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Costruzione di Motori per Aeromobili - Machine Design

Fatigue - Chapter 3

Chapters

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Sect. 1, 2 - From specimen to component properties

While Chapter 2 was devoted to basic material properties as tested in the laboratory on specimens, this Chapter considers all those factors which are typical of components, which undergo all the technological processes and have specific dimensions and shapes. These factors have been investigated through specific testing, and have been represented by correction factors to the basic material properties.

Section 2 deals with fatigue corrections for surface effects produces by machining, roughness, residual stresses, cold and hot surface treatment, either chemical or mechanical.

1. Component fatigue strength: introduction (1/3)

This chapter deals with fatigue design at infinite life.

Parameters for component strength in fatigue are calculated from material fatigue limits $\sigma_{D-1,T}$ and $\tau_{D-1,T}$, which were defined in Sections 4, 5 of Chapter 2, in the special test case of:

- reversed stress, R=-1
- tested in tension-compression
- at a given temperature T

The calculation of component strength at fatigue under normal stress will now be described in a frequent and notable case, the beam or the rod, which is a good example to understand the steps of the procedure.

A caveat: extension to shell structures and welded components requires additional special treatment, which is beyond the scope of this chapter.

1. Component fatigue strength: introduction (2/3)

This section shows how component fatigue limits is obtained through design factors due to additional component features which for better understanding are now grouped in three parts:

1 - surface effects:

- b_0 surface roughness & machining
- K_V , K_S surface treatment and coating

2 - (volume) gradient effects:

- size effect in bending and torsion
- K_f (fatigue) notch factor for component shape

3 - composite stresses, other than those in simple standard cases:

- effect of combined stress components

1. Component fatigue strength: introduction (3/3)

As said, the treatment is limited to non-welded components of bar or rod-like shape; it will proceed by examining the evidence and the effect of one factor at a time, then multiplying factors*.

Step 1: calculates surface effects for fully reversed (alternating) stress, which affect the "component fatigue limit":

$$\sigma_{C,D-1} = \sigma_{D-1,T}\ b_0\ K_V\ K_S$$

Step 2: calculates applied (service) effective stresses in the design point:

$$\sigma_{eff}^{X} = \sigma_{nom}^{X}\ K_f^{x}$$

where the index x qualifies the separate cases:
a: axial load (tension/compression), b: bending, t: torsion

Step 3: will compose tension and shear stresses into an "equivalent stress" σ_{eq} to be introduced in the Haigh diagram (here preferred), or in any equivalent diagram.

* not obvious, it implies assuming no interaction

2. Component strength - Step 1 (1/10): surface finish

In order to account for all the factors below in the analysis of component fatigue, the material fatigue limit is reduced by the surface finish factor b_0.

- Materials as tested in the laboratory are always in a different condition (surface finish, residual stress, etc.) from the materials as they are actually used in the component.
- An important part of the analysis is to “correct” the basic materials data to obtain an estimate of the fatigue limit of the material in the component or structure of interest. Fatigue cracks usually nucleate on the surface so that the condition of the surface plays a major role in the fatigue resistance of a component.
- Test specimens are polished to eliminate the effects of surface finish. The degree of surface damage depends not only on the processing but also on the strength of the material.
- Higher strength materials are more susceptible to surface damage.

2. Component strength - Step 1 (2/10): surface finish

Coefficient b_0 is defined experimentally through tests on "as machined" and "polished" specimens as:

$$b_0 = \frac{\sigma_{D-1}(\text{as machined})}{\sigma_{D-1}(\text{polished})}$$

accounting for the observed reduction in fatigue strength with increasing surface roughness.

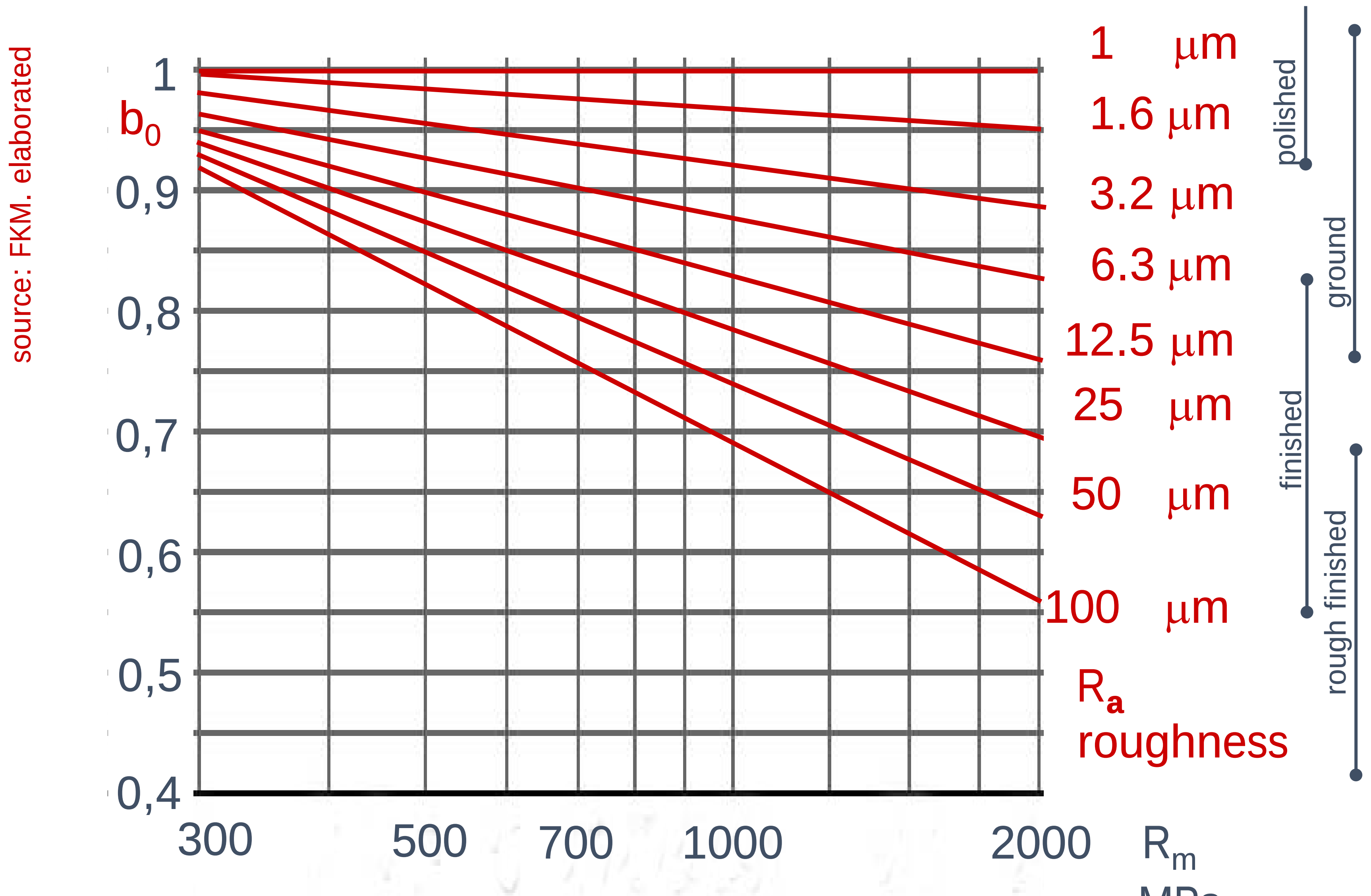
Coefficient b_0 allows for roughness and, implicitly, sub-surface residual stresses originated by machining.

$$\boxed{\sigma_{C,D-1} = \sigma_{D-1,T}\ b_0\ \ldots}$$

-1= reversed i.e. alternating stress

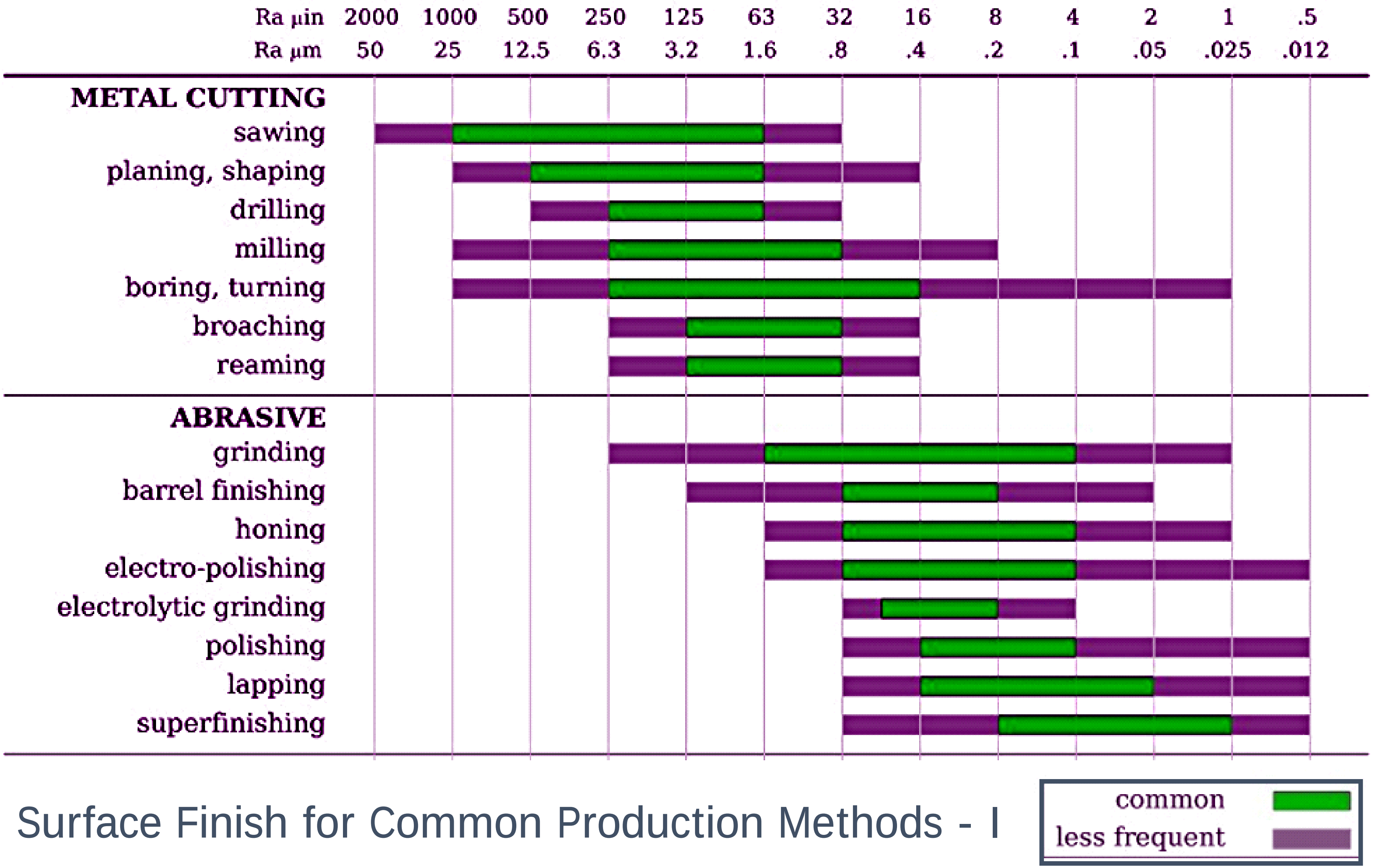
D = fatigue limit - Duration

C = component



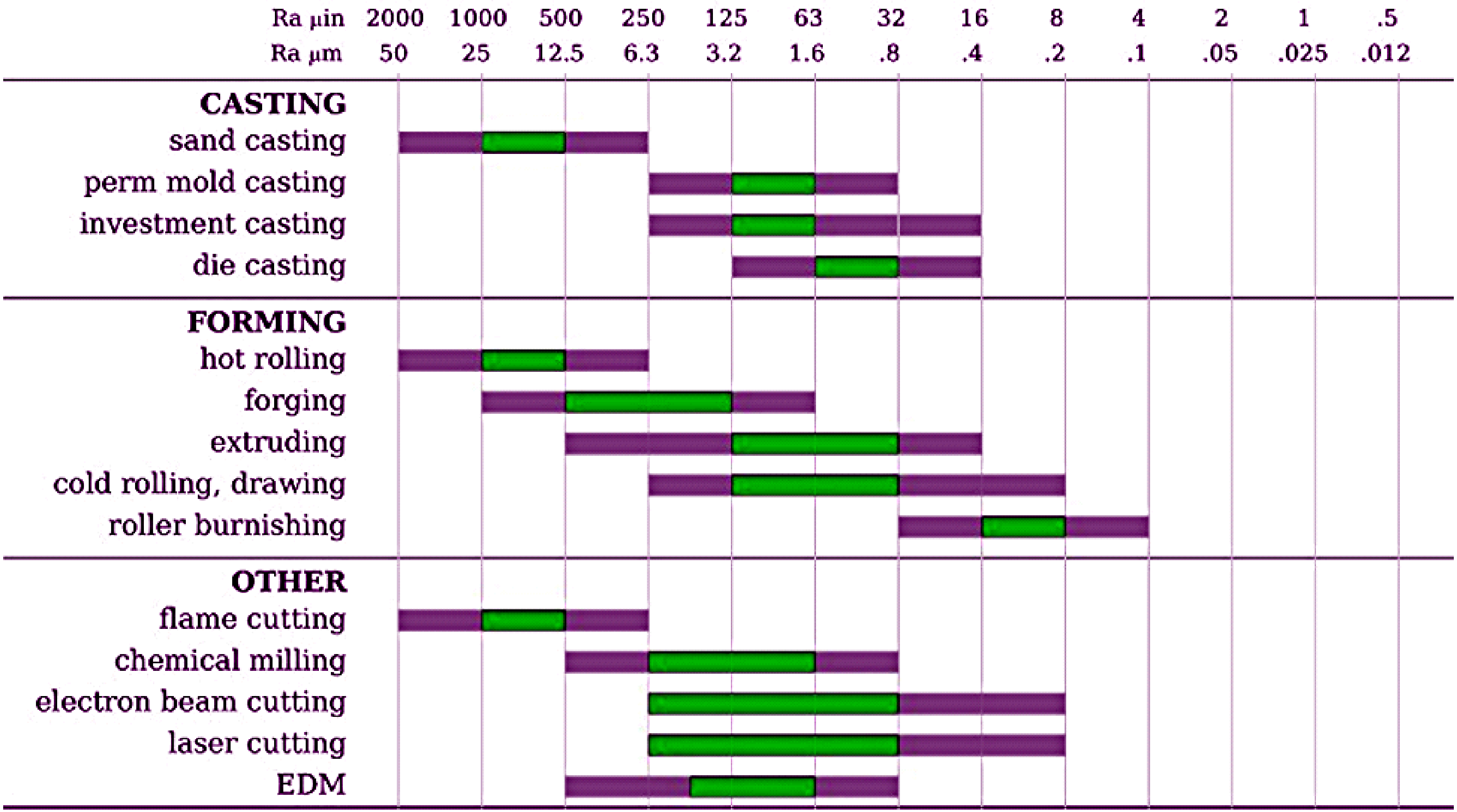
Note: in the FKM standard b_o is named $K_{R,\sigma}$

2. Component strength - Step 1 (4/10): surface finish



Surface Finish for Common Production Methods - I

conversion: 40 µin ≅ 1 µm (2% error)



Surface Finish for Common Production Methods - II

conversion: 40 µin ≅ 1 µm (2% error)

The FKM "surface treatment" factor K_V takes into account chemical, thermal and mechanical treatments which produce both grain structure changes and residual stresses.
The following values from FKM are available*:

surface treatment on steel	un-notched component from-to	notched component from-to
nitriding	1.10÷1.25	1.30÷3.00
case hardening	1.10÷2.00	1.20÷2.50
carbo-nitriding	1.80	/

surface treatment on steel	un-notched component from-to	notched component from-to
cold rolling	1.10÷1.40	1.30÷2.20
shot peening	1.10÷1.30	1.10÷2.50
induct/ flame hardening	1.20÷1.60	1.50÷2.80

values of K_v for specimens 8 to 40 mm diameter*

source: FKM

$$\boxed{\sigma_{C,D-1} = \sigma_{D-1,T} \ldots K_V \ldots}$$

*Disclaimer: these data are reported incompletely and limited to educational usage – For design with a professional responsibility the reader is urged to access the original and adjourned source

Residual stresses

Many processes for increasing component strength or the service life under dynamic loads are based on inducing residual compressive stresses in the boundary layer. These include heat treatments such as case-hardening or nitriding and mechanical treatments such as shot peening and deep rolling.

Thermal treatments are processes which rely on the diffusion of either carbon, carburizing, or nitrogen, nitriding, onto and into the surface of a steel component. They both increase the strength of the steel and, through volumetric changes, cause a compressive residual stress to be left on the surface.

In mechanical treatments although some alteration in the strength of the material occurs as a result of work hardening, the improvement in fatigue strength is due mainly to the compressive surface stress. Chip removal processes such as upcut milling or a well-matched grinding process can also induce significant residual compressive stresses in a component.

An example for shot peening is given in the next slides.

Residual stresses are a very broad and extremely important field. The reader is urged to extend his/her knowledge by referring to the ample literature.

2. Component strength - Step 1 (8/10): surf. treatment

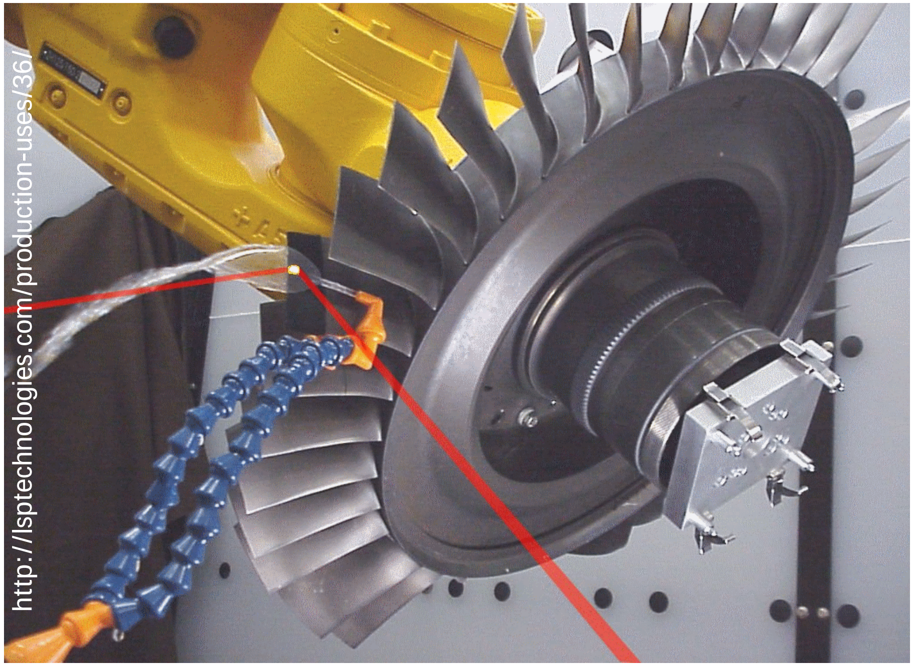
Shot peening

A cold working process used to produce a compressive residual stress layer and modify mechanical properties of metals.

It entails impacting a surface with particles (round metal ones, or glass or ceramics) with an impact force sufficient to spread the metal plastically, causing changes in the mechanical properties of the surface.

Plastic deformation induces a residual compressive stress in a peened surface, along with tensile stress in the interior.

Surface compressive stresses confer resistance to metal fatigue and to some forms of stress corrosion.



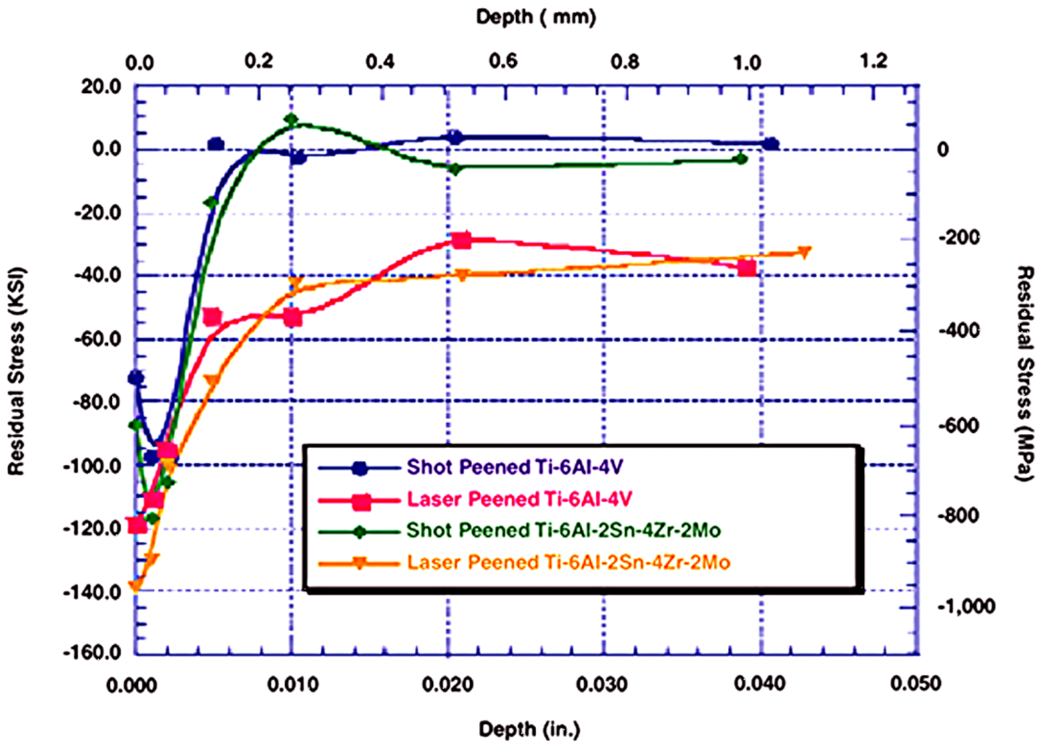
LSP – Laser Shot Peening

LSP uses a series of high-intensity laser pulses to generate deep residual compressive stresses in a specified location.

This process is used in producing a variety of aircraft components.

Aerospace applications of laser peening include many gas turbine engine components, suchas airfoils and integrally bladed rotors (IBRs) because of the enormous benefits of preventing fatigue failures and improving damage tolerance for these critical parts.

The compressive residual stresses that result from conventional shot peening fall to nearly zero at a depth of 0.005 in. The residual stresses from LSP (Laser Shot Peening) are significant, more than -40 ksi at a depth of 0.040 in.



