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Costruzione di Motori per Aeromobili - Machine Design

Bolted connections - Chapter 2

Chapters

1 Prestressed single bolt connections

2 Refinements and special problems

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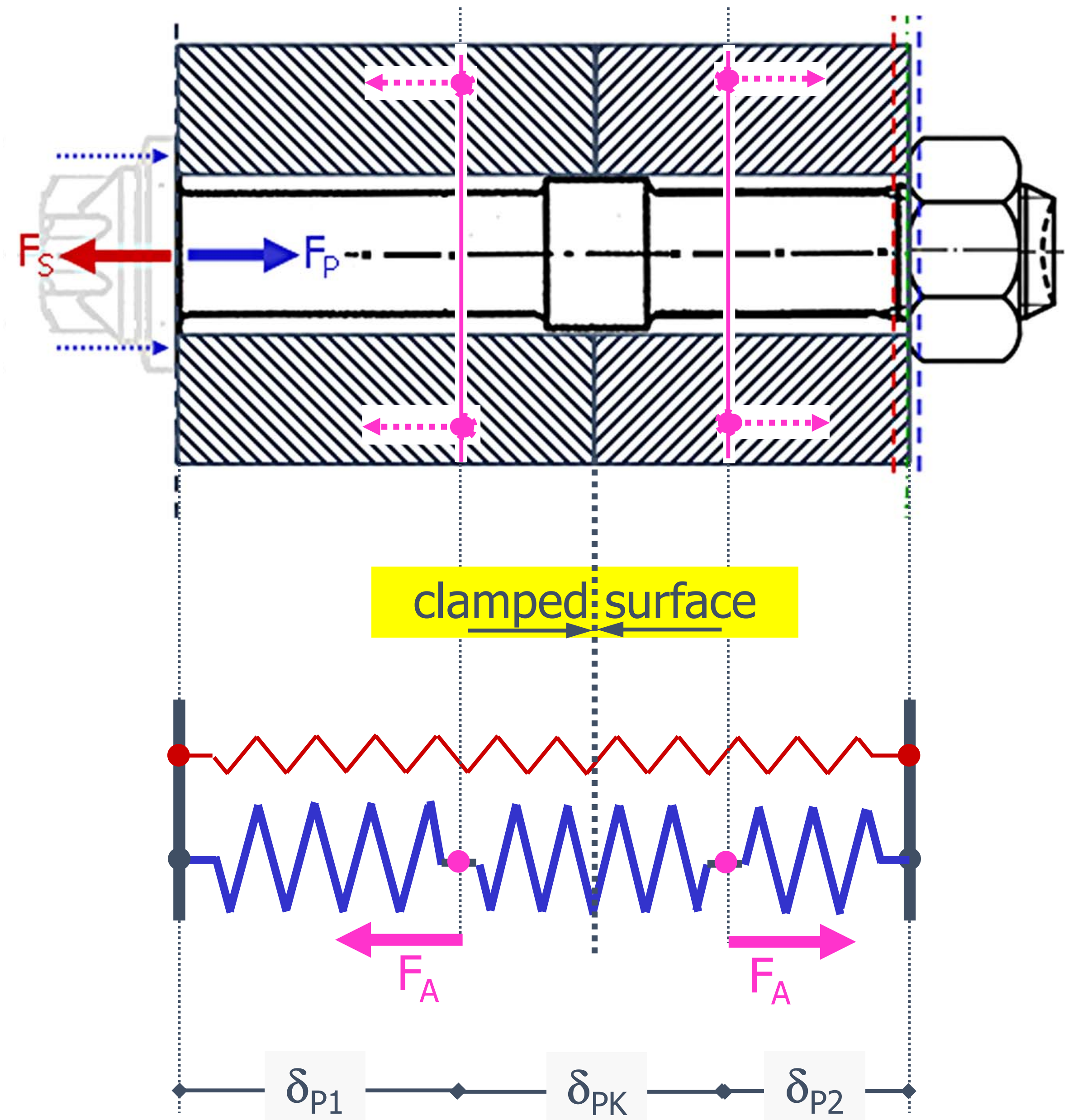
Section 1 -Intermediate loads

This section illustrates the load sharing of a variable load which acts to separate the clamped parts, applied well inside the head-nut span.

It is shown how much additional load is taken by the bolt, showing that it is less than the amount taken under the (simplified) hypothesis of application on the parts just at the separation between respectively nut and head. So, a less severe condition which is advantageous in cases when stresses are particularly high, as if happens in critical machinery components subjected to fatigue.

It is also shown that, on the contrary, this situation is less favourable against separation of the parts at their inner contact surface.

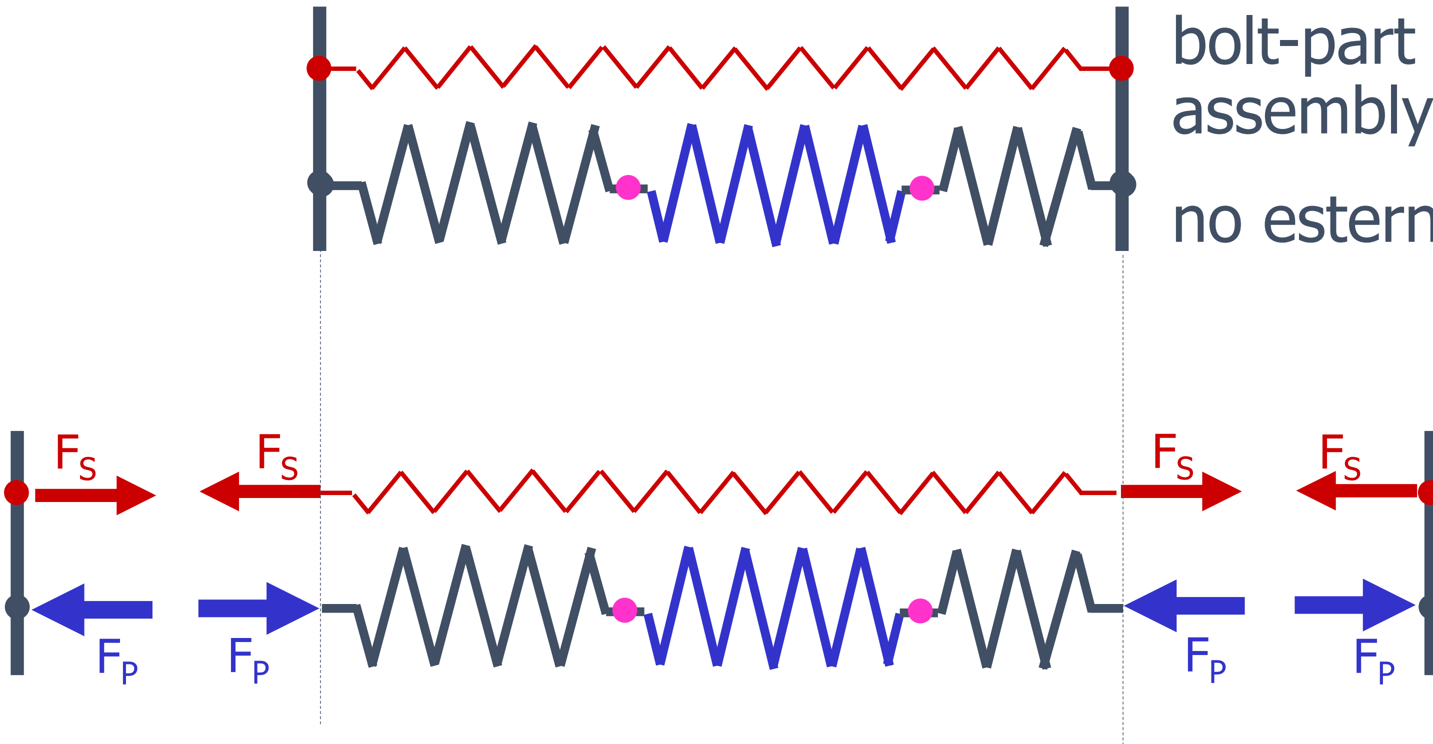
1 - Intermediate loads (1/11)



The figure on the left shows that the external force F_A is applied to the parts at a location which is no longer (as it was in the elementary approach) at the level of the nut/part and head/part contact surface.

The spring model collects force positions and stiffness of the three sections in which parts are now divided, resiliences δ_{P1}, δ_{PK}, δ_{P2}.

1 - Intermediate loads (2/11)



bolt-part system after assembly

no esternal forces applied

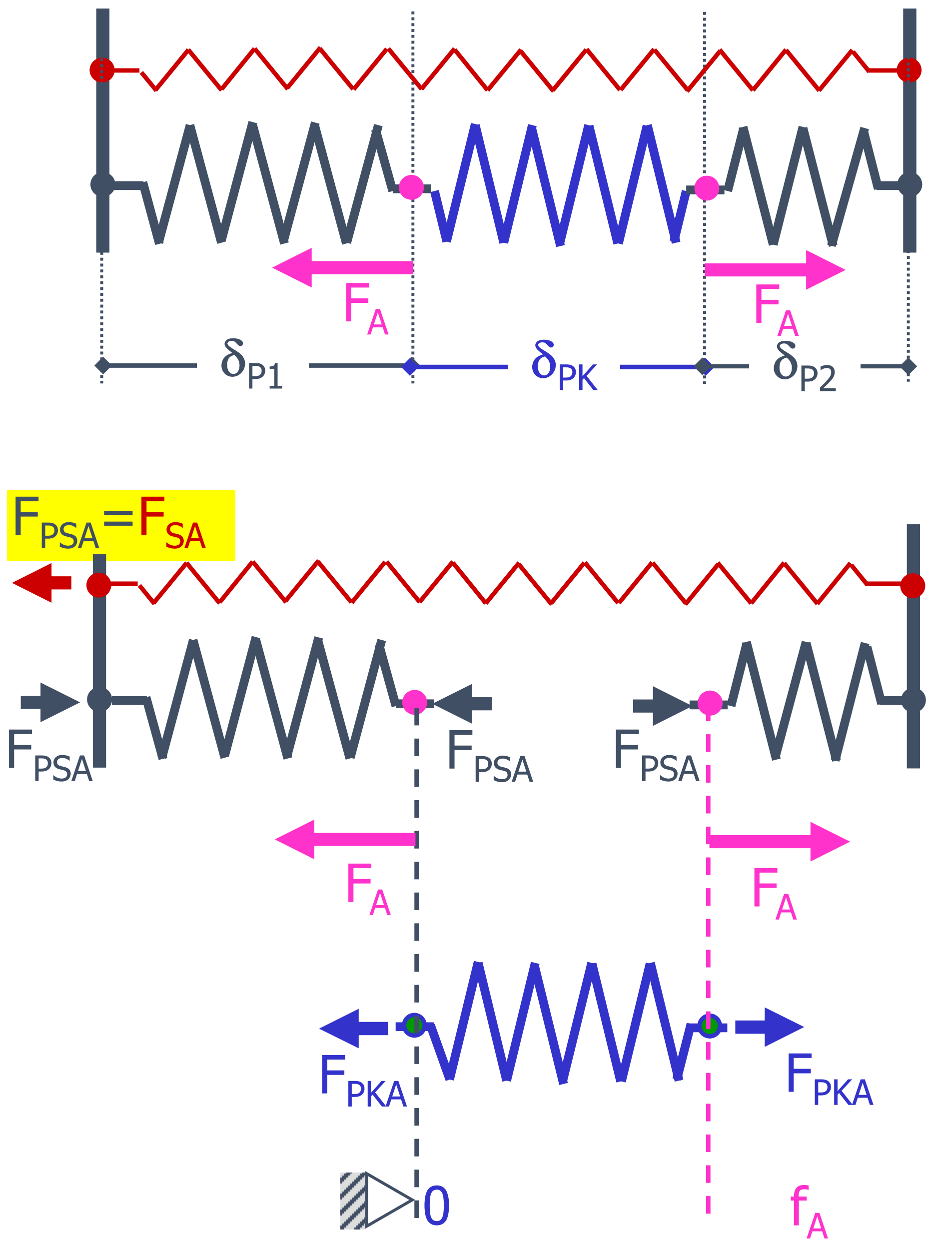
exploded view drawing

inner forces shown

Assembly force equilibrium: $|F_S| = |F_P|$

However, springs seen during assembly are, as in the elementary approach, bolt spring and parts spring in parallel.

The corresponding part of the joint diagram is then the same.



The body loaded by the external force F_A is split into two spring sections:

-) the inner spring, resilience $\delta_{PK}<\delta_P$

-) the series of springs with resiliences δ_{P1} , δ_{P2}, δ_S, total resilience $\delta_{PS}= \delta_{P1}+\delta_{P2}+\delta_S>\delta_S$

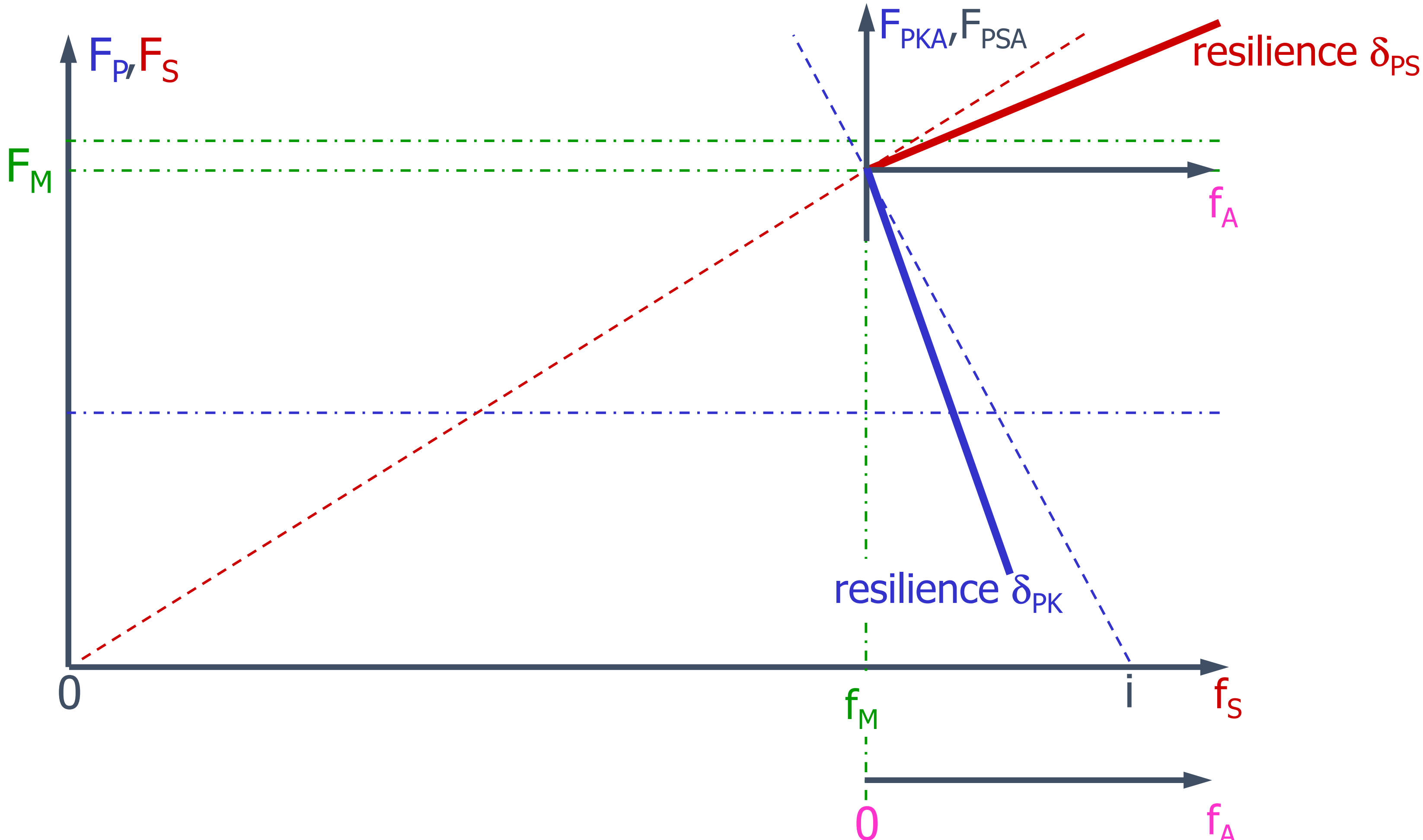
These two spring sections are in parallel, and share the external load F_A with the same mechanism already demonstrated before.

