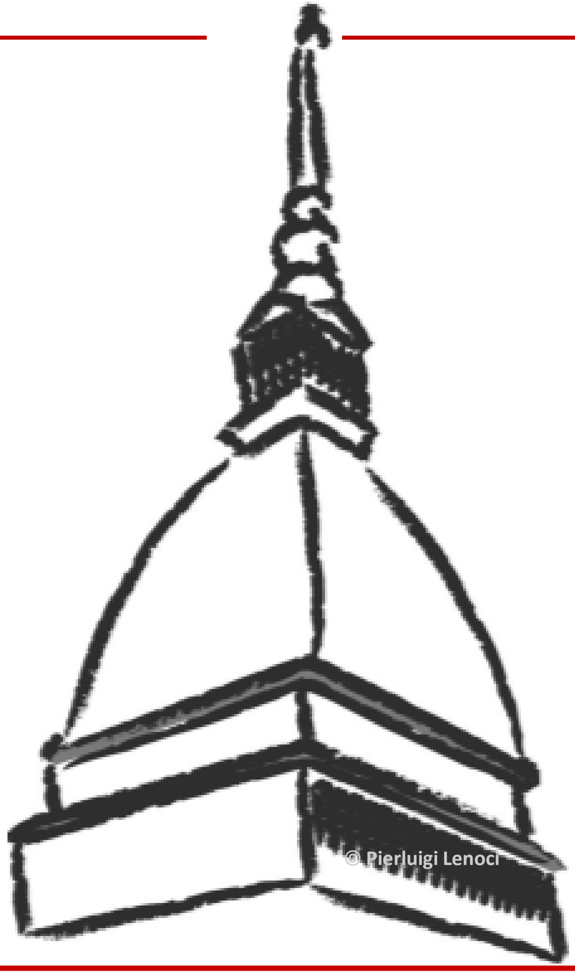
Daniele Botto

Costruzione di Motori per Aeromobili

Hertz contacts & bearings - Chapter 2



Chapters

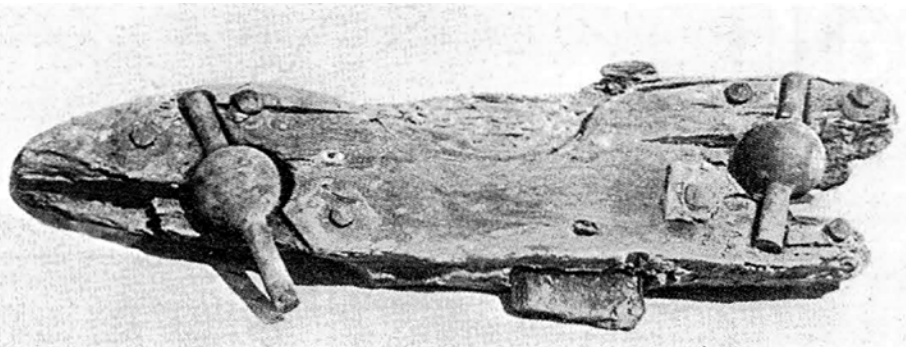
Index of contents

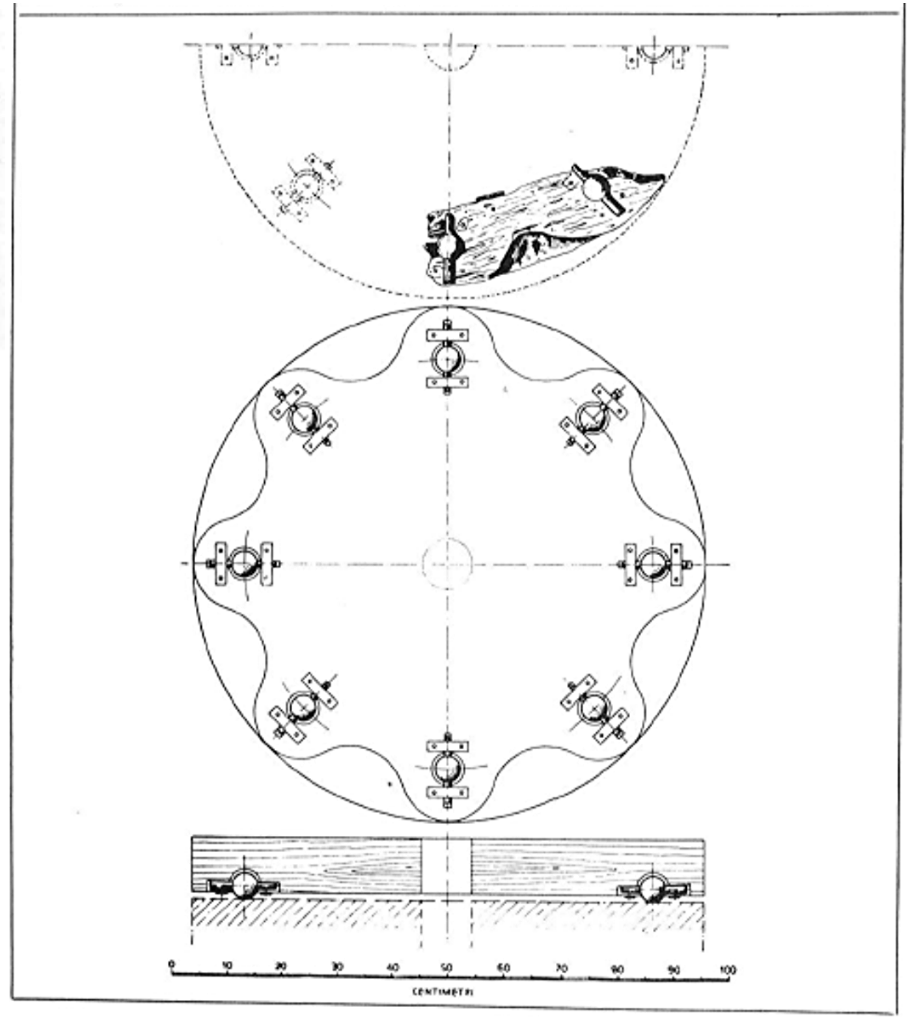
Sections 1, 2 - An introduction to bearing types

Section 1 provides a concise overview of bearing types most commonly in use. Plus some hystorical information.

Section 2 links, with the aid of a practical example, inner bearing geometry with pressure distribution at the rolling body / raceway contacts, inner and outer.

The history of rolling bearing is quite long and worth being known*.



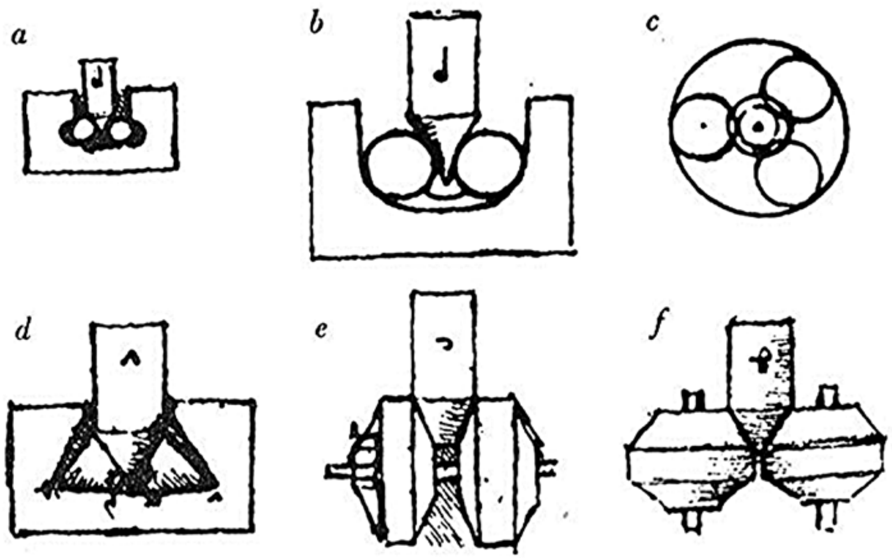


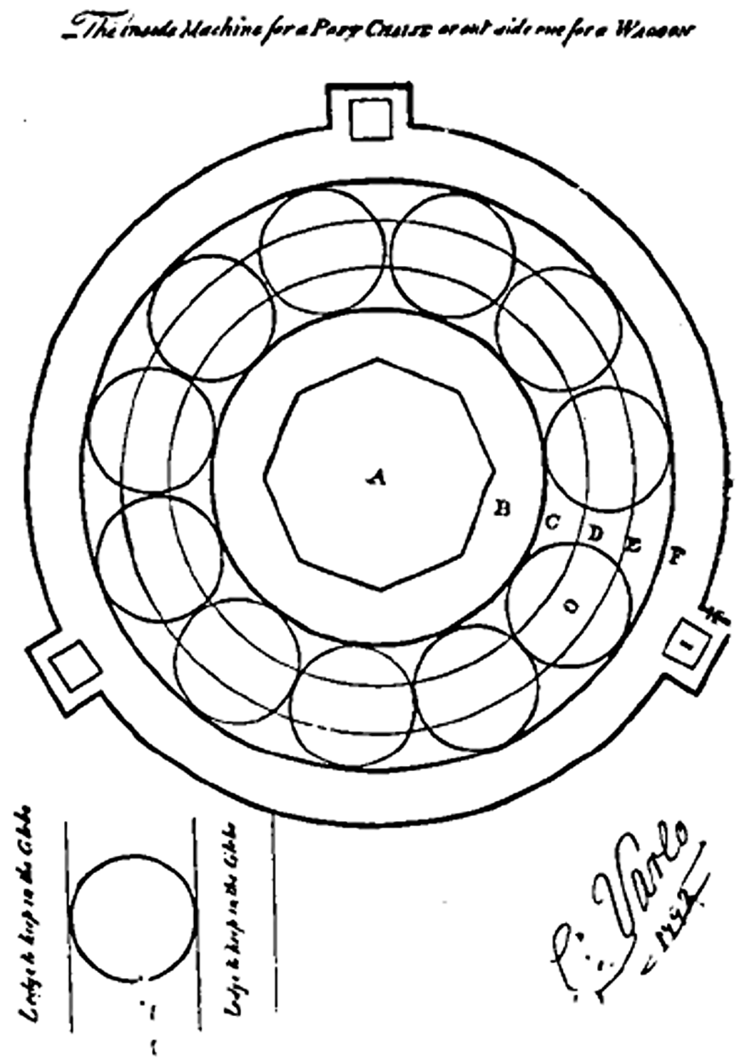
The figure above shows a fragment of revolving wooden platform - in fact a form of thrust bearing - with bronze balls and iron straps (ca. A. D. 50) found on the roman ships in lake Nemi (*recovered in 1932 but destroyed by fire on May 31, 1944 shortly after an exchange of artillery between U.S. and german troops*).

A reconstruction of the whole platform on the right.

* a suggested reading: Dowson D., Hamrock B.J., History of ball bearings, NASA Technical Memorandum 81689, 1981; http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19810009866_1981009866.pdf

Exploring history, we of course find a contribution from Leonardo da Vinci (figure below: ball, cone and roller pivot bearings from Codex Madrid I*).





Then we can jump to patents for wheel carriages, like in the drawing on the right from a patent of C. Varlo, London 1772.

*Dowson D., Hamrock B.J., History of ball bearings, NASA Technical Memorandum 81689, 1981; http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19810009866_1981009866.pdf

Earlier, in 1734, Jacob Rowe was awarded Patent Specification No. 543 by His Majesty King George II.

“ ... in a small booklet printed under Tom's Coffee House in Russell Street, Covent Garden, London, he explained in delightful detail how his "friction-wheels" could be applied to carts, wagons, coaches, watermills, windmills, and horse-operated mills.

... Jacob Rowe was also one of the first men to quantify the economic aspects of tribology, as least as far as friction in bearings was concerned. He argued that if all wagons and carriages were fitted with friction-wheels, they could be drawn by half the 40,000 horses then employed in the United Kingdom. Since the labor of the horse was valued at 1s. 6d. per day, this represented a direct saving of £1500 per day, or £547,500 per annum. Furthermore the cost of keeping a horse was estimated to be £10 per annum, and the savings on this account thus equalled £200,000 per annum. Rowe's estimate of the total potential savings on the operation of wheeled vehicles was thus £747,500 per annum ...

... There is no evidence that this improvement in the accounts of the early eighteenth century exchequer was achieved, but Jacob Rowe's treatise is a fascinating document.” *

* Dowson D., Hamrock B.J., History of ball bearings, NASA Technical Memorandum 81689, 1981; http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19810009866_1981009866.pdf

1. Introduction (4/13)

Contemporary bearing design is based on Hertz contact theory, but basic notions on Hertz contact are not sufficient. Others are also required, i.e., thermal dissipation, friction, lubrication, metallurgy, load sharing between rolling elements ... just to quote the most important ones.

In the following the reader will find only basic “introductory information“; further reading is strongly urged of manufacturers’ publications and of specialised textbooks such as:

T.A. Harris, *“Rolling Bearing Analysis”*, John Wiley & Sons, III ed. 1991 , IV ed. 2001; with Michael N. Kotzalas, CRC Press, 2006

J. Brändlein, *Ball and Roller Bearings: Theory, Design, and Application*, John Wiley & Sons1999

For descriptions of types and for their drawings, see one of the manufacturer’s catalogues. A good collection of free-use drawings for all bearing types:

http://commons.wikimedia.org/wiki/Category:Drawings_of_roller_bearings

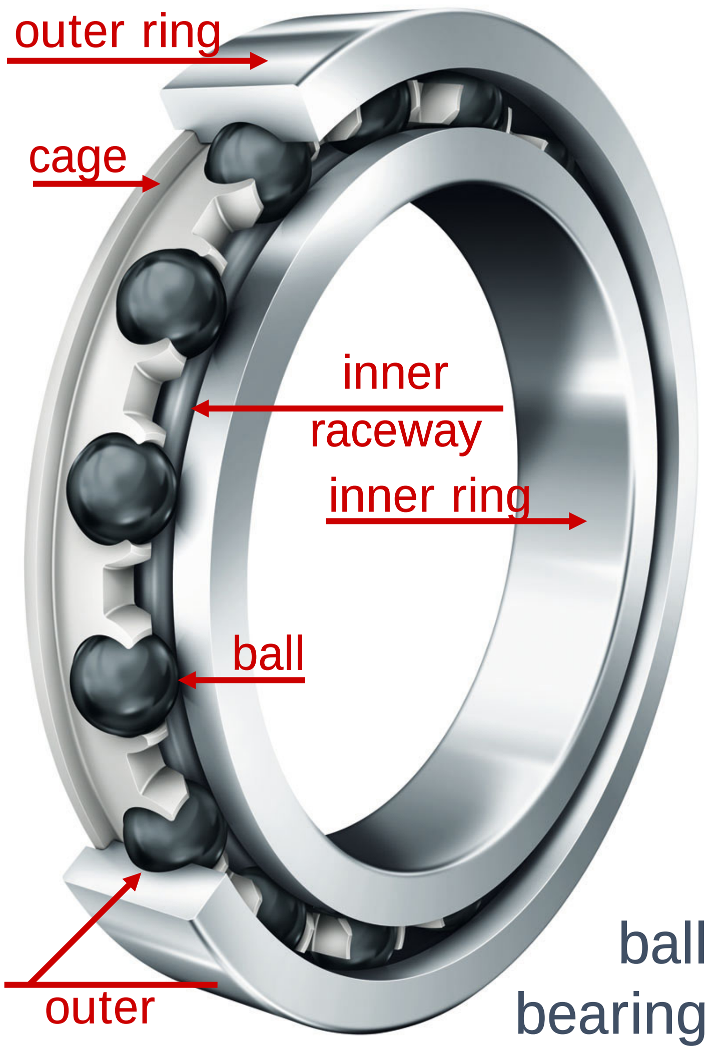
Rolling bearings can be classified into two types: ball and roller.

Balls and rollers roll on an inner and an outer track called “raceway”, and are held together and spaced evenly around the bearing by means of a separator called “cage”.

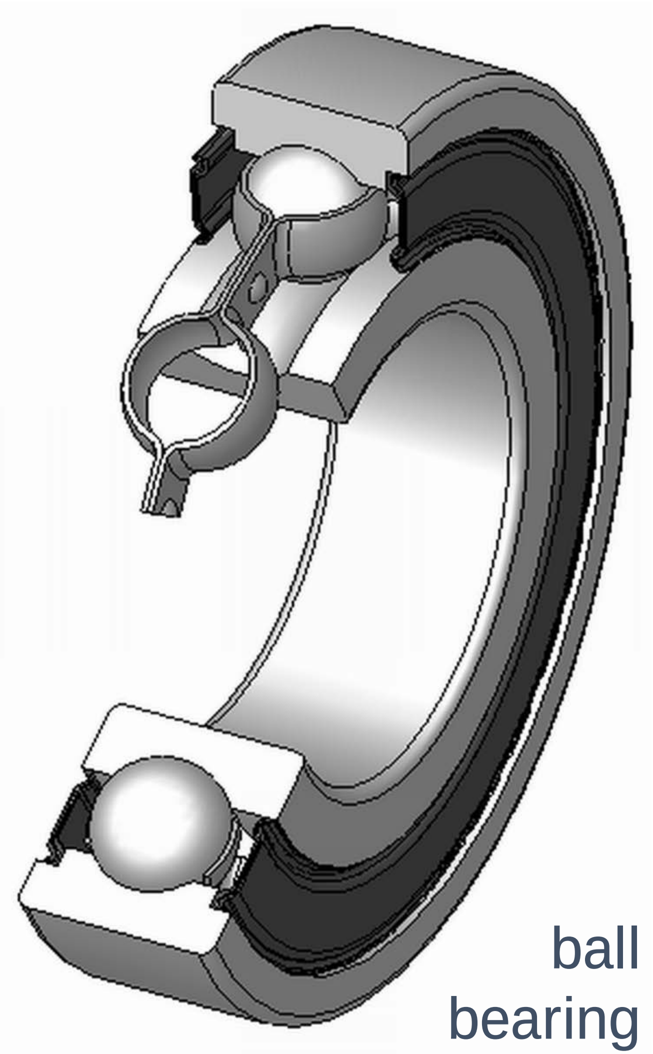
Ball bearings create the least amount of friction of any common bearing because the ball itself is the best antifriction rolling device known. The ball maintains point contact with the surface it rolls on and reduces friction to a minimum. It is, therefore, best suited to high-speed applications.

Roller bearings make use of cylindrical or conical-shaped rollers between the raceways.

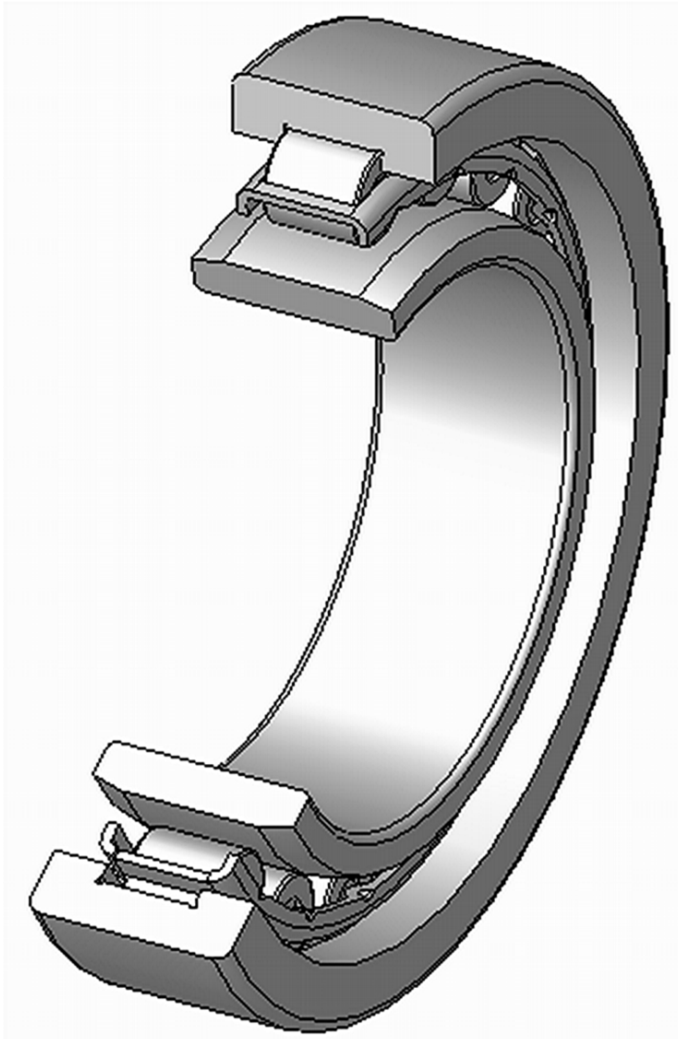
There are several types of rolling bearings; the next slides will show just few main types and some details.



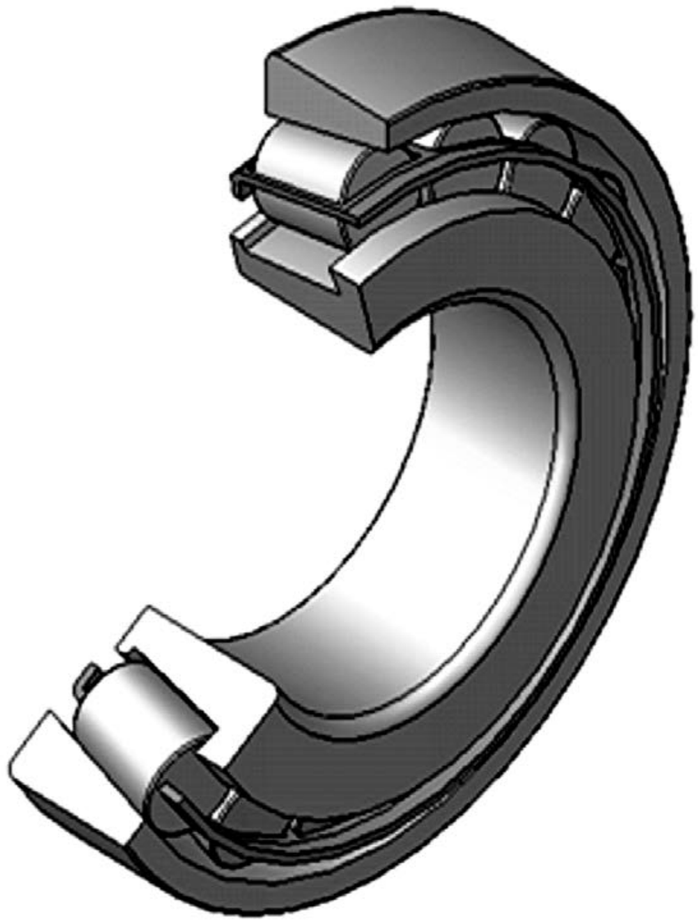
ball
bearing
sinthetic moulded cage



ball
bearing
pressed and riveted cage

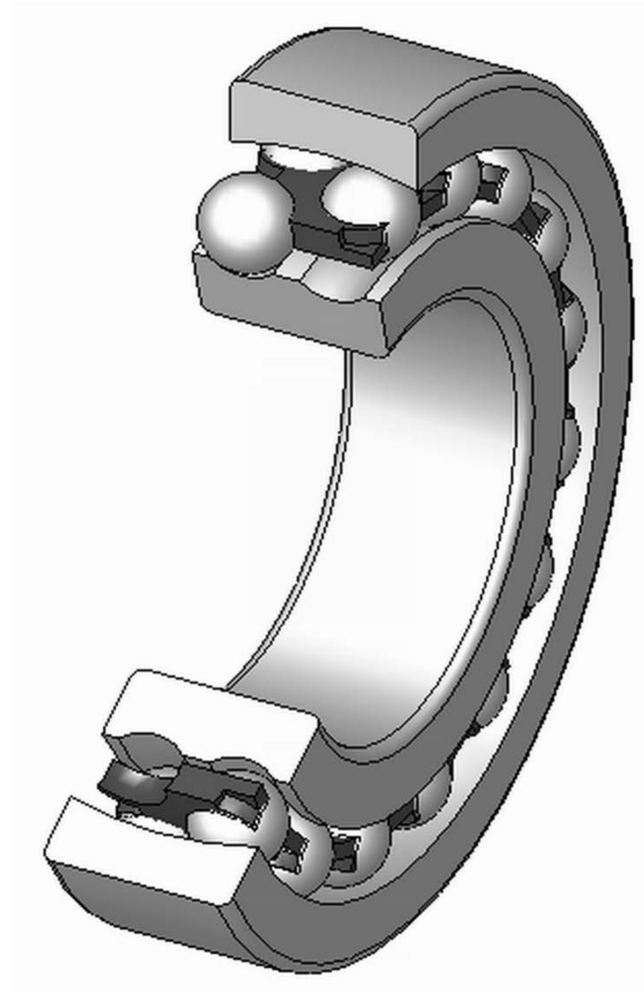


cylindrical roller bearing

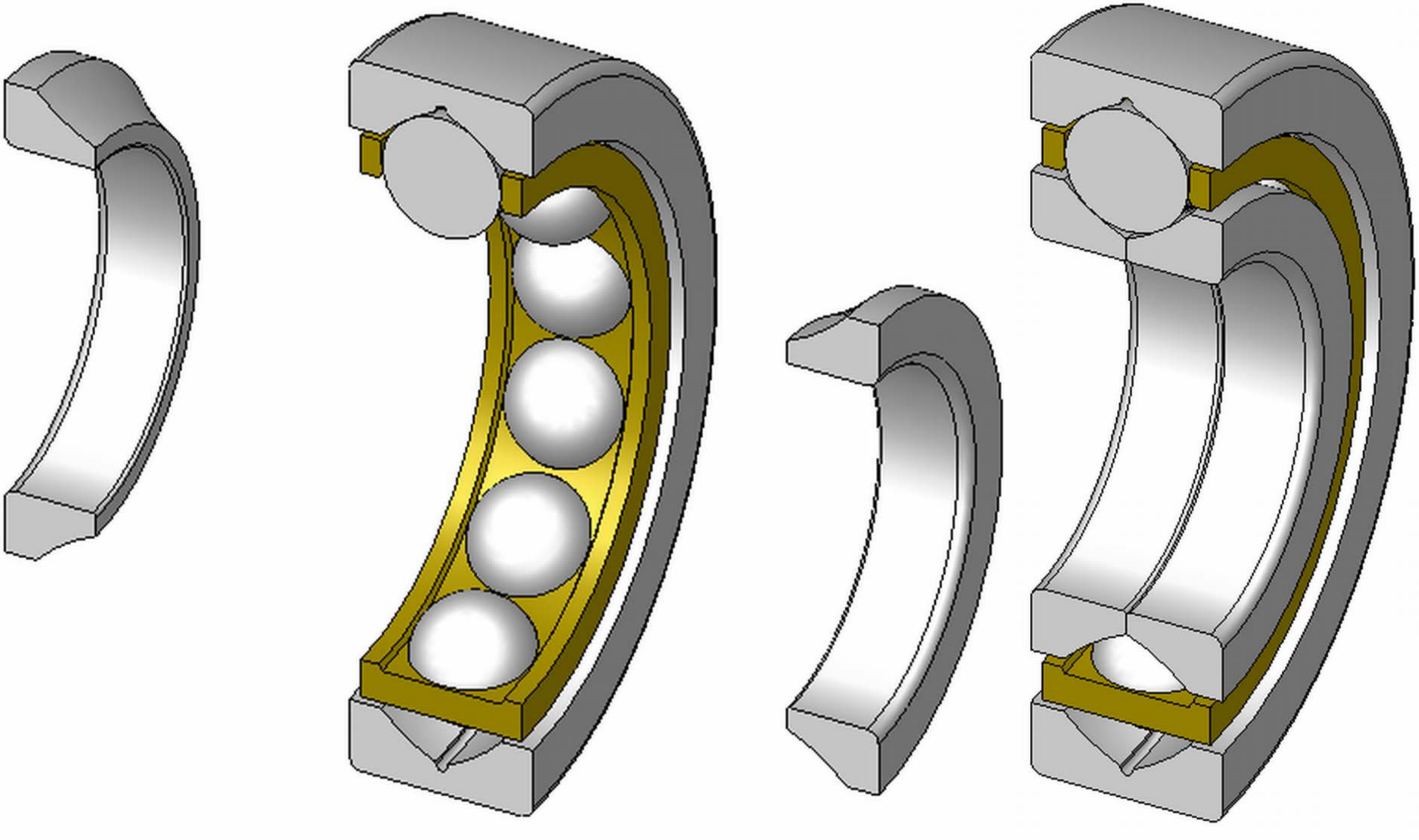


tapered
roller bearing

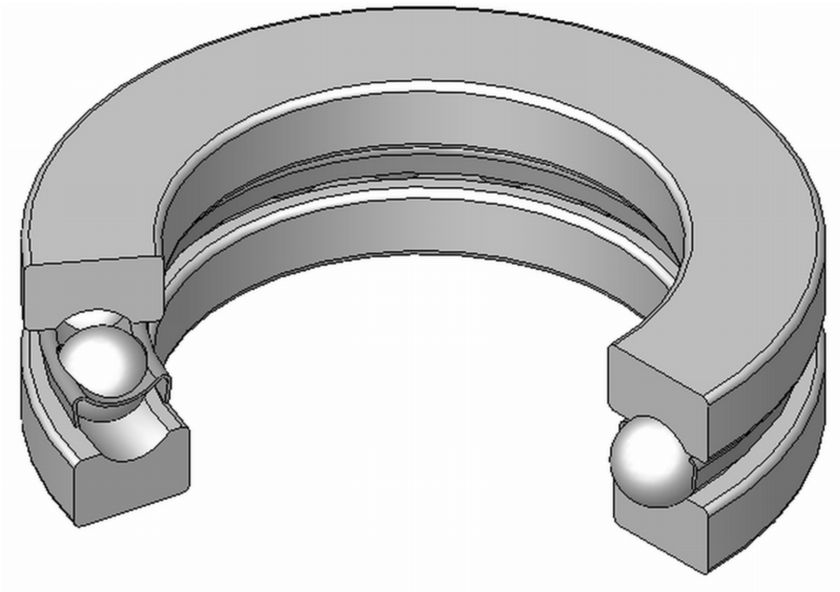
angular contact
ball bearing



spherical ball bearing
double-row

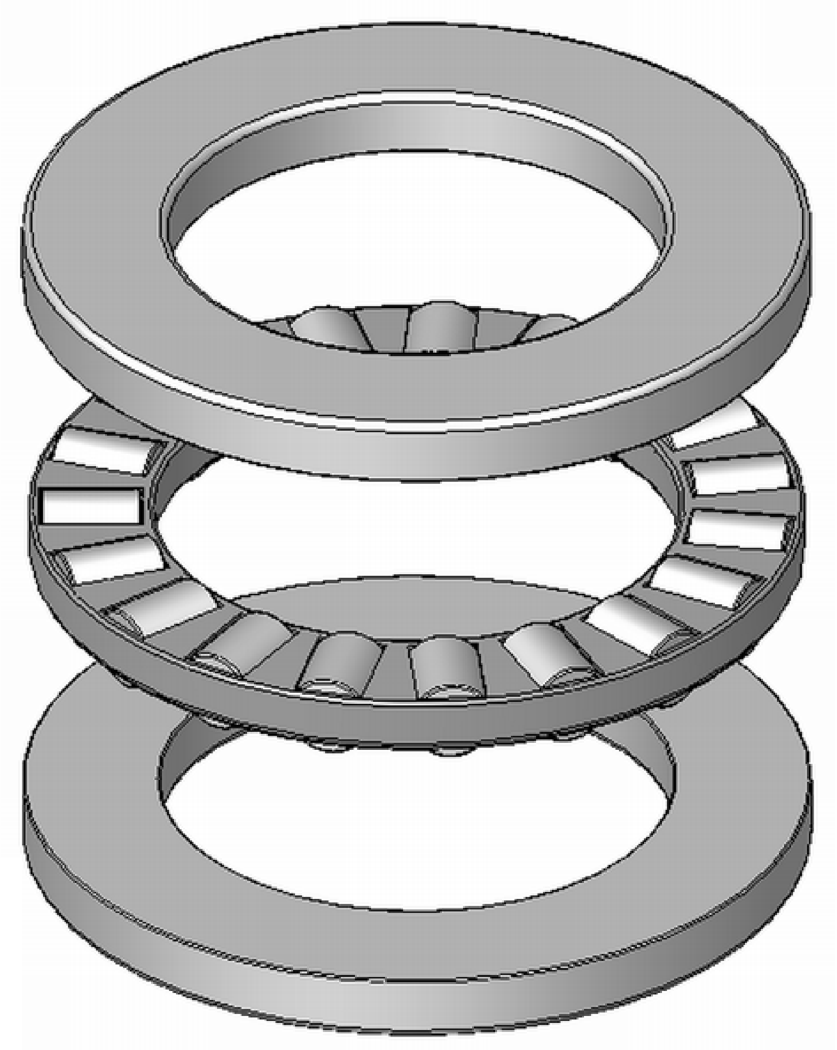


four points contact ball bearing – solid brass machined cage

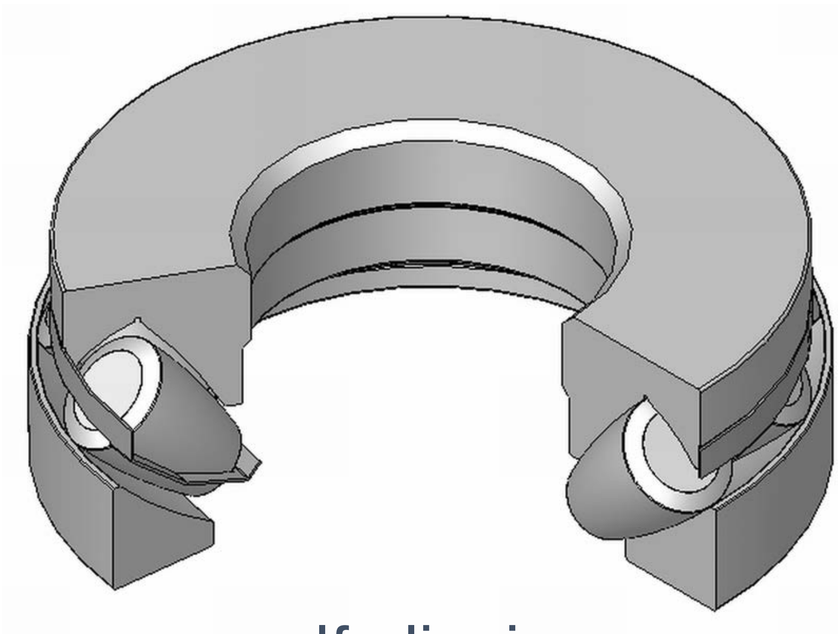


axial thrust ball bearing

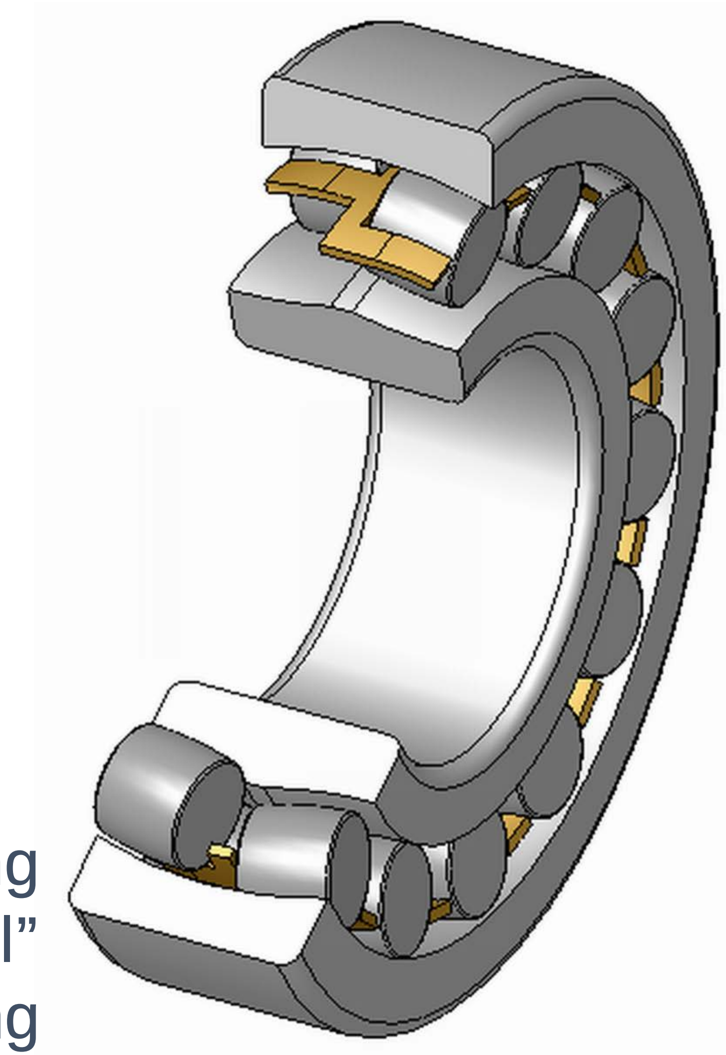
axial thrust cylindrical roller bearing



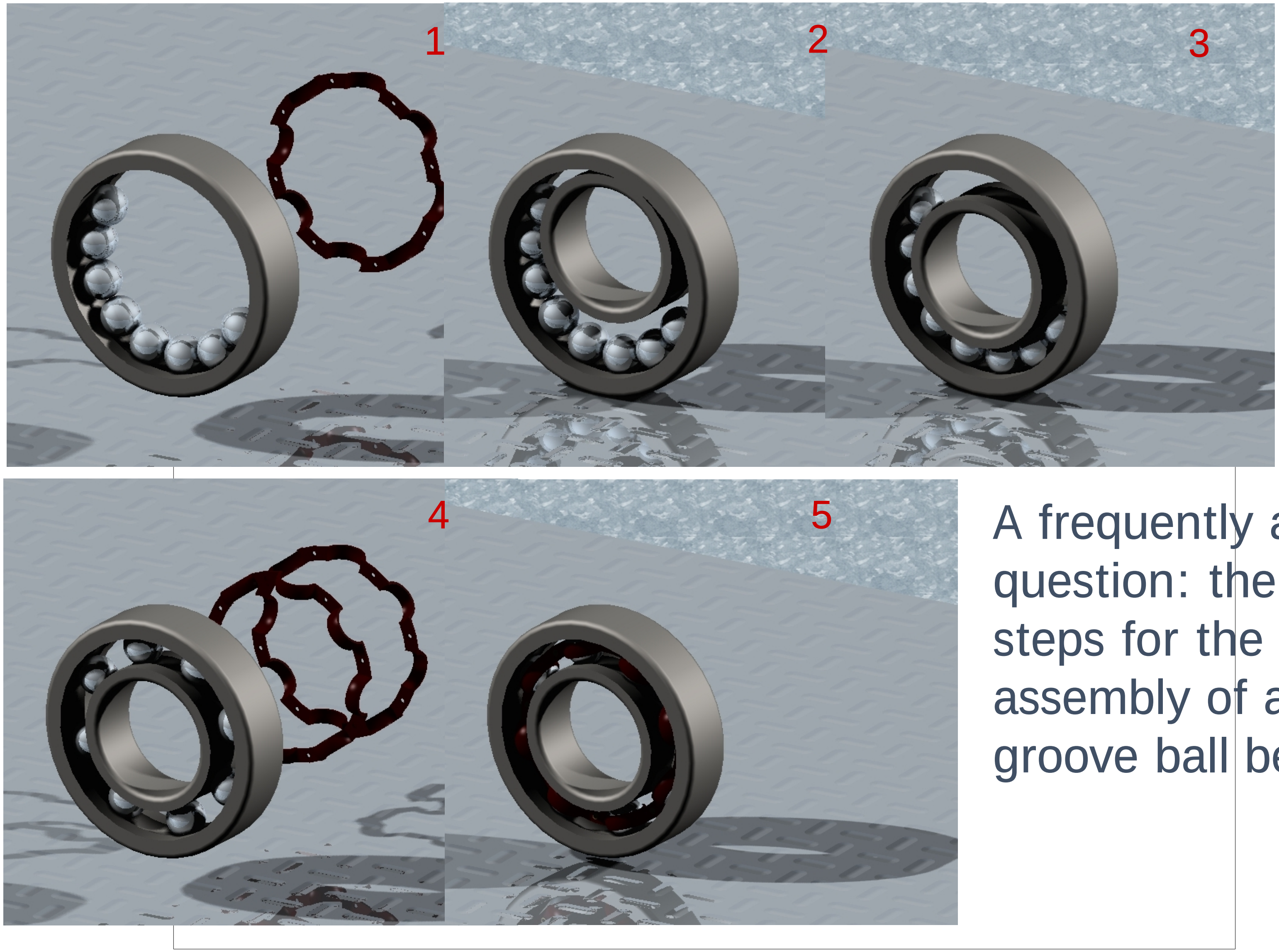
1. Introduction (11/13)



self-aligning
axial thrust roller bearing

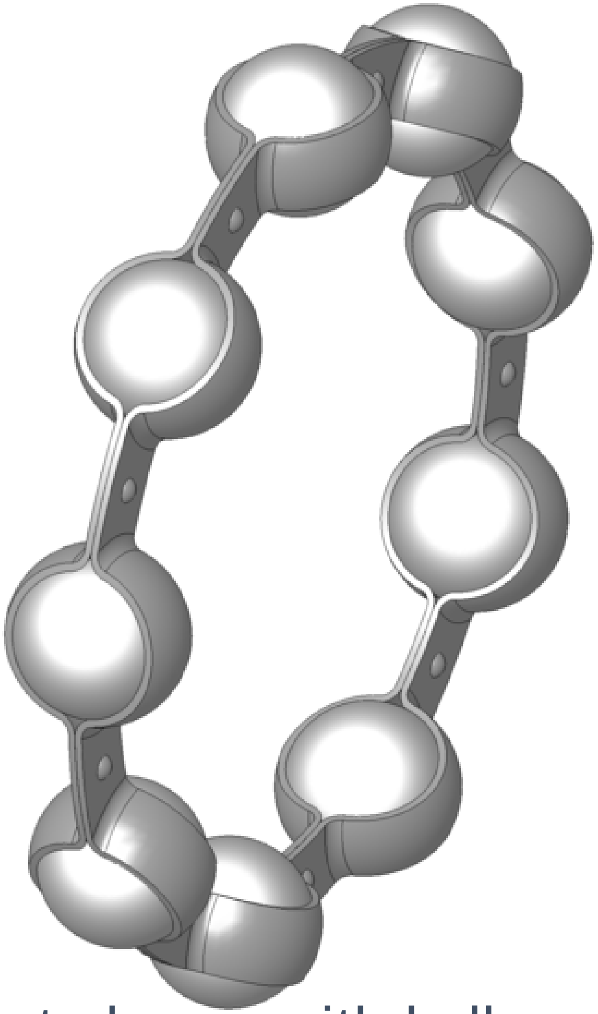


self-aligning
“spherical”
roller bearing



A frequently asked question: the five steps for the factory assembly of a deep groove ball bearing

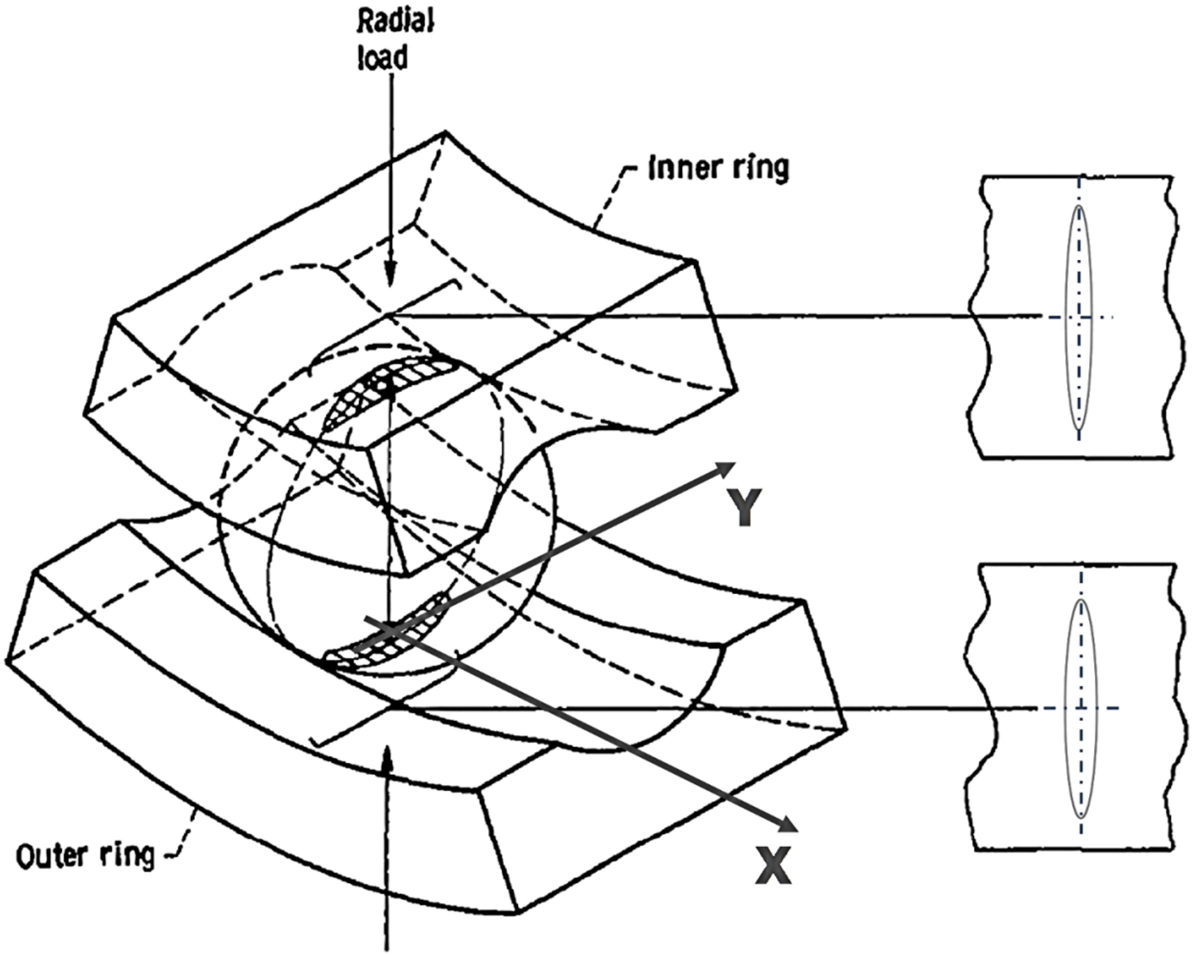
riveted cage



riveted cage with balls

2. Radial ball bearing curvatures and contact (1/3)

In the case of radial ball bearings, as illustrated here below:



Ball:

$$\alpha_X = \alpha_Y = 1/d_s$$

Inner ring:

$$\beta_{Xi} = +1/D_i$$

$$\beta_{Yi} \cong -1/(1{,}05\ d_s)$$

Outer ring:

$$\beta_{Xo} = -1/D_o$$

$$\beta_{Yo} \cong -1/(1{,}05\ d_s)$$

Adapted from: B. J. Hamrock and W. J. Anderson, Rolling-Element Bearings, NASA Ref. Publ. 1105, June 1983

The formula in sect. 3 sl. 18 of this chapter, for $\vartheta = 0$ gives:

$$\cos\tau = \frac{\sqrt{(\alpha_x - \alpha_y)^2 + (\beta_x - \beta_y)^2 + 2(\alpha_x - \alpha_y)(\beta_x - \beta_y)}}{\alpha_x + \alpha_y + \beta_x + \beta_y} =$$

$$= \frac{\left|(\alpha_x - \alpha_y) + (\beta_x - \beta_y)\right|}{\alpha_x + \alpha_y + \beta_x + \beta_y}$$

Inner ring:

$$\alpha_x + \alpha_y + \beta_x + \beta_y =$$

$$= \frac{1}{d_s} + \frac{1}{d_s} - \frac{1}{1{,}05\ d_s} + \frac{1}{D_i} \cong$$

$$\cong \frac{1{,}05}{d_s} + \frac{1}{D_i}$$

Outer ring:

$$\alpha_x + \alpha_y + \beta_x + \beta_y =$$

$$= \frac{1}{d_s} + \frac{1}{d_s} - \frac{1}{1{,}05\ d_s} - \frac{1}{D_o} \cong$$

$$\cong \frac{1{,}05}{d_s} - \frac{1}{D_o}$$

2. Radial ball bearing curvatures and contact (3/3)

Inner ring:

$$\left|\left(\alpha_x - \alpha_y\right) + \left(\beta_x - \beta_y\right)\right| =$$

$$= \left|0 + \left(-\frac{1}{1{,}05\, d_s} - \frac{1}{D_i}\right)\right| \cong$$

$$\cong \frac{0{,}95}{d_s} + \frac{1}{D_i}$$

Outer ring:

$$\left|\left(\alpha_x - \alpha_y\right) + \left(\beta_x - \beta_y\right)\right| =$$

$$= \left|0 + \left(-\frac{1}{1{,}05\, d_s} + \frac{1}{D_o}\right)\right| \cong$$

$$\cong \frac{0{,}95}{d_s} - \frac{1}{D_o}$$

$$\cos\tau = \frac{\dfrac{0{,}95}{d_s} + \dfrac{1}{D_i}}{\dfrac{1{,}05}{d_s} + \dfrac{1}{D_i}}$$

$$\cos\tau = \frac{\dfrac{0{,}95}{d_s} - \dfrac{1}{D_o}}{\dfrac{1{,}05}{d_s} - \dfrac{1}{D_o}} \cong \frac{\dfrac{0{,}95}{d_s} - \dfrac{1}{D_i + 2d_s}}{\dfrac{1{,}05}{d_s} - \dfrac{1}{D_i + 2d_s}}$$

Sections 3, 4, 5 - Load sharing between rolling bodies

Section 2 illustrates the well-known Stribeck solution of the statically indeterminate problem of determining how the load transmitted by the inner ring to the outer ring of a radial bearing produces contact forces between the rings and the rolling elements (spheres or cylinders). The solution gives the value of the most loaded rolling element, the number and position of rolling elements subjects to loading, and information on how much load goes on the rolling elements.

