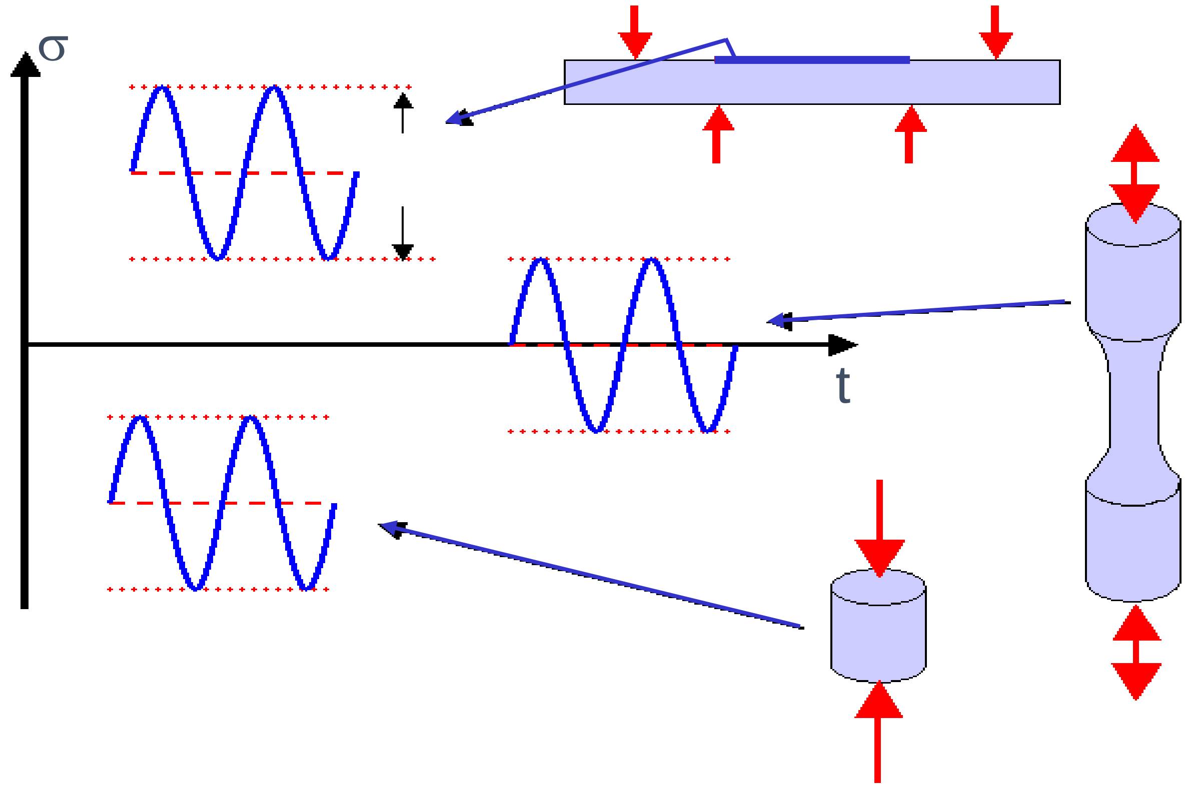
Above this safe stress level failure will occur within a certain "life", measured as number of cycles.

Crack growth is not explicitly accounted for in the Stress-Life method. Because of this, Stress-Life methods are often considered *crack initiation* (or *incubation*) life estimates.

| | | | |
|---|---|---|---|
| Diameter of round specimens: | steel | 6 ÷ 8 mm | |
| | light alloys | 8 ÷13 mm | *frequently used values* |
| Width, rectangular specimens: | steel | 2 ÷ 5 mm | |
| | light alloys | 4 ÷ 8 mm | |

The basic definitions of fatigue diagrams, symbols and names are in Ch. 1 Sect. 5

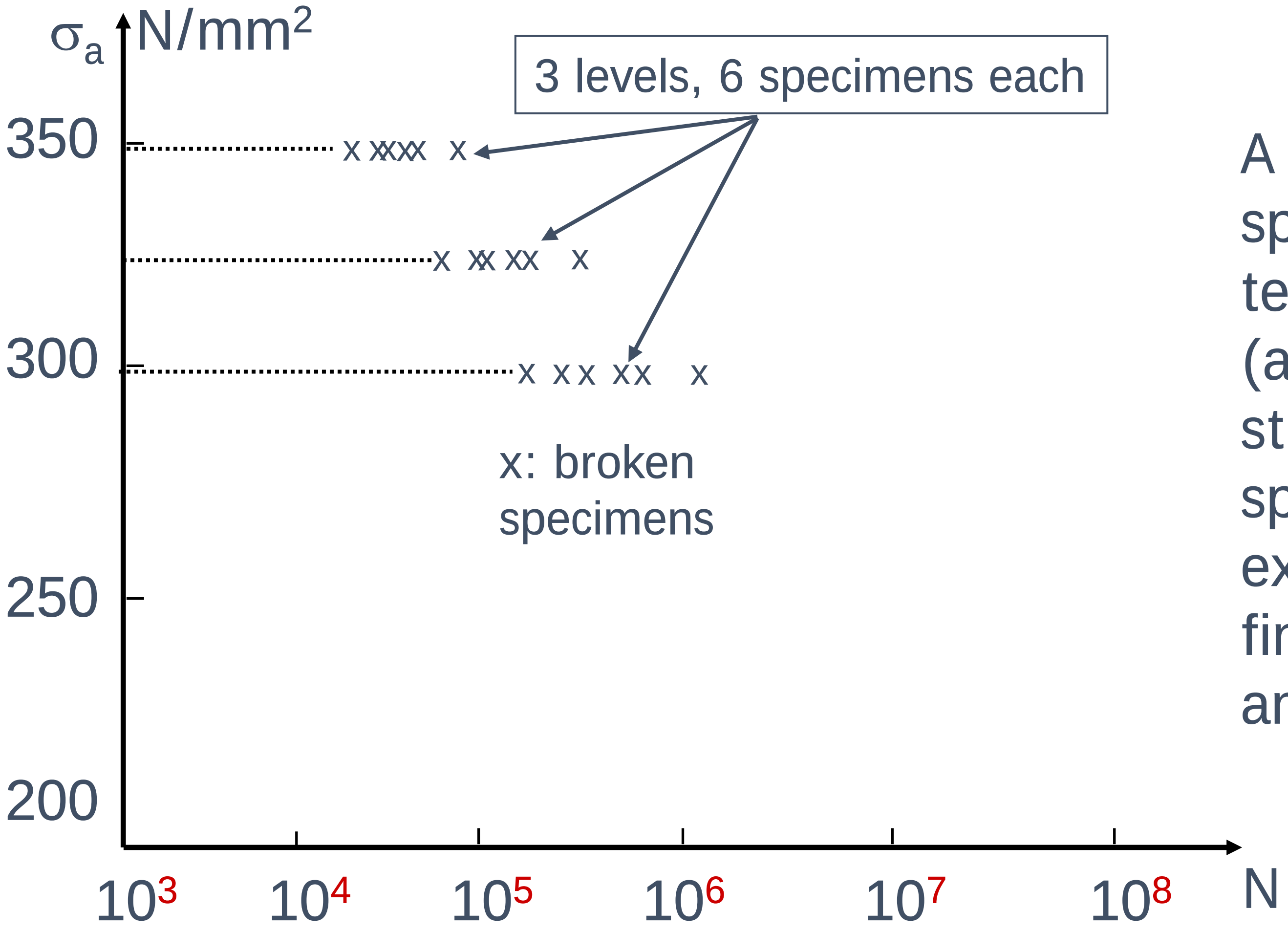
The testing machines are described in Ch. 1 Sect. 10



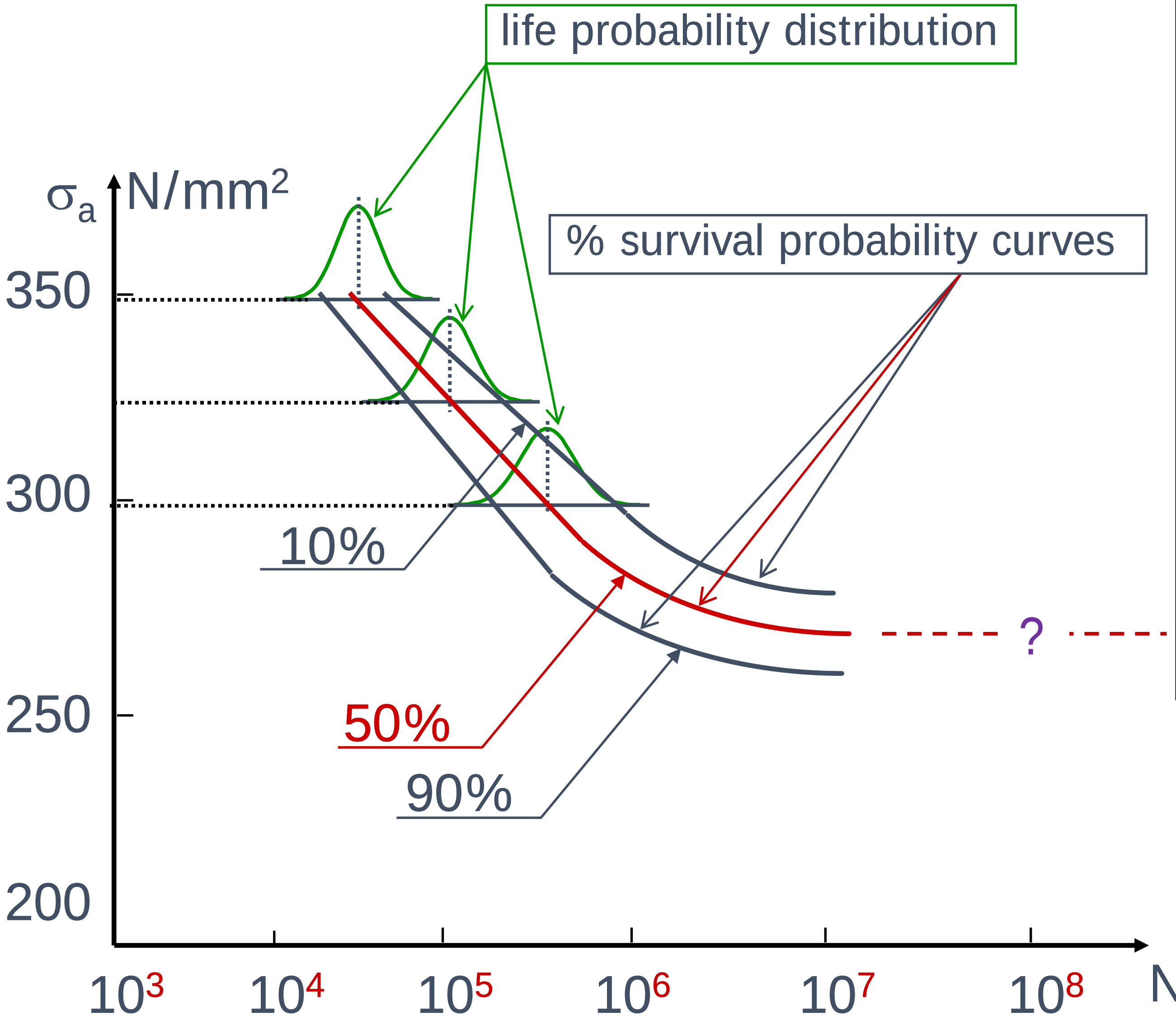

dauskardt.stanford.edu/.../bmenzel_research.html

Experimental determination of Wöhler curves

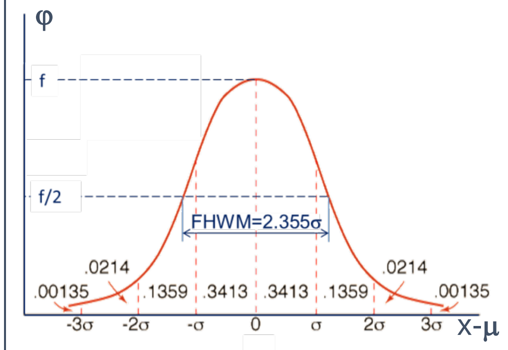
The basic procedures of Wöhler test are outlined in ch. 1 sect. 4




A group of specimens is tested at each of (at least) three stress levels spanning the expected range of finite life stress amplitude




Data are used to obtain the “life probability” curves at each stress level, on which the stress-life curves are then fitted.




$$\varphi = \frac{1}{f} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

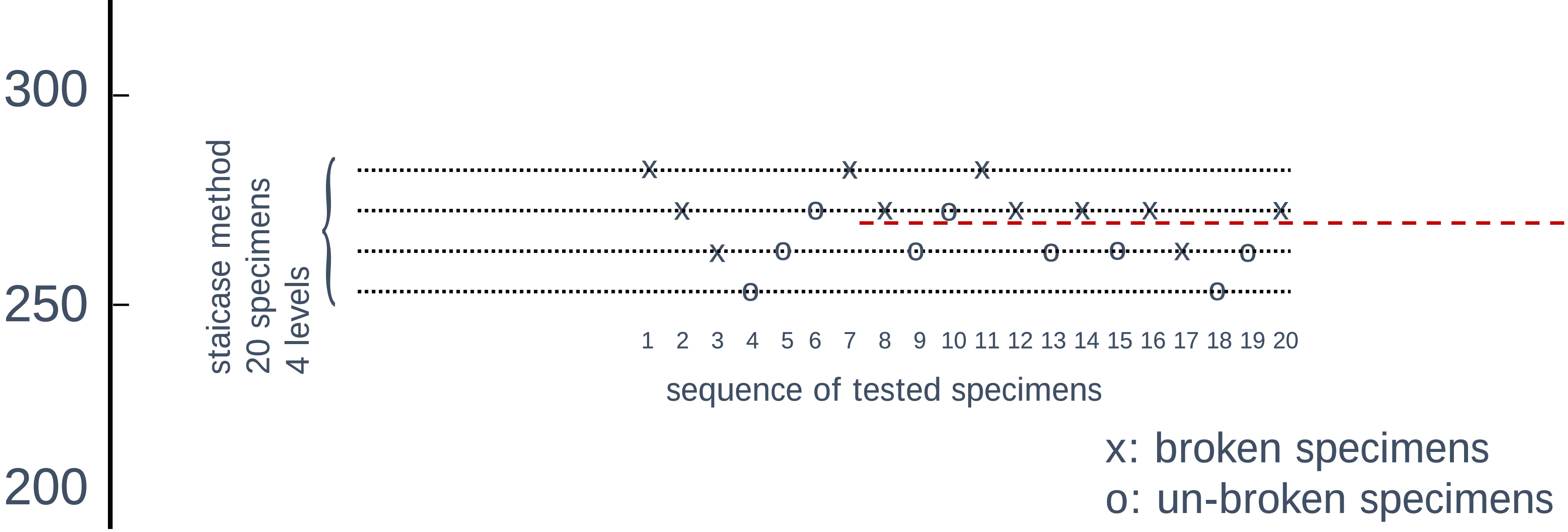
$$f = \sigma\sqrt{2}$$

$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\,\sigma$$

The **fatigue limit** can be determined with the "**staircase**" method. A small number of equally spaced stress levels is set around the expected fatigue limit. The first specimen is tested at the highest stress level. If failure occurs, a further test is carried out at a one-step down stress.

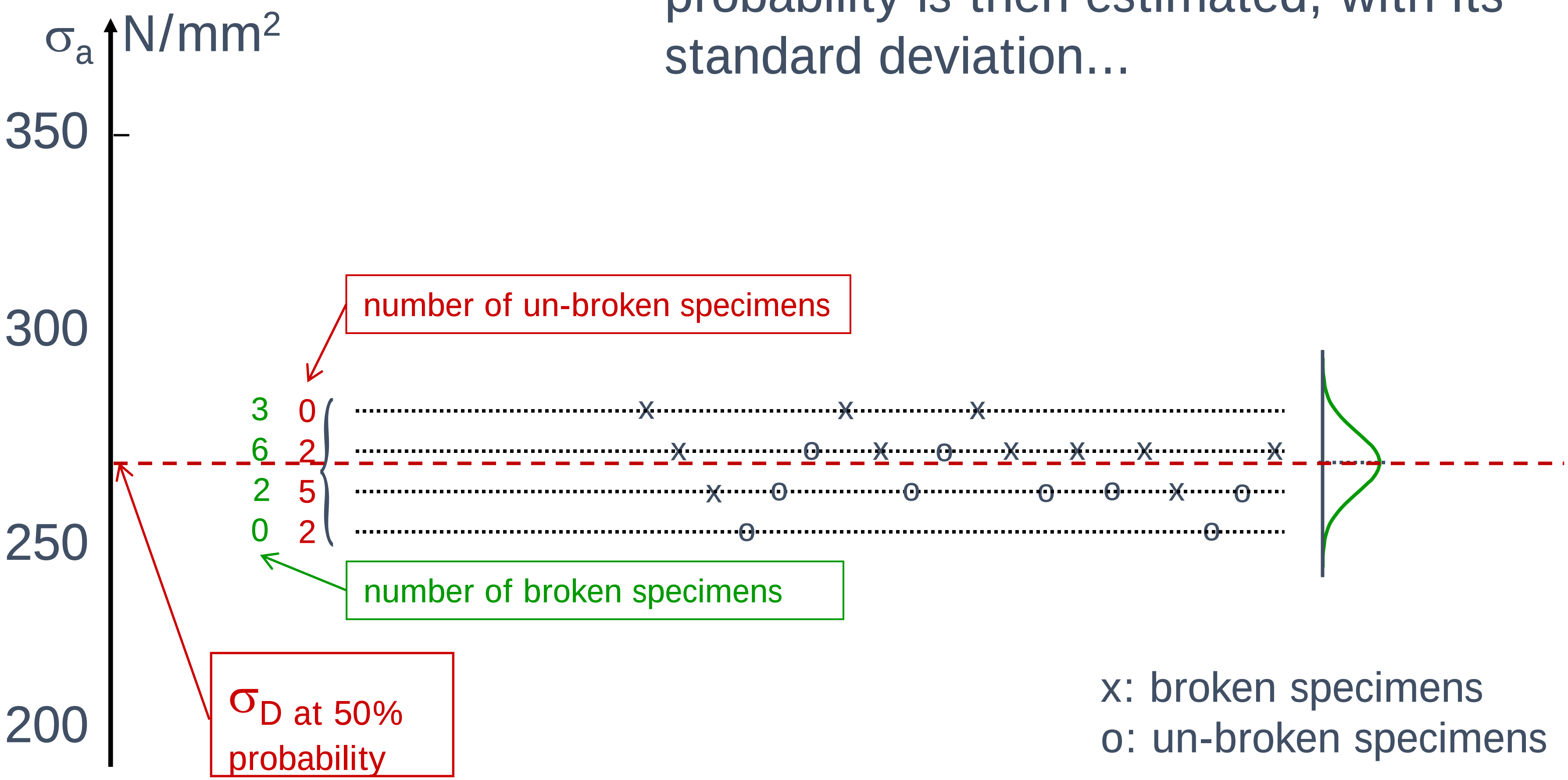
If failure does not occur (say, at $10^7$ cycles) the test is stopped and a further specimen is tested at a one-step up stress level.

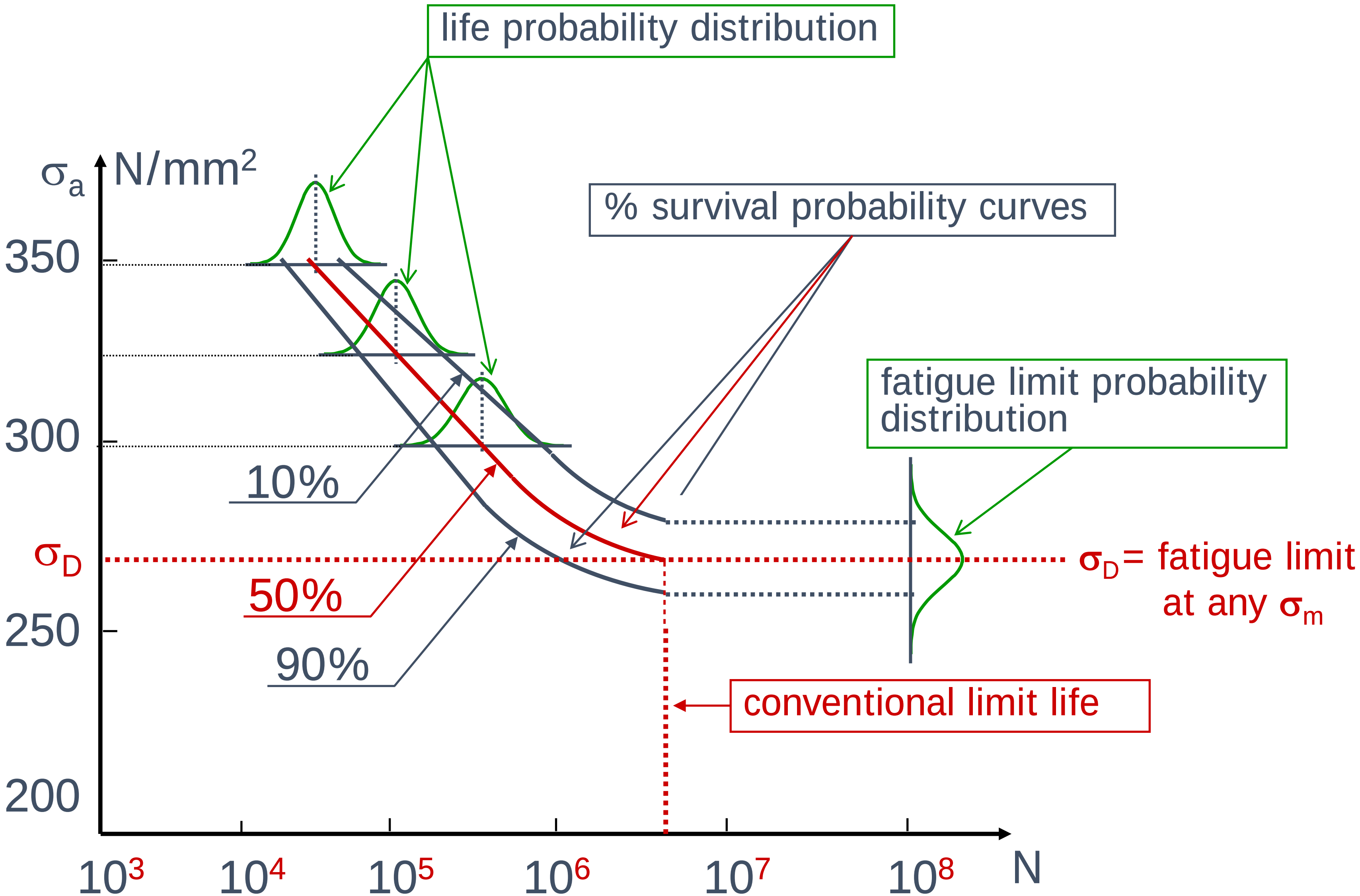
Up to 15÷20 specimens must be employed.