Equilibrium: $F_A=F_{PKA}+F_{PSA}$

Note that : $F_{PSA}=F_{SA}$.

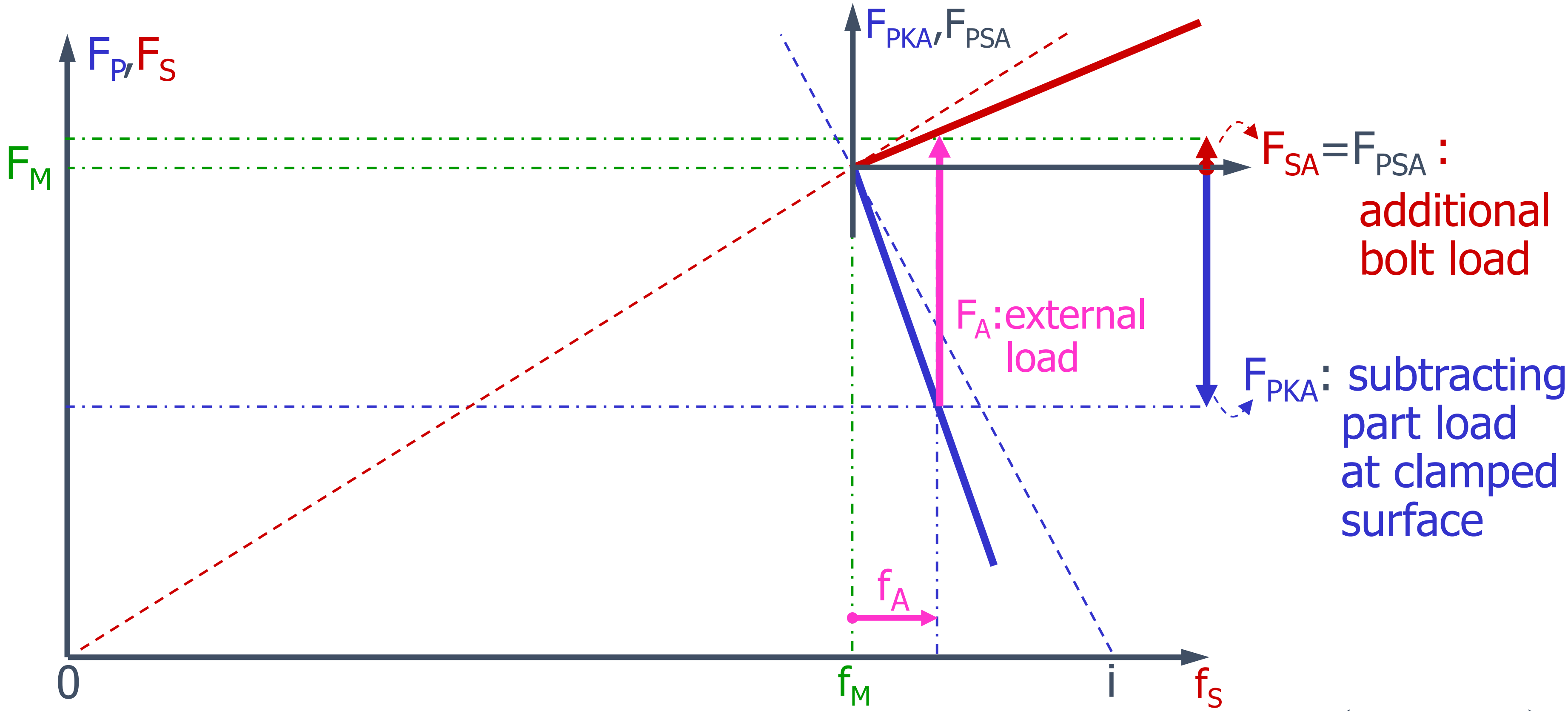
f_A is the extension of inner spring

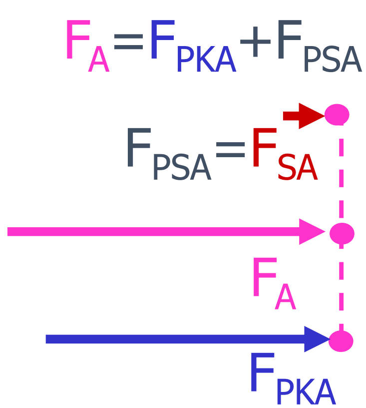
1 - Intermediate loads (4/11)



Remark that this is a combination of the assembly diagram on which the external loading diagram is super-imposed, the latter having origin at the initial tightening point. Variables on axes differ for the two diagrams, in fact they measure forces and displacements at two different points.

Attention! Force F_{PKA} is the part force at the inner interface, where separation may occur. Resiliences δ_{PK} and δ_{PS} are those seen by force F_{PKA}.





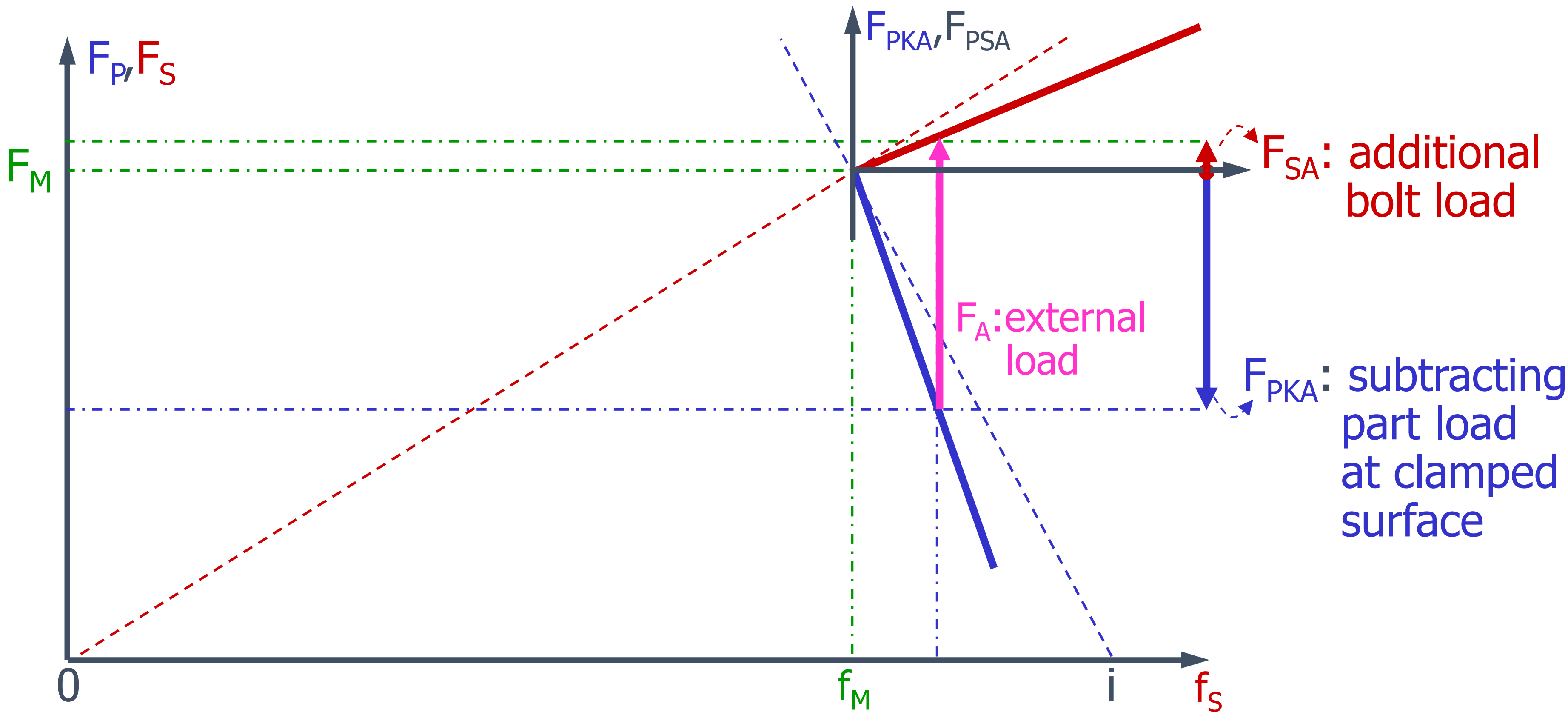
$$\begin{cases} f_A = \delta_{PS}\ F_{PSA} \\ f_A = \delta_{PK}\ F_{PKA} \end{cases}$$

$$F_{PKA} = \frac{\delta_{PS}}{\delta_{PK}} F_{PSA}$$

$$F_A = \frac{\delta_{PS}}{\delta_{PK}} F_{PSA} + F_{PSA} = F_{PSA}\left(\frac{\delta_{PS} + \delta_{PK}}{\delta_{PK}}\right)$$

$$\text{then}: \boxed{F_{PSA} \equiv F_{SA}} = F_A \frac{\delta_{PK}}{\delta_{PS} + \delta_{PK}} =$$

$$= F_A \frac{\delta_{PK}}{(\delta_S + \delta_{P1} + \delta_{P2}) + \delta_{PK}} = \boxed{F_A \frac{\delta_{PK}}{\delta_S + \delta_P}}$$

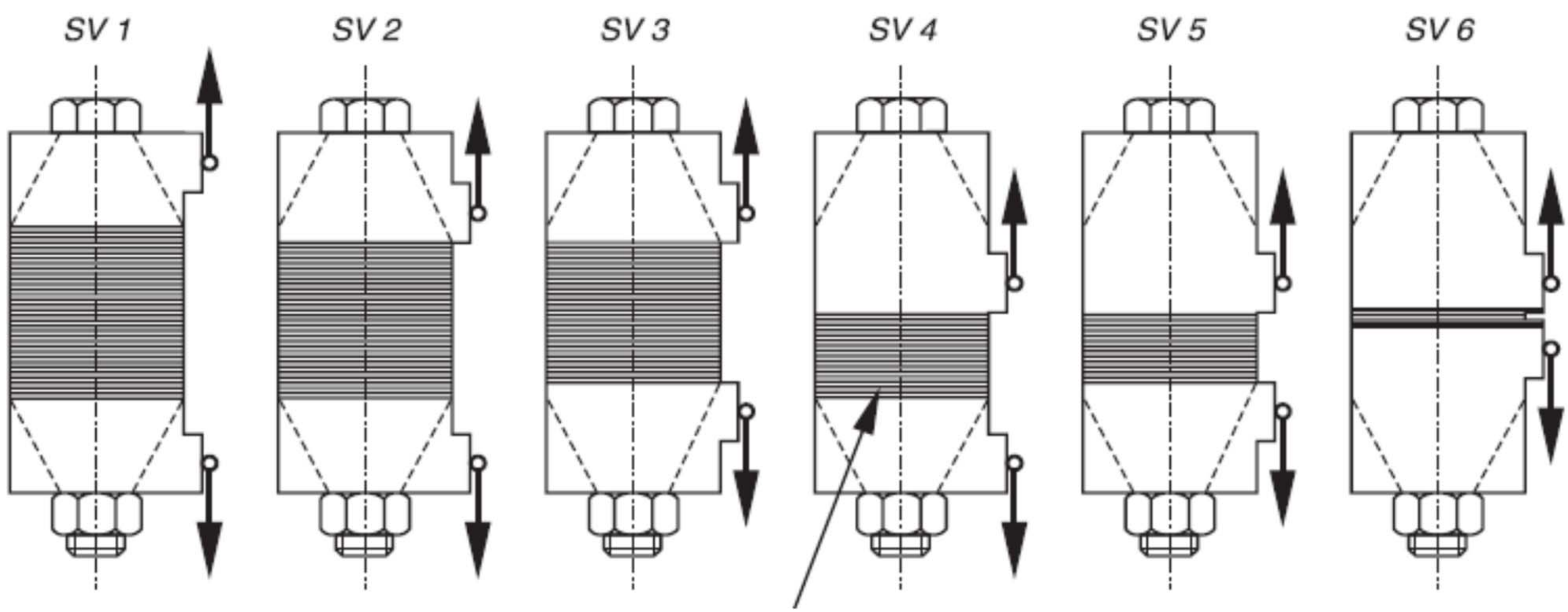


$$F_{SA} = F_A \frac{\delta_{PK}}{\delta_S + \delta_P} = F_A \frac{\delta_{PK}}{\delta_P} \frac{\delta_P}{\delta_S + \delta_P} = F_A\, n\, \frac{\delta_P}{\delta_S + \delta_P}$$

n = load introduction factor

Approximate values for the "Load introduction factor"

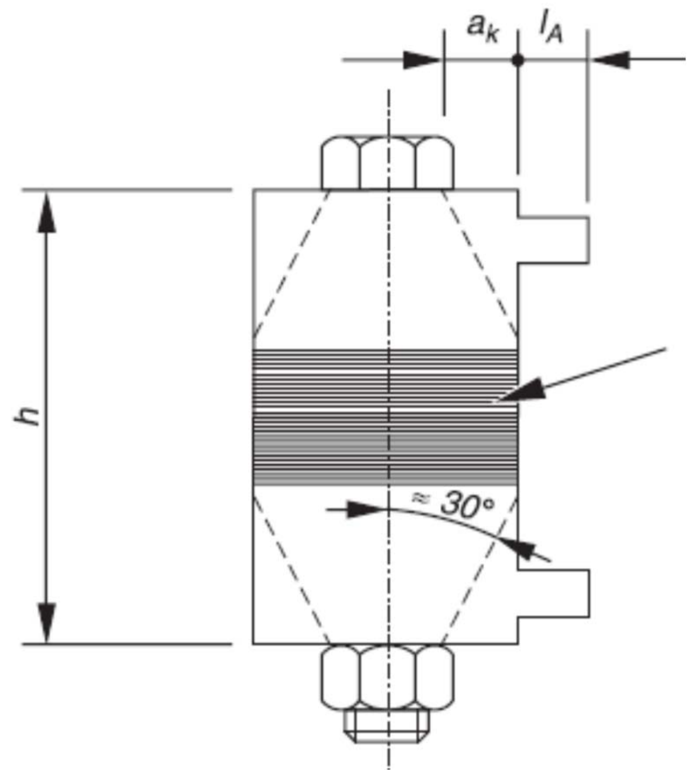
The VDI 2230-2003 defines the load introduction factor with quite complex formulas whose explanation is beyond the scope of these notes. Only the main results are shown here. The joint must be assigned to one of the joint types as in the Figure below, relative the position of the load introduction points.