Section 3 deals with the case of a bearing rotating at a so high speed that the centrifugal forces on the rolling elements cannot be neglected when calculating contact forces.

Section 4 finds shows how an axial force can be supported by a radial bearing, provided a diametral clearance allows an axial displacement between the inner and the outer ring.

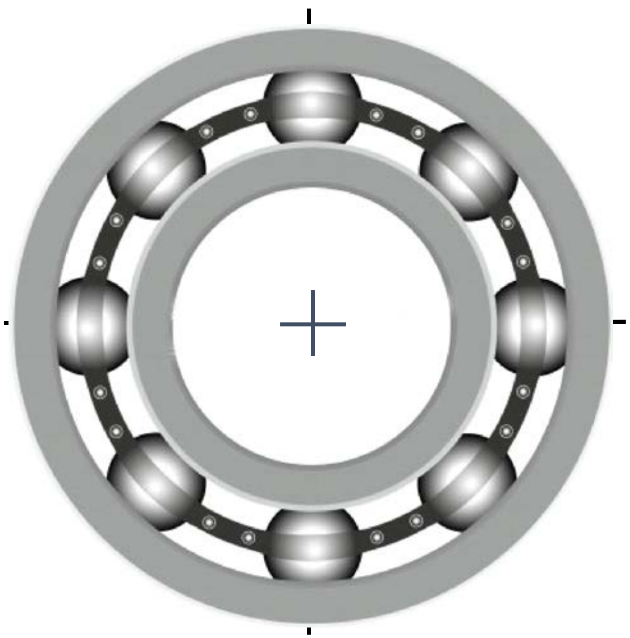
Sphere and roller contacts have the same form for the approach- -force relation (here below for steel on steel):

Sphere: $$\delta = \delta^* \, 3{,}60 \cdot 10^{-4} \sqrt[3]{\sum \frac{1}{\tilde{d}}} \, F^{2/3}$$

for the single contact of a ball on a raceway , Ch.1 Sect.5 sl.1

Cylinder: $$\delta = 3{,}84 \cdot 10^{-5} \frac{F^{0,9}}{L^{0,8}}$$

for the single contact of a roller on a raceway, Palmgren, Ch.1 Sect.6 ; you may use Brändlein, Ch.1 Sect.6 sl. 7



bearing seen from outside

Note that if centrifugal forces are absent or negiglible:

$F_i = F_o \equiv F$,

otherwise it would be $F_o = F_i + F_c$

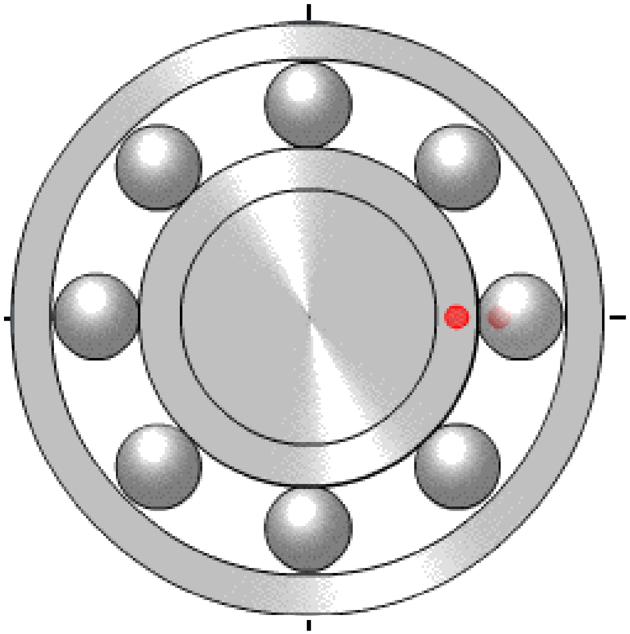
Later we shall need the equation in the form: $F = K\,\delta^n$, so

we define exponent ^{n} at $n=1{,}5$ for spheres and $n \cong 1{,}1$ for cylinders

Sphere and roller contacts have the same form for the approach- -force relation (here below for steel on steel):

Sphere: $\delta = \delta^* \, 3{,}60 \cdot 10^{-4} \sqrt[3]{\sum \frac{1}{\tilde{d}}} \, F^{2/3}$

Cylinder: $\delta = 3{,}84 \cdot 10^{-5} \frac{F^{0,9}}{L^{0,8}}$



view of the raceways profiles

Note that if centrifugal forces are absent or negiglible:

$F_i = F_o \equiv F$,

otherwise it would be $F_o = F_i + F_c$

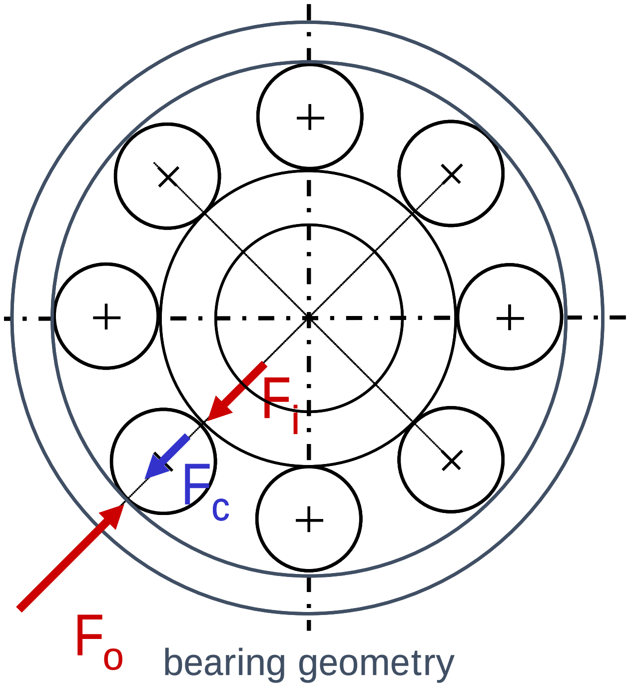
Later we shall need the equation in the form: $F = K \, \delta^n$, so

we define exponent ^{n} at $n=1{,}5$ for spheres and $n \cong 1{,}1$ for cylinders

Sphere and roller contacts have the same form for the approach- -force relation (here below for steel on steel):

Sphere: $\delta = \delta^* \, 3{,}60 \cdot 10^{-4} \sqrt[3]{\sum \frac{1}{\tilde{d}}} \, F^{2/3}$

Cylinder: $\delta = 3{,}84 \cdot 10^{-5} \frac{F^{0,9}}{L^{0,8}}$



bearing geometry

Note that if centrifugal forces are absent or negiglible:

$F_i = F_o \equiv F$,

otherwise it would be $F_o = F_i + F_c$

Later we shall need the equation in the form: $F = K\,\delta^n$, so

we define exponent ^{n} at $n=1{,}5$ for spheres and $n \cong 1{,}1$ for cylinders

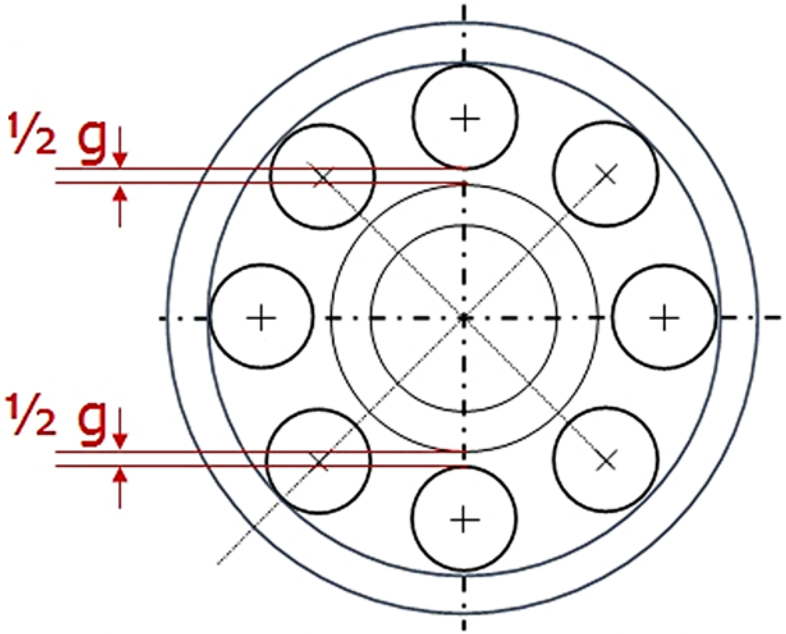
Then $F_o = K_o\,\delta_o^n$, $F_i = K_i\,\delta_i^n$; the total approach is calculated as the sum of inner and outer local displacements with $F_i=F_o$:

$$\delta_{tot} = \delta_i + \delta_o = F^{1/n}\left[\frac{1}{K_i^{1/n}} + \frac{1}{K_o^{1/n}}\right] =$$

$$= \left(\frac{F}{K_{tot}}\right)^{1/n}$$

or : $\boxed{F = K_{tot}\cdot\delta_{tot}^n}$

$$K_{tot} = \left[\frac{1}{K_i^{1/n}} + \frac{1}{K_o^{1/n}}\right]^{-n}$$



In the case of radial bearings we shall now start from the basic configuration where all (circles representing) raceways are concentric, and the rolling elements in contact with the outer raceway.

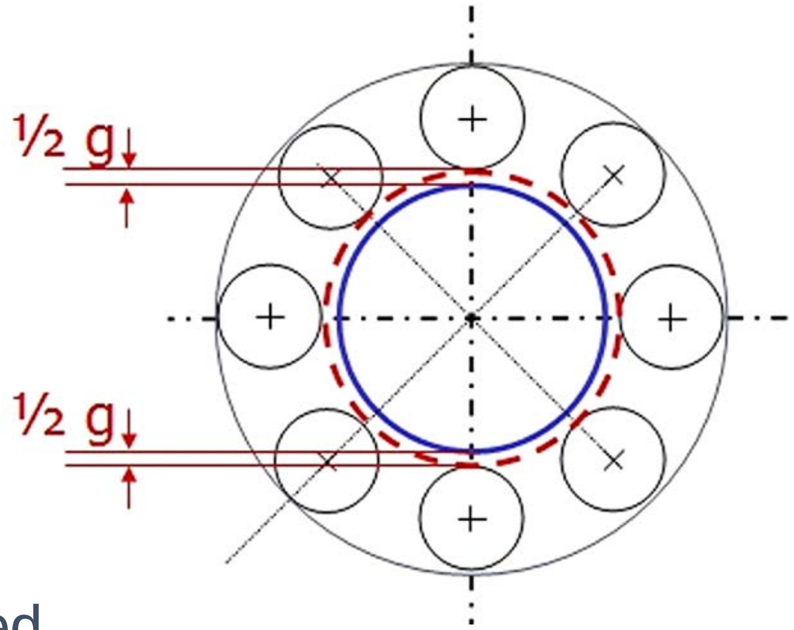
This image differs from that in the previous page: the total "radial clearance" g was introduced.

The following elements have been put into evidence:

Note that "radial clearance" g is the total possible displacement of the inner ring against the outer ring.

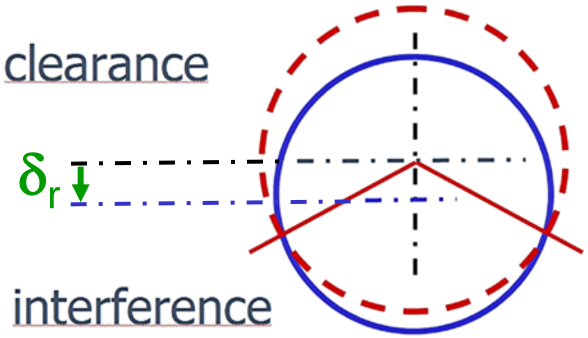
thick solid line: the circle at the lower bottom of the raceway on the inner ring

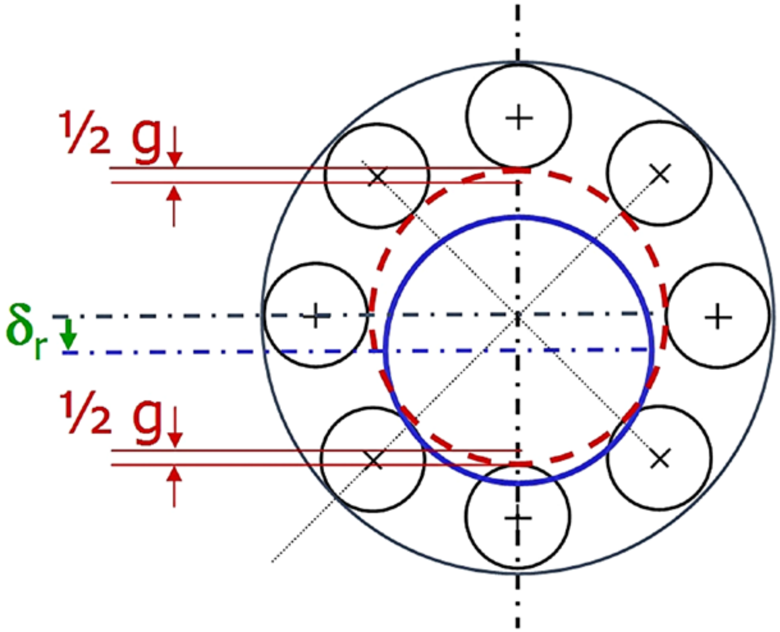
dashed line: the profile of the envelope of undeformed rolling bodies taken in their outer-most possible position, i.e., in contact with the outer raceway.



We assume that both rings are mounted in perfectly rigid supports, i.e., they will remain perfectly circular under load, with the exception of local deformations due to Hertz contacts; we must remember that these are orders of magnitudes lower than the radii of rolling bodies and raceways.

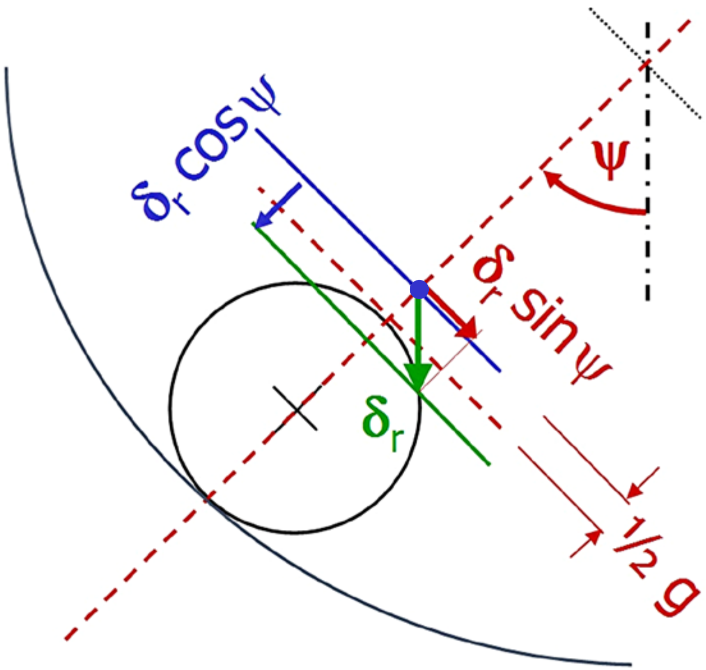
We shall then move the inner ring (and race) relative to all other elements by the "radial approach" δ_r .

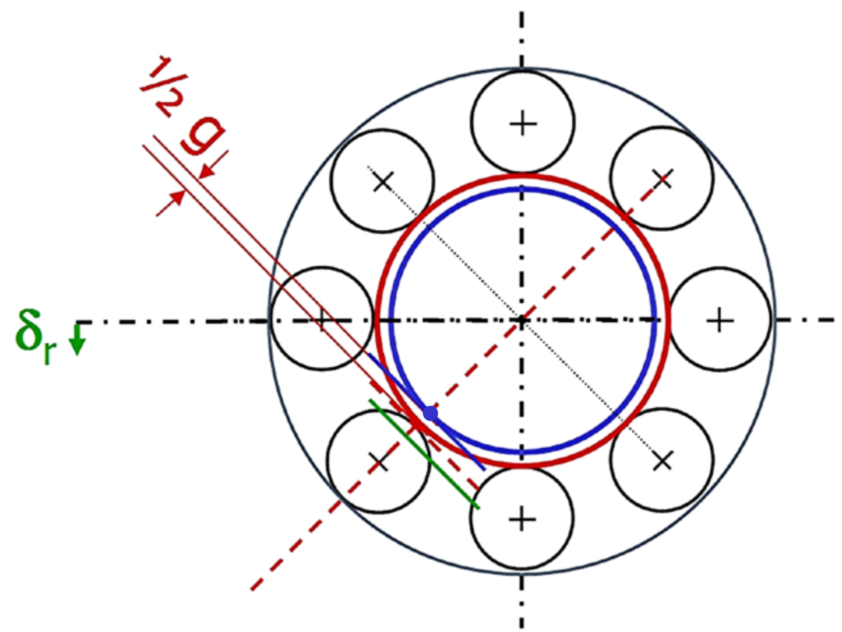




Due to the fact that contact surfaces and local deformations affect dimensions which are very small compared to bodies' diameters, we shall in the following substitute circles with their tangent segments ...

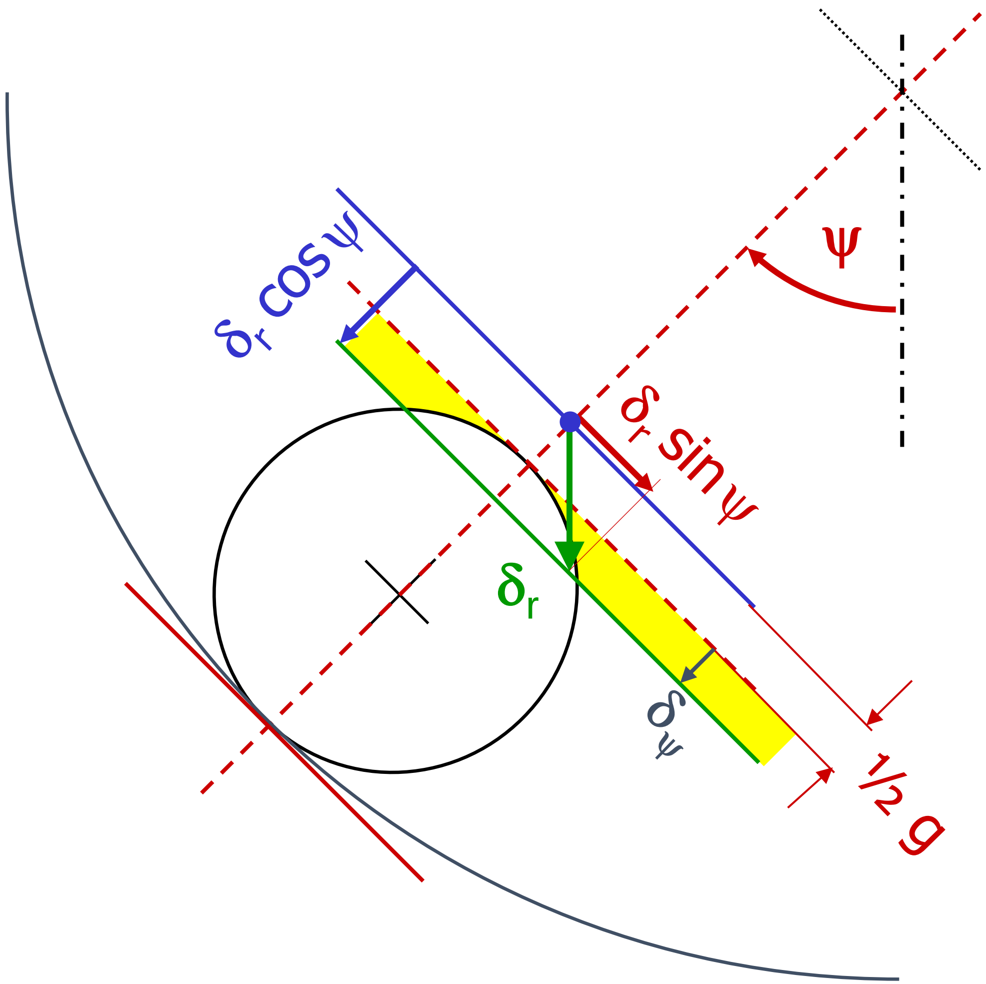
... then, a magnification in the vicinity of the theoretical contact point (or line) shows that the radial (here, vertical) displacement or "approach" δ_r has two components, one of them, $\delta_r \cos\psi$, along the radius through the centre of the rolling body at angle ψ:

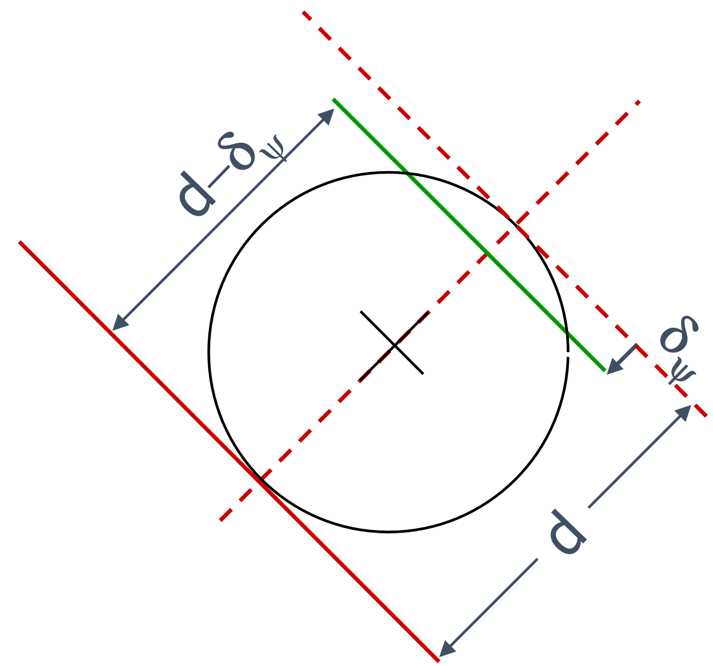




If the radial component $\delta_r \cos\psi$ of δ_r exceeds the radial clearance ½ g, it produces an interference.

The radial interference along the diameter of the rolling body at the angle ψ, $\delta_\psi = \delta_r \cos\psi - \frac{1}{2} g$, sets two parallel lines between which the elastic bodies will accommodate through the local, inner and outer, elastic "approach".





then ...

check : for $\psi = 0$, $\delta_\psi = \delta_r \cdot \cos\psi - \frac{1}{2} g \Rightarrow \delta_0 = \delta_r - \frac{1}{2} g$

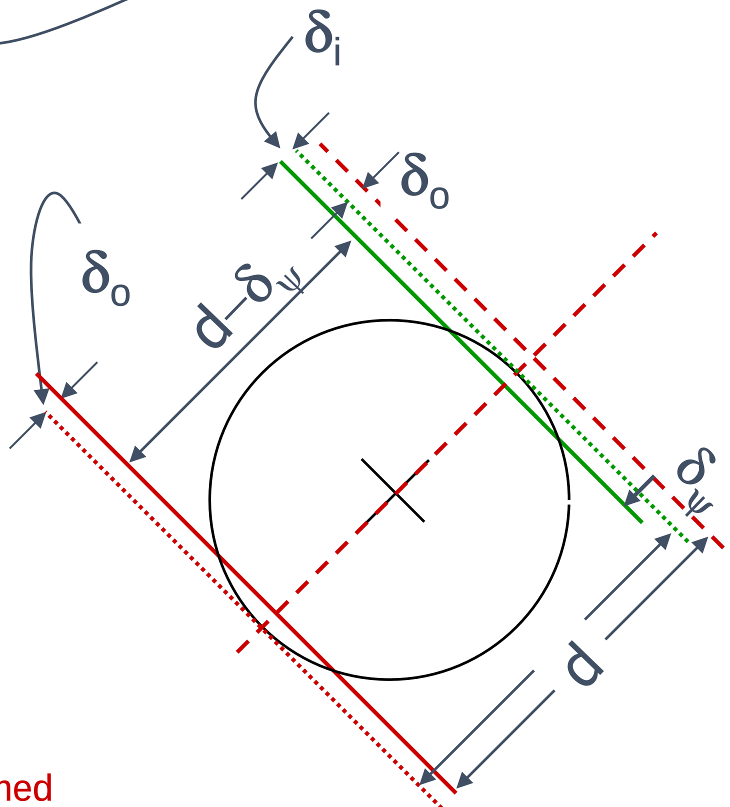
Attention!!
δ_0 is δ_ψ for $\psi=0$
do not misunderstand $_o$ for $_0$

... then: $\delta_\psi = \delta_r \cos\psi - ½\, g = \delta_i + \delta_o$

Attention!! This is δ_o, i.e. the approach at the outer ring

The ratio of "ring-to-ring" approach δ_ψ at angle ψ over the maximum approach δ_0, which occurs at angle 0, is:

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_r \cdot \cos\psi - \frac{1}{2} g}{\delta_r - \frac{1}{2} g} = \frac{\cos\psi - \frac{1}{2}\frac{g}{\delta_r}}{1 - \frac{1}{2}\frac{g}{\delta_r}}$$



$d-\delta_\psi = d-(\delta_i+\delta_o)$

Attention!! This drawing is simplified and may be misleading!.
In the figure it is represented as a penetration of the undeformed rolling body inside the undeformed raceway. The approach is elastically shared between rolling body and raceway. Both deform.

3. Load distribution in radial bearings (7b/16)

check : for $\psi = 0$, $\delta_\psi = \delta_r \cdot \cos\psi - \frac{1}{2} g \Rightarrow \delta_0 = \delta_r - \frac{1}{2} g$

Attention!!
δ_0 is δ_ψ for $\psi=0$
do not misunderstand $_0$ for $_o$

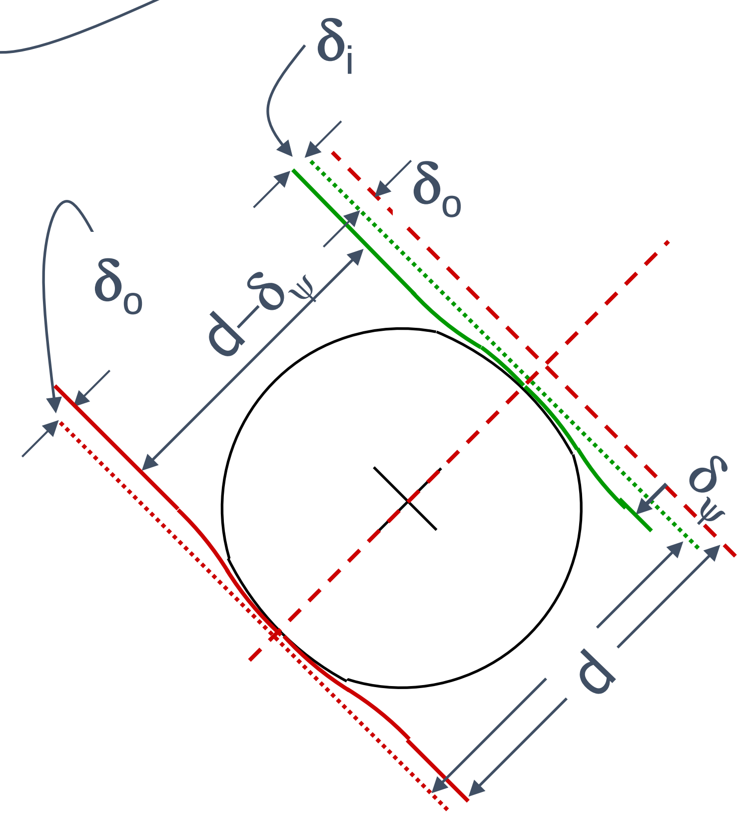
... then: $\delta_\psi = \delta_r \cos\psi - ½\, g = \delta_i + \delta_o$

Attention!! This is δ_o, i.e. the approach at the outer ring

The ratio of "ring-to-ring" approach δ_ψ at angle ψ over the maximum approach δ_0, which occurs at angle 0, is:

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_r \cdot \cos\psi - \frac{1}{2} g}{\delta_r - \frac{1}{2} g} = \frac{\cos\psi - \frac{1}{2}\frac{g}{\delta_r}}{1 - \frac{1}{2}\frac{g}{\delta_r}}$$

In the drawing now it is seen that the approach is elastically shared between rolling body and raceway. Both deform.



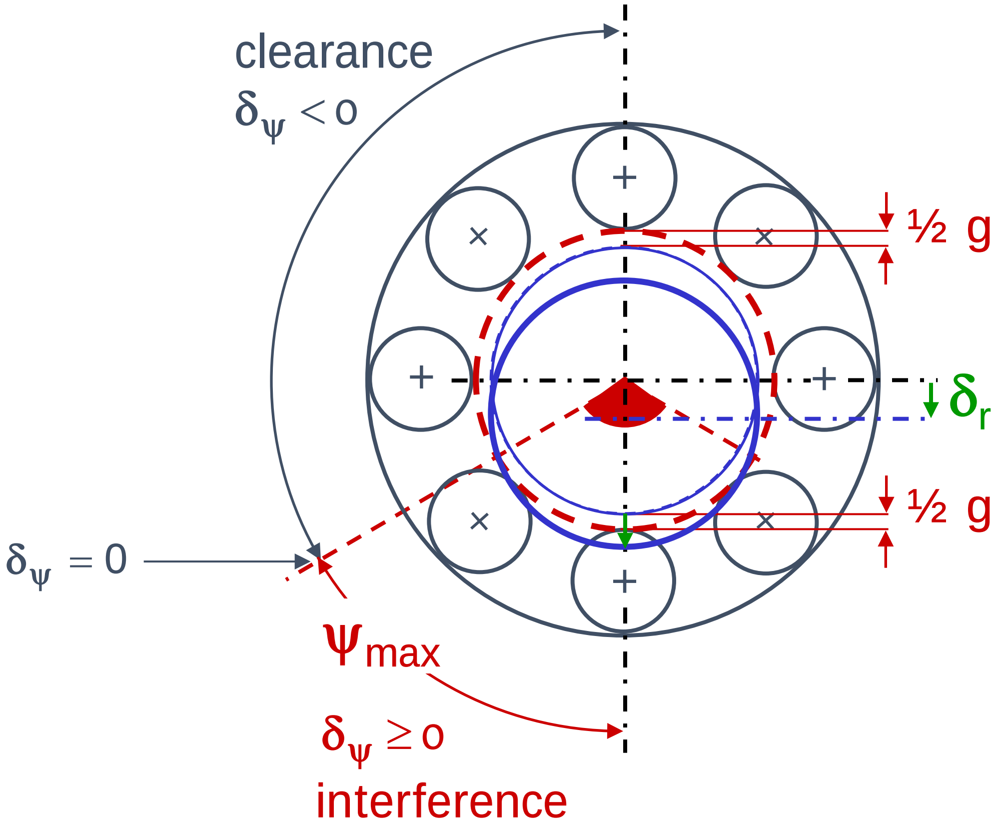
$d-\delta_\psi = d-(\delta_i+\delta_o)$

ψ_{max} is the angular half-opening of the loaded part of the bearing, i.e where $\delta_\psi \geq 0$; outside this angle there is no contact.

Where δ_ψ is zero:

$$0 = \delta_r \cdot \cos\psi_{max} - \frac{1}{2} g \Rightarrow$$

$$\Rightarrow \begin{cases} \cos\psi_{max} = \dfrac{1}{2}\dfrac{g}{\delta_r} \\ \psi_{max} = \arccos\left(\dfrac{g}{2\delta_r}\right) \end{cases}$$



- for g=0 , ψ_{max}= 90°
- for g>0 , ψ_{max}=0 at δ_r=½ g
- for g>0 , $\psi_{max} \Rightarrow 90°$ for $\delta_r \Rightarrow \infty$

We can then write:

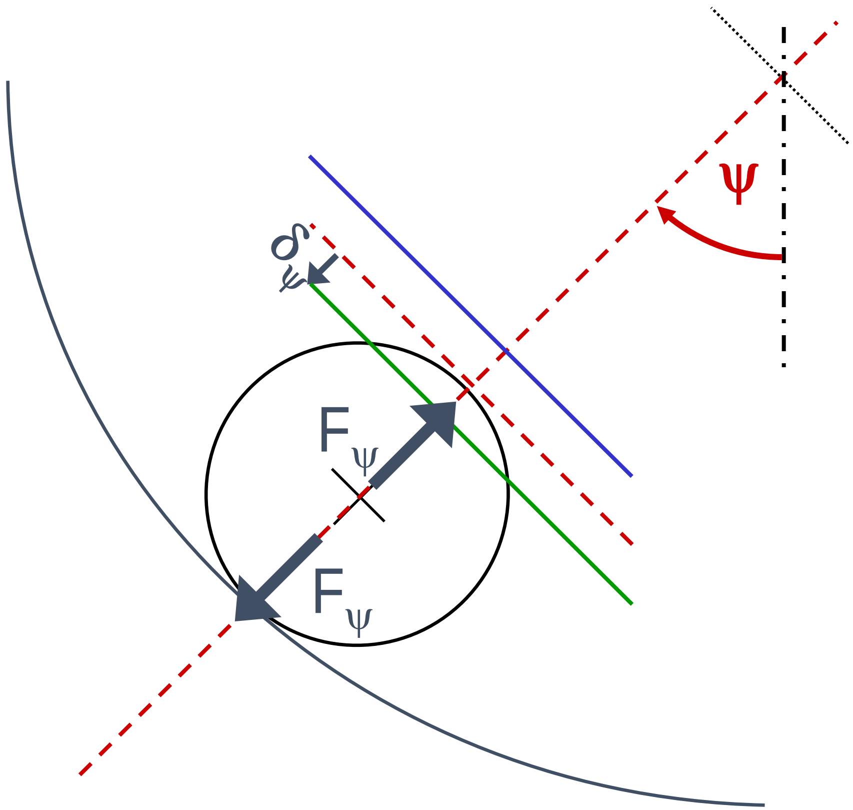
$$\frac{\delta_\psi}{\delta_0} = \frac{\cos\psi - \frac{1}{2}\frac{g}{\delta_r}}{1 - \frac{1}{2}\frac{g}{\delta_r}} = \frac{\cos\psi - \cos\psi_{max}}{1 - \cos\psi_{max}}$$

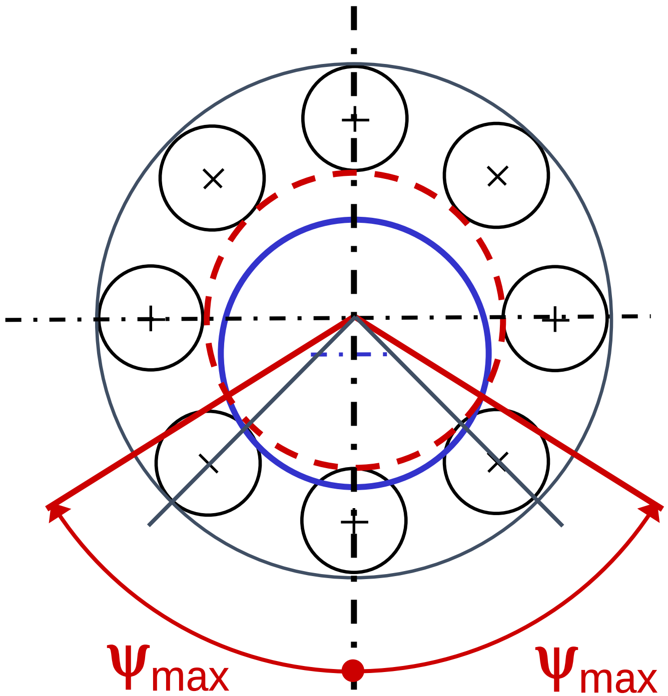
We can remark that the assumption of rigidly translating rings implies that the radial approach of all rolling bodies is a known function of angle ψ . In other words, it depends only on the translational kinematics of the rings/races.

We know that forces developed during a single Hertz approach are a known function of the approach value. Then, all contact forces are expected to be a known function of angle ψ.

From: $F = K_{tot} \cdot \delta_{tot}^{n} \equiv K_{tot} \cdot (\delta_i + \delta_o)^n$ then: $F_\psi = K_{tot} \cdot \delta_\psi^n$ (sl. 2, 7)

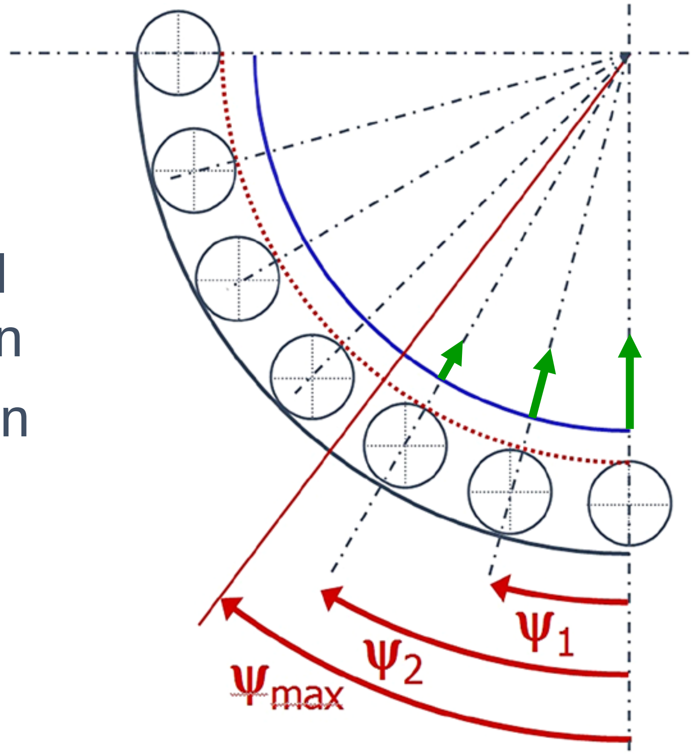
$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^n = \left(\frac{\cos\psi - \cos\psi_{max}}{1 - \cos\psi_{max}}\right)^n$$





The total vertical load R is the sum of the vertical components of all contact forces exchanged between the ball/roller and the inner ring:

$$R = \sum_{-\psi_{max}}^{+\psi_{max}} F_{\psi_i} \cdot \cos\psi_i$$



F_{ψ_i} → in the local approach direction

$\cos\psi_i$ → proiection along the vertical (bearing) approach direction

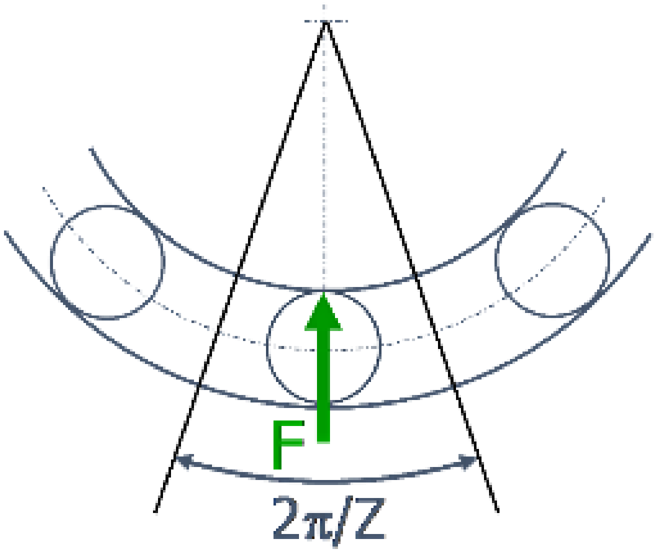
Substituting $F_{\psi i}$:

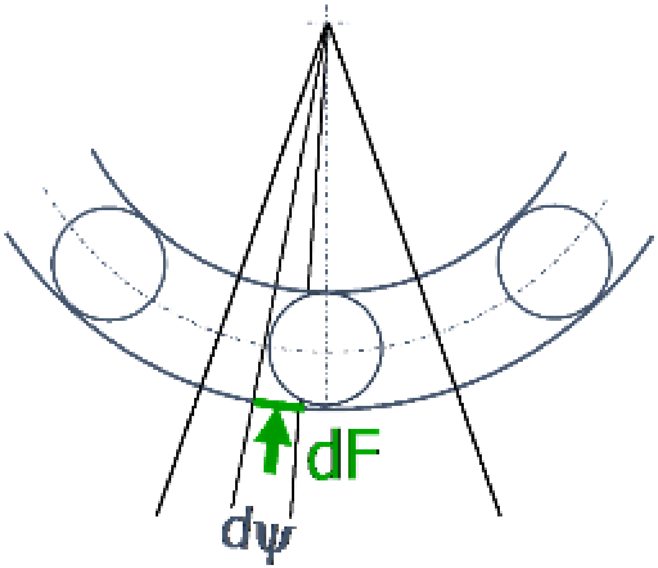
$$R = F_0 \sum_{-\psi_{max}}^{+\psi_{max}} \left(\frac{\cos\psi_i - \cos\psi_{max}}{1 - \cos\psi_{max}} \right)^n \cdot \cos\psi_i$$

The summation is discrete, and extends to all rolling bodies satisfying $\psi_i \leq \psi_{max}$ $(\psi_0 = 0°, \psi_1, \ldots \psi_i \ldots)$

In order to get a result for the most stressed ball or roller (evidently the one at $\psi=0°$) we must perform a finite summation which must be adapted to each case; we can get simpler and more general formulas by considering the stiffness of rolling bodies not concentrated but distributed on an arch $2\pi/Z$ of the raceway, where Z is the total number of rolling bodies :

$$dK_{tot} = \frac{K_{tot}}{2\pi / Z} d\psi \qquad F_\psi = K_{tot}\,\delta_\psi^n \quad \Rightarrow \quad dF_\psi = dK_{tot}\,\delta_\psi^n = K_{tot}\,\delta_\psi^n \frac{Z}{2\pi} d\psi$$





The finite summation is thus transformed into an integral:

$$R = \sum_{-\psi_{max}}^{+\psi_{max}} F_{\psi_i} \cdot \cos\psi_i = \int_{-\psi_{max}}^{+\psi_{max}} \cos\psi \cdot dF_\psi = K_{tot} \frac{Z}{2\pi} \int_{-\psi_{max}}^{+\psi_{max}} \cos\psi \, \delta_\psi^n \, d\psi =$$

$$= K_{tot}\,\delta_0^n \frac{Z}{2\pi} \int_{-\psi_{max}}^{+\psi_{max}} \frac{\delta_\psi^n}{\delta_0^n} \cos\psi \; d\psi =$$

$$= F_0 \cdot Z \cdot \frac{1}{2\pi} \int_{-\psi_{max}}^{+\psi_{max}} \boxed{\left(\frac{\cos\psi - \cos\psi_{max}}{1 - \cos\psi_{max}} \right)^n} \cos\psi \cdot d\psi = F_0 \cdot Z \cdot J_S$$

where J_S is the "Stribeck integral", which is a function of $g/2\delta_r$ through ψ_{max} ; this integral has been evaluated for different values of ψ_{max} , i.e. for different values of the relative clearance $g/2\delta_r$.

3. Load distribution in radial bearings (14/16)

For ball bearings and zero clearance (which means $\psi_{max} = 90°$) J_S=0,229, then the highest roller load (the one at ψ=0), is:

$$F_0 = \frac{4.37 \cdot R}{Z}$$

For roller bearings and zero clearance: $\longrightarrow$ $F_0 = \frac{4.08 \cdot R}{Z}$

When g≠0 the integration limits are dependent on δ_r . Stribeck* took into account the standard clearances of bearings and the radial approach δ_r corresponding to the highest load sustained by the bearing (in numbers ca. $g/2\ \delta_r$=0.30, ψ_{max}=72°) and obtained the celebrated formula:

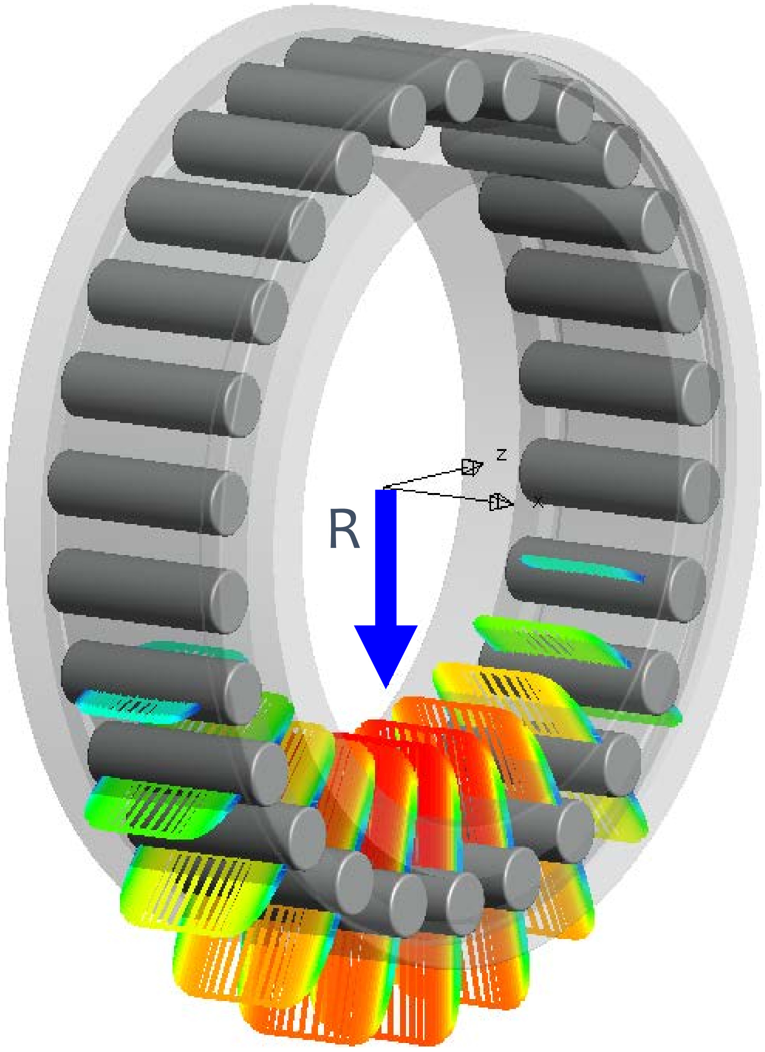
$$F_0 \cong \frac{5 \cdot R}{Z}$$

which according to Harris** can be used today for both ball and roller bearings with the standard clearances.

* R. Stribeck, "Kugellager für Beliebige Belastungen" , Z. Ver. dt.Ing., vol. 45, n. 3, pp. 73-125, 1901

* R. Stribeck, "Ball Bearings for Various Loads", Trans. ASME, 29, pp. 420-463, 1907

**T. H. Harris, Rolling Bearing Analysis, John Wiley & sons, III ed. 1991 , see pages 199 and 201



source: FAG, courtesy Schaeffler Italia S.r.l.

Case study: the figure shows the contact surface pressure on the center of the contact along needle rollers.

This bearing was calculated for null radial play (explain how one can get this conclusion from the figure).

Additional information: the reduction of pressure at roller ends is a clue that “crowning" - a roller radius drop near the roller edges - was adopted as a countermeasure against stress concentrations at the edge (see Ch.1, Sect. 6, sl. 8).

Crowning is particularly beneficial when the bearing works in misaligned conditions, which tend to highly increase edge pressures, this resulting in rapid reduction of bearing lifetime.

The shapes adopted for crowning are proprietary and patented.

Interference

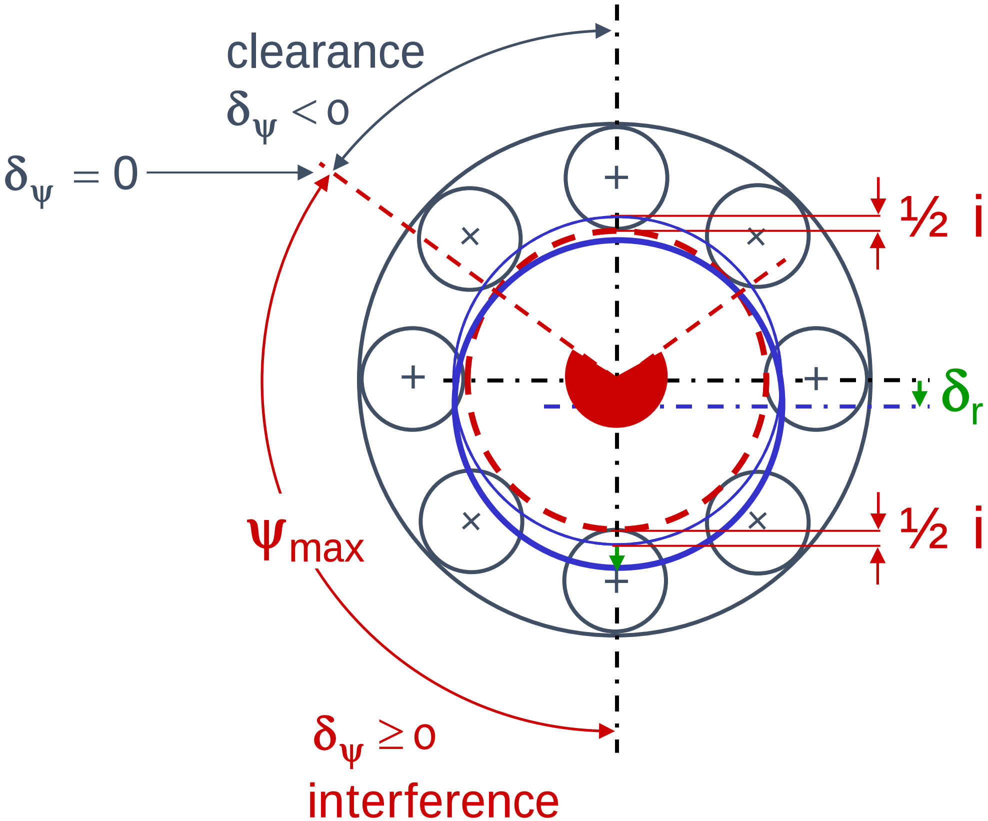
If instead of having a clearance g we have an interference i , i.e., i=-g then:

Angle ψ_{max} where δ_ψ is zero:

$$0 = \delta_r \cdot \cos\psi_{max} + \frac{1}{2} i \Rightarrow$$

$$\Rightarrow \begin{cases} \cos\psi_{max} = -\frac{1}{2}\frac{i}{\delta_r} \\ \psi_{max} = \operatorname{ar\,cos}\left(-\frac{i}{2\delta_r}\right) \end{cases}$$

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_r \cdot \cos\psi + \frac{1}{2} i}{\delta_r + \frac{1}{2} i} = \frac{\cos\psi + \frac{1}{2}\frac{i}{\delta_r}}{1 + \frac{1}{2}\frac{i}{\delta_r}}$$

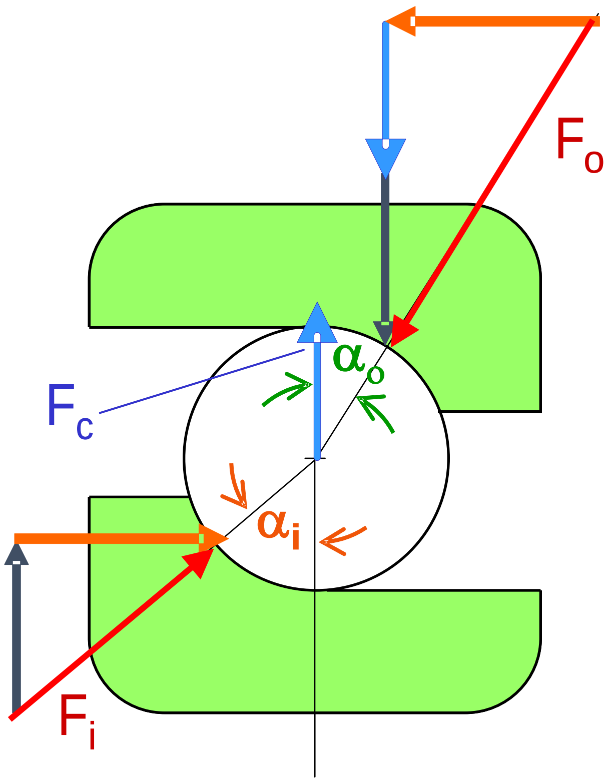


- for i=0 , ψ_{max}= 90°
- for i>0 , ψ_{max}=180° at δ_r=½ i
- for i>0 , $\psi_{max} \Rightarrow$ 90° for $\delta_r \Rightarrow \infty$

4. High speed bearings (1/9)

In this section we shall examine only the case of cylindrical roller bearings.