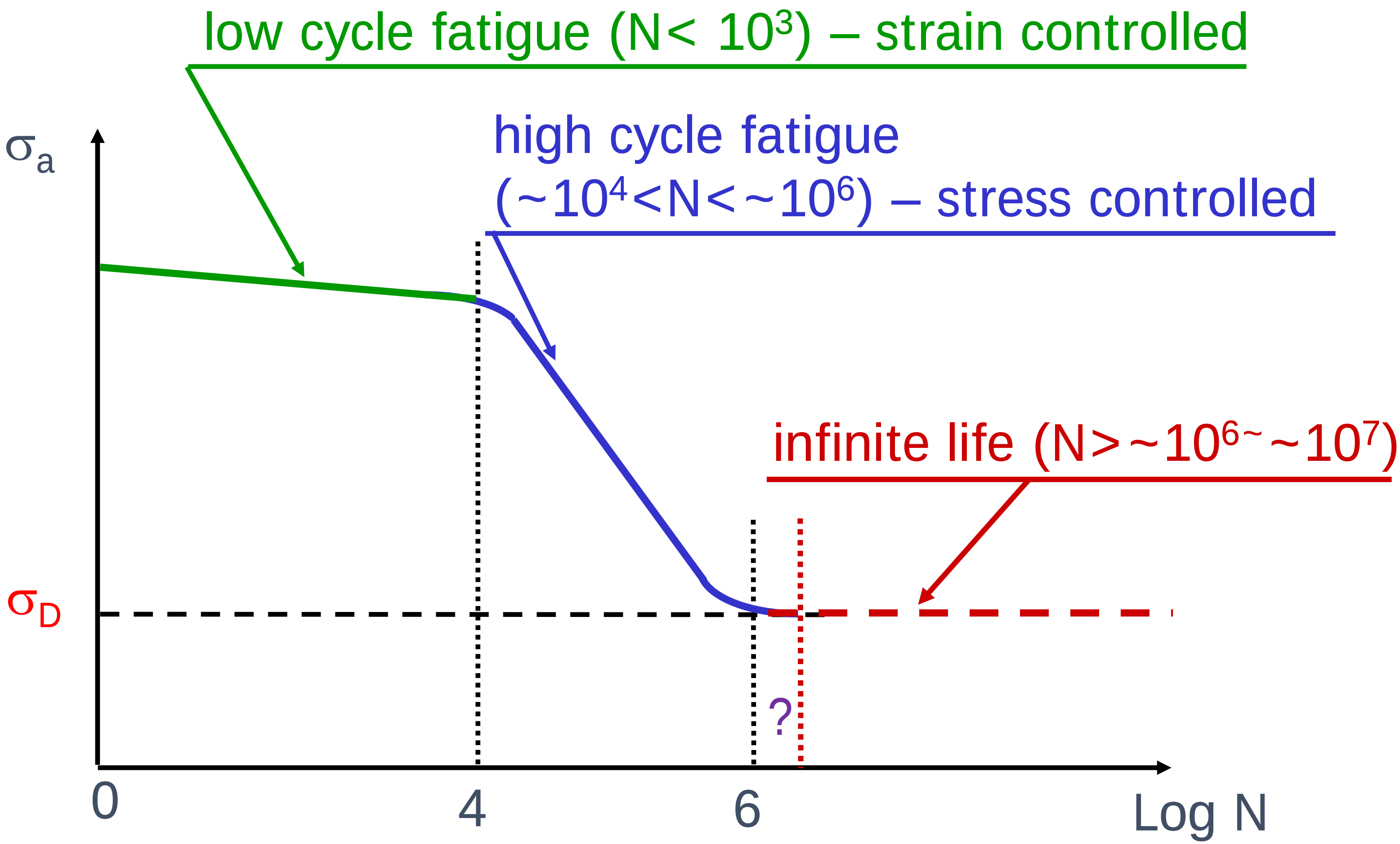

x: broken specimens
o: un-broken specimens

Through the appropriate statistical tools, the fatigue limit at 50% level of probability is then estimated, with its standard deviation…




life probability distribution
% survival probability curves
fatigue limit probability distribution
σa N/mm2
350
300
250
200
σD
10%
50%
90%
σD= fatigue limit at any σm
conventional limit life
10^3
10^4
10^5
10^6
10^7
10^8
N

Wöhler curve can be divided in three parts



Certain materials have a *endurance limit* which represents a stress (amplitude) level below which the material does not fail and can be cycled infinitely. If the applied stress (amplitude) level is below the endurance limit of the material, the structure is said to have an *infinite life*. It is also called *fatigue limit* .

Curve A shows an endurance limit, typical of steel and titanium in benign (non-corrosive) environmental conditions

If a “knee” on the curve is not easily detectable, fatigue limit $\sigma_D$ is defined at $N=10^6$ or $2x10^6$ or $5x10^6$ or even $10^7$ cycles.

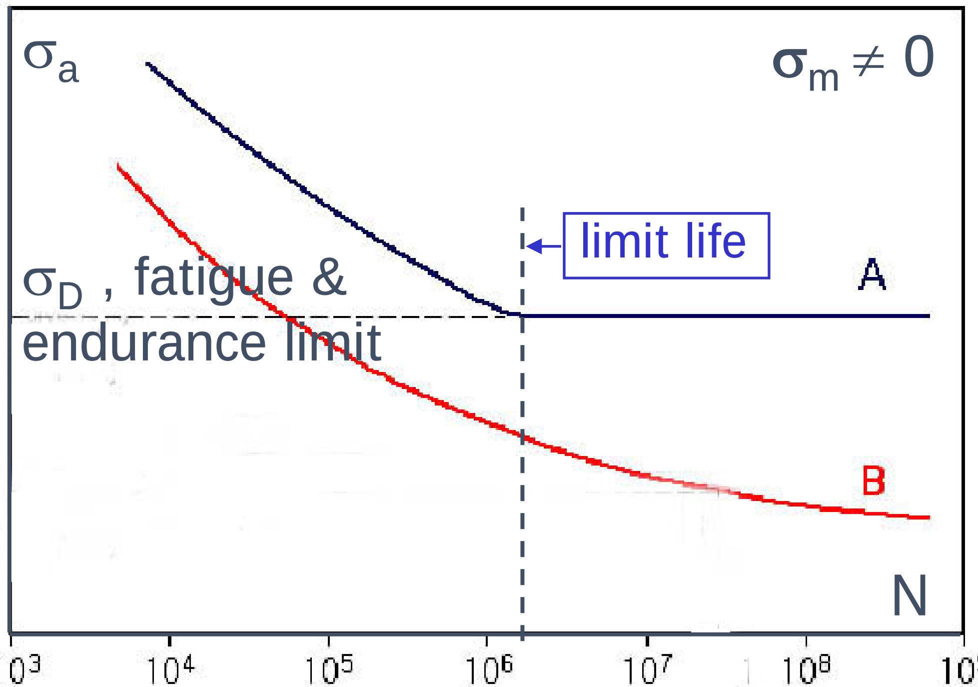



diagram from HugeDomains.com

Many non-ferrous metals and alloys, such as aluminum, magnesium, and copper alloys, do not exhibit well-defined *endurance limits*. These materials instead display a continuously decreasing S-N response, similar to Curve B**.**

In such cases a proper *fatigue limit* $\sigma_D$ for a given number of cycles must be specified.

An effective fatigue limit for these materials is defined as the stress that causes failure at $1 \times 10^8$ or $5 \times 10^8$ loading cycles.

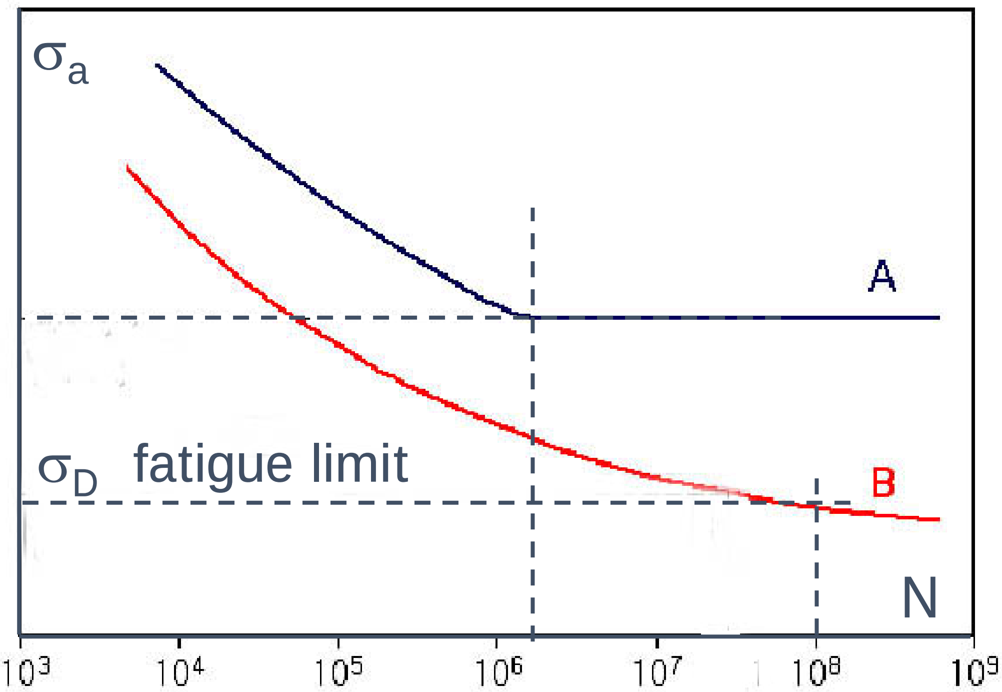



diagram from HugeDomains.com

The main points to be remembered about the FKM-Guideline named "Analytical strength assessment of components in mechanical engineering" are:

Valid for components produced with or without machining or by welding of steel, of iron or of aluminum materials that are intended for use under normal or elevated temperature conditions, and in detail:

- for static loading
- for fatigue loading with more than about $10^4$ cycles (HCF) constant or variable amplitude
- for components with geometrical notches
- for components with welded joints
- for milled or forged steel, also stainless steel, cast iron materials as well as aluminum alloys or cast aluminum alloys
- for components temperatures
  from -40°C to 500°C for steel
  from -25°C to 500°C for cast iron
  from -25°C to 200°C for alum.
- for a non-corrosive environment

# Sect. 2 to 5 - Basic material properties, data and factors

The purpose of Sections 2 to 5 of this chapter is the investigation of the main factors which influence fatigue as they are seen on the specimens which are used to characterise the materials.

They are condensed in the following:

- $R_e$ , $K_{T,D}$ and $R_m$ from monotonic static tests corrected for $d_{eff}$ , "effective diameter" of the semi-finished product (sect. 3 sl. 1,5)
- $\sigma_{D-1,T} = K_{T,D}\, f_{W,s}\, R_m = K_{T,D}\, f_{W,s} \cdot K_{d,m}\, K_A\, R_{m,N}$

  definition of $R_{m,N}$ in sect. 3 sl. 1,2 ; $K_{T,D}$ in sect. 5 sl. 1, 2;
  $f_{W,s}$ in sect. 4 sl. 1,2 ; $K_{d,m}$ in sect. 3 sl. 3; $K_A$ in sect. 3 sl. 4

A selection of data is presented, together with formulas and their coefficients. It is stressed that data and formulas must be connected in a coherent system.

Reference is made here to the FKM-Guideline 5th edition, representing the state-of-the-art knowledge of German origin at the year 2003.

Warning: an updated 6th edition 2012 is now available; variations not of interest here.

# 2. $R_m$ vs. $\sigma_{D-1}$ for steel (1/3)

Experimental observation: the fatigue limit is in a certain relation with static strength values.

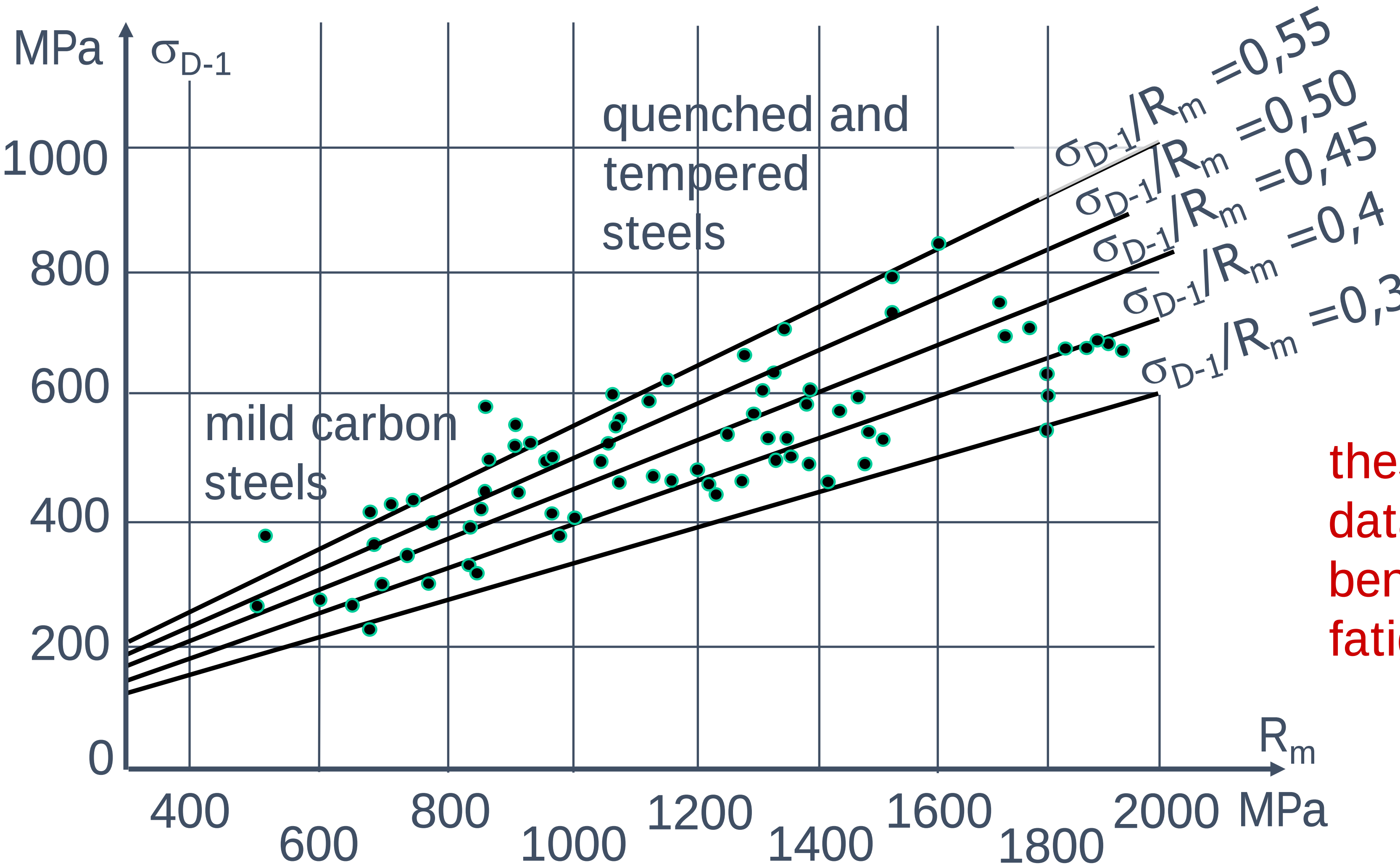
A deep knowledge of the inner reasons for this relation is of great importance for material scientists when looking for evidence and theories aiming at explaining the inner failure mechanisms in different failure modes. This will not be approached in these slides.

However, even an approximate empirical relation is of great practical importance when exact fatigue data are not available, and the fatigue limit (quite expensive to obtain) must be roughly estimated from the (much cheaper) value of tensile strength.

Next slides will show examples from the (ancient) literature.

# 2. $R_m$ vs. $\sigma_{D-1}$ for steel (2/3)

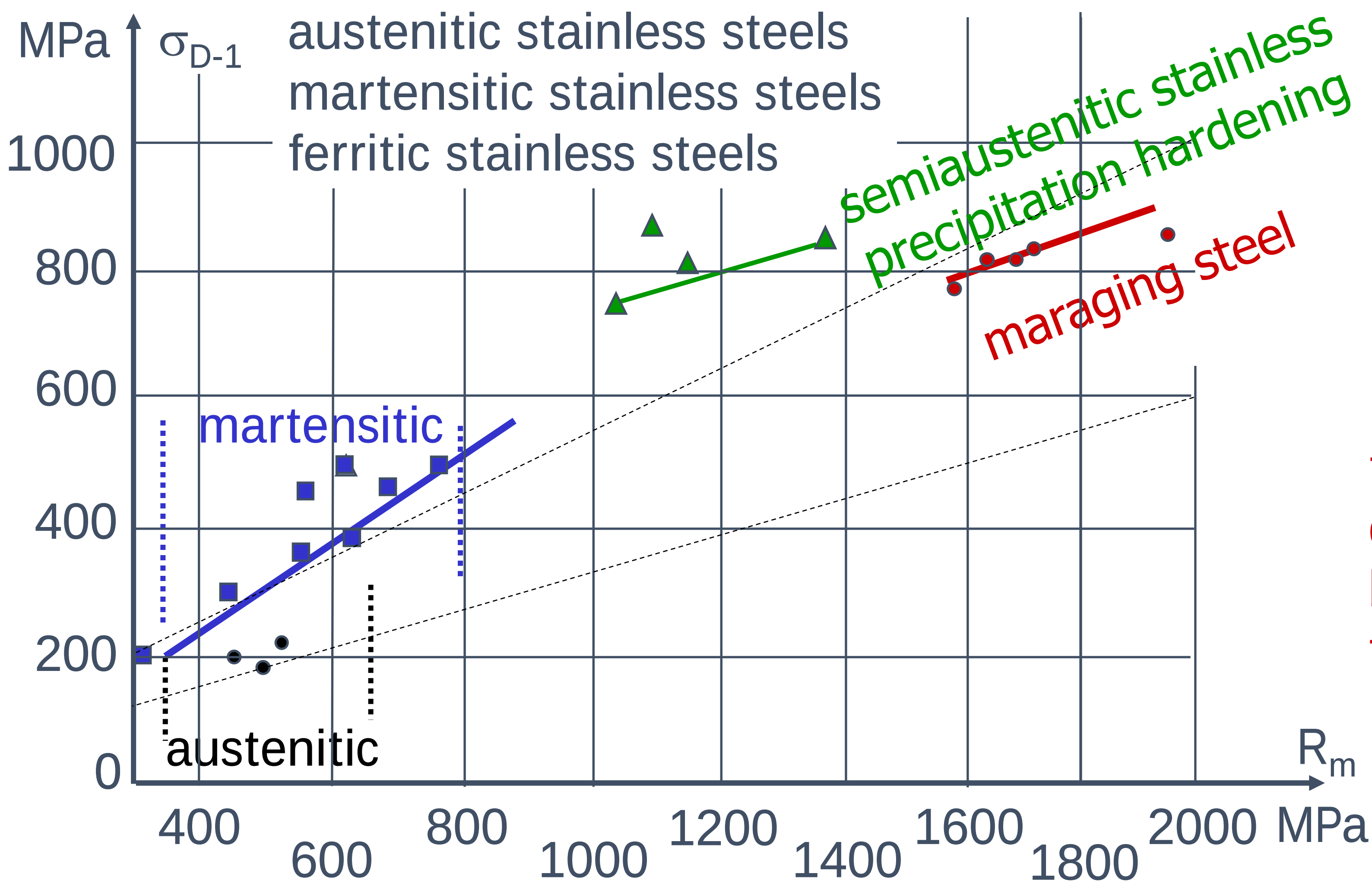
Empirical relation between static strength and fatigue limit




these data for bending fatigue !