1 - Intermediate loads (8/11)

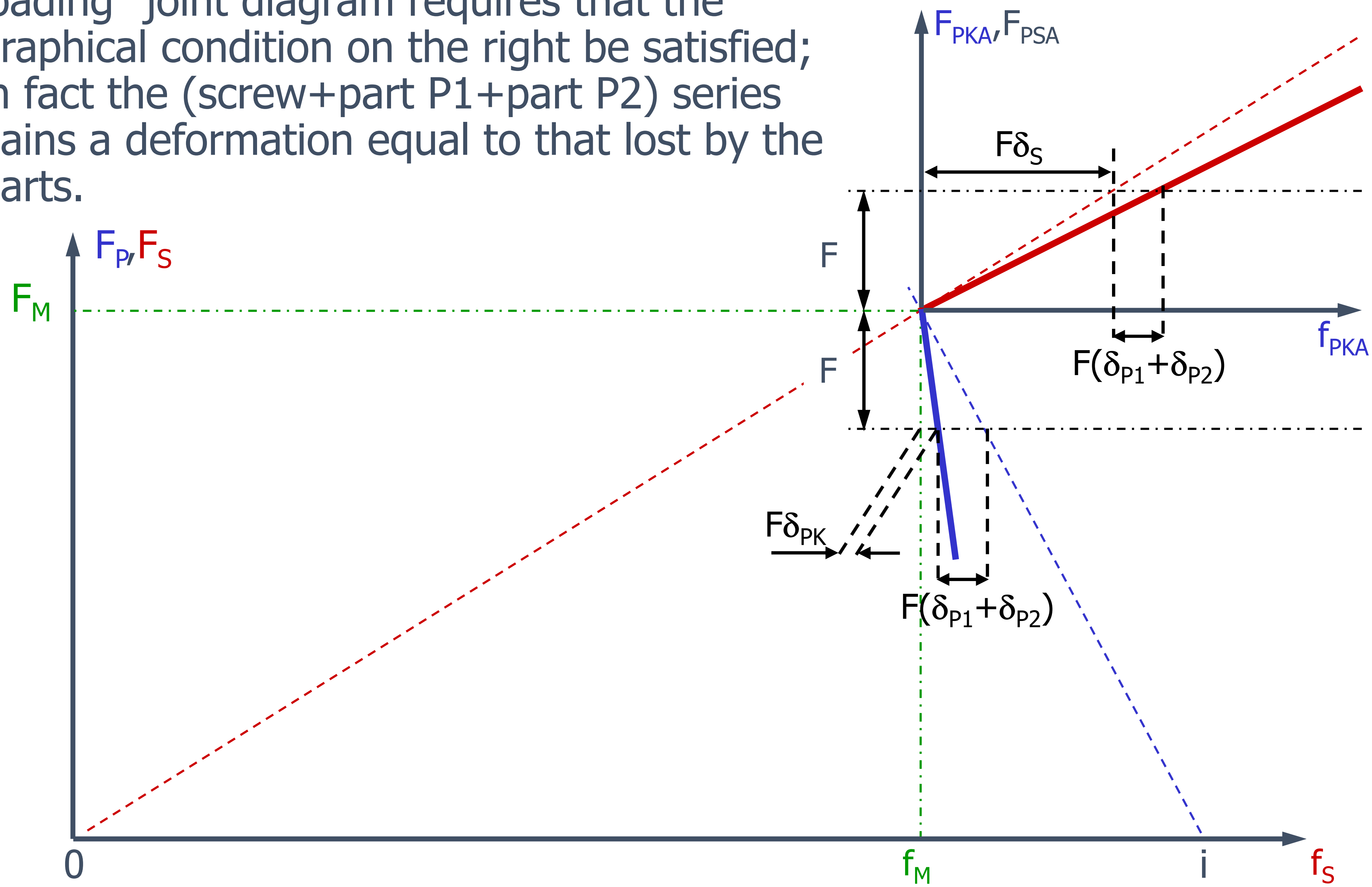
The height h, the distance a_k and the length l_A must be determined from the geometry of the joint as in the Figure. In the case of concentric loading $l_A = 0$

The load introduction factor n can finally be determined either directly or by linear interpolation from the Table



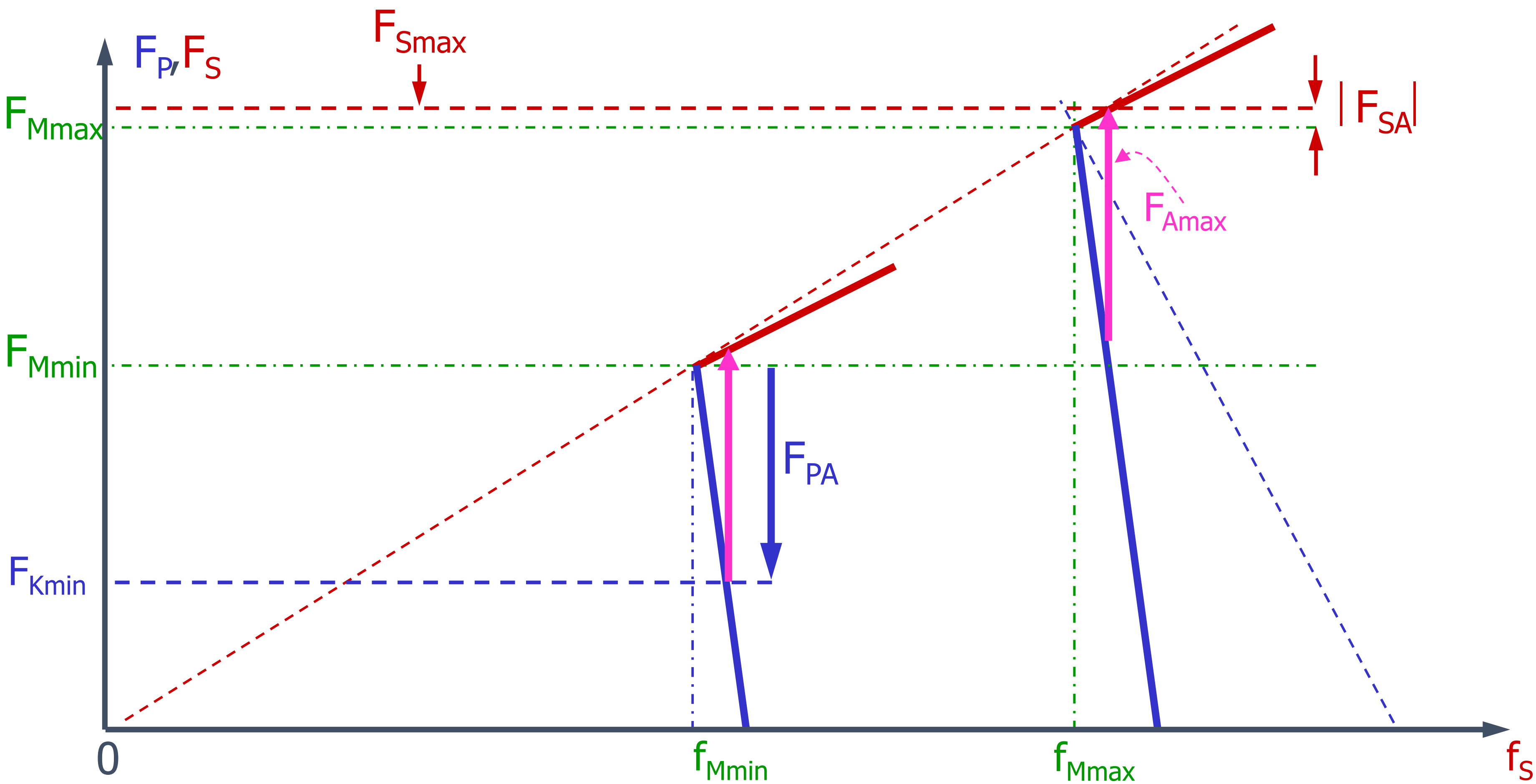
l_A/h	0,00				0,10				0,20				≥ 0,30			
a_k/h	0,00	0,10	0,30	≥ 0,50	0,00	0,10	0,30	≥ 0,50	0,00	0,10	0,30	≥ 0,50	0,00	0,10	0,30	≥ 0,50
SV 1	0,70	0,55	0,30	0,13	0,52	0,41	0,22	0,10	0,34	0,28	0,16	0,07	0,16	0,14	0,12	0,04
SV 2	0,57	0,46	0,30	0,13	0,44	0,36	0,21	0,10	0,30	0,25	0,16	0,07	0,16	0,14	0,12	0,04
SV 3	0,44	0,37	0,26	0,12	0,35	0,30	0,20	0,09	0,26	0,23	0,15	0,07	0,16	0,14	0,12	0,04
SV 4	0,42	0,34	0,25	0,12	0,33	0,27	0,16	0,08	0,23	0,19	0,12	0,06	0,14	0,13	0,10	0,03
SV 5	0,30	0,25	0,22	0,10	0,24	0,21	0,15	0,07	0,19	0,17	0,12	0,06	0,14	0,13	0,10	0,03
SV 6	0,15	0,14	0,14	0,07	0,13	0,12	0,10	0,06	0,11	0,11	0,09	0,06	0,10	0,10	0,08	0,03

The correct construction of the "external loading" joint diagram requires that the graphical condition on the right be satisfied; in fact the (screw+part P1+part P2) series gains a deformation equal to that lost by the parts.

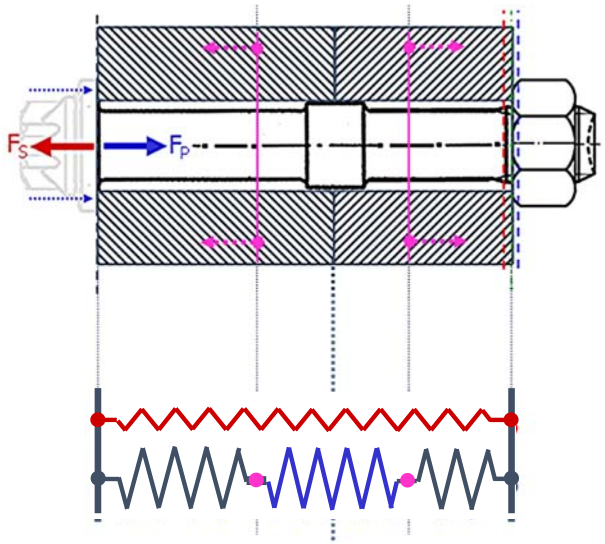


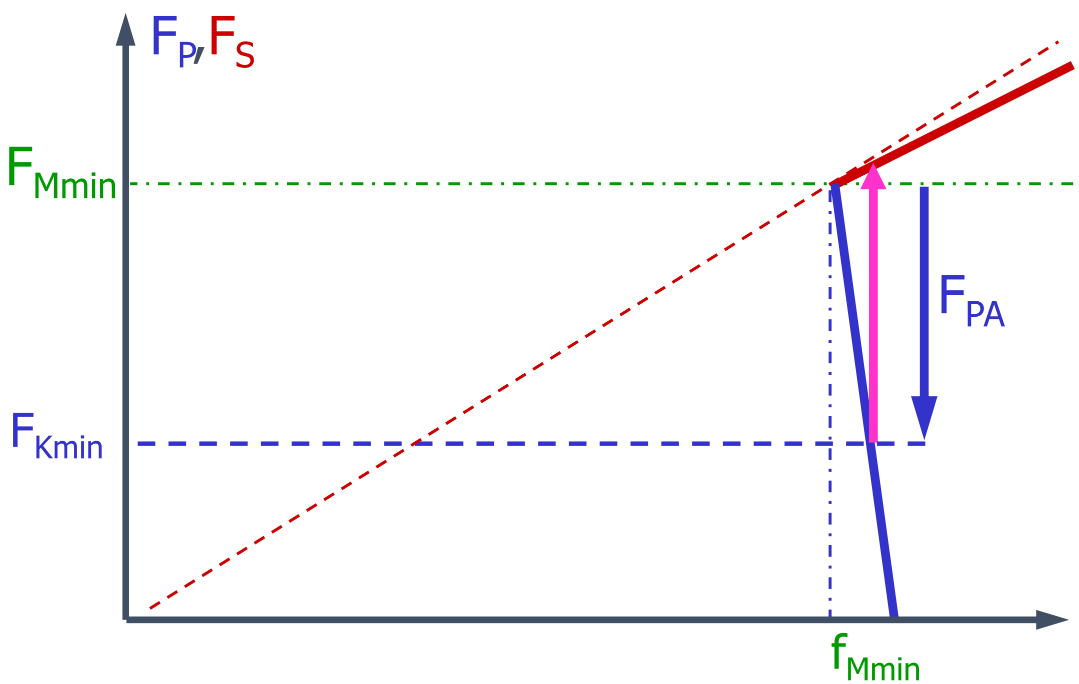
1 - Intermediate loads (10/11)

Useless details may be omitted, and a simple version of the joint diagram is presented as here indicated:



1 - Intermediate loads (11/11)





A final remark: the minimum load on the part F_{Kmin} must be kept under control and guaranteed if there is a separation surface which is within the “core” part “k”, i.e. the one represented by the blue spring.

The outer sections of the part, receive an additional compressive load equal to $|F_{SA}|$, so they do not pose a problem.

Section 2 - Allowable bolt stress

This section justifies a simplified formula of practical use which, under an assumption on the safe side, gives a limit for the additional bolt axial stress in operation at 10% of $R_{p0,2}$.

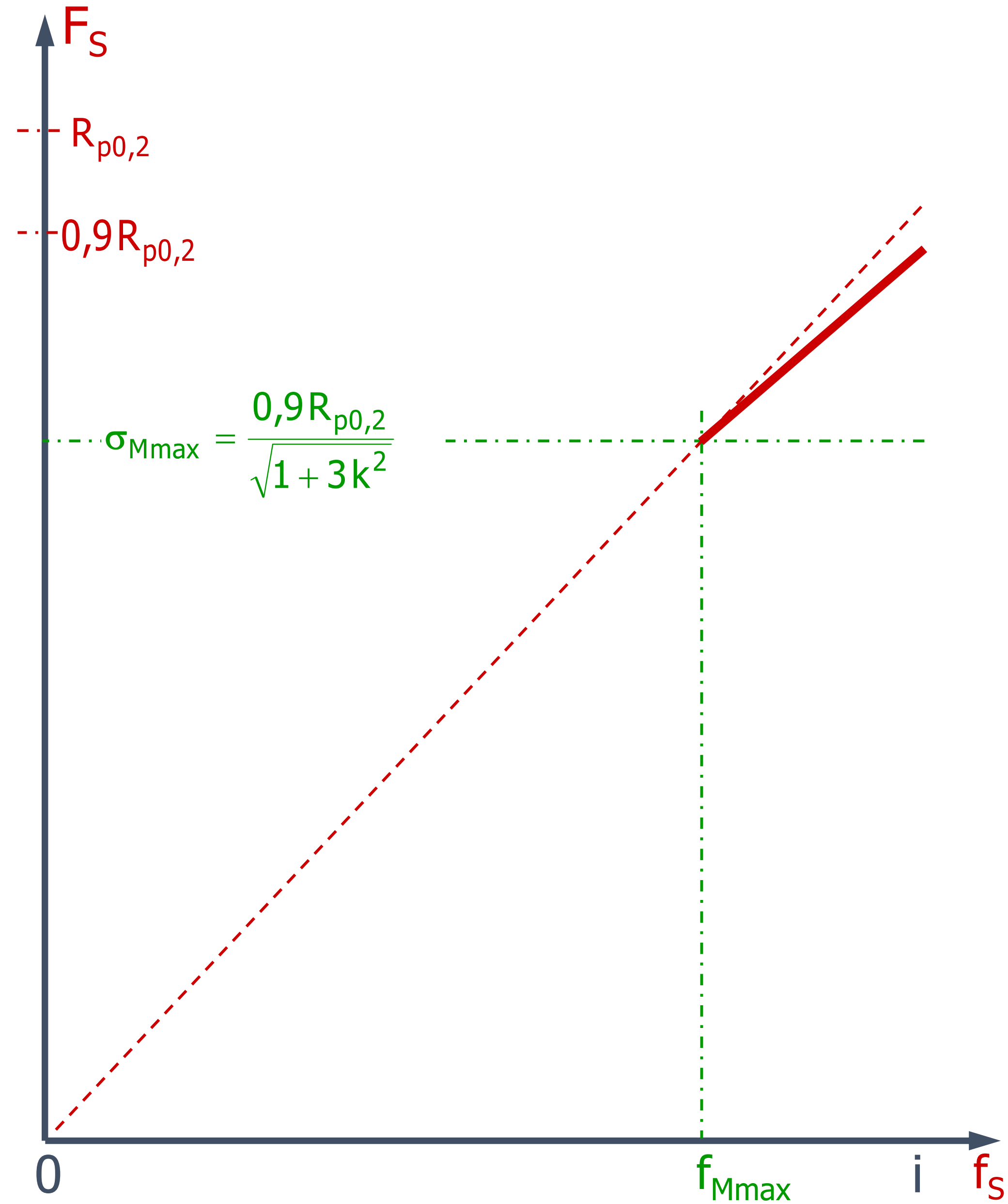
This section also puts forward a refinement in the calculation of bolt stresses taking into account the fact that the amount of torsional stress is considerably less than predicted by torque during assembly, thank to bolt/part springback after the release of applied torque.

2 - Allowable bolt stress (1/6)

The maximum load on the bolt in service was determined in Chapter 1 on the basis of the rigorous composition of the maximum axial stress $\sigma_{tot} = \sigma_{Mmax} + \sigma_{SA}$ and of the tangential stress τ_{Mmax} due to torsion, calculated at the maximum level produced during tightening (see Ch. 1, Sect. 13, sl.9).