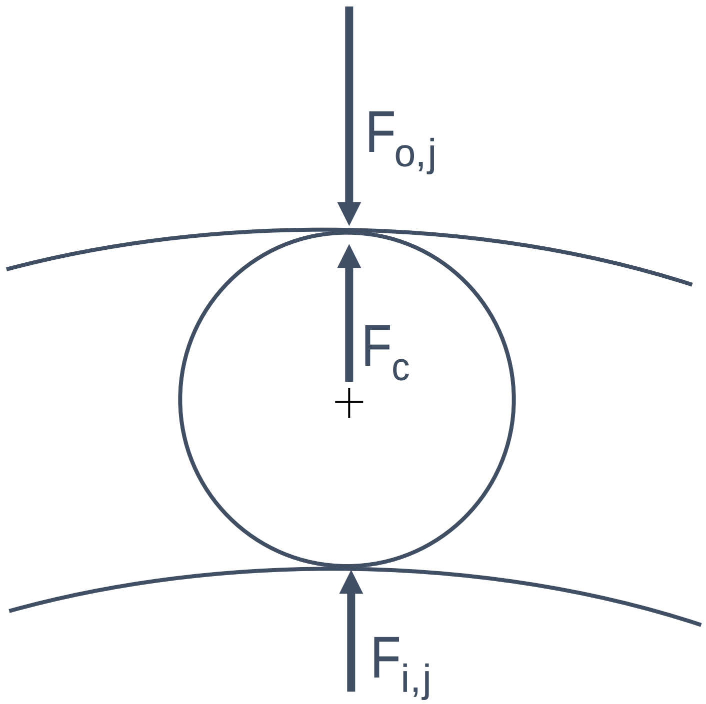
The other types, and in particular those which can sustain axial loads with angular contacts, need quite elaborate equations which then require iterative computer solutions.



The centrifugal force F_c sums up vectorially to contact forces, so that the equilibrium of the inner and outer contact forces F_i and F_o on each ball is given as in the figure on the left.

Then all forces on all balls must be compatible with radial and axial approaches of rings, through local contact deformabilities.

The additional simplification in not considering angular contact bearings is that we must not take into account the effect of gyroscopic moments .

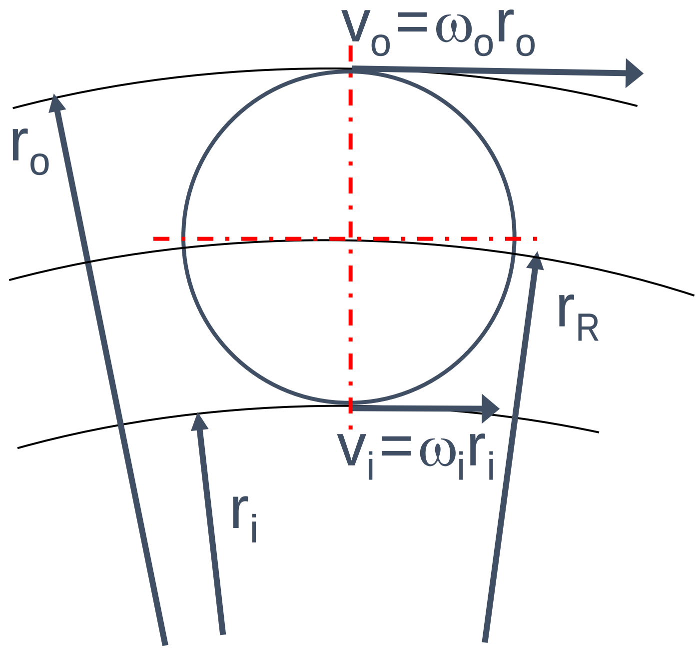


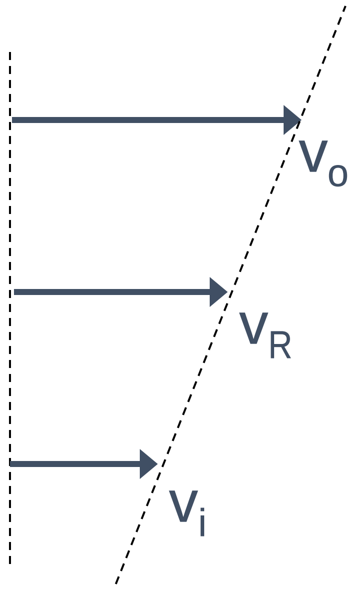
On the left we see forces on a j-th cylindrical roller moving at such a speed that the centrifugal force cannot be neglected.

The angular position of the roller to the radial loading line is ψ_j .

Step 1 is to calculate roller speed:

1) Given the angular speed ω_o of the outer ring, and ω_i of inner ring, then the angular speed of the roller centre – i.e. also the angular speed of the cage – is obtained through the the pheripheral velocities:





$$\omega_R = \frac{v_R}{r_R}; \qquad r_R = \frac{r_i + r_o}{2}$$

2) The centrifugal force on the roller is determined:

$$F_c = m_R \cdot r_R \cdot \omega_R^2$$

3) The radial elastic approach δ_{oc} due to F_c (on the outer raceway):

$$\delta_{oc} = \left(\frac{F_c}{K_o} \right)^{\frac{1}{n}}$$

Where we shall assume that K_o for the "single" cylinder to raceway contact is calculated (Ch. 1, Sect. 6, sl. 6) as:

$$\delta_o = \frac{3{,}84 \cdot 10^{-5}}{L^{0{,}8}} F^{0{,}9} \qquad \text{then: } K_o = \left(\frac{L^{0{,}8}}{3{,}84 \cdot 10^{-5}} \right)^{\frac{10}{9}}$$

[F]=N, [L]=mm, [δ]=mm (remember that: $\delta_i \cong \delta_o$, $K_i \cong K_o$)

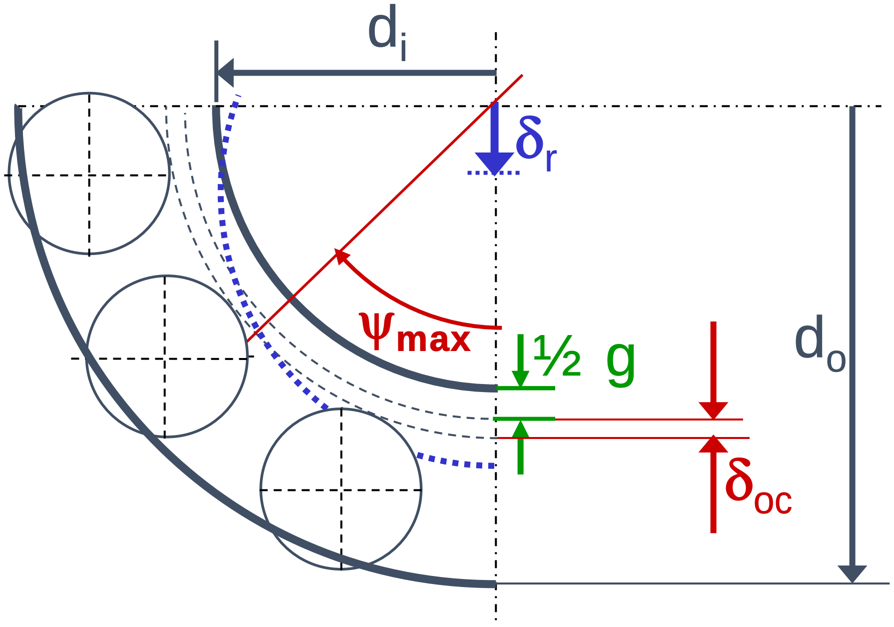
4) The fact that all rollers are compressed against the outer ring produces an increase of the radial play - or clearance - between the inner raceway and the inner envelope of rollers; thenthe total radial play of the bearing increases from g the value:

$$g + 2\delta_{oc}$$

5) During a radial approach δ_r , the maximum contact angle ψ_{max} is the one at which:

$$\delta_r \cos\psi_{max} = \frac{1}{2} g + \delta_{oc}$$

$$\psi_{max} = \arccos\left(\frac{\frac{1}{2} g + \delta_{oc}}{\delta_r}\right)$$



6) The index which up to which to count rollers in contact:

$$j_{max} = \mathrm{int}\left(\frac{\psi_{max}}{360/z}\right)$$

highest index of rollers in contact

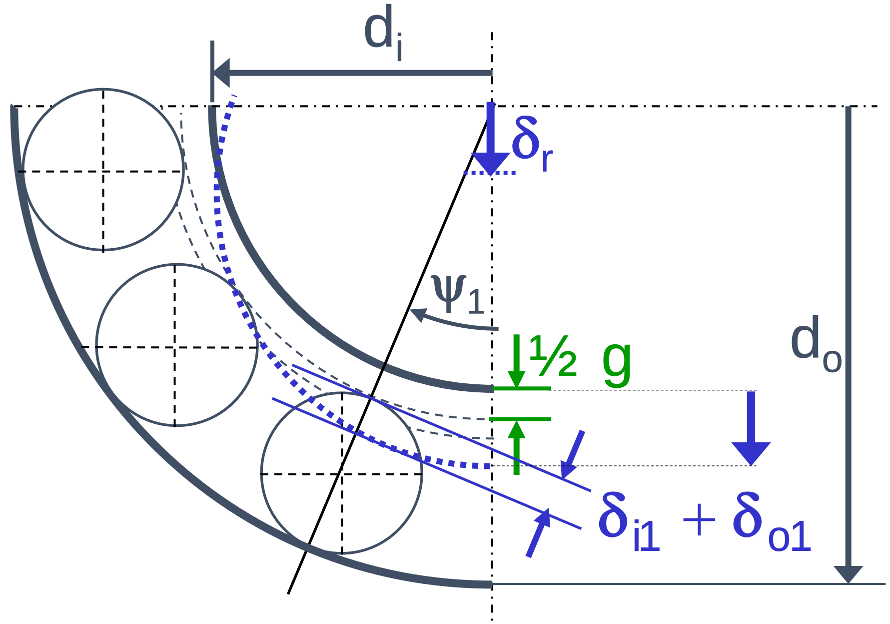
$-j_{max}$,...................., 0 ,......................, $+j_{max}$

roller on the vertical through the center

7) For the j-th roller inside $\pm\psi_{max}$, at angle ψ_j , apply:

$$\delta_r \cos\psi_j = \frac{1}{2}g + \delta_{ij} + \delta_{oj} =$$

$$= \frac{1}{2}g + \left(\frac{F_{ij}}{K_i}\right)^{\frac{1}{n}} + \left(\frac{F_{ij}+F_c}{K_o}\right)^{\frac{1}{n}}$$



4. High speed bearings (7 / 9)

It is convenient to use the total radial approach δ_r as the independent variable, and to calculate forces F_{ij} as the dependent variable for all -jmax $\leq$ j$\leq$+jmax; for a set of δ_r values from zero to $\delta_{r,\,max}$:

$$\delta_{ij} + \delta_{oj} = \boxed{\left(\frac{F_{ij}}{K_i}\right)^{\frac{1}{n}} + \left(\frac{F_{ij} + F_c}{K_o}\right)^{\frac{1}{n}} = \delta_r \cos\psi_j - \frac{1}{2} g} \qquad \left(|j| \leq j_{max} \right)$$

note that when the roller is not by a inner force, i.e., $F_{ij=0}$, which happens for a roller placed at the limit position $\psi=\psi_{max}$, it is:

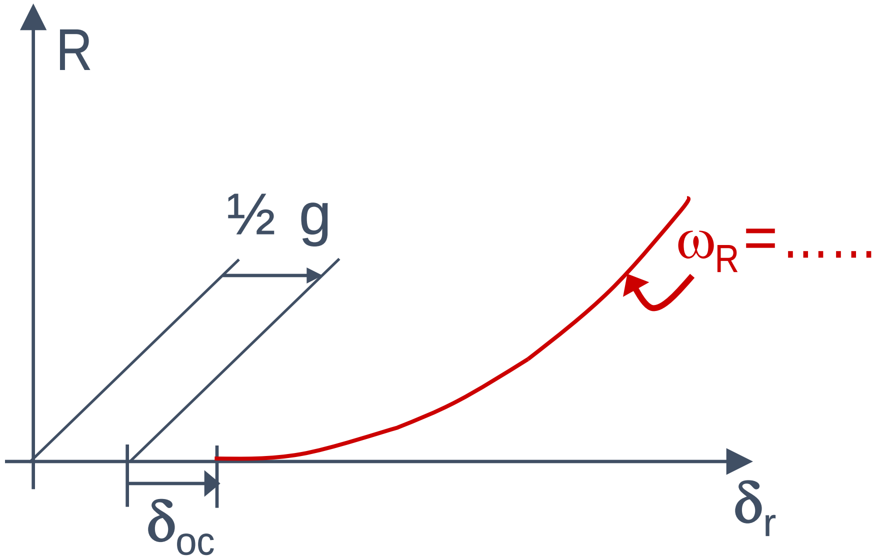
$$\delta_r \cos\psi_{max} = \frac{1}{2} g + \left(\frac{F_c}{K_o}\right)^{\frac{1}{n}} \equiv \frac{1}{2} g + \delta_{oc} \quad \text{then} : \cos\psi_{max} = \frac{\frac{1}{2} g + \delta_{oc}}{\delta_r}$$

8) For each value of δ_r use the formula above to calculate $F_{ij}(\delta_r)$, which is implicit, then an iterative procedure is needed.

9) When all F_{ij} are available for each value of δ_r , then the total radial force on the inner ring:

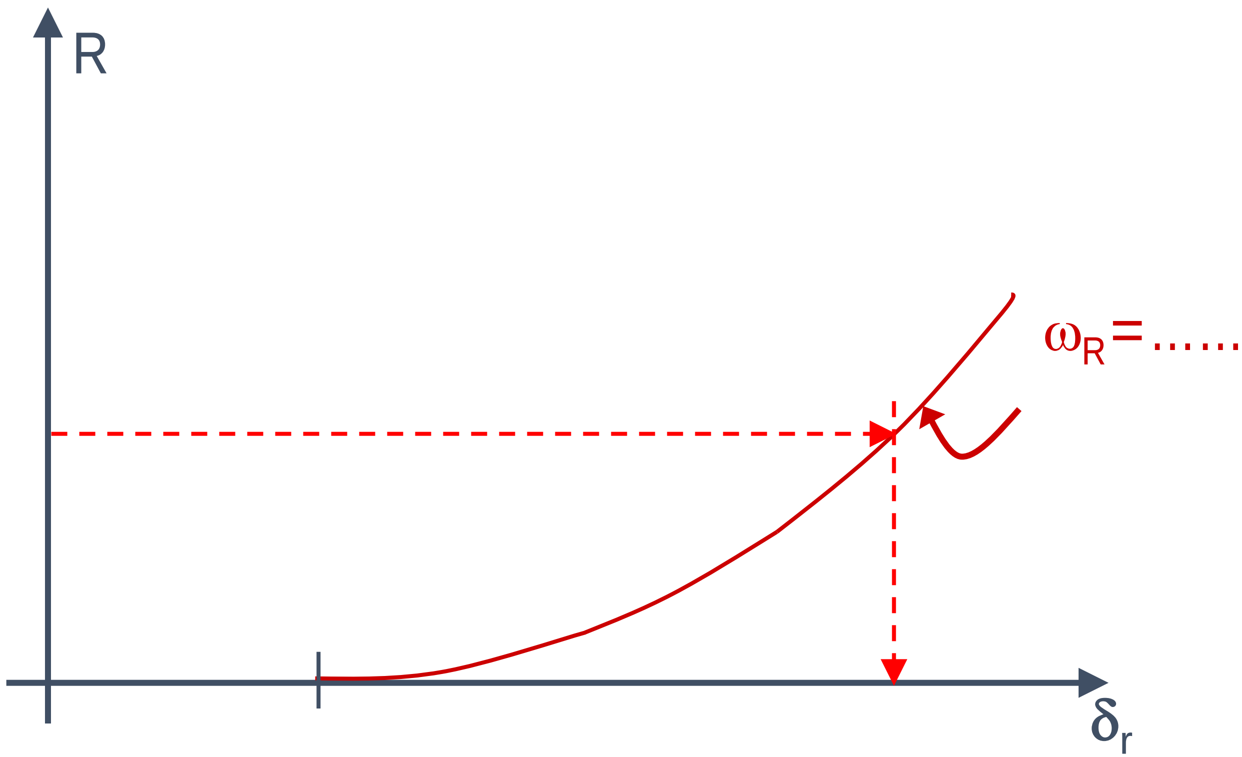
$$R(\delta_r) = \sum_{-j_{\max}}^{+j_{\max}} F_{ij}(\delta_r) \cdot \cos\psi_j$$

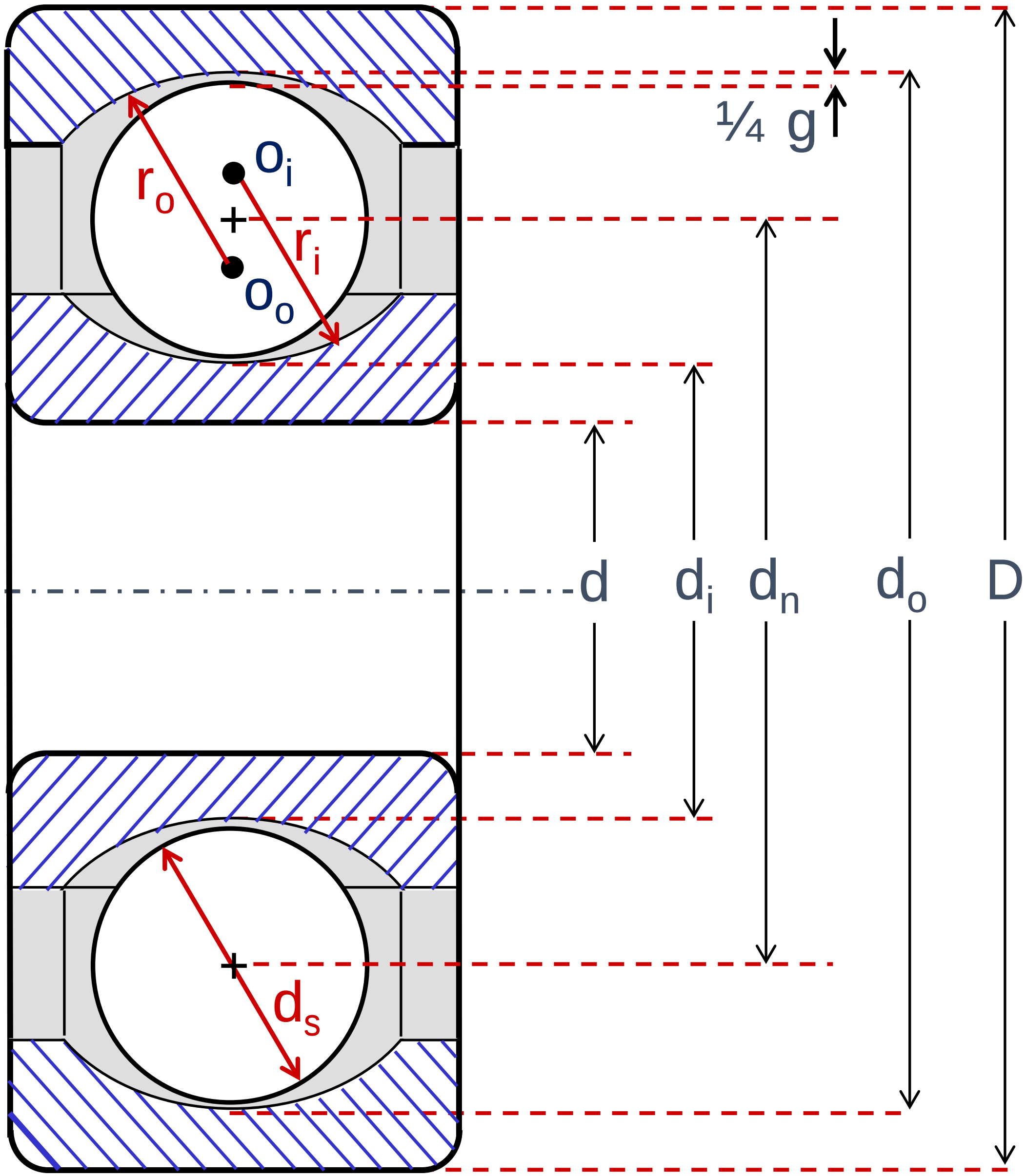
10) The radial force diagram $R(\delta_r)$ can be plotted for any given rotation speed ω_R , which will be the curve parameter:



Initial contact occurs in ψ=0 when δ_r= ½ g + δ_{oc}

11) Once the diagram is available, it is used to choose the radial force R and find the correspondent radial approach; then, knowing δ_r , other values like ψ_{max}, F_{ij} , F_{oj} , and in particular F_{i0} , F_{o0} , hence also the maximum contact pressure are at hand.





The radial rigid ball bearing is meant mainly for radial loads.

d = nominal bore diameter

D = nominal outer diameter

d_i = diameter of inner raceway

d_o = diameter of outer raceway

d_s = ball diameter

d_n = diameter of ball centers

g = clearance or play (*next slide)

r_i = groove radius of inner raceway

r_o = groove radius of outer raceway

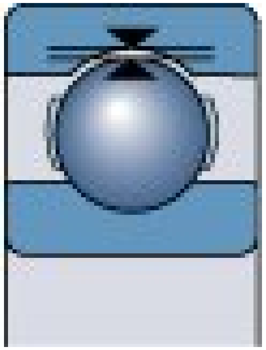
usually for r_i and r_o

$0.515\ d_s < r < 0.530\ d_s$

5. Can a radial bearing take axial load? (2/7)

Table of radial play values g

from SKF catalog



*Bearing internal clearance, or "play", is defined as the **total movement of** one bearing ring against the other in the radial direction (radial internal clearance) or in the axial direction (axial internal clearance).

Bore diameter d		Radial internal clearance C2		Normal		C3		C4		C5	
over	incl.	min	max	min	max	min	max	min	max	min	max
mm		µm									
2,5	**6**	0	7	2	13	8	23	-	-	-	-
6	**10**	0	7	2	13	8	23	14	29	20	37
10	**18**	0	9	3	18	11	25	18	33	25	45
18	**24**	0	10	5	20	13	28	20	36	28	48
24	**30**	1	11	5	20	13	28	23	41	30	53
30	**40**	1	11	6	20	15	33	28	46	40	64
40	**50**	1	11	6	23	18	36	30	51	45	73
50	**65**	1	15	8	28	23	43	38	61	55	90
65	**80**	1	15	10	30	25	51	46	71	65	105
80	**100**	1	18	12	36	30	58	53	84	75	120
100	**120**	2	20	15	41	36	66	61	97	90	140
120	**140**	2	23	18	48	41	81	71	114	105	160
…	…	…	…	…	…	…	…	…	…	…	…

It is a diametral clearance. The image above refers to the total radial movement.

5. Can a radial bearing take axial load? (3/7)

Remarks on bearing clearance

It is necessary to distinguish between the internal clearance of a bearing before mounting and the internal clearance in a mounted bearing, which has reached its operating temperature (operational clearance). The initial internal clearance (before mounting) is greater than the operational clearance because different degrees of interference in the fits and differences in thermal expansion of the bearing rings and the associated components cause the rings to be expanded or compressed.

The radial internal clearance is of considerable importance if satisfactory operation is to be obtained. As a general rule, ball bearings should always have an operational clearance that is virtually zero, or there may be a slight preload. Cylindrical, spherical and CARB toroidal roller bearings, on the other hand, should always have some residual clearance - however small - in operation. The same is true of tapered roller bearings, except in bearing arrangements where stiffness is desired, e.g. pinion bearing arrangements where the bearings are mounted with a certain amount of preload.

The bearing internal clearance referred to as **normal** has been selected so that a suitable operational clearance will be obtained when bearings are mounted with the fits usually recommended and operating conditions are normal. Where operating and mounting conditions differ from the normal ... SKF recommends checking residual clearance in the bearing after it has been mounted.

http://www.skf.com/portal/skf/home/products?newlink=1_0_56&lang=en&maincatalogue=1

5. Can a radial bearing take axial load? (4/7)

Remarks on bearing preload

Depending on the application it may be necessary to have either a positive or a negative operational clearance in the bearing arrangement. In the majority of applications, the operational clearance should be positive, i.e. when in operation, the bearing should have a residual clearance, however slight.

However, there are many cases, e.g. machine tool spindle bearings, pinion bearings in automotive axle drives, bearing arrangements of small electric motors, or bearing arrangements for oscillating movement, where a negative operational clearance, i.e. a preload, is needed to enhance the stiffness of the bearing arrangement or to increase running accuracy.

The application of a preload, e.g. by springs, is also recommended where bearings are to operate without load or under very light load and at high speeds. In these cases, the preload serves to provide a minimum load on the bearing and prevent bearing damage as a result of sliding movements of the rolling elements.

http://www.skf.com/portal/skf/home/products?lang=en&maincatalogue=1&newlink=1_0_83

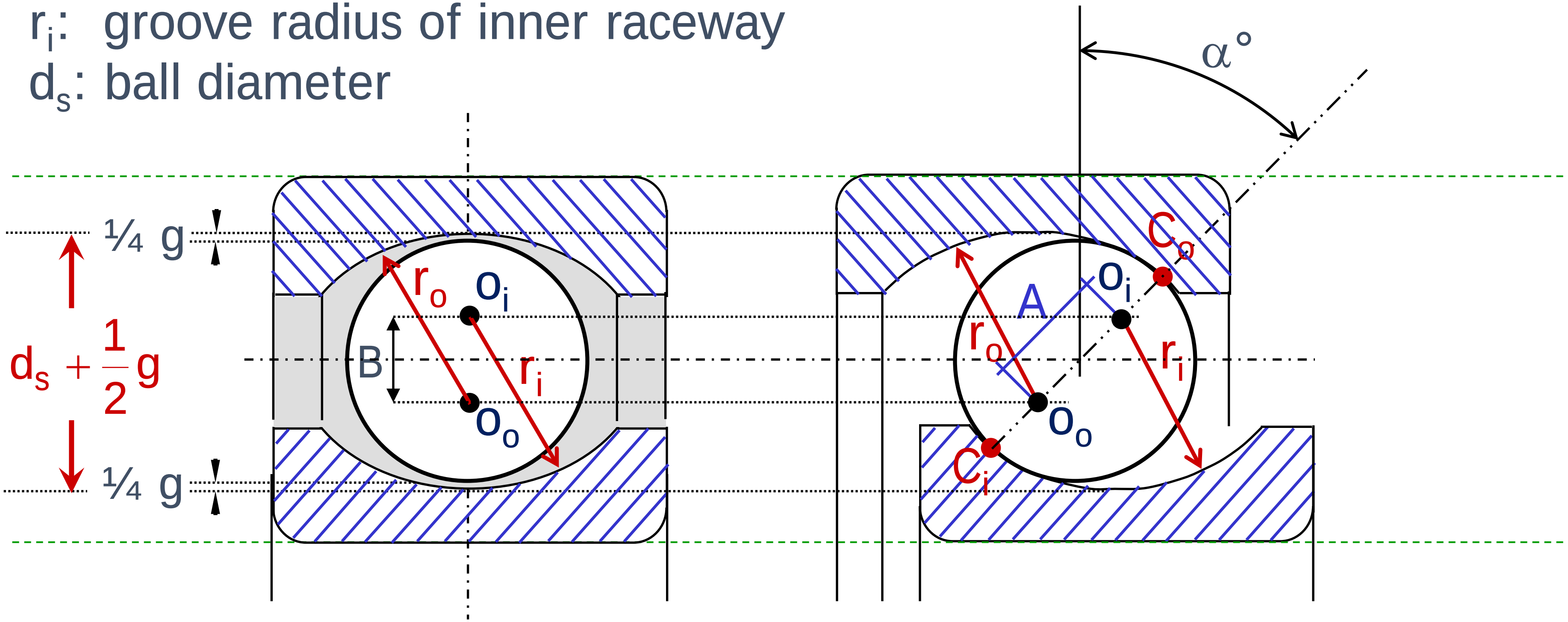
While there is radial play, under the action of an axial thrust the inner ring displaces axially, the ball touches inner and outer raceways respectively in points C_i and C_o through which the the line of transmitted force passes; then, the bearing is enabled to take axial load.

r_o: groove radius of outer raceway

r_i: groove radius of inner raceway

d_s: ball diameter

(case $r_i=r_o$)

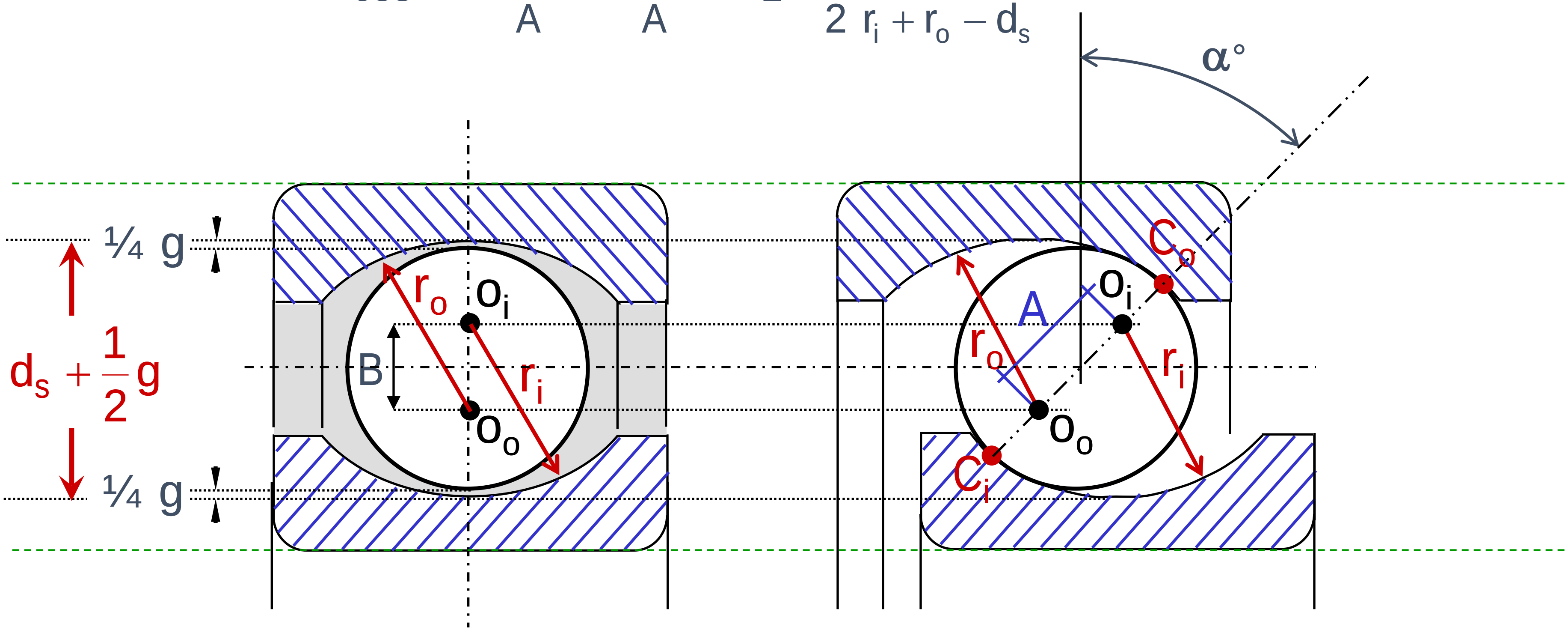


$$\boxed{d_s + \frac{1}{2} g = r_i - B + r_o}$$

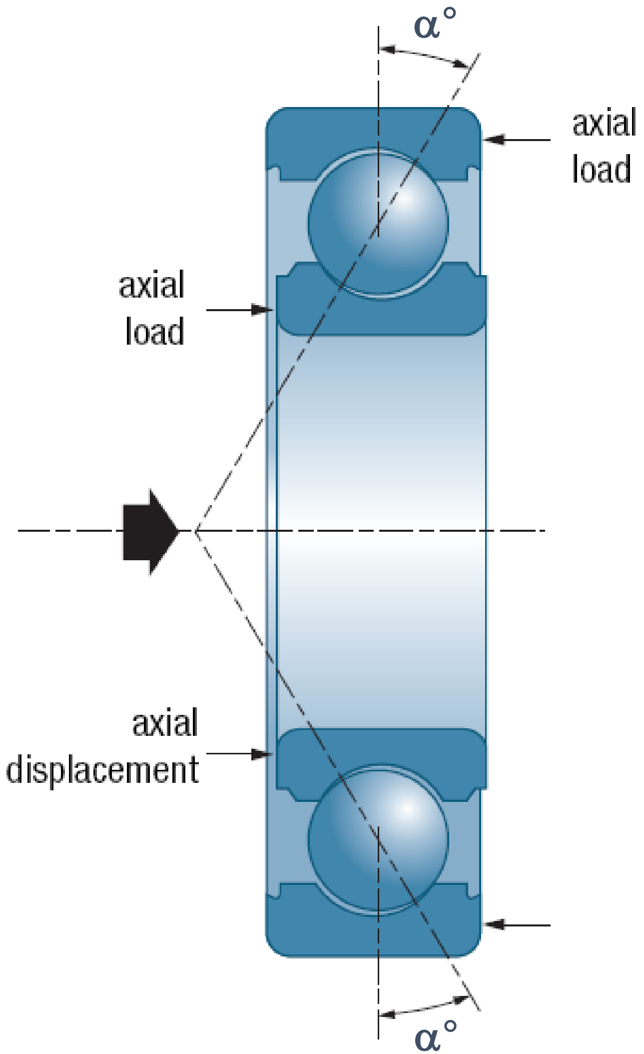
$$\boxed{d_s = r_i - A + r_o}$$

subtracting: $\frac{1}{2} g = A - B \Rightarrow B = A - \frac{1}{2} g$

$$\cos\alpha° = \frac{B}{A} = \frac{A - \frac{1}{2} g}{A} = 1 - \frac{1}{2} \frac{g}{r_i + r_o - d_s}$$



5. Can a radial bearing take axial load? (7 / 7)



An example with values according to current practice:

$r_i = r_o \cong 0.525\ d_s$

$$\cos\alpha^\circ = 1 - \frac{1}{2}\frac{g}{d_s(2\cdot 0.525 - 1)} = 1 - \frac{1}{2}\frac{g}{d_s\ 0.05}$$

For a radial deep groove bearing with bore 30 ÷ 40 mm,

$d_s \cong 10$ mm, g = 20 µm (max “normal” play)

$$\cos\alpha^\circ \cong 1 - \frac{20\cdot 10^{-3}}{10\cdot 2\cdot 0.05} = 1 - \frac{10^{-3}}{5\cdot 10^{-2}} = \frac{98}{100}$$

$$\alpha^\circ = \mathrm{ar}\cos\left(\frac{98}{100}\right) = 11{,}5^\circ$$

Section 6 - The “equivalent” static load

Section 6 provides an introductory treatment to the problem of bearings subjected to a combination of static axial and radial loads. It is shown that different combinations can produce the same maximum stresses or permanent deformations, and therefore are equivalent from this point of view.

The treatment is oversimplified, but it allows to gather the main ideas, thus building the foundations for the next three Sections and justifying the linear formula for the equivalent load.

The static load P_0, either axial or thrust, is used to determine the bearing size when:

- the bearing is stationary or making slight oscillating movements and withstanding continuous or intermittent loads,
- the bearing is subjected to shocks during normal rotation.

The "basic static capacity" C_0 , otherwise said "basic static load rating", of a bearing (either radial or thrust) is the load, (correspondingly, either radial or thrust) applied to the bearing ring that causes a certain "limit" damage i.e. a certain "limit" permanent local deformation at the most heavily loaded contact.

The basic static load rating C0 as defined in ISO 76:2006 corresponds to a calculated contact stress at the centre of the most heavily loaded rolling element / raceway contact. This stress produces a total permanent deformation of the rolling element and raceway, which is approximately 0,0001 of the rolling element diameter. The loads are purely radial for radial bearings and axial, centrically acting for thrust bearings.

http://www.skf.com/group/products/bearings-units-housings/super-precision-bearings/principles/bearing-life-and-load-ratings/permissible-static-loads/index.html

6. The equivalent static radial load I (2/17)

The situation is quite simple if the bearing is purely radial or purely axial: in those cases both C_0 and P_0 are defined as, respectively, radial or axial loads.

However, if a bearing is capable of taking both a radial and an axial load, static bearing loads may include a radial and an axial component: in such case these two load components, simultaneously applied, must be converted into an equivalent static bearing load P_0 .

This P_0 ... *is defined as that hypothetical load (radial for radial bearings and axial for thrust bearings) which, if applied, would cause the same maximum rolling element load in the bearing as the actual loads to which the bearing is subjected.*
http://www.skf.com/group/products/bearings-units-housings/super-precision-bearings/principles/bearing-life-and-load-ratings/permissible-static-loads/index.html

This transformation is necessary because catalogs define the “basic static capacity” of a bearing with only one number C_0.

6. The equivalent static radial load I (3/17)

For all bearings the “ basic static load rating” C_0 (given in kN) is then compared to the “equivalent static load” P_0 applied to the bearing:

$$\frac{C_0}{P_0} \geq S_0$$

$S_0=1,5 \rightarrow$ bearings with “normal” noise

$S_0=0,5 \rightarrow$ bearings where noise requirements are not stringent

C_0 depends on the max contact pressure in the worst conditions:

ISO 76-1987 defines such pressure $p_{max,s}$ as the one which produces a total permanent set of $1/10^4$ times the diameter of the rolling element (see Ch.1, Sect.9, sl. 2, 6) at:

- $p_{max,s}=4600$ MPa: for self aligning ball bearings
- $p_{max,s}=4200$ MPa: for all other ball bearings
- $p_{max,s}=4000$ MPa: for all roller bearings

In lieu of the more rigorous approach, for (radial of thrust) bearings subjected to a radial "R" and axial "A" (combined) loads, the (radial or thrust) equivalent force (under the usual name P_0) can be calculated with the following simplified formula:

$$P_0 = X_0\, R + Y_0\, A$$

If deep groove radial bearings are subjected to both radial and axial loads R and A, an "equivalent" static radial load is defined with the following (catalog) formula:

$$\begin{cases} P_0 = 0{,}6\, R + 0{,}5\, A \\ \text{however} \\ P_0 = R \quad \text{if} \quad P_0 < R \end{cases}$$

(for single bearings and bearing pairs arranged in tandem see Sect. 11)

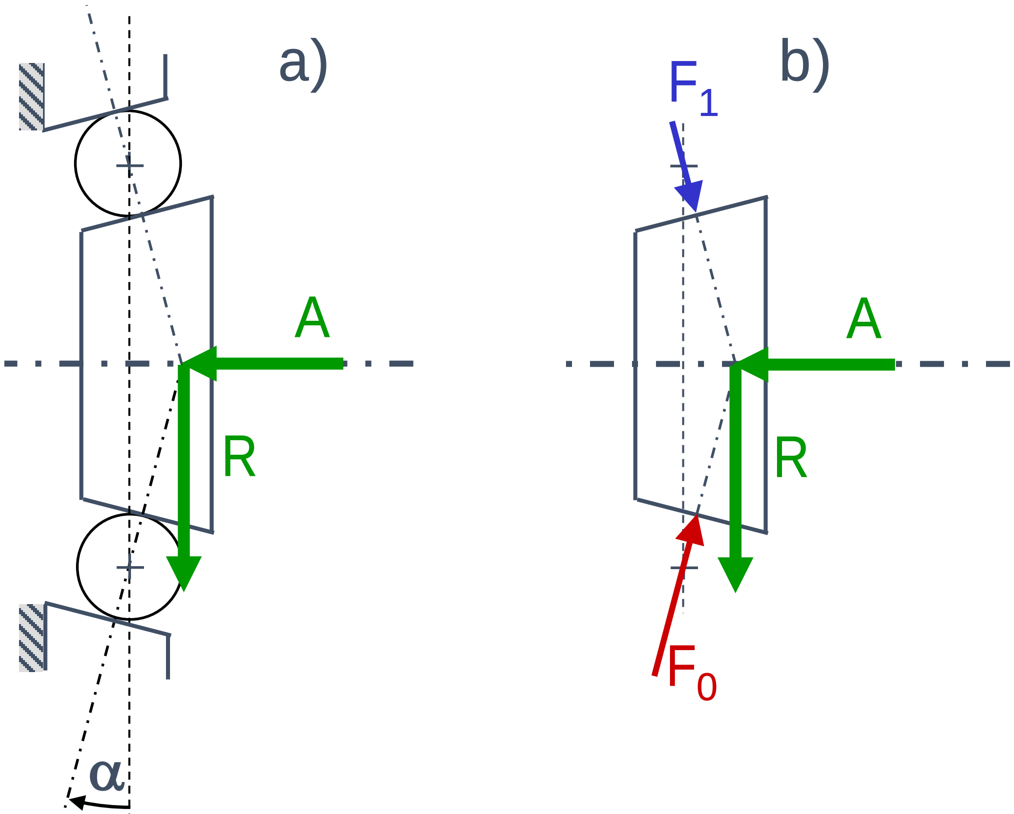
In order to have a qualitative idea about the origin of the linear formula:

$$P_0 = X_0\,R + Y_0\,A$$

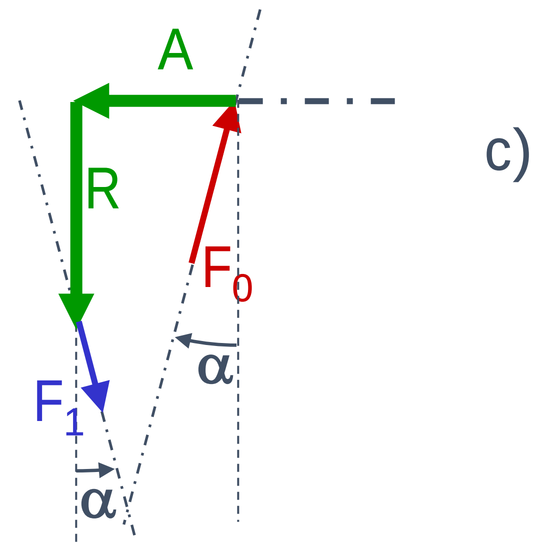
we shall consider first, in the next slides, a drastically simplified model of the bearing behaviour that allows a simple solution by means of just equilibrium equations.

Let us remark once again that here we are dealing with static loads.

The rigourous approach to P_0 is outlined, for radial bearings only, in Sect. 7, 9 of this Chapter.



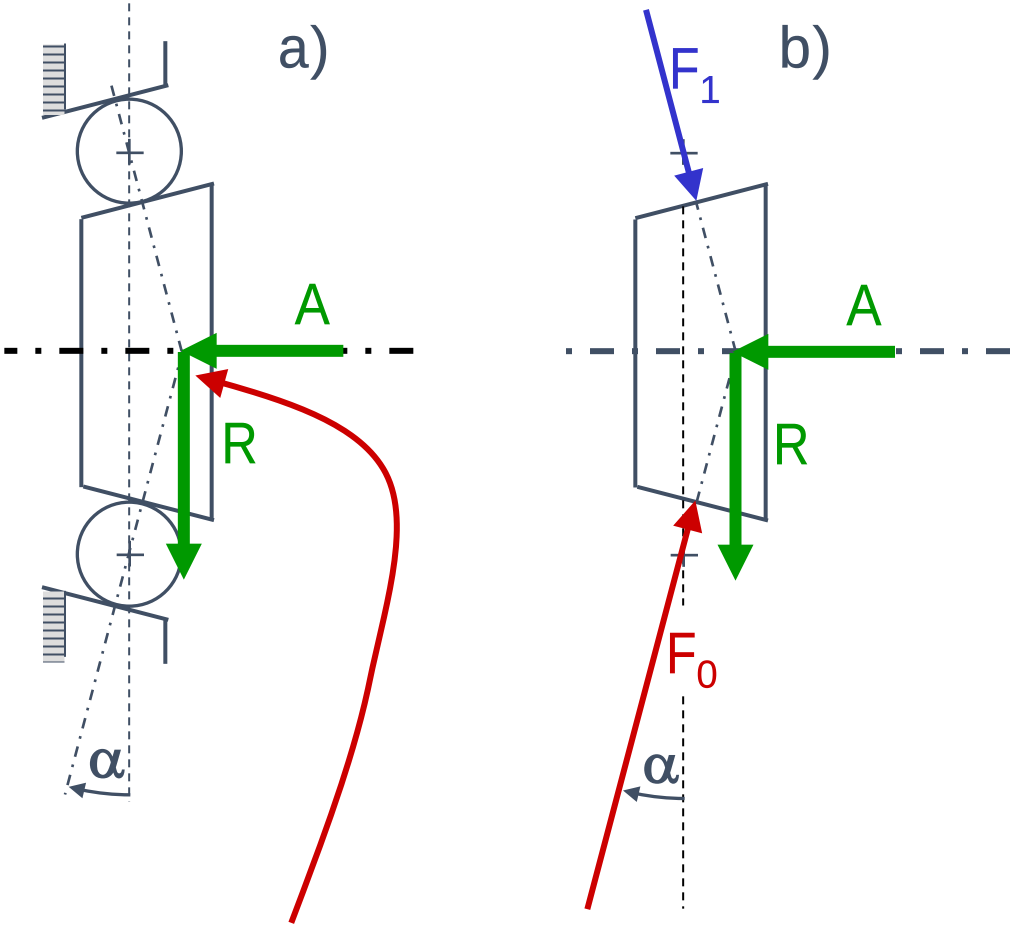
With these values of contact forces F_0 and F_1 equilibrium is evidently not satisfied...



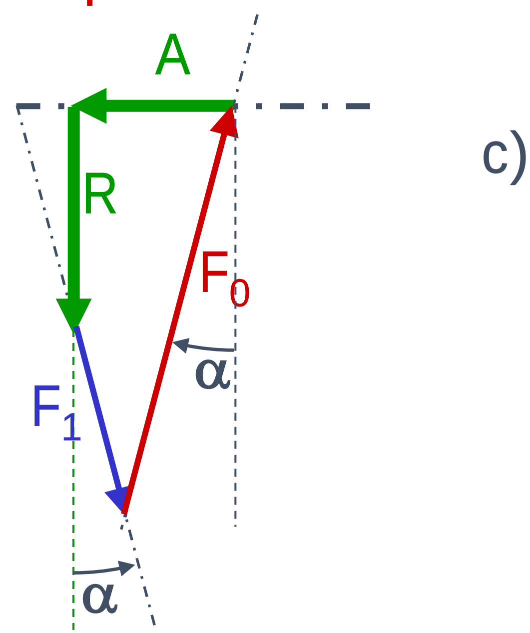
Let us consider a very simple bearing having just two rolling elements, one (1) at the top and one (0) at the bottom.

Both radial R and axial A loads are applied. Raceways are represented by their tangent cones, with the same contact angle.