# 2. $R_m$ vs. $\sigma_{D-1}$ for steel (3 / 3)

Empirical relation between static strength and fatigue limit




these data for bending fatigue !

# 3. Static strength values and material tables (1/11)

Reliable material data must come from an accredited source !!!

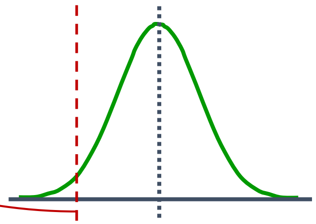
Reference will here be made to those of Analytical Strength Assessment, FKM – Forschungskuratorium Maschinenbau, 5th ed., VDMA Verlag, Frankfurt am Main, 2003 (in what follows, quoted shortly as FKM) which are an ample and coherent standard

$R_{m,N}$ : static failure stress, i.e. , the maximum stress experienced in a tensile test (minimum guaranteed value or else lower boundary of the tensile strength)

$R_{e,N}$ : minimum guaranteed value of yield strength

$\sigma_{D-1,N}$ : fatigue limit for R=-1 (goes along with its statistical definition)

The suffix $_{..N}$ indicates that the values are given in the published tables of the Standard (Norm).



$R_{m,N}$ : tensile strength, according to standard
- 97,5% probability
- tested at room temperature
- tested along the main direction of milling and forging
- in the case of steel it applies to the smallest dimension of a semi-finished product; in the case of cast iron and cast aluminium alloys to the test piece size defined in standards

$R_p$ : yield stress, a generalization of $R_{eH}$ and $R_{p0,2}$
- the same conditions hold as above for $R_{m,N}$

$\sigma_{D-1,N}$ : fatigue limit for R=-1, i.e. fully reversed stress
- however tested in tension-compression (FKM <u>does not</u> use values from rotating bending for round specimens or from alternating bending for flat specimens); the reason for this will be clear later

REMARK: the value given by FKM corresponds to 97,5% probability; FKM safety factors are coherent with this.

# 3. Static strength values and material tables (3/11)

Material strength depends on the specimen/component size.

The material "Norm" parameters are experimentally determined with specimens having the "effective diameter" $d_{eff,N}$ of the semi-finished product or the raw casting.

Different materials have a different $d_{eff,N}$ , specified in each FKM table.

FKM has formulas to modify $R_{m,N}$ and $R_{e,N}$ to pass from $d_{eff,N}$ , the "Norm" (Standard) effective diameter, to the "component" effective diameter $d_{eff}$ through the technological size factor $K_{d,m}$ :

$$\boxed{R_m = K_{d,m}\, K_A\, R_{m,N}}$$

## Definition of effective diameter $d_{eff}$

In the case of a solid round bar the diameter d is $d_{eff}$ .

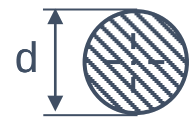
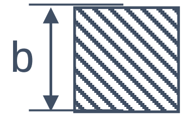
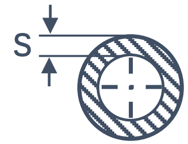
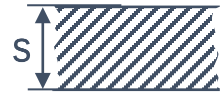
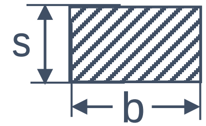
In other cases, $d_{eff}$ is calculated with the formulas defined in the table on the right, which for cases in the first two cases on the right:

$d_{eff}$ = 4 V/O, with:

V: specimen volume

O: lateral surface

In rod-shaped components made of quenched and tempered steel, the effective diameter $d_{eff}$ is the diameter existing while the treatment is performed; if machining follows, $d_{eff}$ is the larger original diameter.

| Cross section | Heat treatable steel (also forgings), case hardening steel, heat treatable cast steel, GGG, GT or GG | Non-alloyed struct. steel (also forgings), fine grained struct. steel, norm. quench. and tempered steel, Aluminium |
|---|---|---|
|   | d | d |
|   | b | b |
|   | 2s | s |
|   | 2s | s |
|   | $\frac{2bs}{b+s}$ | s |

For all steels, except stainless steel, and for cast iron the technological size factor is:

-) $d_{eff} \leq d_{eff,N} \Rightarrow K_{d,m} = 1$

-) $d_{eff,N,m} < d_{eff} \leq d_{eff,max,m} \Rightarrow$ $$K_{d,m} = \frac{1 - 0{,}7686\, a_{d,m} \log(d_{eff} / 7{,}5)}{1 - 0{,}7686\, a_{d,m} \log\left(d_{eff,N,m} / 7{,}5\right)}$$

$d_0 = 7{,}5$ mm

(*0,7686=1/log20*)

mm

-) $d_{eff} \leq d_{eff,max,m} \Rightarrow K_{d,m} = K_{d,m}(d_{eff,max,m})$

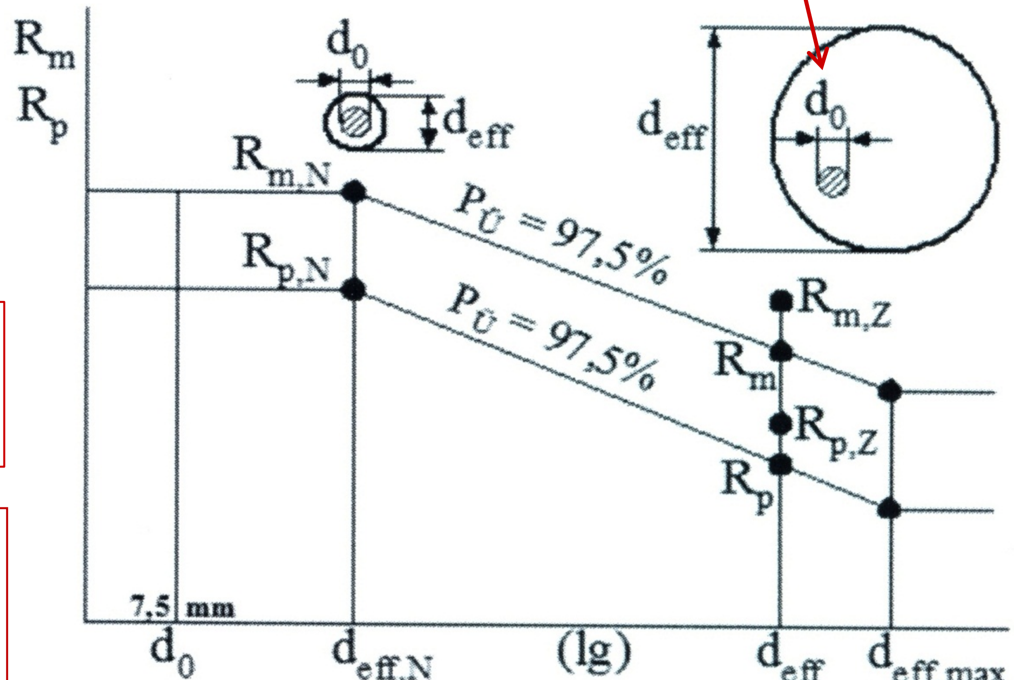
In short

$$\begin{cases} K_{d,m} = 1 \text{ for } d_{eff} \leq d_{eff,N} \\ K_{d,m} < 1 \text{ for } d_{eff} > d_{eff,N} \end{cases}$$

For milled steel $d_{eff,max\ (m\&p)} = 250$ mm

For other steels $d_{eff,max\ (m\&p)} = \infty$

For yield strength here called $R_p$ replace the values $K_{d,m}$ , $d_{eff,N,m}$ , $a_{d,m}$ with $K_{d,p}$ , $d_{eff,N,p}$ , $a_{d,p}$




source: FKM

Examples of ($d_{eff,N}$ , $a_{d,m}$ ) from FKM 5th edition 2003

| | | |
|---|---|---|
| Non-alloy structural steels DIN EN 10025 (1994), grades S185, S235, S275, S355 | $d_{eff,N,m}$=40 mm<br>$d_{eff,N,p}$=40 mm | $a_{d,m}$=0,15<br>$a_{d,p}$=0,3 |
| Weldable fine grain structural steels in normalised condition DIN 17102 (1983) grades StE255 → StE500 | $d_{eff,N,m}$=70 mm<br>$d_{eff,N,p}$=40 mm | $a_{d,m}$=0,2<br>$a_{d,p}$=0,3 |
| Heat treatable steel in quenched and tempered condition DIN EN 10083-1 (1996) grades C22 → C60, 28Mn6 → 51CrV4 | $d_{eff,N,m}$=16 mm<br>$d_{eff,N,p}$=16 mm | $a_{d,m}$=0,3<br>$a_{d,p}$=0,4 |
| Case hardening steels in the blank hardened cond. DIN EN 10084 (1998) grades C10E →14NiCrMo13-4 | $d_{eff,N,m}$=16 mm<br>$d_{eff,N,p}$=16 mm | $a_{d,m}$=0,5<br>$a_{d,p}$=0,5 |
| Spheroidal graphite cast irons DIN EN 1563 (1997) grades GJS 350-22-LT → GJS 900-2 | $d_{eff,N,m}$=60 mm<br>$d_{eff,N,p}$=60 mm | $a_{d,m}$=0,15<br>$a_{d,p}$=0,15 |

# 3. Static strength values and material tables (7/11)

In the case of wrought aluminium alloys FKM tables give “component” values $R_m$ and $R_e$ of the semi-finished product indicated in the table, according to size. Example: strips & sheets, extruded rods & bars, tubes & profiles of "DIN EN” norms.

Norms other than FKM also for steel provide tables with size-dependent values (example, the italian standard UNI-EN 10083.1).

Whatever the method, the important fact is:
that these experimental material parameters are dependent on the size in a way which cannot be neglected, specially on account of the fact that specimens are in the smaller size range, and strength values decrease with increasing size.

# 3. Static strength values and material tables (8/11)

According to FKM, transition from values for static strength coming from "normal" laboratory conditions, i.e. longitudinal test specimen, to "real component" with material anisotropy is done through the multiplying factor:

$K_A$ = anisotropy factor, used in: $R_m = K_{d,m}\ K_A\ R_{m,N}$

which allows for the fact that the strength values of milled steel and forgings are lower transversally to the direction of milling and forging.

For strength in the direction of processing the factor is 1.

For strength trasverse to the main processing direction:

| | | | | |
|---|---|---|---|---|
| ($R_m$ in MPa) Steel | up to 600 $K_A$= 0,90 | from 600 to 900 $K_A$= 0,86 | from 900 to 1200 $K_A$= 0,83 | above 1200 $K_A$= 0,80 |
| ($R_m$ in MPa) Alum. Alloys | up to 200 $K_A$= 1,00 | from 200 to 400 $K_A$= 0,95 | from 400 to 600 $K_A$= 0,90 | source: FKM |

# 3. Static strength values and material tables (9/11)

The following tables sample some values relating static tensile properties with the fatigue limit

$\sigma^{tc}_{D-1,N}$

tested in tension-compression.

| Steel | Type | $R_{m,N}$ | $R_{e,N}$ | $\sigma^{tc}_{D-1,N}$ | $\sigma^{b}_{D-1,N}$ |
|---|---|---|---|---|---|
| Weldable fine grain structural steels * DIN EN 10025/94 DIN 17102 (1983 | S 235 JR | 360 | 235 | 160 | 180 |
| | S 275 JR | 430 | 275 | 195 | 215 |
| | S 355 JR | 510 | 355 | 230 | 255 |
| | StE 255 | 360 | 255 | 160 | 180 |
| | StE 500 | 610 | 500 | 275 | 300 |
| Case hardening steels # blank hardened DIN EN 10084/98 | 16MnCr5 | 1000 | 695 | 400 | 430 |
| | 20MnCr5 | 1200 | 850 | 480 | 510 |
| | 18CrMo4 | 1100 | 775 | 440 | 470 |

Reduced and adapted from FKM

* $d_{eff,N}$ = 40 mm    # $d_{eff,N}$ = 16 mm

(continued)

| Steel | Type | $R_{m,N}$ | $R_{e,N}$ | $\sigma^{tc}_{D-1,N}$ | $\sigma^{b}_{D-1,N}$ |
|---|---|---|---|---|---|
| Quenched and tempered steels in quenched and tempered condition<br>DIN EN 10083/96 | C35 | 630 | 430 | 285 | 310 |
| | C45 | 700 | 490 | 315 | 345 |
| | C60 | 850 | 580 | 385 | 415 |
| | 34Cr4 | 900 | 700 | 405 | 435 |
| | 34CrMo4 | 1000 | 800 | 450 | 480 |
| | 50CrMo4 | 1100 | 900 | 495 | 525 |
| source: FKM | 36NiCrMo16 | 1250 | 1050 | 560 | 595 |

$d_{eff,N}$ = 16 mm

A sample from the FKM material properties for an aluminium alloy

| Alloy | Type / condition | | Thickness from......to | | $R_m$ | $R_e$ | $\sigma^{tc}_{D-1,N}$ | $\sigma^{b}_{D-1,N}$ | $A_{10}\%$ |
|---|---|---|---|---|---|---|---|---|---|
| Wrought aluminium alloys strips sheets plates | EN AW-2014 AlCu4SiMg | T3 | 0.4 | 1.5 | 395 | 245 | 120 | 140 | 14 |
| | | | 1.5 | 6.0 | 400 | 245 | 120 | 140 | 14 |
| | | T4 T451 | 0.4 | 1.5 | 395 | 240 | 120 | 140 | 14 |
| | | | 1.5 | 6.0 | 395 | 240 | 120 | 140 | 14 |
| | | | 6.0 | 12.5 | 400 | 250 | 120 | 140 | 14 |
| | | T451 | 12.5 | 40.0 | 400 | 250 | 120 | 140 | 10 |
| | | | 40.0 | 100.0 | 395 | 250 | 120 | 140 | 7 |
| | | ........................ | | | | | | | |
| DIN EN 485-2 (1995) source: FKM | | T6 T651 | 0.4 | 1.5 | 440 | 390 | 130 | 150 | 6 |
| | | | 1.5 | 6.0 | 440 | 390 | 130 | 150 | 7 |
| | | | 6.0 | 12.5 | 450 | 395 | 135 | 155 | 7 |

Here we see also $\sigma^{b}_{D-1,N}$ = fatigue limit in bending

Remark: $K_{d,m}=1$

$A_{10}\%$ = elongation over a length $11{,}3\sqrt{S_0}$ (i.e. L=10d)

The fatigue limit $\sigma_{D-1}$, in fully reversed stress (R=-1), is either presented in tables, as in the previous section, or expressed through formulas, as it will be shown soon.

The second approach is particularly useful in all those cases where fatigue data are not provided by material tables, which is quite frequent (fatigue data are often proprietary data).

According to FKM the material fatigue strength values presented in their own tables are intended for information only.

FKM requires to determine fatigue limits as described below ($\sigma_{D-1..}$ values here defined are $^{tc}$, i.e. those in tension-compression):

A) $\sigma^{tc}_{D-1} = f_{W,\sigma}\, R_m$ B) $\tau^{tc}_{D-1} = f_{W,\tau}\, \sigma^{tc}_{D-1}$

where coefficients $f_{W,\sigma}$ , $f_{W,\tau}$ are found in the following table:

| Type of material | $f_{W,\sigma}$ * | $f_{W,\tau}$ | Type of material | $f_{W,\sigma}$ | $f_{W,\tau}$ |
|---|---|---|---|---|---|
| Case hardening steel | 0,40 | 0,577# | GT- malleable cast iron | 0,30 | 0,75 |
| Stainless steel | 0,40 | 0,577 | GG - grey cast iron | 0,30 | 0,85 |
| Forging steel | 0,40 | 0,577 | Wrought alum. alloys ** | 0,30 | 0,577 |
| Steels other than these | 0,45 | 0,577 | Cast alum. alloys ** | 0,30 | 0,577 |
| GS – steel castings | 0,34 | 0,577 | * $f_{W,\sigma}$ values for $N=10^6$ cycles | | |
| GGG – spheroidal graphite cast iron | 0,34 | 0,65 | ** does not correspond to the $N=\infty$ limit, but to $N=10^6$ | | |

source: FKM

Compare these values with Sect. 2 Sl. 2

Adjourned symbols today are: GGG ⇒ GJS
GT ⇒ GJM
GG ⇒ GJL

# easy to check that $0{,}577 = 1/\sqrt{3}$, which is related to a Von Mises-like formula

# 5. Temperature factor (1/2)

Fatigue limits are calculated in Sect. 4 for the material in the test specimen form and at room (normal) temperature.

Fatigue limits are lower at increased temperature.

The following are the ranges of "normal temperatures" where no correction is needed (FKM):

| | |
|---|---|
| - fine grain structural steel: | from: – 40°C to 60°C |
| - other kinds of steel: | from: – 40°C to 100°C |
| - cast iron materials: | from: – 25°C to 100°C |
| - age hardening alum. alloys: | from: – 25°C to 50°C |
| - non age-hard. alum. alloys: | from: – 25°C to 100°C |

For temperatures outside the range:

$\sigma_{D-1,T} = K_{T,D}\ \sigma_{D-1}$ $\qquad$ $\tau_{D-1,T} = K_{T,D}\ \tau_{D-1}$ $\qquad$ … … …

(for brevity the apex $^{tc}$ is here omitted)

# 5. Temperature factor (2/2)

source: FKM

... ... ... where empirical interpolation formulas give:

at "normal" temperature: $K_{T,D} = 1$

at low temperature: it is necessary to check specific guidelines

at high temperature (T is in °C):

- -fine grain structural steel: $K_{T,D} = 1 - 10^{-3}\, T$ *
- -other kinds of steel: $K_{T,D} = 1 - 1.4\ 10^{-3}\, (T-100)$ *
- -GS: $K_{T,D} = 1 - 1.2\ 10^{-3}\, (T-100)$ *
- -GGG, GT, GG: $K_{T,D} = 1 - a_{T,D}\, 10^{-3}\, (T-100)$ *
- -aluminium alloys: $K_{T,D} = 1 - 1.2\ 10^{-3}\, (T-50)$ **

| material | GGG | GT | GG |
|---|---|---|---|
| $a_{T,D}$ | 1,6 | 1.3 | 1.0 |

* valid up to 500 °C

** valid up to 200 °C

# Sections 6 to 9 - Mean stress and Haigh diagram

Section 6 shows how the experimental Haigh diagram, for all lives, can be derived from Wöhler curves.

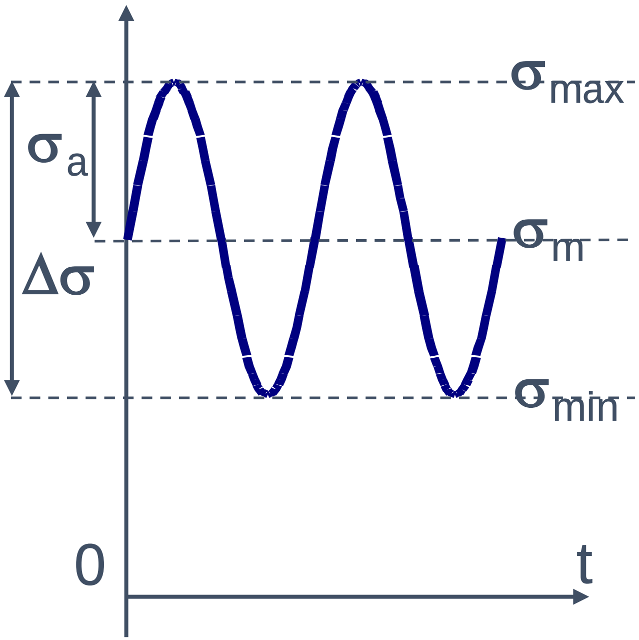
Section 7 shows the “master diagram”, a popular diagram for design purposes which accumulates fatigue data under different mean stresses and presents each line as the fatigue life under the net of maximum and minimum stresses in addition to mean stress and alternating stress. Moreover, a “universal representation allows to collect a family of material under one diagram.

Section 8 must be understood as an interpolation exercise, whith under certain assumption permits to draw the Haigh diagram with approximate curves that are convenient for later formula manipulations.