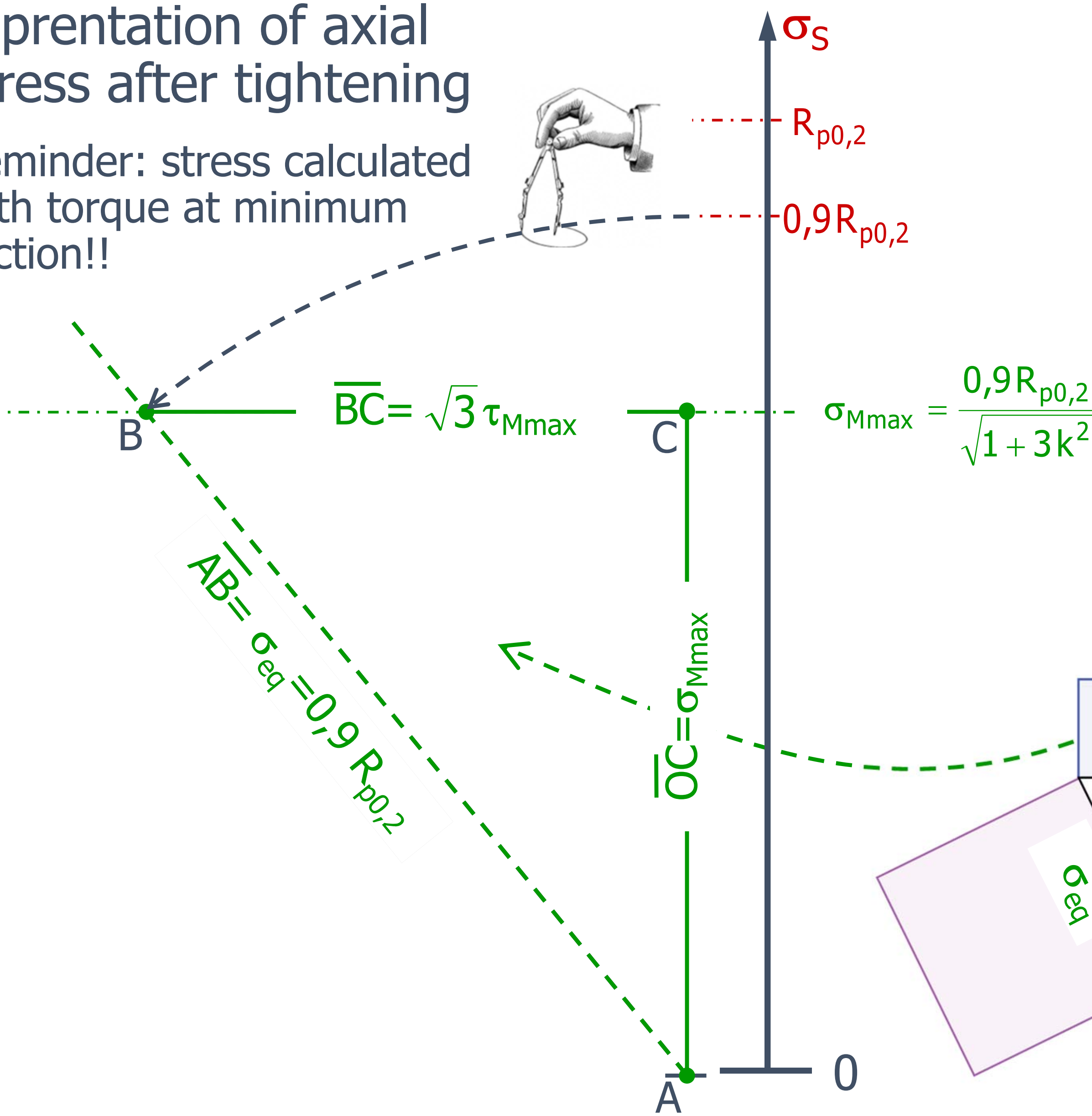
On the right the F_S-f_S lines for the bolt are shown.

Dividing forces by A_0 , corresponding stresses are represented on the vertical axis, as shown in next slide.



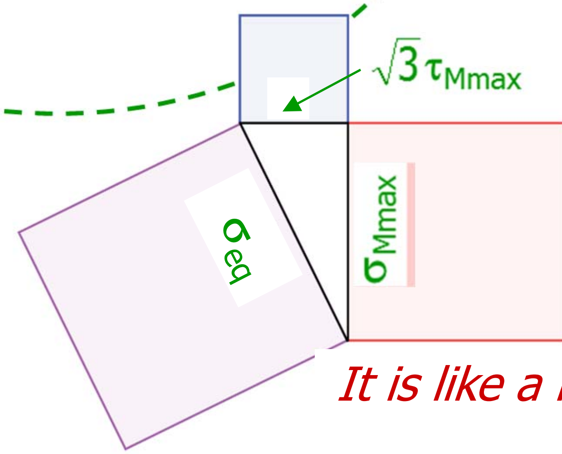
This is a graphical reprentation of axial stress after tightening

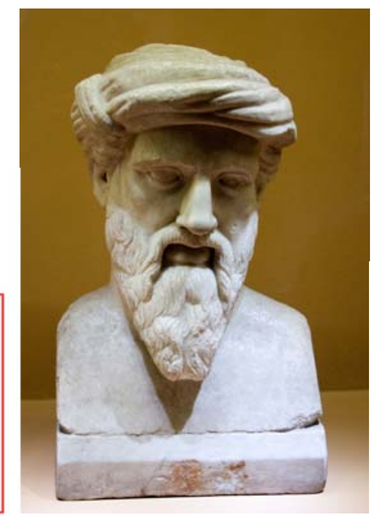
Reminder: stress calculated with torque at minimum friction!!



"yield" allowable bolt axial stress Ch.1 Sect. 13 sl. 9

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = \frac{F_{Mmax}}{A_0}\sqrt{1+3k^2}$$

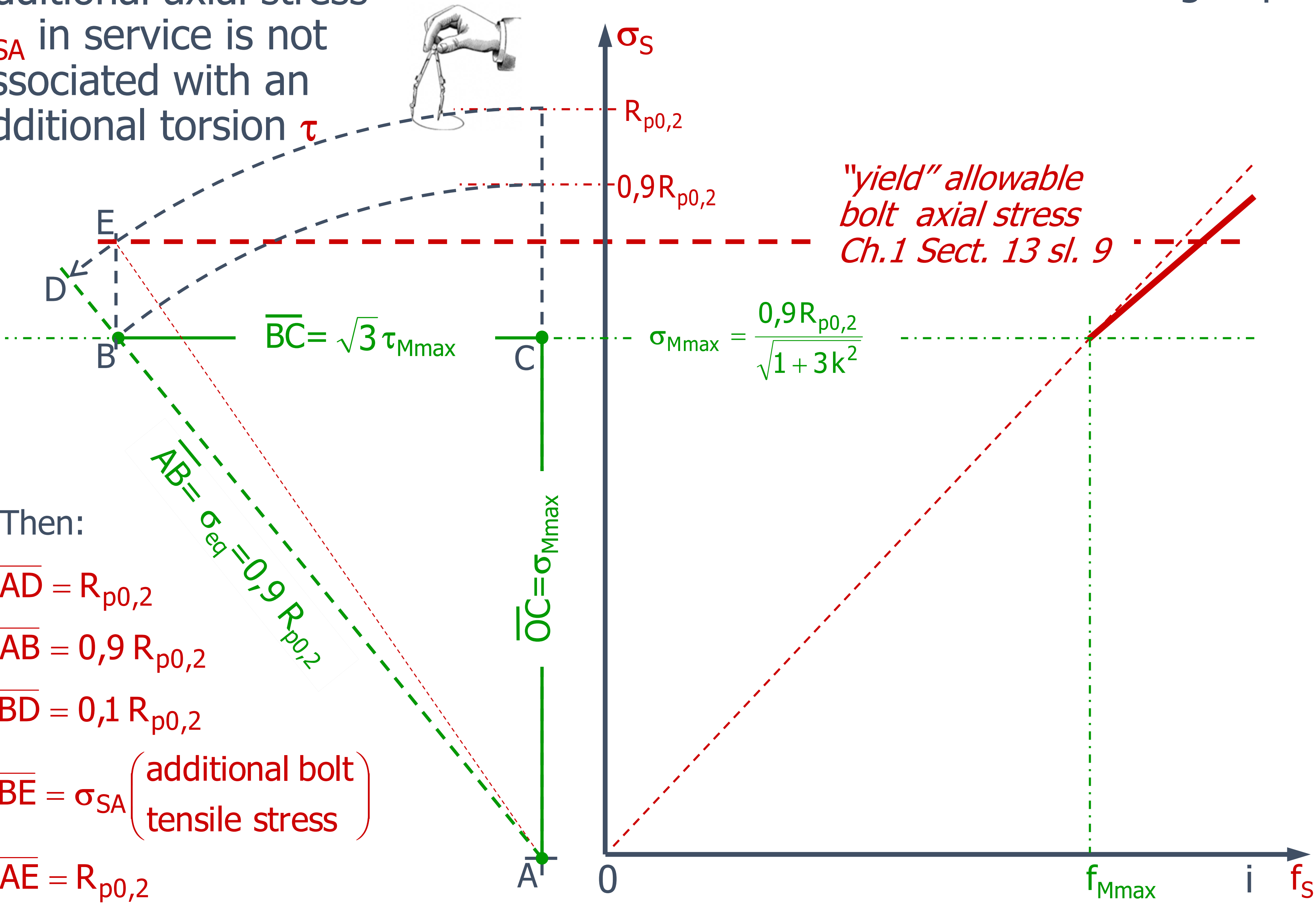




It is like a Pythagoras theorem

Additional axial stress σ_{SA} in service is not associated with an additional torsion τ

Then total axial stress can go up to E



Then:

$\overline{AD}=R_{p0,2}$

$\overline{AB}=0{,}9\,R_{p0,2}$

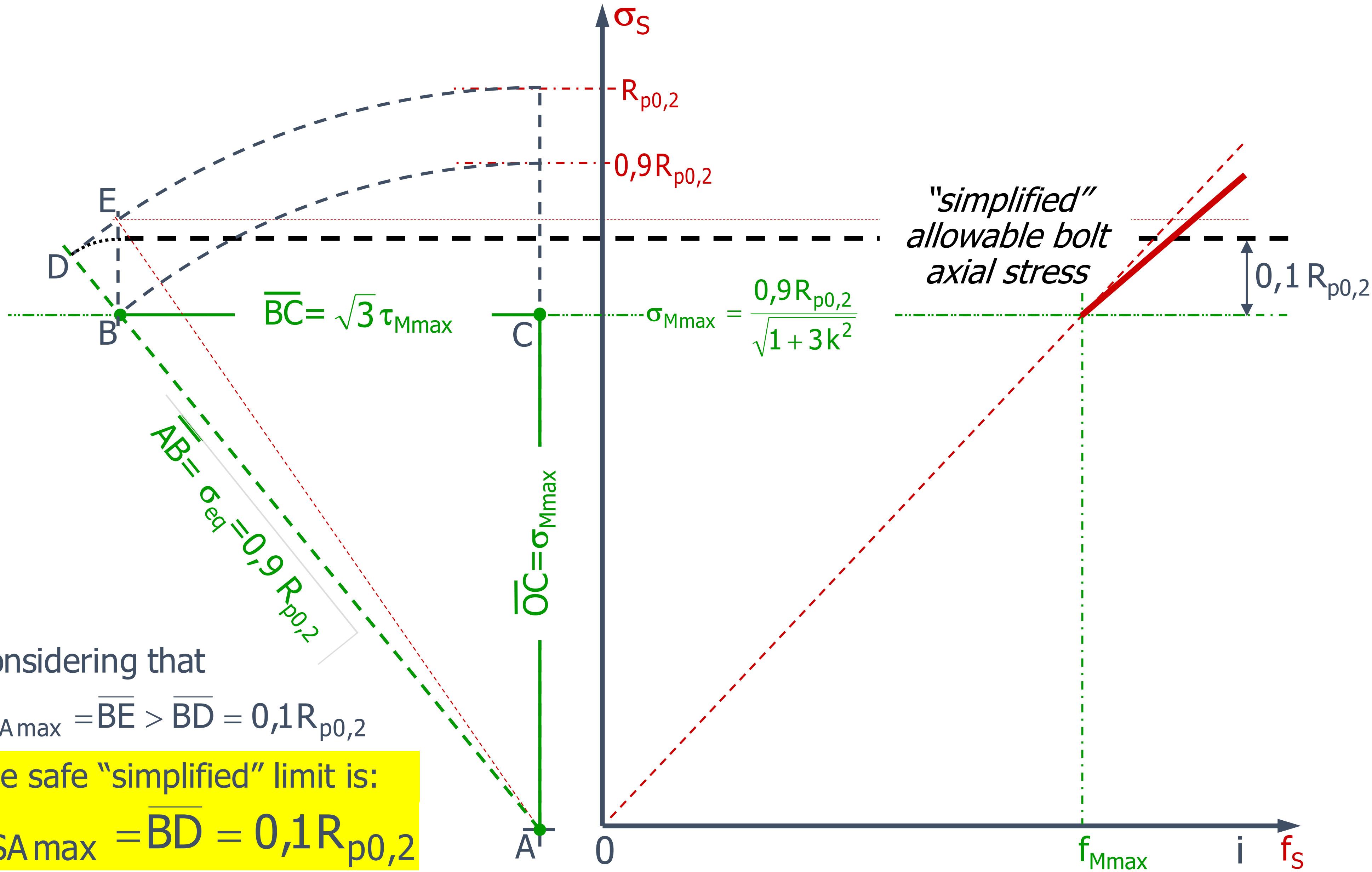
$\overline{BD}=0{,}1\,R_{p0,2}$

$\overline{BE}=\sigma_{SA}\left(\begin{array}{l}\text{additional bolt}\\ \text{tensile stress}\end{array}\right)$

$\overline{AE}=R_{p0,2}$

2 - Allowable bolt stress (4/6)

Simplified procedure to determine maximum bolt (axial additional) stress σ_{SA}



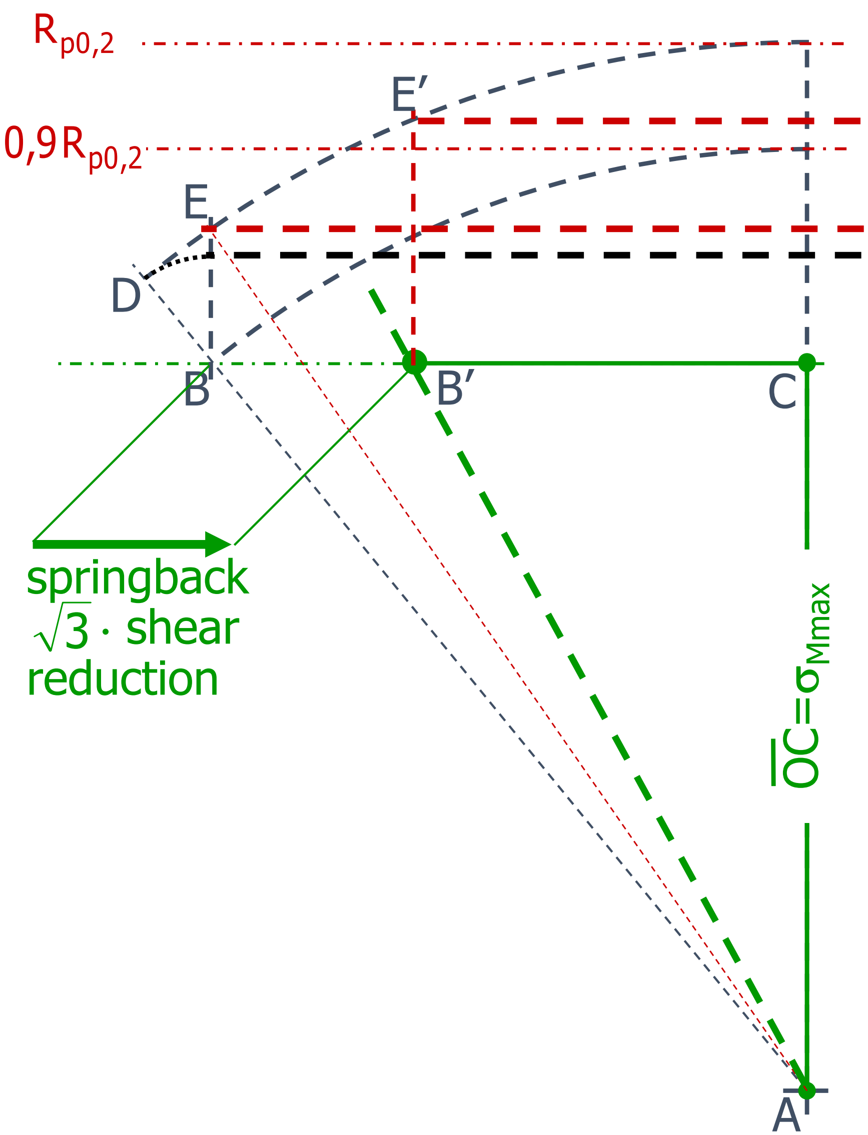
Considering that

$$\sigma_{SA\,max} = \overline{BE} > \overline{BD} = 0{,}1 R_{p0,2}$$

the safe "simplified" limit is:

$$\sigma_{SA\,max} = \overline{BD} = 0{,}1 R_{p0,2}$$

2 - Allowable bolt stress (5/6)



The effective safety margin against yield is even higher ($\overline{B'E'}$ vs. $\overline{DB}$), should we consider that the effective torsional stress in the bolt "after" the tightening process is less that the one present while nut threads are sliding over the bolt threads.

