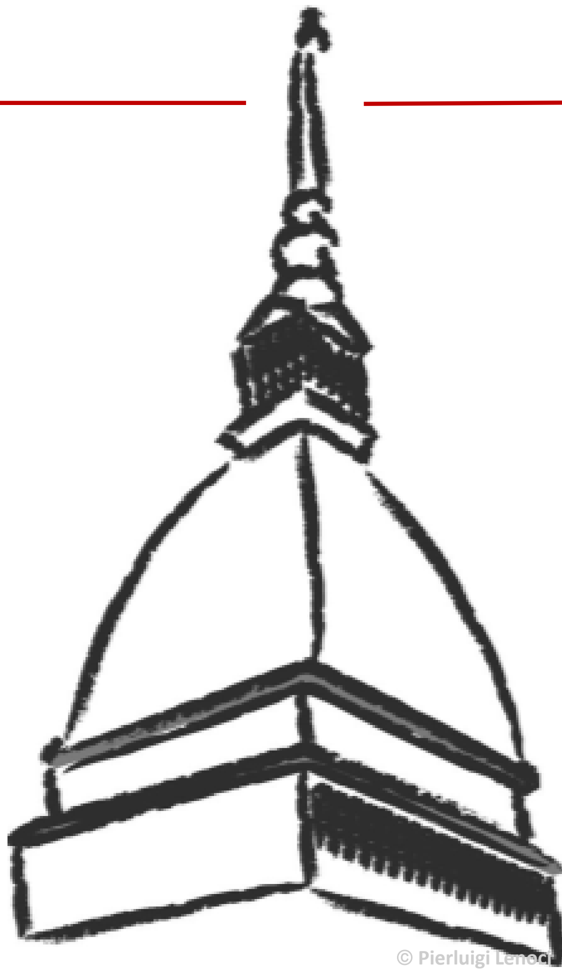
Daniele Botto

Costruzione di Motori per Aeromobili - Machine Design

Fatigue - Chapter 4



Chapters

Finite life

1. Linear Damage or Miner's Rule
2. Cycle counting
3. Finite life and stress spectra
4. Finite life and the exceedance diagram
5. Appendix: the Haibach extension

Sections 1, 2 - Finite life

These sections refer to one-dimensional tensile stresses (mainly tensile, but compressive stress could be included as well without difficulty). They deal with variable stresses that sooner or later will produce a failure, i.e. life is finite.

Section 1 defines the Miner's rule for cumulative damage at multiple stress levels applied during the component life. A numerical example is worked out for better clarity.

First the concept of "equivalent stress amplitude" is introduced. Then it is shown how the simplified interpolation formulas introduced in Ch. 2 Sect. 8 can extend the power of this concept.

Section 2 shows one "cycle counting" method (the bath-tub version of the rain-flow), and its practical application when a stress spectrum is to be obtained from a time signal.

The “linear damage rule” was first proposed by Palmgren* in 1924 and was further developed by Miner** in 1945. Today the method is commonly known as Miner's Rule.

The linear damage rule is based on the concept of cumulative damage.

* A. Palmgren, “Die Dauerhaftigkeit der Kugellager,” *Z. Ver. Deutsch. Ing.*, **68** (1924)

** M. A. Miner, “Cumulative damage in fatigue,” *J. Appl. Mech.*, **12** (1945)

Read also:

M. A. Miner, “Estimation of fatigue life with Particular Emphasis on Cumulative Damage , ch. 12 in ” *Metal Fatigue”, G. Sines and J.L. Waisman editors,* Mc Graw Hill, USA, (1959)

1. Linear Damage or Miner's Rule (2/11)

The cumulative damage concept requires a “damage fraction” w_i as the fraction of the total damage W necessary to produce a crack.

Failure is predicted to occur when
the sum of the w_i's reaches the limit W: $\sum_i w_i = W$

i : index of each set of applied load cycles at constant stress level σ_i

w_i : damage fraction accumulated during the load cycles of interval i at constant stress level σ_i

W : total damage constant

The linear damage rule states that the damage fraction at a given constant stress level σ_i is equal to a mathematical fraction:
the number n_i of applied cycles at stress level σ_i
divided by the fatigue life N_i at stress level σ_i , i.e. : $w_i = \dfrac{n_i}{N_i}$

The most natural assumption seems to be:

$$\sum_i \frac{n_i}{N_i} = 1$$

When there is only one level σ_i , i=1, final failure occurs for $n_i = N_i$ (of course...what else?): then, according to Miner, the damage constant W is assumed to be equal to 1.