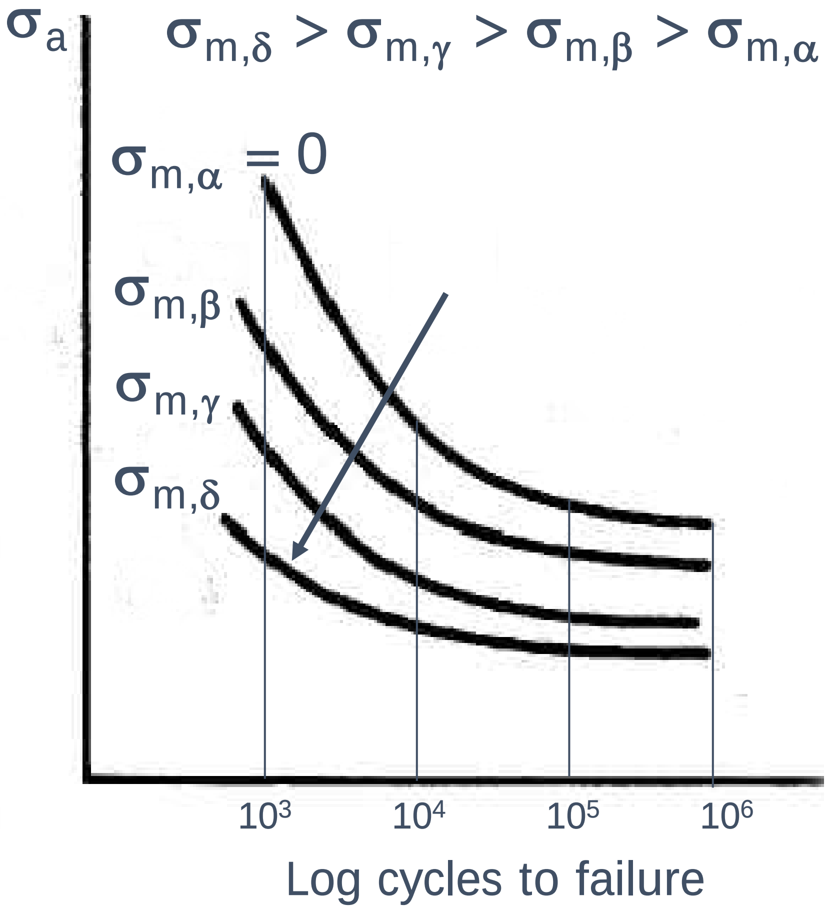
Section 9 the linear simplifications of the Haigh curve at infinite life, in the way they are currently used by design standards.

# 6. Influence of mean stress on fatigue (1/4)

When mean stress $\sigma_m$ is different from zero Wöhler curves will be different from those obtained for $\sigma_m = 0$ or stress ratio R =-1.



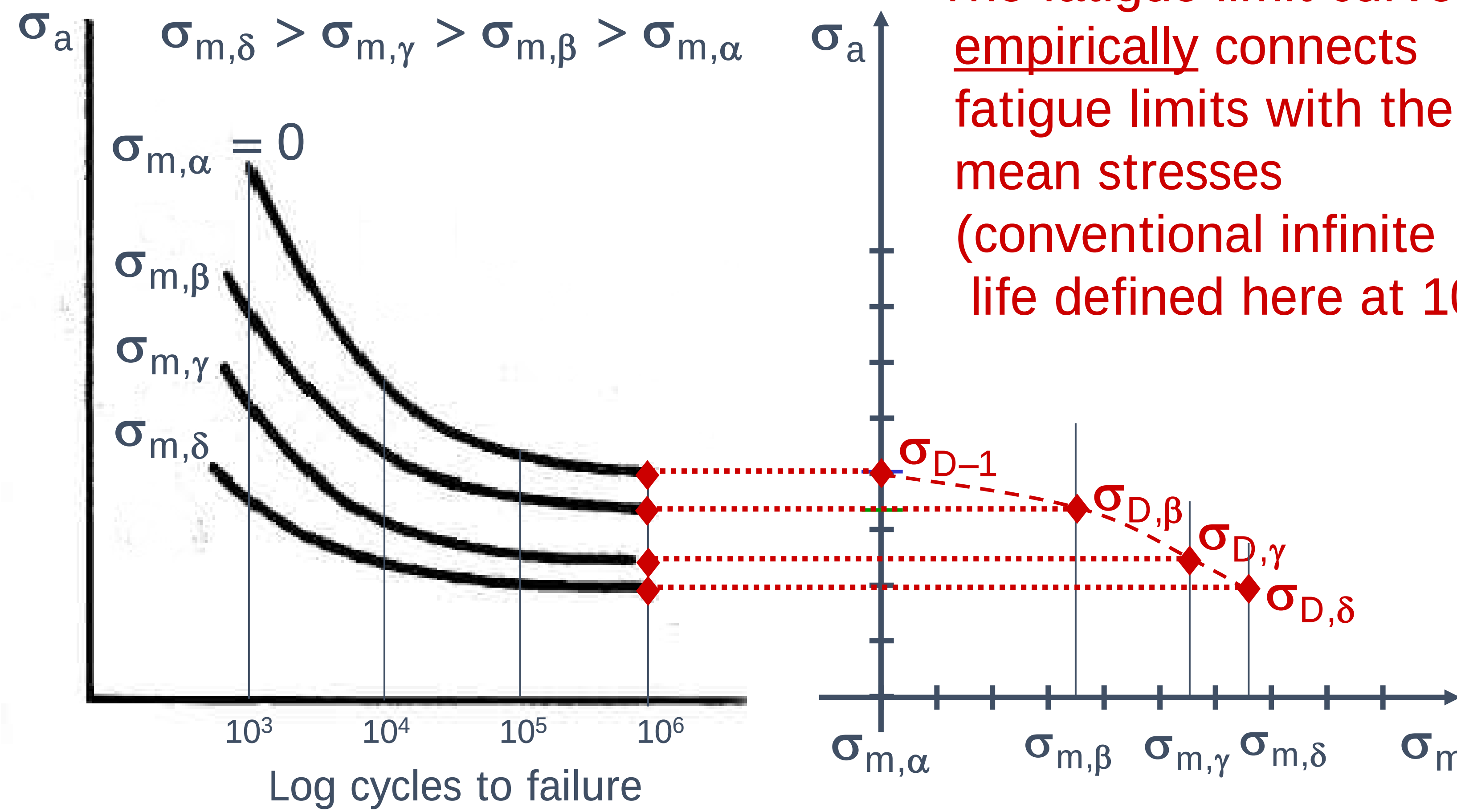

stress ratio $R = \dfrac{\sigma_{min}}{\sigma_{max}}$



The diagrams below qualitatively show the effect of mean stress $\sigma_m$ on fatigue limits $\sigma_D$ .

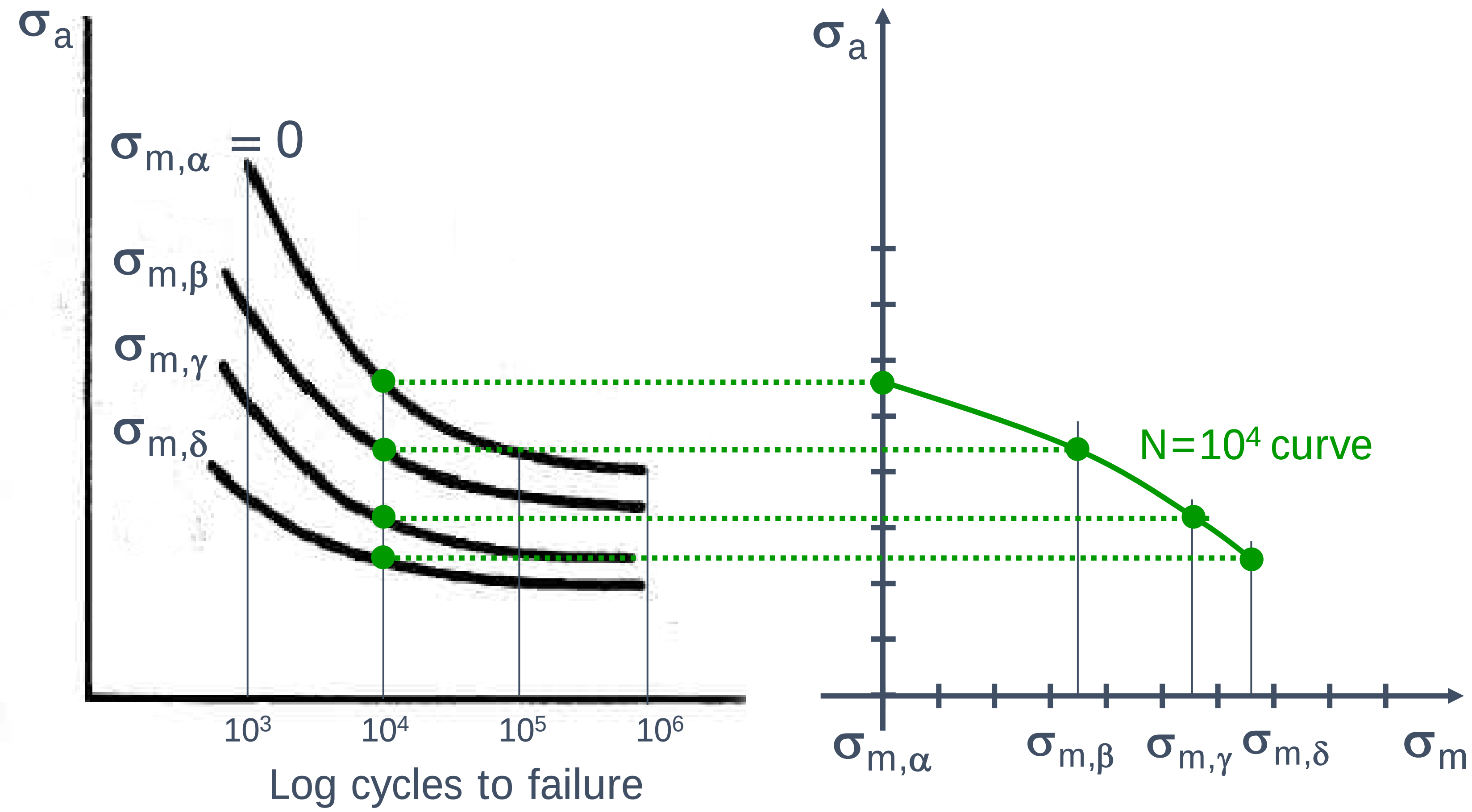
The diagram on the right is called “Haigh” diagram.

The fatigue limit curve <u>empirically</u> connects fatigue limits with their mean stresses (conventional infinite life defined here at $10^6$)

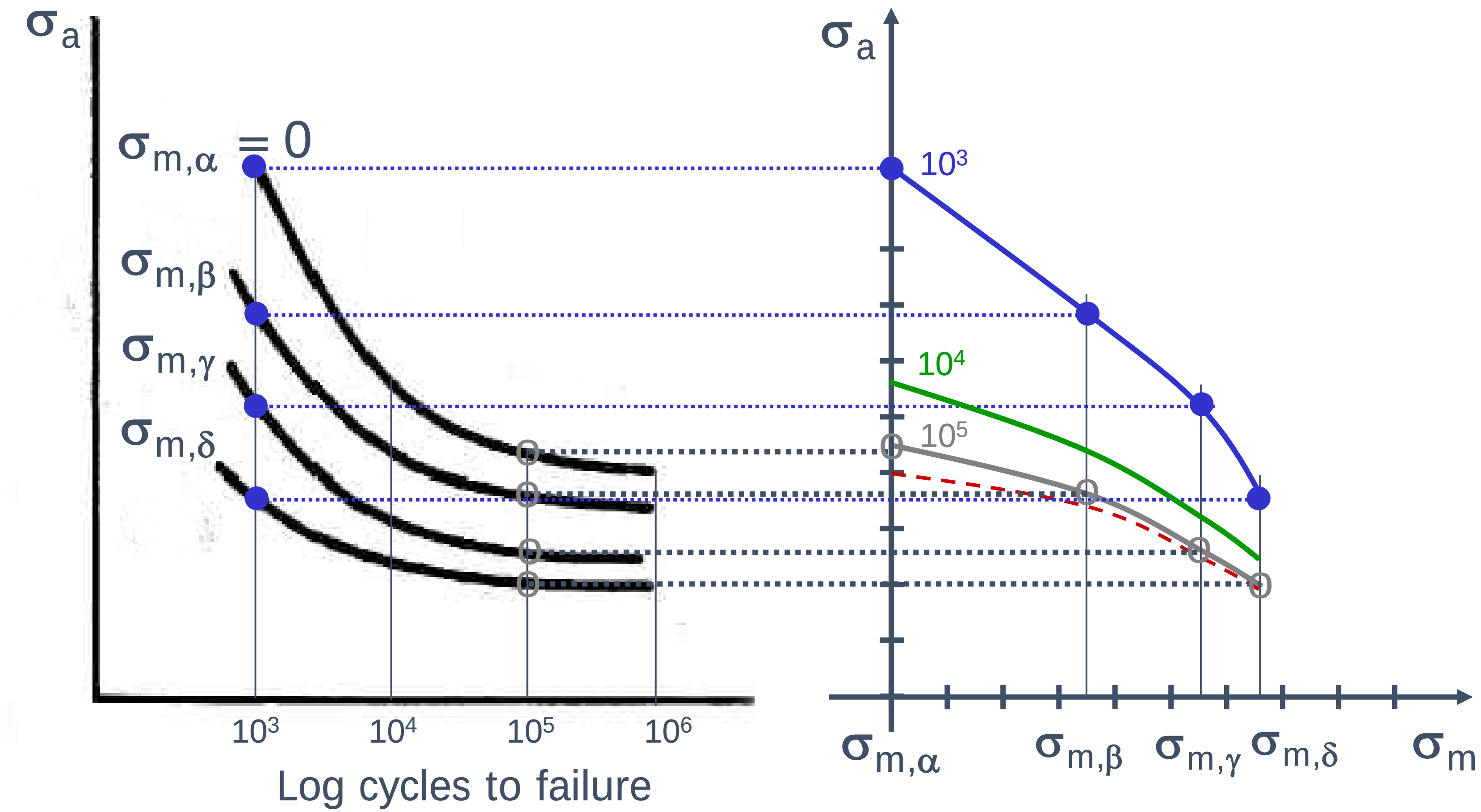


# 6. Influence of mean stress on fatigue (3/4)

As shown below, a constant life curve is generated from a set of Wöhler curves for different “mean stress” values.

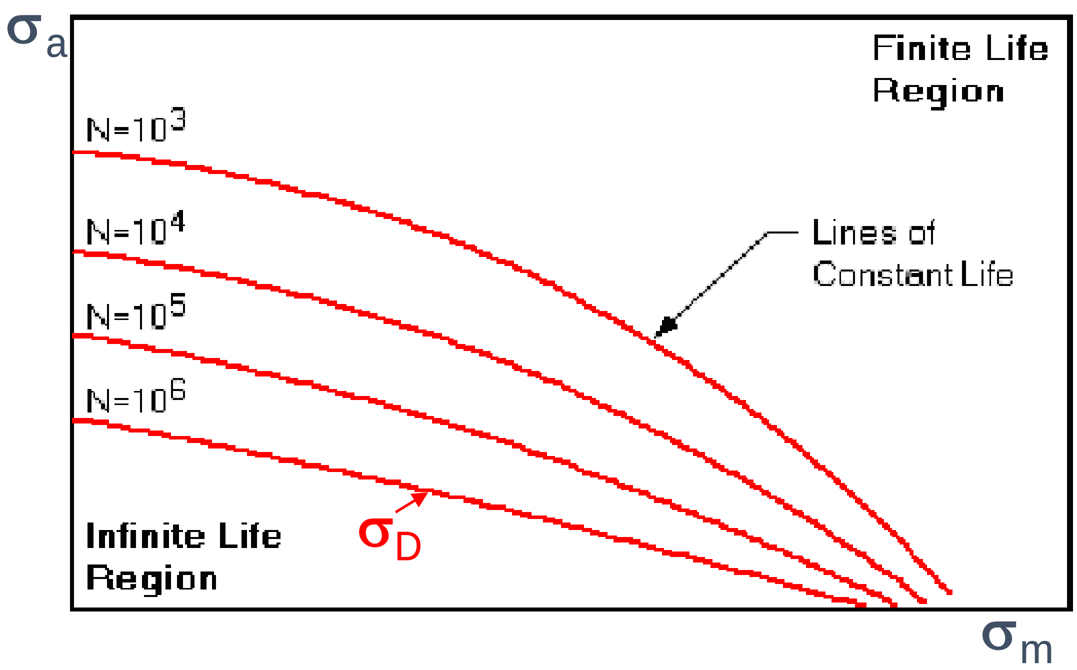


Here the curves for three finite lives ($10^3$, $10^4$, $10^5$) plus the fatigue limit ($N \geq 10^6$) are obtained from Wöhler curves and represented in the Haigh diagram.



# 7. The Haigh plot: experimental data (1/6)

A Haigh diagram plots the mean stress, mostly in the tensile range, along the x-axis and the oscillatory stress amplitude along the y-axis. It sometimes called also “Goodman” diagram.

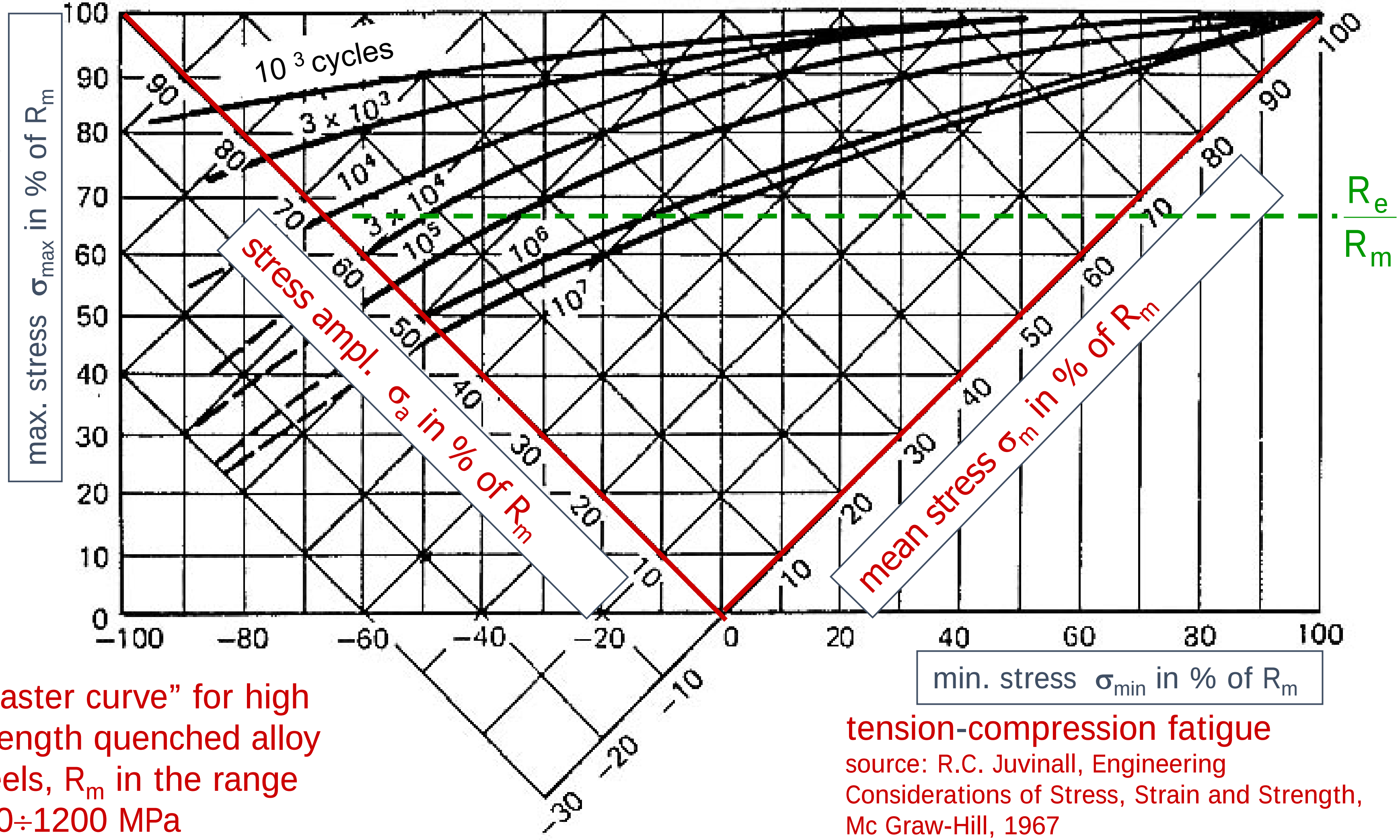



The next slide will show a “master diagram” i.e. a Haigh diagram normalised expressing them in % of $R_m$. Why? Because it is expected that such curves will be valid for a family of materials:

in this case AISI 4340 ≡ 34CrNiMo6 AISI 2330 ≡ 60SiMn5
AISI 4130 ≡ 25CrMo4 AISI 8630 ≡ 30NiCrMo22

a sample of steel alloy data

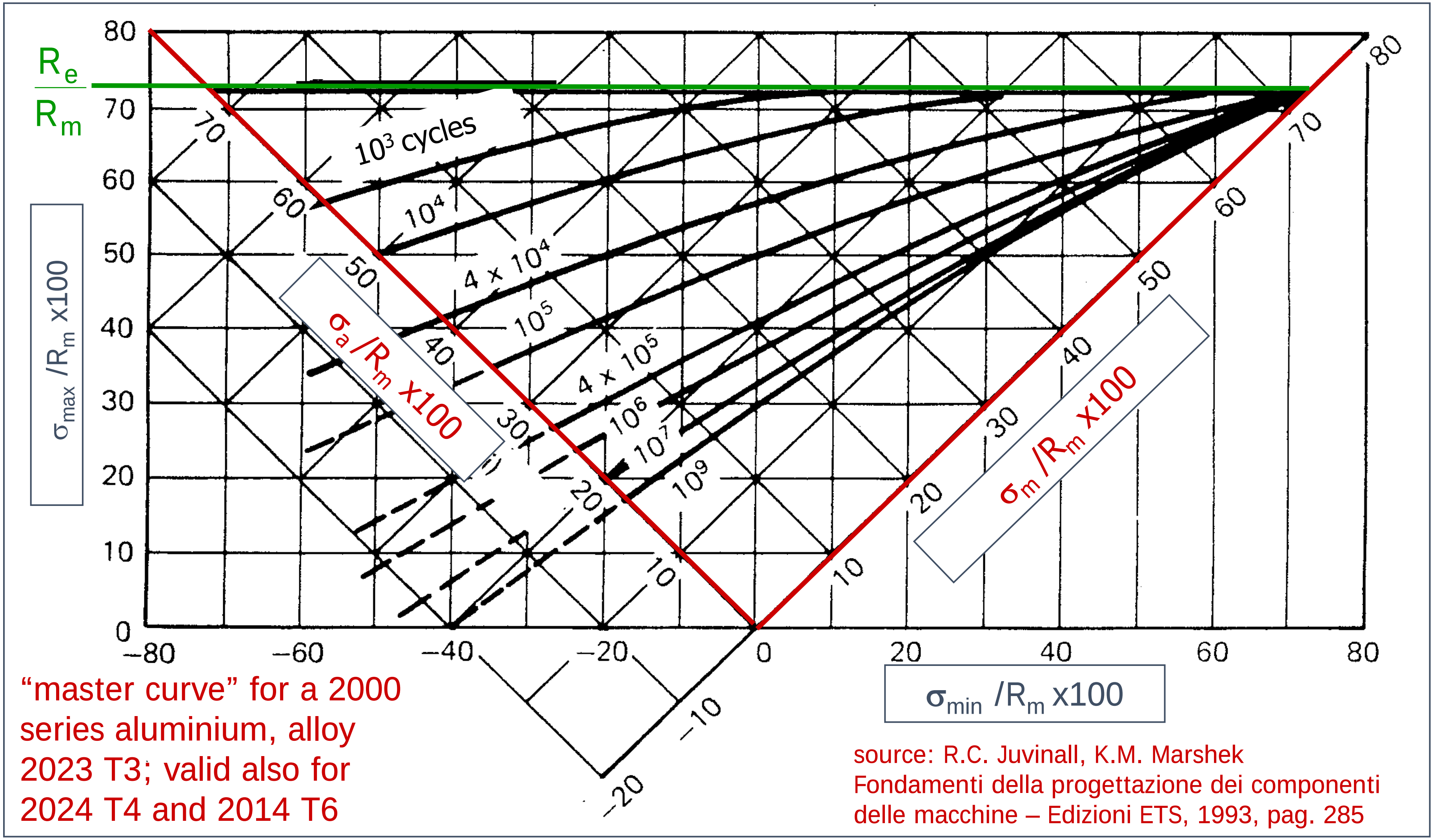
units: non-dimensional




"master curve" for high strength quenched alloy steels, $R_m$ in the range 850÷1200 MPa