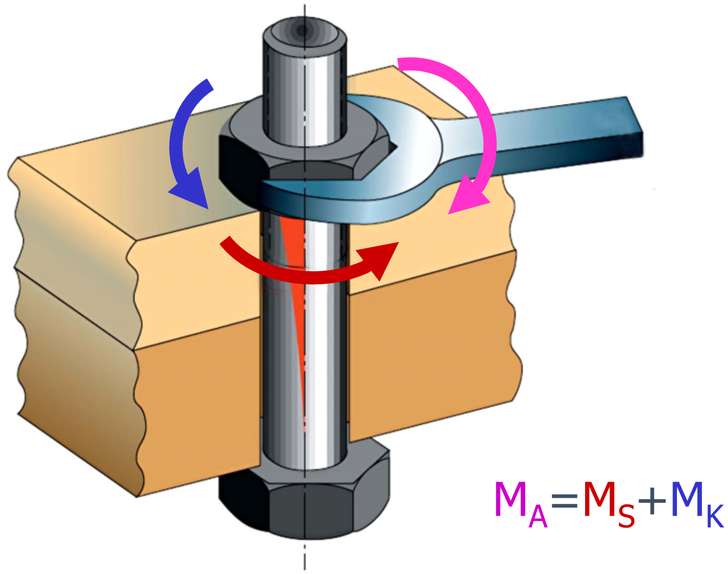
Some of the torsion load on a bolt, acquired when applying a preload, may be released by springback when the wrenching torque is removed.

The amount of relaxation depends on the friction under the bolt head or nut, and on the torsional spring stiffness of the bolt and of the head-to-part contact.

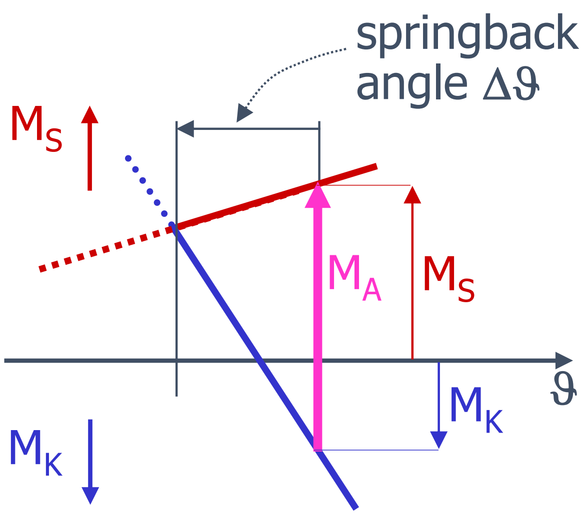
Since within the small strain linear theory torsion does not affect the bolt length, torsional relaxation should not affect axial bolt load.

2 - Allowable bolt stress (6/6)

SK, Bolt tightening Handbook, 2001



- M_A: wrench torque on nut
- M_S: elastic screw-to-nut reaction
- M_K: elastic part-to-nut reaction



$\Leftarrow$

The whole thing is complex and quite controversial. According to Bickford: "*Many people, in fact, will insist that torsional stress disappears immediately and completely when the wrench is removed from the fastener. Others find that it doesn't disappear until a breakaway torque is applied*".*

However, calculation and control of the amount of springback is difficult. So, unless precise data are available, it is safer to assume that shear stress in the bolt is at its upper limit, calculated according to sliding head-to-part and nut-to-screw contacts with the friction model.

The qualitative figure on the left assumes that during springback there is no sliding between nut and part, nut and bolt.

*J. H. Bickford, Introduction to the Design and Behavior of Bolted Joints, Non-Gasketed Joints, CRC Press 2008

Section 3 - Stress distribution along the thread

This section shows that the total axial load transmitted between bolt and nut along the thread varies with an exponential law.

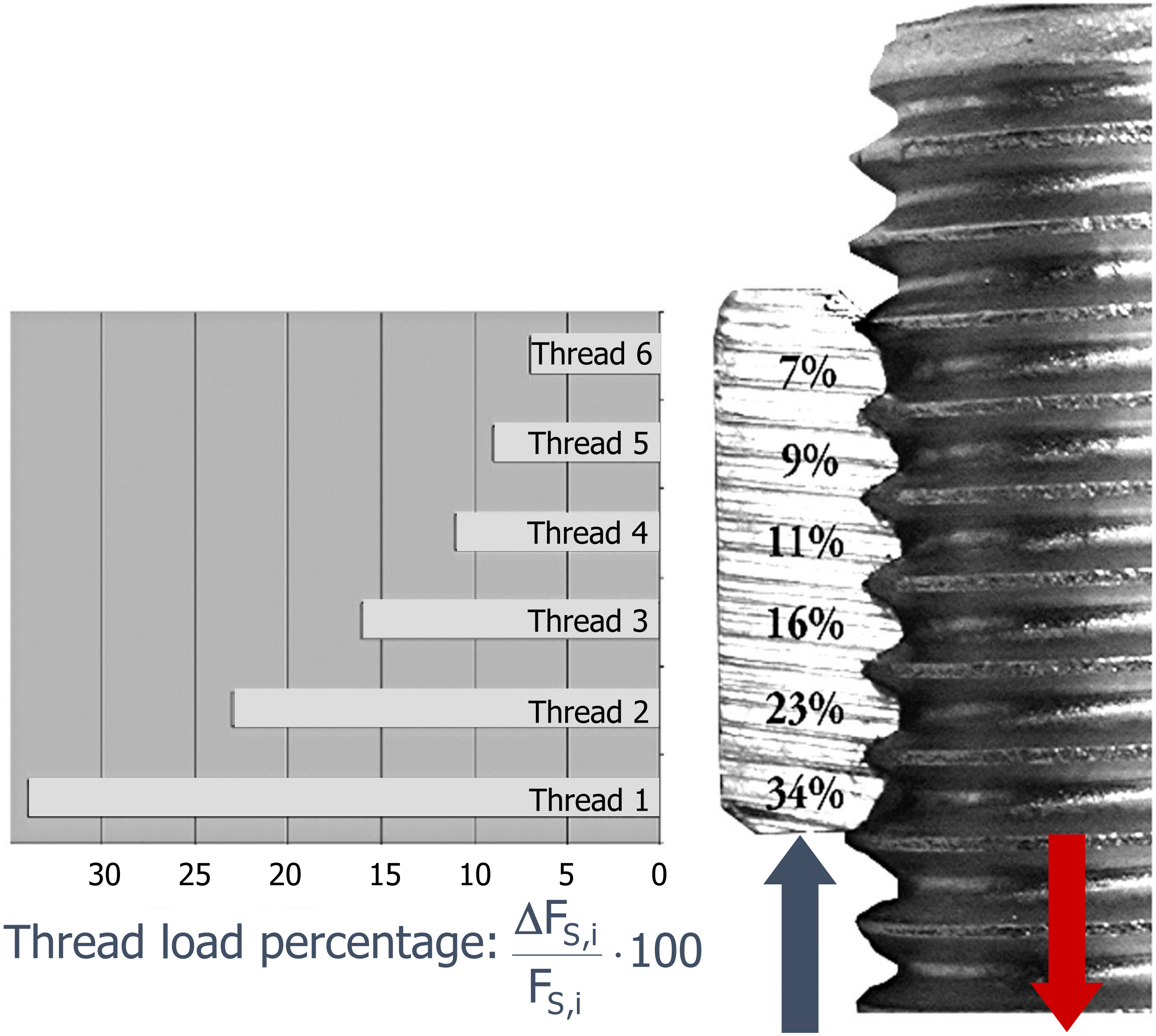
It also shows that for ordinary nuts, which are fully in compression, the first engaged thread - toward the clamped part - takes over 1/3 of the total bolt load.

This has consequences on the maximum stress at the root radius of the first thread, where there is then the highest danger to fatigue produced by the variable part of the load.

The section shows some solutions which have been historically developed to mitigate such vary local, and very dangerous, fatigue stresses.

3 - Load distribution on threads (1/12)

http://www.fastenal.com/web/services.ex?action=engineering&article=ScrewThreadDesign

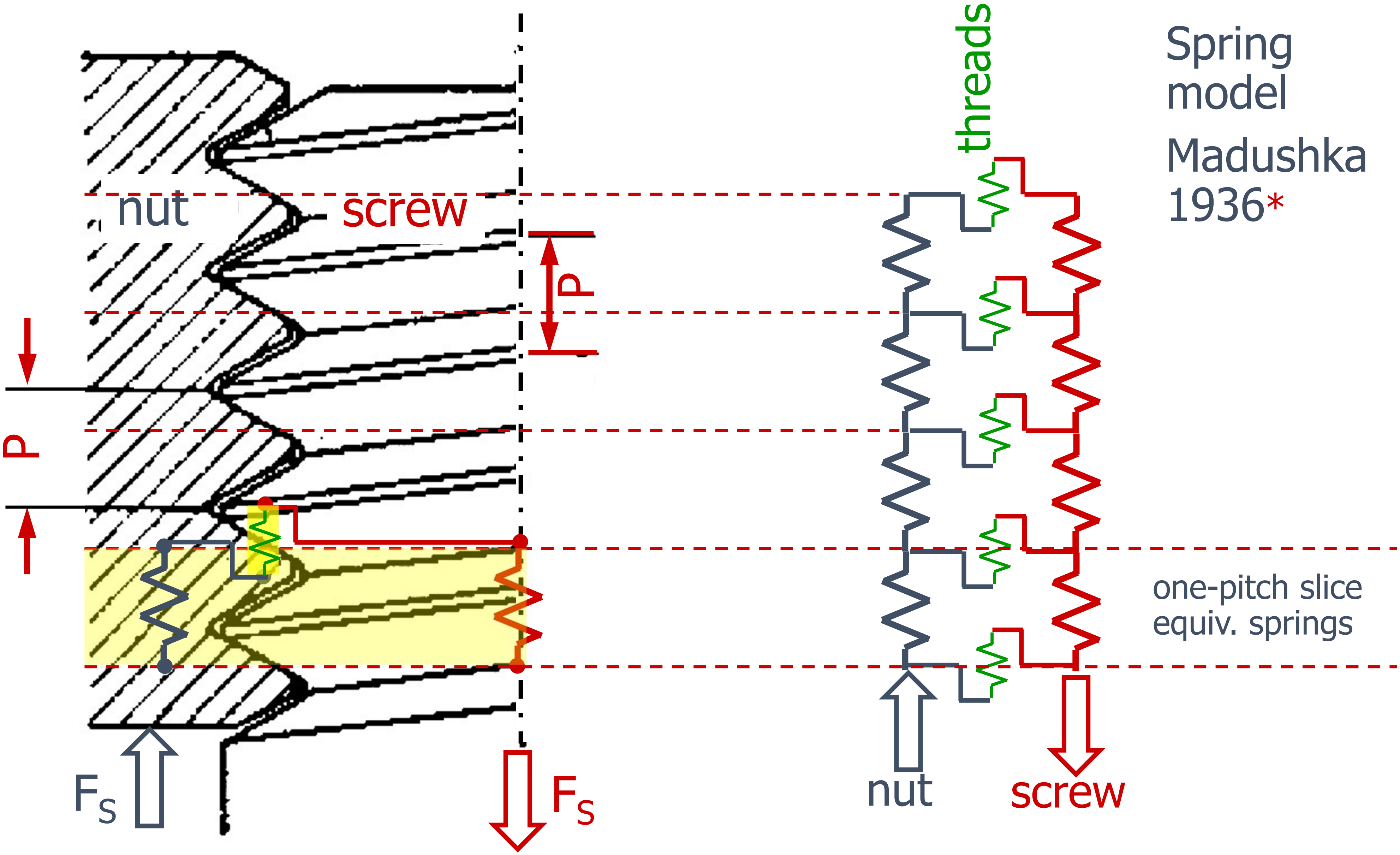


Thread load percentage: $\frac{\Delta F_{S,i}}{F_{S,i}} \cdot 100$

This figure introduces the problem of load distribution of the screw load on the threads.

It is clearly seen that the first engaging thread takes about 1/3 of the total bolt load.

3 - Load distribution on threads (2/12)

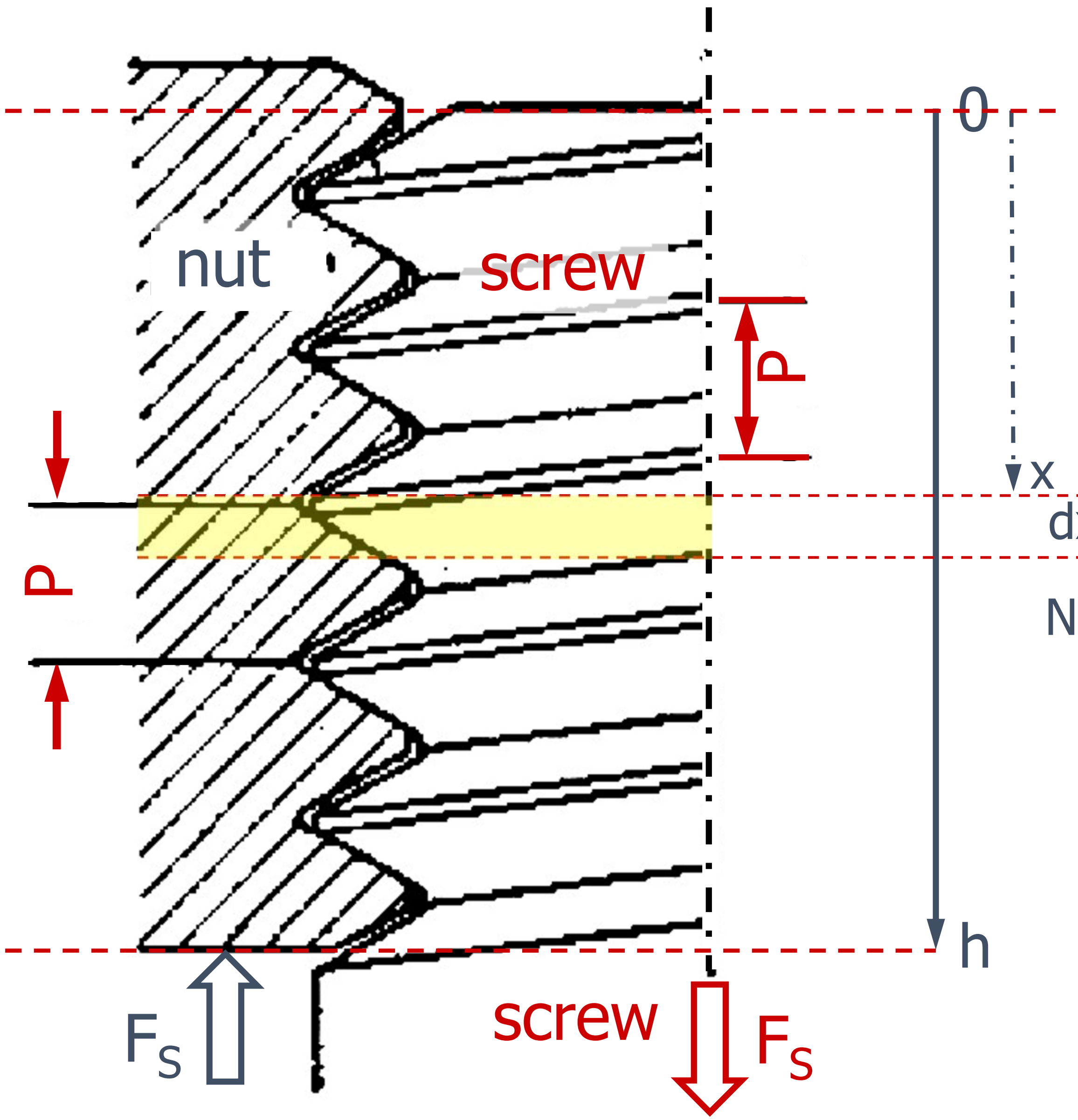


Spring model

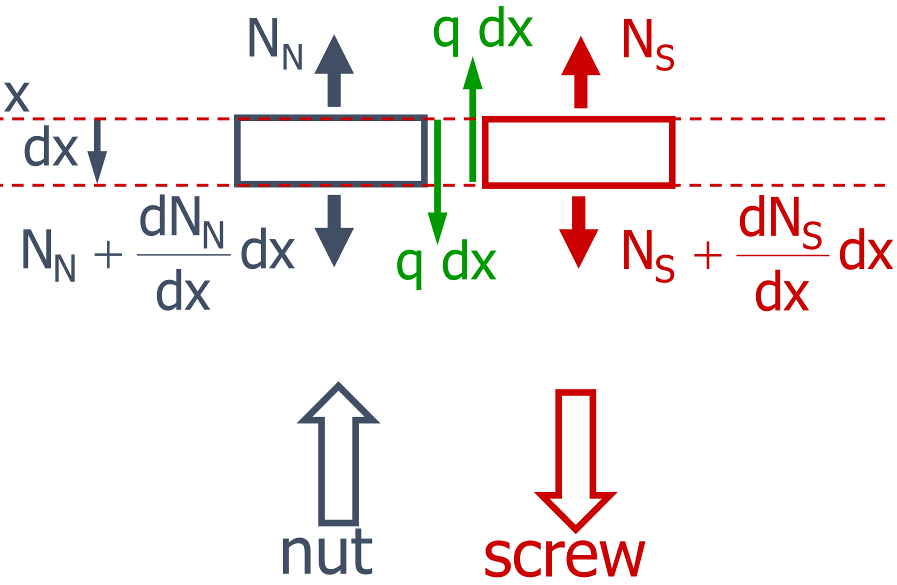
Madushka 1936*

* Madushka, L., "'Beanspruchung von Schraubenverbindungen und zweckmassige gestaltung der gewindetrager" Forschung, 7, 299-305 (1936)