6. The equivalent static radial load I (7/17)

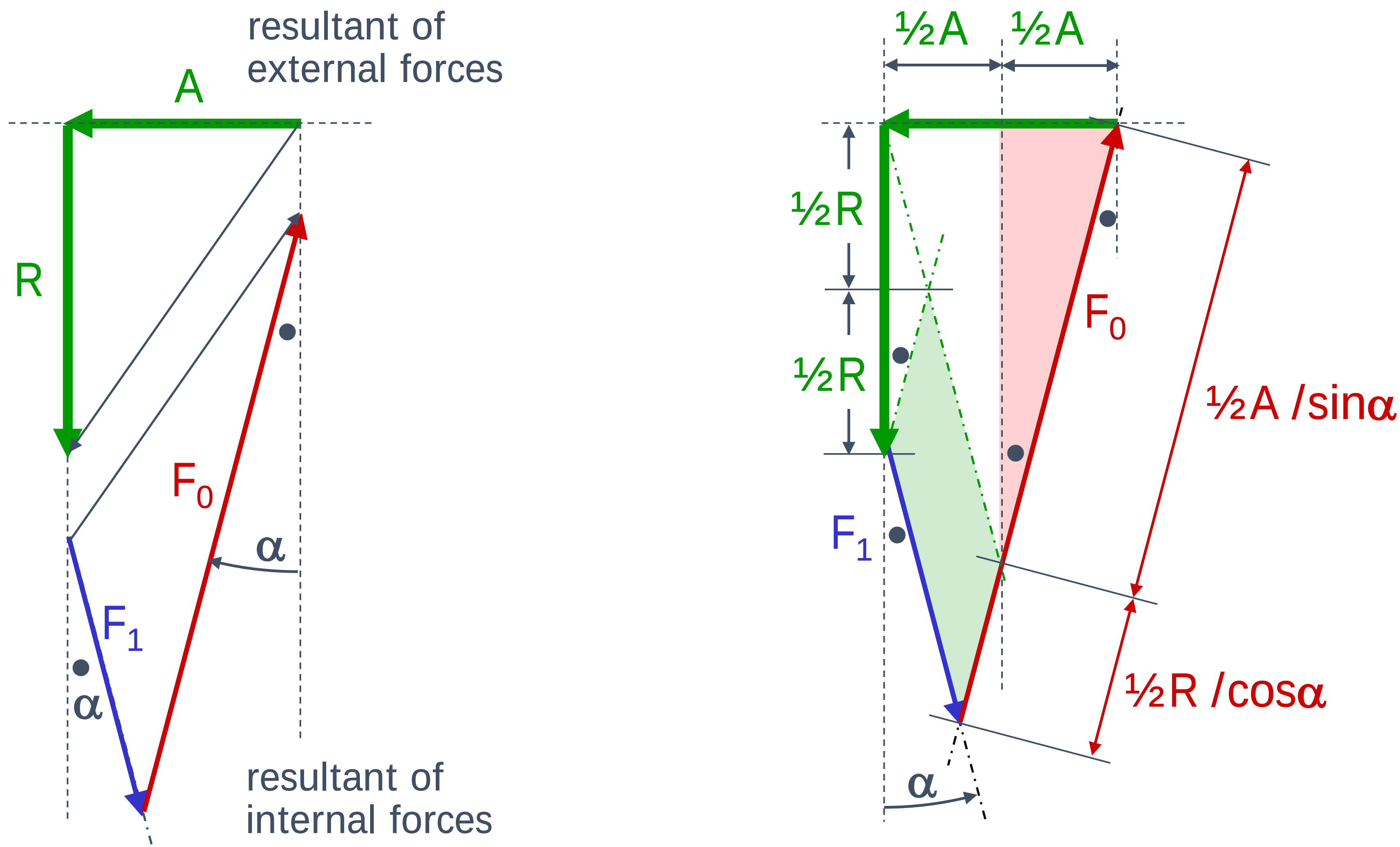


... while here contact forces F_0 and F_1 evidently satisfy equilibrium...



Remark: all forces through one point = equilibrium to rotation !

... as they "close" the polygon of forces applied to the inner ring.



This picture shows how F_0 is expressed as the sum of two segments ...

Recipes for the formal geometrical demonstration

Divide A in two equal parts, draw the orthogonal to A at mid-point M

Intersect F_0 at V, then connect N to V

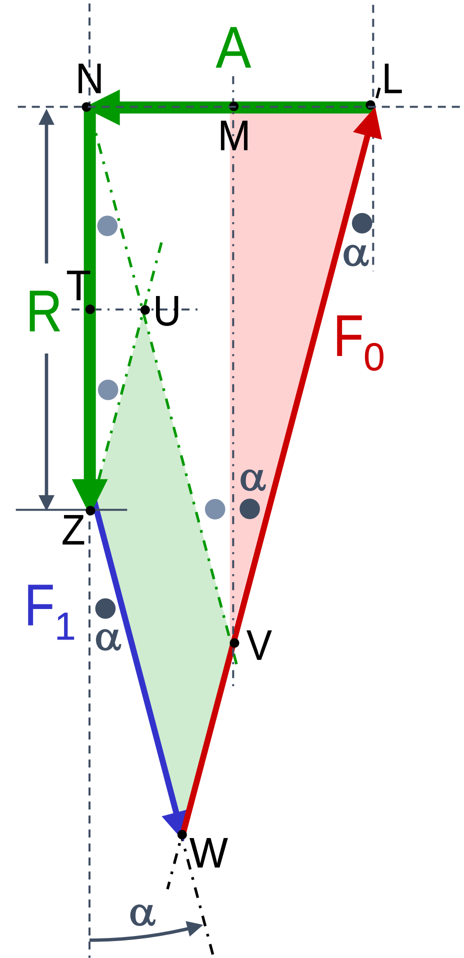
Triangles LMV and NMV are equal (right – angled triangles with same bases and height); then LNV is isosceles; then angle NVM equal to α ; then angle VTZ equal to α

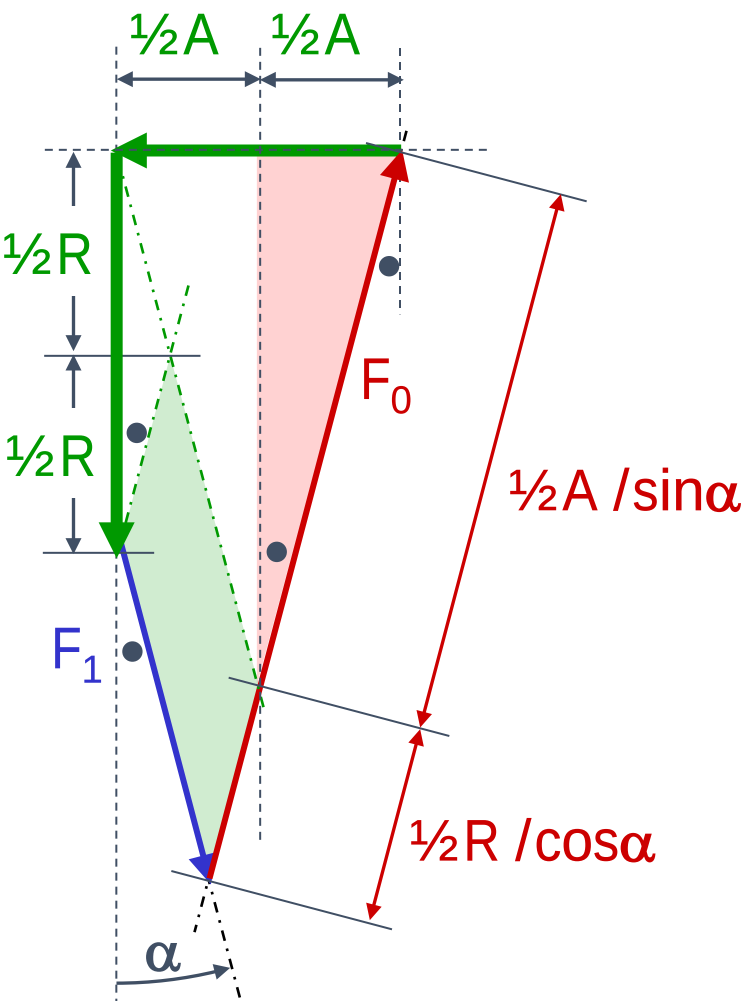
Then NV parallel to F_1

Divide R in two equal parts at mid-point T, then draw the orthogonal to R , intersecting segment NV at U

Triangles NTU and ZTU are equal (right – angled triangles with same bases and height); then angle TZU equal to TNU=α

Then segment ZU parallel to F_0 ; then ZUVW is a parallelogram; then UZ=VW.





The lower, and the most loaded, rolling element takes the highest load, calculated as:

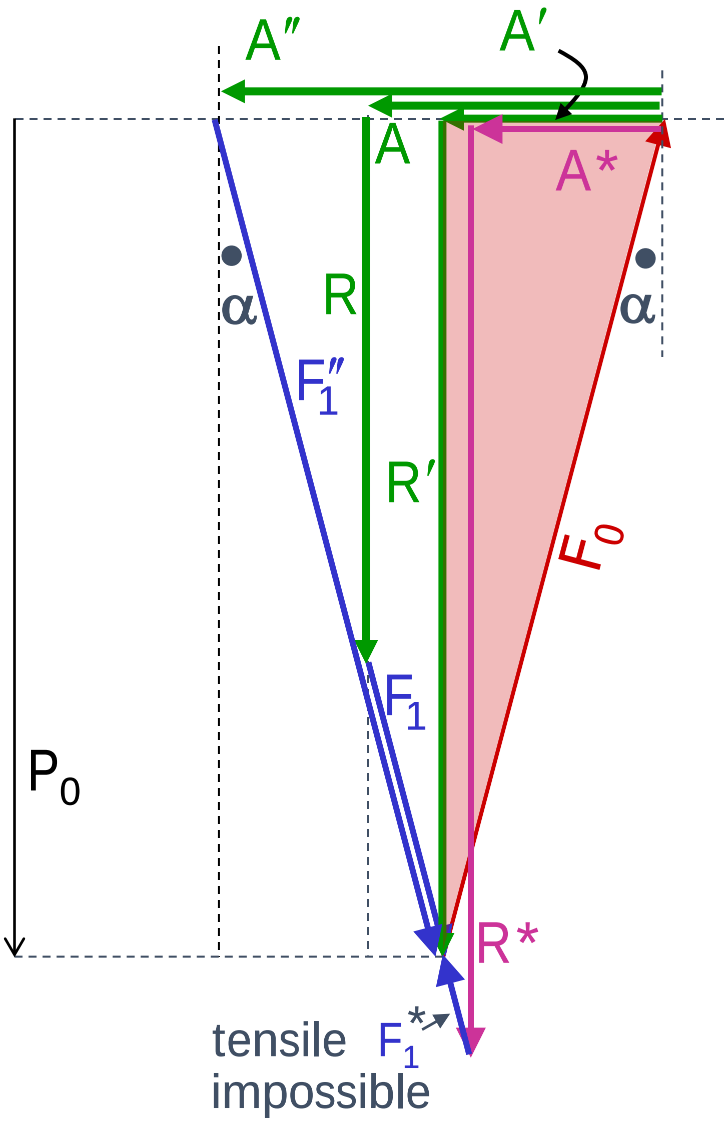
$$F_0 = \frac{1}{2}\frac{R}{\cos\alpha} + \frac{1}{2}\frac{A}{\sin\alpha}$$

which is to be compared with the limit load $F_{0,lim}$ determined by the maximum allowable pressure.

Example for ball bearings:

$$p_{max} = \zeta \sqrt[3]{F_0} \;\Rightarrow\; F_{0,lim} = \left(\frac{p_{max,s}}{\zeta}\right)^3$$

where ζ is a constant determined according to the formulas in Ch.1, Sect.5, sl.1 .



There is a variety of R , A combinations that produce the same F_0 and the same p_{max}, i.e., that are "equivalent":

R , A : any intermediate case

A'' : purely axial, R'' =0 (the lowest R/A)

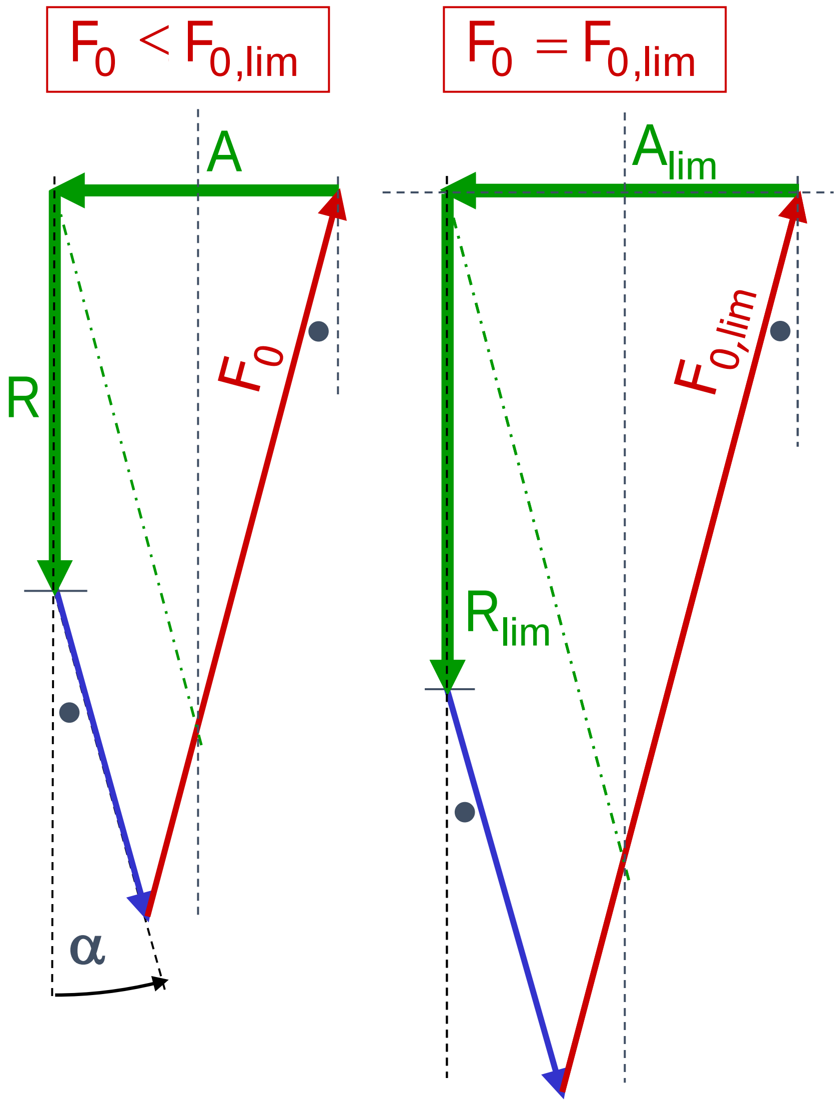
R' , A' : the highest* possible R/A ratio

All these cases have the same P_0 , the radial component of F_0

$$P_0 = \frac{1}{2} R + \frac{1}{2\,\mathrm{tg}\alpha} A \geq R$$

* Combinations with higher R/A , like R*-A*, are impossibile as they require a tensile contact force F_1^*.

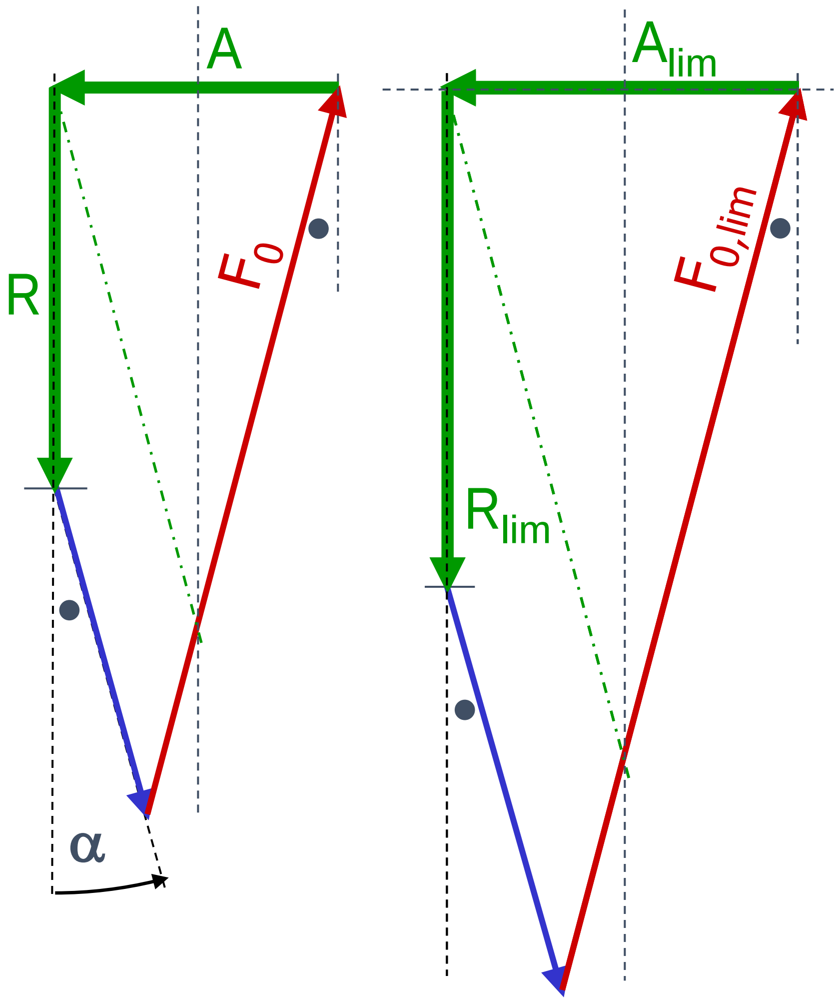
A convenient mathematical check against the impossible condition is: $P_0 \geq R$



If for any given (possible) ratio A/R external loads A and R grow proportionally, then load F_0 grows in the same proportion and finally reaches a limit value $F_{0,lim}$ which marks the onset of failure.

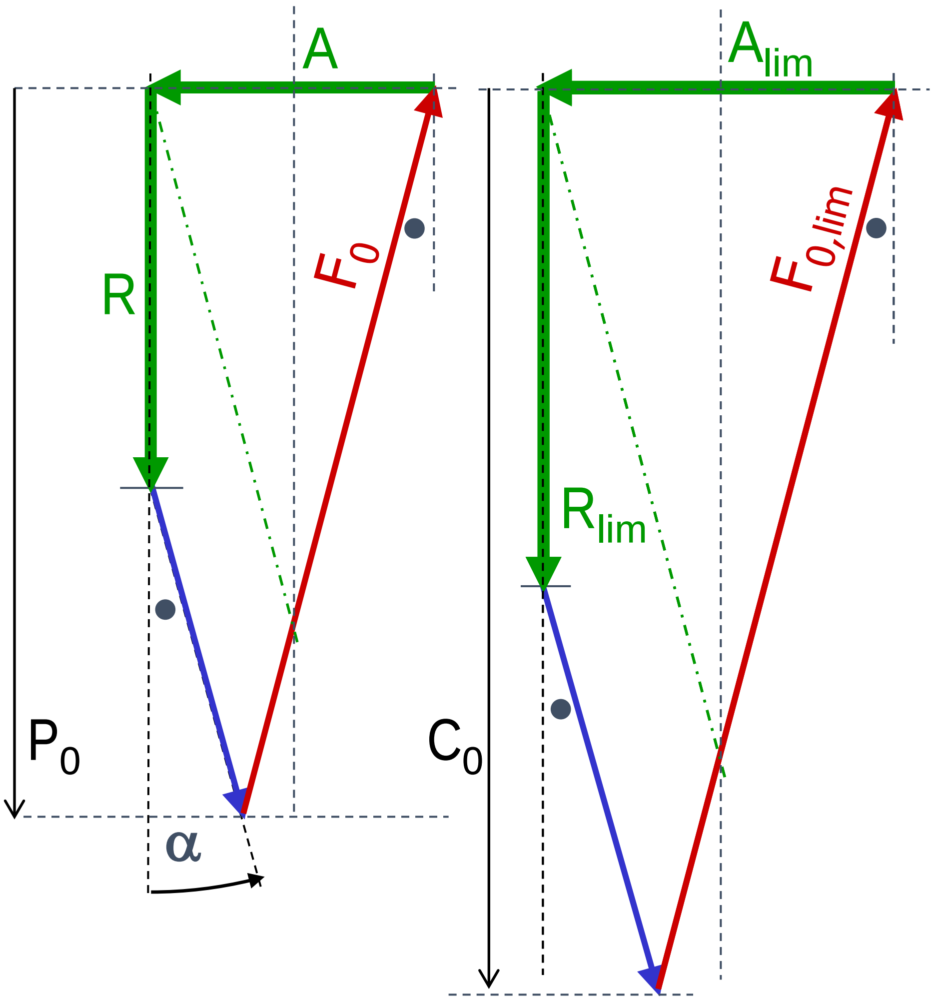
$$\frac{1}{2}\frac{R_{lim}}{\cos\alpha} + \frac{1}{2}\frac{A_{lim}}{\sin\alpha} = F_{0,lim}$$

Limiting the contact force: $F_0 = \frac{1}{2}\frac{R}{\cos\alpha} + \frac{1}{2}\frac{A}{\sin\alpha} \le F_{0,lim} \equiv \frac{1}{2}\frac{R_{lim}}{\cos\alpha} + \frac{1}{2}\frac{A_{lim}}{\sin\alpha}$



The problem with this procedure is that it requires the knowledge of inner geometry to determine F_0 , while we would like to use just the forces R,A that we see from the outside of the bearing.

If we project F_0 and $F_{0,lim}$ on the radial direction we get P_0 and C_0:



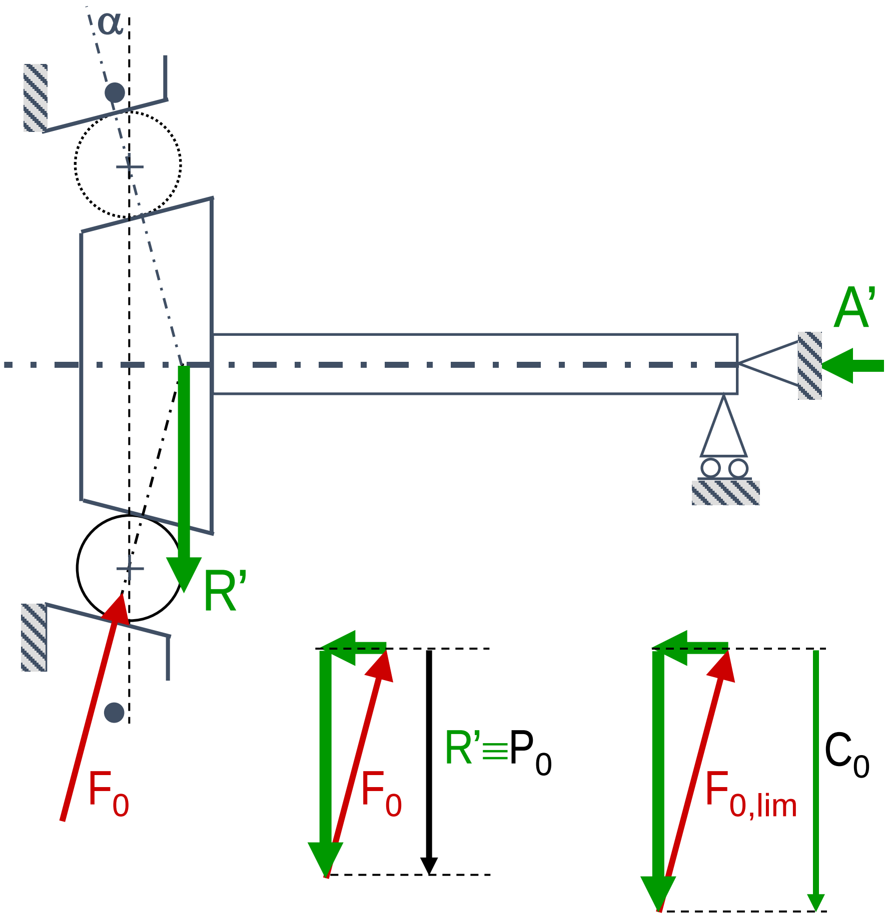
$$P_0 = \frac{1}{2}R + \frac{1}{2}\frac{A}{\tan\alpha} \leq C_0$$

$$C_0 = \frac{1}{2}R_{lim} + \frac{1}{2}\frac{A_{lim}}{\tan\alpha}$$

We may give C_0 the meaning of a real external radial force instead of a projection of an internal force through a reference case.

In sl. 11 of this section we have seen this to be the one described as (R′, A′) with:
A’=R’ tanα .

The reference case



$$C_0 = F_{0,lim} \cos\alpha$$

Imagine an experiment on the equipment on the left, where the inner ring is just lightly approached to the balls, then the radial force R' is applied.

The shaft is axially constrained.

When R' is applied the upper ball looses contact, and the horizontal component of F_0 determines force A', which is applied to the shaft end by the constraint ensuring axial equilibrium.

As seen before, this is the limit case where $A'/R' = \tan\alpha$.

When $F_0 = F_{0,lim}$ then R' reaches its limit R_{lim}, which is called C_0:

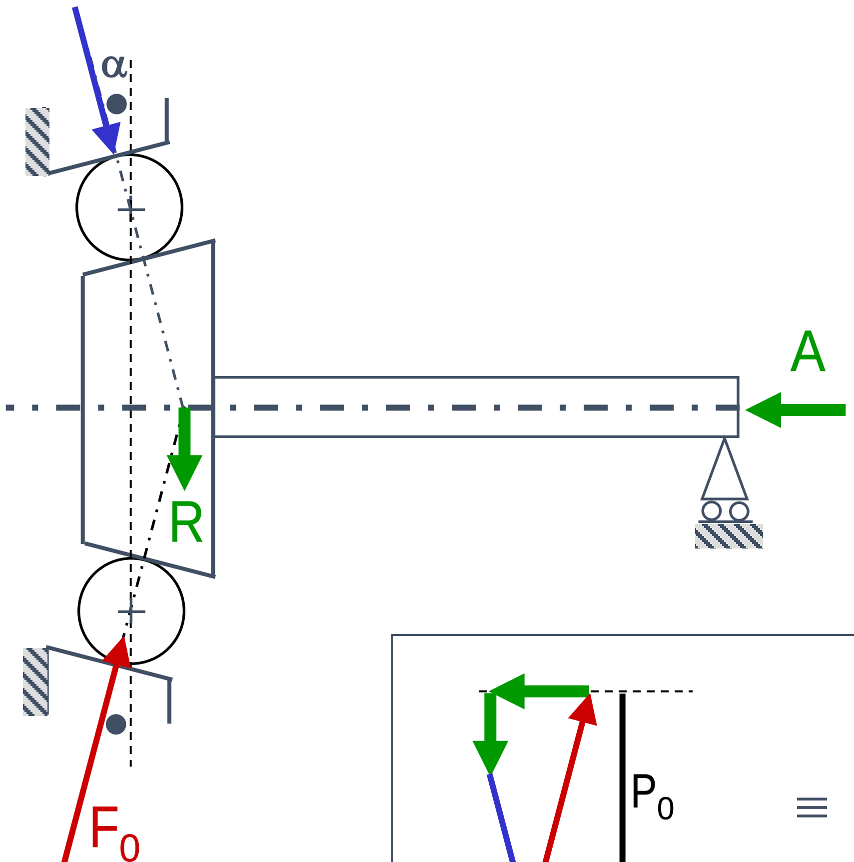
"basic static capacity"

or

"basic static load rating".

In all other (A, R) cases:

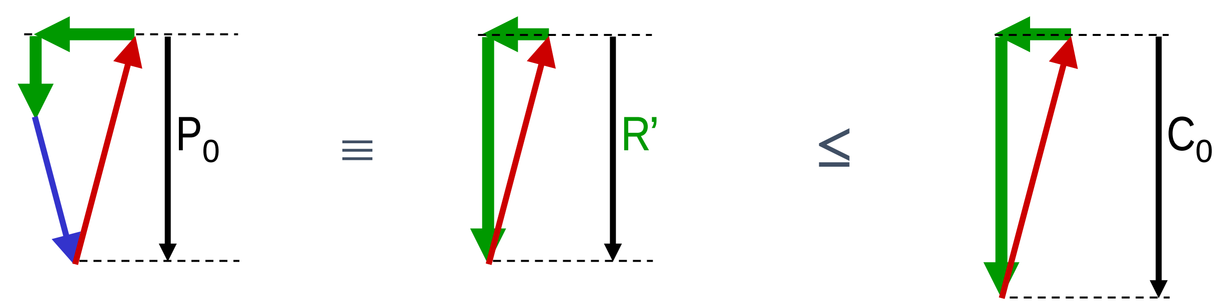
$$F_0 \equiv \frac{R}{2\cos\alpha} + \frac{A}{2\sin\alpha} \le C_0 \frac{1}{\cos\alpha} \equiv F_{0,lim}$$



and projecting radially:

$$\underbrace{P_0 = \frac{1}{2}R + \frac{1}{2}\frac{A}{\tan\alpha}}_{\text{equivalent radial load}} \le \overbrace{C_0}^{\text{basic static capacity}}$$

In this simple case the equivalent radial load P_0 has the physical meaning of "projection of F_0 along the radius".



Then P_0 ... equivalent to R' ... must be lower than C_0

6. The equivalent static radial load I (17/17)

This simple model, statically determinate, serves the purpose of qualitatively justifying the structure of a formula for the equivalent radial load which is found on rolling bearing manuals, and has the structure:

$$P_0 = X_0 R + Y_0 A \qquad \text{(in our example it would be : } X_0 = \frac{1}{2}\ ;\ Y_0 = \frac{1}{2\,\mathrm{tg}\alpha}\text{)}$$

However, reality is much more complex already for the static case, due to load distribution among several rolling elements; let alone in the case of fatigue !

In Sect.7 we see the real play of forces in the case of an oblique or angular ball bearing under simultaneous axial A and radial R forces. As in the Stribeck problem for purely radial bearings, we must deduce radial and axial forces from given radial and axial displacements. This is imposed by the static indeterminacy of the problem.

Sections 7, 8, 9 - The “equivalent” static load

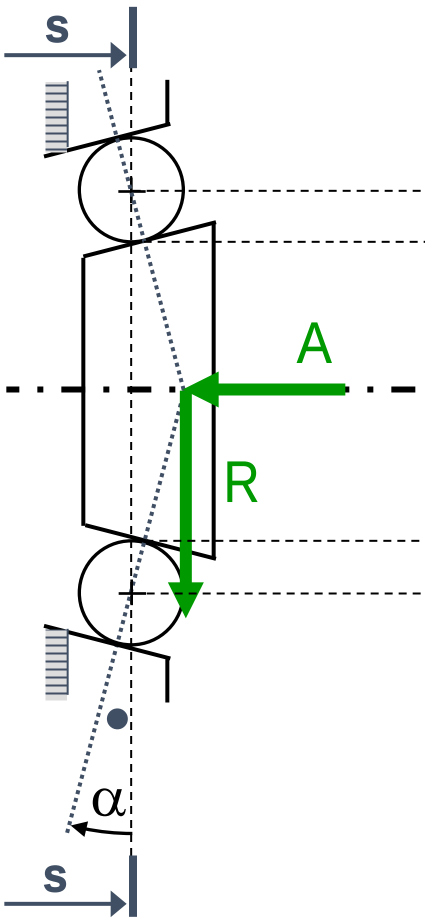
Section 7 develops the relation between the simultaneous axial and radial approach on an angular ball bearing, the forces transmitted through the rolling elements, the axial and radial force transmitted between the inner and the outer ring. The treatment presents the case where centrifugal forces are negligible.

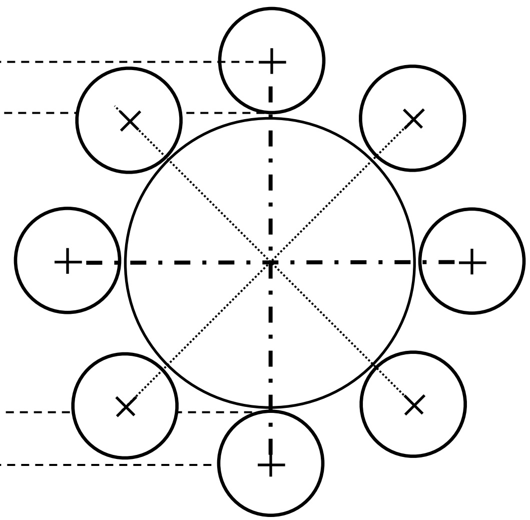
The sections 8, 9 which follow are more advanced, and are normally not presented in lectures as they require more concentration than can be normally obtained in a classroom. They are meant for those who like to become specialists in this field; reading and understanding requires some concentration, and the individual study of the material here provided is, by and large, the best approach.

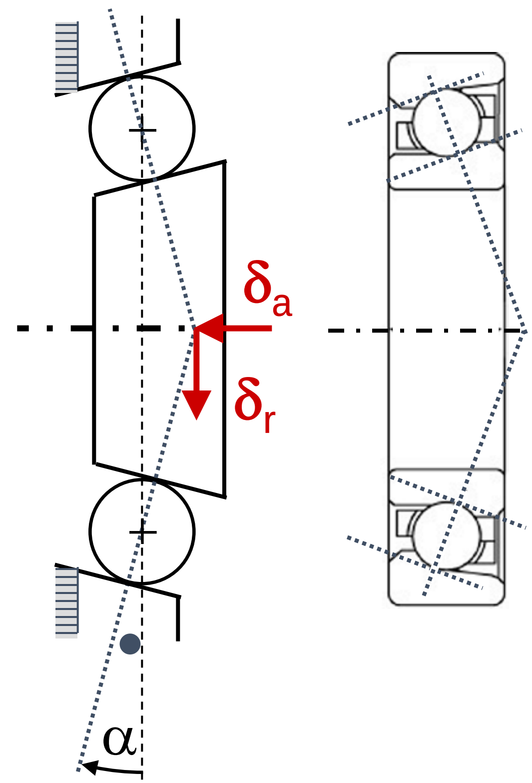
Section 8 shows how contact forces are distributed, how they can be graphically represented and how they sum up vectorially to compose axial and radial total forces on the bearing.

Section 9 re-develops, now in the final rigourous way, the concept of equivalent radial force already explored in Section 6. To those well-versed in the subject the study will provide some satisfaction.

Raceways are represented by their tangent cones, with the same “initial” contact angle, i.e. at zero force .







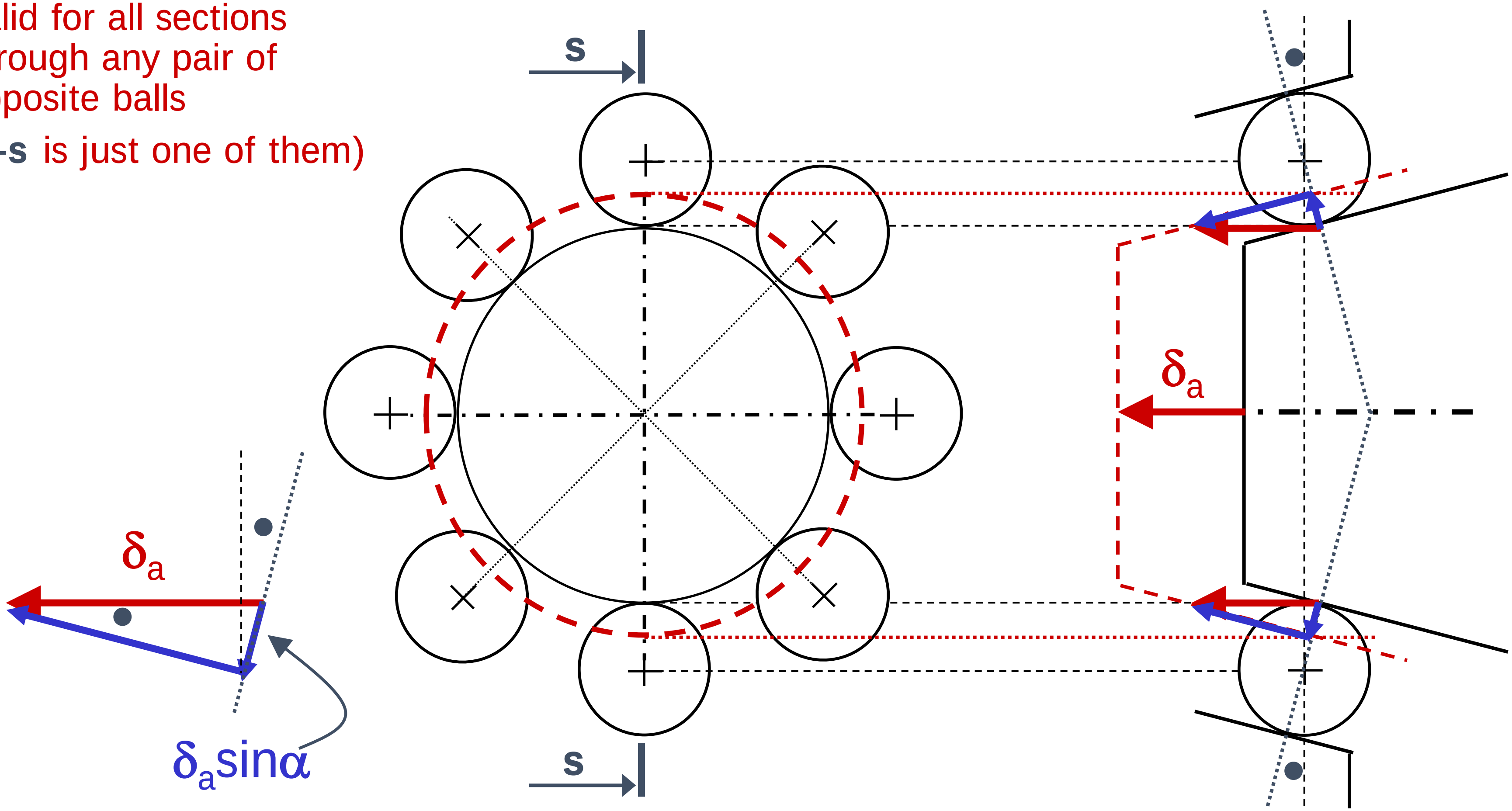
Axial cross section

Diametral section through plane **s-s**

Approaches of the inner ring (outer ring fixed)

Valid for all sections through any pair of opposite balls

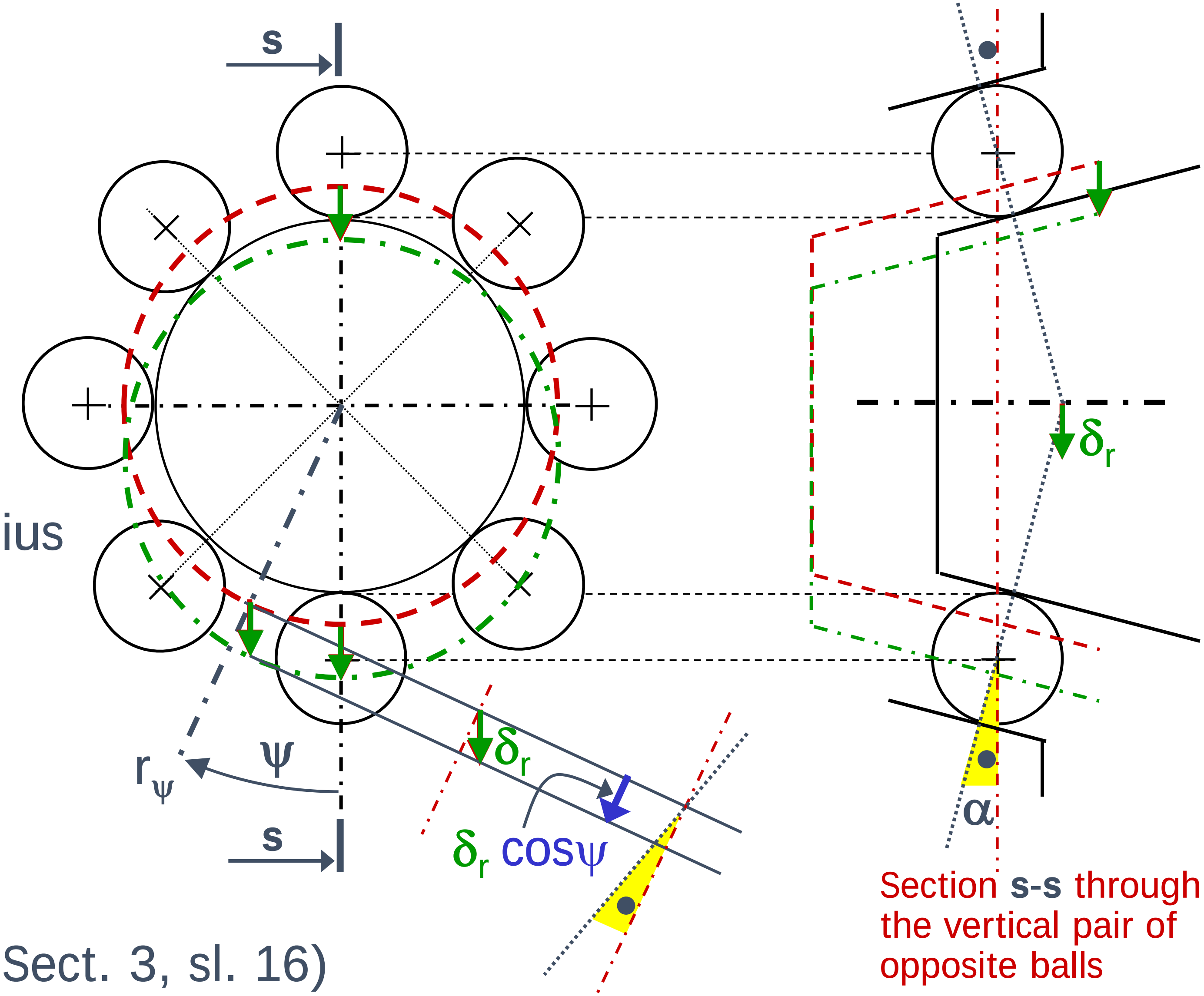
(**s**-**s** is just one of them)



Axial approach δ_a produces a uniform interference on all balls; the value of interference along the normal to the contact is $\delta_a \sin\alpha$, i.e. the component of δ_a along the normal to the contact.

Radial approach δ_r produces a vertical movement downwards, whose component along the radius at angle ψ is:

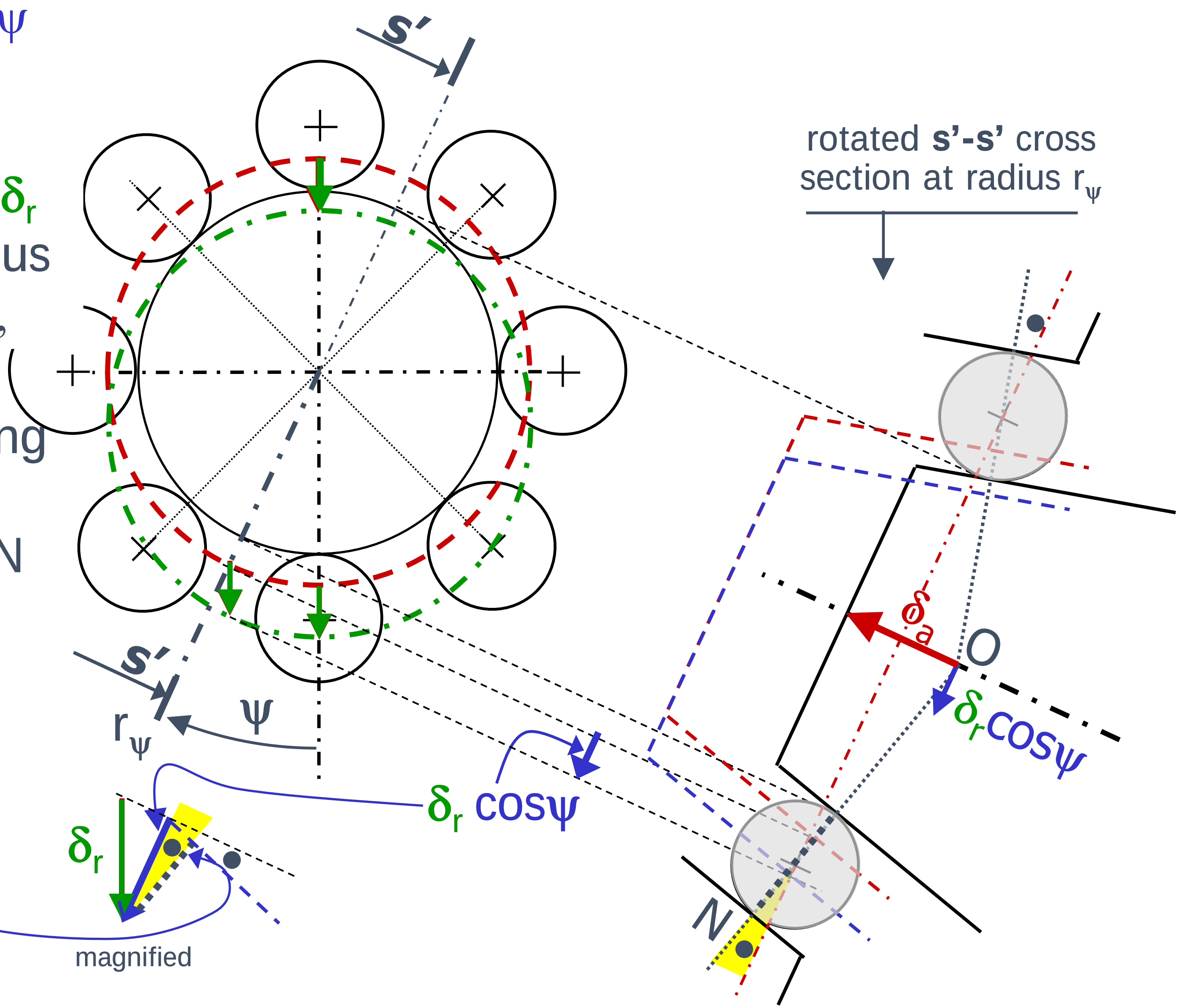
$\delta_r \cos\psi$



(compare with Sect. 3, sl. 16)

In fact, $\delta_r \cos\psi$ is the component of displacement δ_r along the radius r_ψ at angle ψ, while its projection along the normal to the contact ON is

$\delta_r \cos\psi \cos\alpha$



The total displacement along the normal to the contact is then:

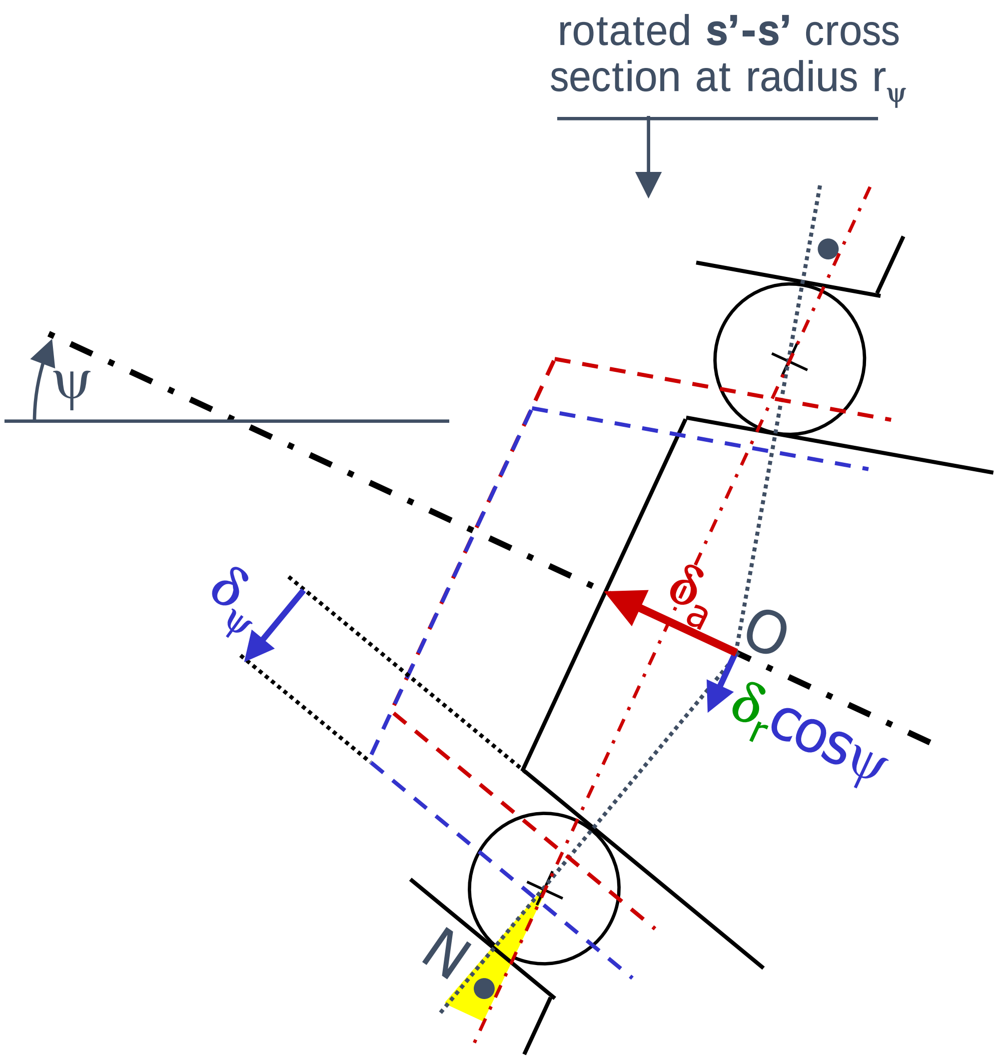
$\delta_\psi = \delta_r \cos\psi \cos\alpha + \delta_a \sin\alpha$

At $\psi = 0$:

$\delta_0 = \delta_r \cos\alpha + \delta_a \sin\alpha$

therefore the displacement ratio:

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_a \sin\alpha + \delta_r \cos\psi \cos\alpha}{\delta_a \sin\alpha + \delta_r \cos\alpha}$$



By writing:

$$\frac{\delta_\psi}{\delta_0} = \frac{\delta_r \cos\psi \cos\alpha + \delta_a \sin\alpha}{\delta_r \cos\alpha + \delta_a \sin\alpha} = \frac{\cos\psi + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}{1 + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}$$

By comparing with the
equation for the purely radial bearing,

$$\longrightarrow \quad \frac{\delta_\psi}{\delta_0} = \frac{\cos\psi - \dfrac{1}{2}\dfrac{g}{\delta_r}}{1 - \dfrac{1}{2}\dfrac{g}{\delta_r}}$$

However, note that the oblique bearing behaves like a radial bearing with an interference "i" (i.e. a negative clearance i=-g) produced by the axial displacement δ_a:

$$\frac{\delta_\psi}{\delta_0} = \frac{\cos\psi + \dfrac{1}{2}\dfrac{i}{\delta_r}}{1 + \dfrac{1}{2}\dfrac{i}{\delta_r}} \quad \ldots \text{where } \frac{i}{2} \text{ is : } \delta_a \mathrm{tg}\alpha$$

see this Chapter, Sect. 3. sl 16

For : $\delta_\psi = 0 \quad \psi = \psi_{max}$, i.e. :

$\delta_r \cos\psi_{max} \cdot \cos\alpha + \delta_a \sin\alpha = 0$, then if: $\dfrac{\delta_a}{\delta_r} \cdot \mathrm{tg}\alpha \le 1$

$$\cos\psi_{max} = -\frac{\delta_a}{\delta_r} \cdot \mathrm{tg}\alpha \quad \text{(negative !)}$$

therefore :

$$\frac{\delta_\psi}{\delta_0} = \frac{\cos\psi + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}{1 + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha} = \frac{\cos\psi - \cos\psi_{max}}{1 - \cos\psi_{max}}$$

Compare with Sect. 3 sl. 9 !