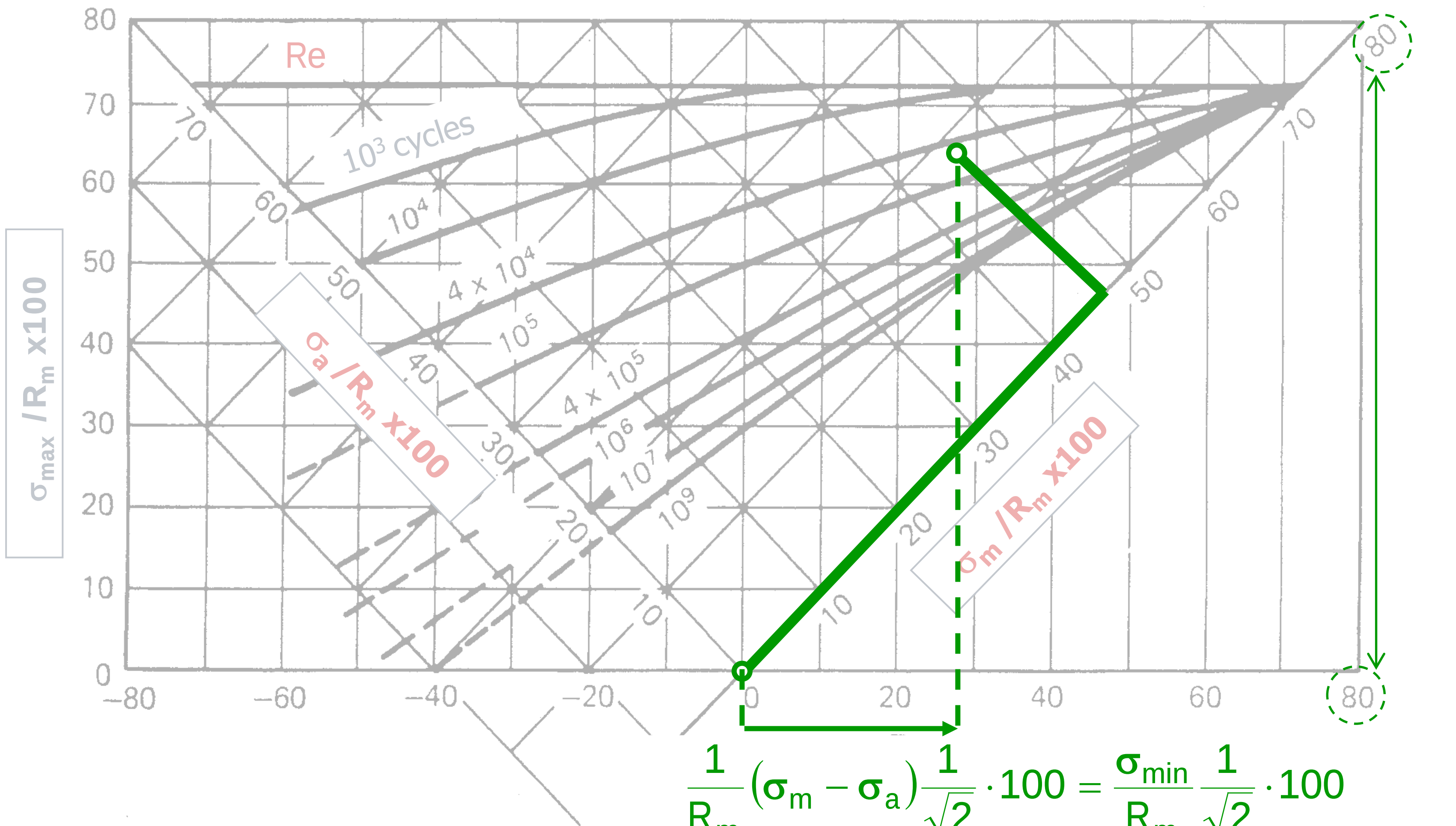
tension-compression fatigue

source: R.C. Juvinall, Engineering Considerations of Stress, Strain and Strength, Mc Graw-Hill, 1967 a sample of aluminium alloy data    units: non-dimensional



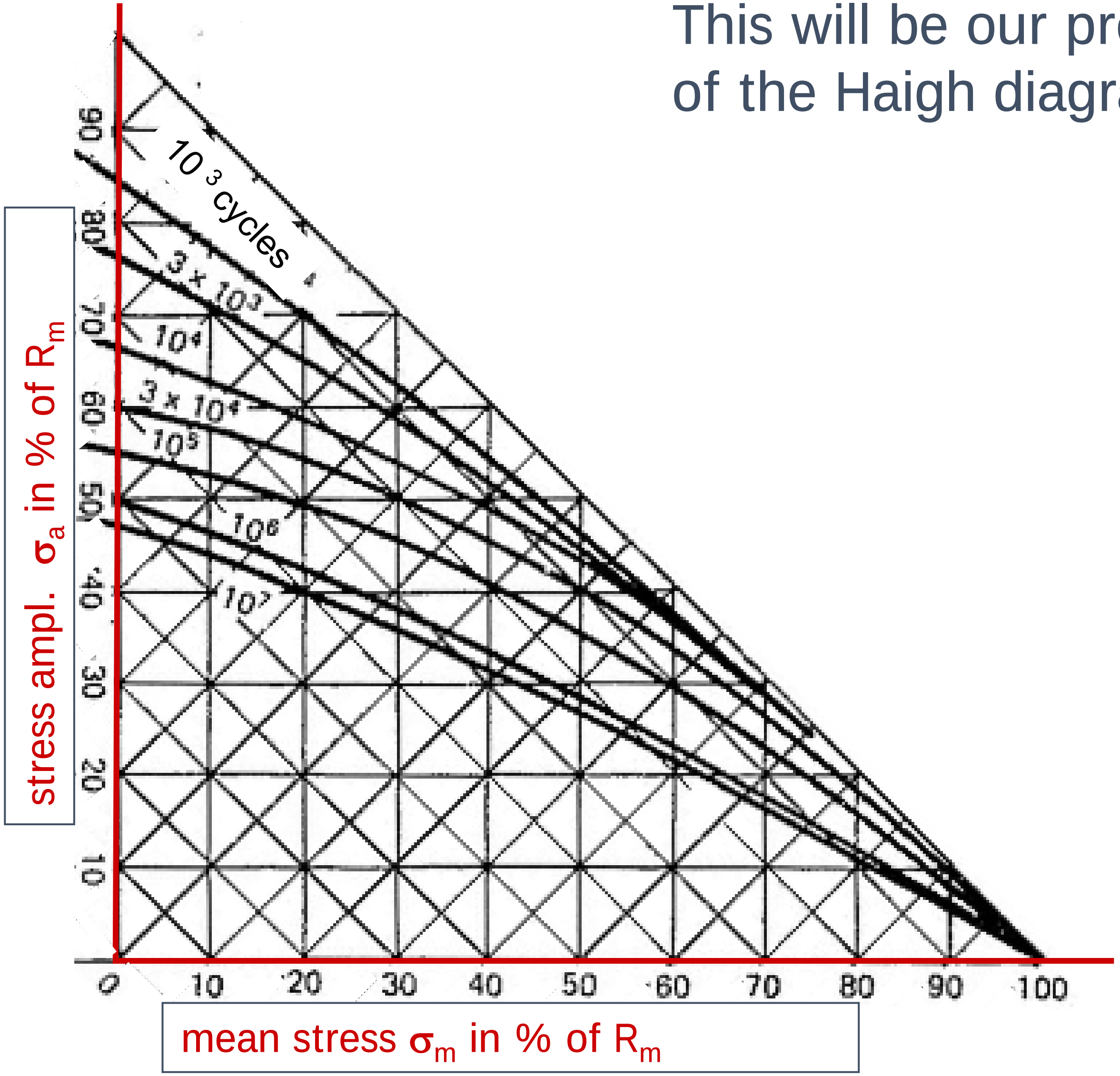

"master curve" for a 2000 series aluminium, alloy 2023 T3; valid also for 2024 T4 and 2014 T6

source: R.C. Juvinall, K.M. Marshek Fondamenti della progettazione dei componenti delle macchine – Edizioni ETS, 1993, pag. 285




$$\frac{1}{R_m}(\sigma_m - \sigma_a)\frac{1}{\sqrt{2}}\cdot 100 = \frac{\sigma_{min}}{R_m}\frac{1}{\sqrt{2}}\cdot 100$$

changing scale on the horizontal axis read $\sigma_{min}/R_m$




This will be our preferred version of the Haigh diagram

Some data for your reference

| AISI 4340 or DIN 34NiCrMo6 | | 2024 T3 or DIN AlCuMg2 | |
|---|---|---|---|
| C 0.38 - 0.43 %<br>Cr 0.4 - 0.6 %<br>Mn 0.6 - 0.8 %<br>Mo 0.2 - 0.3 %<br>Ni 1.65 – 2 % | mechanical properties when normalized at 870°C, measured at T=25° C: | Al 90.7 - 94.7 %<br>Cr max 0.1 %<br>Cu 3.8 - 4.9 %<br>Fe max 0.5 %<br>Mg 1.2 - 1.8 %<br>Mn 0.3 - 0.9 %<br>Si max 0.5 %<br>Ti max 0.15 %<br>Zn max 0.25 % | mechanical properties:<br><br>E (GPa): 73<br>$R_m$ (MPa): 480<br>$R_e$ (MPa): 345<br>A (%): 18 |
| heat treatable, low alloy steel known for its toughness and capability of developing high strength in the heat treated condition while retaining good fatigue strength | E (GPa): 190-210<br>$R_m$ (MPa): 1275<br>$R_e$ (MPa): 860<br>A (%): 22 | aircraft fittings, gears and shafts, bolts, clock parts, computer parts, couplings, fuse parts, hydraulic valve bodies | |

# 8. The Haigh plot: data approximation (1/9)

A very substantial amount of testing is required to generate a Haigh diagram, and it is usually impractical to develop curves for all combinations of mean and alternating stresses. Several empirical relationships that relate alternating stress to mean stress have been developed to address this difficulty.

The two most widely accepted methods are those of Goodman and Gerber. Experience has shown that experimental points fall between the Goodman and Gerber curves.

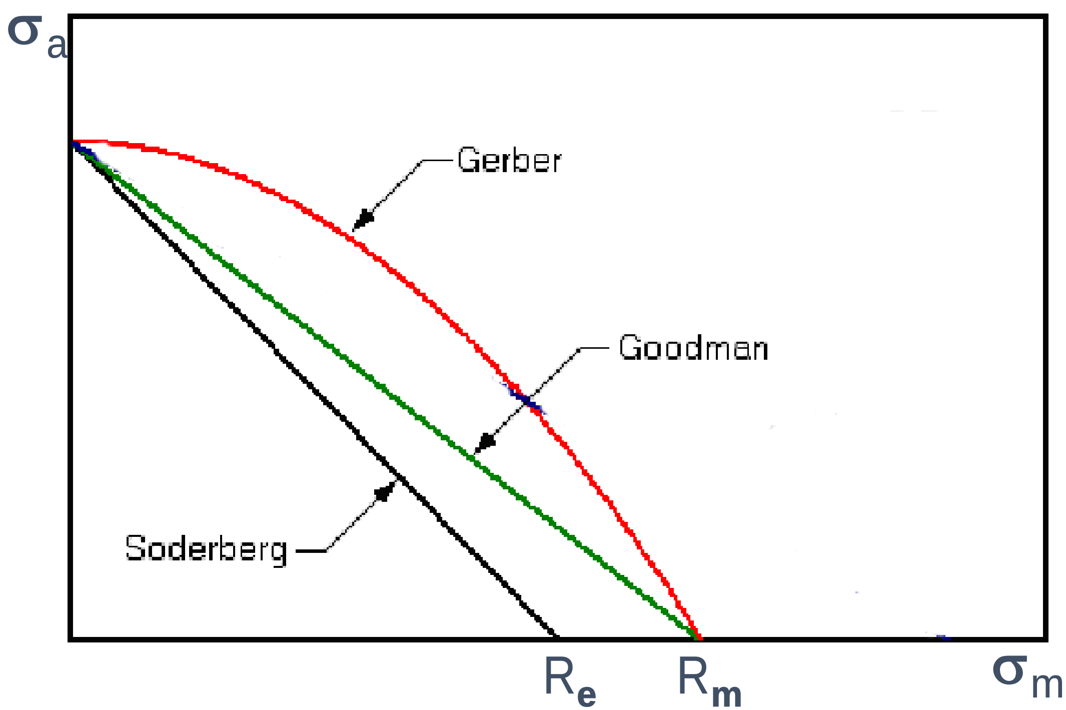
| | |
|---|---|
| Goodman<br>England, 1899 | $\frac{\sigma_D}{\sigma_{D-1}} + \frac{\sigma_m}{R_m} = 1$ |
| Gerber<br>Germany, 1874 | $\frac{\sigma_D}{\sigma_{D-1}} + \left(\frac{\sigma_m}{R_m}\right)^2 = 1$ |
| Soderberg<br>USA, 1930 | $\frac{\sigma_D}{\sigma_{D-1}} + \frac{\sigma_m}{R_e} = 1$ |

Goodman straight line is often used due to mathematical simplicity and slightly more conservative values.

Soderberg is over-conservative.

A graphical comparison of these equations is shown below.

All methods should only be used for <u>tensile</u> mean stress values. For cases where the mean stress is small relative to the alternating stress (R << 1), there is little difference in the methods. The Soderberg method is conservative in excess.




As R approaches 1, ($\sigma_a \Rightarrow 0$) static limit to yield predominates.

This typically the operation area of cyclically loaded high strength bolts and of vibrating turbine or compressor blades.

# 8. The Haigh plot: data approximation (3/9)

When all the necessary Wöhler curves are not available to deduce the complete Haigh diagram as it was shown in Sect. 6 - sl. 2, 3, 4 the problem is to find a rule: 1) to interpolate data or 2) to reproduce curves from a minimum set of data.

In the next slides the second case is presented through a worked out example.

Please keep in mind that this reproduction is an assumption which is justified on the basis of its ability to predict the behavior of real data to a certain degree of acceptable approximation. As such, the method needs a confirmation based on experimental validation, then it must be used with caution.

Another thing to underline is that the particular choice of formulas may be of advantage for data treatment (sl. 9 of this section and its later consequences).

# 8. The Haigh plot: data approximation (4 / 9)

It may be expedient to interpolate first the Haigh curve for the limit life taken at N=$10^6$ cycles (however, some standards do it for 2*$10^6$ or 5*$10^6$). In the next slide we see a parabola through three points, chosen at $\sigma_m/R_m \cdot 100 = 0, 40, 100$).

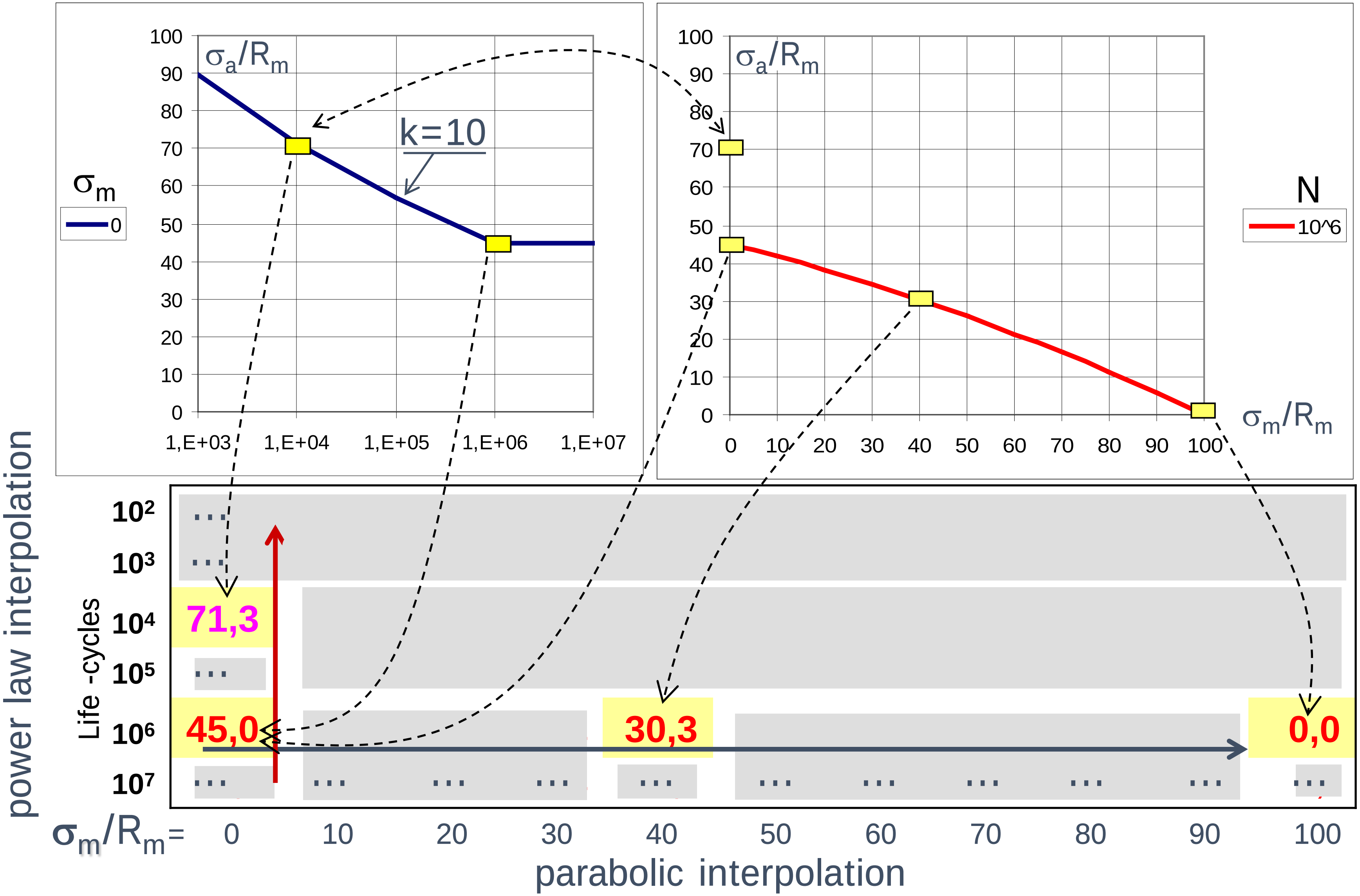
The linear interpolation will be introduced later.