3 - Load distribution on threads (3/12)



Continuous model
Birger 1950*



infinitesimal slice of **nut** and **screw** in contact at the **thread** mean diameter

* Birger, IA, (1944) Distribution of Load in a Screw Thread, (in russian) Vestnik Mashinostroeniya 11

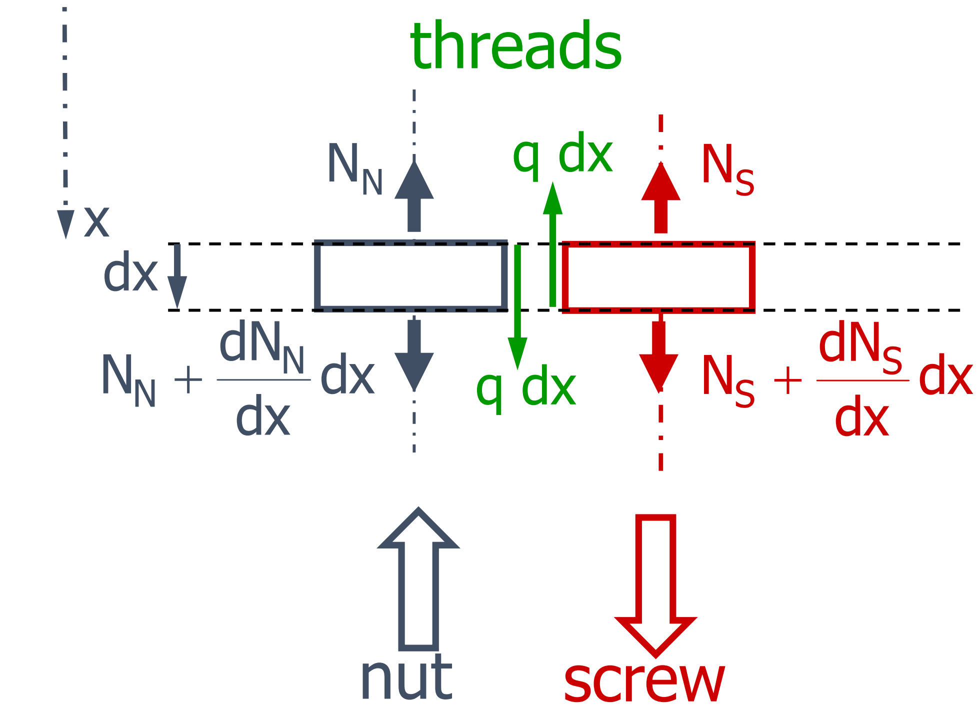
Neglecting the fact that the thread is wound helicoidally around the pitch cylinder, a purely axial model is studied where an infinitesimal slice dx of the bolt is in contact with the corresponding slice of nut through the thread.

q : load per unit length (axial) exchanged between threads

N_S: load (axial) on the screw cross section

N_N: load (axial) on the nut cross section

Side by side representation actually coaxial



infinitesimal slice of **nut** and **screw** in contact at the **thread** mean diameter

Bolt equilibrium:

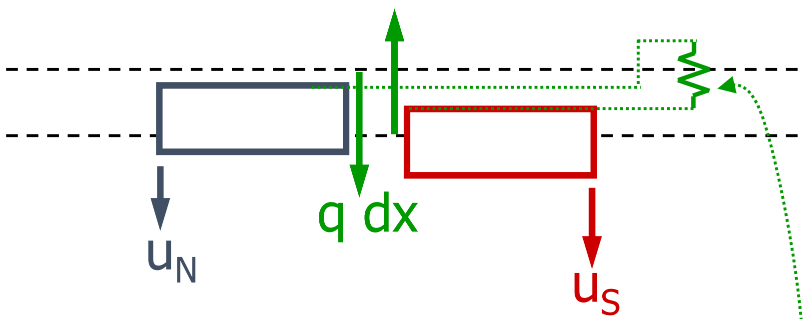
$$\frac{dN_S}{dx} = q$$

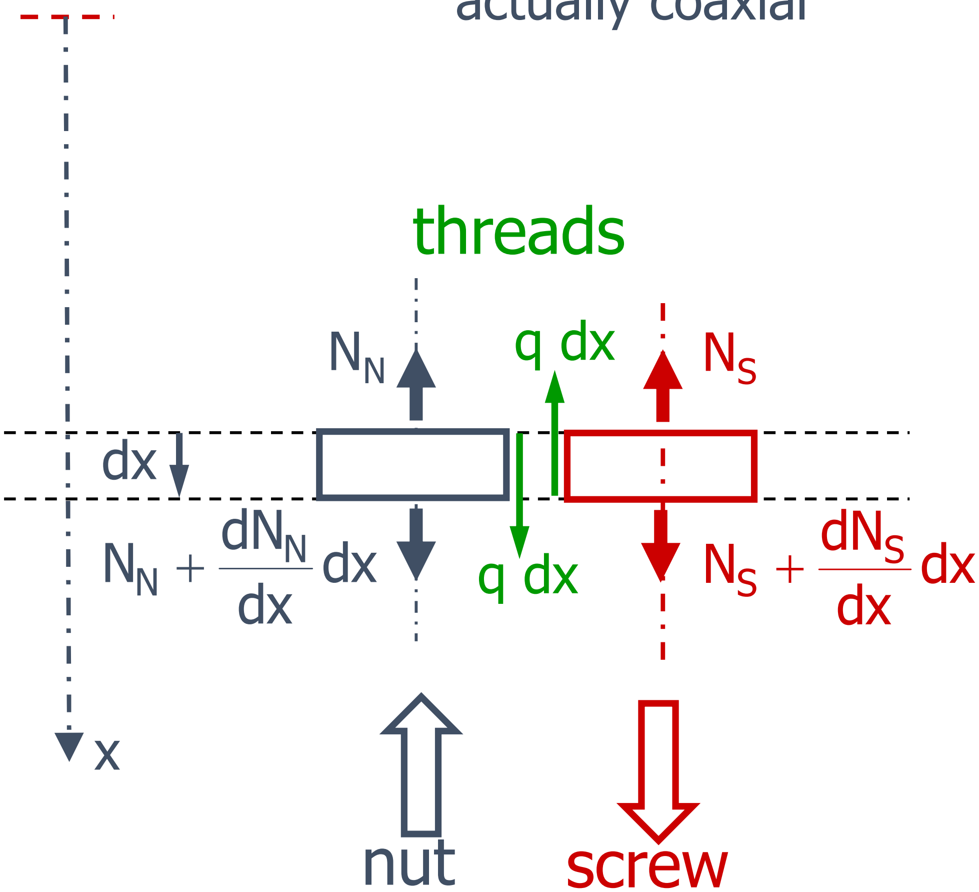
Nut equilibrium:

$$\frac{dN_N}{dx} = -q$$

Side by side representation actually coaxial

Displacements u_N, u_S under load:





Thread distributed load q is due to the elastic reaction between bolt and nut threads, which deform as short plane strain beams in bending and shear, plus local contact elasticity:

$$q = k(u_S - u_N)$$

3 - Load distribution on threads (6/12)

$$\frac{du_S}{dx} = \varepsilon_S = \frac{N_S}{E_S A_S} \qquad \frac{du_N}{dx} = \varepsilon_N = \frac{N_N}{E_N A_N}$$

Bolt and nut stress-strain relations

$$q = k(u_S - u_N)$$: thread load-displacement relation

$$\frac{dq}{dx} = k\left(\frac{du_S}{dx} - \frac{du_N}{dx}\right) = k\left(\frac{N_S}{E_S A_S} - \frac{N_N}{E_N A_N}\right)$$

$$\frac{d^2q}{dx^2} = k\left(\frac{1}{E_S A_S}\frac{dN_S}{dx} - \frac{1}{E_N A_N}\frac{dN_N}{dx}\right) = q\left(\frac{k}{E_S A_S} + \frac{k}{E_N A_N}\right)$$

Equilibrium, stress-strain and load-displacement equations are combined in order to produce one differential equation of the second order with variable q

$$\left.\begin{array}{l} \dfrac{d^2q}{dx^2} - \lambda^2 q = 0 \\ with: \quad \lambda^2 = \left(\dfrac{k}{E_S A_S} + \dfrac{k}{E_N A_N}\right) \end{array}\right\} \Rightarrow \quad q = A_1 e^{\lambda x} + A_2 e^{-\lambda x}$$

3 - Load distribution on threads (7/12)

Boundary conditions for the "compressed" nut:

$$\frac{dq}{dx} = \lambda\left(A_1 e^{\lambda x} - A_2 e^{-\lambda x}\right) = k\left(\frac{N_S}{E_S A_S} - \frac{N_N}{E_N A_N}\right)$$

$$\begin{cases} \text{a) } x=0 \Rightarrow N_N=0,\ N_S=0 \\ \text{b) } x=h \Rightarrow N_N=-F_S,\ N_S=F_S \end{cases}$$

Condition a) produces:

$$\lambda(A_1 - A_2) = 0 \rightleftarrows \quad \Rightarrow \quad A_1 = A_2 \ \Rightarrow A$$

Then: $q = A\left(e^{\lambda x} + e^{-\lambda x}\right)$

Condition b) produces: $\lambda A\left(e^{\lambda h} - e^{-\lambda h}\right) = F_S\left(\frac{k}{E_S A_S} + - \frac{k}{E_N A_N}\right) \equiv F_S \lambda^2$

It then follows: $A = \dfrac{\lambda}{e^{\lambda h} - e^{-\lambda h}} F_S$ hence: $q = \lambda \dfrac{e^{\lambda x} + e^{-\lambda x}}{e^{\lambda h} - e^{-\lambda h}} F_S$

3 - Load distribution on threads (8/12)

This is also written: $q = F_S \lambda \dfrac{ch(\lambda x)}{sh(\lambda h)}$

The integral of q dx over the height of one thread turn P has the meaning of "fraction of screw load taken by one turn of thread".

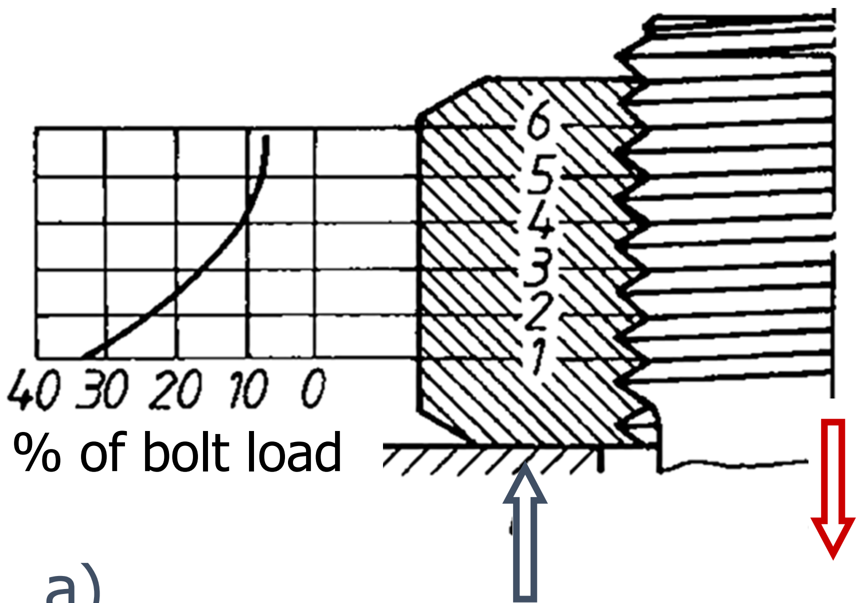
$$Q(x) = F_S \int_X^{X+P} \lambda \frac{ch(\lambda x)}{sh(\lambda h)}\, dx = \frac{F_S}{sh(\lambda h)} \int_X^{X+P} [\lambda ch(\lambda x)]\, dx =$$

$$= \frac{F_S}{sh(\lambda h)} \int_X^{X+P} d[sh(\lambda x)] = \frac{F_S}{sh(\lambda h)} \left[sh\big(\lambda(X+P)\big) - sh(\lambda X) \right]$$

This is normally given for:

Thread 1: from X=h to X=h-P
Thread 2: from X=h-P to X=h-2P
etc., up to the last thread at the top of the nut

3 - Load distribution on threads (9/12)

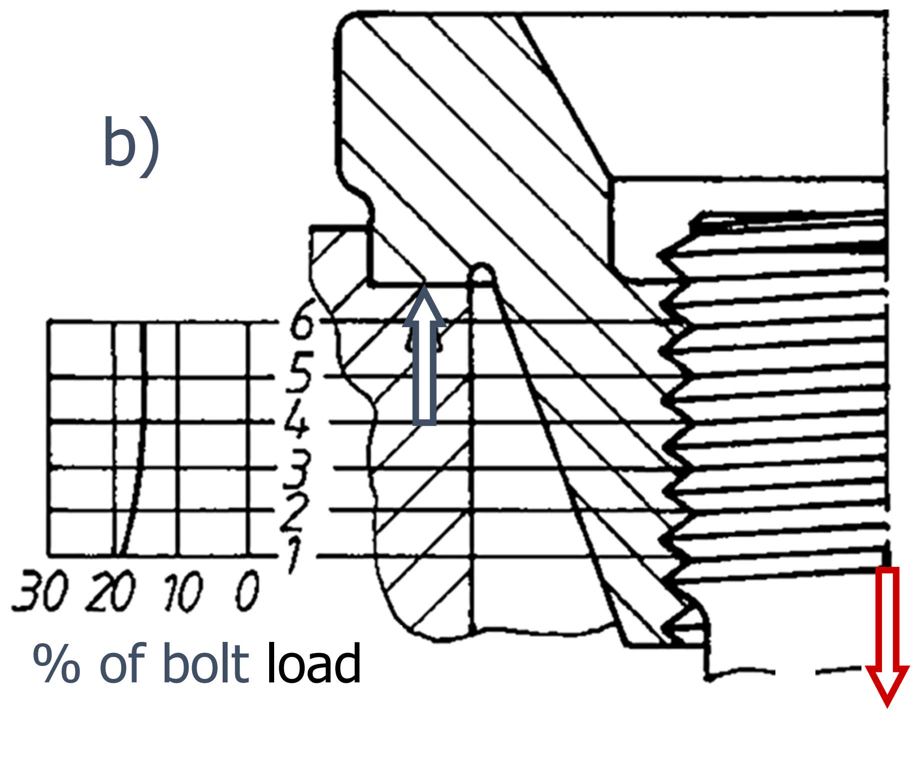


a)

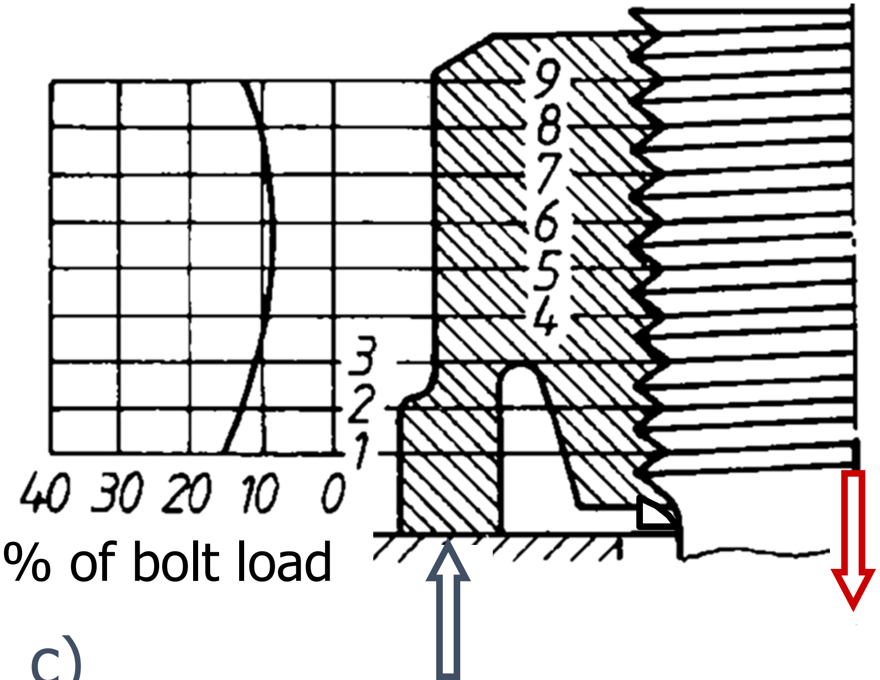
The figure below shows the load distribution on the six engaged thread in the case of "nut in tension" (boundary conditions differ from case a) It is much more uniform.