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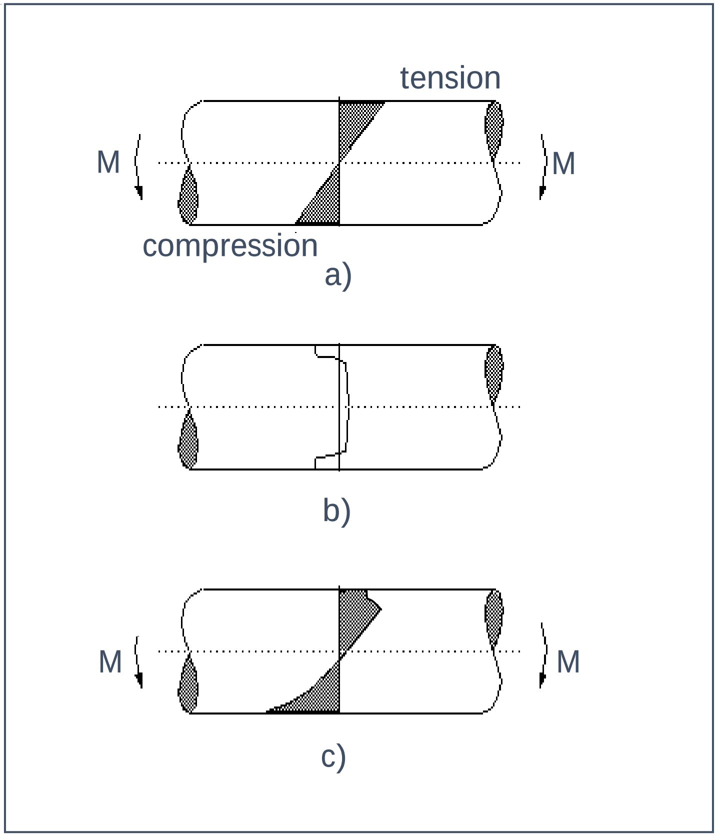


a) : stresses due to bending

b) : residual stresses due to shot-peening

c) : superposition of a) and b); the highest tensile total stress is reduced

The FKM “surface coating” factor K_S allows for the influence of coating on the fatigue strength of a component made of aluminium alloy; it is given as follows:

-for steel and cast iron materials: $K_S=1$*

-for aluminium alloys without coating: $K_S=1$

-for aluminium alloys with an anodic coating of thickness x:

$$K_S = 1 - 0.27\ \mathrm{Log}(x) \qquad (x \text{ in } \mu m)$$

$$\text{example: with } x = 30\ \mu m : K_S = 0{,}60$$

$$\text{with } x = 1\ \mu m : K_S = 1{,}00$$

$$\boxed{\sigma_{C,D-1} = \sigma_{D-1,T} \ldots\ldots K_S}$$

* WARNING, the picture above is incomplete. Please be aware that the chrome and nickel plating of steel components can more than halve the endurance limit due to the creation of tensile residual stresses at the surface. These tensile stresses are a direct result of the plating process itself. Electroplating may also cause a shortening of the fatigue life through the mechanism of hydrogen embrittlement.

Sect. 3, 4 - From specimen to component properties

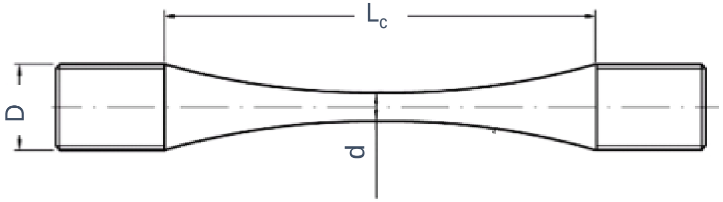
Section 3 examines the size effect and the gradient theory. Special reference is made to notches and their great importance to the considerable local increase of stress. This quite long Section is concluded with FKM formulas incorporating the relative gradient theory into the notch effect.

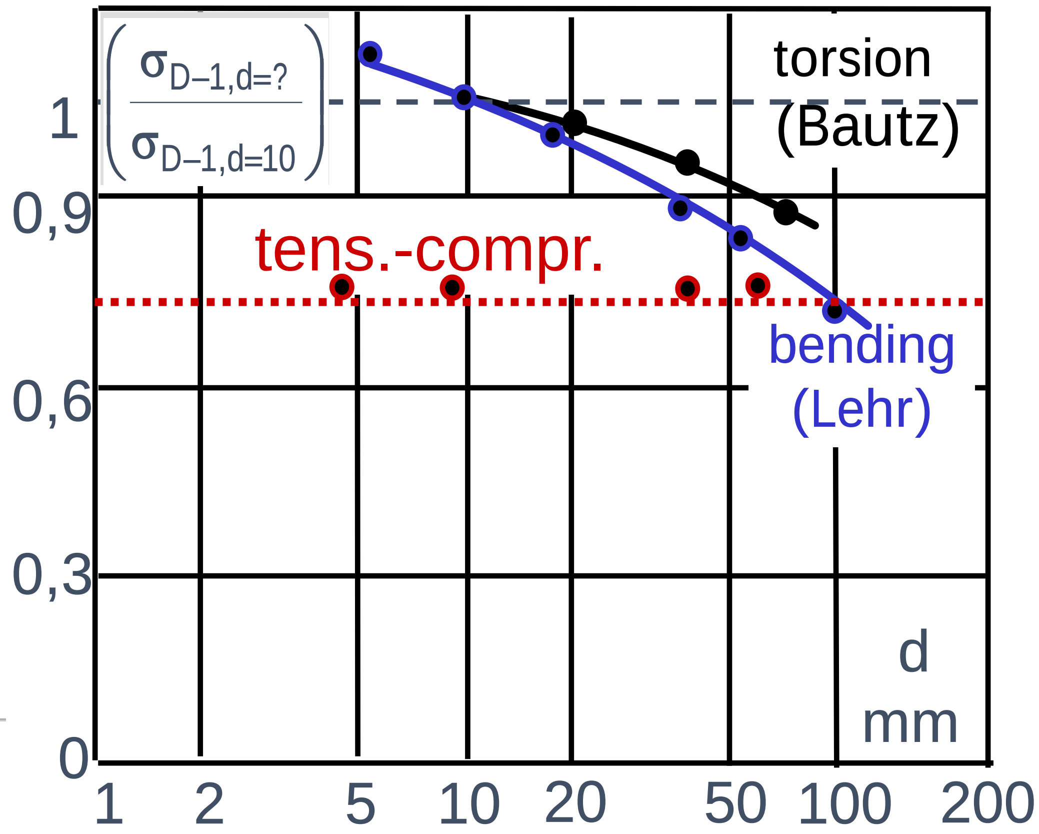
Section 4 shows how to treat two-dimensional fatigue stresses, composition of normal, σ , and shear, τ , with special reference to the case of the shafts. The final formula which defines the alternating equivalent tensile stress σ_a, and the mean equivalent tensile stress, σ_m, to be used in connection with Haigh diagram, is finally presented.

When σ_{D-1} is found on specimens otherwise equal except the diameter, and when the test is in bending or torsion fatigue (substitute τ for σ), then it is observed that the fatigue limit decreases with increasing diameter. This is called "size effect".

It is due to the presence of a gradient.

In fact, tests in tension -compression do not show such dependence of fatigue limit on diameter.

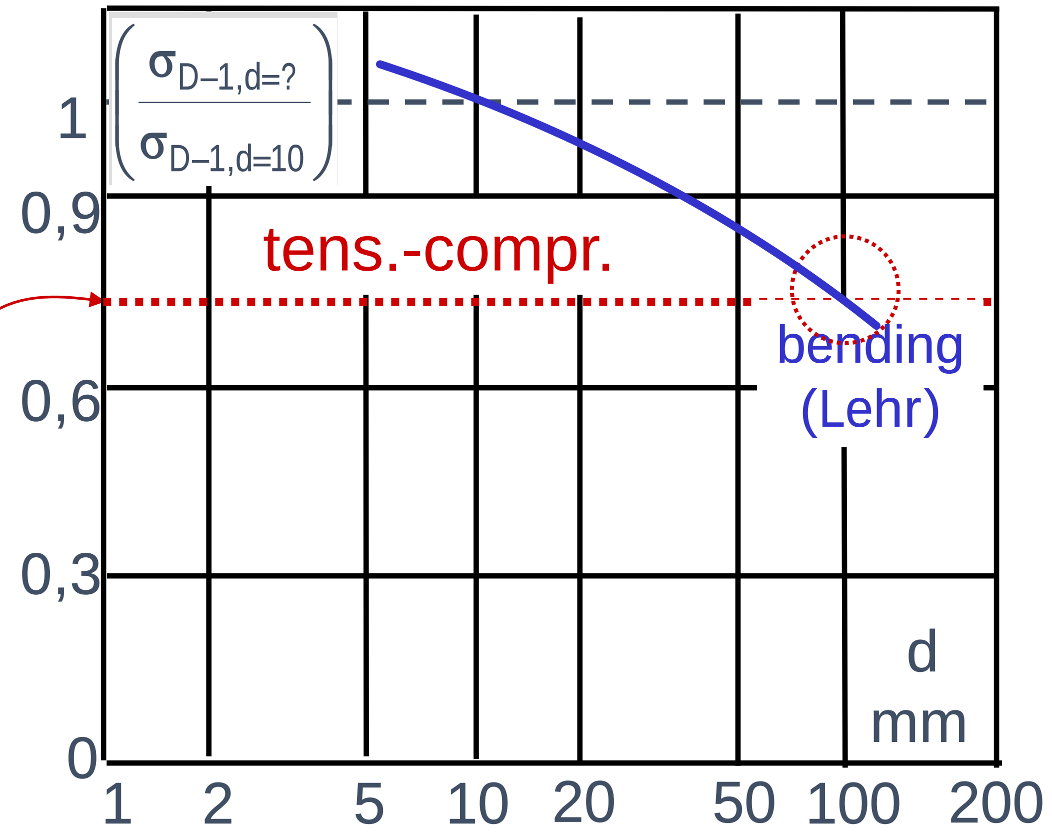




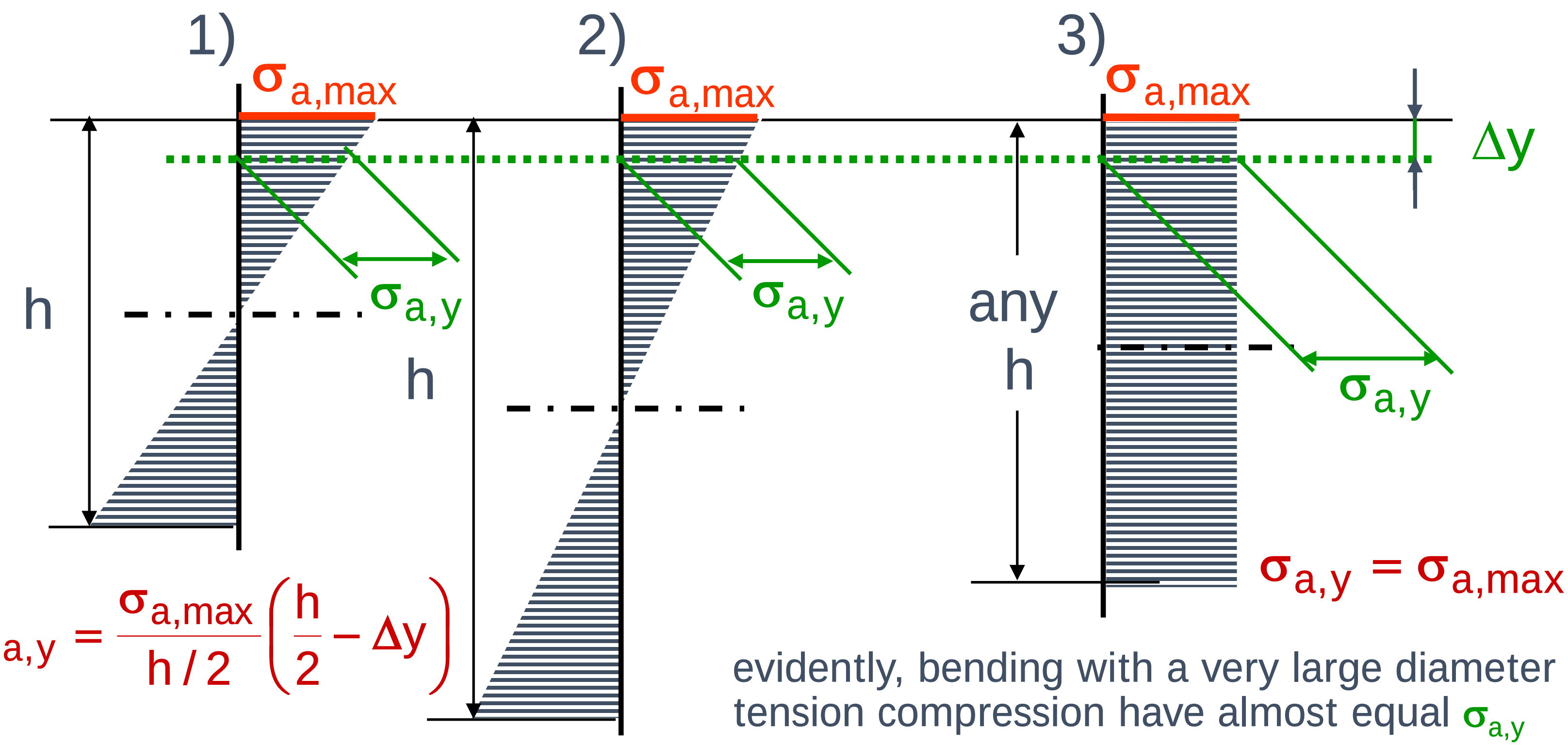
It is found that the ratio of fatigue limit in bending for high diameters: $\dfrac{\sigma_{D-1,d=100}}{\sigma_{D-1,d=10}}$ (bending !) and the ratio of the constant tension-compression limit (any diameter) to the bending fatigue limit at d=10 mm

$$\frac{\sigma_{D-1,d=any\ (tc)}}{\sigma_{D-1,d=10\ (bending)}}$$

are almost equal, approximately at a value of 0,7 .



In the case of bending, the stress varies along the height of the cross section and has a gradient, while for tension compression the gradient is zero. If we take the same maximum stress $\sigma_{a,max}$ (the alternating stress in R=-1 fatigue), the stress $\sigma_{a,y}$ at depth Δy will increase with decreasing gradient: $\sigma_{a,y}$ grows from 1) to 3).



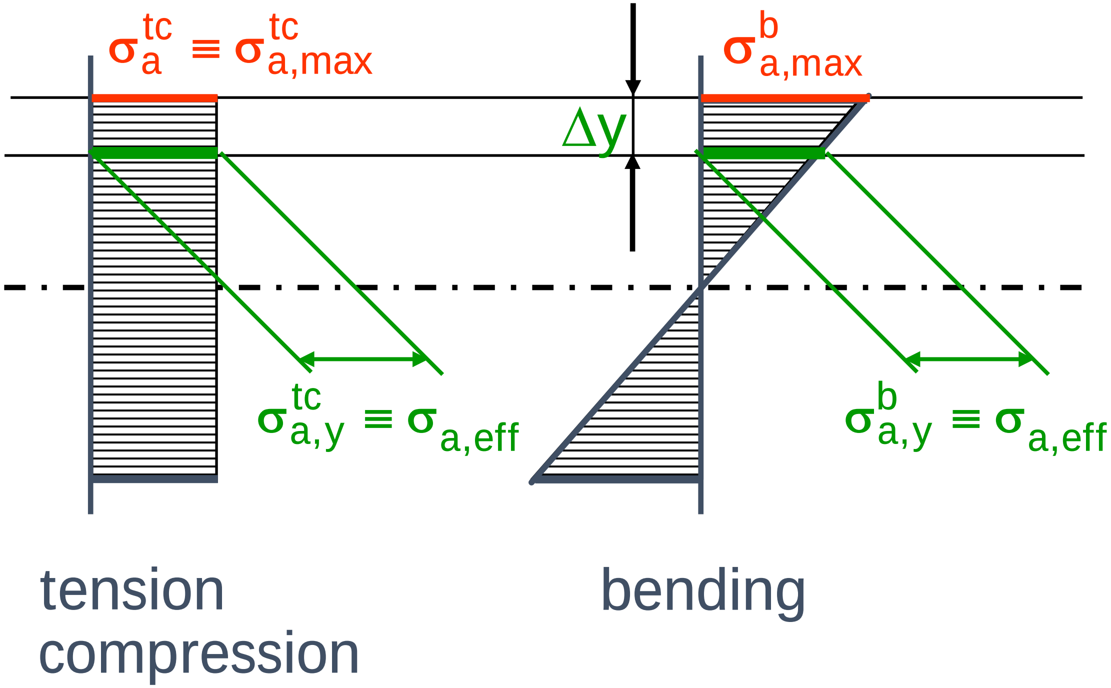
$$\sigma_{a,y} = \frac{\sigma_{a,max}}{h/2}\left(\frac{h}{2} - \Delta y\right)$$

evidently, bending with a very large diameter and tension compression have almost equal $\sigma_{a,y}$

The experimental results shown in sl. 2 of this Section are explained if we assume that the alternating stress $\sigma_{a,y}$ at depth Δy is the "effective" stress $\sigma_{a,eff}$ which governs fatigue damage.

The explanation proceeds from this slide to sl. 7

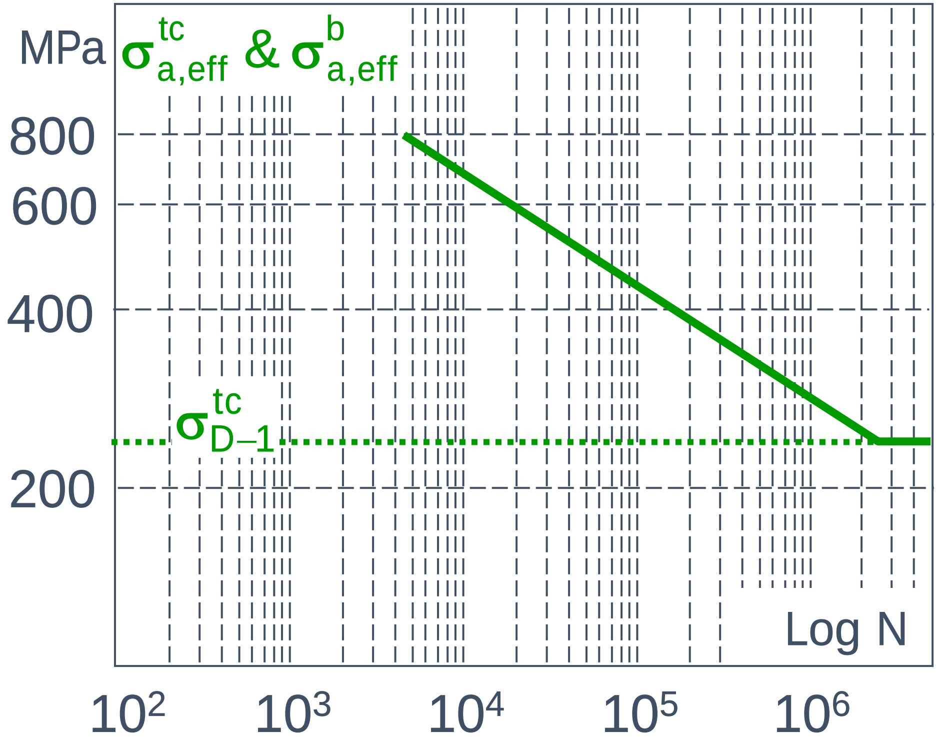
In fact, if we take the same $\sigma_{a,eff}$: $\sigma^{b}_{a,max} > \sigma^{tc}_{a,max}$



In the simplified terms of this lesson, stress $\sigma_{a,y}$ at depth Δy may be considered a sort of mean "effective" stress inside the elementary block (or volume) which determines fatigue damage and crack nucleation.

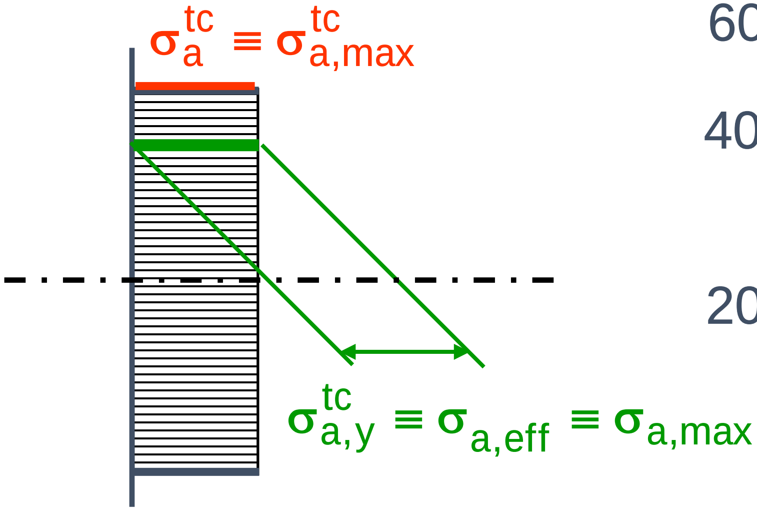
The "similarity principle" assumes then that the parameter controlling fatigue is this "mean stress".

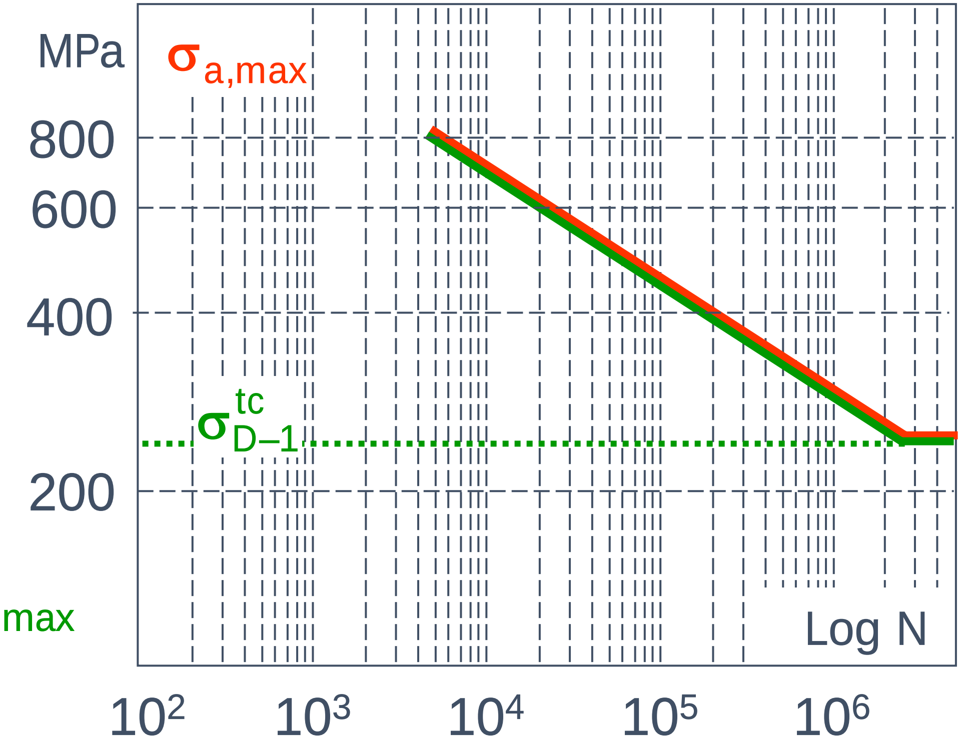
Consequence: a fatigue curve has to be the same for the two cases "tc" or "b" if Wöhler diagram are drawn in terms of $\sigma_{a,eff}$.