For different lives the power law $\sigma_a^k N = \text{const}$ is used to interpolate the Wöhler curve at $\sigma_m=0$, with the exponent k chosen so to pass through the two points at lives $10^6$ and $10^4$

(in the case of this example we find: $k = \dfrac{2}{\text{Log}\,71{,}3 - \text{Log}\,45} \cong 10$ )

The through data are the five numbers in yellow boxes seen in the next two slides; the mathematics of interpolation is quite trivial and is omitted

Finally, the power law with the exponent determined as above is used to generate Woehler curves for all other $\sigma_m$ . This is going to have useful consequences for data manipulation.



σa/Rm
σm
0
k=10
1,E+03
1,E+04
1,E+05
1,E+06
1,E+07
N
10^6
σm/Rm
power law interpolation
Life -cycles
10²
10³
10⁴
10⁵
10⁶
10⁷
71,3
45,0
30,3
0,0
σm/Rm=
0
10
20
30
40
50
60
70
80
90
100
parabolic interpolation


48

... 2 – Values for $\sigma_{a,i}$ at all $N_i$ for $\sigma_m$=10% of $R_m$ are obtained through:

$$\frac{\sigma_{a,i}}{R_m} = 41{,}7\left(\frac{10^6}{N_i}\right)^{1/10}$$

etc. for all $\sigma_m$, one k for all

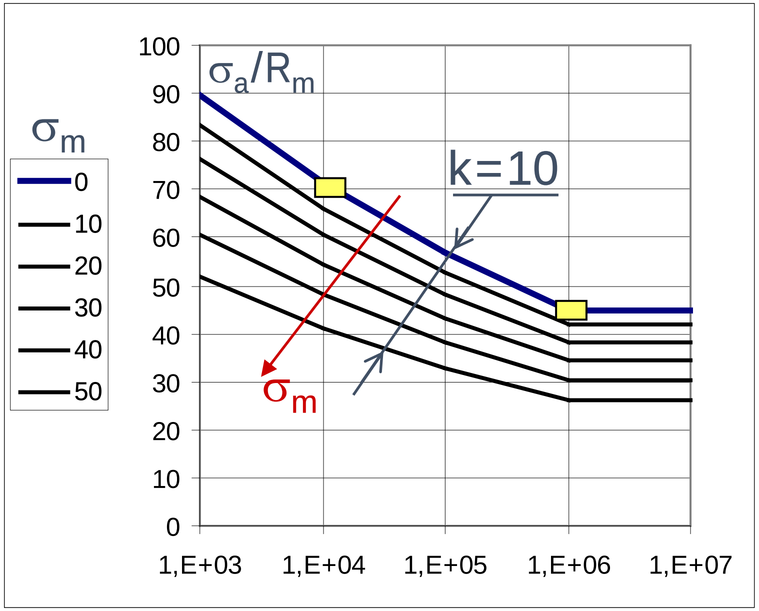
1 - Values for $\sigma_D = f(\sigma_m)$ at $n=10^6$ are interpolated through a parabola ...

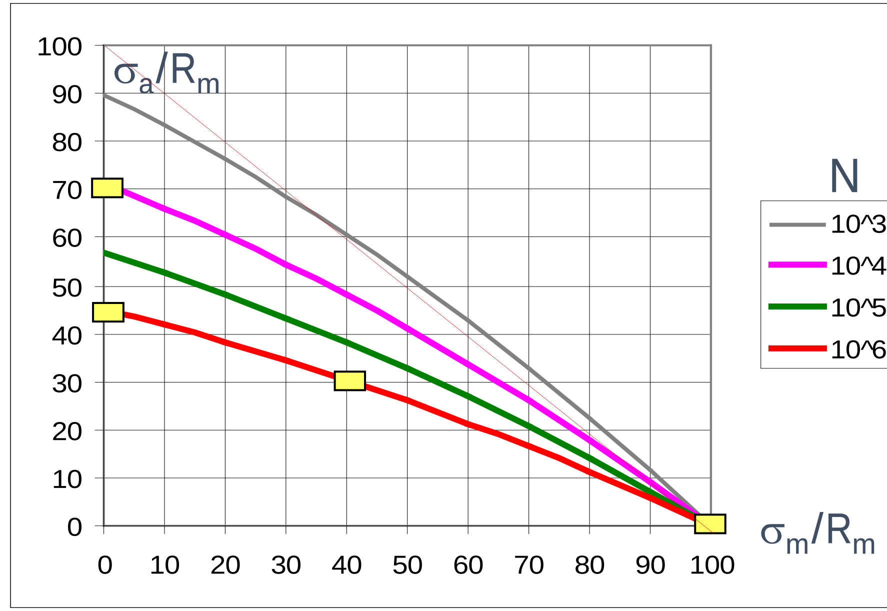
3 - These $\sigma_a$ are taken equal to $R_m - \sigma_m$, i.e. now $100-\sigma_m/R_m$

power law interpolation

| Life -cycles | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $10^2$ | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 0 |
| $10^3$ | 89,8 | 83,3 | | | | | | | | | |
| $10^4$ | 71,3 | 66,2 | | | | | | | | | |
| $10^5$ | 56,7 | 52,6 | | | | | | | | | |
| $10^6$ | 45,0 | 41,7 | 38,2 | 34,4 | 30,3 | 26,0 | 21,3 | 16,4 | 11,2 | 5,7 | 0,0 |
| $10^7$ | 45,0 | 41,7 | 38,2 | 34,4 | 30,3 | 26,0 | 21,3 | 16,4 | 11,2 | 5,7 | 0,0 |
| $\sigma_m/R_m$= | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

parabolic interpolation

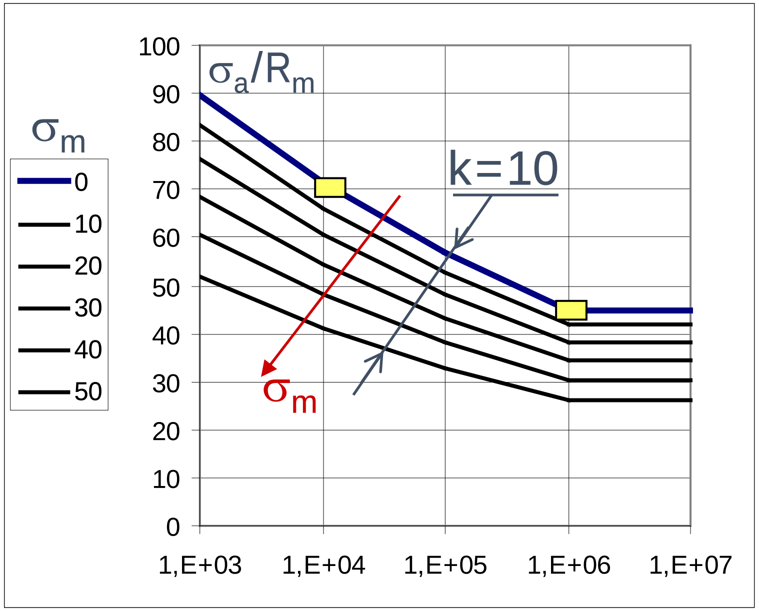


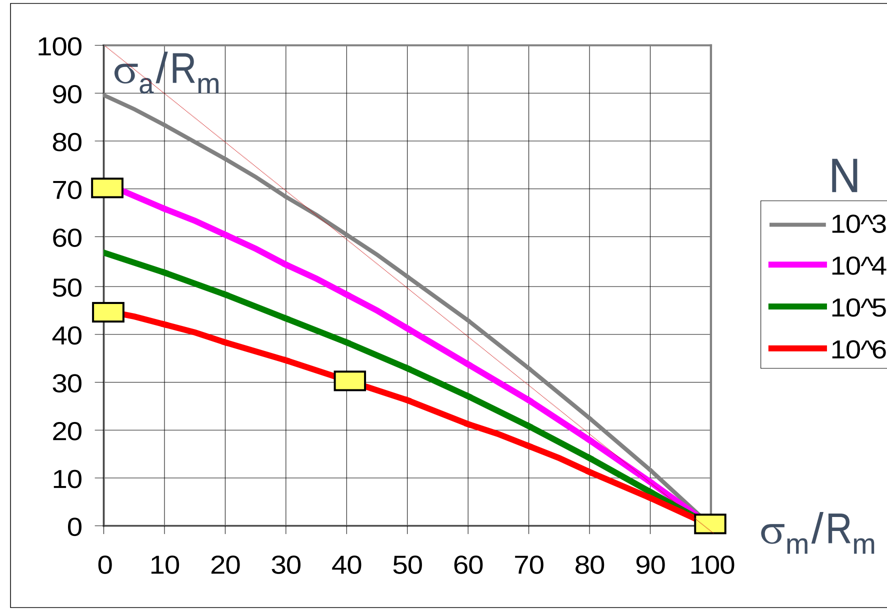




Life -cycles

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $10^2$ | **100** | **90** | **80** | **70** | **60** | **50** | **40** | **30** | **20** | **10** | **0** |
| $10^3$ | 89,8 | 83,3 | 76,3 | 68,7 | 60,5 | 51,8 | 42,6 | 32,7 | 22,4 | 11,5 | 0,0 |
| $10^4$ | **71,3** | 66,2 | 60,6 | 54,5 | 48,1 | 41,2 | 33,8 | 26,0 | 17,8 | 9,1 | 0,0 |
| $10^5$ | 56,7 | 52,6 | 48,1 | 43,3 | 38,2 | 32,7 | 26,9 | 20,7 | 14,1 | 7,2 | 0,0 |
| $10^6$ | **45,0** | 41,7 | 38,2 | 34,4 | **30,3** | 26,0 | 21,3 | 16,4 | 11,2 | 5,7 | **0,0** |
| $10^7$ | 45,0 | 41,7 | 38,2 | 34,4 | 30,3 | 26,0 | 21,3 | 16,4 | 11,2 | 5,7 | 0,0 |
| $\sigma_m/R_m$= | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

# 8. The Haigh plot: data approximation (8 / 9)




Life -cycles

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $10^2$ | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | 0 |
| $10^3$ | | | | | | | | | | | |
| $10^4$ | | | | | | | | | | | |
| $10^5$ | | | | | | | | | | | |
| $10^6$ | | | | | | | | | | | |
| $10^7$ | 45,0 | 41,7 | 38,2 | 34,4 | 30,3 | 26,0 | 21,3 | 16,4 | 11,2 | 5,7 | 0,0 |
| $\sigma_m/R_m$= | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

exercise, find an explanation for the values above at life $N=10^2$ cycles , and for the values below at life $N=10^7$

check the curve for $10^3$ cycles ... any problem? all is right? sure?

The Haigh curve for fatigue limit is: $\sigma_D = \sigma_D(\sigma_m)$

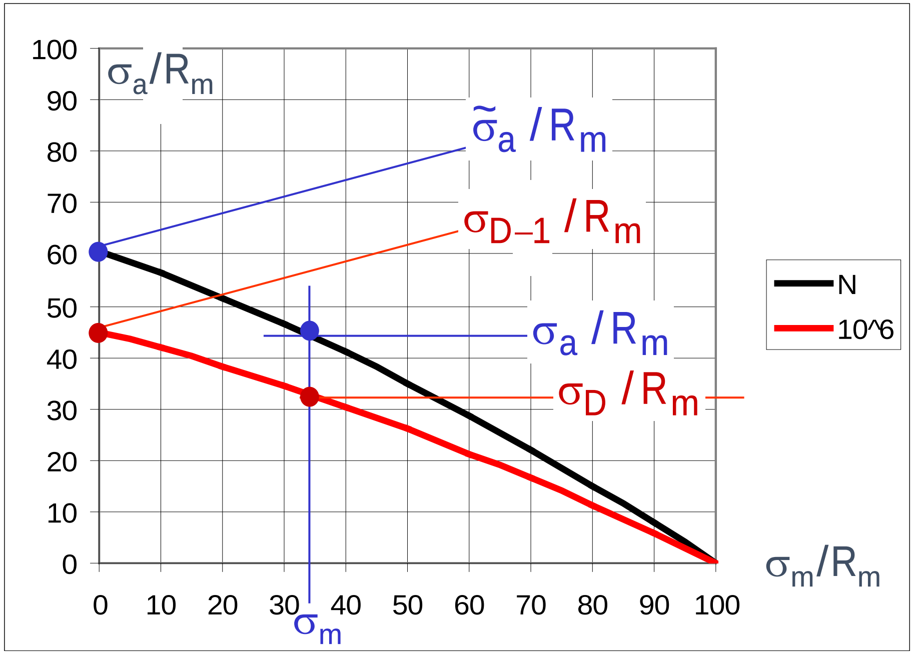
Omitting reference to $R_m$, power law interpolation with the same value of k for all values of $\sigma_m$ produces:

$$\left[\sigma_a(\sigma_m)\right]^k N = \left[\sigma_D(\sigma_m)\right]^k 10^6$$

This means that independent on the value $\sigma_m$:

$$\frac{\sigma_a(\sigma_m)}{\sigma_D(\sigma_m)} = \left(\frac{10^6}{N}\right)^{1/k}$$

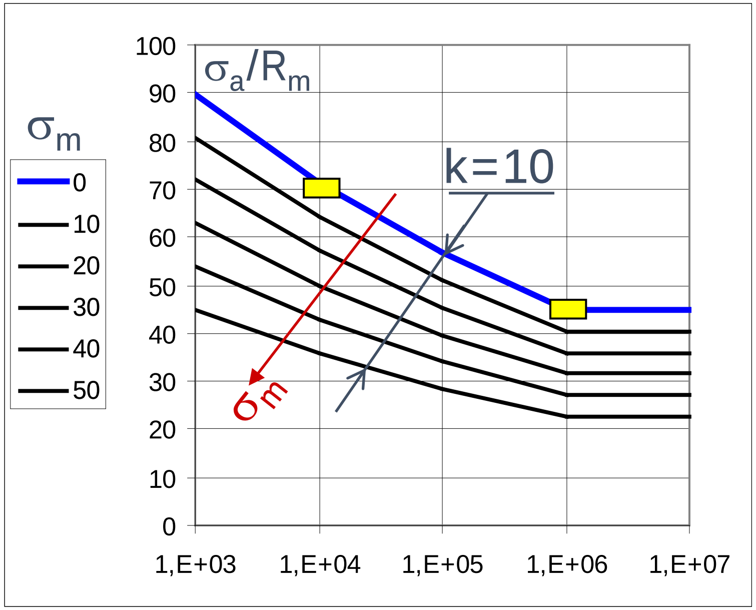
or, any Haigh curve $(\sigma_a, \sigma_m)$ for a given life N is proportional to the curve $(\sigma_D, \sigma_m)$ i.e. to the one for life $10^6$ .

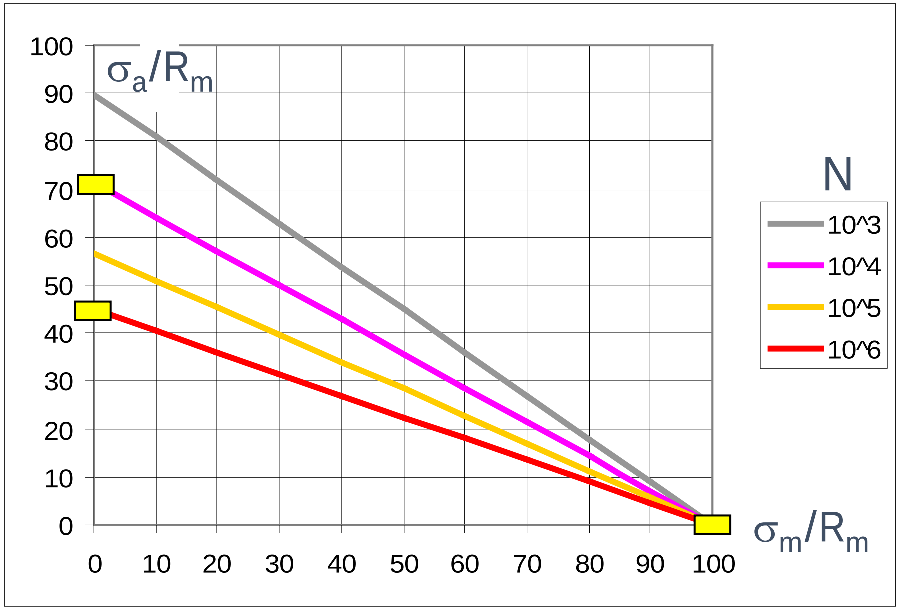



The consequence is:

$$\boxed{\frac{\tilde{\sigma}_a}{\sigma_{D-1}} = \frac{\sigma_a(\sigma_m)}{\sigma_D(\sigma_m)} \quad \text{for any N}}$$

Later we shall also write: $\begin{cases} s_a = \sigma_a / R_m \\ s_m = \sigma_m / R_m \end{cases} \Rightarrow \frac{\tilde{s}_a}{s_{D-1}} = \frac{s_a(s_m)}{s_D(s_m)}$



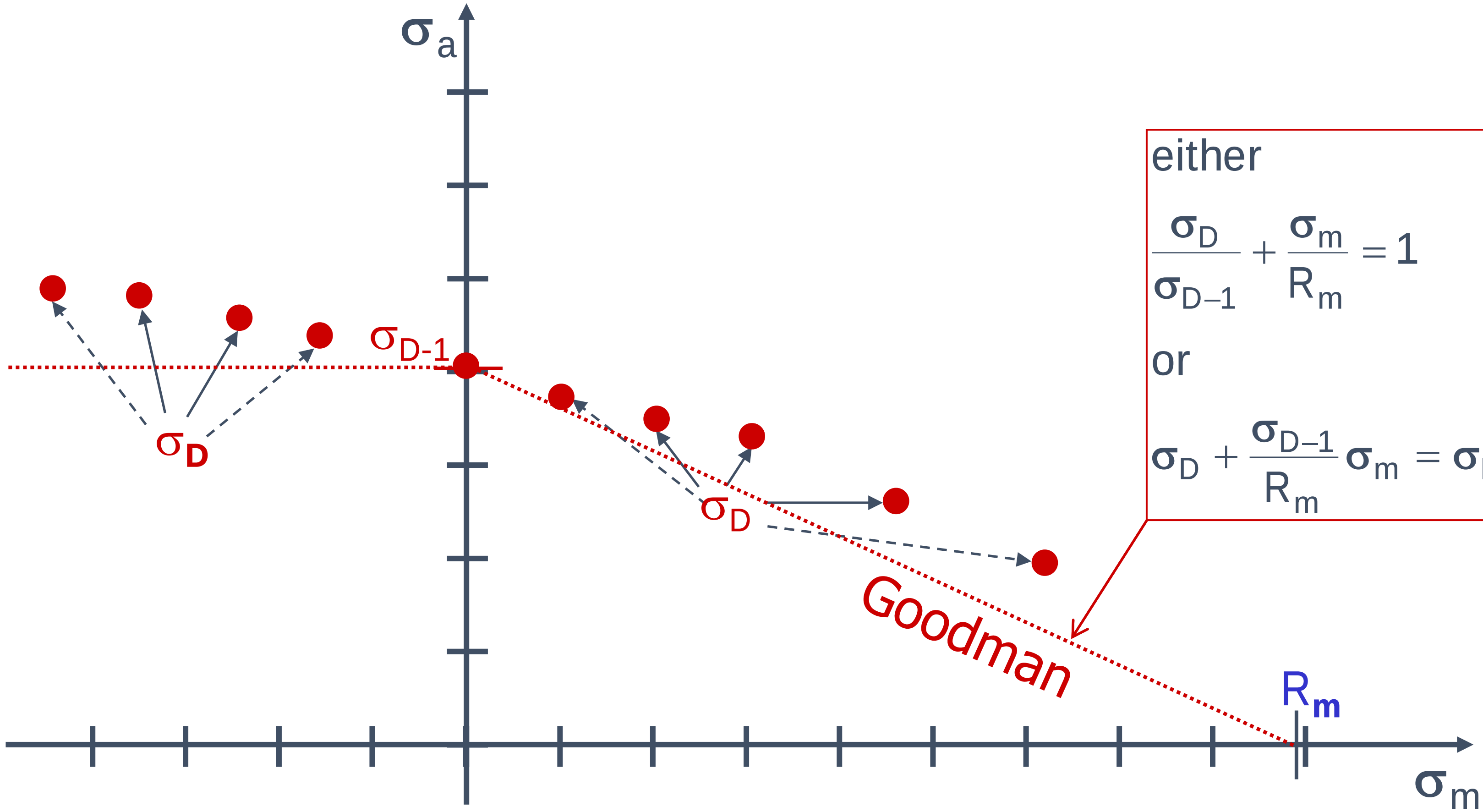




In many practical instances, the curves in the Haigh plot are further simplified as straight lines, as indicated above.

Only three values are needed: $\sigma_{D-1}$, $R_m$ plus one point on the $10^6$ Wöhler curve, or in alternative the value of exponent k.

The equation for "Goodman" line $\sigma_D$ is: $\sigma_D = \sigma_{D-1}\left(1 - \frac{\sigma_m}{R_m}\right)$

A realistic plot of values for the empirical function $\sigma_D = f(\sigma_m)$ is shown below with the Goodman line; also negative $\sigma_m$ have been represented, where the Goodman line does not hold.