The figure above shows the load distribution on the six engaged thread in the case of "nut in compression", i.e., the case solved before.
It is markedly non uniform.

b)



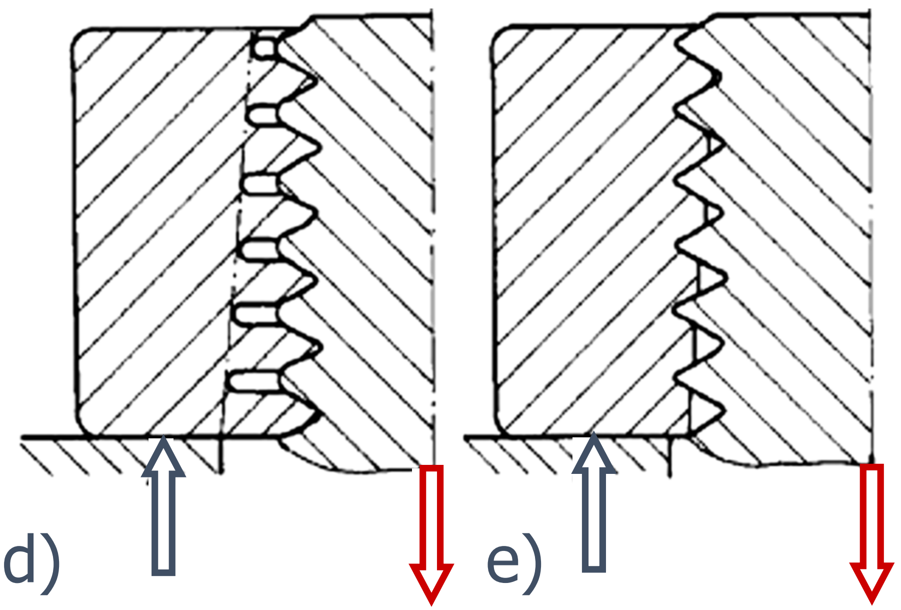
3 - Load distribution on threads (10/12)



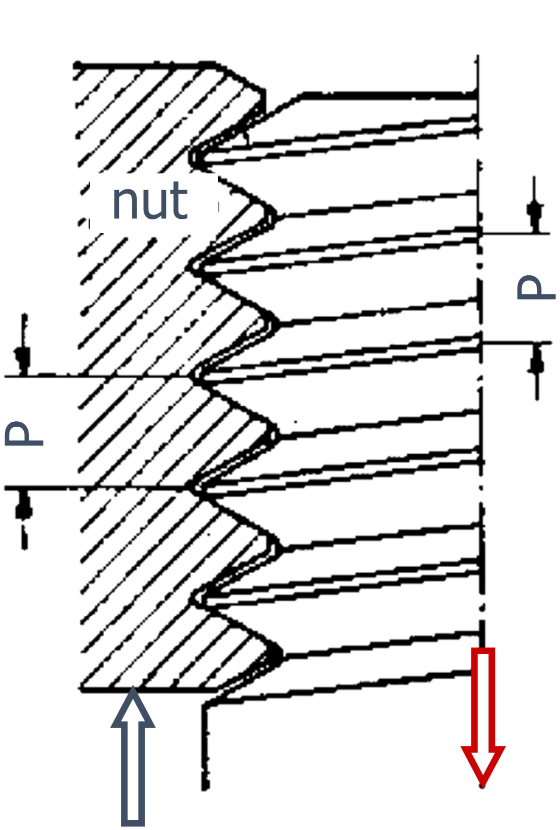
c)

This is a compromise solution, where the lower part of the nut is "in tension"
Load distribution is quite satisfactory.

To better distribute load, solution d) employs flexible threads with variable elasticity.
Solution e) tapers the inner part of the nut, producing a conical thread, thus allowing higher flexibility of bolt threads at the nut support level.

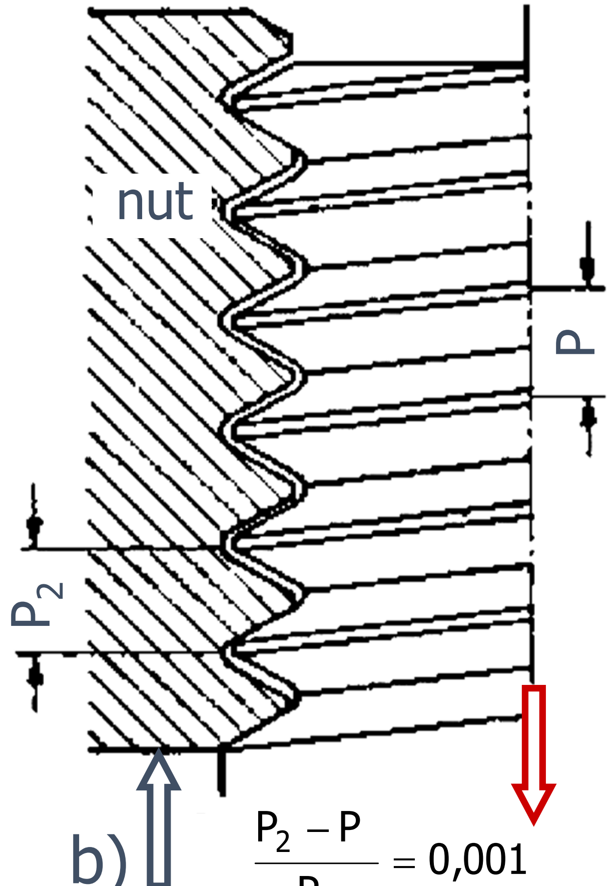


d) e)



a)

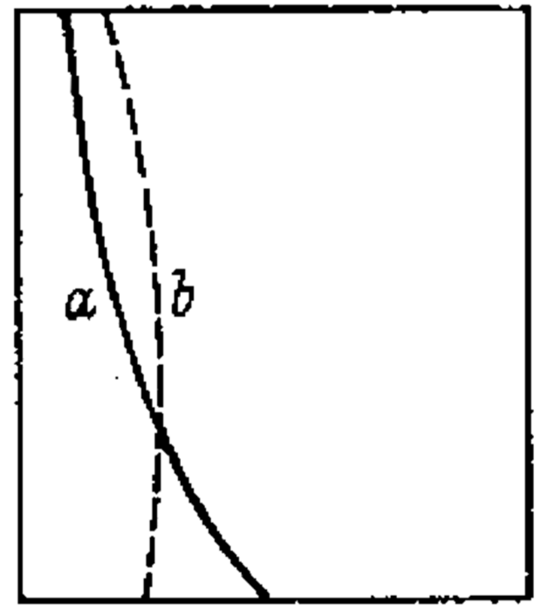
This figure shows the normal engagement between a screw and a “compressed” nut



b)

$$\frac{P_2 - P}{P} = 0{,}001$$

There is here a small difference between bolt and nut pitch; the first thread takes load only after some bolt elongation

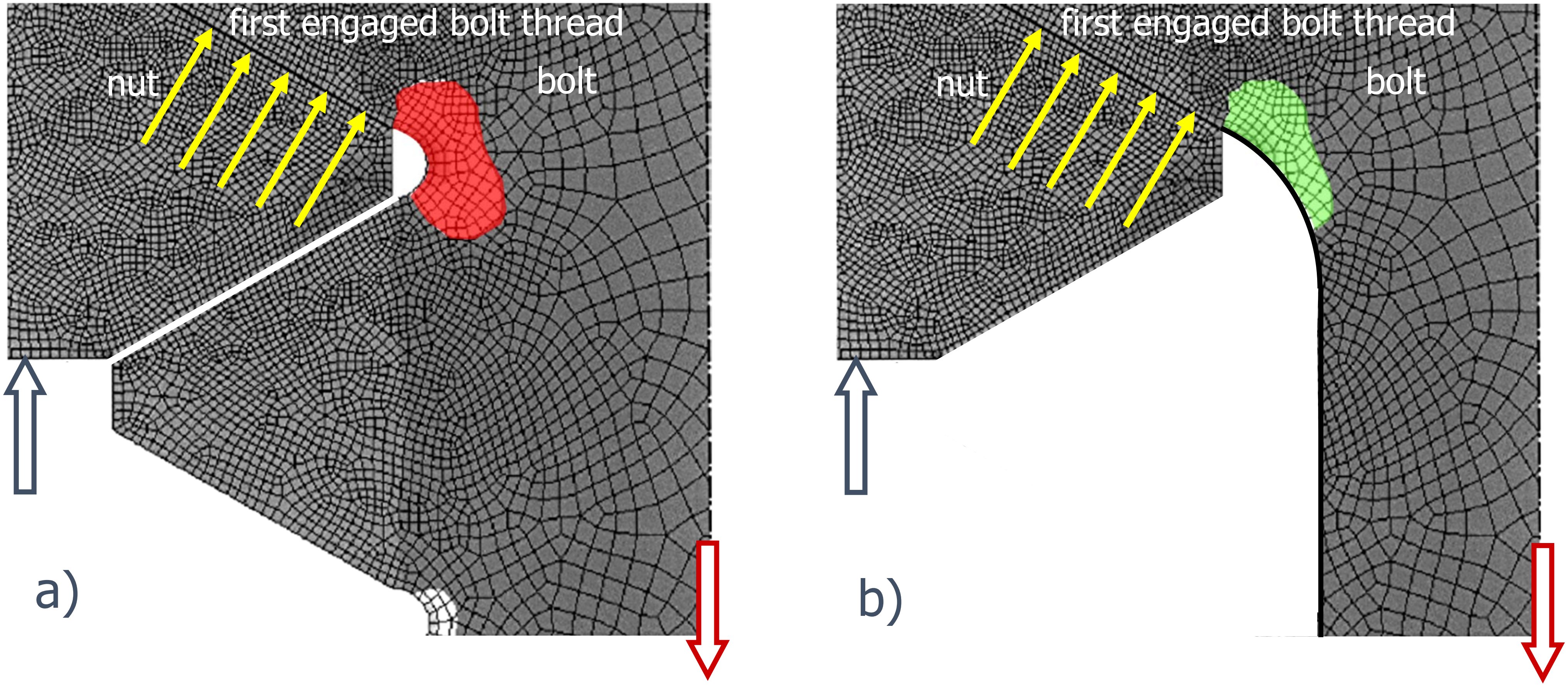


% of bolt load on engaged threads

c)

This figure shows the benefit of solution b) against a). However, the shape of curve b) depends on load.

3 - Load distribution on threads (12/12)



In Sect. 3 sl. 9 the reduced shank bolt was one pitch inside the nut, this being case b); case a) is when this is not done or when the bolt has no reduced shank.
In both cases the first engaged thread takes the same share of the bolt load.
But in case b) the bending of the first engaged thread generates at the root a maximum stress which is lower because the fillet radius is much higher than in case a), where the much smaller thread bottom radius is a much more severe notch. In a) local elastic stresses are higher, affecting fatigue.

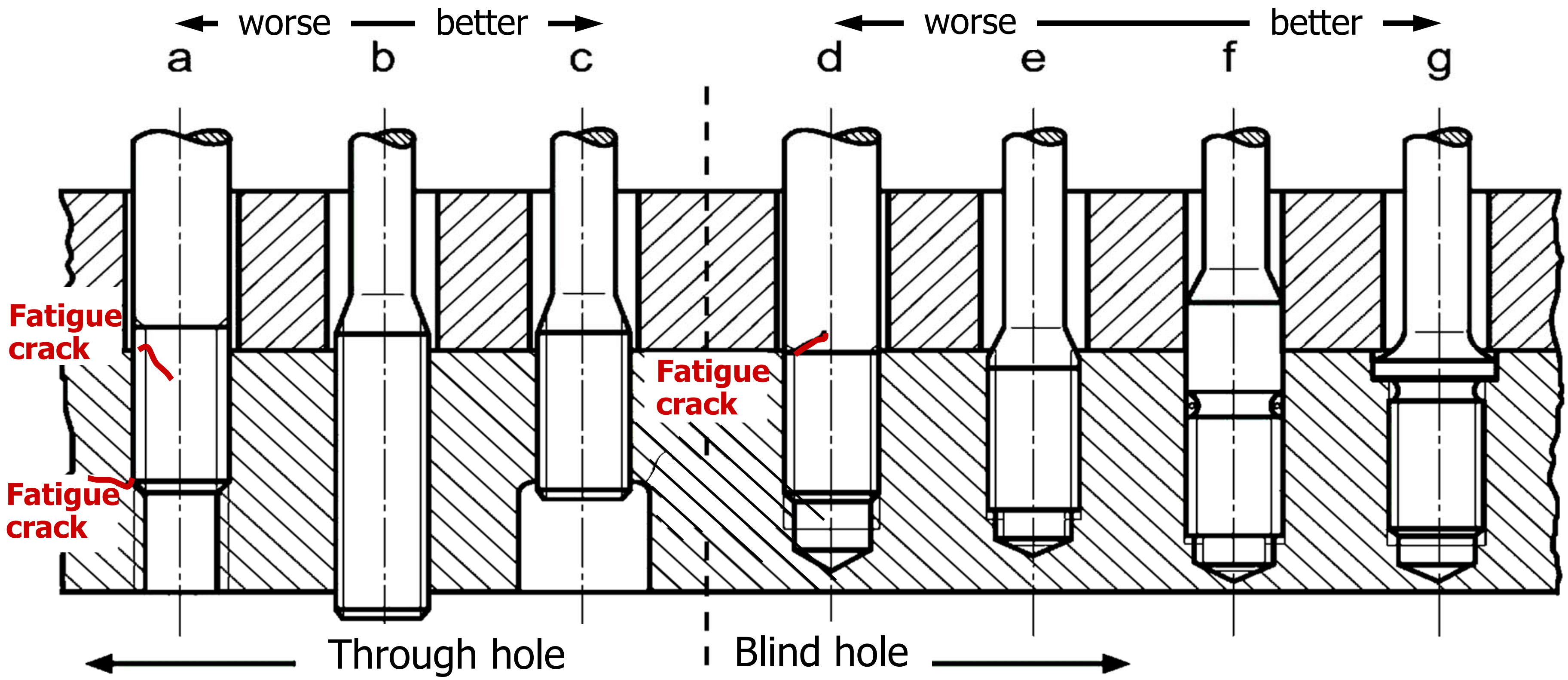
Section 4 - Bolt refinements

This section compares different bolt shapes according to their capability of mitigating fatigue stresses and, therefore, improve their load capability.