The results of Miner's original tests showed that the damage constant W corresponding to failure ranged from 0.61 to 1.45,

other researchers have shown variations as large as 0.18 to 23.0, with most results tending to fall between 0.5 and 2.0,

it seems to be true that the average value is close to Miner's proposed value of 1; however, the dispersion is rather discouraging.

1. Linear Damage or Miner's Rule (4/11)

By far the largest effort in this respect, however, was spent by Kotte and Eulitz of the Technical University of Dresden ... who collected hundreds of test programmes, i.e. many thousands of variable-amplitude and corresponding constant-amplitude tests and carried out new Miner calculations.

One outstanding result of all these checks was that Miner's rule generally proved wildly unconservative; that is, the specimens -- in the case of the IABG programme, the components -- failed long before their predicted lives; factors of 10 and more on the unconservative side were not uncommon.

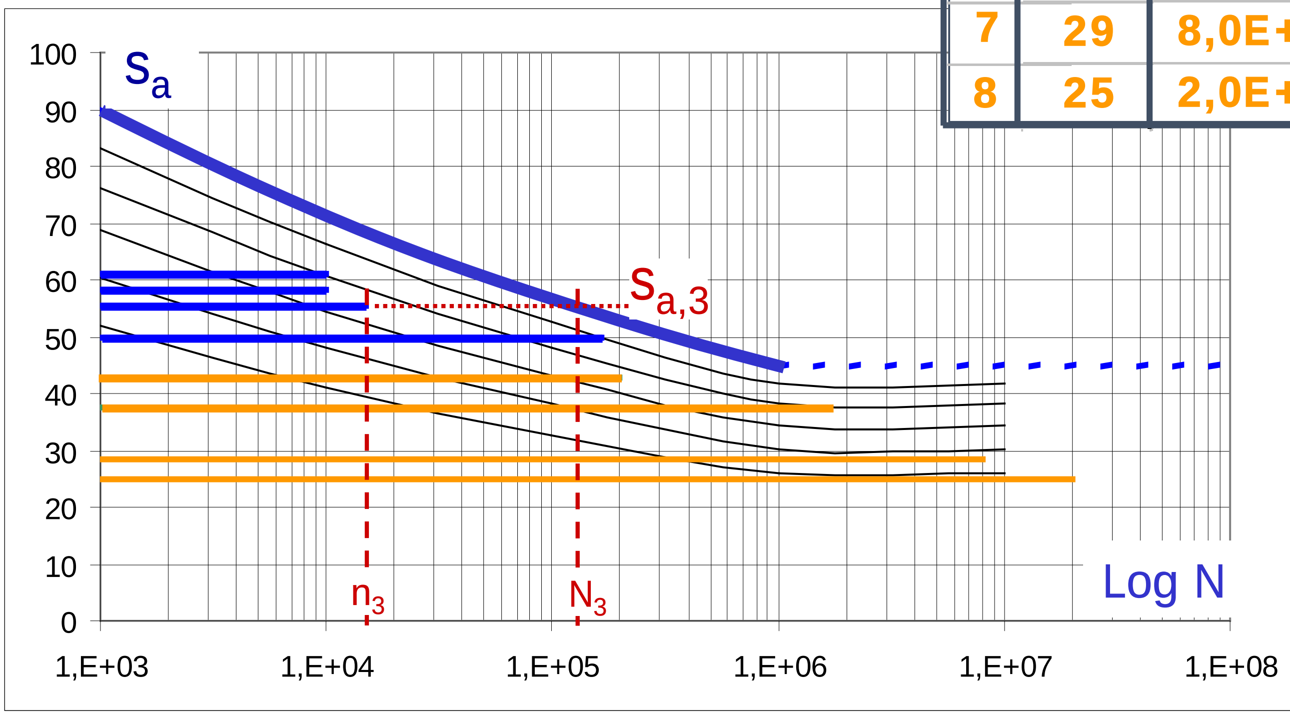
from: Walter Schütz, A History of Fatigue, *Engineering Fracture Mechanics* Vol. 54, No. 2, pp. 263-300, 1996

Despite these limitations, the linear damage rule is still widely used. However, for conservative estimates of the life of a structure a damage constant less than 1 prescribed.