This formula is valid only when $\delta_a \mathrm{tg}\alpha \le \delta_r$, and the contact extends over the angle: $\dfrac{\pi}{2} \le \psi_{max} \le \pi$

This is case I of next slide.

NOTE: when: $\psi_{max} = \dfrac{\pi}{2}$ for $\delta_a = 0$ only balls in the lower-half are in contact.

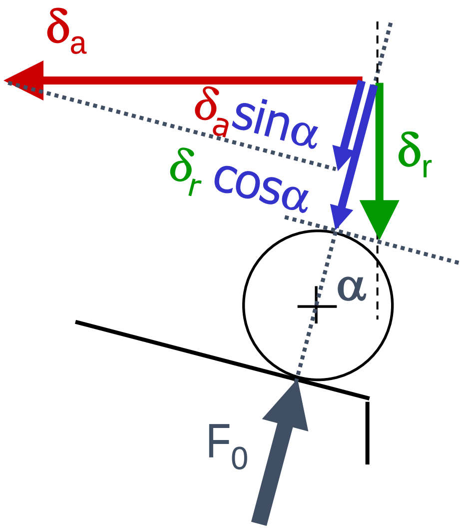
Case I - When $\pi/2 \le \psi_{max} \le \pi$, then $\cos\psi_{max}$ is defined, i.e., the force ratio can written trigonometrically:

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^n = \left(\frac{\cos\psi - \cos\psi_{max}}{1-\cos\psi_{max}}\right)^n$$

Case II - When $\delta_a \mathrm{tg}\alpha > \delta_r$ (which implies $\psi_{max}=\pi$) then the formula must be written:

$$\frac{F_\psi}{F_0} = \left(\frac{\delta_\psi}{\delta_0}\right)^n = \left(\frac{\cos\psi + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}{1 + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}\right)^n$$

On the bottom ball, at $\psi=0$:



$$F_0 = K_{tot} \cdot (\delta_a \sin\alpha + \delta_r \cos\alpha)^n$$

$$\begin{cases} n = 1{,}5: & \text{point contact} \\ n \cong 1{,}1: & \text{line contact} \end{cases}$$

K_{tot} is the total stiffness of the two contacts, Sect. 3 sl. 10.

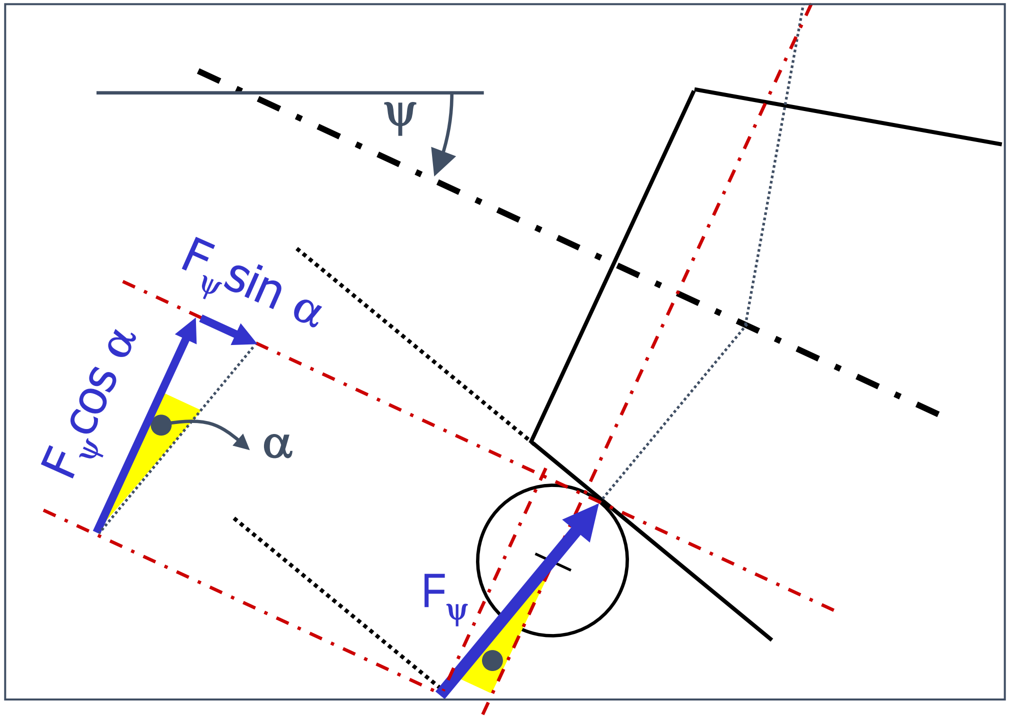
In case I the forces are calculated as:

$$A = \sum_{-\psi_{max}}^{+\psi_{max}} F_{\psi_i} \cdot \sin\alpha \quad = \quad F_0 \sin\alpha \sum_{-\psi_{max}}^{+\psi_{max}} \left(\frac{\cos\psi_i - \cos\psi_{max}}{1 - \cos\psi_{max}} \right)^n$$

$$R = \sum_{-\psi_{max}}^{+\psi_{max}} F_{\psi_i} \cdot \cos\alpha \cdot \cos\psi_i = F_0 \cos\alpha \sum_{-\psi_{max}}^{+\psi_{max}} \left(\frac{\cos\psi_i - \cos\psi_{max}}{1 - \cos\psi_{max}} \right)^n \cos\psi_i$$

$$\psi_{max} = \mathrm{ar}\cos\left(-\frac{\delta_a}{\delta_r} \mathrm{tg}\alpha \right)$$

For a given δ_a/δ_r , ψ_{max} is given, then the ratio A/R follows. It is interesting to check that this ratio has just a quite small variability for varying Z.



Case I-a: $\delta_a = 0$

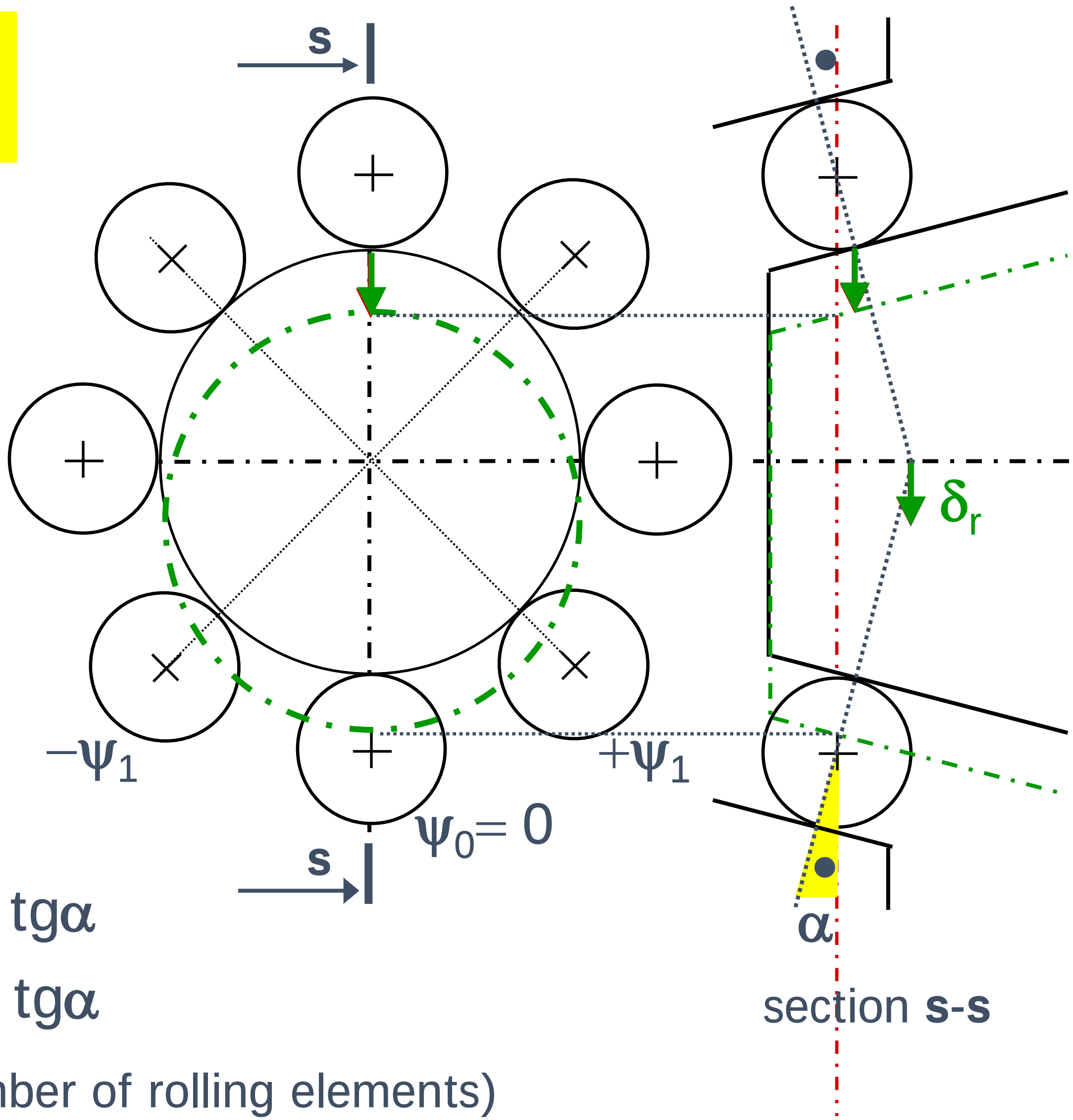
$\cos\psi_{max} = 0,\ \psi_{max} = \pi/2$

$$\frac{A}{R} = tg\alpha \frac{\sum_{-\pi/2}^{+\pi/2} (\cos\psi_i)^n}{\sum_{-\pi/2}^{+\pi/2} (\cos\psi_i)^{n+1}}$$

A representative range of values :

$$\begin{cases} Z = 10 \rightarrow (A/R) = 1.23 \cdot tg\alpha \\ Z = 20 \rightarrow (A/R) = 1.21 \cdot tg\alpha \end{cases}$$

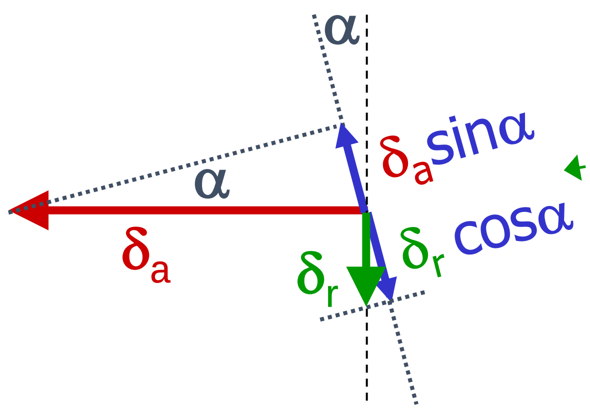
(reminder: Z is the total number of rolling elements)

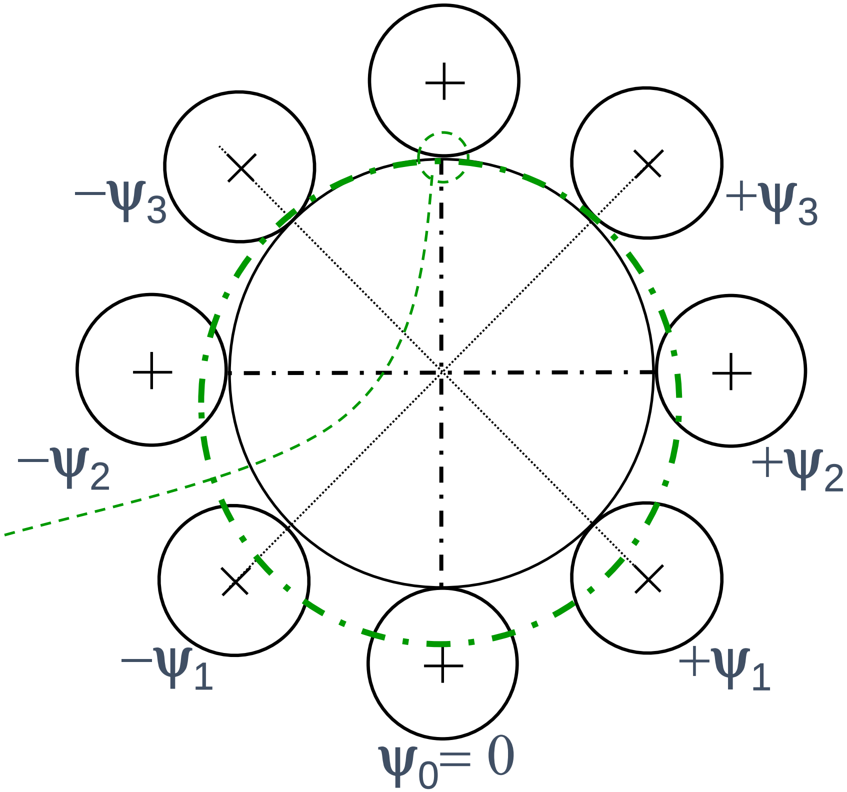


Case I-b: the maximum value for the ratio A/R is found at:

$$\delta_a = \frac{\delta_r}{tg\alpha} \quad \text{i.e.} \quad \delta_a \sin\alpha = \delta_r \cos\alpha$$

$$(\cos\psi_{max} = -1 \rightarrow \psi_{max} = \pi)$$





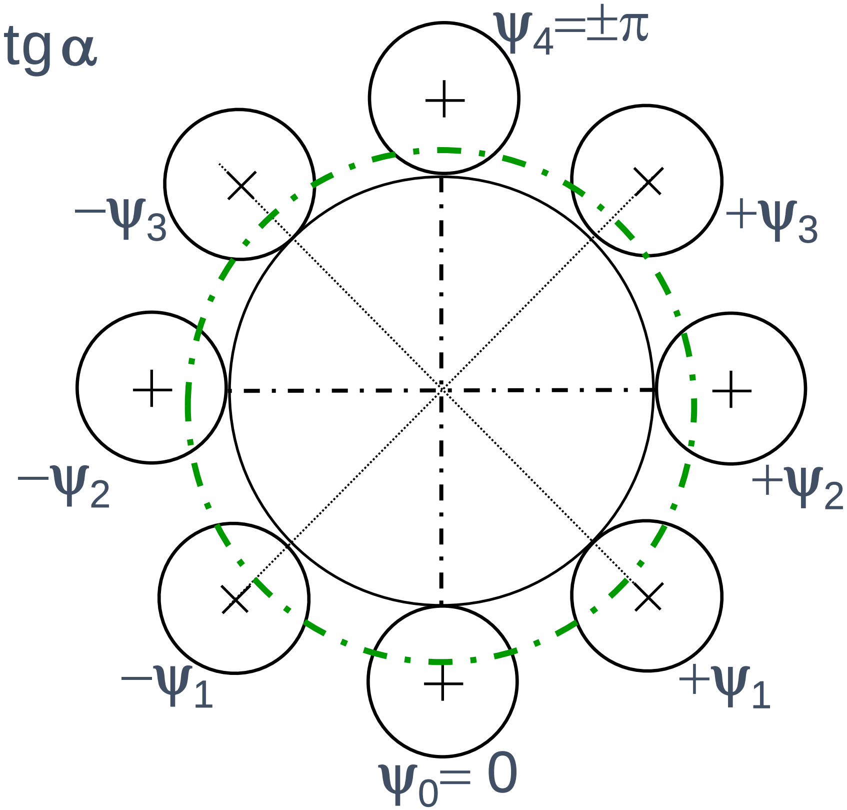
$$\begin{cases} Z = 10 \rightarrow (A/R) = 1.67 \cdot tg\alpha \\ Z = 20 \rightarrow (A/R) = 1.67 \cdot tg\alpha \end{cases}$$

$$\frac{A}{R} = tg\alpha \cdot \frac{\sum_{-\pi}^{+\pi} (1 + \cos\psi_i)^n}{\sum_{-\pi}^{+\pi} \cos\psi_i (1 + \cos\psi_i)^n}$$

In case II i.e. when δ_a exceeds $\delta_r/\mathrm{tg}\,\alpha$

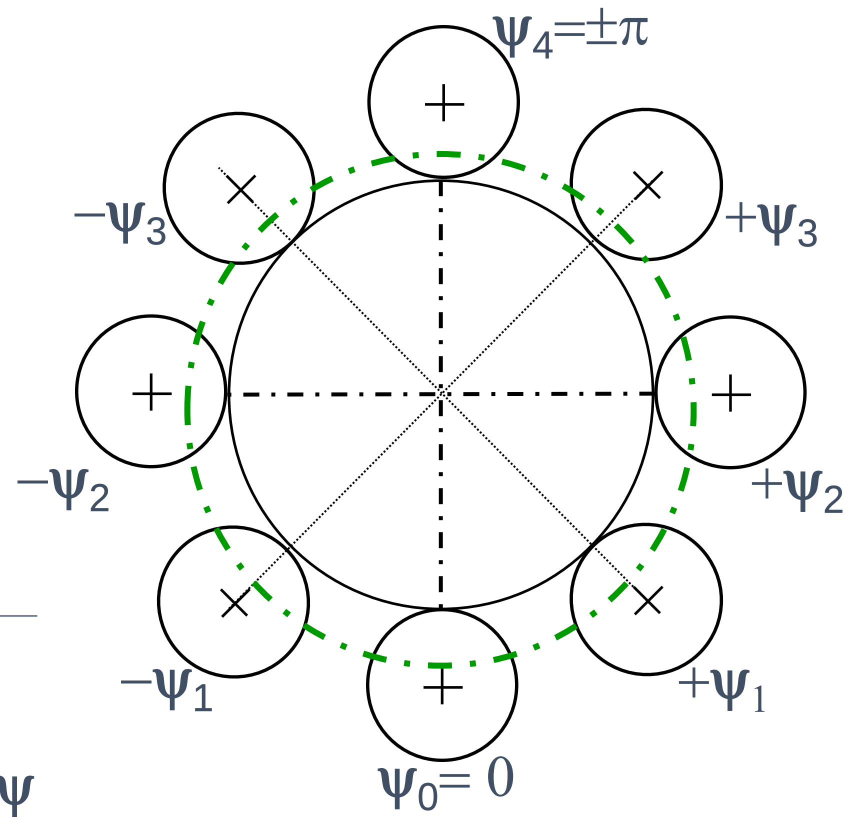
$$\begin{cases} A = \sum_{-\pi}^{+\pi} F_{\psi_i} \cdot \sin\alpha \\ R = \sum_{-\pi}^{+\pi} F_{\psi_i} \cdot \cos\psi_i \cdot \cos\alpha \end{cases}$$

$$\begin{cases} A = F_0 \sin\alpha \sum_{-\pi}^{+\pi} \left(\dfrac{\cos\psi_i + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}{1 + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha} \right)^n \\ R = F_0 \cos\alpha \sum_{-\pi}^{+\pi} \left(\dfrac{\cos\psi_i + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha}{1 + \dfrac{\delta_a}{\delta_r}\mathrm{tg}\alpha} \right)^n \cos\psi_i \end{cases}$$



... then in this case II :

$$\frac{A}{R} = \mathrm{tg}\alpha \cdot \frac{\displaystyle\sum_{-\pi}^{+\pi} \left(\frac{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} \cos\psi + 1}{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} + 1} \right)^n}{\displaystyle\sum_{-\pi}^{+\pi} \left(\frac{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} \cos\psi + 1}{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} + 1} \right)^n \cos\psi}$$



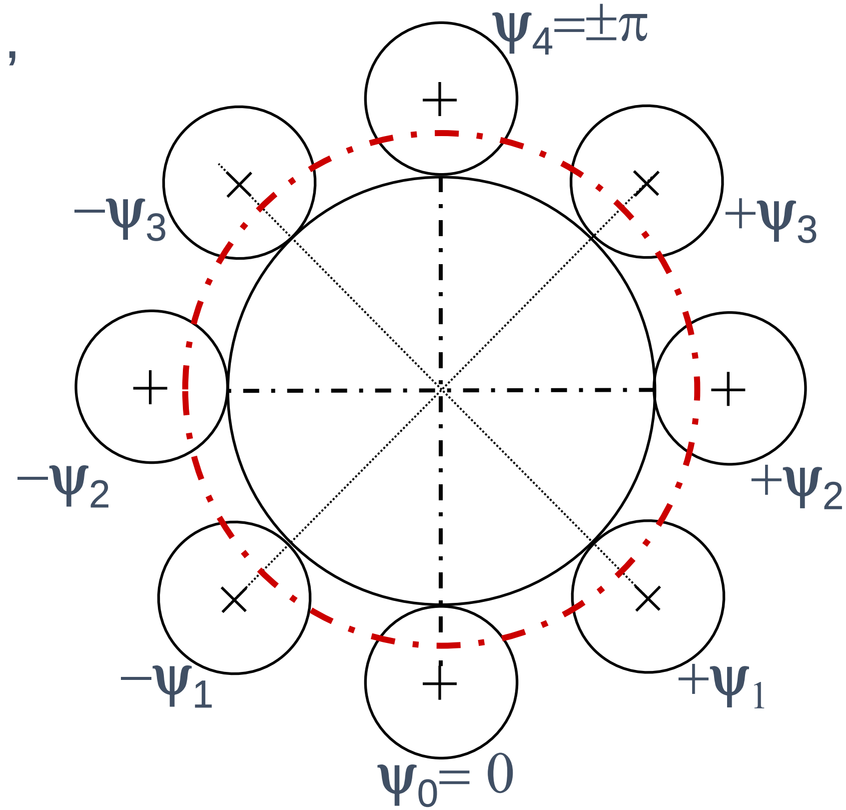
For $\delta_r/\delta_a \Rightarrow 0$: $A/R \Rightarrow \infty$

When the approach is purely axial, i.e., $\delta_r / \delta_a = 0$, then:

$$A = F_0 \sin\alpha \sum_{-\pi}^{+\pi} \left(\frac{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} \cos\psi + 1}{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} + 1} \right)^n$$

$$\sum_{-\pi}^{+\pi} 1 \Rightarrow Z$$

$$A = Z\, F_0 \sin\alpha$$



while:

$$R = F_0 \cos\alpha \sum_{-\pi}^{+\pi} \left(\frac{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} \cos\psi_i + 1}{\dfrac{\delta_r}{\delta_a \mathrm{tg}\alpha} + 1} \right)^n \cos\psi_i = F_0 \cos\alpha \sum_{-\pi}^{+\pi} \cos\psi_i = 0$$

This Sections aims at showing how the total radial and axial force components build up by summing the projections of contact forces for $\psi_0 = 0, \pm\psi_1, \pm\psi_2, \pm\psi_3$ ($\Delta\psi$=22,5°)

rotated view of cross section at angle ψ_1

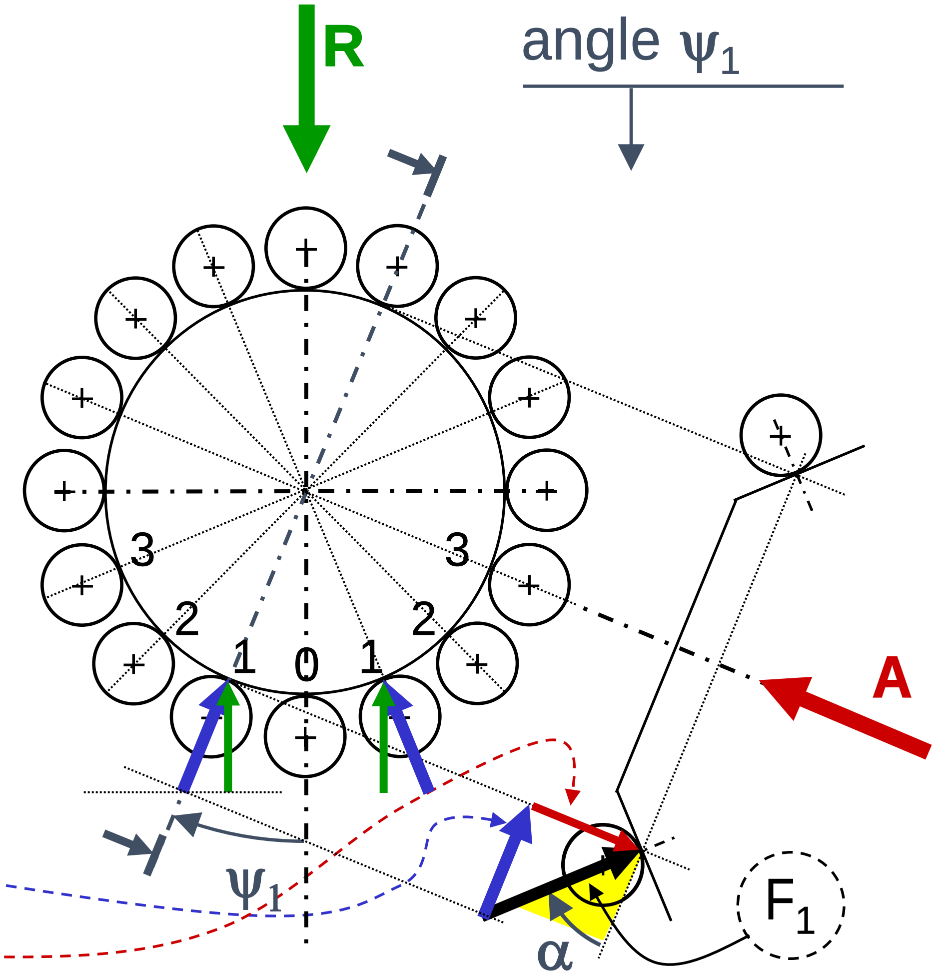
Case: Z=16, α=45°, and pure radial approach:

i.e. $\frac{\delta_a}{\delta_r} \tan\alpha = 0$

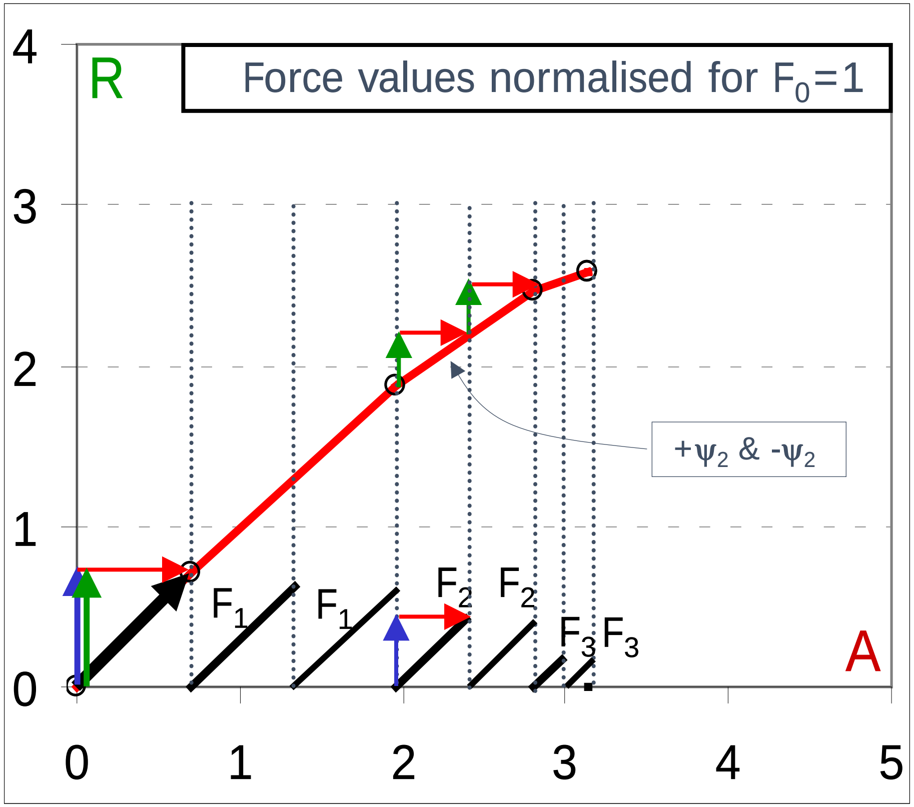
force normal to contact: F_1

R component: $F_1 \cos\alpha \cos\psi$

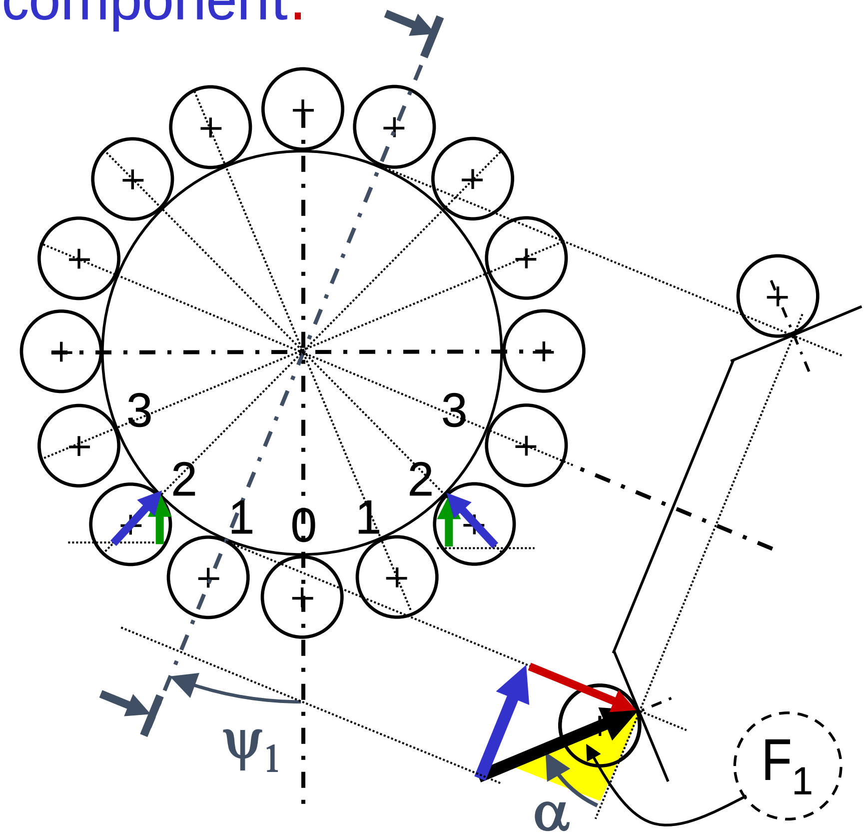
A component: $F_1 \sin\alpha$



8. Forces in angular contact ball bearings II (2/6)



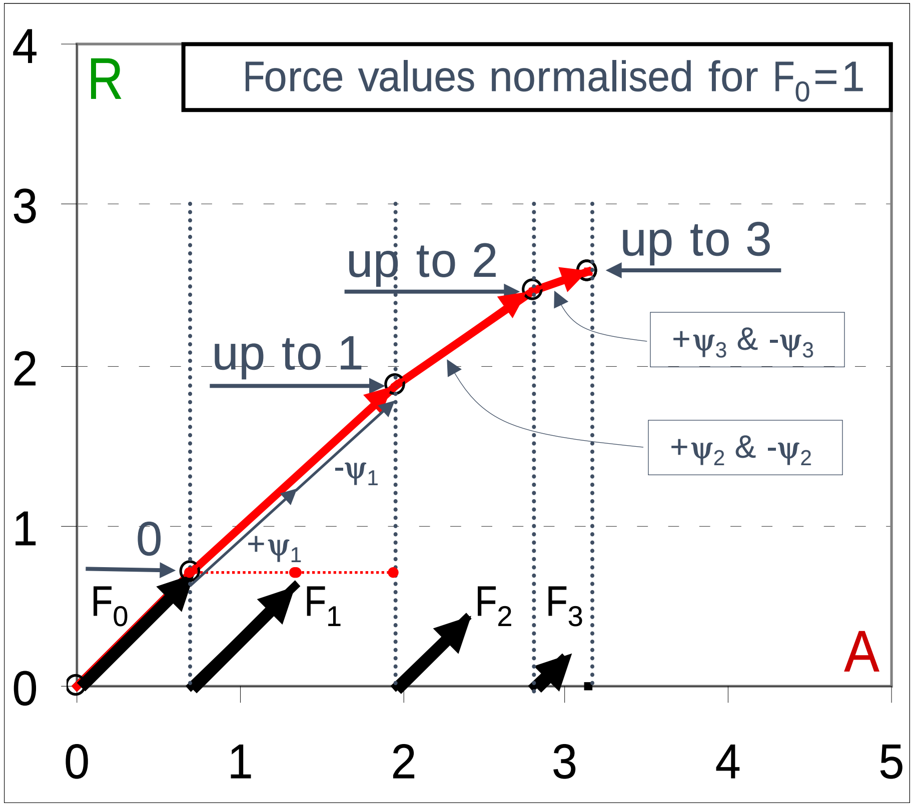
Example of construction: contribution of balls at $\psi=0$ and $\psi=\pm\psi_2$ to (bearing) radial R component, axial A component and local contact radial component.



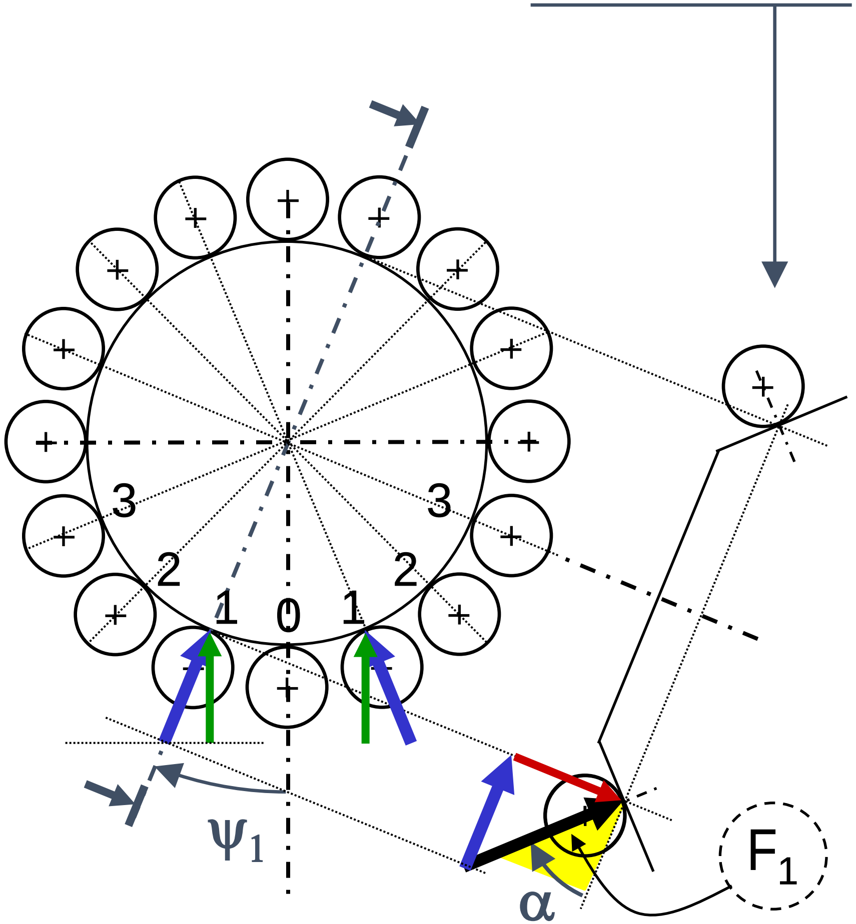
Black arrows: normal contact force F_i on a sphere at angle ψ_i as seen in their own section plane.

Green arrow: R component

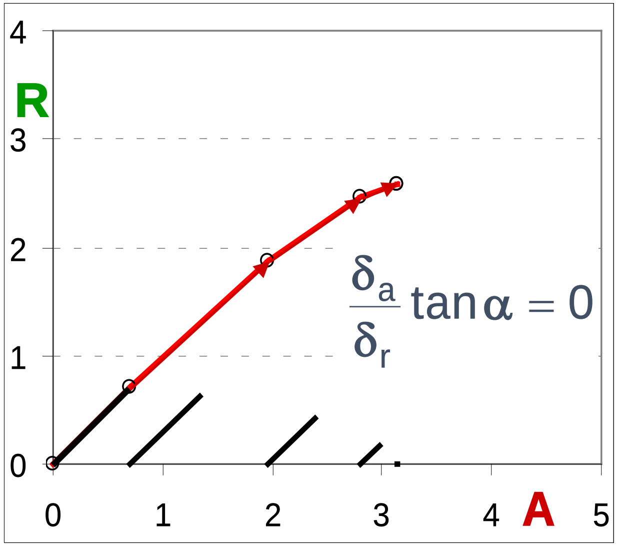
Red arrow: A component

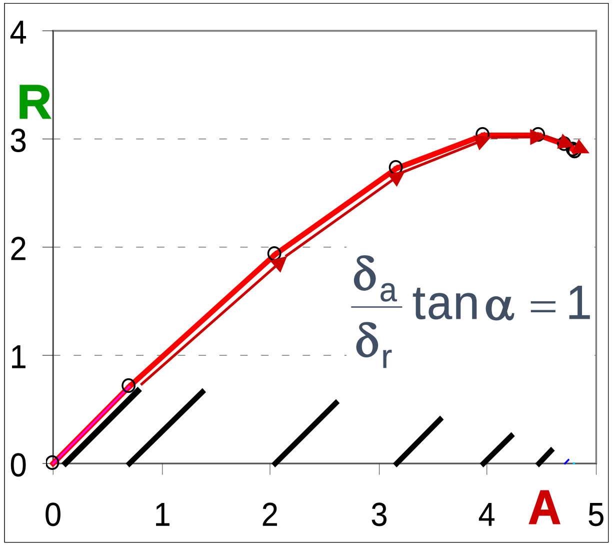


Red line: progressive vector sum of radial R and axial A components of contact forces. as seen in the $\psi_1=0$ section plane



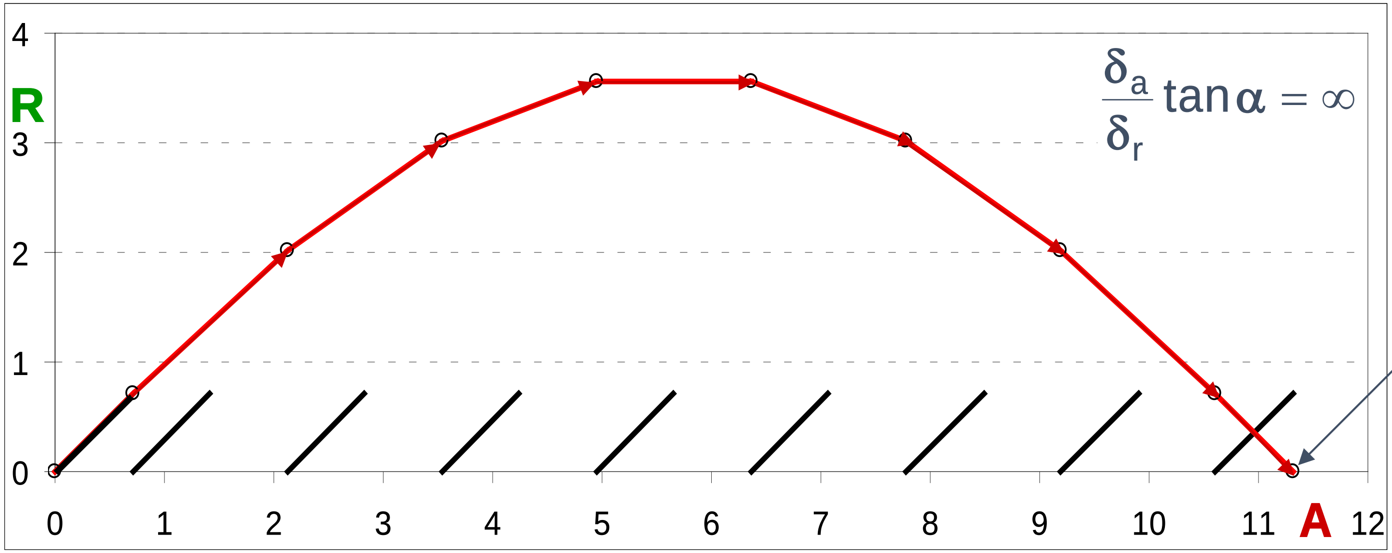
$$\frac{\delta_a}{\delta_r}\tan\alpha = 0$$





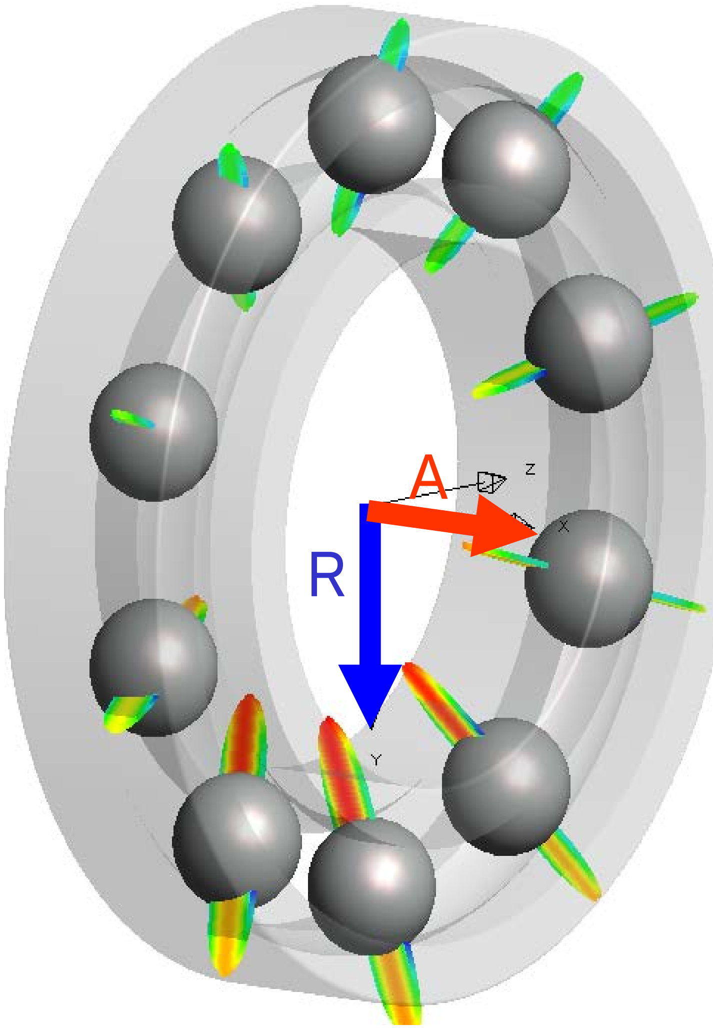
Buildup of radial **R** and axial **A** force contributed by contacts, starting from $\mathbf{F_0}$.

Three cases for Z=16, α=45°



A=16·sin45° = 11,31

(F_0=1)



source: FAG, courtesy Schaeffler Italia S.r.l.

Case study: this figure visualises contact surface pressure along the long axis of the contact ellipse, for a case of combined radial and axial load.

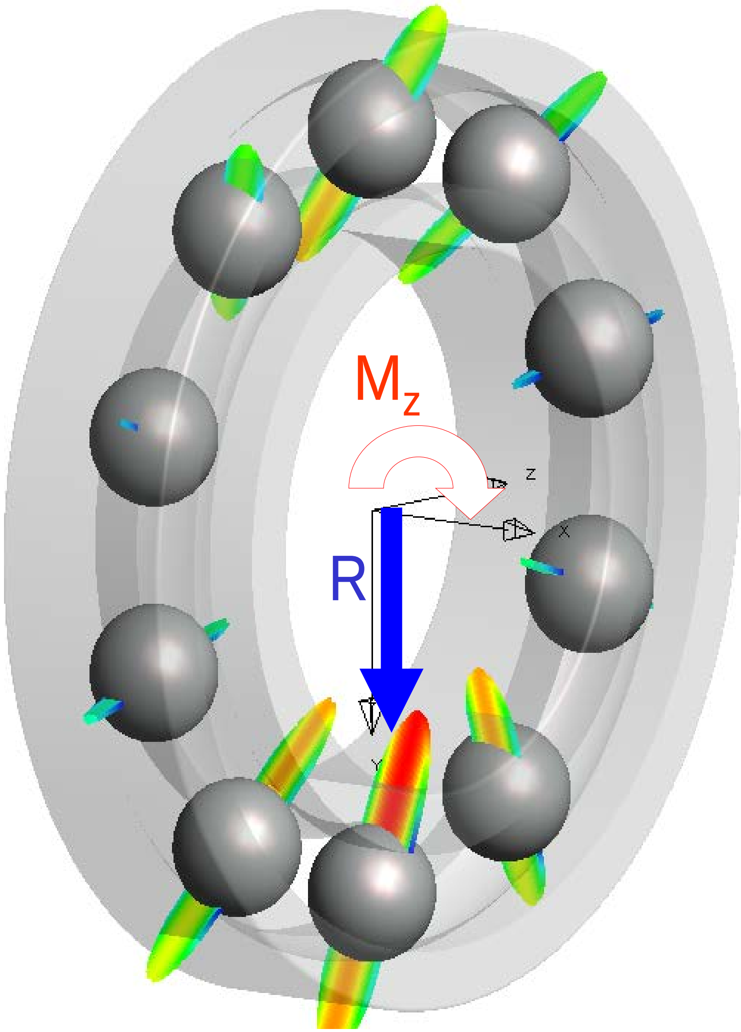
You can deduce the distribution of forces on balls.

Explain why colours representing pressure are different at the inner and outer contacts of the same ball.

Can you figure out qualitatively which colours are associated to the highest and lowest stresses?

Is there a redundancy in the way pressures are represented?

You may remark that this is a radial bearing; however, see Sect. 5 of this Chapter, an axial force recovers the radial clearance and the contact becomes inclined, although of a small angle.



source: FAG, courtesy Schaeffler Italia S.r.l.

Case study: this figure visualises contact surface pressure along the long axis of the contact ellipse, for a case of combined radial force and tilting moment.

You can deduce the distribution of forces on balls. Compare this figure with the one in the previous slide, and explain why force inclinations are different at certain locations.

Please note that the calculation method presented in the ISO 281:2007 does not consider the single bearing under tilting moment; a loading condition which is normally a non-recommended practice.

In both cases I and II, axial A and radial R loads can be written:

$$A = \sum_{-\psi_{max}}^{+\psi_{max}} F_\psi \cdot \sin\alpha \qquad = F_0 \sin\alpha \; Z\, \tilde{a}$$

$$R = \sum_{-\psi_{max}}^{+\psi_{max}} F_\psi \cdot \cos\psi \cdot \cos\alpha = F_0 \cos\alpha \; Z\, \tilde{r}$$

takes into account only the proportion of load sharing among rolling bodies

takes into account contact angle

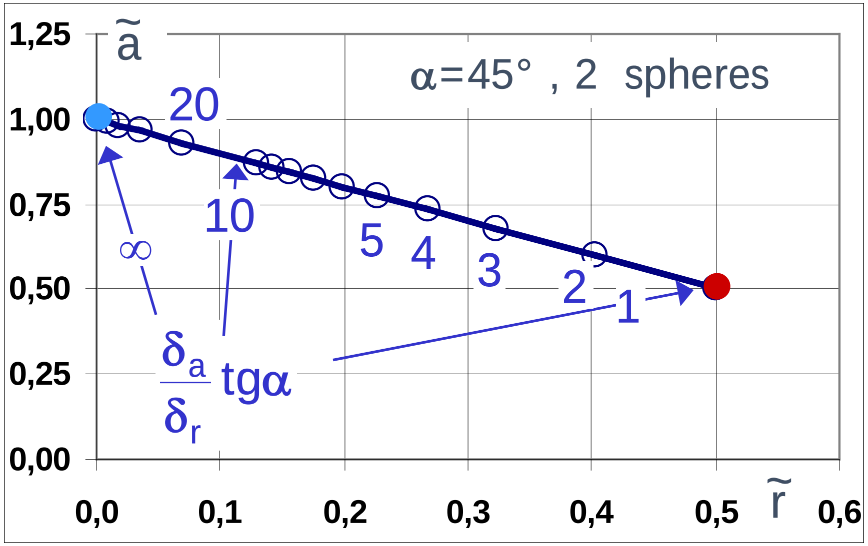
takes into account total roller number

It is now interesting to take $\frac{\delta_a}{\delta_r} \mathrm{tg}\alpha$ as a parameter and calculate $\tilde{a}$, $\tilde{r}$.

The diagrams in the next slides show the result for four representative cases:

2 spheres (for comparison with results of section 6)

8, 12, 16 spheres (the effect of a different number of rolling bodies)



$$\tilde{a} = \frac{1}{2}\left[\left(\frac{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}\right)^n + \left(\frac{-1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}\right)^n\right]$$

$$\tilde{r} = \frac{1}{2}\left[\left(\frac{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}\right)^n + (-1)\left(\frac{-1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}{1+\frac{\delta_a}{\delta_r}\,\mathrm{tg}\alpha}\right)^n\right]$$

Let us calculate for Z=2, using formulas of Sect. 7 sl. 12.

In the case of two spheres $\tilde{a}$, $\tilde{r}$ stay on a straight line.