If we instead - as it is usually done- put on the vertical axis the maximum stresses $\sigma_{a,max}$ calculated at the surface, then:

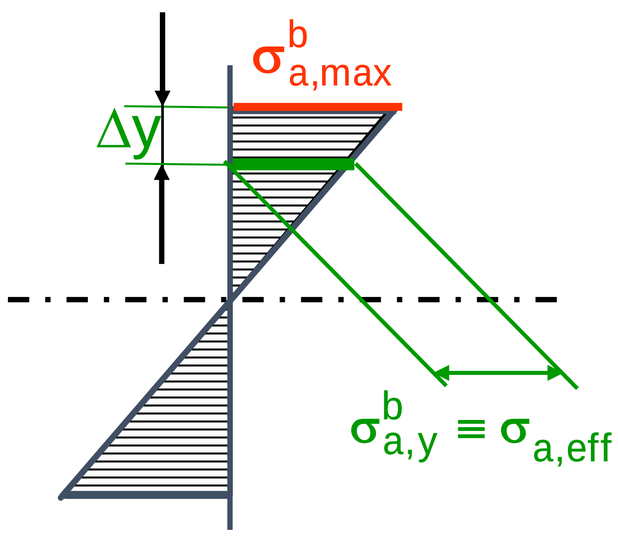
-) the curve of max stress for tc (tension-compression) obviously coincides with the curve for effective stress at (any) depth Δy:





-) but it is not so for bending ...

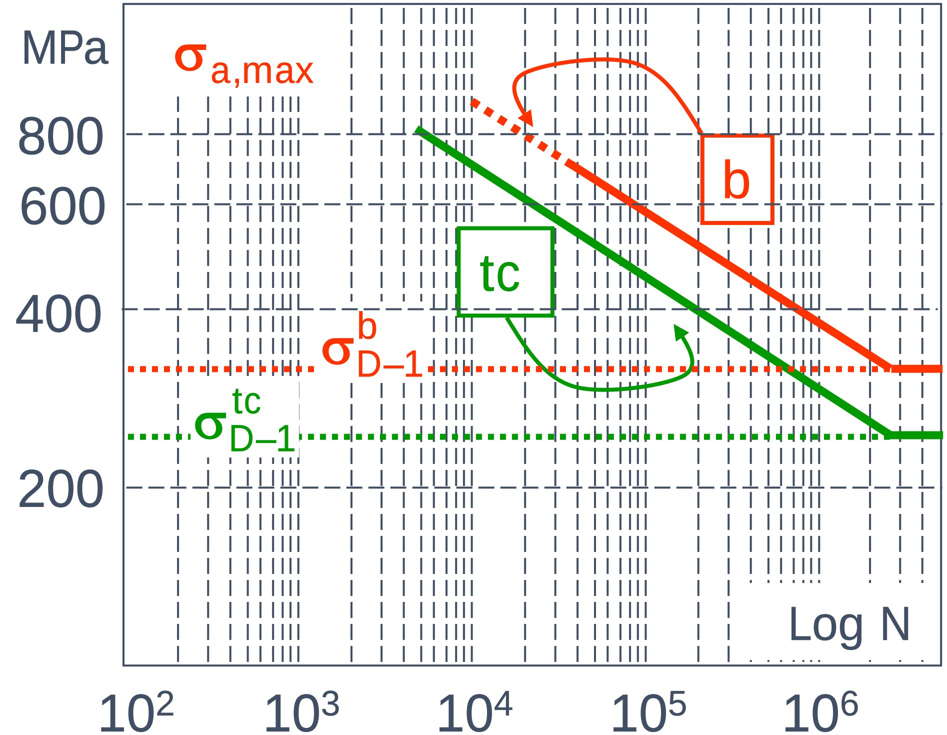
$$\left\{ \begin{array}{l} \sigma^b_{a,max} = \dfrac{\sigma^b_{a,max}}{\sigma_{a,eff}} \sigma_{a,eff} \\ \downarrow \qquad\qquad\qquad\qquad \downarrow \\ \sigma^b_{D-1} = \dfrac{\sigma^b_{a,max}}{\sigma_{a,eff}} \sigma^{tc}_{D-1} \end{array} \right.$$



according to sl. 5 of this section

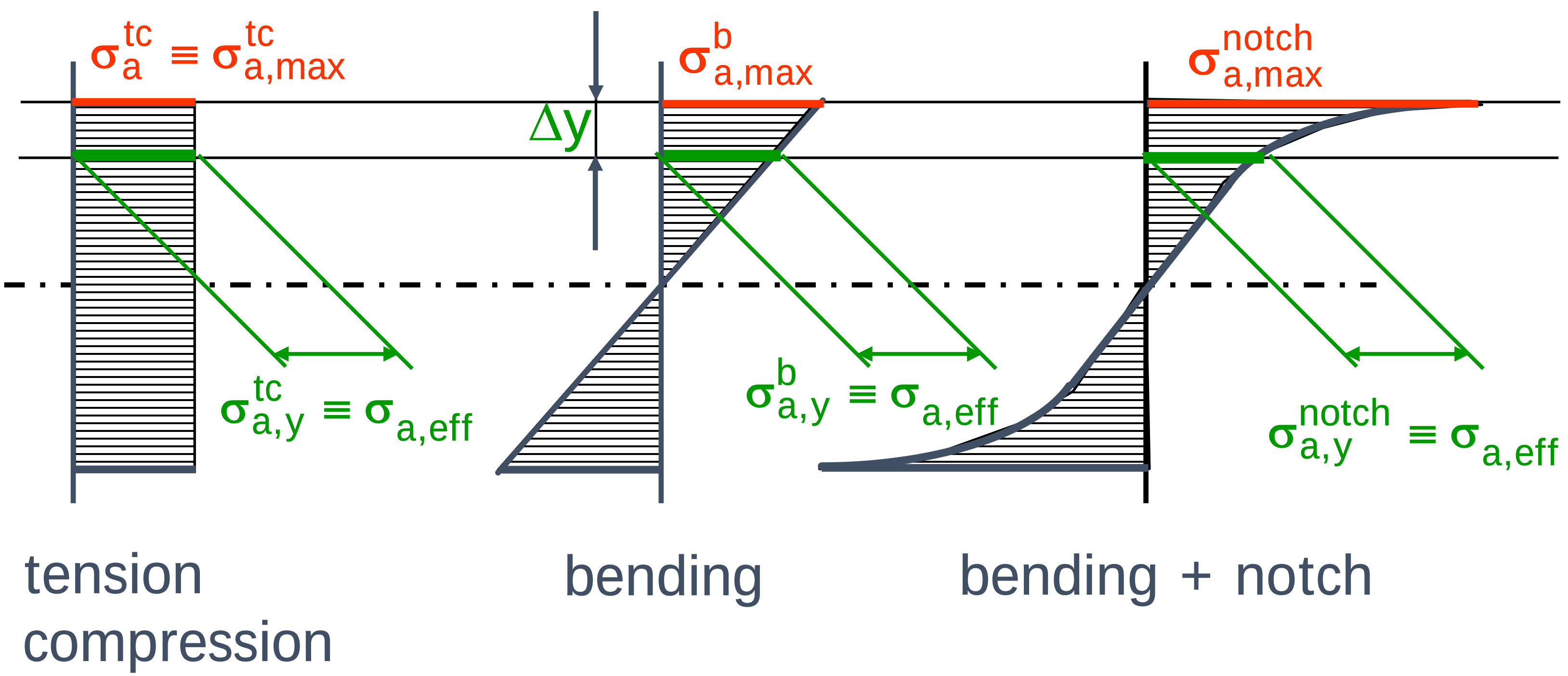
This explains the size effect for bending and torsion fatigue specimens.

Because we represent the max stress and not the effective stress.



Let us now compare three cases with the same effective alternating stress $\sigma_{a,y}$ at depth Δy. We anticipate here a qualitative sketch of stresses under a notch in bending, which produces the so called "stress concentration" and requires some explanation:

$$\sigma^{notch}_{a,max} > \sigma^{b}_{a,max} > \sigma^{tc}_{a} \equiv \sigma_{a,eff}$$



What is the meaning of depth Δy ?

The relation between different tests on specimens and observations on components is based on the "similarity principle": *Similar conditions, applied to similar systems, should* produce the same consequences" . For fatigue of notched elements the similarity concept implies: similar stress cycles applied to an unnotched specimen and to the material at the root of a notch in a notched specimen will give the same crack initiation life.

However: what parameters can be used to judge "similarity"? Is it enough that the maximum stress be the same in both cases? Results say that the same maximum stress is not a similarity criterion.

Says Schijve* "*A significant aspect is that the stress cycle in the un-notched specimen is present in a large volume of the material with a relatively large area of surface material. In the notched specimen, the stress cycle of the peak stress (K_t nominal stress) at the notch root is present in a relatively small volume of the material with a relatively small surface area*."

* Schijve J., Fatigue of Structures and Materials in the 20th Century and the State of the Art, Materials Science, Vol. 39, n. 3, 2003

3. Component strength - Step 2 (10/28): Δy

Adds Schijve "*Neuber published a famous book on calculations of stresses around notches in 1937* * *. He found it rather disappointing to see that the reduction factor of the fatigue limit of notched steel specimens was much less than the* K_t *value obtained with his calculations*."

Neuber argued that the peak stress is not entirely responsible of the notch effect, specially for sharp notches which have steep gradients. He also underlined that materials have a non-homogeneous granular structure.

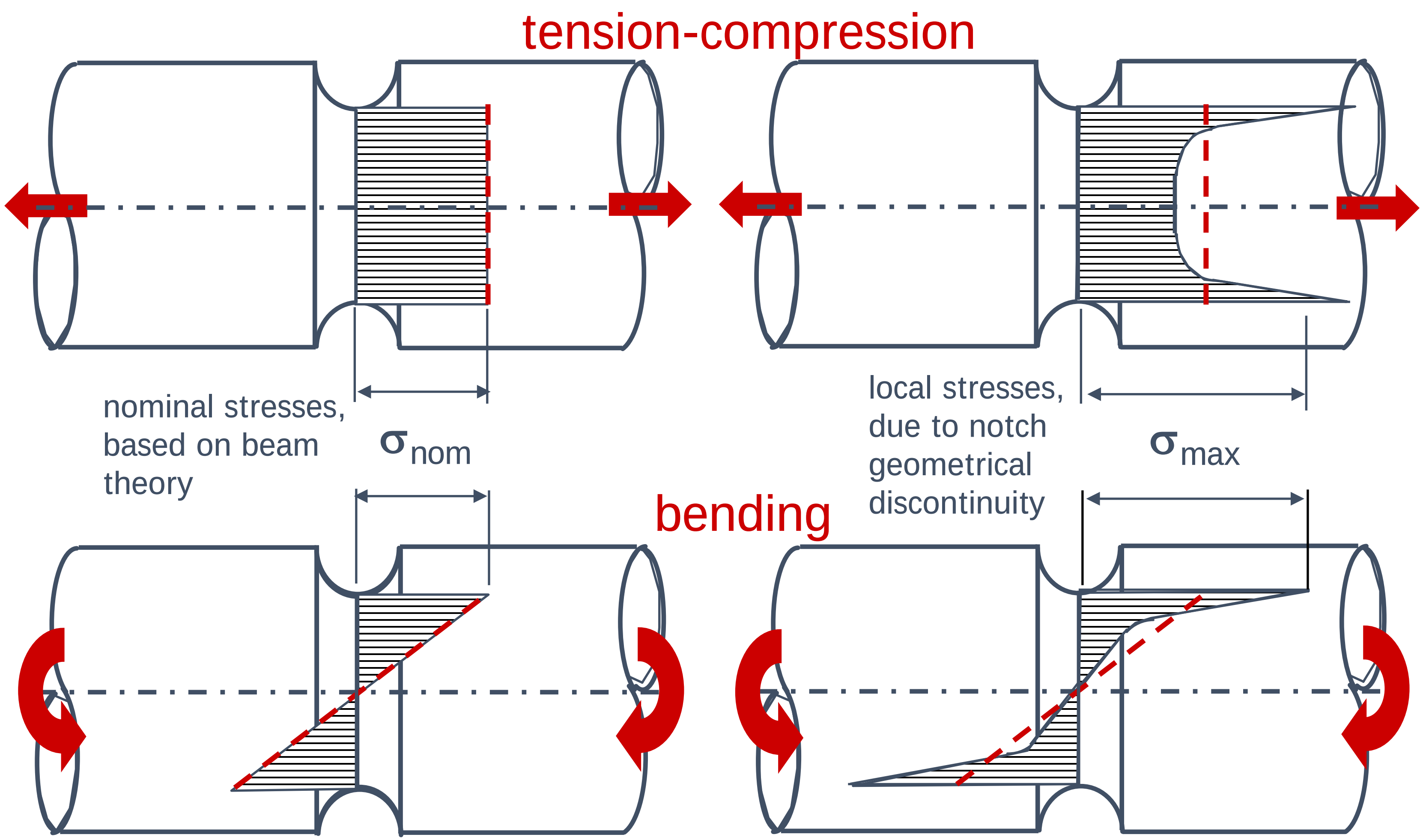
In addition we know that crack incubation requires the formation of slip lines over a volume (not at a point, which has no dimensions).

We can then understand why Neuber introduced* stress gradient effects at notches and the elementary block concept, which states that the average stress over a small volume at the root of the notch is what counts, and not the peak stress at the notch.

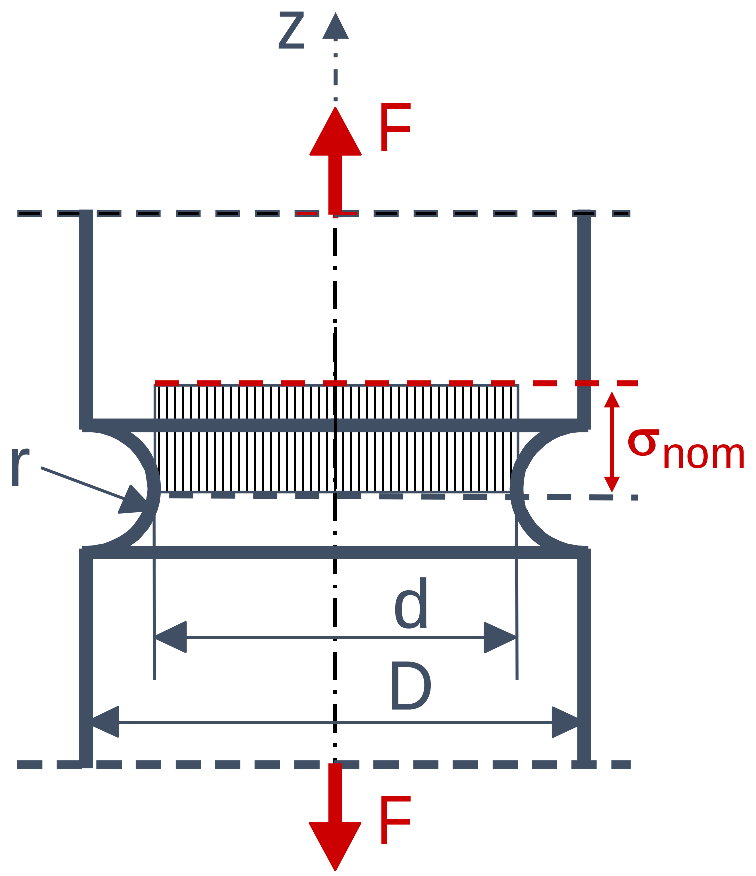
Later (1968) Neuber proposed a method wherein an equivalent larger radius is used to provide a lower K factor. The increment to the radius is dependent on the stress state, the kind of material, and its tensile strength.

* Neuber, H. (1937) *Kerbspannungslehre.* Springer, Berlin. Translation (1946) *Theory of Notch Stresses.* J.W. Edwards, Ann Arbor, Michigan

Notches produce marked deviations from the linear pattern, and quite high peaks:



Stress concentrations are one of the most important factors affecting the fatigue life of any component or structure. These stress concentrations may be intentional in the design or unintentional, such as deep machining marks or other processing related flaws.

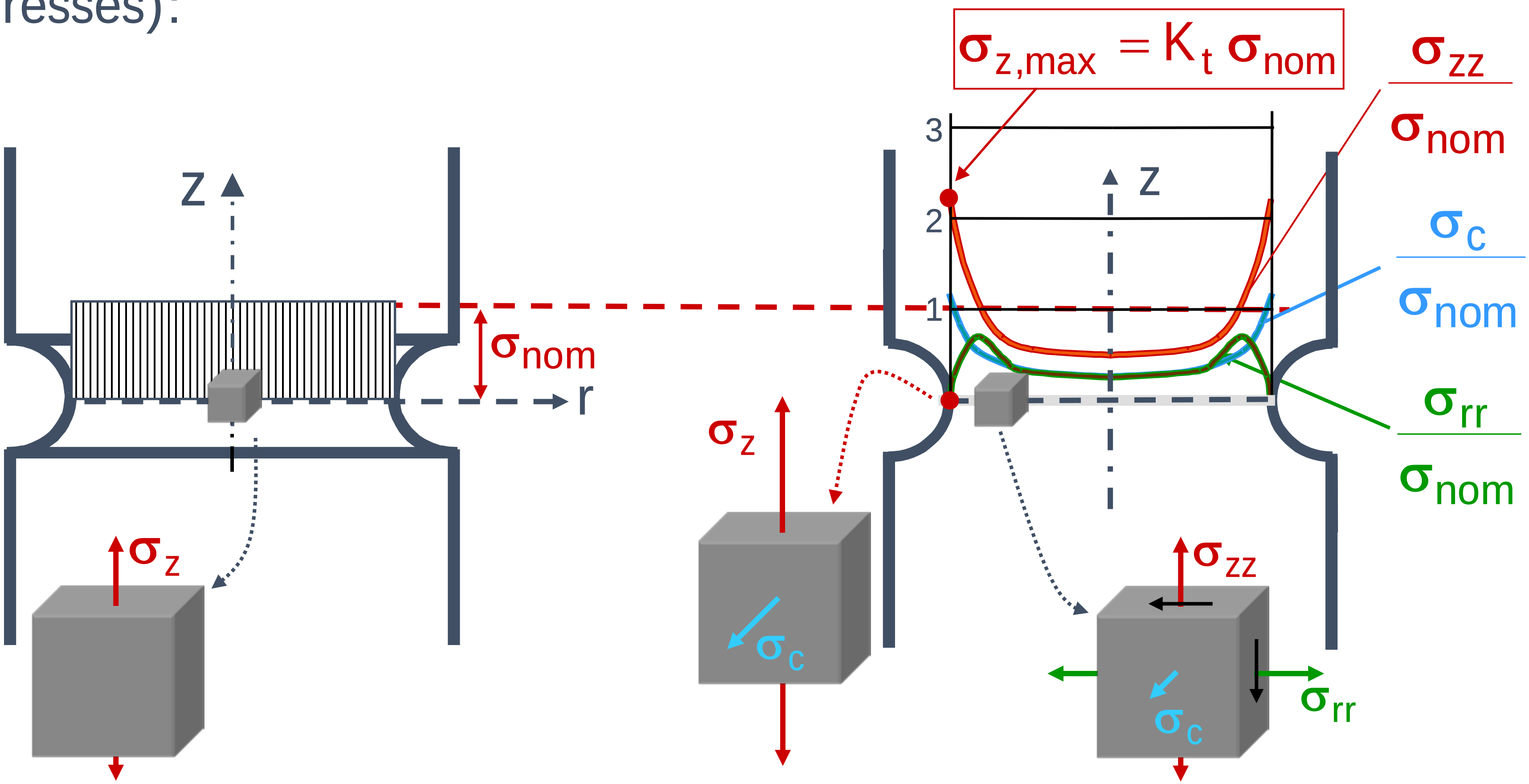


An example: groove on a round bar; in the smallest section the unnotched "nominal stress" σ_{nom} is calculated as:

$$\sigma_{nom} = \frac{4F}{\pi d^2} = \frac{1}{A}\int_A \sigma_z \, dA$$

The stress state in the un-notched bar in tension is one-dimensional, its axial stress σ_z is principal (i.e. non shear stresses):

The stress at the notch section is three dimensional, and no longer constant; shear stresses are present:



3. Component strength - Step 2 (14/28): K_t

The first mathematical treatments of stress concentrations were published in 1937 (Neuber).

Closed form solutions are available for the very simple geometries.

However, for more complex cases, experimental methods for measuring highly localized stresses (photoelastic tests, precision strain gage tests, membrane analogy for torsion, etc.) and computerized finite element solutions have been used.

Neuber, H. (1937) "Theory of Notch Stresses," J. W. Edwards (Ann Arbor, MI), [original:1937, Kerbspannungslehre, Springer, Berlin]

Neuber, H. (1946) Theory of Notch Stresses: Principle for Exact Stress Calculations, J.W. Edwards (Ann Arbor, MI)

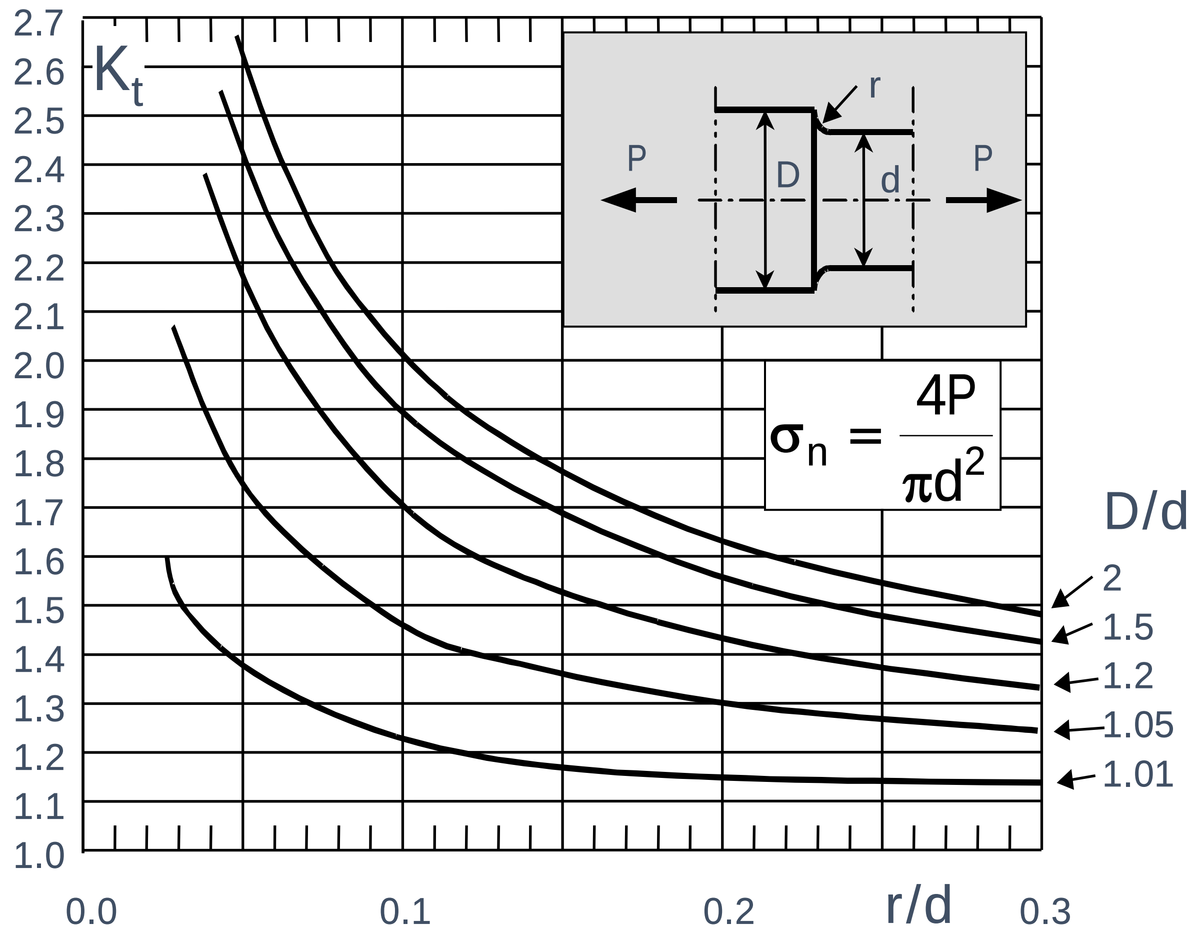
Peterson, R. E. (1953) Stress Concentration Design Factors, John Wiley and Sons (New York)

Peterson, R. E. (1974) Stress Concentration Factors, John Wiley and Sons (New York)

The "(theoretical) stress concentration factor": $K_t = \dfrac{\sigma_{z,max}}{\sigma_{nom}}$ is found in diagrams like the one below:

It is a "shape factor" as it does nor depend on absolute dimensions, but only on ratios.