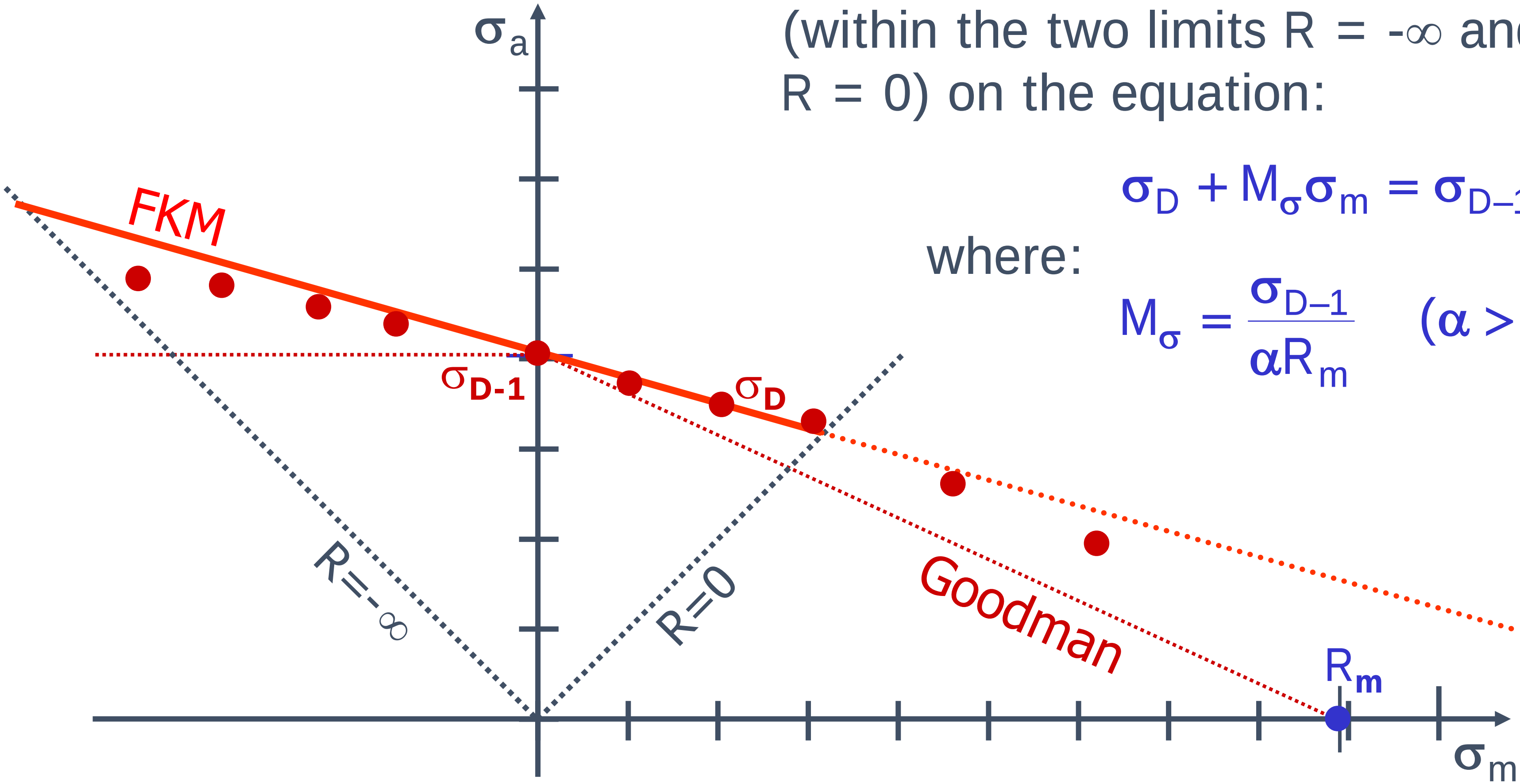

either

$$\frac{\sigma_D}{\sigma_{D-1}} + \frac{\sigma_m}{R_m} = 1$$

or

$$\sigma_D + \frac{\sigma_{D-1}}{R_m}\sigma_m = \sigma_{D-1}$$

FKM refinement




The alternative, less restrictive, simplified representation according to FKM is based (within the two limits R = -∞ and R = 0) on the equation:

$$\sigma_D + M_\sigma \sigma_m = \sigma_{D-1}$$

where:

$$M_\sigma = \frac{\sigma_{D-1}}{\alpha R_m} \qquad (\alpha > 1)$$

# 9. The Haigh plot: linear simplifications (4/4)

The values of $M_\sigma$ for different materials are:

$$M_\sigma = a_M \, 10^{-3} \, R_m + b_M \qquad (R_m \text{ in MPa})$$

Note: $R_m$ is here the component tensile strength as defined in Ch.2 Sect.3 sl.3

| material | steel* | GS | GGG | GT | GG | wrought Al-alloys | cast Al-alloys |
|---|---|---|---|---|---|---|---|
| $a_M$ | 0,35 | 0,35 | 0,35 | 0,35 | 0 | 1.0 | 1.0 |
| $b_M$ | -0.1 | 0.05 | 0.08 | 0.13 | 0.5 | -0.04 | 0.2 |

Source FKM

*including stainless steel

Example for steel: $R_m$= 1000 MPa $\Rightarrow M_\sigma$=0,25

$$M_\sigma = 0.35 - 0.1 = 0.25 = \frac{\sigma_{D-1}}{\alpha R_m} = \frac{0.45 R_m}{\alpha R_m} \Rightarrow \alpha = \frac{0.45}{0.25} = 1.8$$

According to FKM, in the case of torsion: $M_\tau = f_{W,\tau} \, M_\sigma$ , where values $f_{W,\tau}$ are in the table of section 4 slide 2 of this chapter.

# Sections 10, 11, 12, 13 - Infinite life

These Sections elaborate on the Haigh diagram in the case of one-dimensional tensile (and compressive) stresses. Two-and-three-dimensional stresses will be seen in Chapter 3.

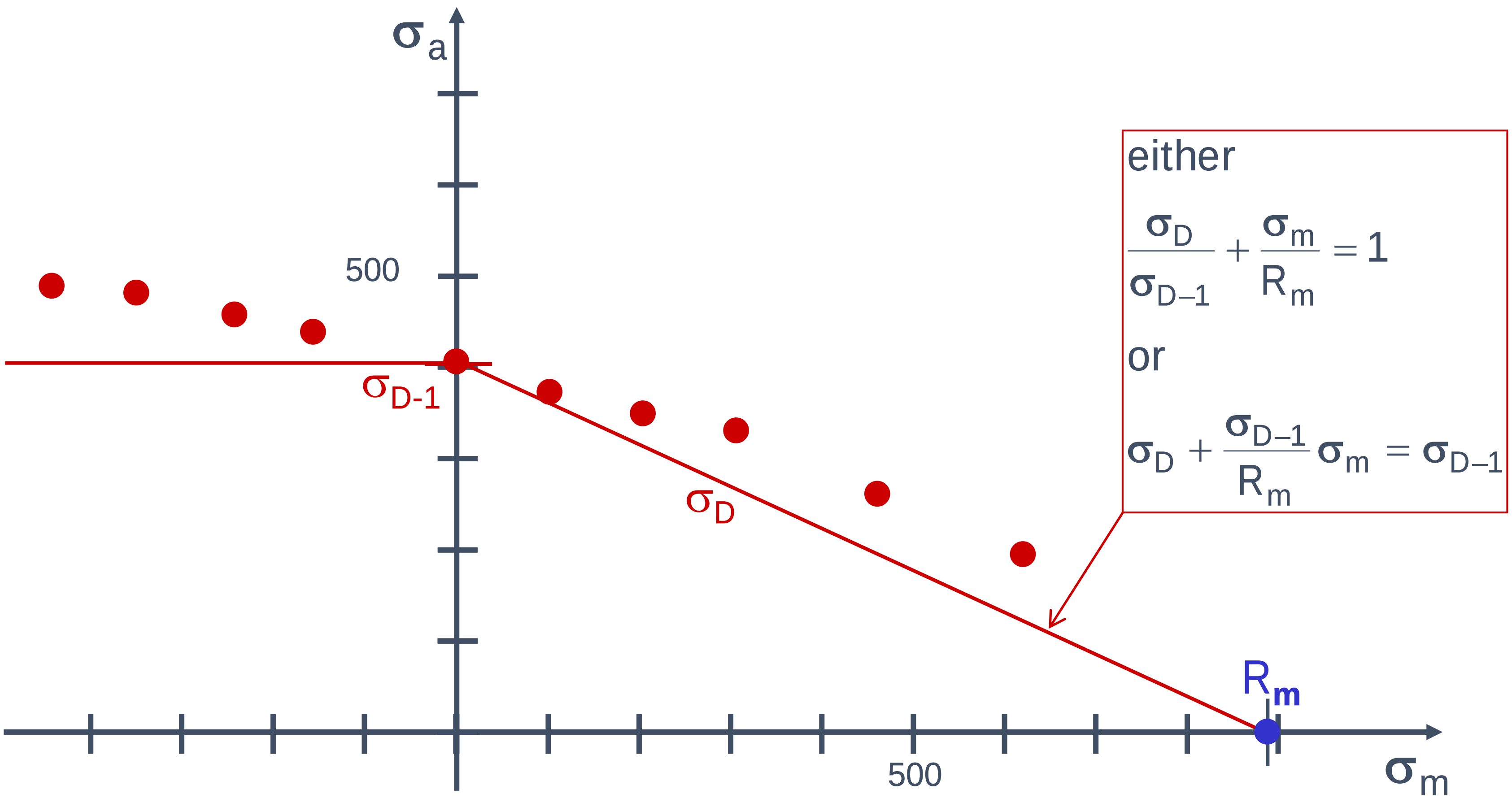
Section 10 shows how the Haigh linearised plot for infinite life limit (for fatigue) and the yield limit (for maximum or peak load) can be plotted on the same diagram. Then how from limit curves a limit area can be defined, and then an admissible area.

Section 11 illustrated the relation between the Haigh diagram and other equivalent graphical representations which are frequently used as well.

Sections 12 and 13 show how the limit and admissible areas are defined by the FKM standard, and may be used to assess the capability to withstand fatigue stresses at infinite life and peak stresses at yield.

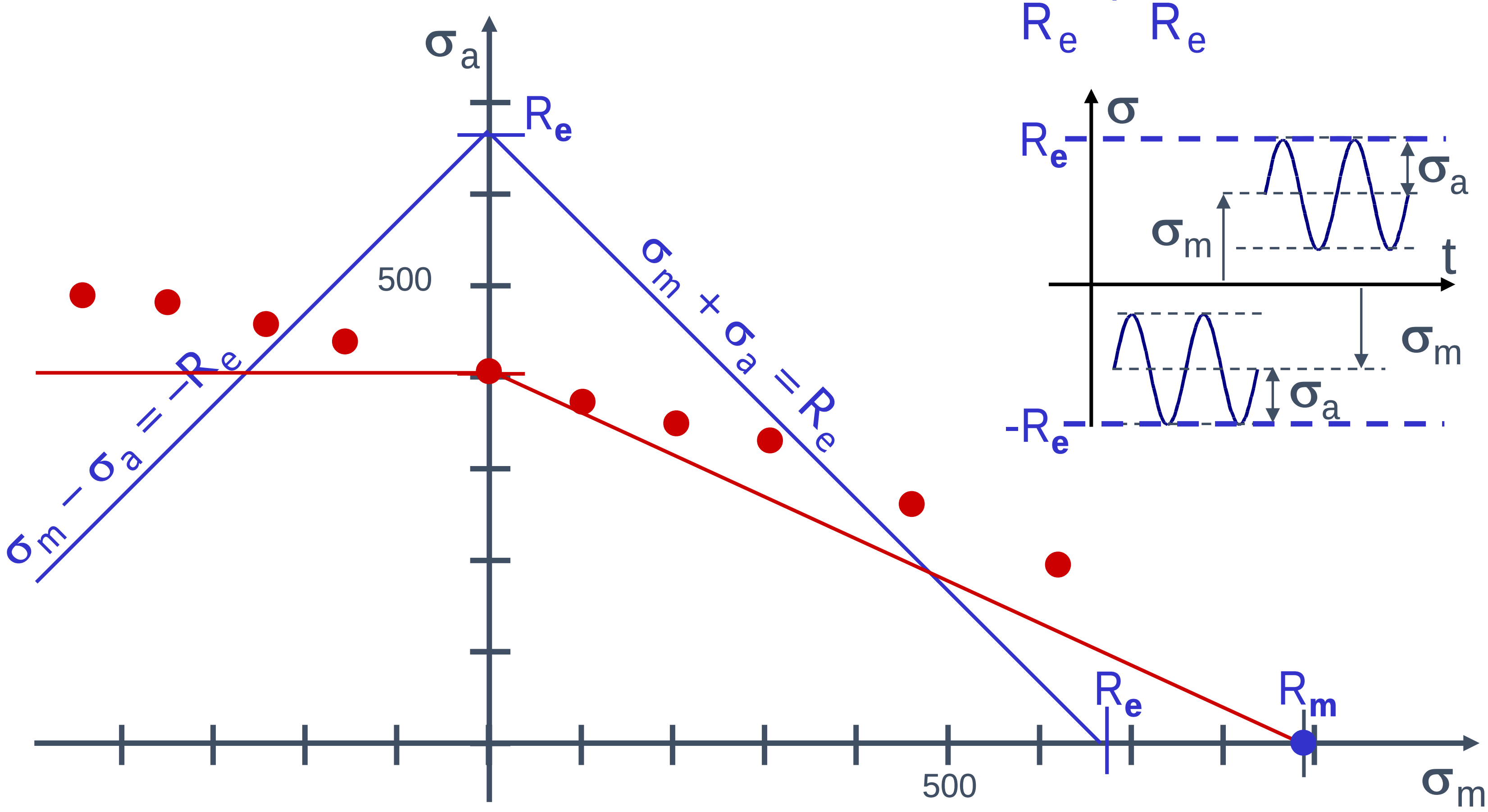
# 10. The Haigh plot: the limit Goodman area (1/3)

As we have already seen before, the most conservative simplified representation of $\sigma_D$ values is given by the red lines (Goodman):

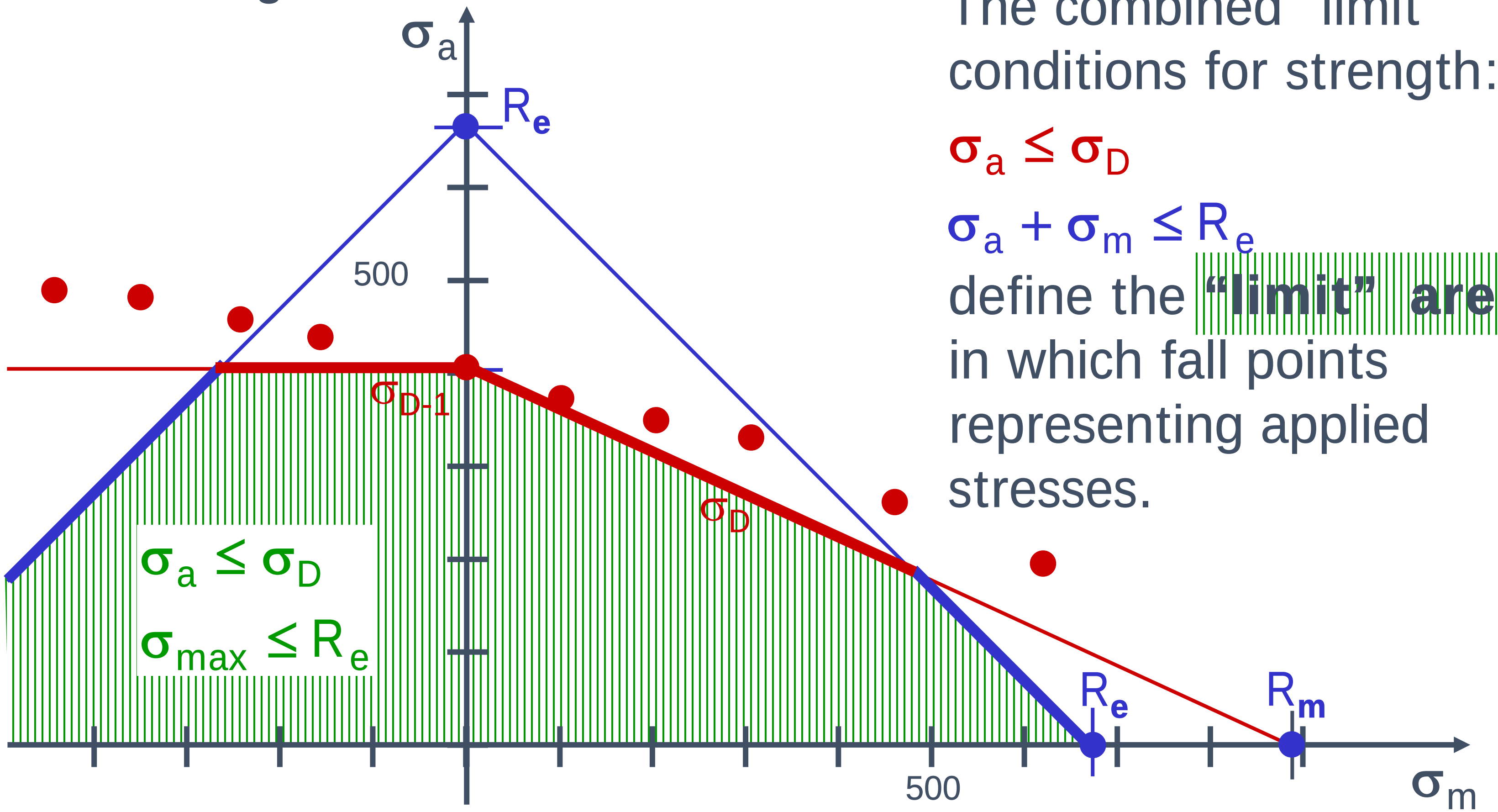


The blue lines represents the yield condition: $\sigma_a + \sigma_m = R_e$

$$\frac{\sigma_a}{R_e} + \frac{\sigma_m}{R_e} = 1$$



The simplified representation allows a conservative estimate for design, without the great expense of a full set of experimentally determined fatigue limits.

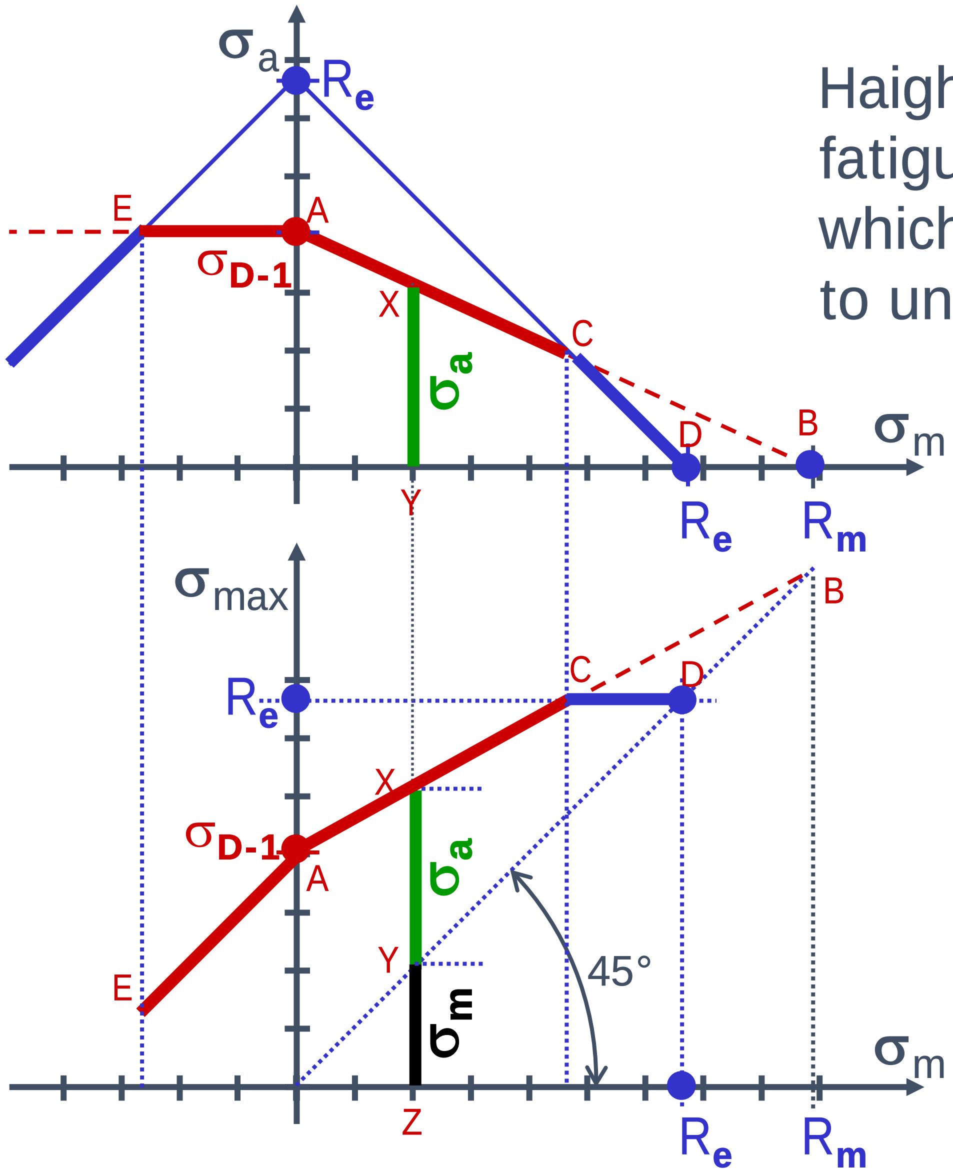



The combined "limit" conditions for strength:

$\sigma_a \leq \sigma_D$

$\sigma_a + \sigma_m \leq R_e$

define the **"limit" area** in which fall points representing applied stresses.