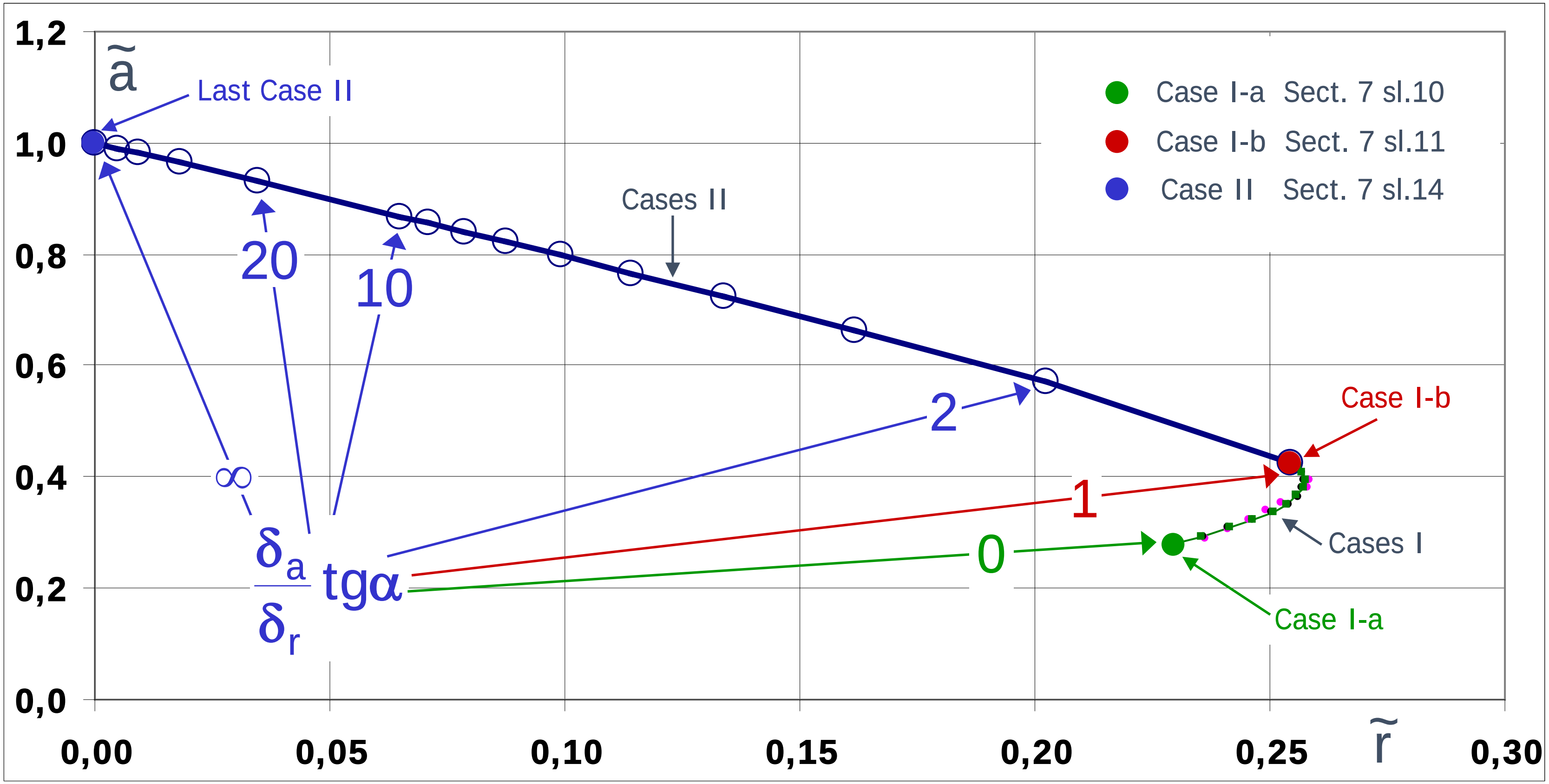
It is easy to check that the formulas on the bottom left of this page satisfy:

$$\tilde{r} + \tilde{a} = 1$$

and we find:

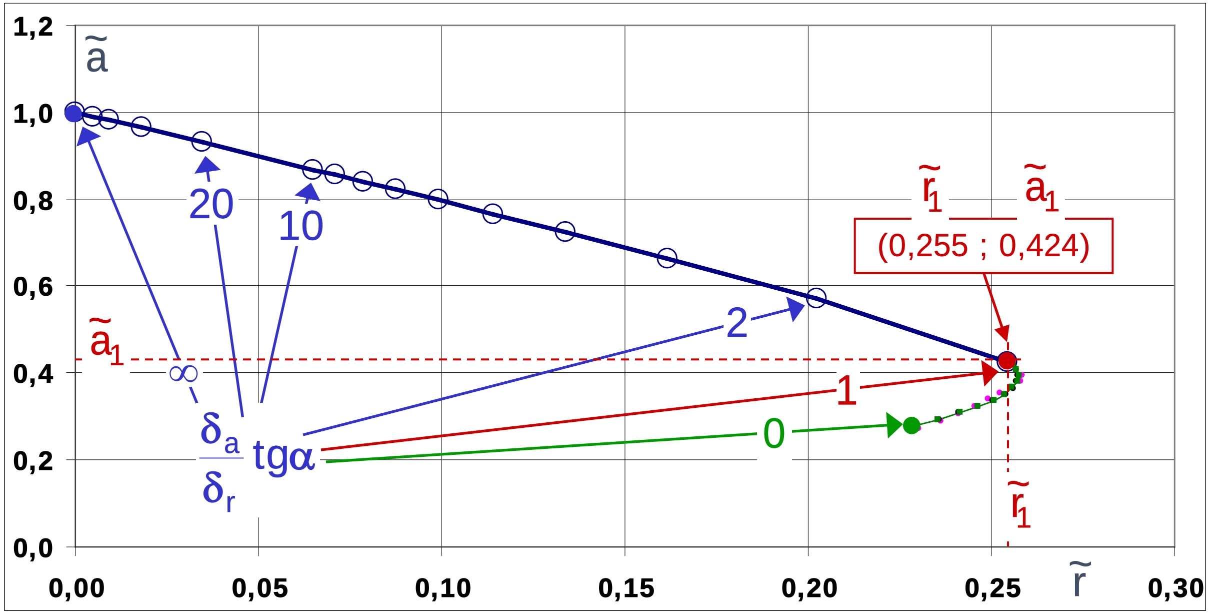
$$\frac{R}{2\cos\alpha} + \frac{A}{2\sin\alpha} = F_0$$

as in Sect. 6 sl. 10 of this Chapter



This figure shows that diagrams obtained for 8, 12 and 16 spheres are almost perfectly superimposed; extremely slight differences are visible only in the section between 0 and 1 (magnify and see dots of different colours).

The curve relative to full contact (blue line) is not a straight line.



However, within 5%,

for: $1 \le \dfrac{\delta_a}{\delta_r} \tan\alpha \le \infty$

it is approximated by a linear equation:

$2{,}26\,\tilde{r} + \tilde{a} = 1$ * → Substituting the following into it:

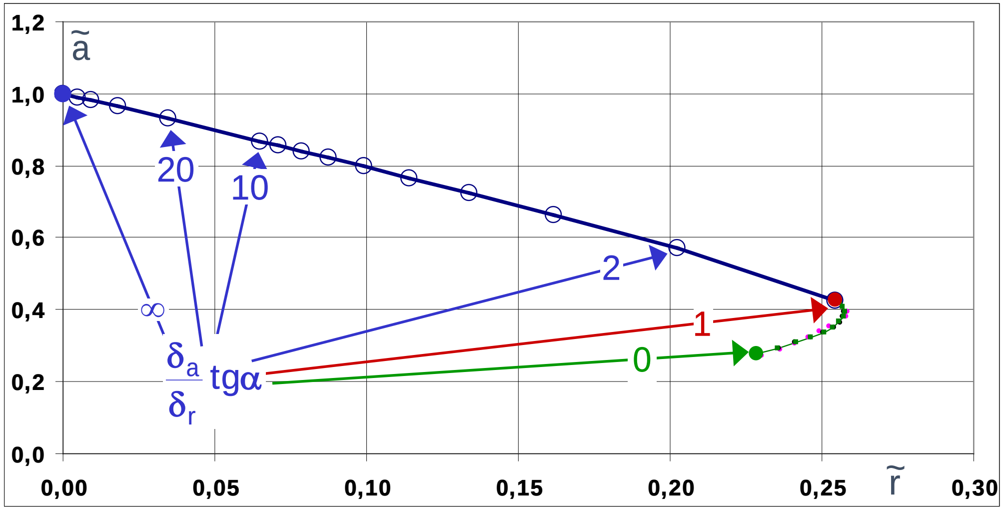
joining the • and • dots.

$A = F_0 \sin\alpha\, Z \cdot \tilde{a}$; $R = F_0 \cos\alpha\, Z \cdot \tilde{r}$

then: $\boxed{\dfrac{2{,}26}{Z\cos\alpha} R + \dfrac{A}{Z\sin\alpha} = F_0}$

$$* \quad \tilde{r}\,\frac{(1-\tilde{a}_1)}{\tilde{r}_1} + \tilde{a} = 1 \;;\; \tilde{r}\,\frac{(1-0{,}424)}{0{,}255} + \tilde{a} = 1$$

$$2{,}26 = \frac{(1-\tilde{a}_1)}{\tilde{r}_1}$$



Once again: in angular contact bearings a radial load is always associated to an axial load, i.e. is never present alone.

However, we want to express the couple through a single number, i.e., a radial load applied to the bearing.

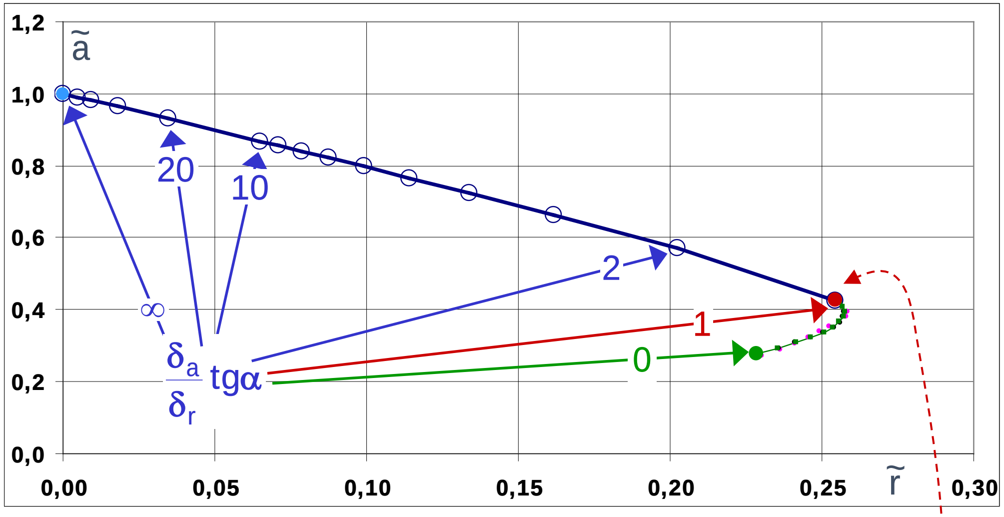
From $F_0 \leq F_{0,lim} \Rightarrow \frac{2{,}26\,R}{Z\cos\alpha} + \frac{A}{Z\sin\alpha} \leq \frac{2{,}26\,R_{lim}}{Z\cos\alpha} + \frac{A_{lim}}{Z\sin\alpha}$

The limit condition could be calculated for infinite pairs (A_{lim} ; R_{lim}), but we want set a limit only on R_{lim} ; then it is convenient to choose the reference case where the ratio A_{lim}/R_{lim} is fixed, and is the special value marked by the red dot •

for: $\frac{\delta_a}{\delta_r} tg\alpha = 1 \Rightarrow \psi_{max} = \pi$

The reference condition is used to set the limit.

When F_0 reaches its limit $F_{0,lim}$ value , R_{lim} reaches its limit value which we name C_0:



$$F_{0,lim} = \frac{C_0}{Z}\left(\frac{2{,}26}{\cos\alpha} + \frac{A_{lim}}{R_{lim}}\frac{1}{\sin\alpha}\right)$$

$$\frac{A_{lim}}{R_{lim}} = \tan\alpha\,\frac{\tilde{a}_1}{\tilde{r}_1}$$

Then:

$$F_0 = \frac{R}{Z}\left(\frac{2{,}26}{\cos\alpha} + \frac{A}{R}\frac{1}{\sin\alpha}\right) \le F_{0,lim} = \frac{C_0}{Z}\left(\frac{2{,}26}{\cos\alpha} + \frac{\tilde{a}_1}{\tilde{r}_1}\frac{1}{\cos\alpha}\right) \equiv \frac{C_0}{Z\cos\alpha}\frac{1}{\tilde{r}_1}$$

remember: $2{,}26 = \frac{(1-\tilde{a}_1)}{\tilde{r}_1}$

then: $(1-\tilde{a}_1)R + \frac{\tilde{r}_1}{\tan\alpha}A \le C_0$

9. The equivalent static radial load II (7/8)

Solution 1

Indipendently of the number of spheres (see Ch.2 Sect. 7 sl. 11):

$$A_{lim} / R_{lim} \cong 1{,}67 \text{ tg}\alpha \; ^*$$

Then, by way of example, in the case α=45°:

$$\frac{R}{Z}\left(\frac{2{,}26}{\cos\alpha} + \frac{A}{R}\frac{1}{\sin\alpha}\right) \leq \frac{C_0}{Z}\left(\frac{2{,}26}{\cos\alpha} + \frac{A_{lim}}{R_{lim}}\frac{1}{\sin\alpha}\right)$$

We get:

$$2{,}26R + \frac{1}{\text{tg}\alpha}A \leq C_0\left(2{,}26 + \frac{1}{\cancel{\text{tg}}\alpha}1{,}67\ \cancel{\text{tg}}\alpha\right) \equiv C_0\, 3{,}93$$

$$\underline{0{,}58\ R + 0{,}26\ A} \leq C_0$$

→ limit radial load (for static loads this is called C_0)

→ equivalent radial load, called P_0, so named because it compares with a limit radial load

* This was an approximated value

9. The equivalent static radial load II (8/8)

Solution 2

The same solution is now reworked using the more general final formula * :

$$\left(1-\tilde{a}_1\right)R+\frac{\tilde{r}_1}{\tan\alpha}A \le C_0$$

Then, by way of example, in the case $\alpha=45°$:

$$\left(0.576\right)R+\frac{0.255}{\tan\alpha}A \le C_0$$

We get again:

$$0{,}58\ R+0{,}26\ A \le C_0$$

The simplified procedure, now shown, produces a formula which compares quite well with the catalog formula for static equivalent (radial) load of angular bearings:

$$P_0 \equiv 0{,}5R+0{,}26\ A \le C_0$$

which however includes, in addition, the effect of the angle increase explained in the next section

* In this case the approximation was in the linearisation of the $\left(\tilde{r},\tilde{a}\right)$ curve, independent of the number of spheres

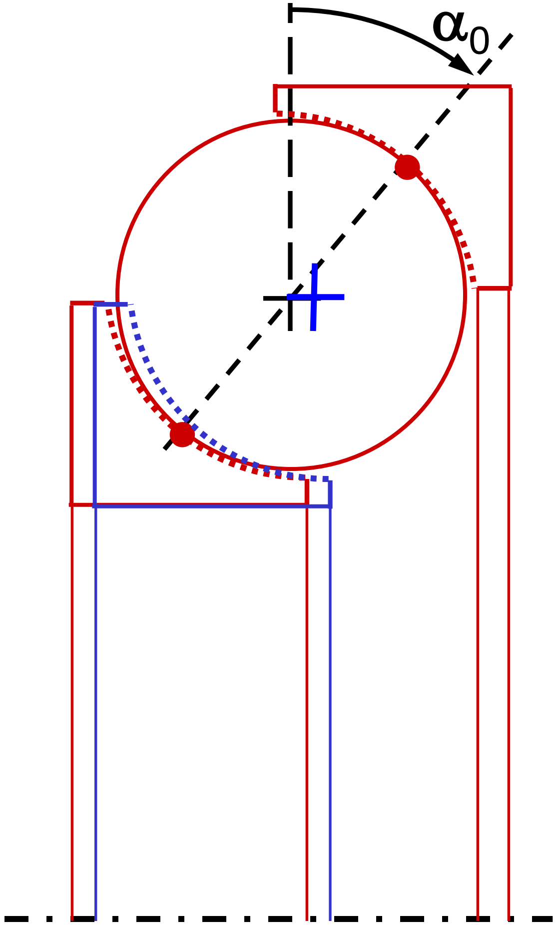
Sections 10, 11, 12 - Axial preload and axial stiffness

Section 10 deals with a non negligible complement to contact problems in angular ball bearings and provides formulas for angle correction and normal approach on radial and angular ball bearings under axial load.

Attention is paid to the fact that, in particular, in the deep groove radial bearings when they are submitted to an axial load the total normal approach cannot be calculated with the simple formula of Sect.7 sl.2 .

Section 11 discusses three main examples of assemblies with coupled ball bearings which require axial preload. With the aid of two examples, the effect of axial preload is discussed in detail.

Section 12 shows axial stiffness diagrams and how the axial load is shared between a pair of preloaded bearings.



In angular contact bearings the contact angle is determined by construction and varies a little as a function of the combined loads; then the effective angle α under load differs from the construction angle α_0.

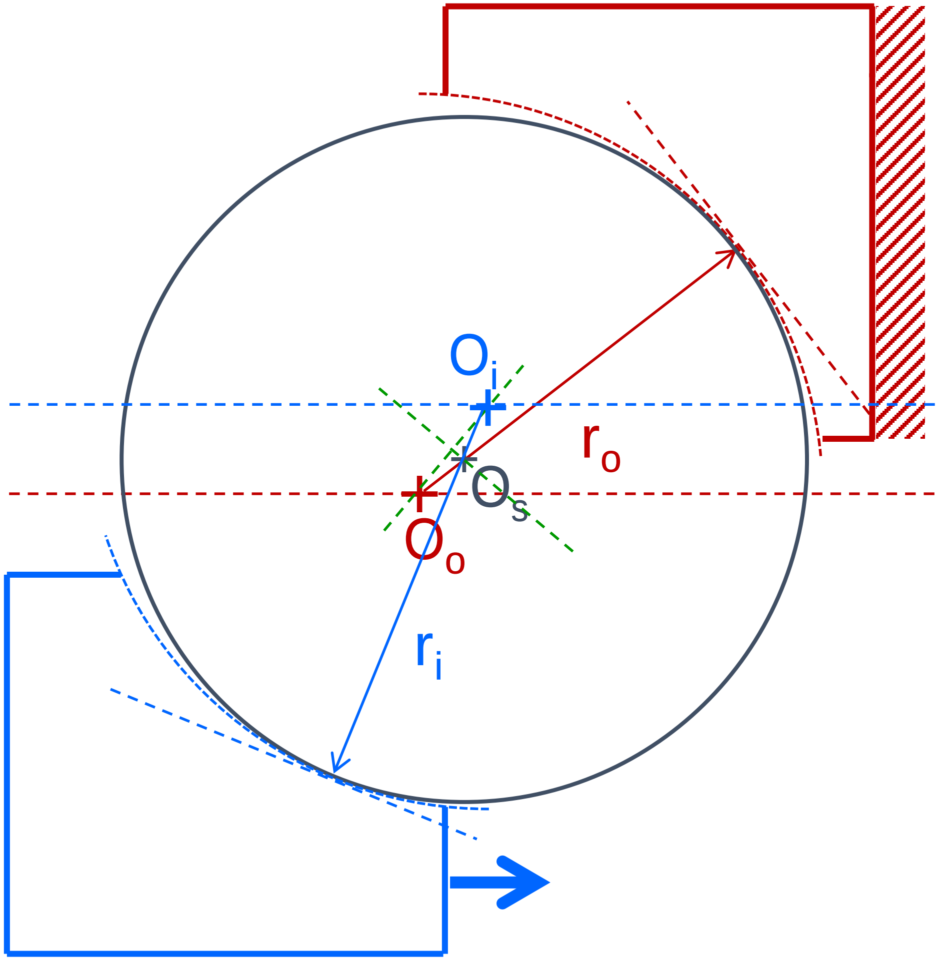
The problem will now be studied in the absence of centrifugal forces, and only the very simple case of purely axial force (and axial approach) will be examined.

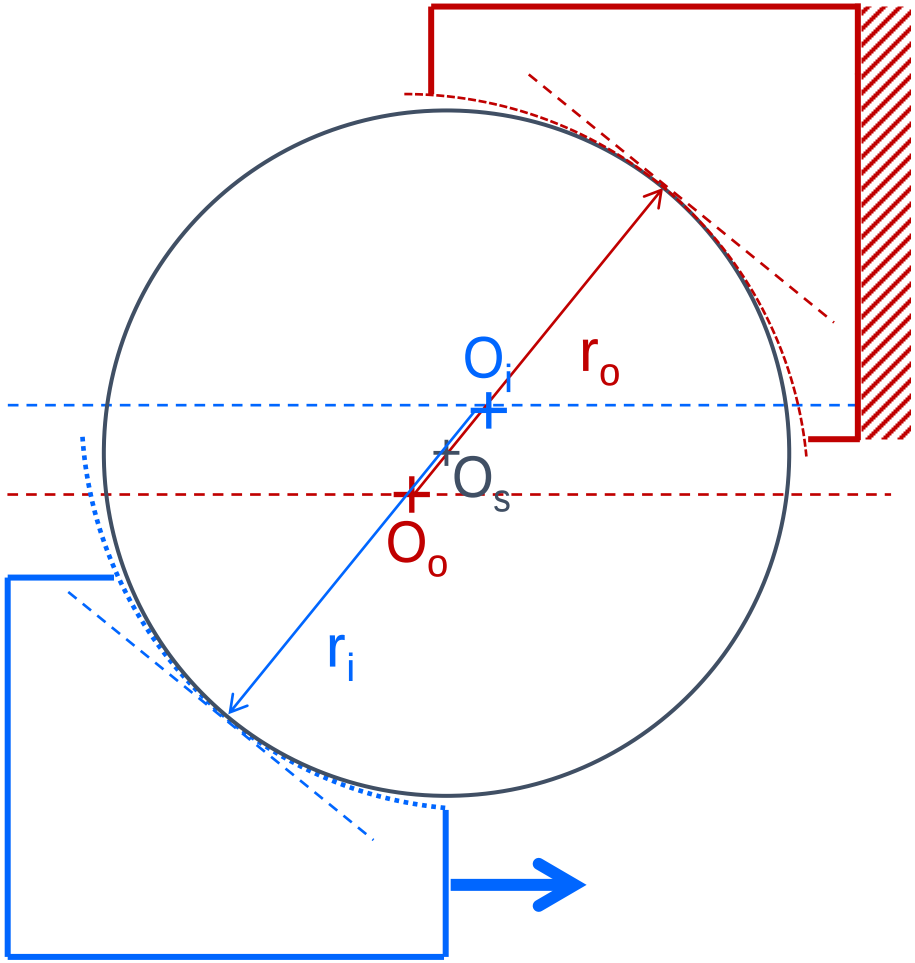
Relative motion is what matters, which is here represented by moving the inner ring to the right, while keeping the outer ring fixed*.

*Also the ball moves, however for ease of graphical representation its displaced position is not seen in this figure.

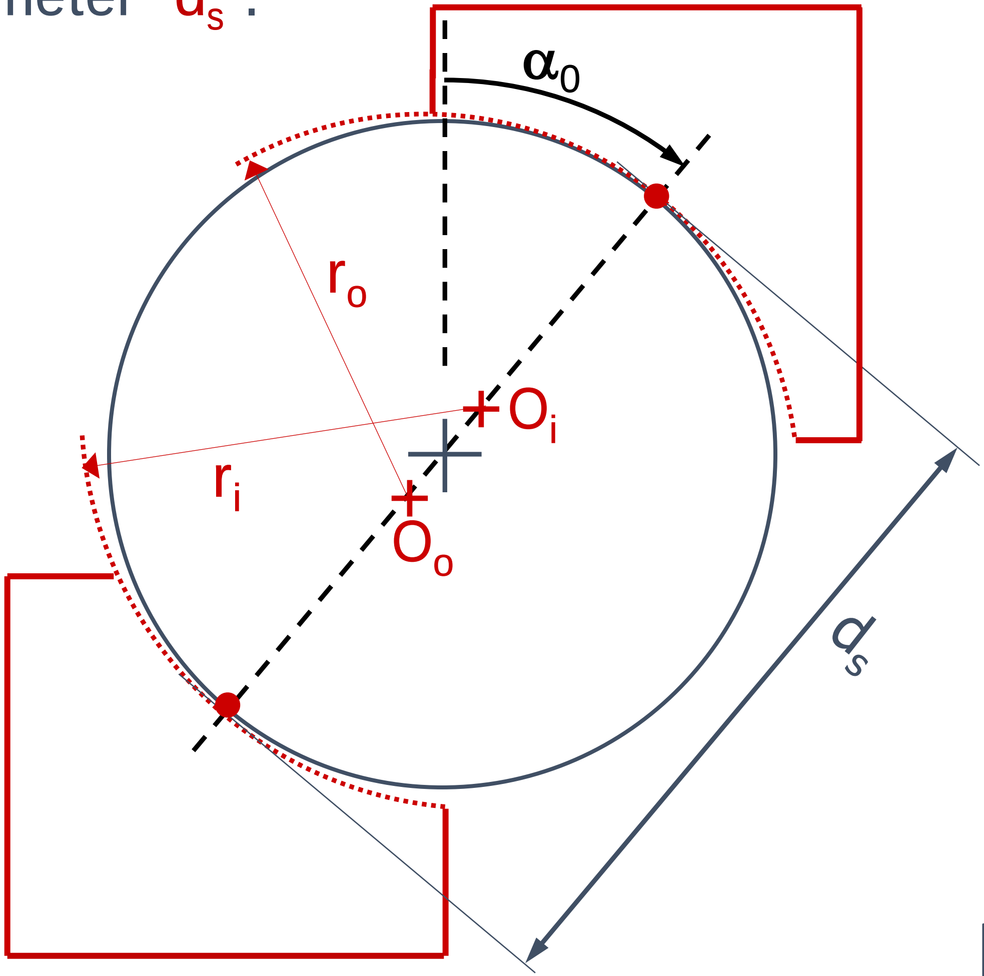
Ball center O_s must always lie on a raceway section radius; its distances from the contacts are equal, so it must stay on the axis of O_oO_i

The maximum possibile approach with no load aligns the ball diameter with the two radii of the raceways





Before the application of an axial force, which produces the axial approach, the two contact points are on an undeformed sphere with diameter d_s .

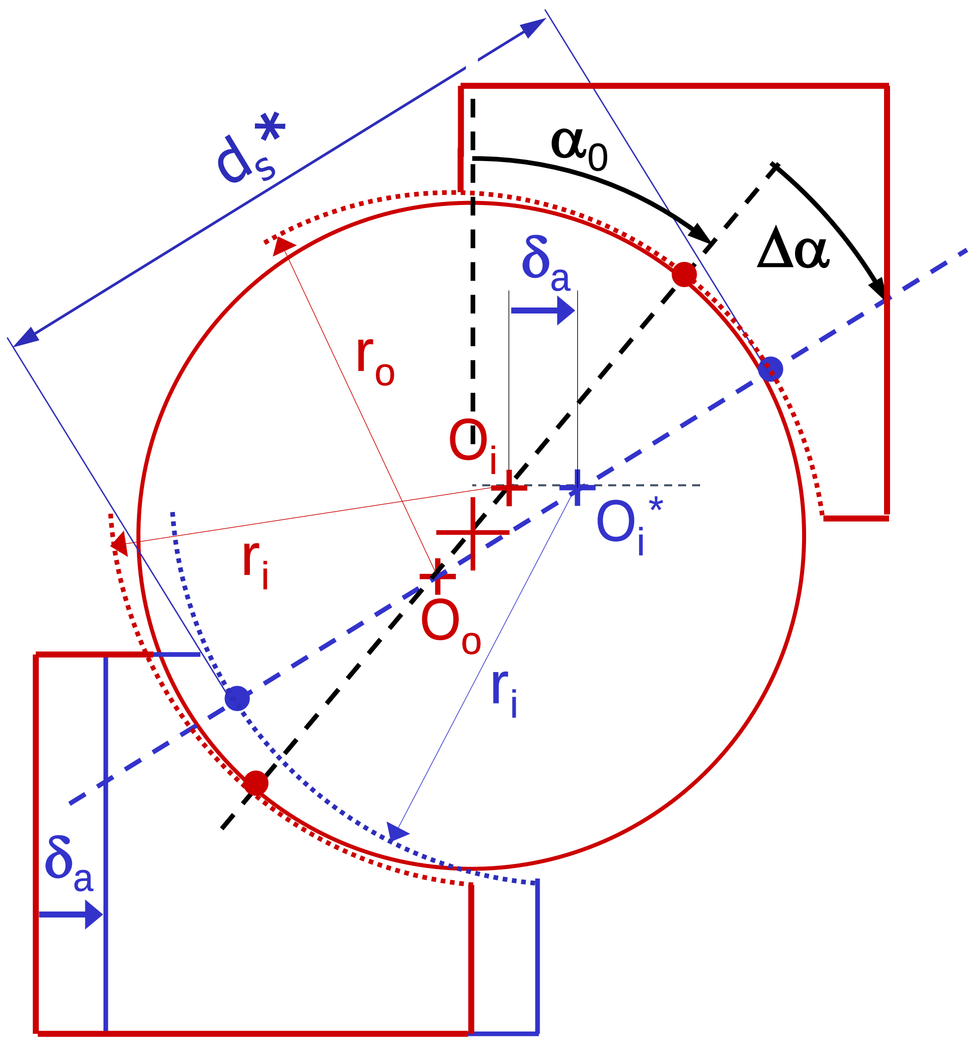


The centers of the outer and inner raceways, O_o and O_i , lie on a ball radius because the normal of ball and each raceway is common, and the radii are aligned on the same ball diameter because this allows the maximum axial relative approach with still no compressive load.

Then, before the further compressing axial approach and load:

$$\boxed{d_s = r_i - \overline{O_o O_i} + r_o}$$

The center O_i moves with the inner ring of the amount δ_a .



The axial approach is:

$$\overline{O_iO_i^*} = \delta_a$$

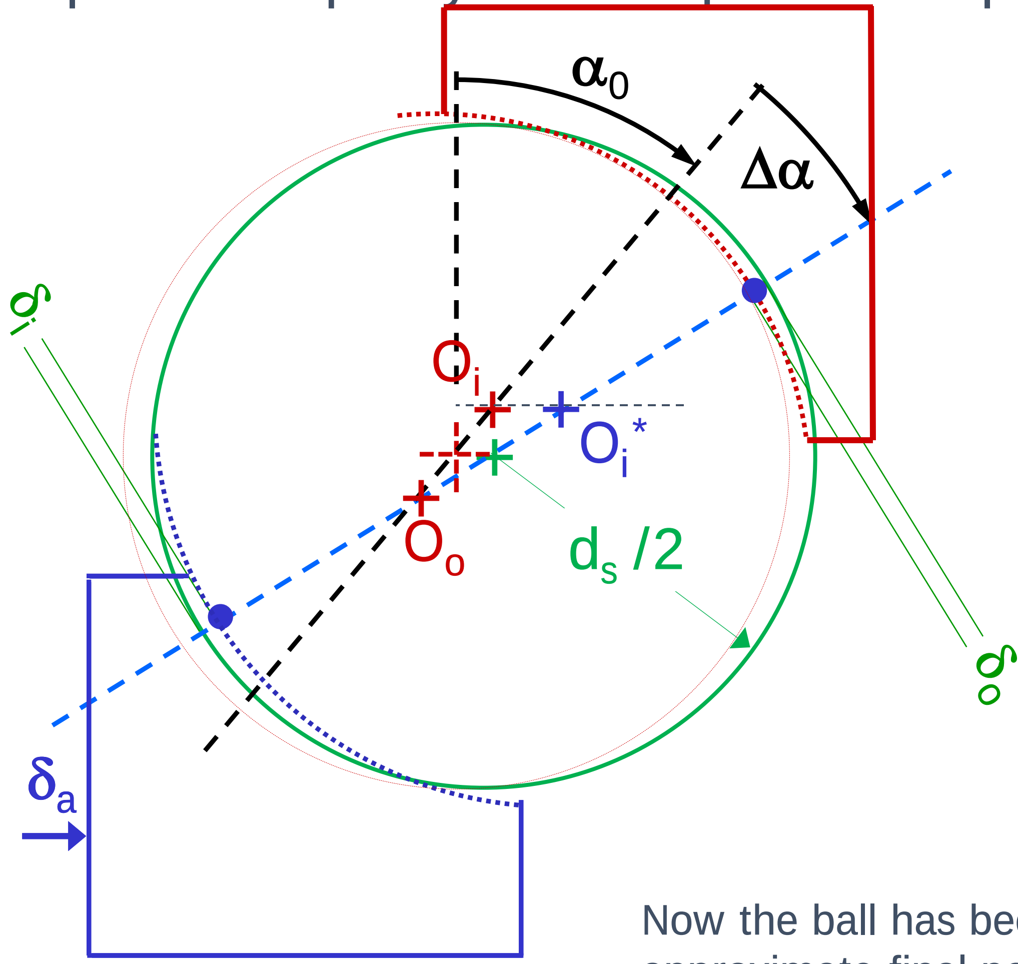
After the displacement: the contact line passes through O_o and O_i^* ; on this segment also the sphere center must lie.

After the displacement: d_s^* is the distance available to contain the (deformed) sphere.

$$\boxed{d_s^* = r_i - \overline{O_oO_i^*} + r_o}$$

In this figure the ball is shown in its initial position, the displaced position is not shown.

… the difference $\delta_n = d_s - d_s^*$ (diametral approach on the sphere) is the sum of the inner (δ_i) and outer (δ_o) contact elastic approaches, each produced partly on the sphere and partly on the raceway.



$$\delta_n \equiv \delta_i + \delta_o = d_s - d_s^*$$

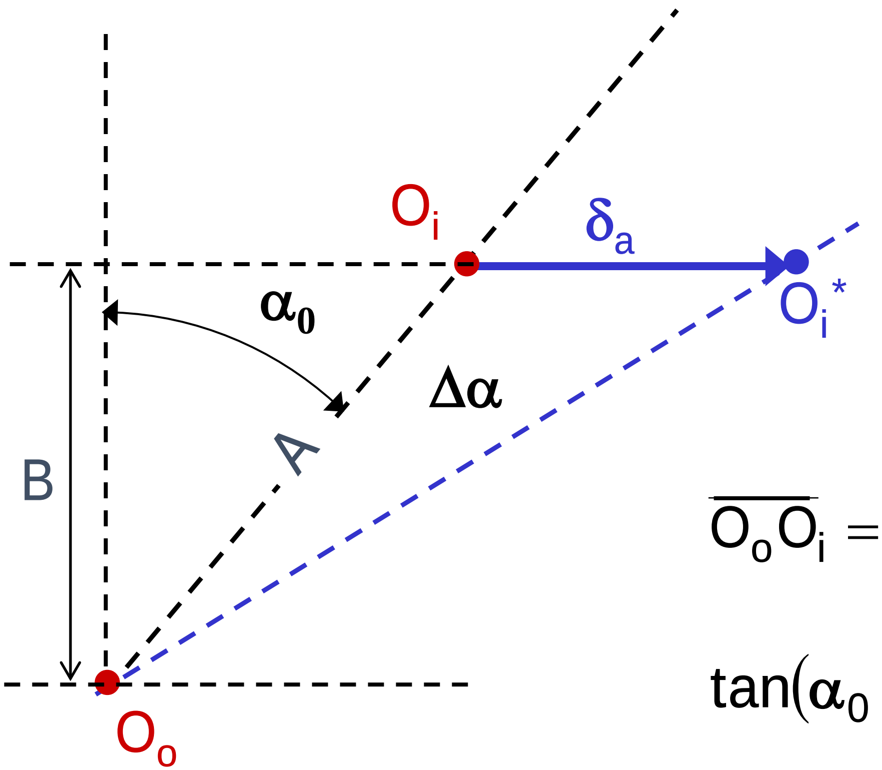
$$\boxed{d_s = r_i - \overline{O_oO_i} + r_e}$$

$$\boxed{d_s^* = r_i - \overline{O_oO_i^*} + r_e}$$

$$d_s - d_s^* = \overline{O_oO_i^*} - \overline{O_oO_i}$$

Now the ball has been represented with its center in the approximate final position + .

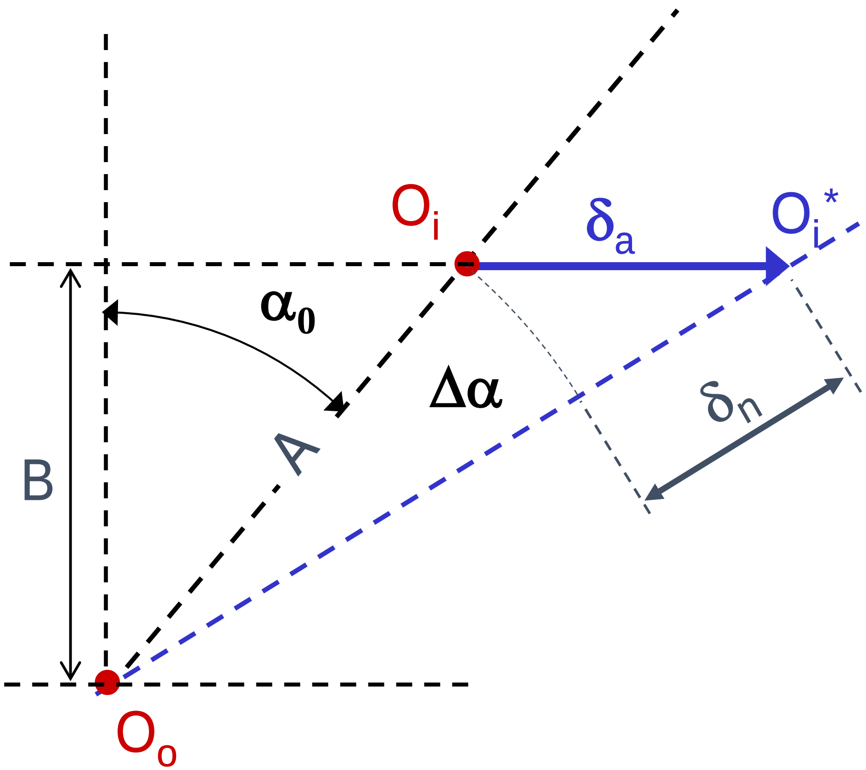
1 - The angle increment $\Delta\alpha$



$$\overline{O_o O_i} = A \qquad \cos\alpha_o = \frac{B}{A}$$

$$\tan(\alpha_0 + \Delta\alpha) = \left(\frac{B\tan\alpha_0 + \delta_a}{B}\right) \equiv \tan\alpha_0 + \frac{\delta_a}{B}$$

$$\Delta\alpha = -\alpha_0 + \arctan\left(\tan\alpha_0 + \frac{\delta_a}{B}\right)$$



2 - The total approach

$$\delta_n = \delta_i + \delta_o = d_s - d_s^* = \overline{O_oO_i}^* - \overline{O_oO_i}$$

$$\delta_n = \overline{O_oO_i}^* - \overline{O_oO_i} \equiv \overline{O_oO_i}^* - A =$$

$$= \frac{B}{\cos(\alpha_0 + \Delta\alpha)} - A$$

Please remark that B and A determine α_0 , and that with the usual radii for the raceways r_i and r_o approx. equal to the ball radius r_s times 1,05 then: $d_s = r_i - \overline{O_oO_i} + r_o \Rightarrow 2\,r_s \cong 2\,r_s\,1{,}05 - \overline{O_oO_i}$ hence: $\overline{O_oO_i} \cong r_s\,0{,}1$ Then α_0 is determined by choosing B.

Warning

In radial ball bearings the contact angle is zero under a purely radial load, and rises to a small α_0 under axial approach as shown in Sect. 5 of this Chapter. Under an axial load, the local deformations from ball-to-raceway contact cause relative axial displacement of the two rings and non negligible increase of the contact angle as a function of the applied axial load. The ratio A/C_0 is used to determine the value of Y and therefore take into account the modification of contact angle due to the axial force.

In angular contact bearings the contact angle is determined by construction and varies little, in relative terms, as a function of the combined loads. The axial load factor Y is therefore considered, in an initial approximation, as being constant. The angular contact ball bearings with an identical contact angle are calculated with the same load factor Y.

With tapered roller bearings, Y varies according to the series and dimension.

11. Angular bearings and axial pre-load (1/21)

Normally, bearings are used with a slight internal clearance under operating conditions. However, in some applications, bearings are given an initial load; this means that the bearings' internal clearance is negative before operation.

This is called "preload" and is commonly applied to angular ball bearings and tapered roller bearings, in which the clearance (and the interference) can be adjusted during assembly.

The main purposes and some typical applications of preloaded bearings are:

- To maintain the bearings in exact position both radially and axially and to suppress shaft runout
- To increase bearing rigidity
- To minimize noise due to axial vibration and resonance
- To prevent sliding between the rolling elements and raceways due to gyroscopic moments
- To maintain the rolling elements in their proper position with the bearing rings

Preloading Methods

1-Fixed position preload

A fixed position (or “located”) preload is achieved by fixing two axially opposed bearings in such a way that a preload is imposed on them. Their position, once fixed, remain unchanged in operation.

Three main methods are used to obtain a fixed position preload:

- By installing a duplex bearing set with previously adjusted ring axial dimensions and clearance
- By using a spacer or shim of proper size to obtain the required spacing and preload
- By utilizing bolts or nuts to allow adjustment of the axial preload; in this case, the torque should be measured to verify the proper preload

2-Constant-pressure preload

A constant pressure preload is achieved using springs to impose a controlled preload. In contrast with the other method this one is practically insensitive to thermal expansions of shaft and supports.

Paired or duplex bearings

Paired (or Duplex) single-row bearings, either or taper-roller, are bearing units supplied ready-to-mount. Paired (angular) bearings are used in applications where either the load capacity of single bearings does not meet the requirements, or a defined axial play has to overcome the time-consuming adjustment of bearing assemblies when multiple mounting is foreseen.

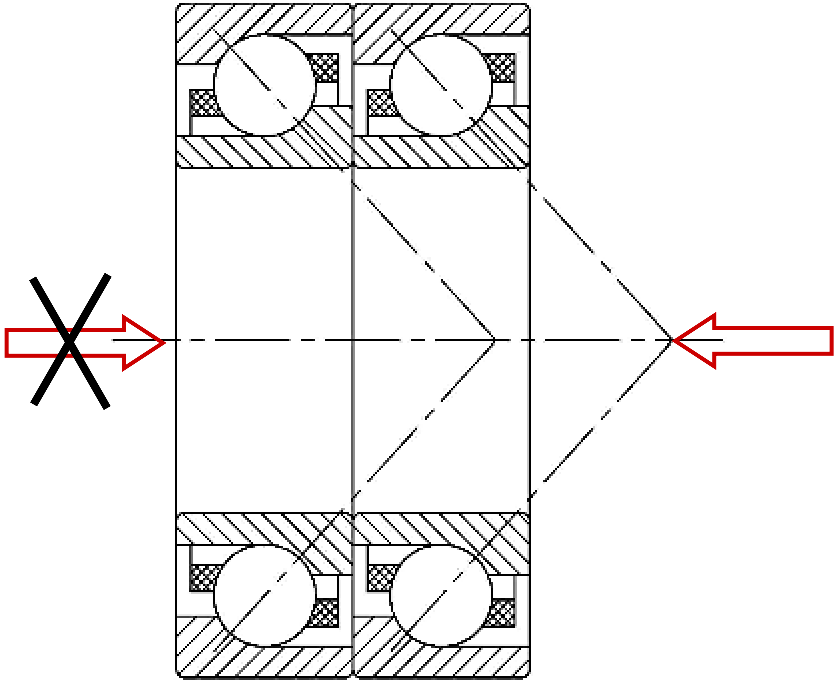
Paired bearings in tandem arrangements (DT: Duplex Tandem) are used where the thrust force exceeds the load capacity of a single bearing.

Single angular contact bearings or paired tandem (DT) units must always be adjusted against another bearing (or tandem unit) mounted in the opposite direction so that it can take either the axial force produce by a radial load on the bearing or a shaft axial force in the opposite direction.

This is done in two ways: back-to-back (“O” or “DB”) or face-to-face (“X” or “DF”).

11. Angular bearings and axial pre-load (4/21)

http://npb.cc/html/1855.html



The tandem bearing arrangement (also known as “DT arrangement”) is a type of duplex bearing arrangement where the two bearings are placed so that contact angle lines are parallel.

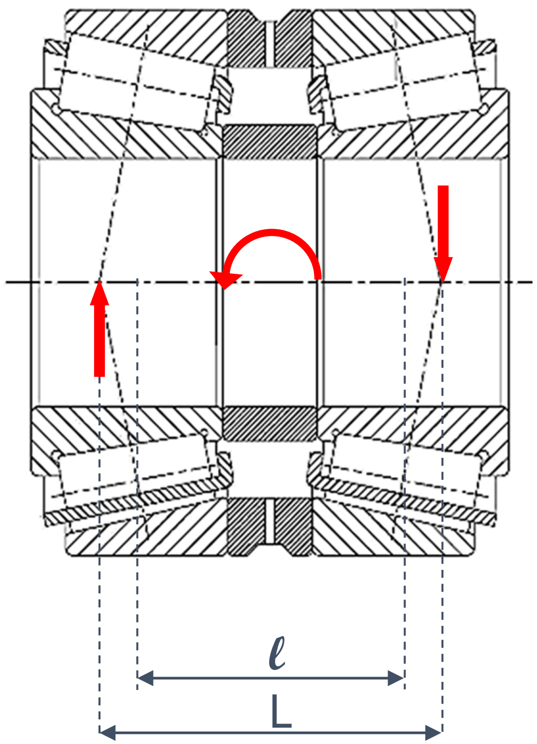
This arrangement makes a very heavy duty thrust bearing arrangement which accepts thrust loading in only one direction then, as said before, in case of thrust reverse or of a radial load*, it must be coupled with another bearing or another bearing set that takes the axial loads in the opposite direction.

*which produces a reverse axial load

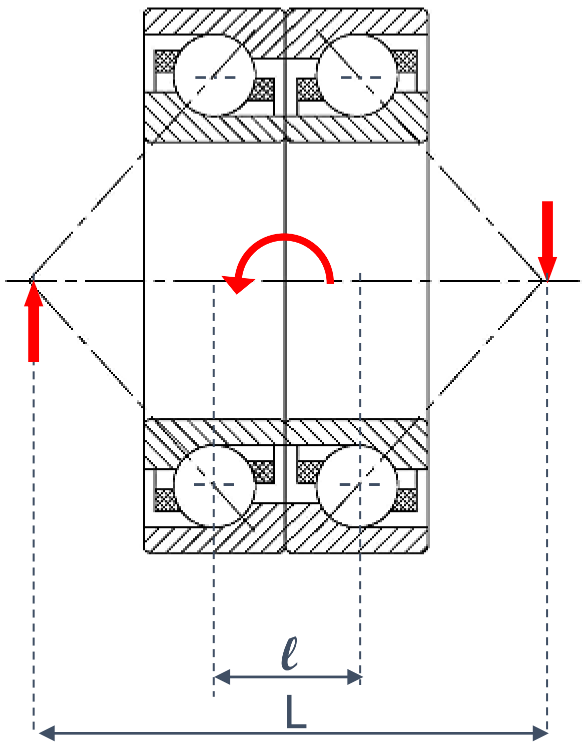
In DB arrangements, for both tapered roller and angular contact ball bearings, the distance L between the pressure centres is longer than the distance ℓ between the bearing centres.

Therefore, bearings arranged back-to-back are more suitable to support tilting moments.

http://npb.cc/html/1849.html



http://npb.cc/html/1855.html



They are unsuitable to compensate for misalignments. Additional forces due to misalignment will shorten the service life of taper roller bearings significantly.

The temperature increase of the shaft and inner rings will cause either an increase of axial clearance or a decrease in axial preload.

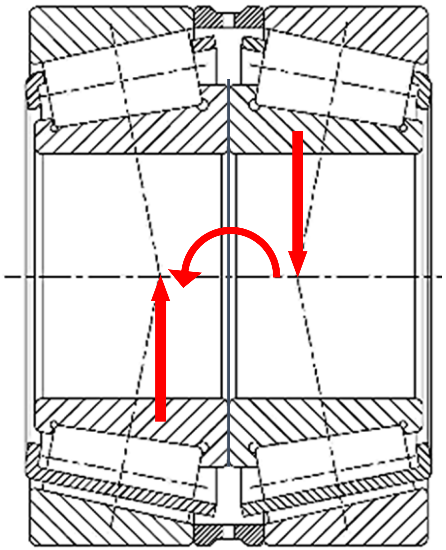
In DF arrangements the distance between the pressure centres is shorter than in DB assemblies.

Therefore face-to-face assemblies are less suitable to support tilting moments.