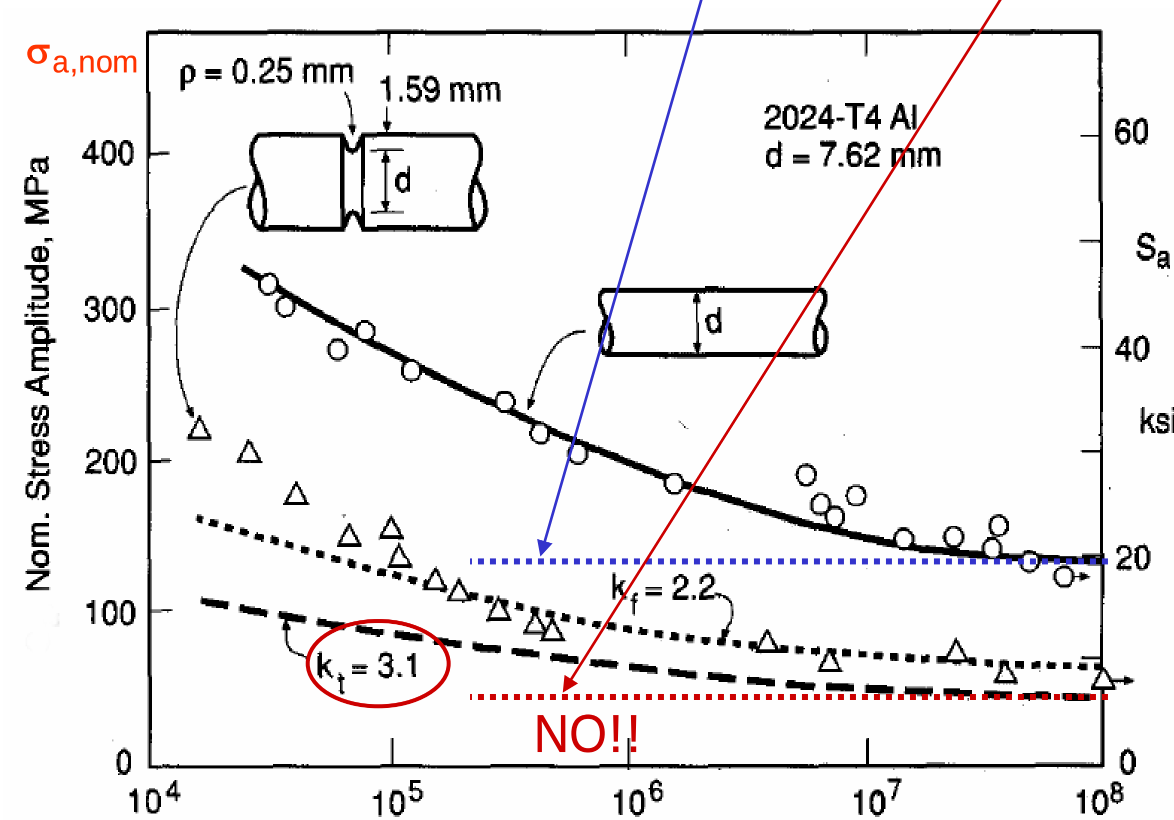
The figure to the right shows one of the many possible cases (shoulder on a round shaft, axial load).



If $\sigma_{a,max}^{notch} \equiv \sigma_{a,z,max}$ were the stress responsible for fatigue, it would be (wrongly) expected that:

$$\sigma_{a,max}^{notch} = \sigma_{a,nom} K_t \leq \sigma_{D-1}^{tc} \Rightarrow \sigma_{a,nom} \leq \frac{\sigma_{D-1}^{tc}}{K_t}$$

The figure shows that for the same min. diameter d the notched bar does not have a fatigue limit (expressed in nominal stresses) K_t=3.1 times lower but only K_f=2.2 times lower, i.e. nominal stresses are higher.



(Δ: experimental points)

Mechanical Behavior of Materials, N. Dowling, 1993

It means that also in this case, where a robust gradient is present, it is not the maximum stress which governs fatigue, but some “effective stress” , which has a lower value.

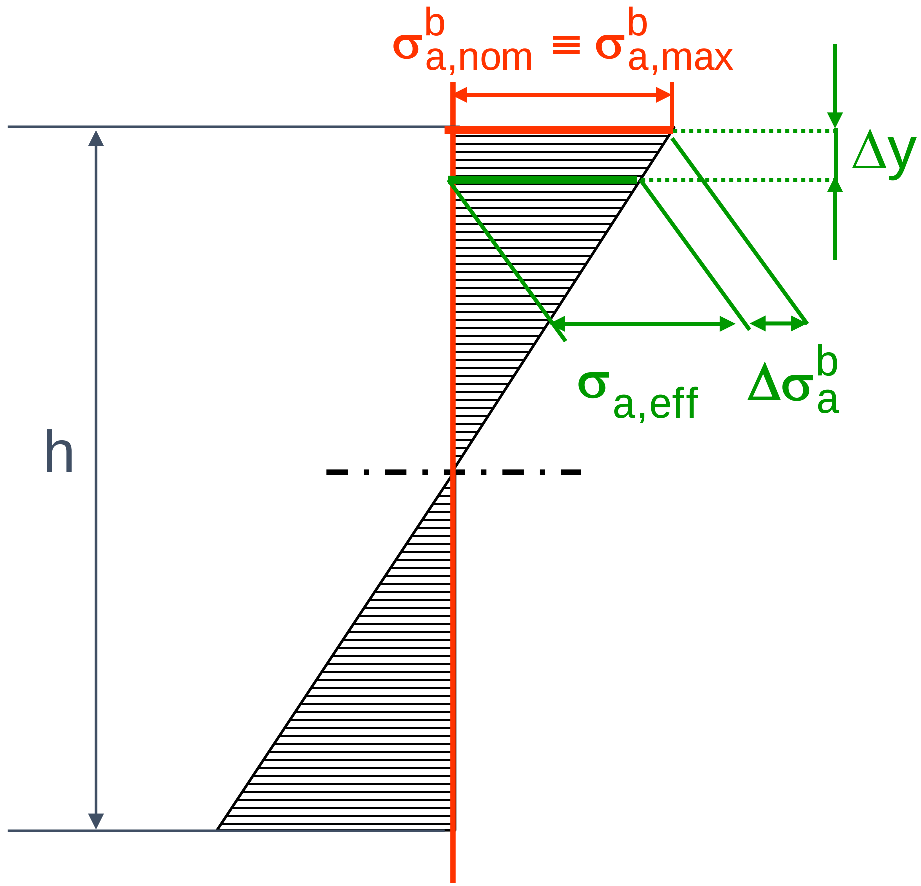
It is therefore reasonable to apply to the case of notch effect the same treatment which was used to find an explanation for the size effect.

It is what we shall do in the next slides. But before going to them, a word of warning.

The nominal stress is defined as the maximum stress calculated by the traction, or bending, or torsion formulas, for the section of lowest area, without notch (beam formula).

$$\sigma_{a,nom}^{tc} = \frac{4F}{\pi d^2} \qquad \sigma_{a,nom}^{b} = \frac{M_b}{W_b} \qquad \tau_{a,nom}^{t} = \frac{M_t}{W_t}$$

In the case of pure bending, the stress varies linearly along the height of the cross section; it is convenient to define the relative stress gradient χ [mm^{-1}] :

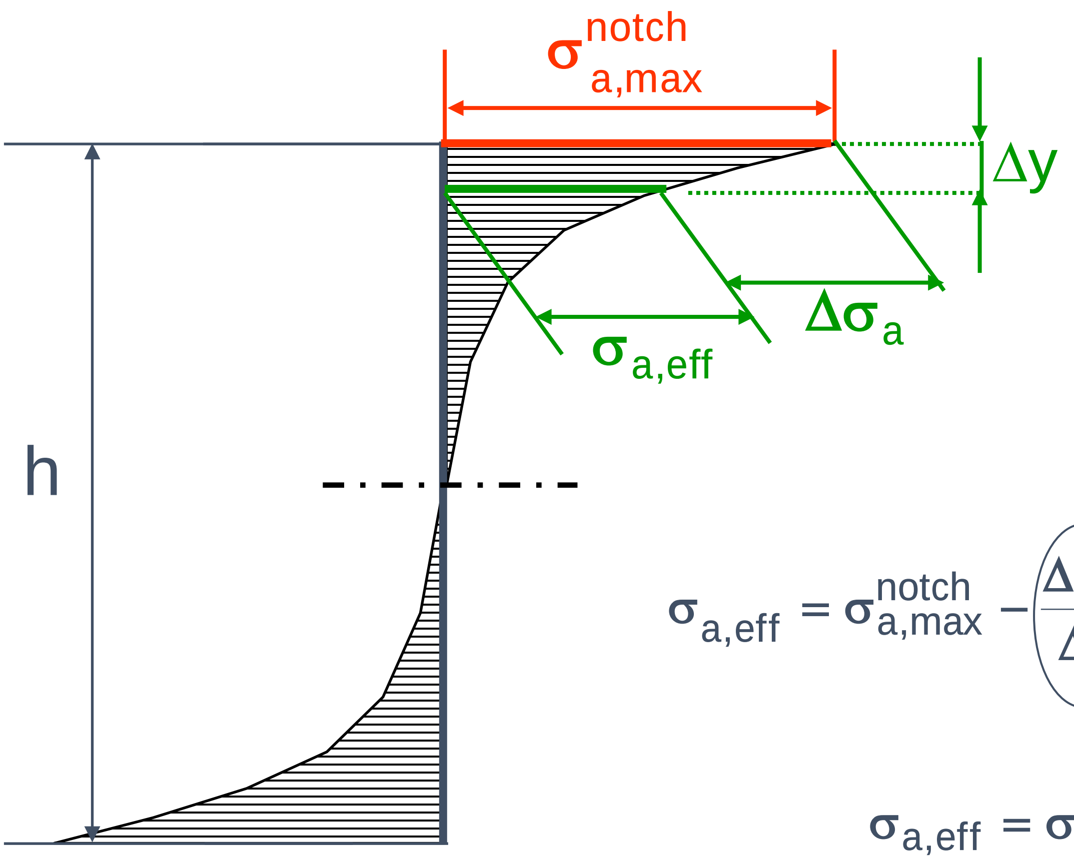


$$\chi = \frac{d\sigma}{dy}\,\frac{1}{\sigma^b_{a,max}} = \frac{\sigma^b_{a,max}}{h/2}\,\frac{1}{\sigma^b_{a,max}} = \frac{2}{h}$$

$$\sigma_{a,eff} = \sigma^b_{a,max} - \frac{d\sigma^b_a}{dy}\,\Delta y$$

$$\sigma_{a,eff} = \sigma^b_{max}\left(1 - \chi\,\Delta y\right) \cong \frac{\sigma^b_{max}}{\left(1 + \chi\,\Delta y\right)}$$

The relative stress gradient χ is equal for all stress gradient forms, i.e. it does not change if stresses are multiplied by a factor:

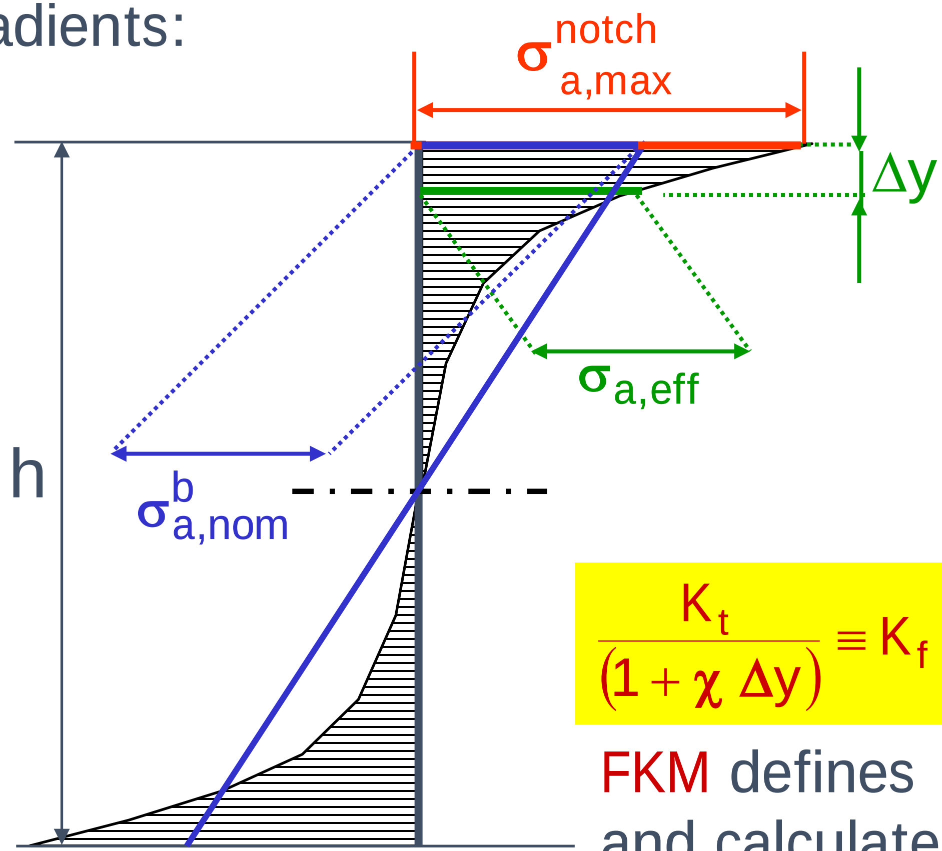


$$\chi \cong \frac{\Delta\sigma_a}{\Delta y}\frac{1}{\sigma^{notch}_{a,max}}$$

$$\sigma_{a,eff} = \sigma^{notch}_{a,max} - \frac{\Delta\sigma_a}{\Delta y}\Delta y \Rightarrow \frac{\sigma_{a,eff}}{\sigma^{notch}_{a,max}} = 1 - \chi\Delta y$$

$$\sigma_{a,eff} = \sigma^{notch}_{a,max}\left(1 - \chi\,\Delta y\right) \cong \frac{\sigma^{notch}_{a,max}}{\left(1 + \chi\,\Delta y\right)}$$

Putting together items discussed so far concerning notch gradients:



Maximum local stress:

$$\sigma^{notch}_{a,max} = K^b_t \, \sigma_{nom}$$

nominal stress

Effective local stress:

$$\sigma_{a,eff} = \sigma^{notch}_{a,max}(1 - \chi\,\Delta y) \cong$$

$$\cong \frac{\sigma^{notch}_{a,max}}{(1+\chi\,\Delta y)} = \frac{K_t}{(1+\chi\,\Delta y)}\sigma_{nom}$$

$$\frac{K_t}{(1+\chi\,\Delta y)} \equiv K_f$$ is the (fatigue) notch factor

FKM defines $(1+\chi\Delta y)$ with the symbol n_σ and calculates it, according to Siebel and Stieler, with a modified formula (see later).

Since, as shown before, it is $\sigma_{a,eff}$ that governs fatigue,

$$\sigma_{a,eff} = \frac{\sigma_{a,max}^{notch}}{(1+\chi\,\Delta y)} = \frac{K_t \sigma_{a,nom}}{(1+\chi\,\Delta y)} \le \sigma_{D-1}^{tc} \quad \Rightarrow \quad \sigma_{a,nom} \frac{K_t}{(1+\chi\,\Delta y)} \le \sigma_{D-1}^{tc}$$

i.e.

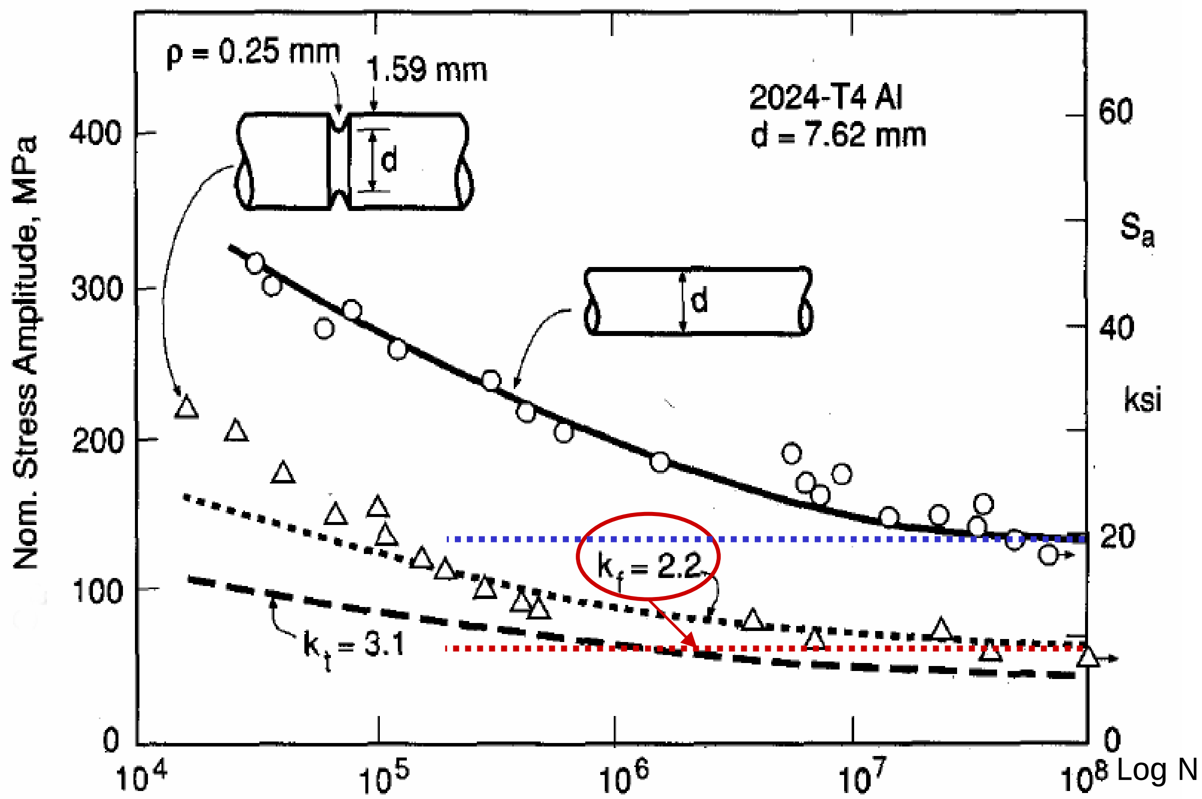
$$\sigma_{a,nom} K_f \le \sigma_{D-1}^{tc}$$

with:

$$\boxed{K_f = \frac{K_t}{1+\chi\Delta y}}$$

Remark!!

$$1 \le K_f \le K_t$$



FKM has a more rational approach, based on the relative gradient, which includes both notch and component size effects.

However it must be stressed that both the notch sensitivity and the stress gradient approach need provision of graphs or formulas to calculate the stress concentration factors K_t . These are commonly available with no difficulty in a number of books and manuals. FKM itself provides both graphs and formulas.

In the case of rods and bars, it must be kept in mind that K_t is different for:

tc: tension-compression or axial load

b : bending

t : torsion

Therefore it is necessary to define different values for: K_t^{tc}, K_t^{b}, K_t^{t}

We shall here cover the case of rods and bars, which is general enough to understand how to use the method.