Haigh representation of material fatigue is flanked by others, which are in use; it is necessary to understand how they relate.

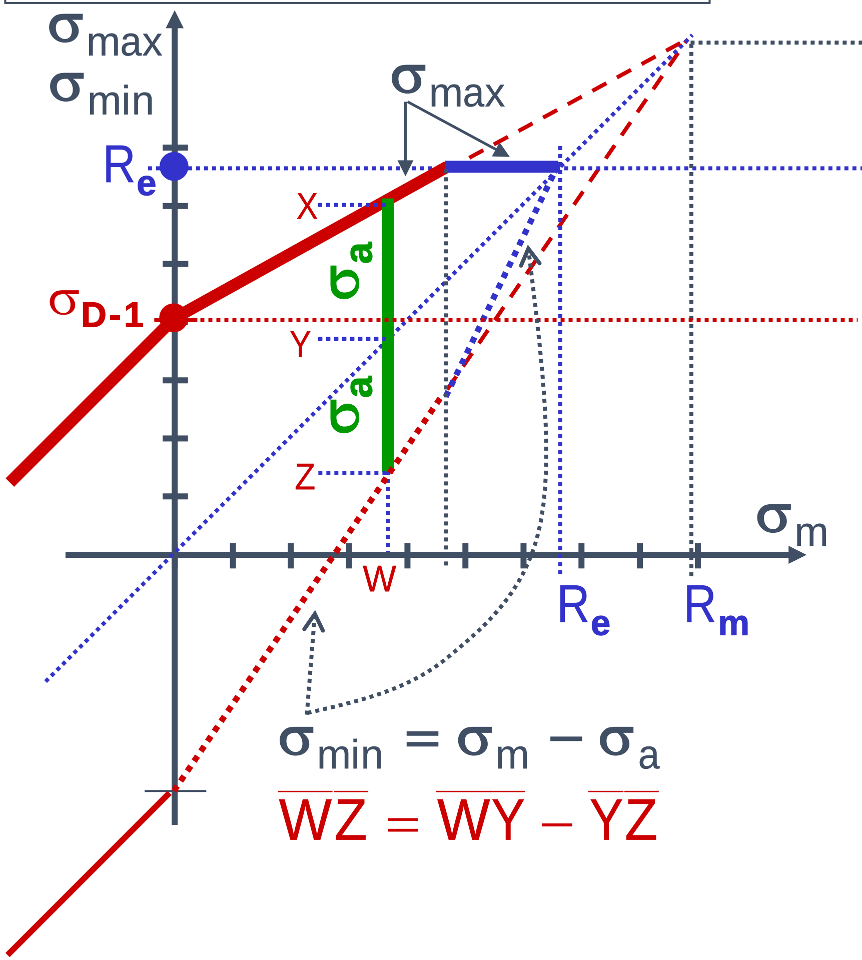
$$\boldsymbol{\sigma}_{max} = \boldsymbol{\sigma}_a + \boldsymbol{\sigma}_m = \overline{XY} + \overline{YZ}$$

Haigh diagram
$\sigma_m$ on abscissas
$\sigma_a$ on ordinates

↓

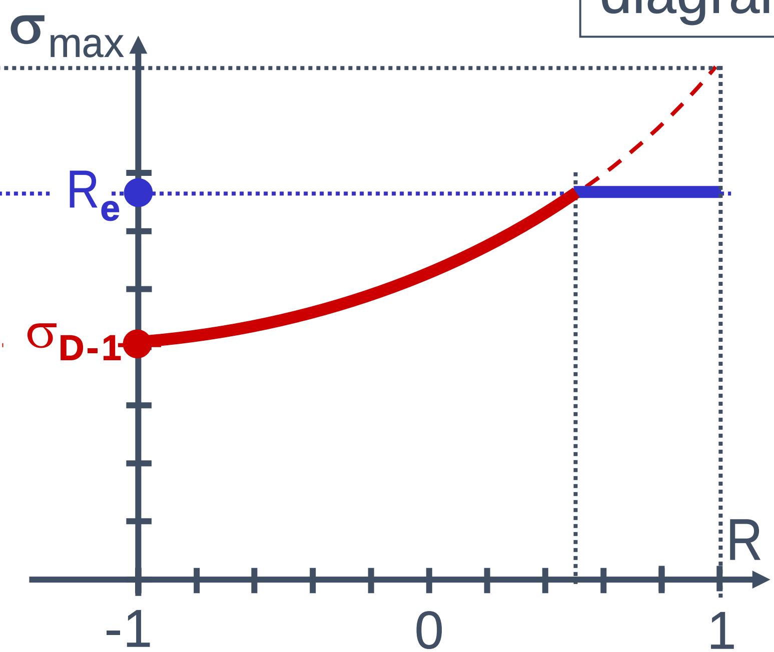
Goodman-Smith Diagram
$\sigma_m$ on abscissas
$\sigma_{max}$ on ordinates

Goodman-Smith diagram (positive $\sigma_m$) → Moore diagram




$$\sigma_{min} = \sigma_m - \sigma_a$$

$$\overline{WZ} = \overline{WY} - \overline{YZ}$$

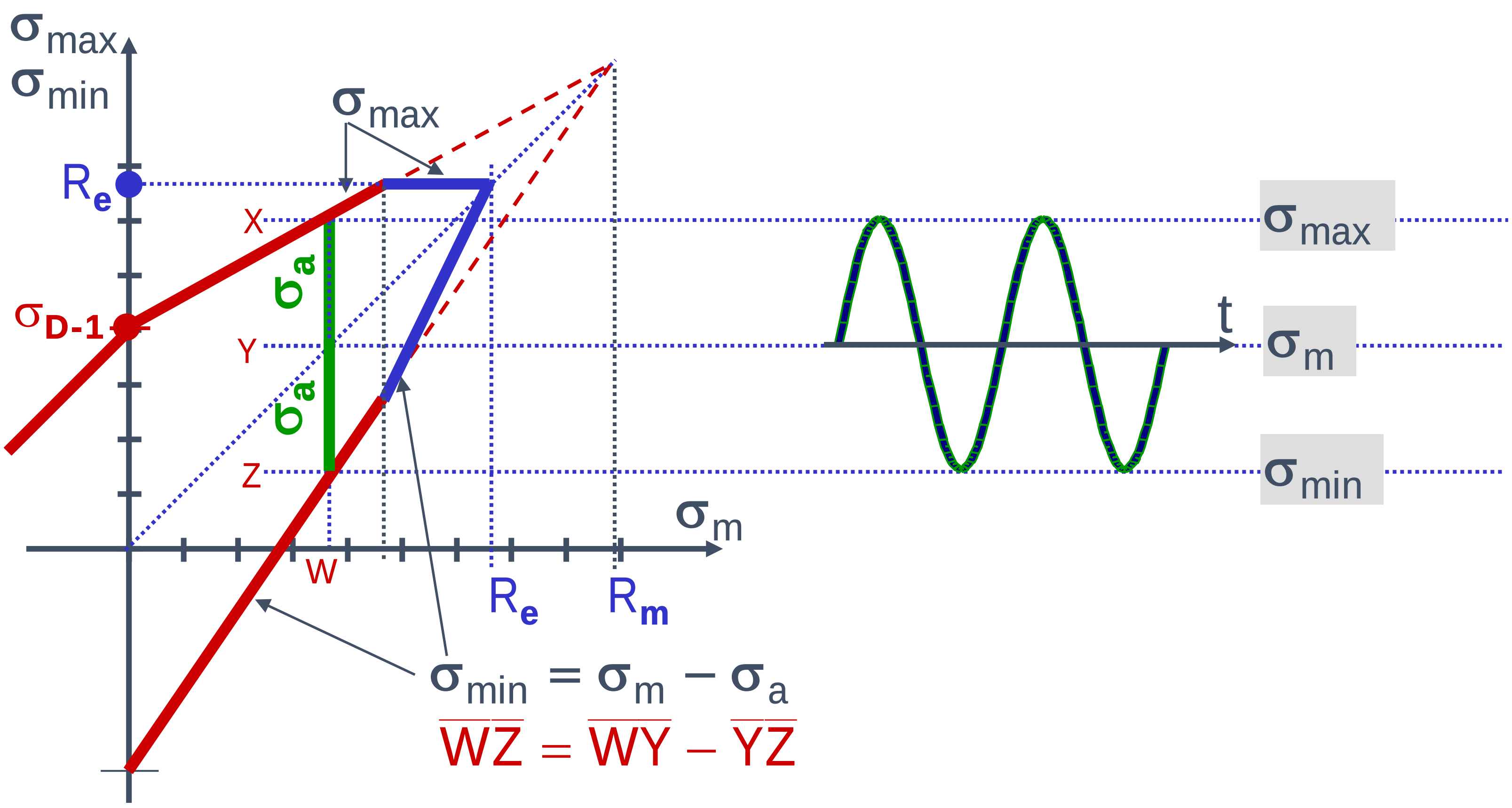



1) $R = \dfrac{\sigma_{min}}{\sigma_{max}} \;;\; \begin{cases} \sigma_{max} = \sigma_m + \sigma_a \\ \sigma_{min} = R\,\sigma_{max} = \sigma_m - \sigma_a \end{cases}$

2) $\sigma_m = \sigma_{max}\dfrac{1+R}{2} \;;\; \sigma_a = \sigma_{max}\dfrac{1-R}{2}$

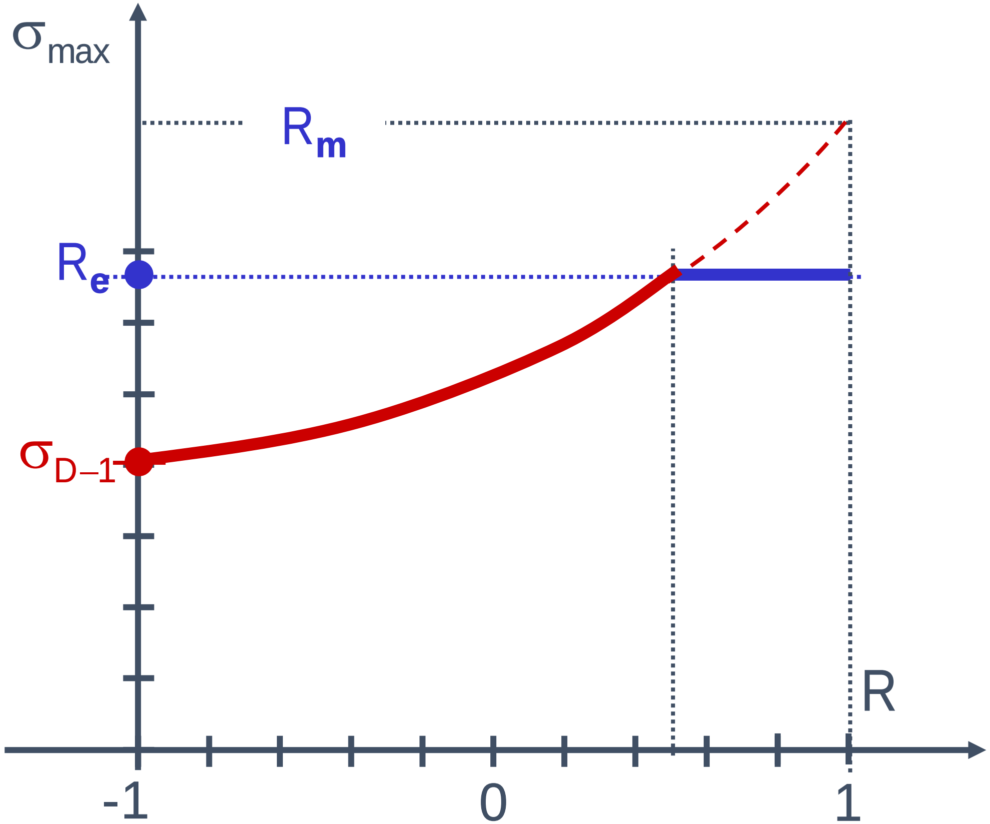
3) $\dfrac{\sigma_a}{\sigma_{D-1}} + \dfrac{\sigma_m}{R_m} = 1 \Rightarrow \dfrac{\sigma_{max}}{2}\left[\dfrac{1-R}{\sigma_{D-1}} + \dfrac{1+R}{R_m}\right] = 1$

The Goodman-Smith diagram lends itself to an easy visualisation of stresses in time:




$$\sigma_{min} = \sigma_m - \sigma_a$$

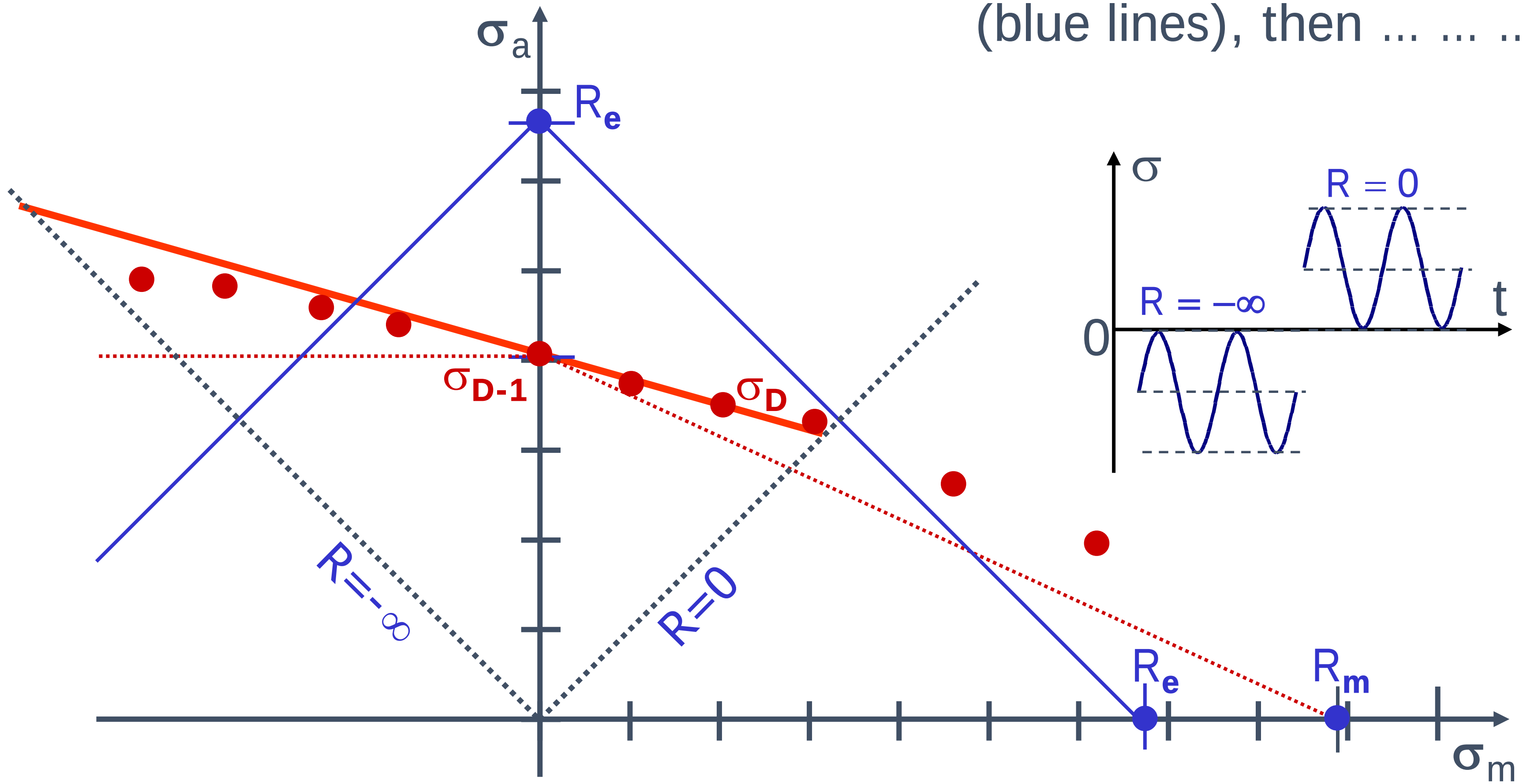
$$\overline{WZ} = \overline{WY} - \overline{YZ}$$



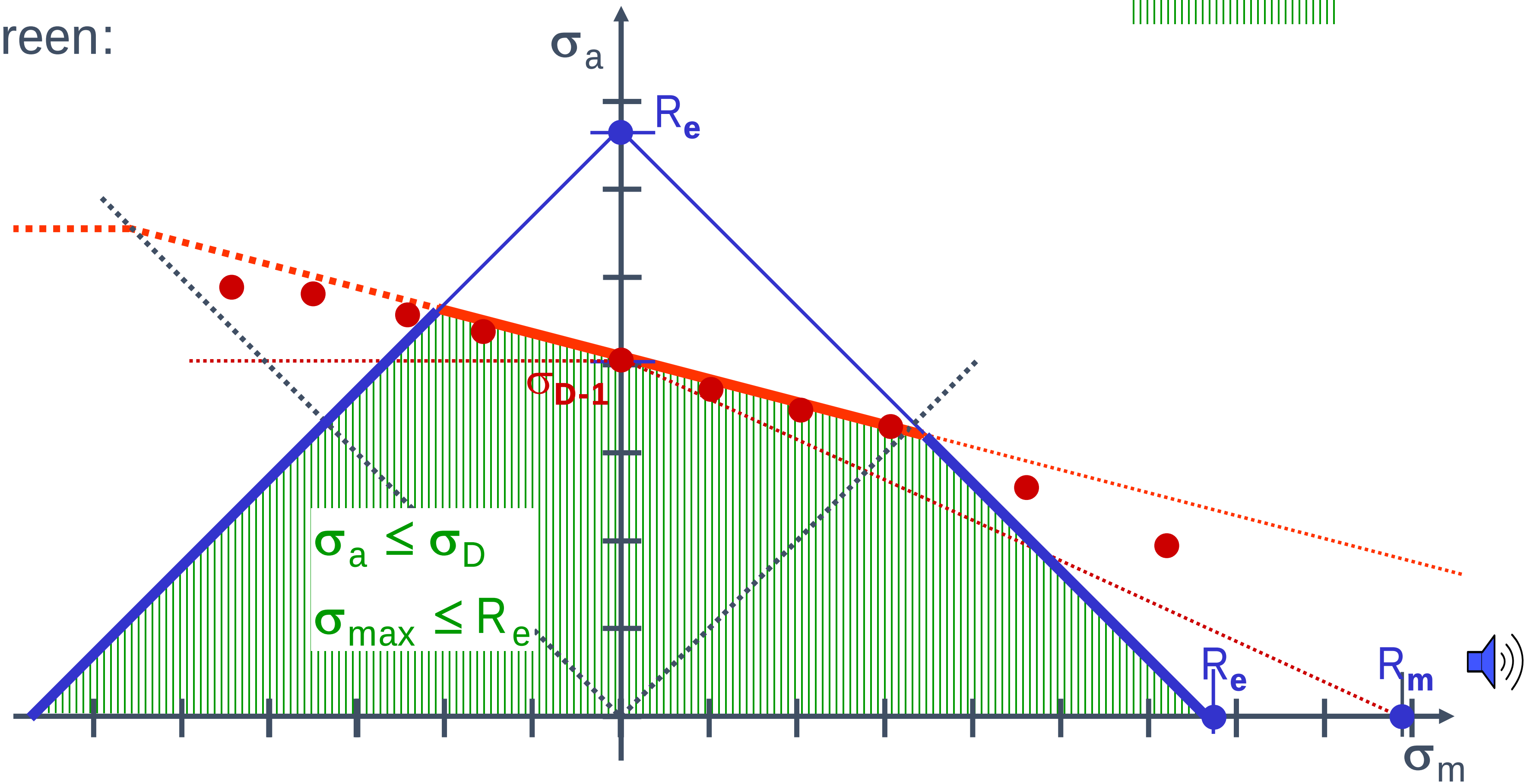

The Moore diagram puts into evidence the maximum stresses, which are the main worry in design against static failure, and takes fatigue into account by means of the stress ratio R.

$$\frac{\sigma_a}{\sigma_{D-1}} + \frac{\sigma_m}{R_m} = 1 \Rightarrow \frac{\sigma_{max}}{2}\left[\frac{1-R}{\sigma_{D-1}} + \frac{1+R}{R_m}\right] = 1$$

If, in alternative to the Goodman line, the FKM line is used (orange line, valid only within the two limits of the stress ratio R=-∞ and R=0), and simultaneously we consider also the yield limits (blue lines), then ... ... ...



... then we get the FKM limit area. FKM description is slightly more complex than here represented, but for the purpose of these lessons we shall be satisfied. The limit area is hatched in green:



Once the “limit” Haigh thresholds and area have been decided according to a standard, for a given material and for a certain component, appropriate safety limits are applied (we shall define them later), producing the “admissible” area hatched in red. Applied stress ⊗ satisfies design requirements when falls inside that area.