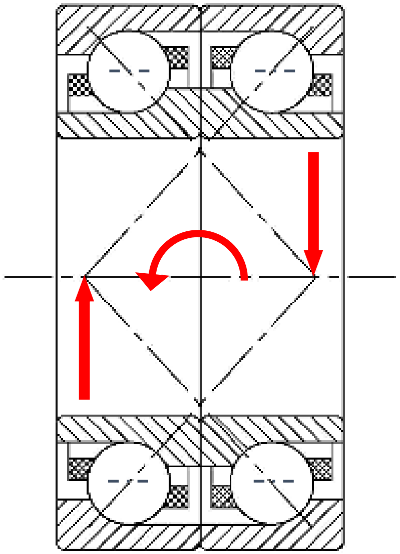
They have a lower angular stiffness.*

In exchange, they are more suitable to compensate for any misalignment.

http://npb.cc/html/1849.html

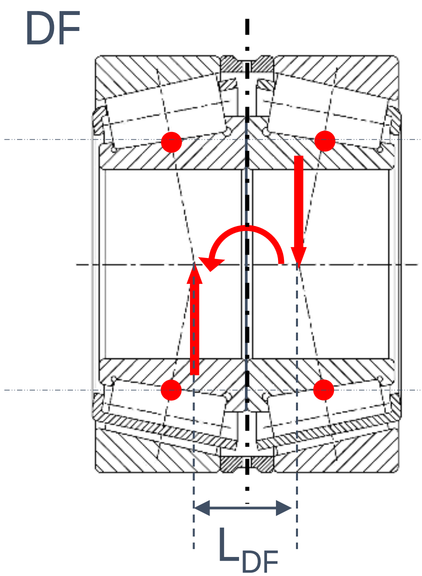


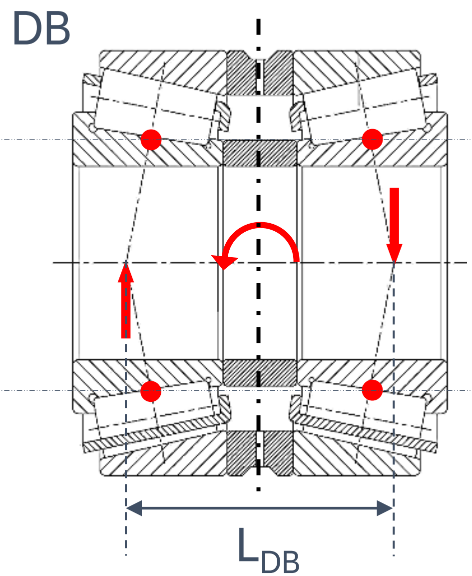
http://npb.cc/html/1855.html

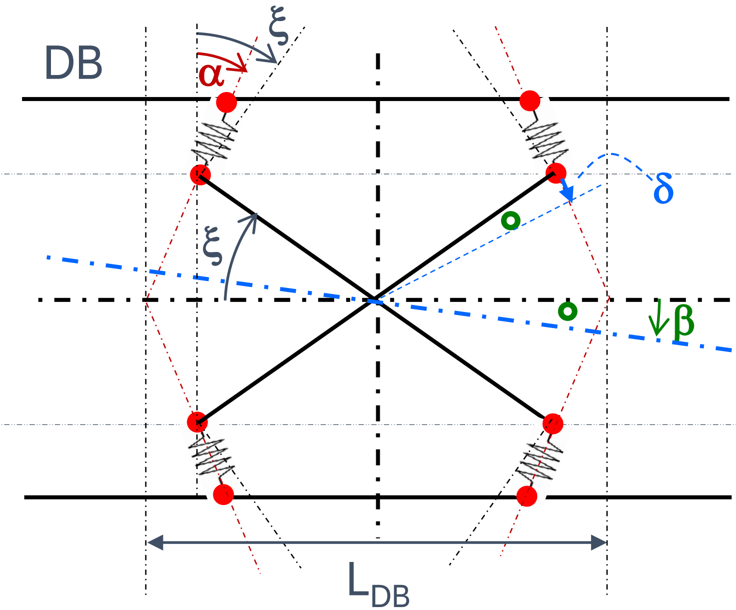


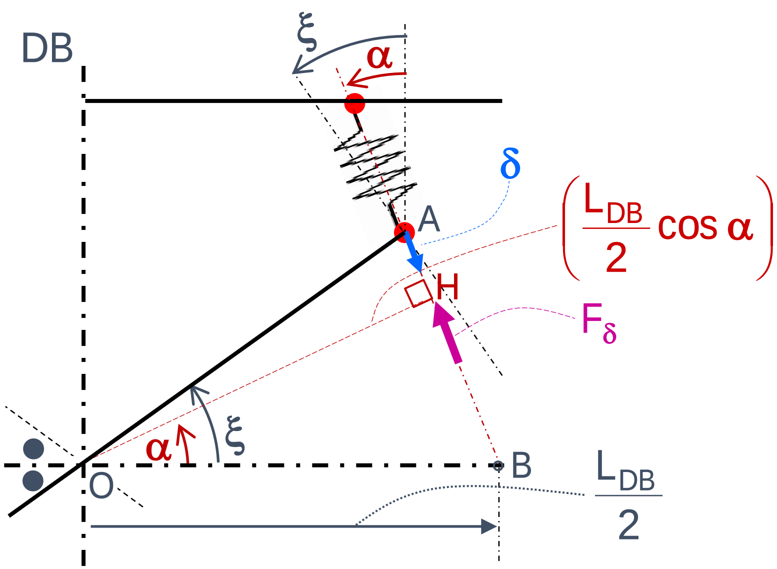
The temperature increase of the shaft and inner rings will cause either a reduction of axial internal clearance or an increase in axial preload.

* Bearing stiffness is evaluated comparatively for DB and DF in sl. 7, 8 of this Section.







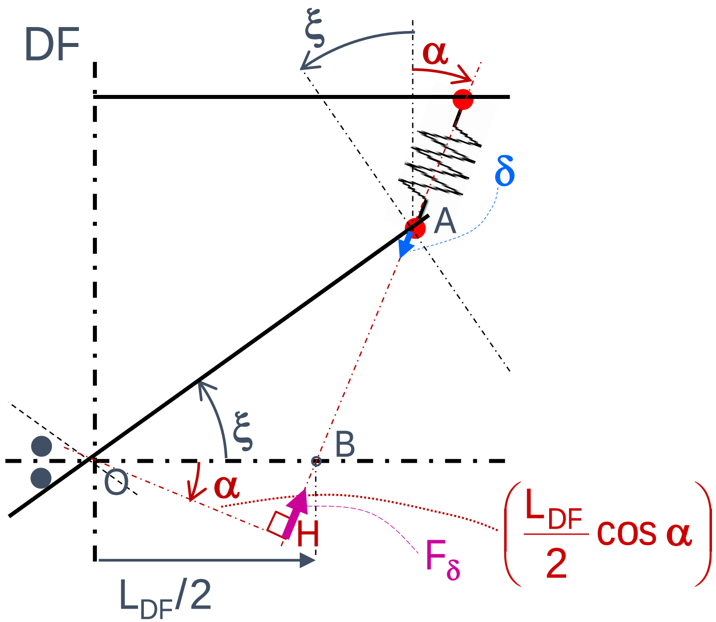


When line OA rotates (with the inner ring and the shaft) about point O by the very small amount β the component of the displacement along the spring axis AB is $\overline{OH}\cdot\beta$, equal to the total normal approach δ of the inner ring against the outer ring:

$$\delta = \beta\left(\frac{L_{DB}}{2}\cos\alpha\right)$$

and given the total roller/raceway stiffness **K**, the force F_δ variation at the single contact and the total tilting moment of four contacts are:

$$F_\delta = \mathbf{K}\,\delta \quad \text{and} \quad M = 4\,\mathbf{K}\left(\frac{L_{DB}}{2}\cos\alpha\right)\delta$$



The same formulas are obtained also for the DF case, consider the figure on the left, then for the same total roller/raceways stiffness **K**, the force F_δ variation at the single contact and the total tilting moment of four contacts are:

$$\delta = \beta\left(\frac{L_{DF}}{2}\cos\alpha\right)$$ approach-tilting angle

$$F = K\,\delta$$ force-approach

$$M = 4\,K\left(\frac{L_{DF}}{2}\cos\alpha\right)\delta$$ total tilting moment

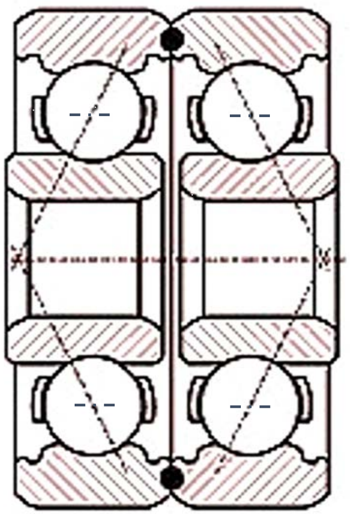
$$\chi = M/\beta = 4\,K\left(\frac{L_{DF}}{2}\cos\alpha\right)^2$$ tilting stiffness

If the bearings in both cases DB and DF have the same preload, the local stiffness **K** about that local contact force will be the same.

Moreover, the proportionality factor between the moment due to the reaction of the four contacts considered in the figure and the total moment due to all rollers will be the same for cases DB and DF.

The result boils down to something very simple: $\dfrac{\chi_{DB}}{\chi_{DF}} = \left(\dfrac{L_{DB}}{L_{DF}}\right)^2$

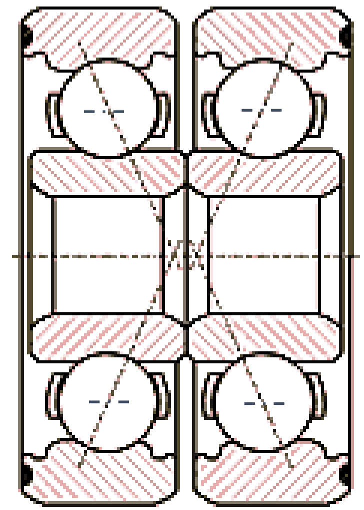
Deep groove ball bearings are also generally preloaded axially, to do so, the bearings should have a greater radial internal clearance than “Normal” (e.g. C3) so that, as with angular contact ball bearings, a contact angle which is greater than zero will be produced.



DB: the two bearings are placed so that contact angle lines of the bearing diverge inwardly.

These contact angle lines form ‘O’ shape, hence called:

“O arrangement”.



DF: the two bearings are placed so that contact angle lines of the bearing converge inwardly.

These contact angle lines form ‘X’ shape, hence called:

“X arrangement”.

Terminology explanation: the «face» is where the contact forces meet, the «back» is the opposite

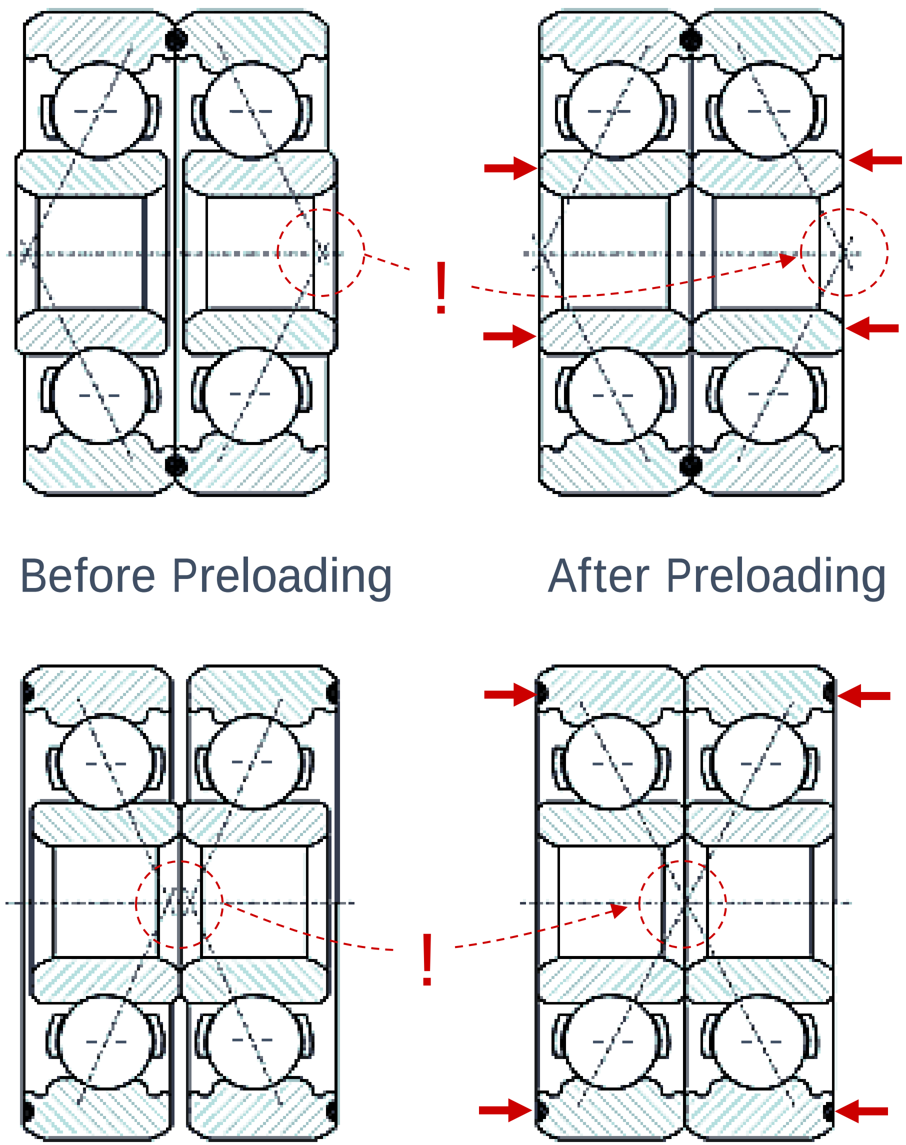
http://www.meadinfo.org/2013/01/duplex-bearing-arrangements-back-to-back-face-to-face-tandem.html

In this case the distance between the pressure centres is so small (in this case it becomes zero after application of the pre-load) that the bearing pair behaves like a hinge.

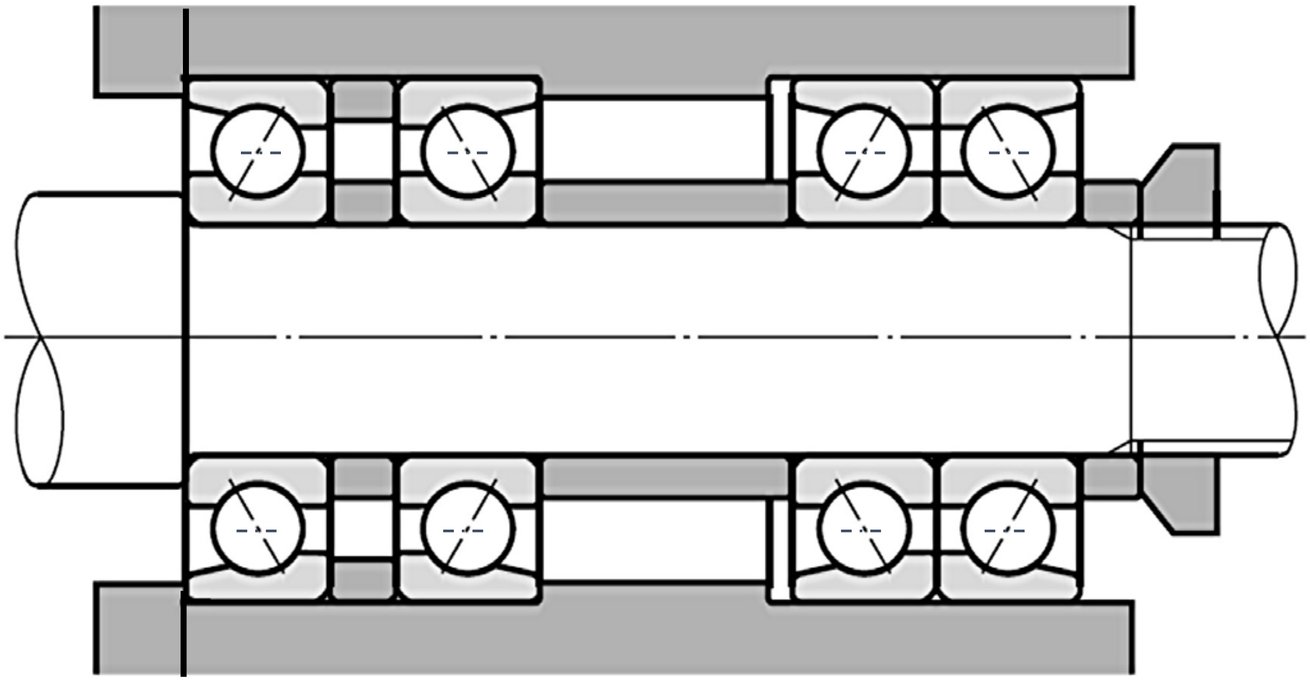
For deep-groove ball bearings, these assembly configurations are used for applications where no bearing clearance can be tolerated and where a high degree of stiffness of the whole assembly is required.

Pairing is achieved with bearings of identical tolerance classes and by applying an axial preload to the inner rings for the O arrangement and on the outer rings for the X arrangement.

Axial tolerances on inner or outer rings will regulate the amount of pre-set preload when rings are pressed one against another during assembly. In the cases of sl.3, 4 of this Section the same effect was obtained through calibrated spacing rings.



From http://www.wib-bearings.com/en/tech/tech_13.htm adapted

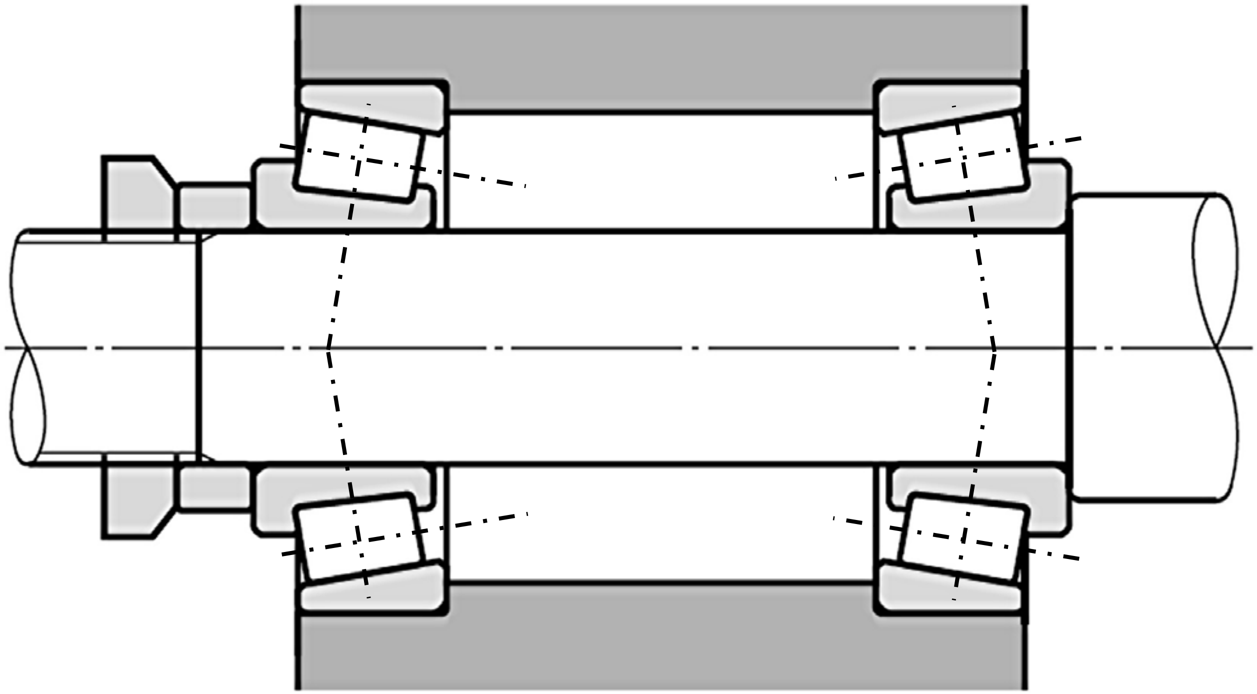


Fixed position preload

In this example of assembly the pair of angular bearing on the right is a preloaded DB duplex unit based on a predetermined offset of the rings.

The pair on the left is preloaded by adjusting the thickness of the spacers.

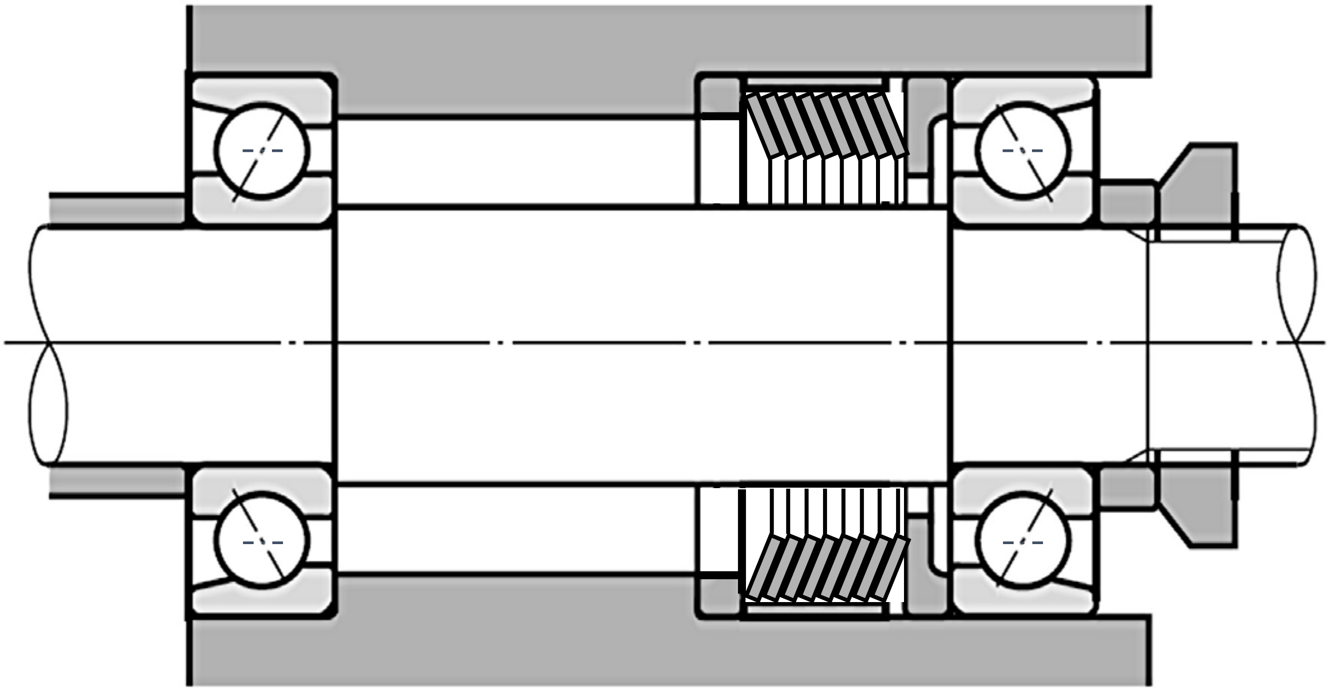
Figures in sl.9, 10, 11, 12 from Chapter 8 “Bearing Internal Clearance and Preload” at http://www.ntn.co.jp/english/products/catalog/bearing/rolling/index.html



Fixed position preload

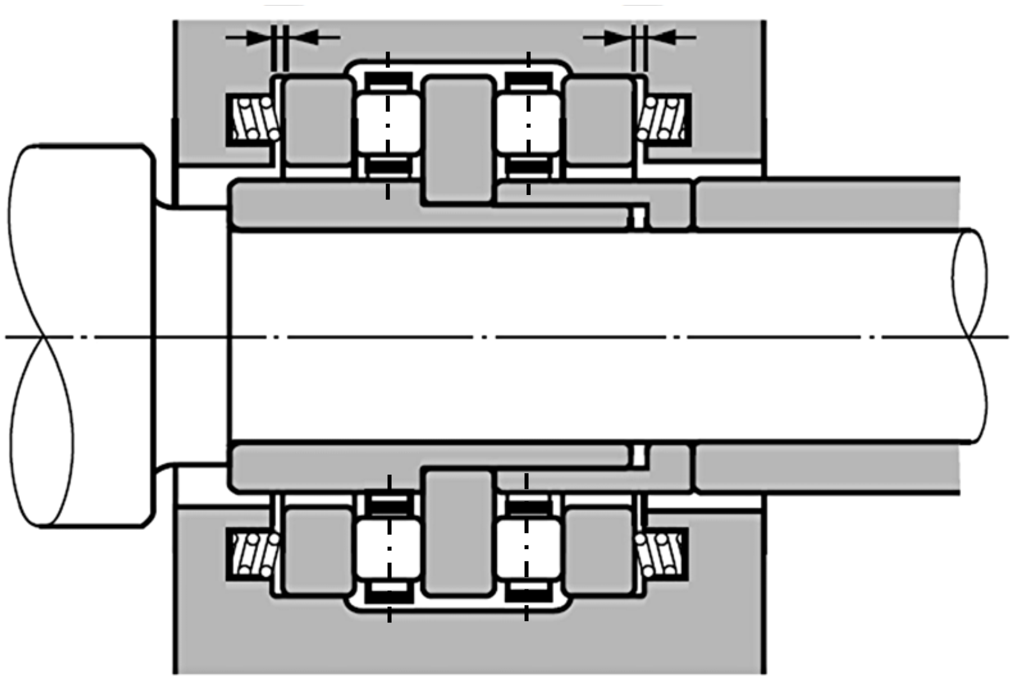
In this example of a tapered roller bearings the desired amount of preload is applied by adjusting a threaded screw, and by measuring the torque or the axial displacement.

Attention! Applying excessive preload could result in reduction of life, abnormal heating, increase in turning torque.



Constant pressure preload

In this example the two opposite angular ball bearings, in a O-type arrangement, the desired amount of preload is applied by a set of Belleville springs. The high compliance of the springs keeps the axial preload constant even in the case of severe axial thermal expansions. The disadvantages of these solutions are that the designs are more complex and normally have lower stiffnesses.



Constant pressure preload

This example for cylindrical roller thrust bearings, is also applied to spherical roller thrust bearings and thrust ball bearings.

Preload is applied by a set of helical coil springs contained in appropriate recesses. Preload is used to prevent smearing of one bearing (due to sliding) when the axial load is taken by the other.

Amount of axial pre-load: it is controlled through the axial displacement allowed by a pre-determined axial gap between inner rings (DB pairs) or outer rings (DF pairs). Its determination requires the knowledge of the non-linear (axial) force-displacement curve.

The calculation algorithm runs as follows:

1 – preparatory work

$$\left.\begin{array}{l}\alpha_X,\ \alpha_Y,\ \beta_X,\ \beta_Y \\ E,\ \nu\end{array}\right\} \quad \alpha_X,\ \alpha_Y,\ \beta_X,\ \beta_Y \Rightarrow \cos\tau = \frac{\left|(\alpha_X - \alpha_Y) + (\beta_X - \beta_Y)\right|}{\alpha_X + \alpha_Y + \beta_X + \beta_Y} \Rightarrow \boxed{a^*,\ b^*,\ \delta^*}$$

(Ch.1 Sect.4 sl.9) or (Ch.1 Sect.10 sl.4)

$$\delta_o = \boxed{\left[\delta_o^*(\alpha_X + \alpha_Y + \beta_X + \beta_Y)_o^{1/3}\left(\frac{3}{2}\frac{1-\nu^2}{E}\right)^{2/3}\right]} F^{2/3} \equiv \boxed{K_o^{-2/3}} F^{2/3}$$

$$\delta_i = \boxed{\left[\delta_i^*(\alpha_X + \alpha_Y + \beta_X + \beta_Y)_i^{1/3}\left(\frac{3}{2}\frac{1-\nu^2}{E}\right)^{2/3}\right]} F^{2/3} \equiv \boxed{K_i^{-2/3}} F^{2/3}$$

(Ch.1 Sect.4 sl.8)

$$\boxed{K_{tot}} = \left[K_i^{-2/3} + K_o^{-2/3}\right]^{-3/2} \quad \text{(Ch.2 Sect.3 sl.2)} \quad \Rightarrow \quad F = K_{tot} \cdot \delta_{tot}^{3/2}$$

$$\left.\begin{aligned} a_i &= \boxed{a_i^* \left(\frac{3}{2(\alpha_X + \alpha_Y + \beta_X + \beta_Y)_i} \frac{1-\nu^2}{E} \right)^{1/3}} F^{1/3} \\ b_i &= \boxed{b_i^* \left(\frac{3}{2(\alpha_X + \alpha_Y + \beta_X + \beta_Y)_i} \frac{1-\nu^2}{E} \right)^{1/3}} F^{1/3} \end{aligned}\right\} \quad \text{(Ch.1 Sect.4 sl.8)}$$

2 - force-displacement curve

$$\begin{cases} \delta_a = \text{assign value} \\ \delta_r = 0 \end{cases}$$

$\delta_0 = f(\delta_a)$ (Ch.2 Sect.10 sl.7)

$F_0 = K_{tot} \cdot \delta_0^{3/2}$ (Ch.2 Sect.3 sl.2)

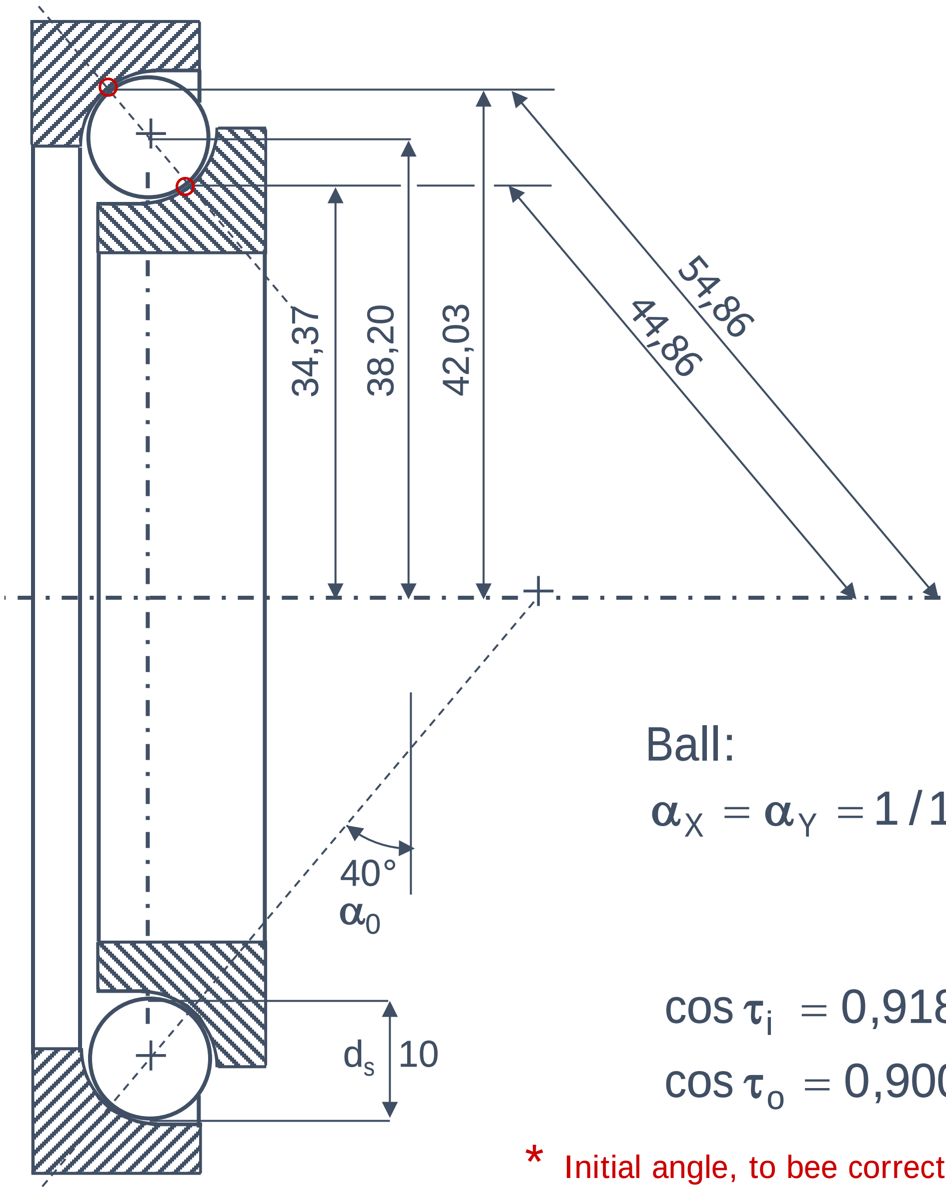
$$\begin{cases} A = Z\, F_0 \sin\alpha \\ R = 0 \end{cases}$$ (Ch.2 Sect.7 sl.14)

3 - contact pressure p_{max}

$$p_{max,i} = \frac{3}{2} \frac{F_0}{\pi a_i b_i}$$ (Ch.1 Sect.4 sl.7)

at the most stressed contact i.e. on the inner ring

Warning: contact angle and ensuing contact radii must be corrected with the axial approach according to Sect.10 sl.6 of this Chapter



Geometrical parameters are defined as in Sect.2 with the data shown for the example ball bearing on the left.

$Z=12$ number of balls

$\alpha_0=40°$ *

$E= 2 \cdot 10^5$ MPa

$\nu= 0,3$

Ball:

$\alpha_X = \alpha_Y = 1/10$

Inner ring:

$\beta_{Xi} = +1/89,73$

$\beta_{Yi} \cong -1/10,50$

Outer ring:

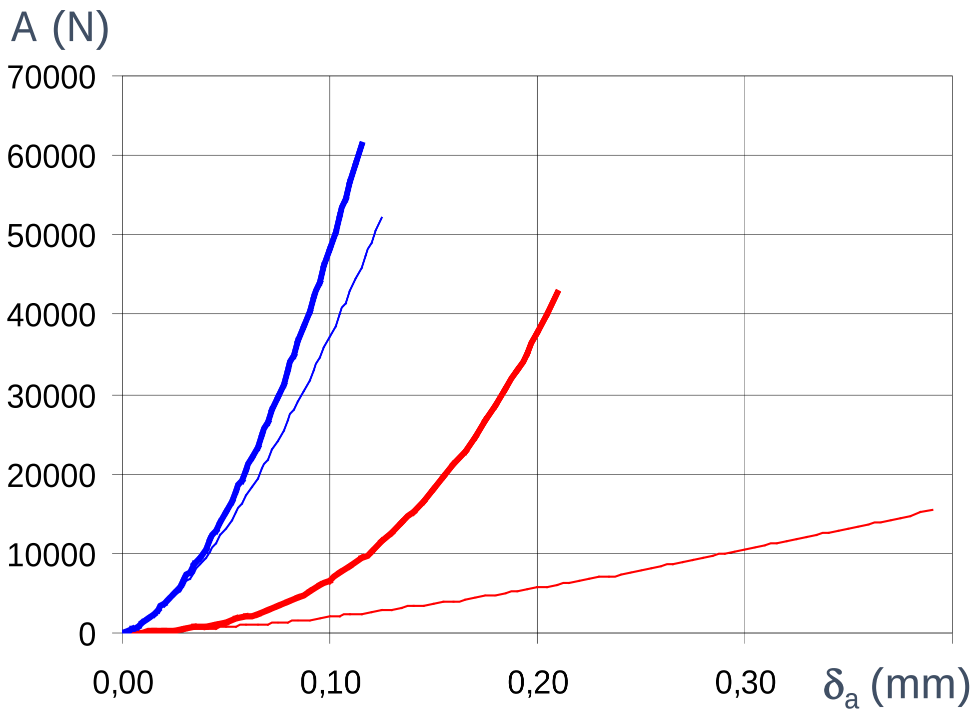
$\beta_{Xo} = -1/109,7$

$\beta_{Yi} \cong -1/10,50$

$\cos\tau_i = 0,918$ $\quad a^* = 3,37$ $\quad b^* = 0,440$ $\quad \delta^* = 0,646$

$\cos\tau_o = 0,900$ $\quad a^* = 3,10$ $\quad b^* = 0,460$ $\quad \delta^* = 0,678$

* Initial angle, to bee corrected with axial approach according to Sect.10 sl.6 of this Chapter



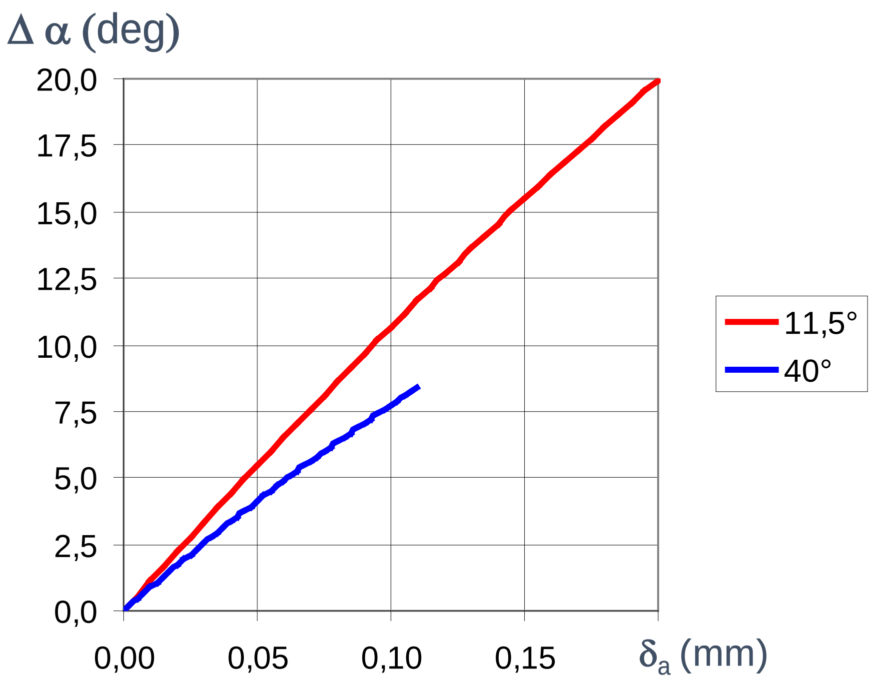
The figures on this page compare two bearings: the angular contact bearing with α_0=40° of the previous page and the deep groove radial bearing of Sect.5 sl.7 of this Chapter, α_0=11,5° .

Solid lines are calculated taking into account the angle increase $\Delta\alpha$ according to Sect.10.

Neglecting this angle increase produces a lower axial stiffness than in reality. The effect is more evident when the initial angle is lower.

All curves are calculated up to a maximum contact pressure p_{max}=4000 MPa, corresponding to the last point of the curve.

This figure shows the axial force A vs. the axial displacement of inner ring agaist the outer ring. It is clear that, particularly for radial deep groove bearings, the solution without angle correction lead to wrong results (thin lines).

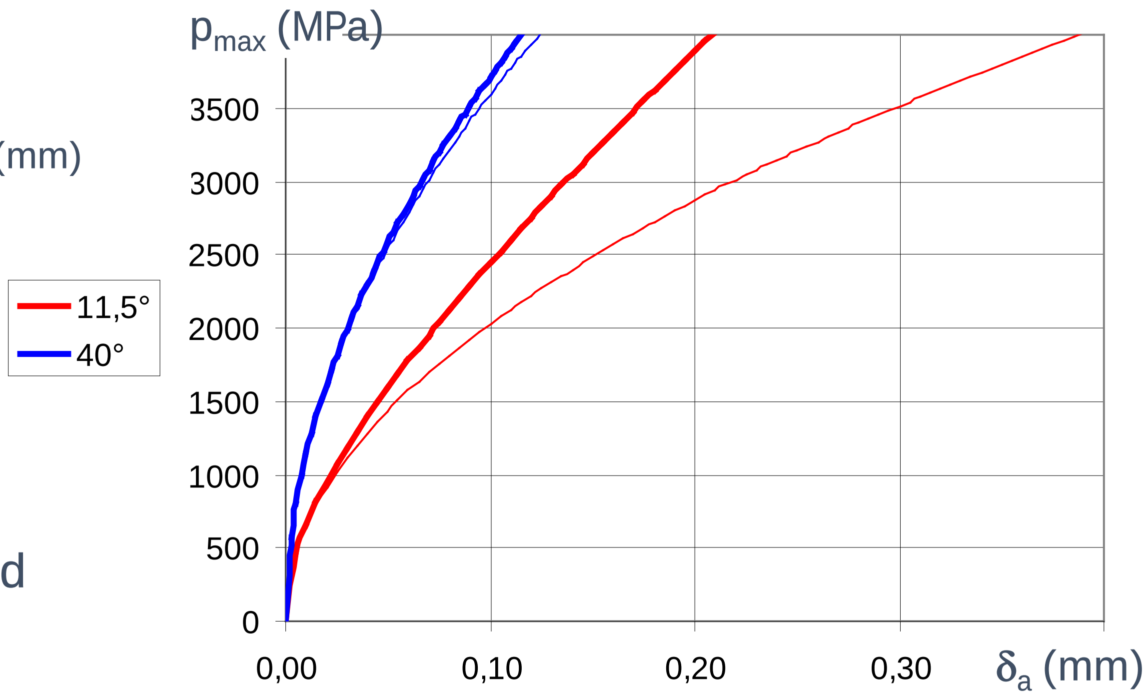


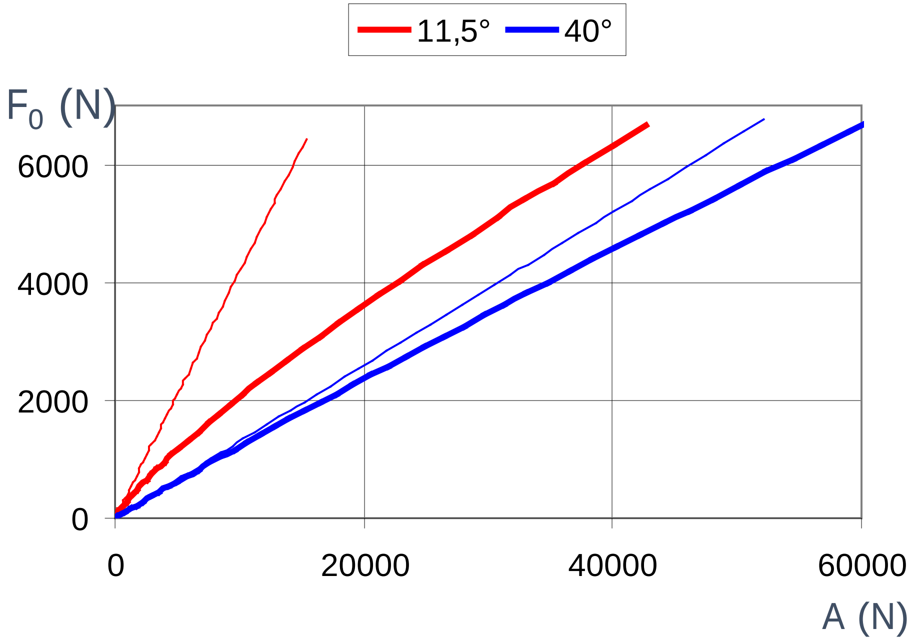
The figure on the left shows the increase $\Delta\alpha$ of the initial contact angle α_0 due to the axial approach as in Sect.10.

The large angle variation justifies the results of the previous slide: the increase of angle from α_0 to $\alpha_0+\Delta\alpha$ makes the bearings axially stiffer.

On the right we see the maximum contact pressure at the inner ring contacts vs. the axial approach of inner ring against outer ring.

Once again, the thin lines are incorrect (no angle change), and are shown to indicate the error due to omission of change.





This figure shows the relation between the force F_0 at the contacts and the total axial force A.

The thin lines show the linear relation according to Sect.7 sl.2.

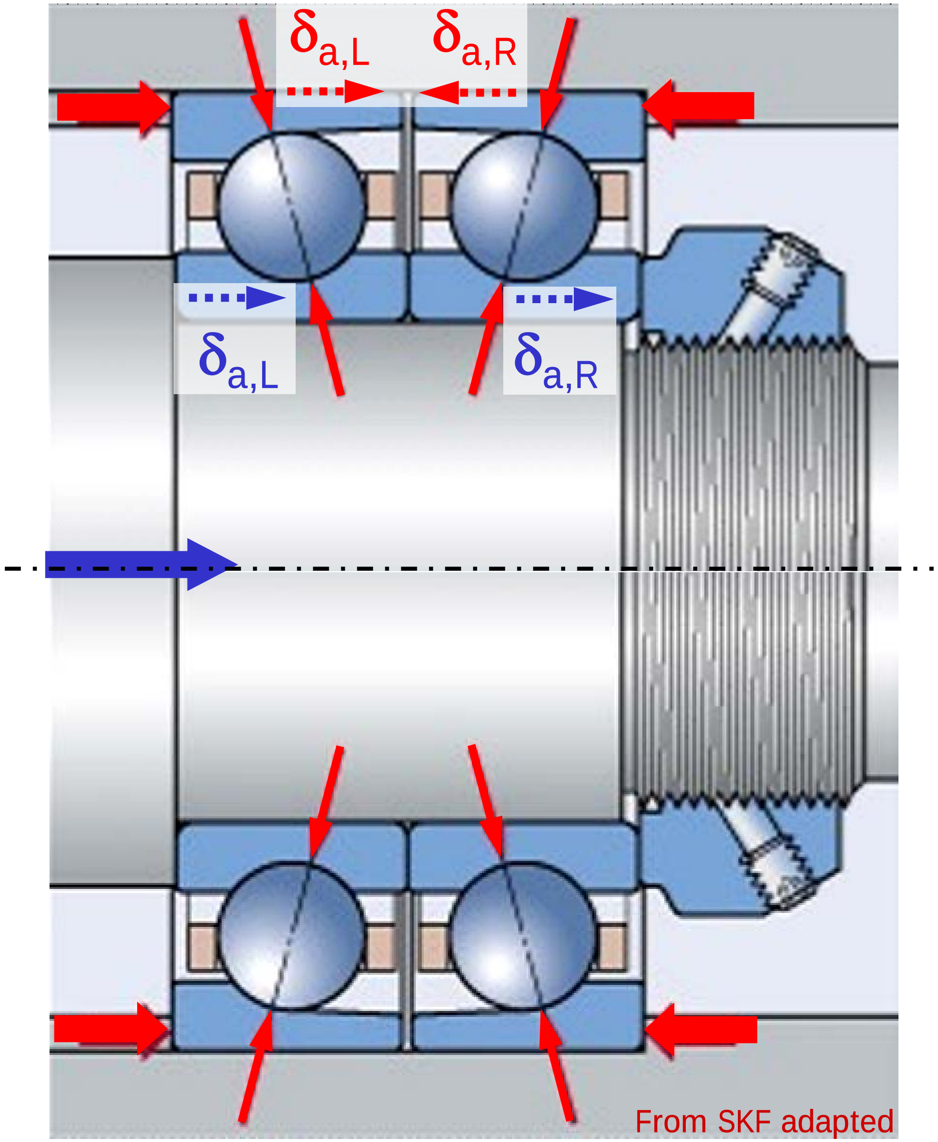
However, the increase of contact angle with axial force and approach produces a deviation, shown by thick lines, which is particularly large for deep groove bearings starting with a lower α_0.

Optimum Preload

Optimum Preload is normally recommended after calculating the optimum operating surface stress at the contact ellipse between the ball and raceway.

- If the noise life requirement is over 10,000 hours, the preload can be calculated based on an optimum surface contact (mean) stress that does not exceed 800 MPa.
- For general applications with a noise life requirement between 5,000 and 10,000 hours, the optimum preload can be calculated using a contact (mean) stress that does not exceed 1000 MPa.
- For stiffness critical applications requiring an operating noise life of less than 5,000 hours, a mean surface stress of less than 1500 MPa should be used.*

* source: Bearing Preload Facts and Information – NMB http://www.nmbtc.com/bearings/engineering/preload.html



In this Section we shall learn the rationale behind preloading angular bearings, either ball or tapered rollers.

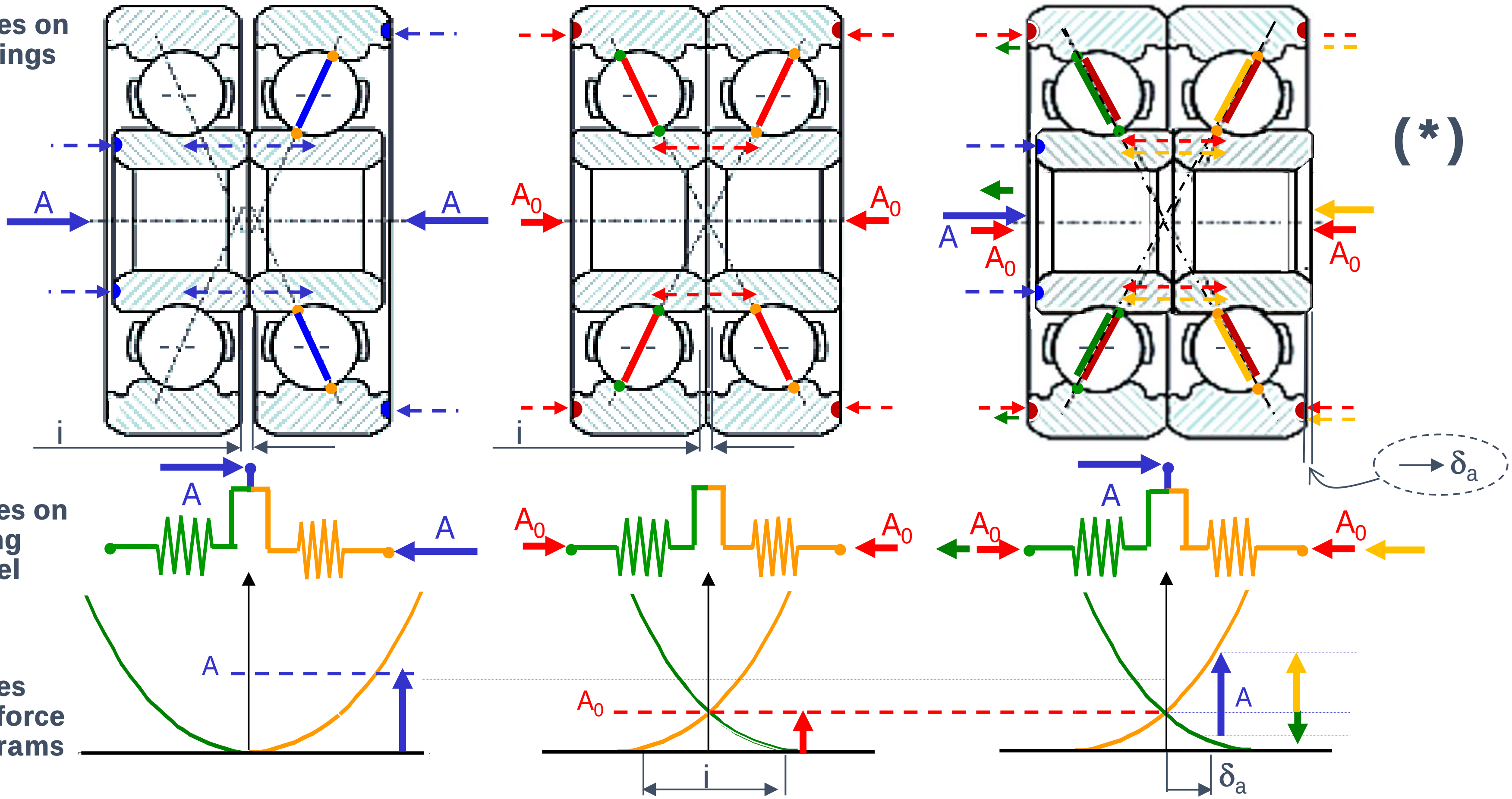
The worked out example concerns the case of “X” or “DF” assembly and “fixed position” preloading, where the preload A_0 is applied by driving into contact the outer rings through opposite displacements $\delta_{a,L} = \delta_{a,R} \equiv \delta_a$

The shaft shoulder transmits an axial force A to the pair of inner rings, which move together axially by the same amount $\delta_{a,L} = \delta_{a,R} \equiv \delta_a$.