Once the appropriate K_t has been obtained, according to FKM K_f is calculated as*:

$$K_f^{tc} = \frac{K_t^{tc}}{n_\sigma(r)}$$

$$K_f^{b} = \frac{K_t^{b}}{n_\sigma(r)\; n_\sigma(d)}$$

$$K_f^{t} = \frac{K_t^{t}}{n_\tau(r)\; n_\tau(d)}$$

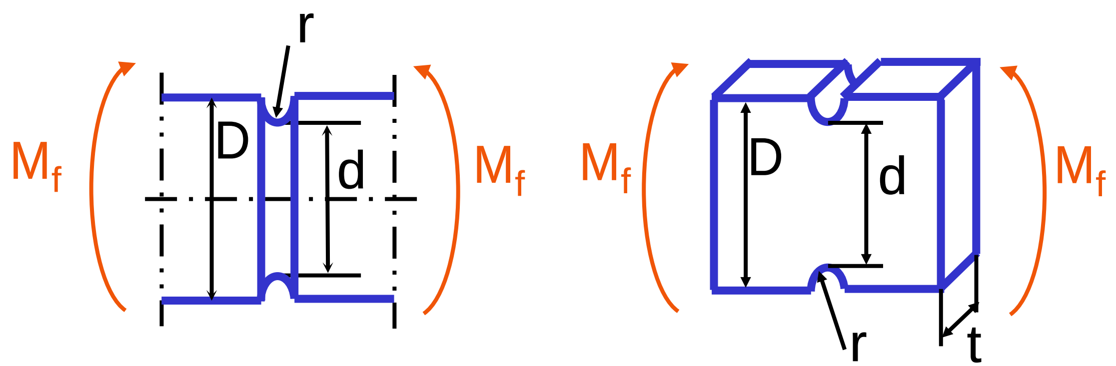
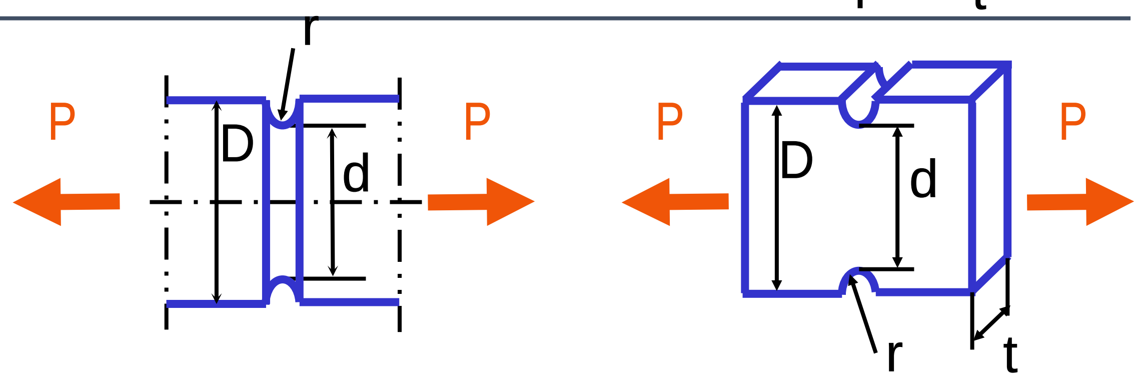
where the denominators mean:

$n_\sigma(r), n_\tau(r)$: factors, for σ , τ due to gradient at notch radius

$n_\sigma(d), n_\tau(d)$: factors, for σ or τ, due to component-size-related gradient (bending, torsion)

* adapted from FKM

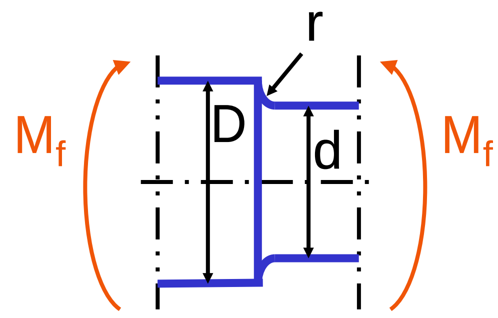
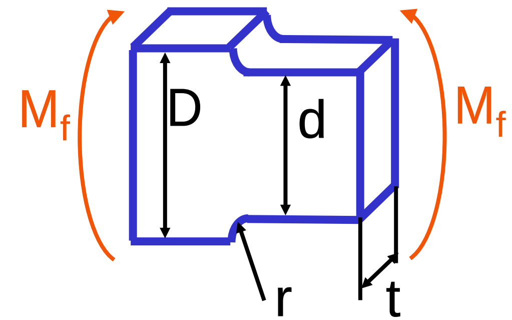
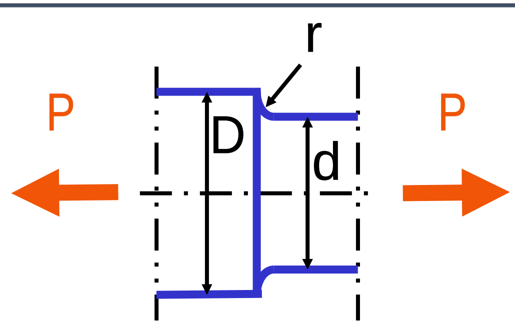
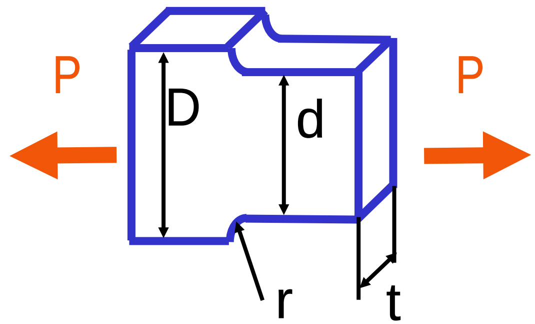
FKM first calculates separately relative gradients for notch readius (r) and component size (d)* effects:

$\chi_\sigma(r)$	$\chi_\sigma(d)$	
$\frac{2}{r}$	$\frac{2}{d}$	
$\frac{2}{r}$	0	

a) groove on shaft and flat bar, normal stress σ (*continues*)

* adapted from FKM

... (*continued*) relative gradients for (r) and (d)*...

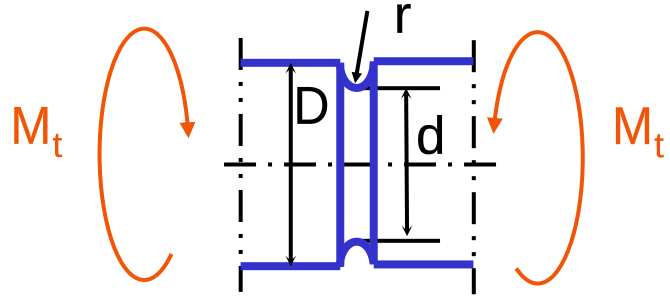
$\chi_\sigma(r)$	$\chi_\sigma(d)$		
$\frac{2.3}{r}$	$\frac{2}{d}$		
$\frac{2.3}{r}$	0		

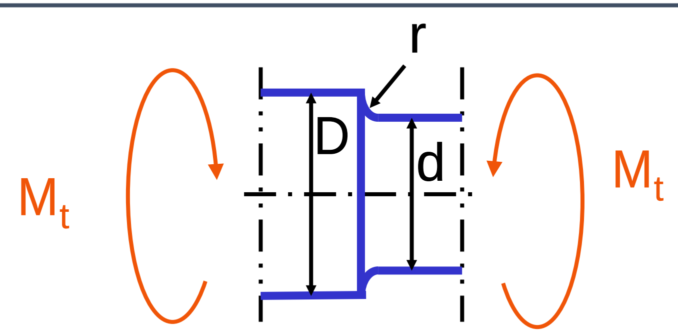
b) shoulder on shaft and flat bar, normal stress σ * (*continues*)

* these values are a selection from the original FKM values

... (*continued*) relative gradients for (r) and (d)*

$\chi_\tau(r)$	$\chi_\tau(d)$
$\frac{1}{r}$	$\frac{2}{d}$
$\frac{1.15}{r}$	$\frac{2}{d}$





c) groove and shoulder on shaft, shear stress τ* (*continues*)

* these values are reduced from the original FKM values

... (*continued*) then $n_{\sigma,\tau}(r)$ and $n_{\sigma,\tau}(d)$ are calculated with the following formulas:

$$\text{for}: \chi \le 0{,}1\ \text{mm}^{-1}$$

$$n_{\sigma,\tau} = 1 + \chi_{\sigma,\tau}\ 10^{-\left(a_G - 0{,}5 + \frac{R_m}{b_G}\right)}$$

$$\text{for}: 0{,}1\ \text{mm}^{-1} \le \chi \le 1\ \text{mm}^{-1}$$

$$n_{\sigma,\tau} = 1 + \sqrt{\chi_{\sigma,\tau}}\ 10^{-\left(a_G + \frac{R_m}{b_G}\right)}$$

$$\text{for}: 1\ \text{mm}^{-1} \le \chi \le 100\ \text{mm}^{-1}$$

$$n_{\sigma,\tau} = 1 + \sqrt[4]{\chi_{\sigma,\tau}}\ 10^{-\left(a_G + \frac{R_m}{b_G}\right)}$$

WARNING: when r= 0, i.e. the notch does not exist, use χ (r) =0

	stainless steel	other steels	GS	GGG	GT	GG	wrought Al-alloys	cast Al-alloys
a_G	0,40	0,50	0,25	0,05	-0,05	-0,05	0,05	-0,05
b_G	2400	2700	2000	3200	3200	3200	850	3200

In summary,

the "effective" (for fatigue) applied time-varying stresses are for each stress case:

- axial: $\sigma^{tc}_{eff}(t) = \sigma^{tc}_{n}(t)\, K^{tc}_{f}$ $\qquad K^{tc}_{f} = \dfrac{K^{tc}_{t}}{n_\sigma(r)}$

- bending: $\sigma^{b}_{eff}(t) = \sigma^{b}_{n}(t)\, K^{b}_{f}$ $\qquad K^{b}_{f} = \dfrac{K^{b}_{t}}{n_\sigma(r)\; n_\sigma(d)}$

- torsion: $\tau^{t}_{eff}(t) = \tau^{t}_{n}(t)\, K^{t}_{f}$ $\qquad K^{t}_{f} = \dfrac{K^{t}_{t}}{n_\tau(r)\; n_\tau(d)}$

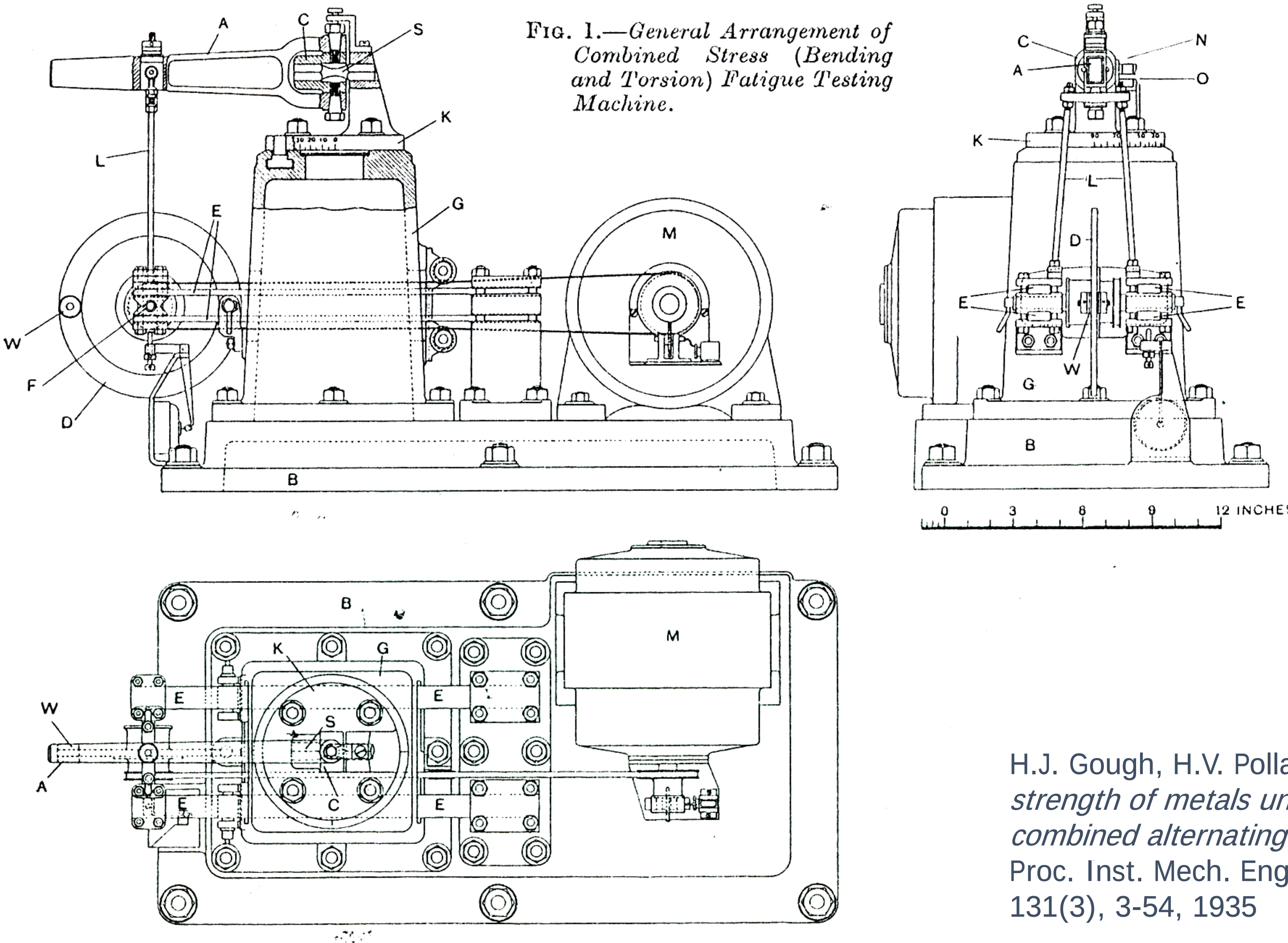
nominal stresses from un-notched beam and rod formulas

The normal and tangential stresses, either at a notch or simply on an un-notched component, can be applied either individually or, as it most often happens, combined.

In this latter case a rule must be found to reduce the combination of a combination of fatigue stresses of different nature to one equivalent tensile stress in fatigue.

Gough and Pollard's works on biaxial fatigue (from the 1920's to the 50's) produced their "rule" for combined bending and torsion, which is an empirical relation found through laboratory experiments. This was done combining normal stresses due to bending with tangential stresses due to torsion.

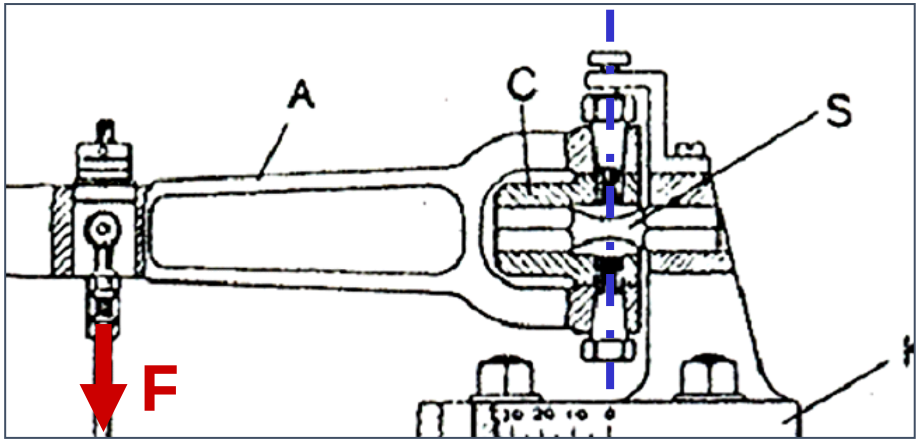
4. Component strength - Step 3 (2/12): σ_a and τ_a

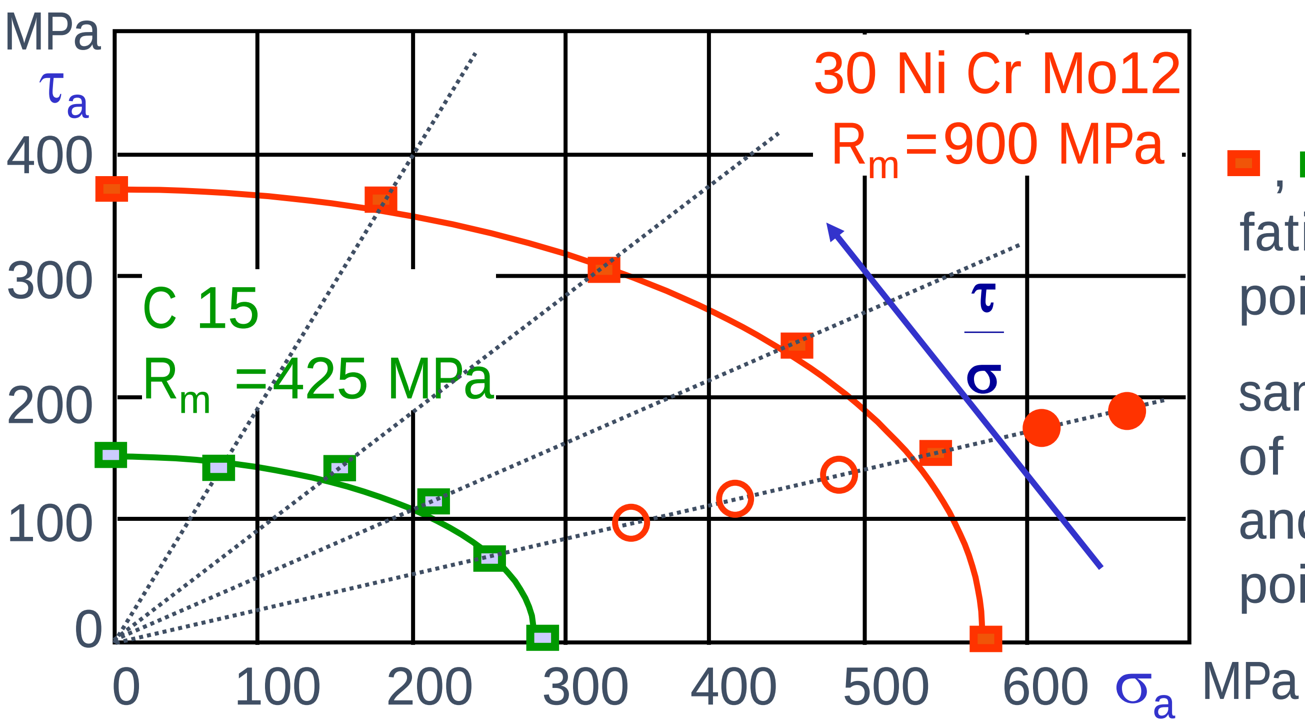


H.J. Gough, H.V. Pollard, *The strength of metals under combined alternating stresses*, Proc. Inst. Mech. Engrs 131(3), 3-54, 1935

4. Component strength - Step 3 (3/12): σ_a and τ_a

On their special testing machine they could set a given τ/σ ratio just by pivoting the pull arm A about axis — · — · — and thus applied alternating σ_a and τ_a, in phase (proportional stresses);
by increasing the pulling force **F** for a given combination they produced:





■, ■: combined fatigue limit points;

sample of infinite life ○ and finite life ● points

Ductile Materials : elliptical interpolation of experimental data

$$\left(\frac{\sigma_a^b}{\sigma_{D-1}^b}\right)^2 + \left(\frac{\tau_a^t}{\tau_{D-1}^t}\right)^2 \le 1 \qquad {\sigma_a^b}^2 + \left(\frac{\sigma_{D-1}^b}{\tau_{D-1}^t}\right)^2 {\tau_a^t}^2 \le {\sigma_{D-1}^b}^2$$

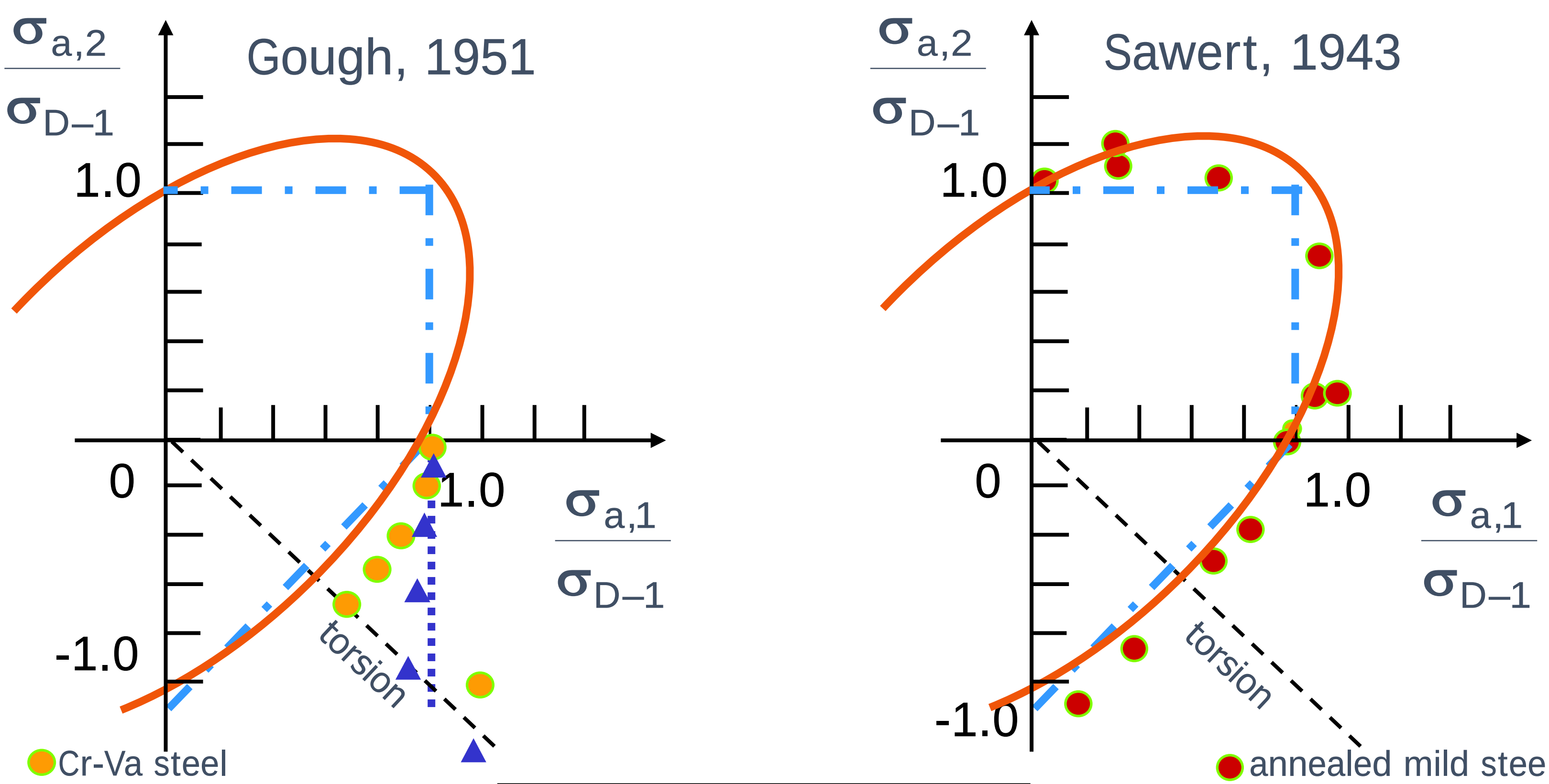
From experimental evidence

$$\boxed{\tau_{D-1}^t \cong 0.6 \cdot \sigma_{D-1}^b \cong \frac{\sigma_{D-1}^b}{\sqrt{3}}} \longrightarrow \underbrace{\sqrt{{\sigma_a^b}^2 + 3{\tau_a^t}^2}}_{\sigma_{a,eq}} \le \sigma_{D-1}^b$$

which is a Von Mises-like formula

For brittle materials the interpolation was a bit more complex:

$$\left(\frac{\tau_a^t}{\tau_{D-1}^t}\right)^2 + \left(\frac{\tau_a^t}{\sigma_a^b} - 1\right)\left(\frac{\sigma_a^b}{\sigma_{D-1}^b}\right)^2 + \left(2 - \frac{\sigma_{D-1}^b}{\tau_{D-1}^t}\right)\left(\frac{\sigma_a^b}{\sigma_{D-1}^b}\right) = 1$$



$\sigma_{1m}=\sigma_{2m}=0$; 10^7 cycles

Experimental evidence from two different independent sources confirm the von Mises formula for the biaxial case of two alternating principal stresses. (bending and torsion apex omitted)

4. Component strength - Step 3 (6/12): Sines formula

Sines (1953) generalised Gough and Pollard results into a three-dimensional formula after the observation of similarity between the empirical Gough ellipse and the von Mises theoretical formula at the onset of yield.
He then stated, in principal axes, the "equivalent" alternating (tensile) stress as:

$$\boldsymbol{\sigma}_{a,eq} = \frac{1}{\sqrt{2}}\sqrt{\left(\boldsymbol{\sigma}_{a,1} - \boldsymbol{\sigma}_{a,2}\right)^2 + \left(\boldsymbol{\sigma}_{a,1} - \boldsymbol{\sigma}_{a,3}\right)^2 + \left(\boldsymbol{\sigma}_{a,2} - \boldsymbol{\sigma}_{a,3}\right)^2}$$

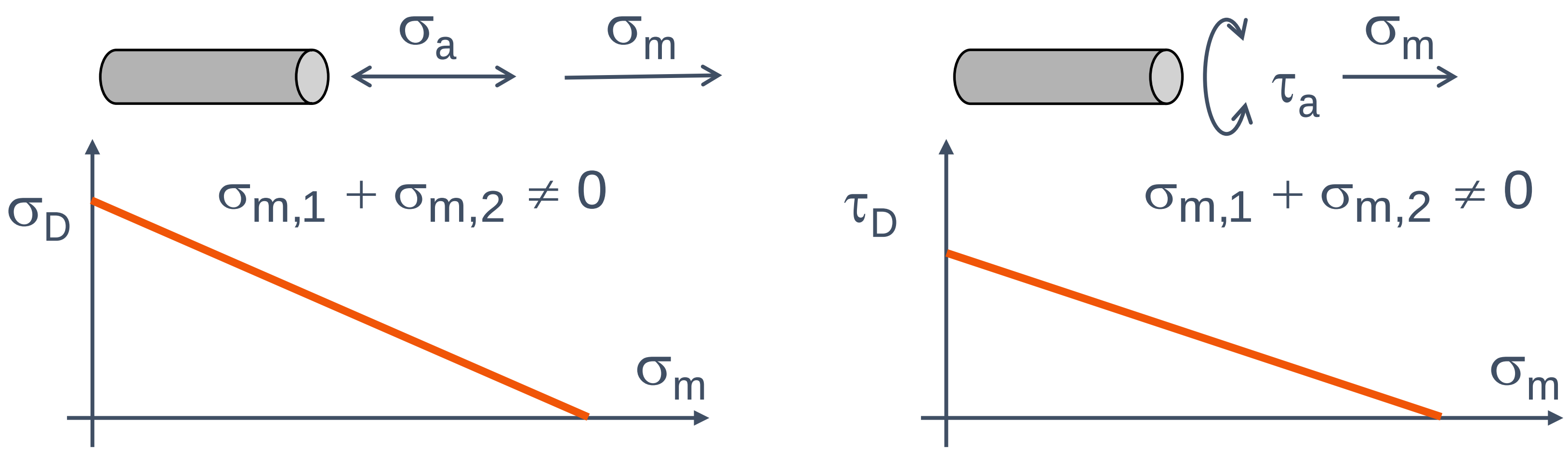
However, he warned that this was confirmed by experiment only in one-dimensional and two-dimensional cases.