Two interesting cases are examined:

- studs
- through hole bolts for highly stressed conrod applications

Different solutions for studs



a) Risk of crack in the first bolt thread and in the hole thread

b) Risk reduction:
  -) in hole, thanks to through shaft
  -) in bolt, thank to more flexible shank

c) Risk reduction thanks to through shaft provided a well rounded fillet is made

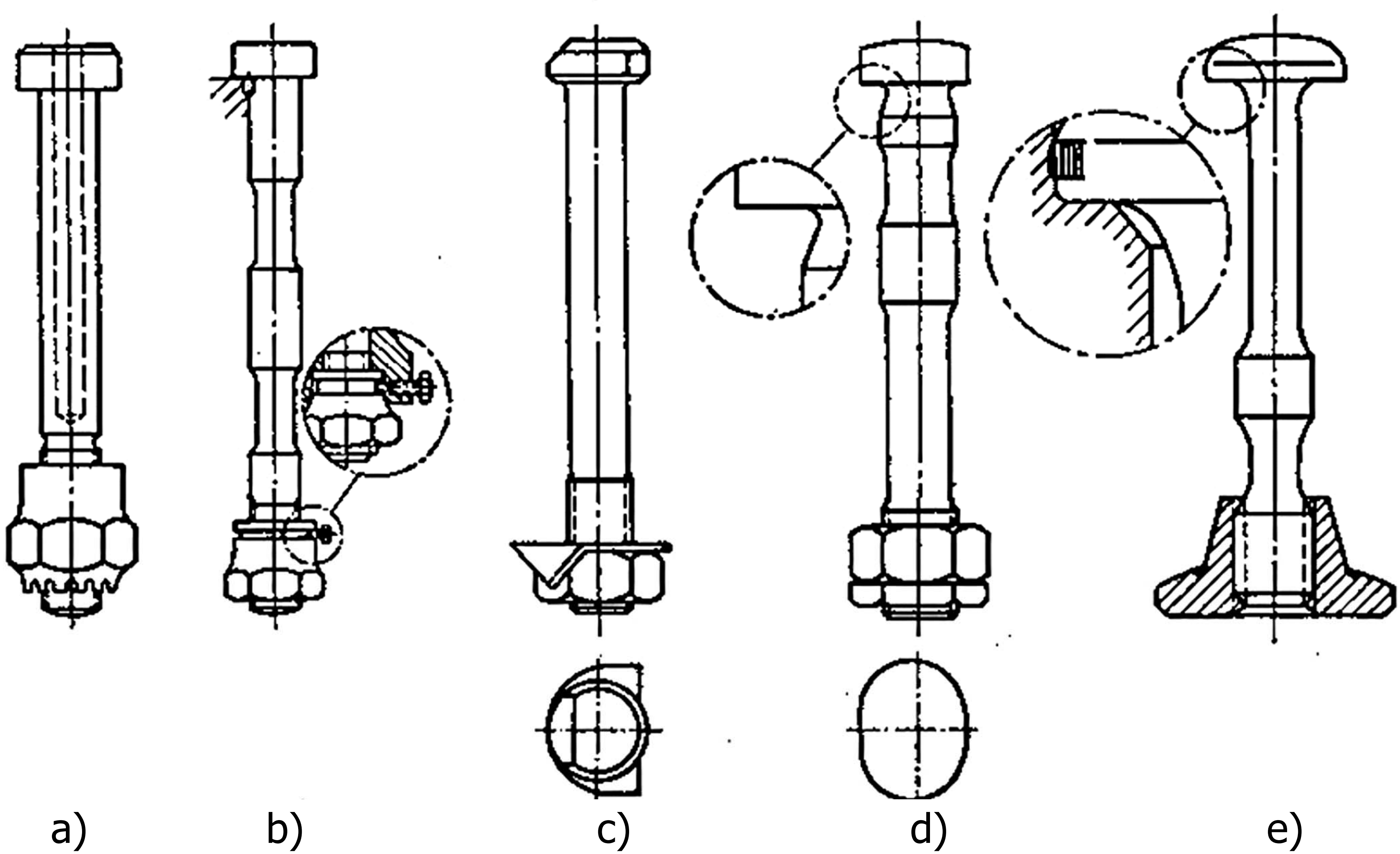
d) Risk of crack where the bolt thread run out point is forced into the hole thread

e) Risk reduction thanks to reduced shank and to check of bolt against bottom end

f) Risk reduction: calibrated centre avoids bending at thread and groove relief stress

g) Flange bears bending and preloads thread

4 - Mitigation of fatigue problems in bolts (2/5)



Hans-Christof Klein, Hochwertige Schraubenverbindungen – Einige Gestaltungsprinzipien und Neuenentwicklungen
Konstruktion, 1959, Heft 6 201-219 & Heft 7 259-264

4 - Mitigation of fatigue problems in bolts (3/5)

a) This solution is presented in the book "Schnellaufenden Dieselmaschinen" (High speed Diesel engines) by Föppl, 1922; a bore along the bolt shaft increases its axial deformability, without using a "waisted " or "reduced shank" solution; compared to the latter, this approach has the shortcoming of a higher bending stiffness and higher stress peaks at the thread runout; a splined nut is a safety against unlocking, however it is doubtful whether this is effective against loss of pre-load; the high nut is due to the use of a fine thread; bolt rotation during tightening is prevented by a wrench applied to the head; the thread correctly ends in a relief groove

b) Proposed in 1927 by Rötscher (Die Maschinenelemente, Springer); it is a first example of "waisted" shank, even though the nominal diameter fitting cylinders take the most of the shaft length; inconveniently, the thread ends into the fitting part without a relief groove; bolt rotation during tightening is prevented by a dowel (pin) under the head

c) This shows a flattened bolt head (against a matching surface on the conrod) to prevent bolt rotation during tightening; this simple bolt is not waisted

d) This has an eccentric circular-arc head to prevent rotation during tightening, easier to manufacture than in c); the shank is mostly waisted, i.e. more compliant both axially and in bending; a larger fillet under the head reduces peak stresses; contrary to case b), the thread ends at shank diameter reduction, thus minimizing local stress peaks

4 - Mitigation of fatigue problems in bolts (4/5)

e) This solution embodies the most advanced developments.

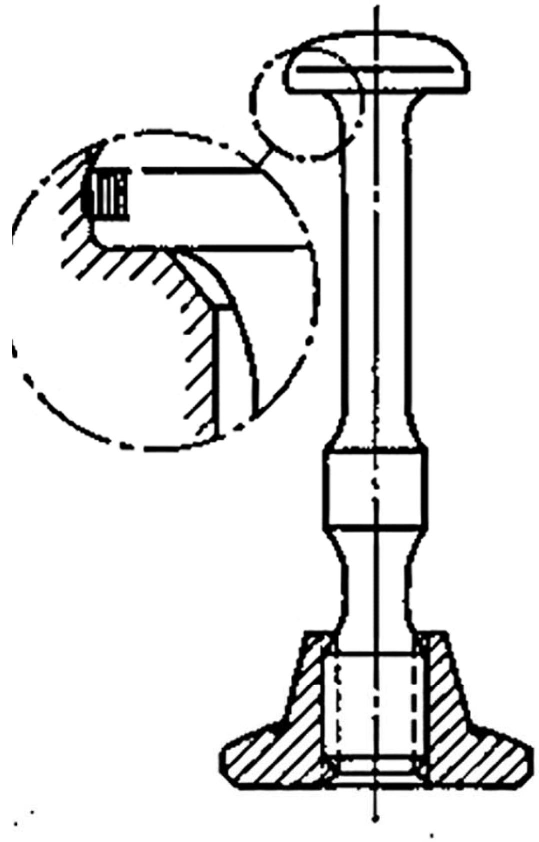
A larger head allows the adoption of a much larger fillet radius between head and (reduced) shank, which is very beneficial to fatigue: in order to accommodate the fillet, the edge of the through hole edge on the conrod must be generously beveled, which requires a larger bolt head.

Like in solution d) the thread ends flush with the reduced shank.

The peak load at the first thread engaging into the nut is reduced by adopting a partly "suspended nut" (as opposed to the "supported nuts" adopted in solutions a, b, c, d) and by inserting the bolt thread one pitch inside the nut.

The shank reduction is quite pronounced: shank dia. 10% to 20% lower than the threaded minor dia.

Use of a coarse thread is convenient due to the fact that the bottom radius is larger, then more favourable to fatigue.



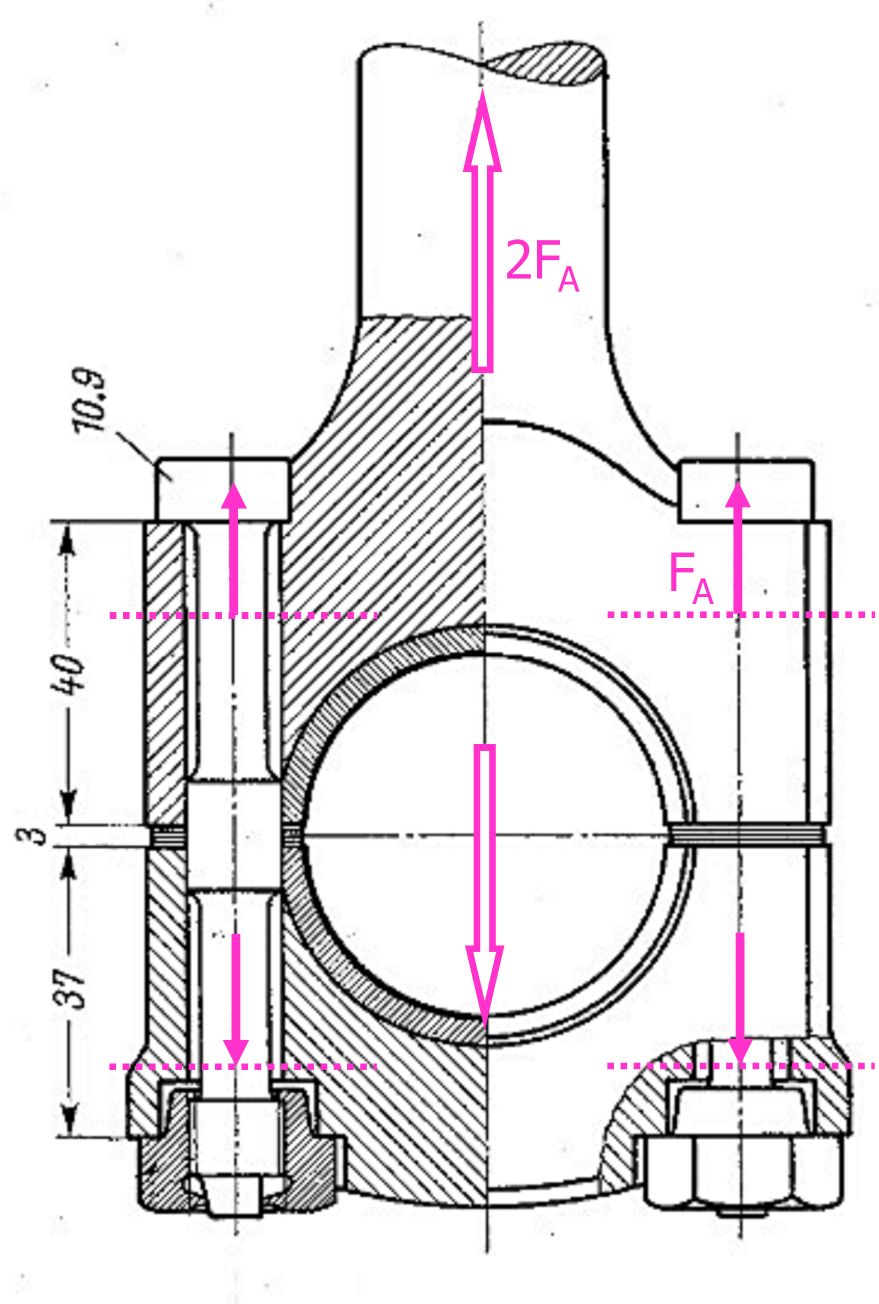
e)

4 - Mitigation of fatigue problems in bolts (5/5)

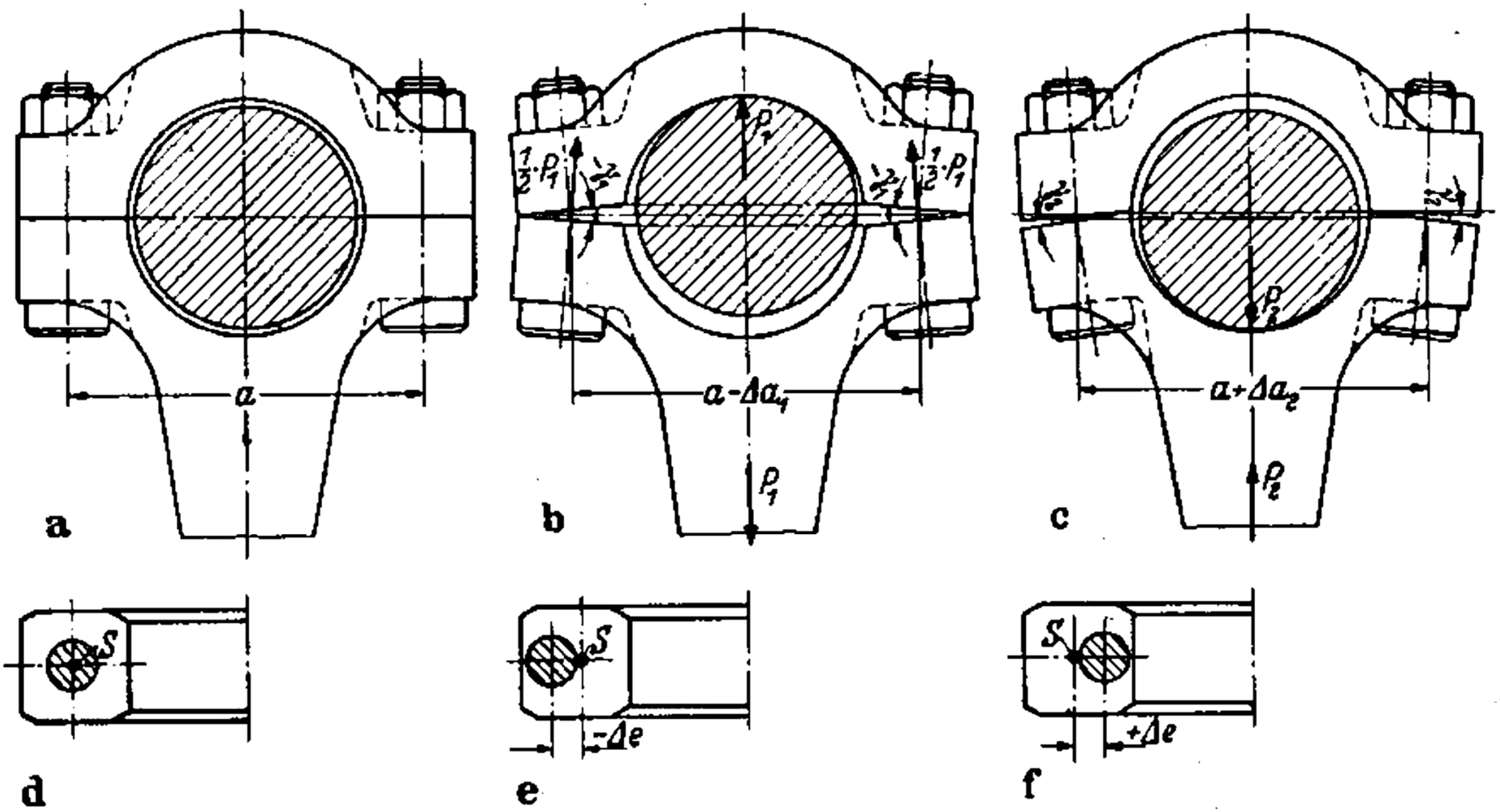
The figure on the right shows a modern application of the advanced bolt on a conrod:

- flat head against rotation during tightening
- waisted shank
- partially “suspended” nut
- first bolt thread one pitch inside the bolt (sect. 3, sl. 9 and 12)

Violet forces applied to the rod and to its cap indicate qualitatively the need to evaluate the position of the part force at each bolt location.



5 - Bending of clamped assemblies (1/x)



Under development

we are working for you

App. 1 - Coarse or fine thread?

Fine threads

- Higher load carrying capacity: they have a slightly larger stress area
- Can tap better in thin walled components
- Permit closer adjustment accuracy due to the lower helix angle
- Have higher resistance to vibration loosening
- Less tolerant to damage
- Easily cross-threaded, therefore require careful assembly

Coarse threads

- Greater stripping strength for the same engagement length
- Quicker assembly and disassembly
- Less likely to cross-thread, easier to assemble, need less care
- Larger allowances, can accomodate thicker coatings and platings without adjustments