Forces on bearings

Forces on spring model

Forces and force diagrams

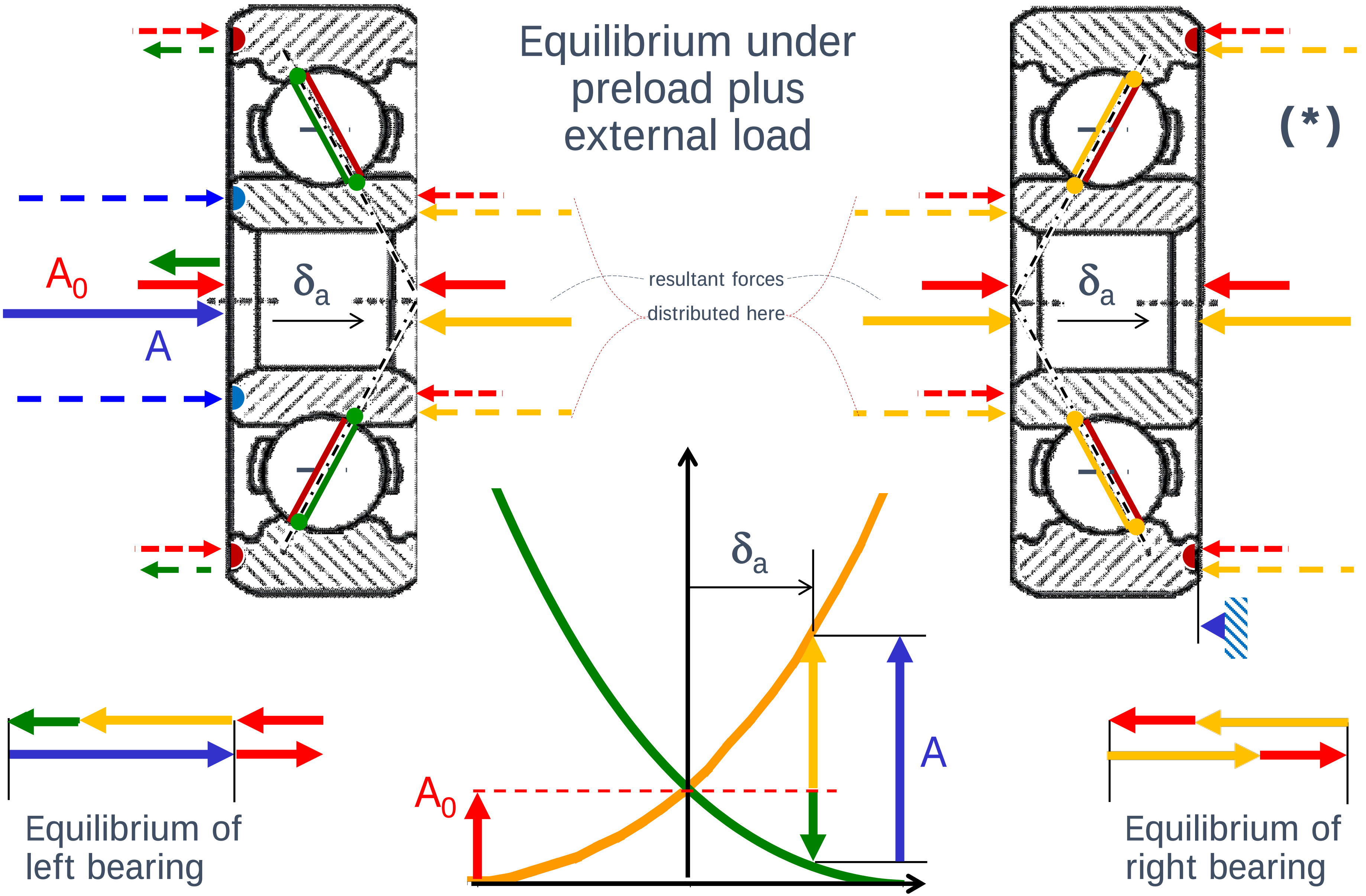


Here the controlled gap "i" between the outer rings will produce the interference when pressed. The force A on the inner rings loads only the right bearing balls.

Here the gap is closed by compressing the outer rings. The preload A_0 passes through the right and left balls and loads the inner rings one against the other.

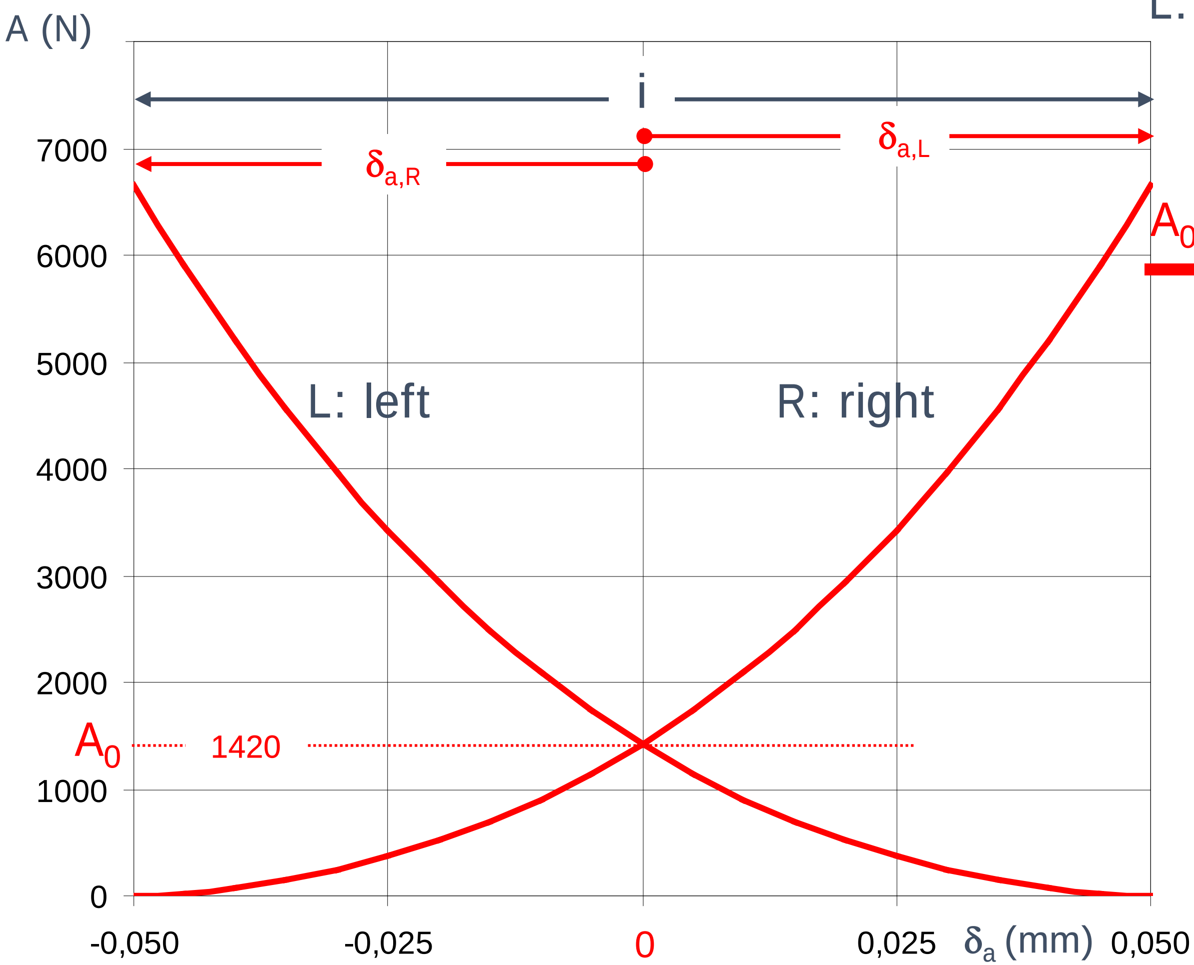
Here the force A applied to the inner rings is taken by the right and left rows of balls. They take also the axial preload A_0. Notice the inner rings moving to the right.

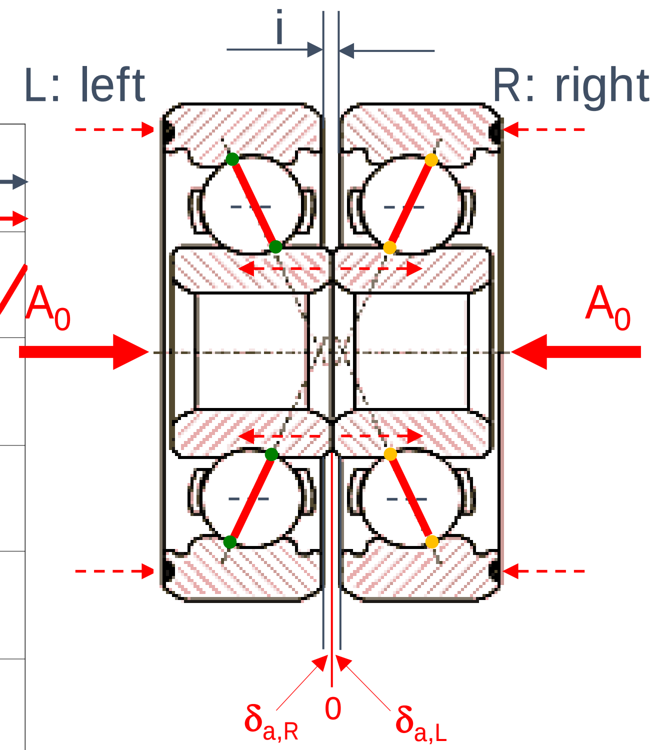
Equilibrium under preload plus external load



Equilibrium of left bearing

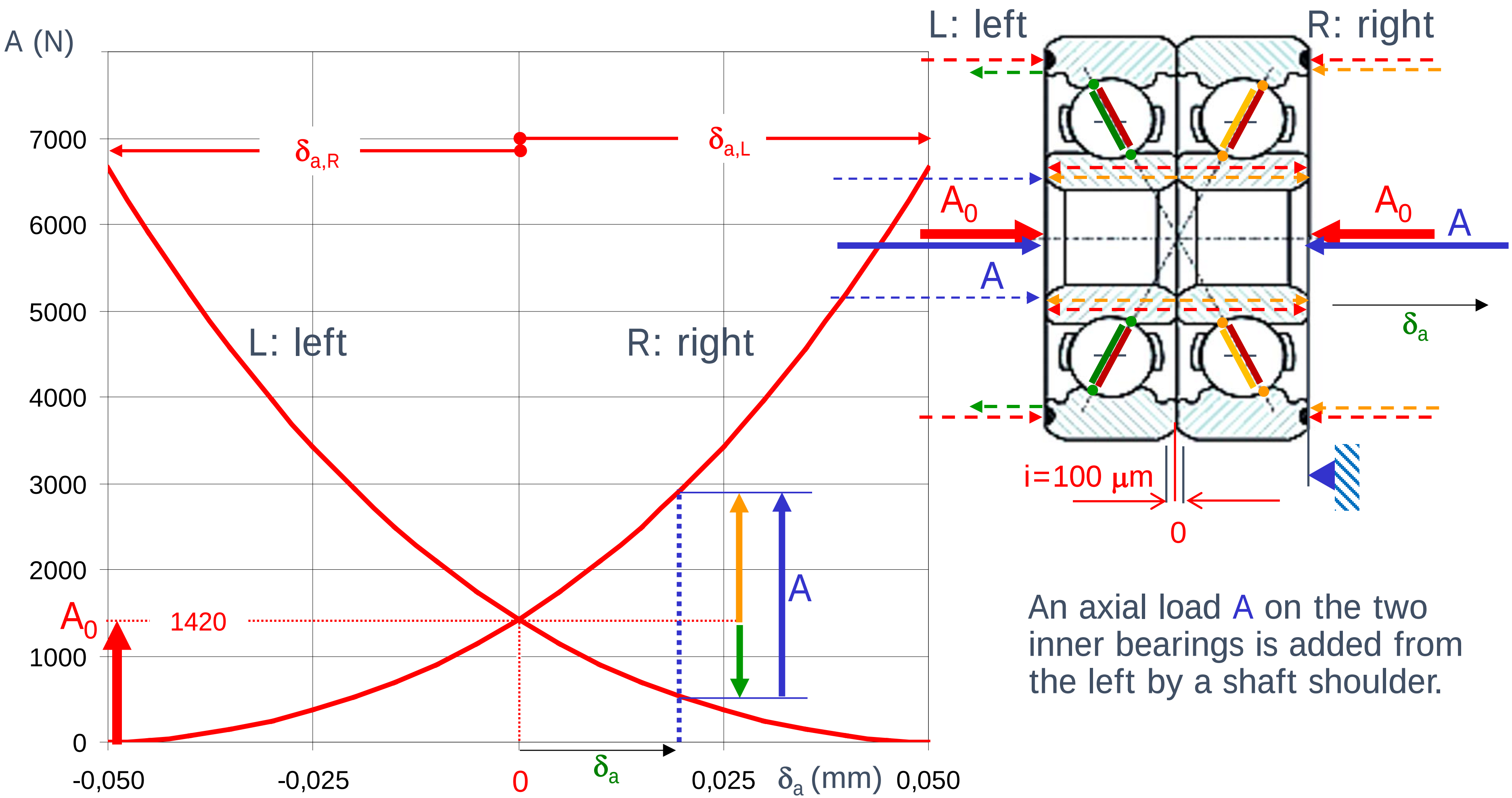
Equilibrium of right bearing





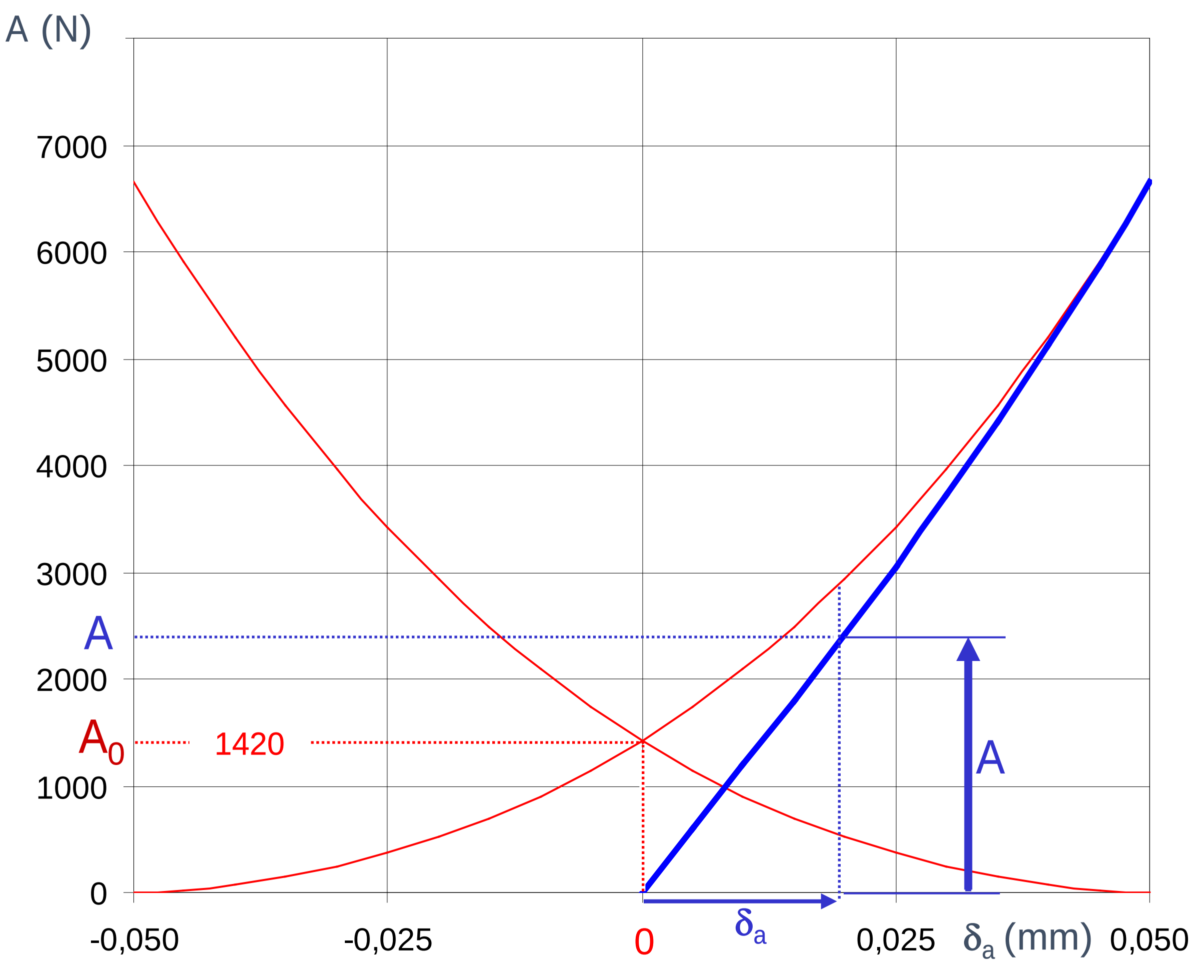
Preload A_0 is applied to the outer rings, in this DF pair; however, the resultant force vector is on the axis.

The pre-determined offset is an “interference”, which we name “i”. In the graph above the axial load A is represented for the two bearings. Here i=0,01 mm



An axial load A on the two inner bearings is added from the left by a shaft shoulder.

Starting from the initial preload A_0=1420 N corresponding to $\delta_{a,L}=\delta_{a,R}$=50 µm we add a common inner ring displacement δ_a by applying a force A to the shaft.



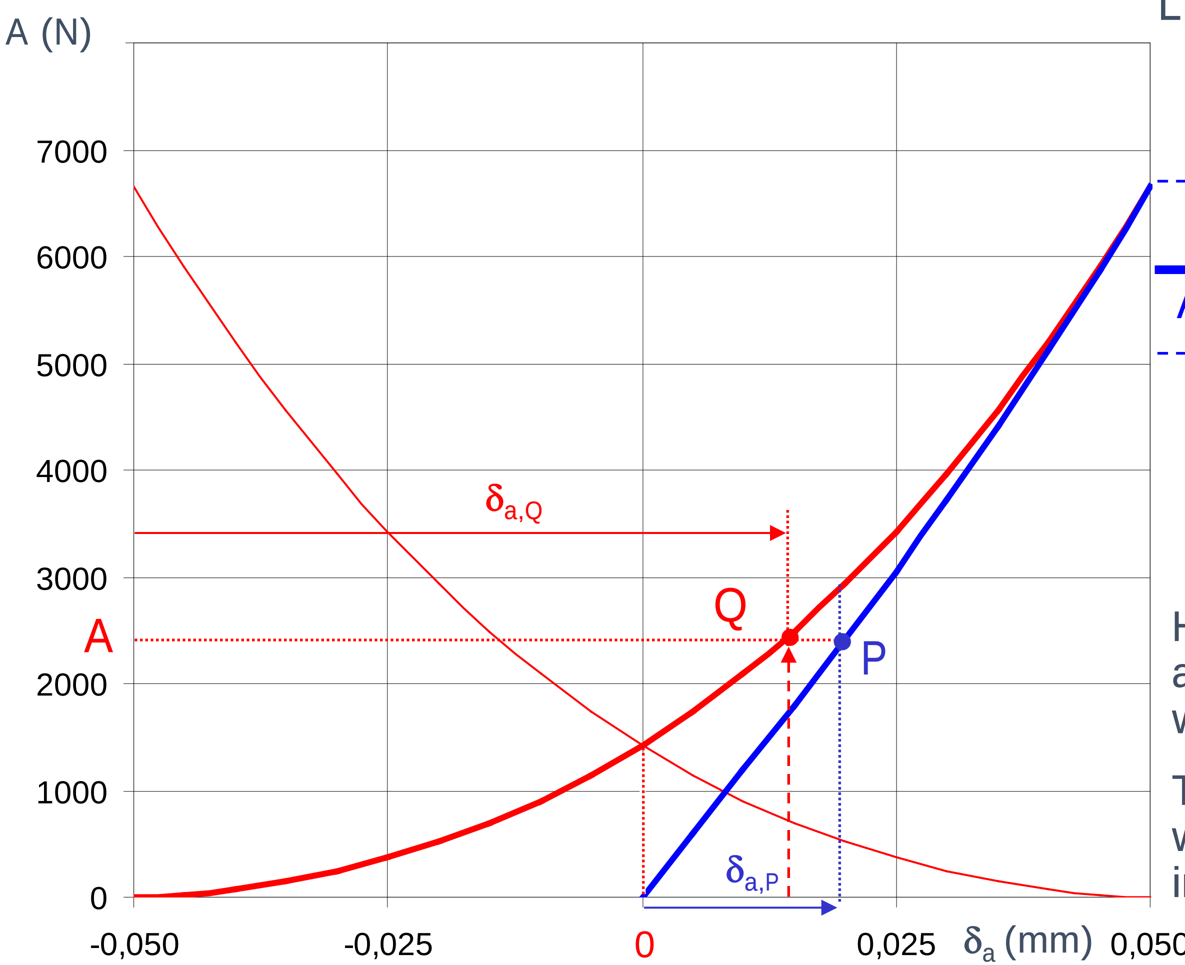
The blue line represents the additional axial force A applied to the shaft and to the two inner rings vs. their common additional displacement δ_a .

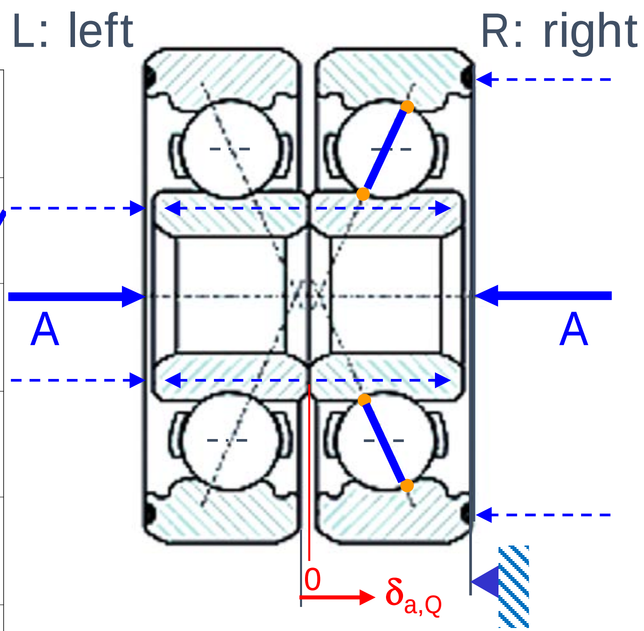
It is clear that the stiffness of the preloaded pair of bearings produces a stiffness which is much higher for two reasons:

1 – they work from a point where their individual stiffness is higher than from zero

2 – the slope depends on the difference of the two curves, then it is higher than each one separately.

The fact that the A-δ_a relation becomes “almost” linear is an additional bonus.





Had we just the right bearing under axial load A , the bearing force would follow the thick red line.

Therefore for the same A we would reach point Q from -0,050 instead of point P from 0 .

The (secant) stiffness seen from the shaft is, in this example, around 3.4 times smaller for the single bearing (i.e. double but not preloaded) compared to the double preloaded, as it is easy to observe that $\delta_{a,Q}$ is about 3.4 times $\delta_{a,P}$.

Closing remarks on axial stiffness

1. One should take into account that the only source of elastic displacements normal to the contact was the local Hertz approach. However, the rings may contribute with their own dilatation under load. Therefore actual axial deformation may not be entirely predicted by the formulas provided, and may vary according to the bearing mounting conditions, such as the material and thickness of the shaft and housing, and bearing fitting.

2. Fixed position preload was at the roots of the case worked out as an example. In the case of constant-pressure preload, which is given by one or more sprigs, it must be borne in mind that their stiffness is, as a rule, much lower than the one of a bearing. Therefore under the magnitude of axial displacement allowed by the bearing, the spring force A_0 will remain practically constant, hence the name. The result is that the stiffness under shaft load A will be that of a single bearing starting from A_0 .

Appendix: load / preload on slewing bearings

Slewing bearings (rings) are special ball or roller bearings of large radial dimensions designed to absorb axial and radial forces plus tilting moments in a single self-retaining and ready-to-install bearing unit.

Ball slewing ring with gear

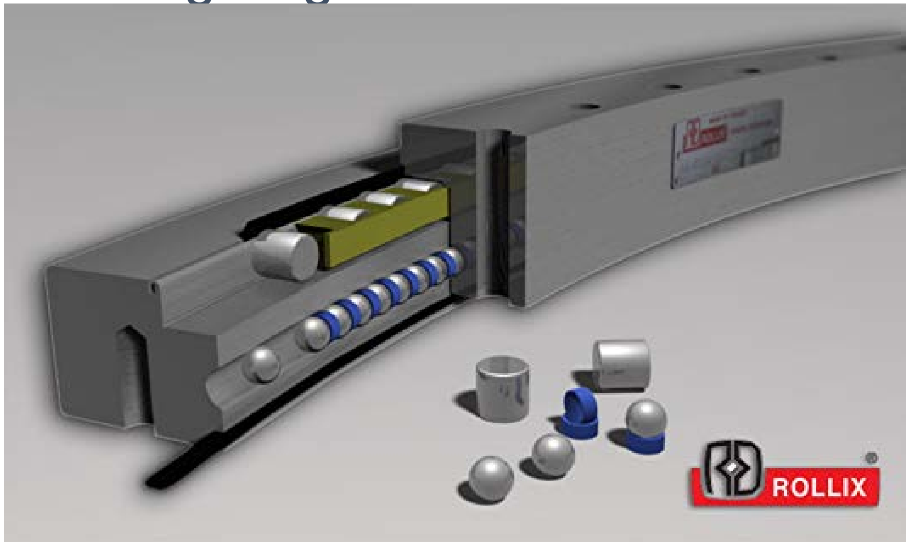
Ball slewing ring with flange

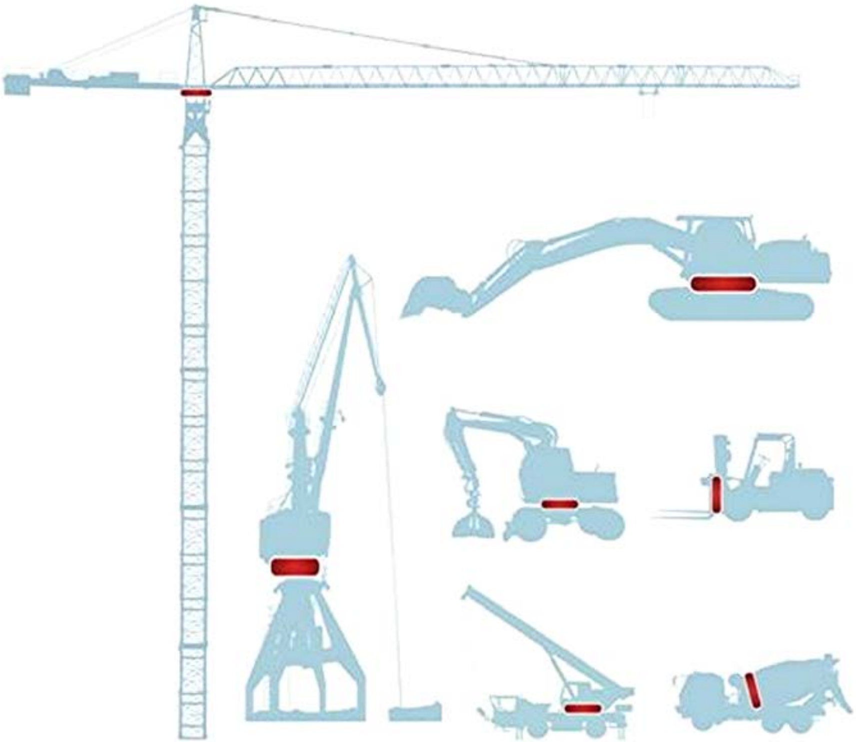
There are several different ball and roller combinations (single and multiple rows).

We shall explore here a particular version of single row ball bearing.

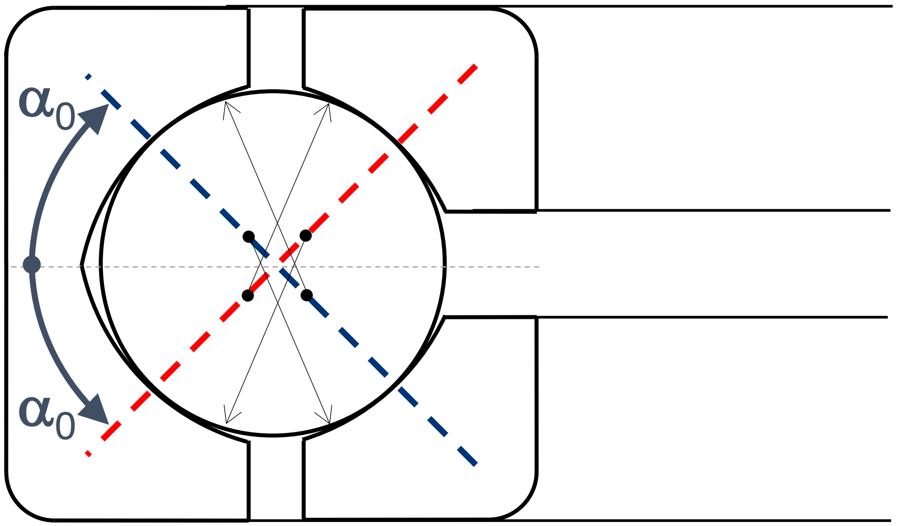
Crossed roller slewing ring carrying a gear on the outer ring.

Two row, roller and ball, slewing ring



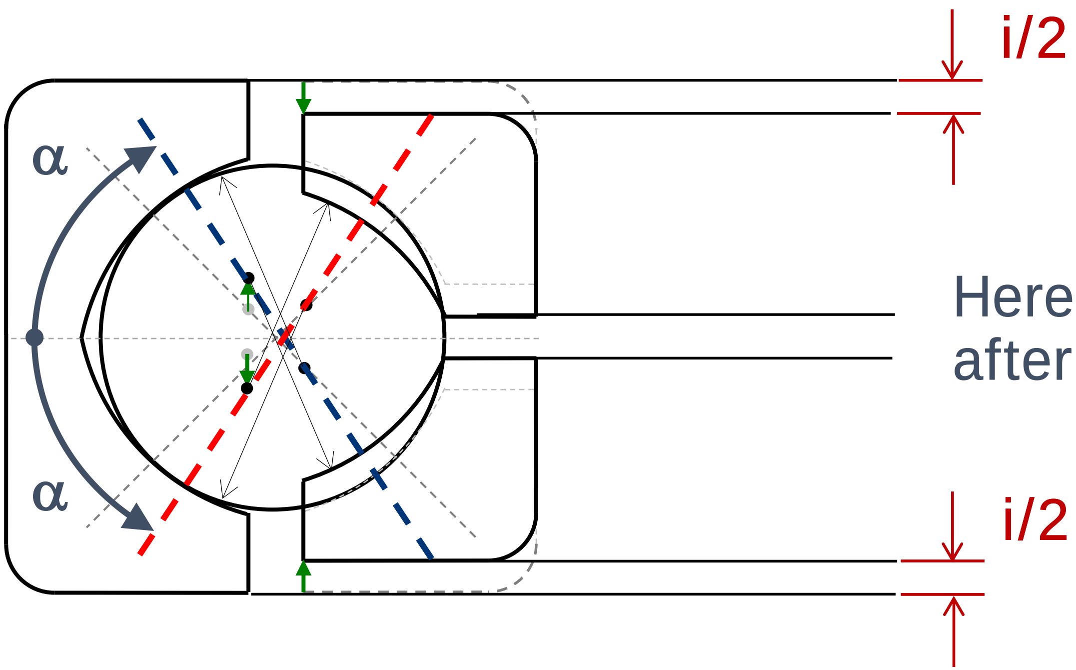


To "slew" means to turn without change of place; a "slewing" bearing is a rotational rolling-element bearing that typically supports a heavy but slow-turning or slow-oscillating load, often a horizontal platform such as a conventional crane, a swing yarder, or the wind-facing platform of a horizontal-axis windmill, a tank turret ... ...

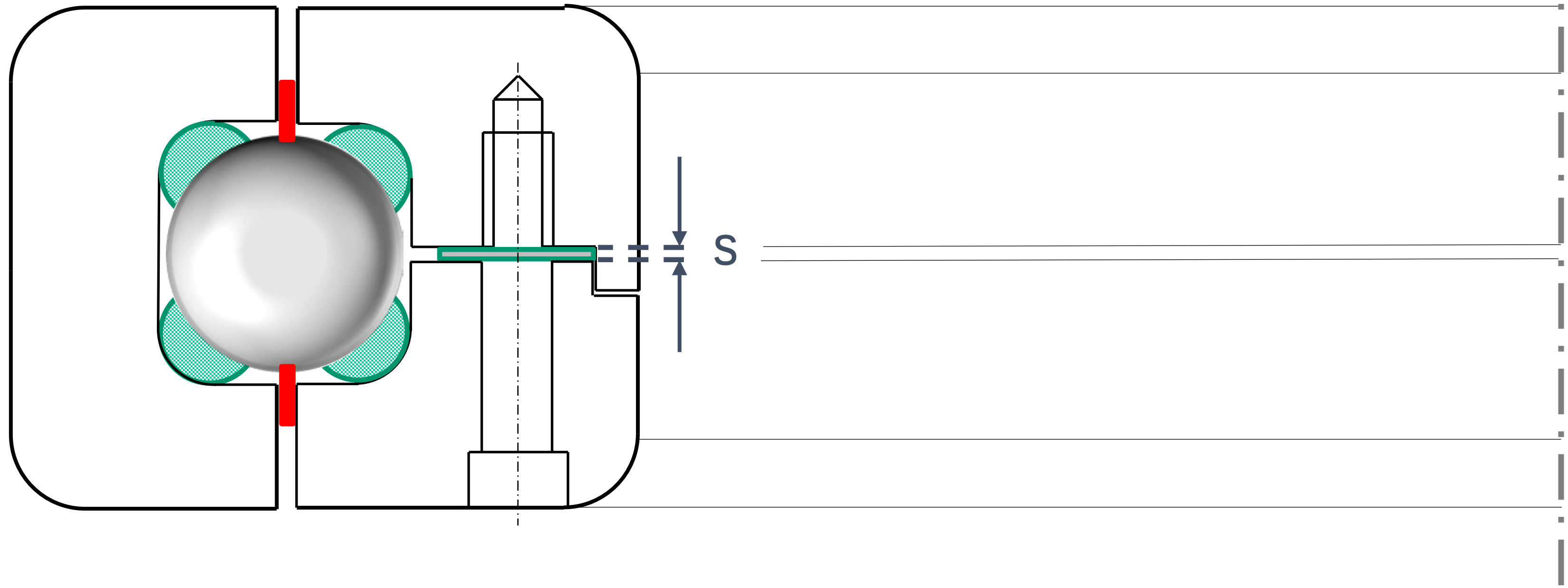


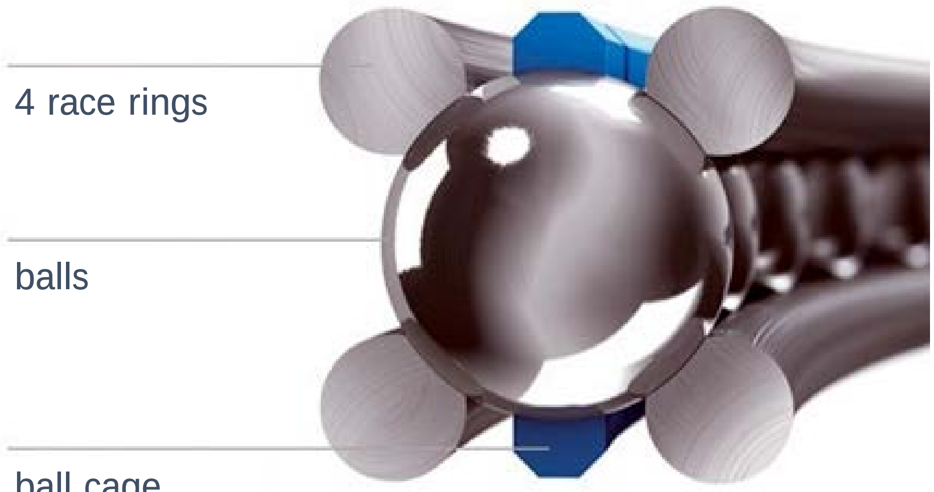
This is a four point contact ball bearing, designed to support axial loads in both directions.

The inner ring is a “split ring”, here shown before preload.



Here the split ring is shown after preload.





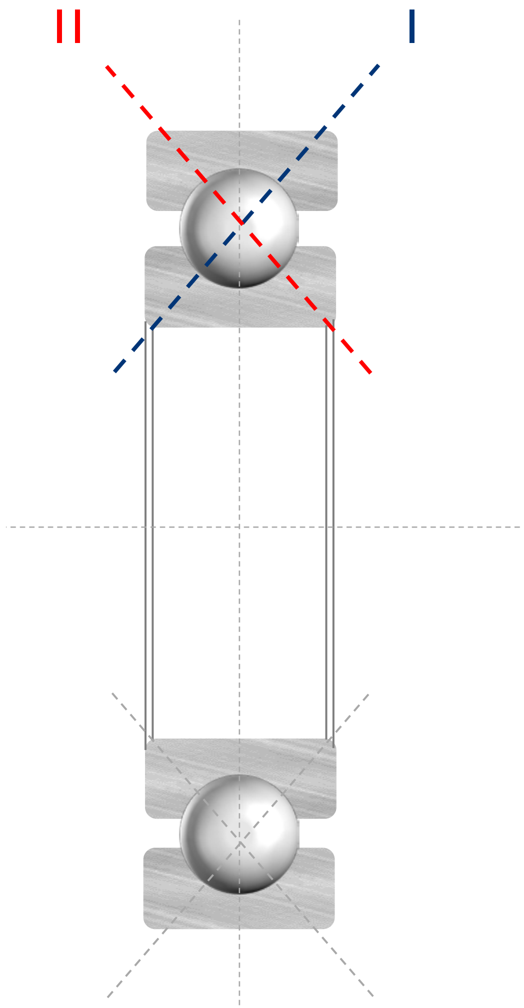
In this case the balls in one row are preloaded by tightening the two split inner rings against a spacer (shim) of thickness ‘s’ .

In this special bearing, the rings are in aluminium but the raceways are made of steel wires.

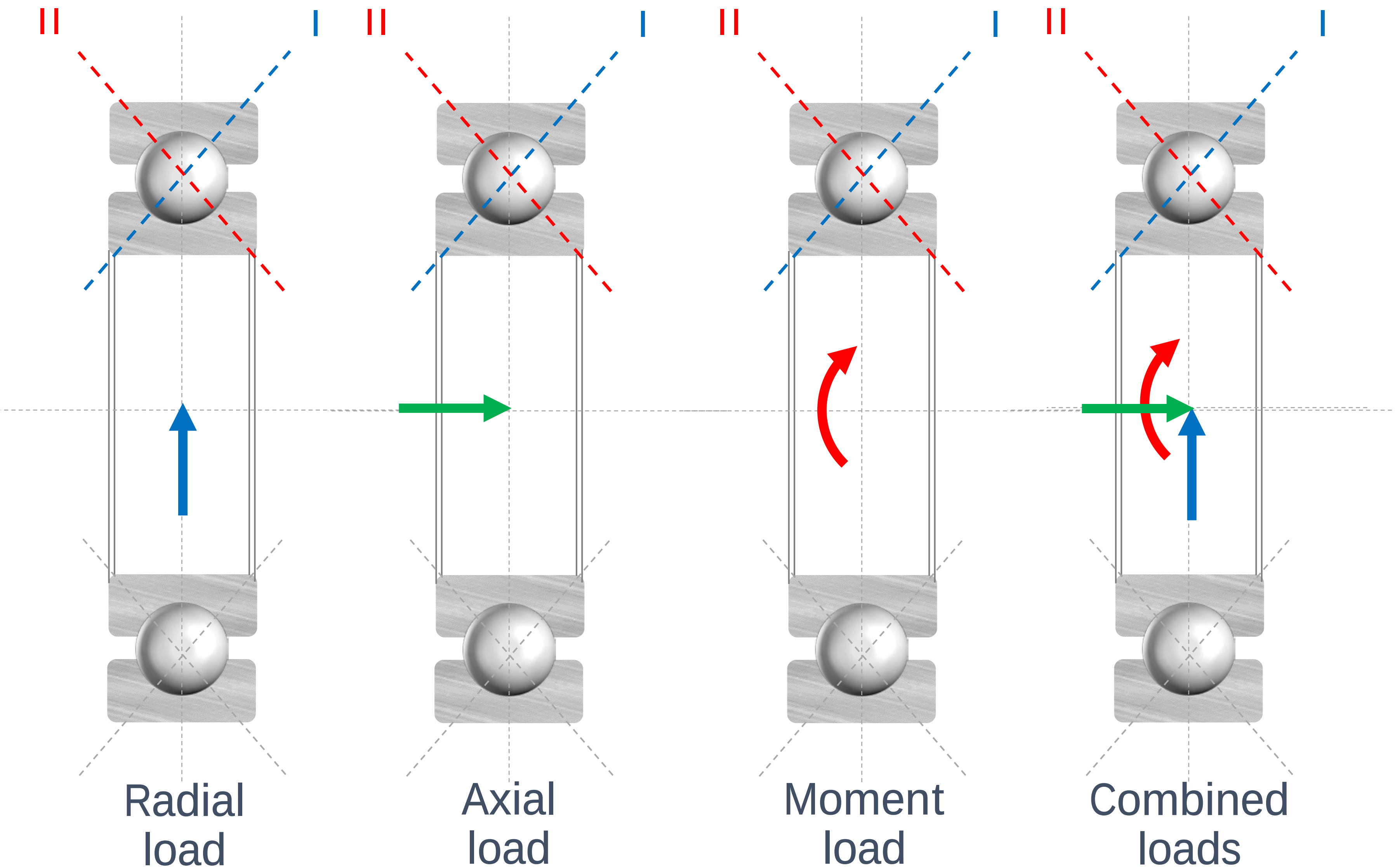
Example

Variable name	Symbol	Value	Unit
Bearing pitch diameter	d_m	1000	mm
Number of rolling elements	Z	48	/
Rolling element diameter	D	40	mm
Curvature radii ratio	$2r/D$	1,05	/
Nominal contact angle	α_0	45	°
Modulus of elasticity	E	210	GPa
Poisson's ratio	υ	0,3	/

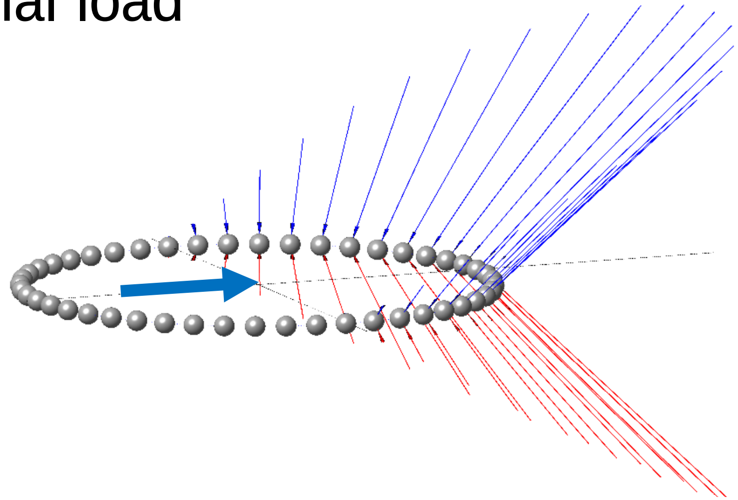
External load	Symbol	Value	Unit
Axial force	F_a	100	kN
Radial force	F_r	100	kN
Tilting moment	M	300	$kN \cdot m$



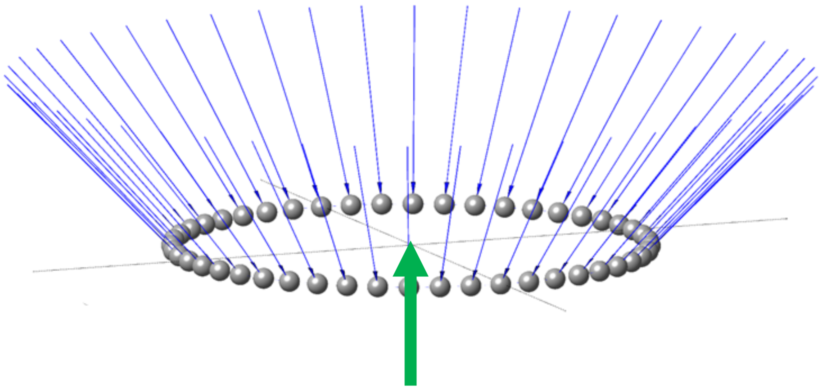
Calculations and figures from: Edoardo Pomarolli, Master Thesis "Load distribution and design stresses in small and large slewing ball bearings", Politecnico di Torino, Oct. 2014


II
I
II
I
II
I
II
I
Radial load
Axial load
Moment load
Combined loads

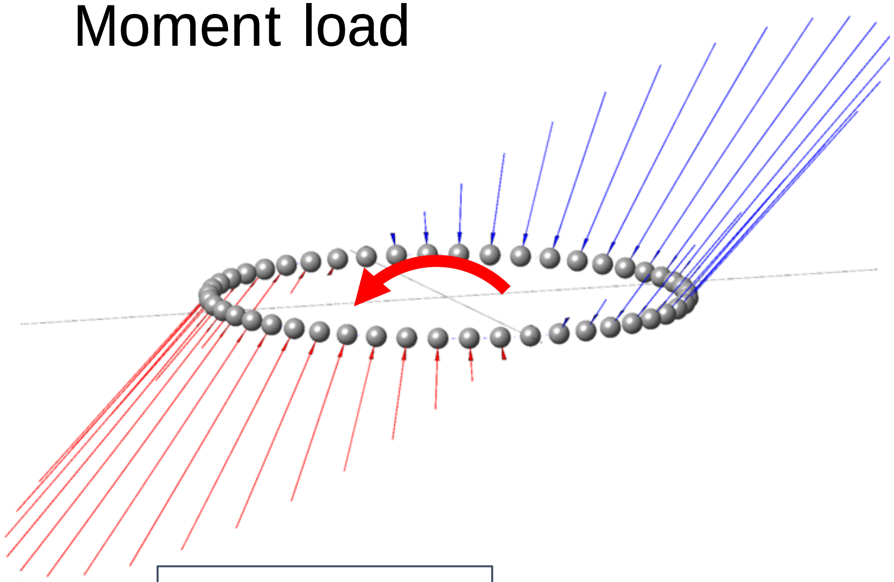
Radial load



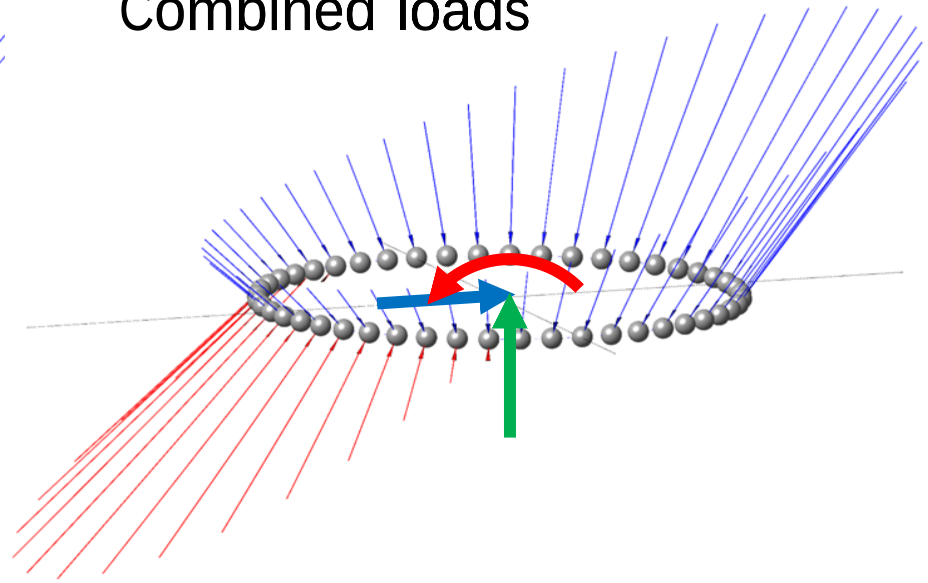
Axial load



Moment load



Combined loads

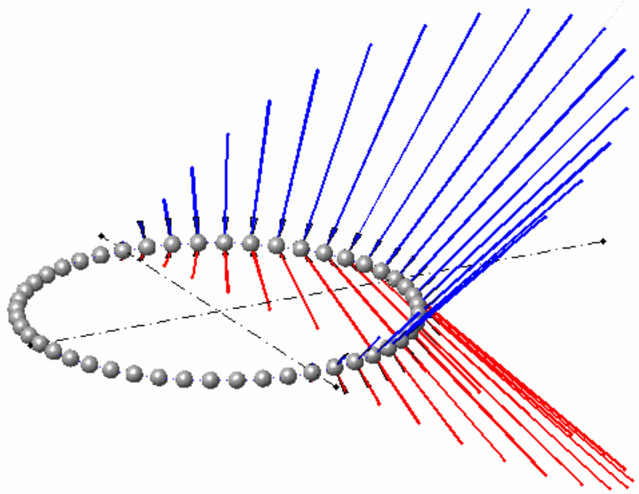


no preload

The slewing ring is subjected to a fixed radial load and, in addition, an interference:

varying axial preload

0 **i** (mm) interference



Radial load 100 kN