For beam-like and rod-like structures we normally use the following version of the von Mises formula; it gives the "equivalent" (combined) value of "effective" tensile and tangential stresses:

$$\boldsymbol{\sigma}_{a,eq,eff} = \sqrt{\left(\boldsymbol{\sigma}_{a,nom}^{tc} K_f^{tc} + \boldsymbol{\sigma}_{a,nom}^{b} K_f^{b}\right)^2 + 3\left(\boldsymbol{\tau}_{a,nom}^{t} K_f^{t}\right)^2}$$

Up to now stresses have been taken alternating (R=-1).

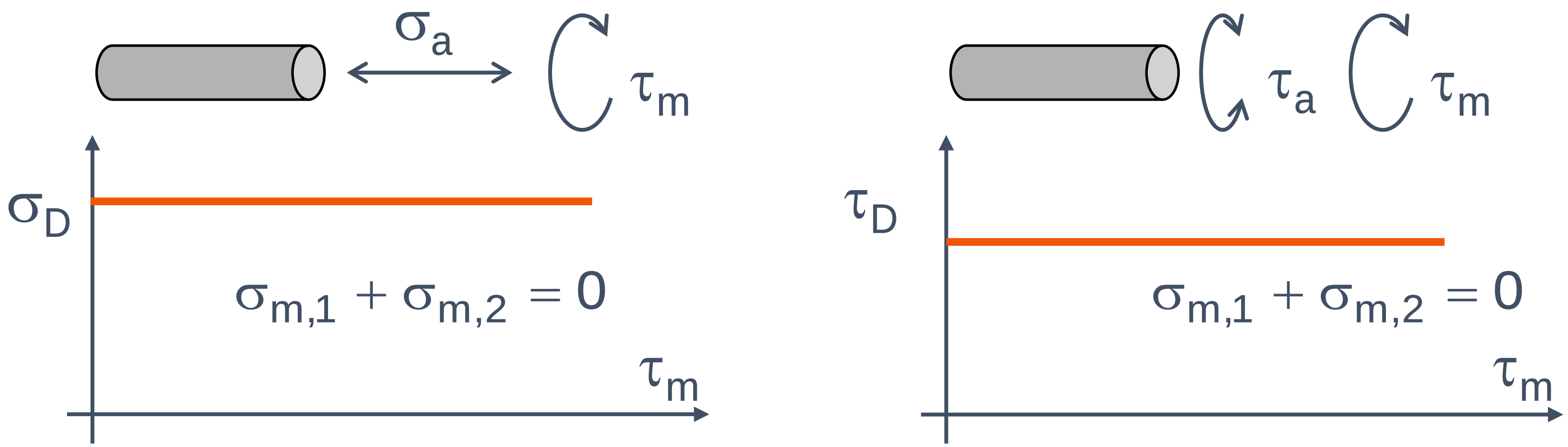
Evidence * shows that the presence of a mean stress σ_m strongly influences the value of σ_D , τ_D .



This is a uniaxial case ... ... this is biaxial !

(bending an torsion indications omitted)

* Sines G., Behavior of Metals under Complex Static and Alternating Stresses, ch. 7 in ”Metal Fatigue”, G. Sines and J.L. Waisman editors, Mc Graw Hill, USA, (1959)



A different case (Sines, 1959) is shown above: for the biaxial case of mean torsion (where, notice, the two principal stresses are equal in value and opposite in sign) increasing values of τ_m do not (or do very little) influence the value of σ_D , τ_D . Of course, the figure above is an extreme simplification of experimental evidence.

(bending an torsion apexes omitted)

4. Component strength - Step 3 (9/12): effect of τ_m

The difference between the first two examples can be made more general, in order to produce a formula which will represent all possible cases.

In the case of mean torsion, the sum of principal stresses (i.e. the first invariant of the “mean stress” tensor, i.e. the trace of the “mean stress” matrix) is zero.

In the case of pure tension, the sum of principal stresses is non-zero, i.e. σ_m.

We might remain just happy with this: the mean tensile stress influences Haigh diagram, the mean torsional shear stress does not.

However, in the case of not just one, but two principal tensile stresses, or in a mixed tension-torsion case we may infer that what matters is the sum of principal stresses, i.e., the first invariant, and develop a formula accordingly.

Then, generalising all these results, the “uniaxial formula”, i.e. the expression of the fatigue limit when stress is uniaxial:

$$\sigma_a + M_\sigma \sigma_m \le \sigma_{C,D-1}$$

becomes in three dimensions (in principal axes):

Sines formula

$$\sigma_{a,eq} + M_\sigma \cdot \sigma_{m,eq} =$$

$$= \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{a,1} - \sigma_{a,2}\right)^2 + \left(\sigma_{a,1} - \sigma_{a,3}\right)^2 + \left(\sigma_{a,2} - \sigma_{a,3}\right)^2} +$$

$$+ M_\sigma \cdot \underbrace{\left(\sigma_{m,1} + \sigma_{m,2} + \sigma_{m,3}\right)}_{\text{proportional to the 1st invariant of the mean stress tensor}} \le \sigma_{C,D-1}$$

In the case of bar and rods, where the biaxial stress state is given by axial+bending normal stresses and by torsional shear stresses, the available evidence supports the following formula written in non-principal axes:

$$\sigma_{a,eq,eff} + M_\sigma \, \sigma_{m,eq,eff} \le \sigma_{C,D-1}$$

$$\sqrt{\left(\sigma_a^{tc} K_f^{tc} + \sigma_a^{b} K_f^{b}\right)^2 + 3\left(\tau_a^{t} K_f^{t}\right)^2} + M_\sigma \left(\sigma_m^{tc} K_f^{tc} + \sigma_m^{b} K_f^{b}\right) \le \sigma_{C,D-1}$$

nominal stresses, suffixes $\ldots_{,nom}$ omitted for brevity

Equivalent and effective alternating stresses, effective mean stresses are put respectively on the vertical and horizontal axes of a Haigh diagram.

An additional comment on the value of M_σ .

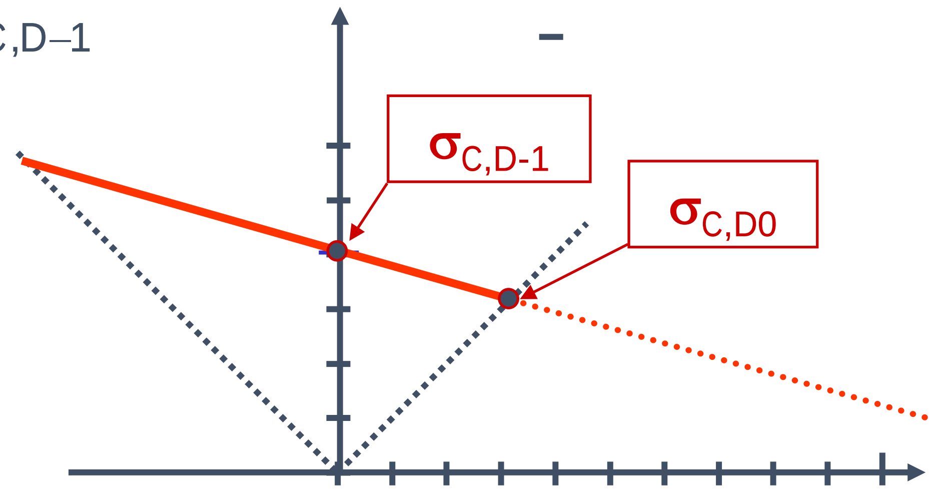
The formula: $\sigma_a + M_\sigma \, \sigma_m = \sigma_{C,D-1}$ is evidently valid for the case:

... where obviously: $\sigma_m = 0 \;\Rightarrow\; \sigma_{a,eq} \le \sigma_{C,D-1}$

as well as for the case: $\sigma_{min} = 0 \rightarrow \sigma_a = \sigma_m$

which at failure gives: $\sigma_a = \sigma_m = \sigma_{C,D0}$ (stress ratio R=0)

where: $\sigma_{C,D0} + M_\sigma \, \sigma_{C,D0} = \sigma_{C,D-1}$



This has the consequence:

$M_\sigma = \dfrac{\sigma_{D-1}}{\sigma_{D0}} - 1$; compare

with Ch.2-Sect.9-Sl.3

Sections 5, 6 - How to design for endurance to fatigue

Section 5 collects all the concepts developed so far and shows how a fatigue safety factor is chosen and used in the context of the Haigh graphical representation.

Section 6 shows how the same concepts are used by FKM in tw-dimensional cases, in particular a different treatment of the shear stress due to torsion.

Section 7 , an Appendix, gives a sample of K_t (stress concentration factor) diagrams.

5. Final endurance assessment (1/4): safety factor

When the fatigue limit is known with an average probability of survival at 50%, the customary safety factor is $S_D=3$.

According to FKM:

- if material data are known with 97.5% probability of survival,
- if design loads are reliably determined on the safe side,

then the basic safety factor for design at σ_D is: $S_D=1.5$

However, it can be reduced under favorable conditions, as shown in the table below:

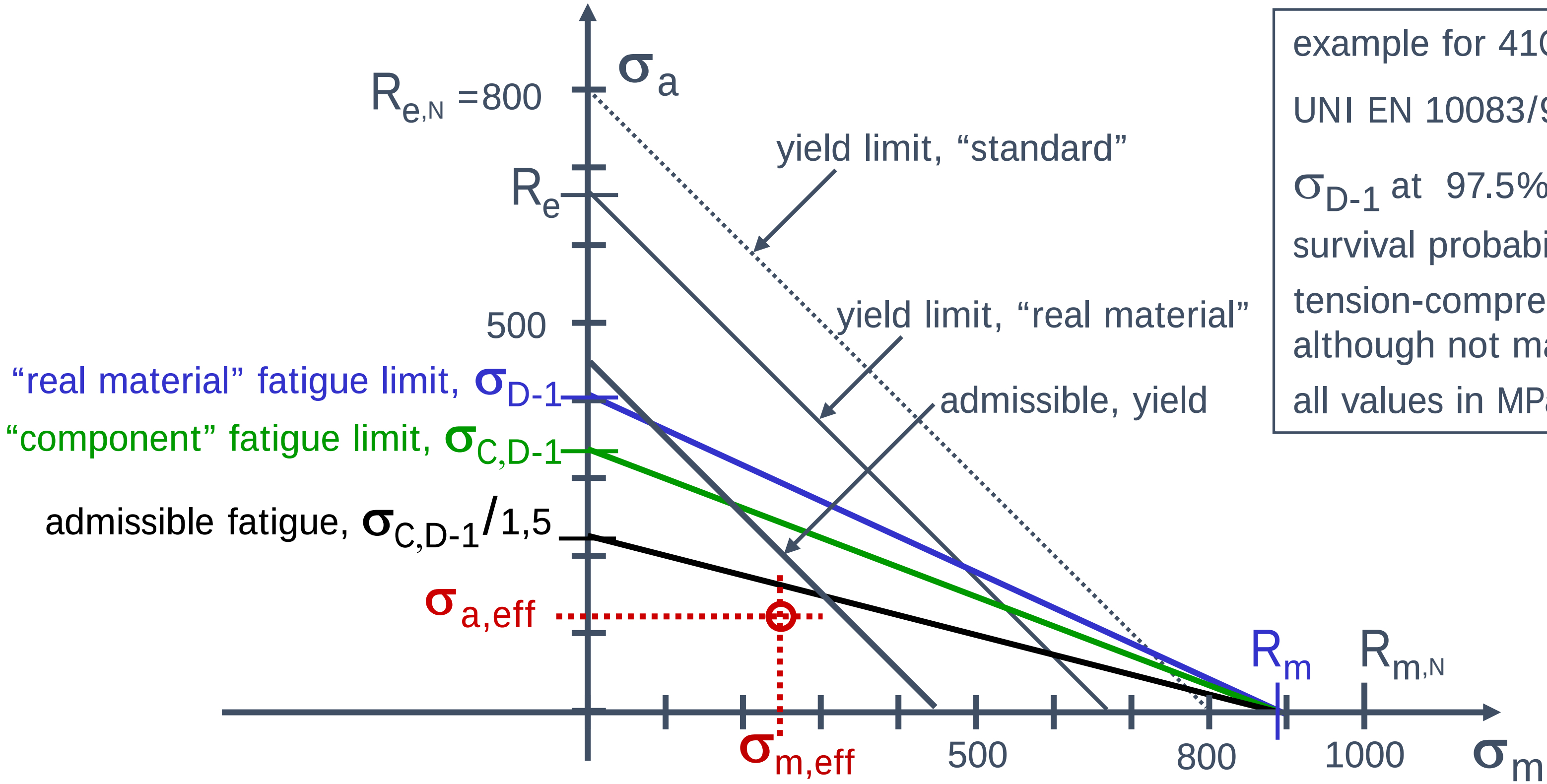
S_D		consequences of failure	
		severe	moderate
regular inspections	no	1.5	1.3
	yes	1.35	1.2

source: FKM

S_D divides the limit stress to produce the “admissible” or “allowable” stress:

$$\sigma_{adm} = \sigma_{lim}/S_D$$

In the simple uniaxial case, $\sigma_{a,eff}$ and $\sigma_{m,eff}$ are to be put respectively on the vertical and on the horizontal axis of the Haigh diagram constructed on the basis of fatigue data for tension-compression (tc, no gradient); they represent the "effective" stresses to be compared to uniaxial fatigue tc limits.



example for 41Cr4

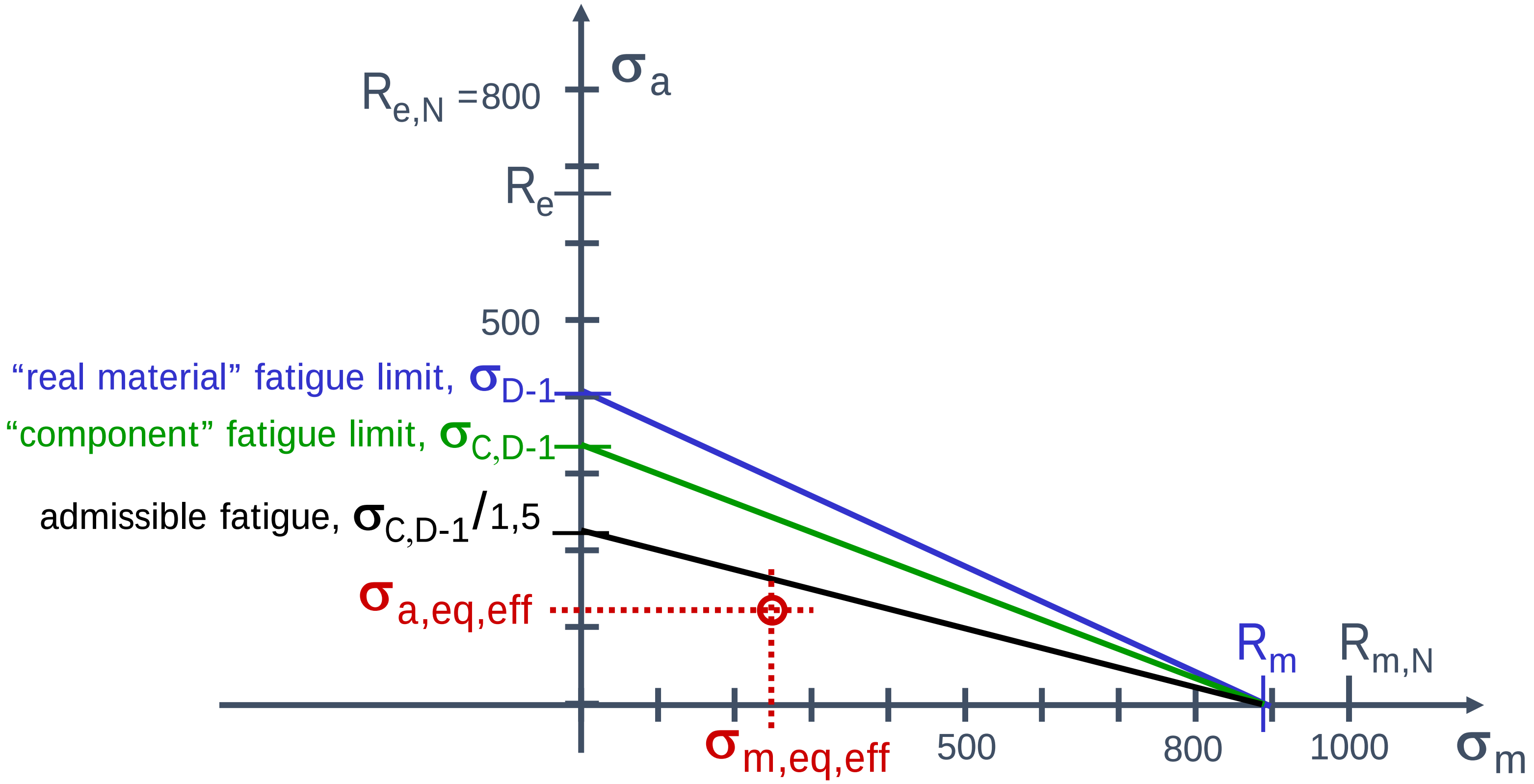
UNI EN 10083/96

σ_{D-1} at 97.5% survival probability;

tension-compression, although not marked;

all values in MPa

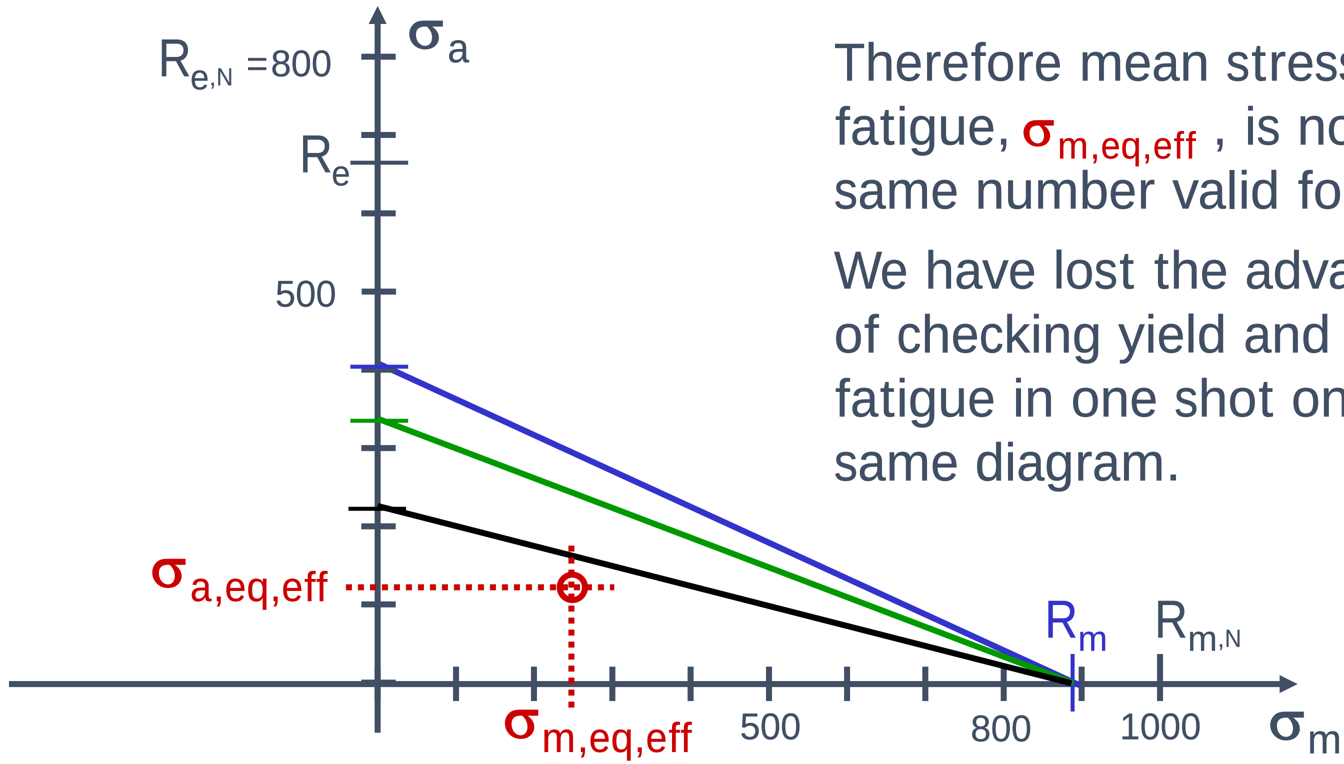
Also in the biaxial case, $\sigma_{a,eff}$ and $\sigma_{m,eff}$ are to be put respectively on the vertical and on the horizontal axis of the same Haigh diagram as before; however, now they represent the "equivalent" and "effective" stresses to be compared to uniaxial fatigue tc limits.



5. Final endurance assessment (4/4): biaxial

In the Haigh diagram for “combined” equivalent stresses the yield curves have no longer been represented: this is the consequence of the fact that the mean equivalent stress in the Haigh diagram is not calculated with a “von Mises like” formula: in fact no influence of τ_m on fatigue was included.

On the contrary, yield does require the effect of shear stresses.



Therefore mean stress for fatigue, $\sigma_{m,eq,eff}$, is not the same number valid for yield.

We have lost the advantage of checking yield and fatigue in one shot on the same diagram.

6. The FKM approach σ_m (1/2)

It is worth, at this point, to underline the FKM has a different approach to the approximation of the effect of mean stresses to the fatigue limit. This approach is on the safe side.

For the case of **steel and wrought aluminium alloys**, where $\tau_D = 0.577\ \sigma_D \equiv \sigma_D/\sqrt{3}$, the equivalent stress in the case of normal (tension plus bending) and shear (torsion) stresses is:

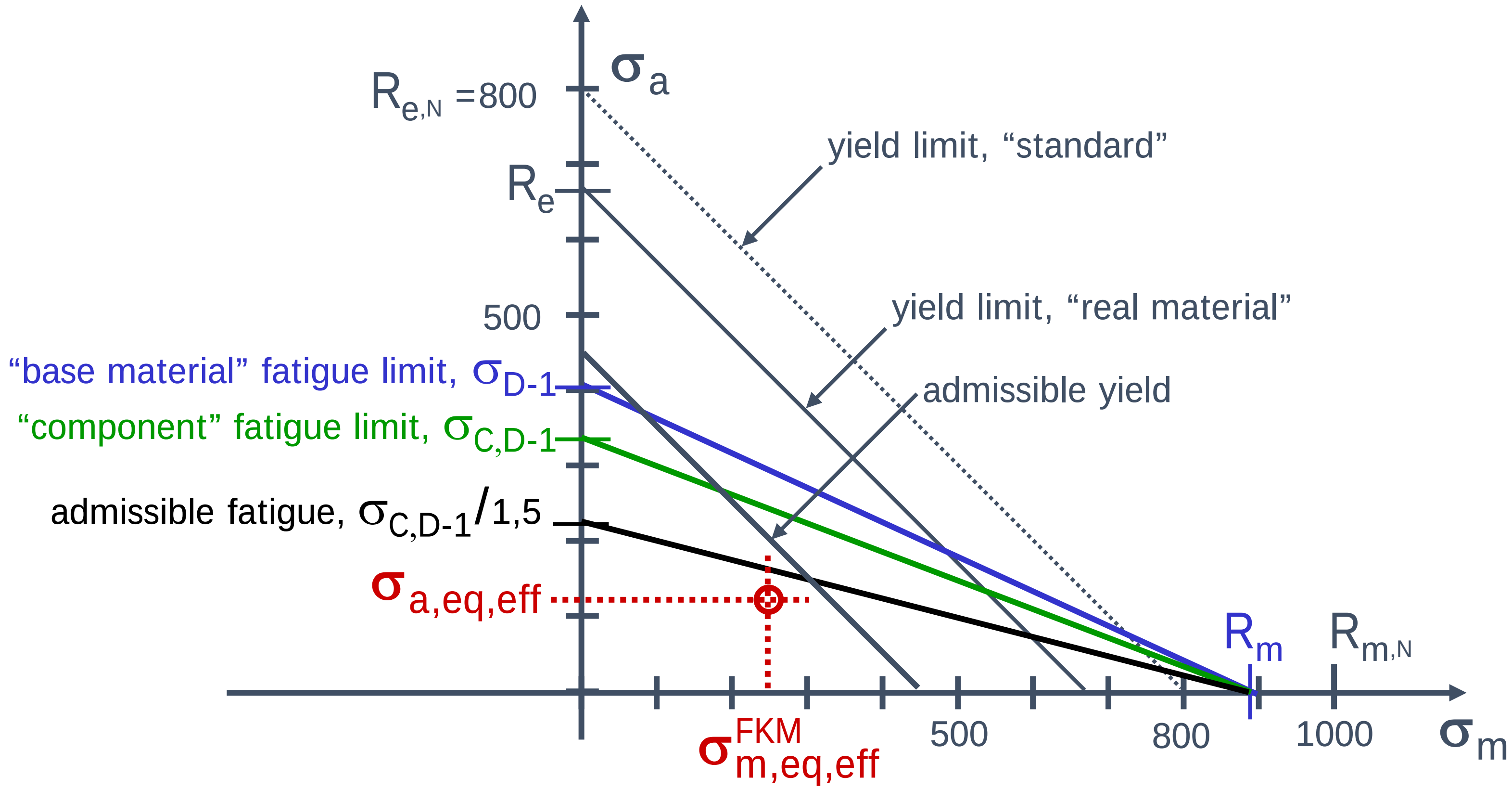
$$\sigma_{m,eq} = \sqrt{\sigma_m^2 + 3\tau_m^2}$$

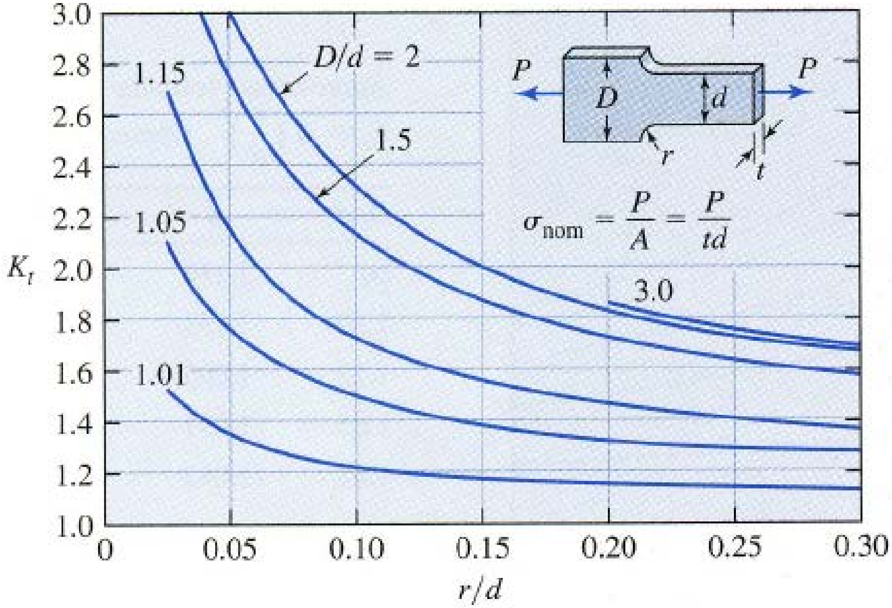
or, at the "effective" depth:

$$\sigma_{m,eq,eff}^{FKM} = \sqrt{\left(\sigma_m^{tc} K_f^{tc} + \sigma_m^{b} K_f^{b}\right)^2 + 3\left(\tau_m^{t} K_f^{t}\right)^2}$$

For other materials, more complex formulas are used; the reader is urged to refer to the original FKM source for details.

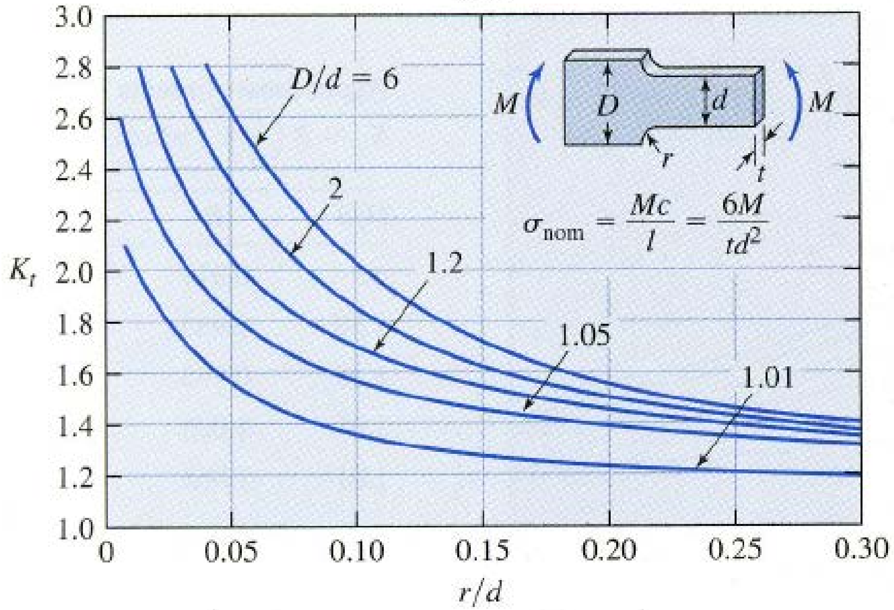
This is the same formula used to calculate von Mises equivalent stresses for yield. Therefore, with this approach both the fatigue and the yield assessment can be done on the same diagram with one applied stress point $(\sigma_{a,eq,eff}\ ;\ \sigma^{FKM}_{m,eq,eff})$.





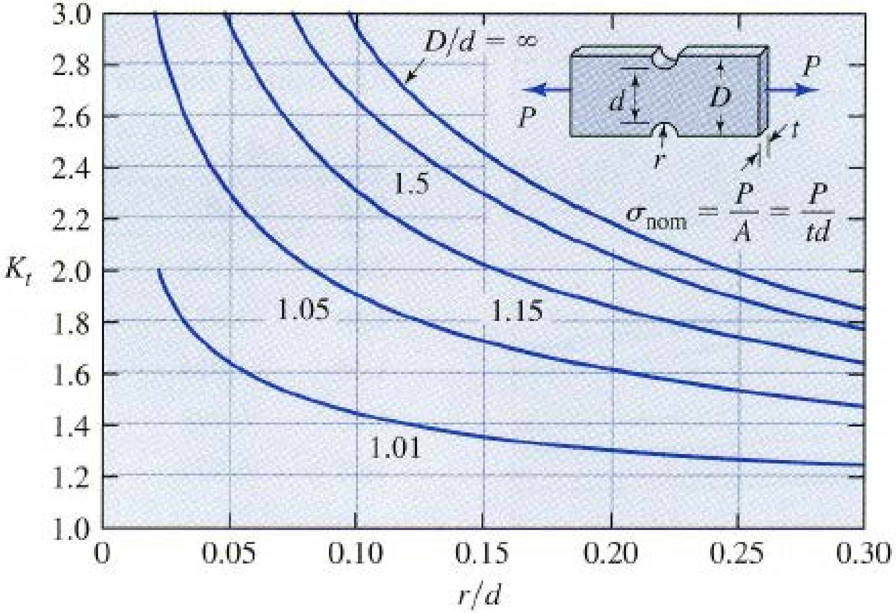
Approximate formula
$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

D/d	B	a
2.00	1.100	−0.321
1.50	1.077	−0.296
1.15	1.014	−0.239
1.05	0.998	−0.138
1.01	0.977	−0.107



Approximate formula
$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

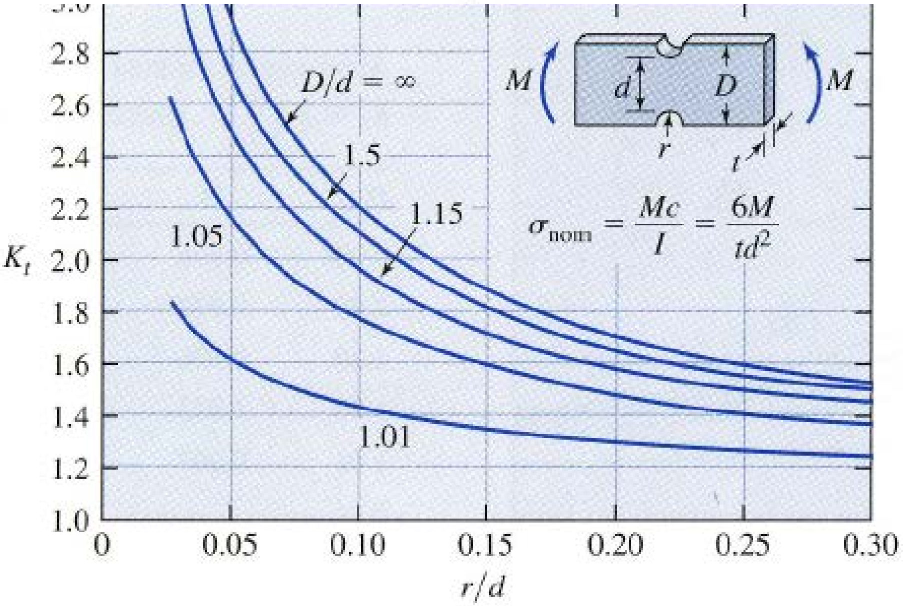
D/d	B	a
6.00	0.896	−0.358
2.00	0.932	−0.303
1.20	0.996	−0.238
1.05	1.023	−0.192
1.01	0.967	−0.154



Approximate formula

$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

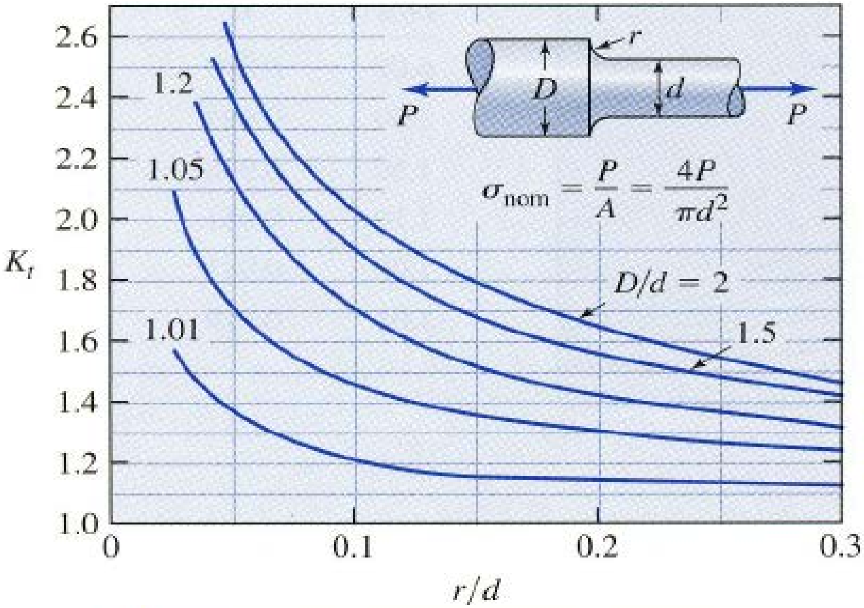
D/d	B	a
∞	1.110	−0.417
1.50	1.133	−0.366
1.15	1.095	−0.325
1.05	1.091	−0.242
1.01	1.043	−0.142



Approximate formula

$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

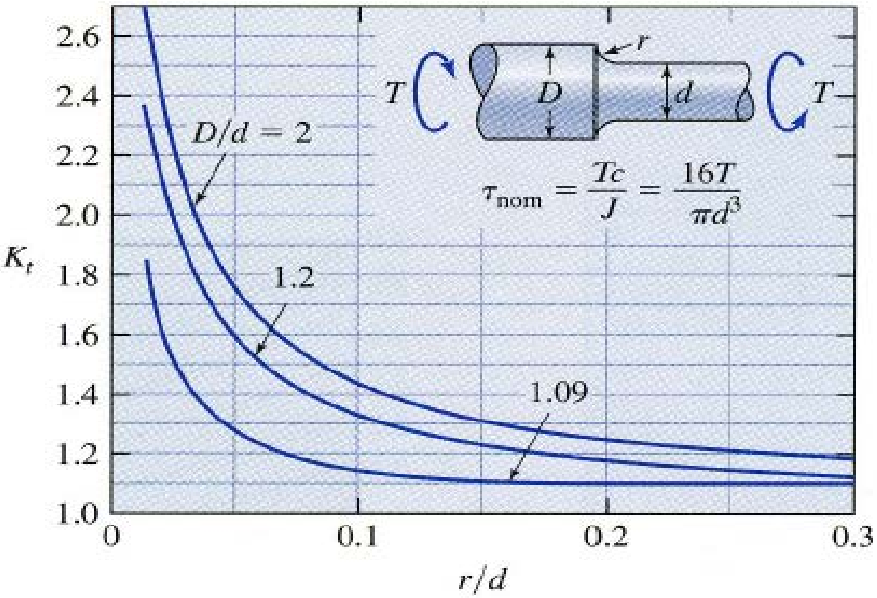
D/d	B	a
∞	0.971	−0.357
1.50	0.983	−0.334
1.15	0.993	−0.303
1.05	1.025	−0.240
1.01	1.061	−0.134



Approximate formula

$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

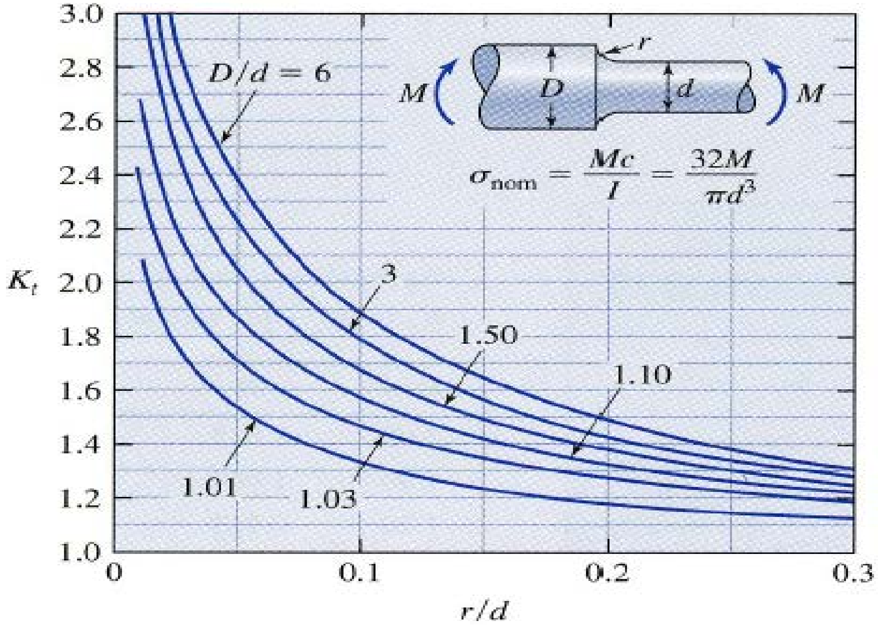
D/d	B	a
2.00	1.015	−0.300
1.50	1.000	−0.282
1.20	0.963	−0.255
1.05	1.005	−0.171
1.01	0.984	−0.105



Approximate formula

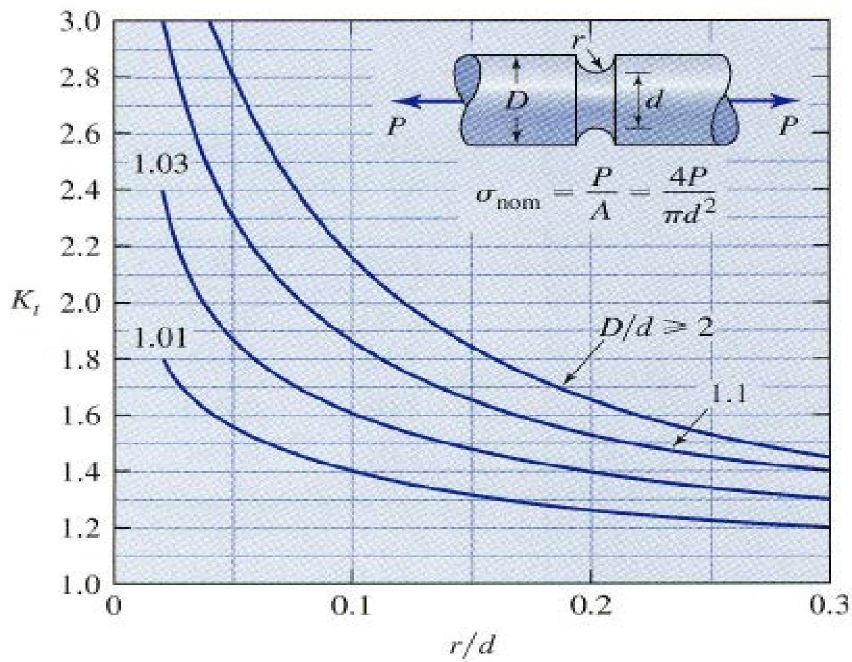
$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

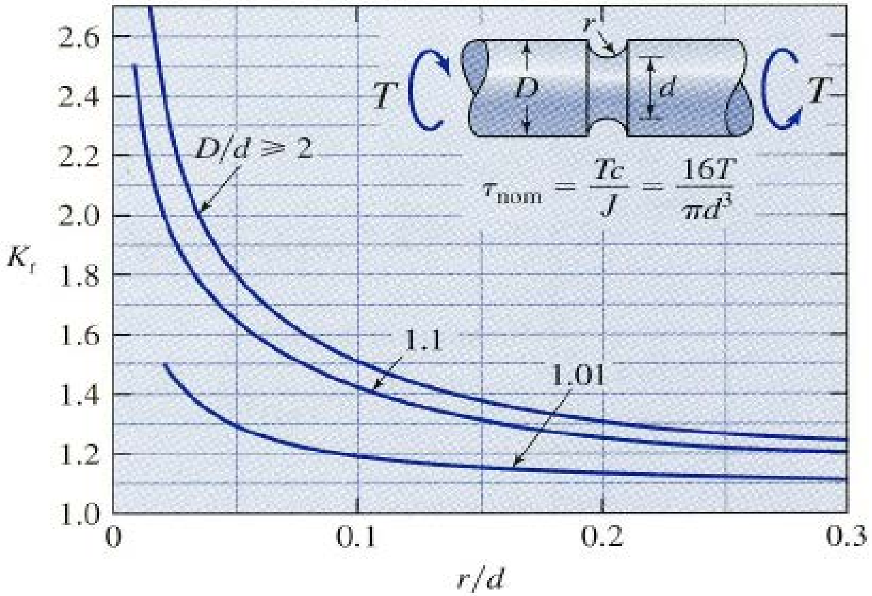
D/d	B	a
2.00	0.863	−0.239
1.20	0.833	−0.216
1.09	0.903	−0.127



Approximate formula
$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

D/d	B	a
6.00	0.879	−0.332
3.00	0.893	−0.309
1.50	0.938	−0.258
1.10	0.951	−0.238
1.03	0.981	−0.184
1.01	0.919	−0.170

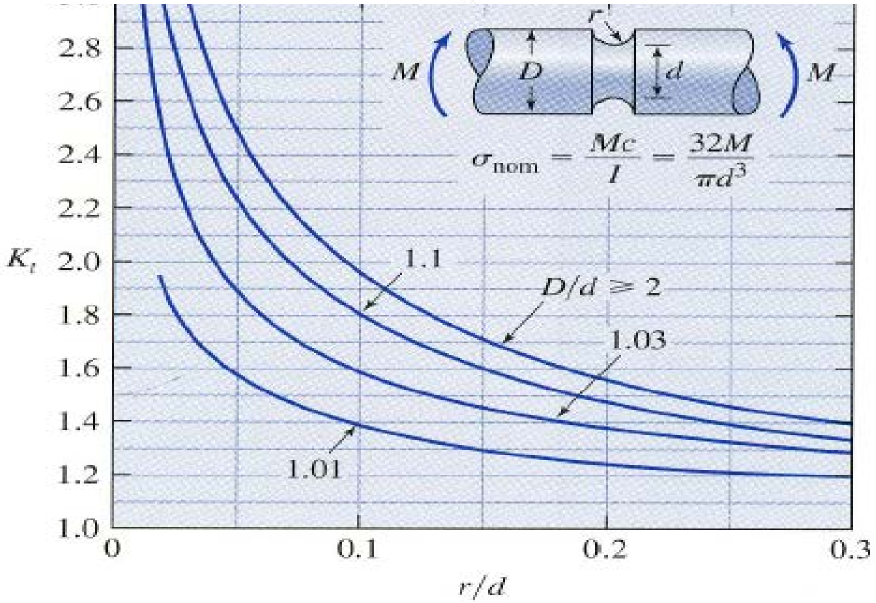




Approximate formula

$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

D/d	B	a
2.00	0.890	−0.241
1.10	0.923	−0.197
1.01	0.972	−0.102



Approximate formula

$K_t \approx B\left(\frac{r}{d}\right)^a$, where:

D/d	B	a
2.00	0.936	−0.331
1.10	0.955	−0.283
1.03	0.990	−0.215
1.01	0.994	−0.152

7. Appendix 1: some K_t diagrams (6/6)

The diagrams in the previous two slides have been presented as a quick reference to most common cases in machine design.

For an ample review of available diagrams, and their origin, please refer to the latest edition of the classical Peterson’s book:

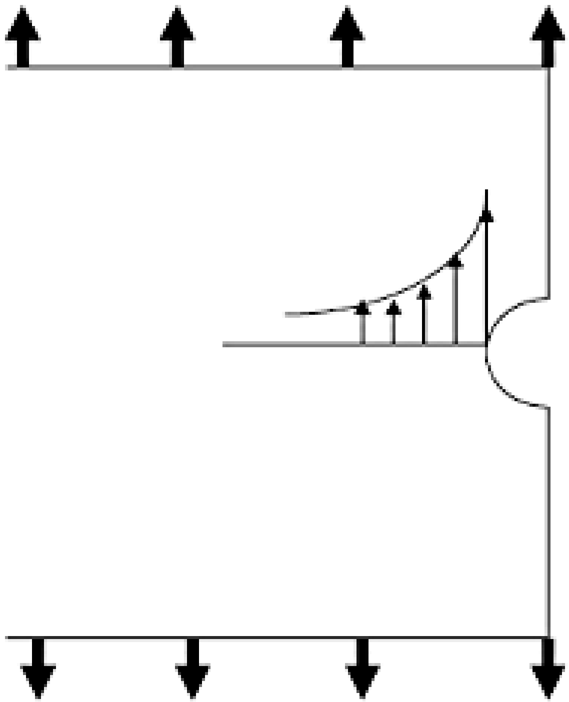
D. W. Pilkey, D. F. Pilkey, “Peterson's stress concentration factors” - 3rd ed., 2008, ISBN 0470048247, ISBN 978-0470048245

A suggested further reading:

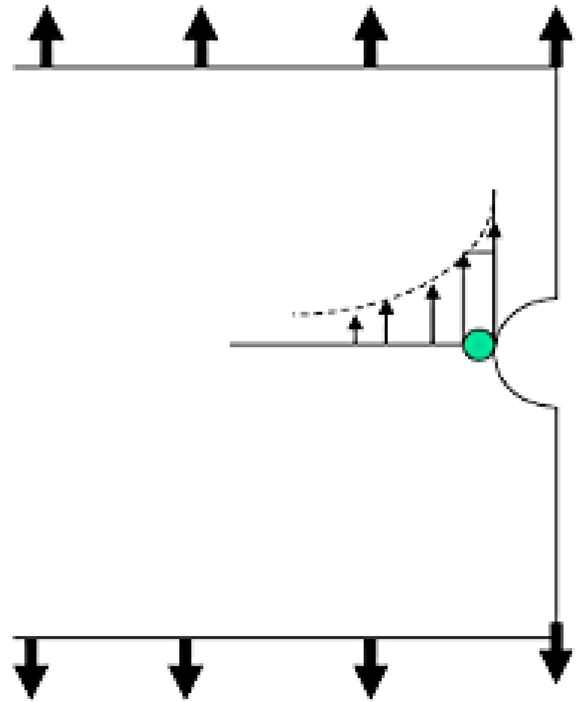
R. E. Peterson, “Notch sensitivity”, ch. 13 in ” *Metal Fatigue”, G. Sines and J.L. Waisman editors,* Mc Graw Hill, USA, (1959) The reason for the difference between K_f and K_t is twofold:

1 - Stress gradient

- The notch stress controlling the fatigue life is not the maximum stress on the surface of the notch root, but an average stress acting over a finite volume of the material at the notch root. This average stress is lower than the maximum surface stress, calculated from K_t .
- When small cracks nucleate at the notch root, they grow into regions where, due to the stress gradient, stresses are lower. Notch sensitivity is therefore decreased.
- Larger notch radii result in lower stress gradients near the notch, and more material is subjected to higher stresses. Notch sensitivity in fatigue is therefore increased.



Elastic stresses



Local plastic stresses

2 - Localized plastic deformation at the notch root

- The localized plastic deformation and notch blunting effect due to yielding at the notch root reduces the notch root stress.

8. Appendix 2: more on stress gradients (3/9)

Unlike the stress concentration factor K_t, which depends only on notch shape, the fatigue notch factor K_f is dependent also on the type of material and on the notch size, because different material have different "process volumes" or effective Δy depths.

Two approaches are available for the (K_t ; K_f) relation: the FKM method in based on the **stress gradient** theory, an older range of methods is based on the **notch sensitivity factor**.

The notch sensitivity factor q is: $q = \dfrac{K_f - 1}{K_t - 1} \Rightarrow K_f = 1 + q(K_t - 1)$

A value of $q=0$ (or $K_f = 1$) indicates no notch sensitivity, whereas a value of $q=1$ (or $K_f = K_t$) indicates full notch sensitivity.

Material	q
C-30 steel-annealed	0.18
C-30 steel-heat treated and drawn at 480ºC	0.49
C-50 steel-annealed	0.26
C-50 steel-heat treated and drawn at 480ºC	0.50
C-85 steel-heat treated and drawn at 480ºC	0.57
Stainless steel-annealed	0.16
Cast iron-annealed	0.00-0.05
Copper-annealed	0.07
Duraluminium-annealed	0.05-0.13

Material	q
S 275/S 460 steels	0.4÷0.7
quench/temp. steel (900*)	0.8÷0.9
quench/temp. steel (1100*)	0.9÷1
austenitic steel	0.1÷0.4
lamellar cast iron	0.1÷0.2

*R_m in MPa

notch sensitivity of some engineering materials.

For a rough first-order determination of the material effect the notch sensitivity factor will be considered as in the table above, where the dependence on notch size (radius) is neglected.

A number of researchers have proposed analytical relations for the determination of q, based on correlation to experimental data.

The most common relations are those proposed by Peterson and Neuber:

$$q = \frac{1}{1 + \frac{a}{r}}$$

Peterson equation

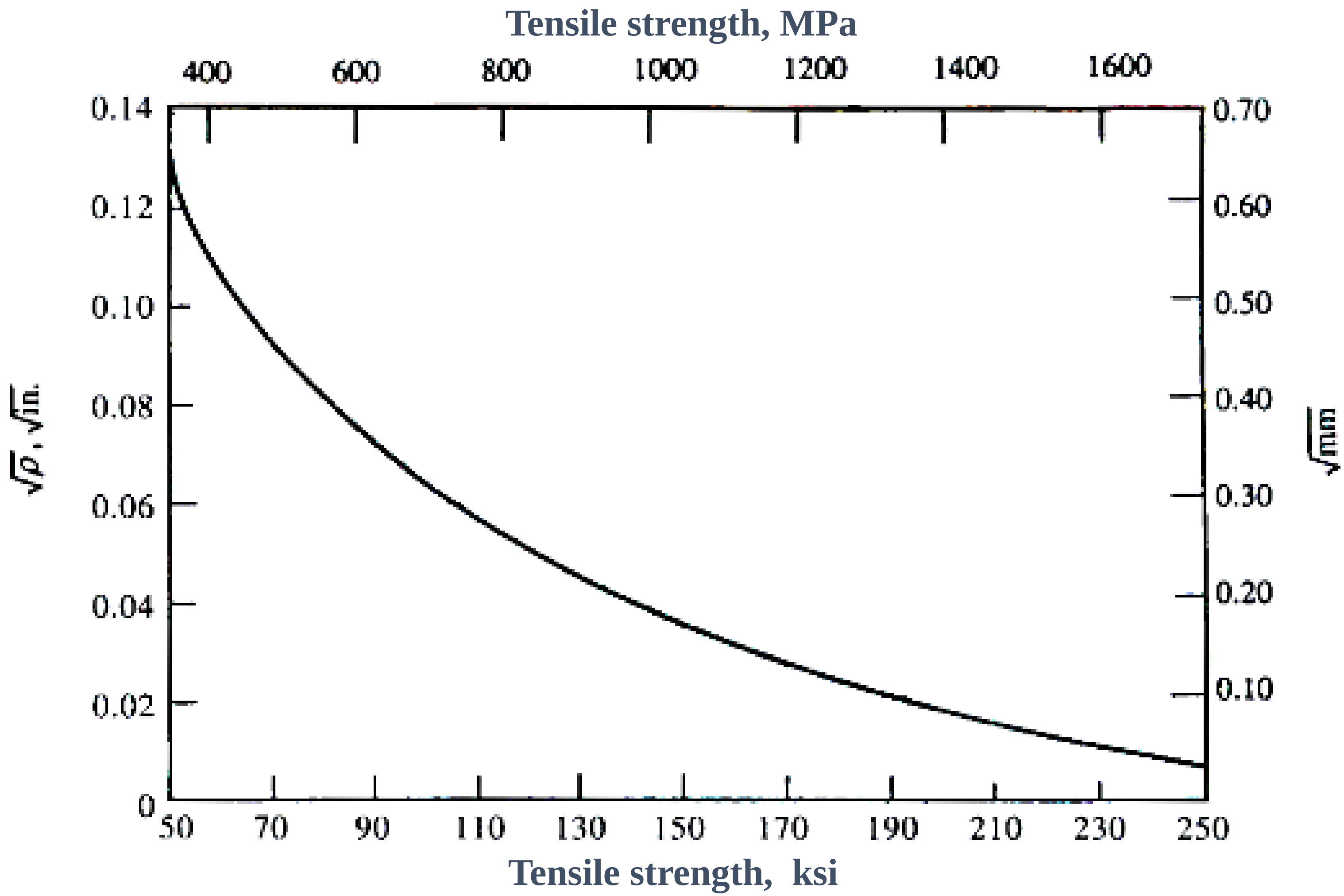
$$q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}}$$

Neuber equation

r: notch root radius
a, ρ: material constants

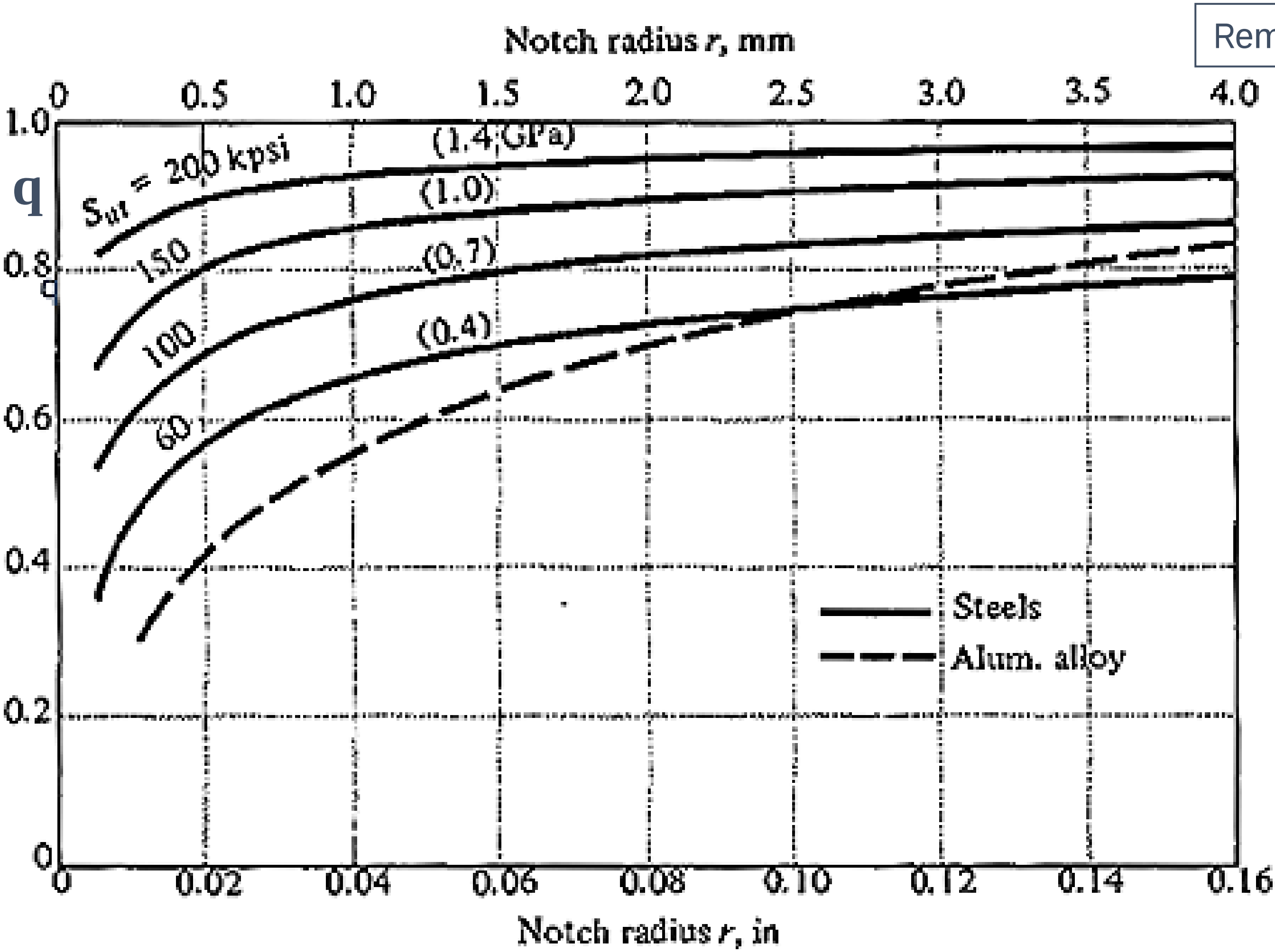
These empirical formulas express the fact that for large notches with large radii we must expect K_f to be almost equal to K_t, but for small sharp notches we may find $K_f << K_t$ (small notch effect) for metals with ductile behavior, while K_f remains near to K_t for high strength metals.

Neuber characteristic length ρ vs material strength



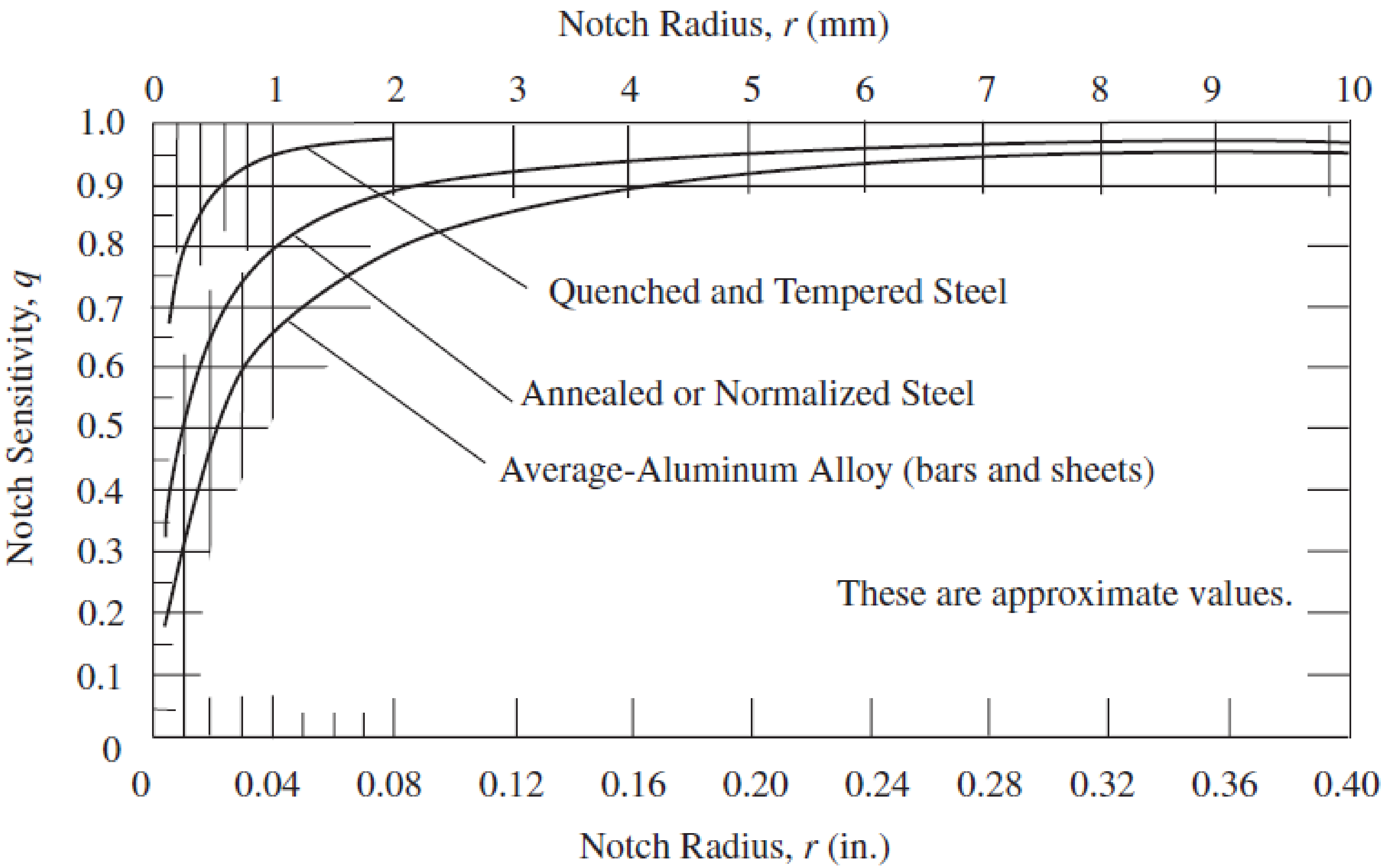
Source: Metal Fatigue in Engineering, Ralph I. Stephens ,Ali Fatemi, Robert R. Stephens, Henry O. Fuchs, John Wiley & sons, 2001

Reminder: 1 ksi=6,89 MPa



from: Metal Fatigue, G. Sines and J.L. Waisman, Mc Graw Hill, New York, 1959

This figure shows values of q for notches on specimens in reversed bending or axial load.



Another figure showing values of q for notches.

Practical consequences

Because of the low sensitivity of small notch radii, the extremely high theoretical stress concentration factors predicted for very sharp notches and scratches are not actually realized.

The notch sensitivity of quenched and tempered steels is higher than that of lower-strength, coarser-grained alloys.

As a consequence, for notched members the strength advantage of high-grade steels over other materials may be lost.

Students who have followed, during these same years, first level courses of Mechanical engineering at Politecnico di Torino may have seen different formulas.

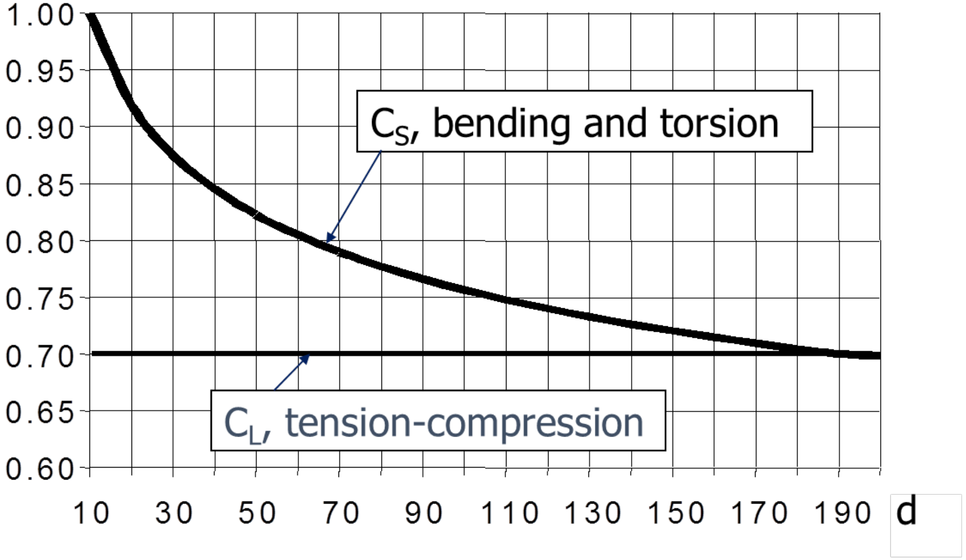
However, they should be aware that elements taken into consideration are the same as in the treatment of this chapter.

The formula to compare is: $\sigma_{D-1}^{C} = \dfrac{C_L \cdot C_S \cdot C_F}{K_f} \sigma_{D-1}$

Where:

- σ_{D-1}^{C} : component fatigue limit
- σ_{D-1} : bending specimen fatigue limit, ≈ 10 mm diameter
- C_L : Load type factor ≈ 1 for plane bending, ≈ 0.7 for tension-compression
- C_S : size factor, replaces C_L for bending only in the case of dia.<10 mm
- C_F : surface finish factor (roughness, machining etc.)
- K_f : (fatigue) notch effect

9. Appendix 3: interfacing FKM with other formulations



On the left, the diagram proposed in that case for coefficient C_S

Other factors mentioned were:

- surface treatments
- operating temperature
- corrosive environment

The most evident limitation of this approach is in that it cannot be extended to the composition of different stress types (e.g., tension plus bending plus torsion) without heavy simplifications, such as taking one, the same and therefore the highest, K_f for all.

Moreover, K_f is seen as a strength reduction rather than, as it is, an applied stress increase.