Data on the right show a simple case for $\sigma_m=0$ and eight blocks of stresses.

We shall employ non dimensional values $s_a=\sigma_a/R_m*100$ for stresses.

	$s_{a,i}$	n_i	N_i	
1	61	1,0E+04	4,8E+04	
2	58	1,0E+04	7,5E+04	
3	55	1,5E+04	1,3E+05	
4	50	1,7E+05	3,5E+05	
5	43	2,0E+05		
6	38	1,6E+06		
7	29	8,0E+06		
8	25	2,0E+07		



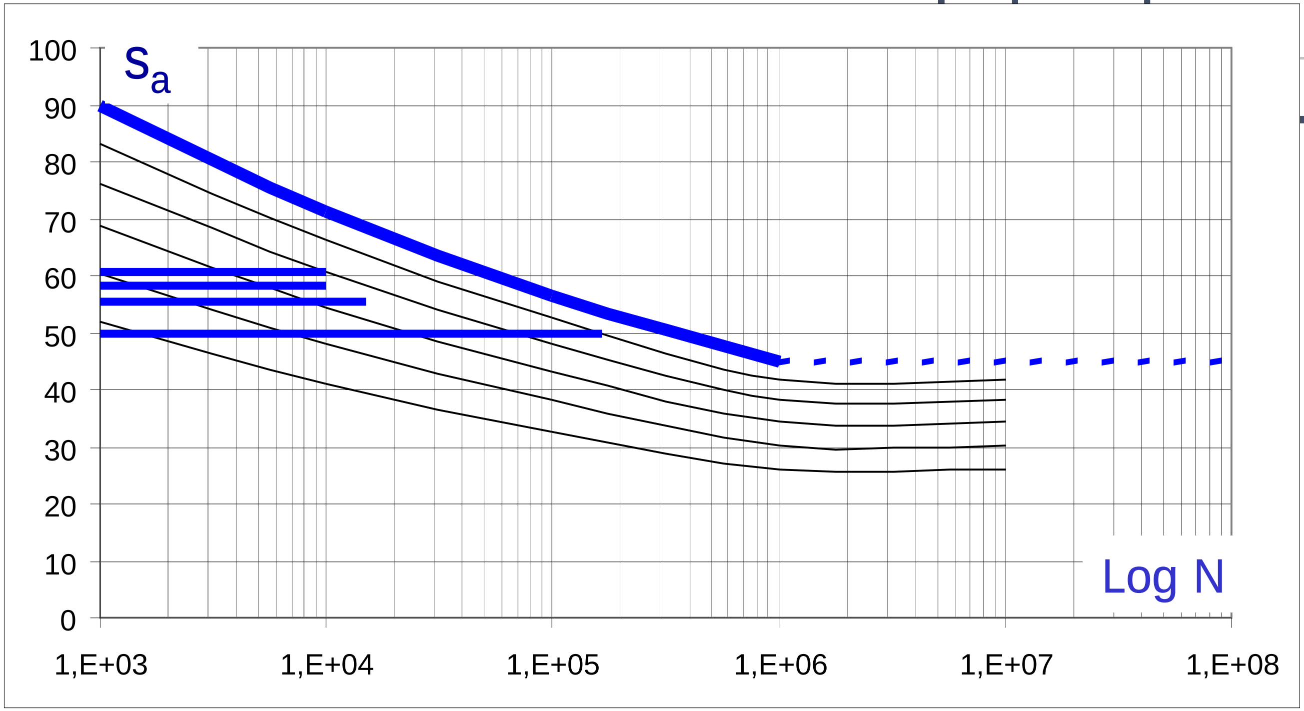
The cycles to be taken into account are only the blue ones, i=1 to 4 for which N_i exists.

Then the damage fractions w_i can be calculated and summed.

In this case the sum is **0,93**.

Then no failure should occur.

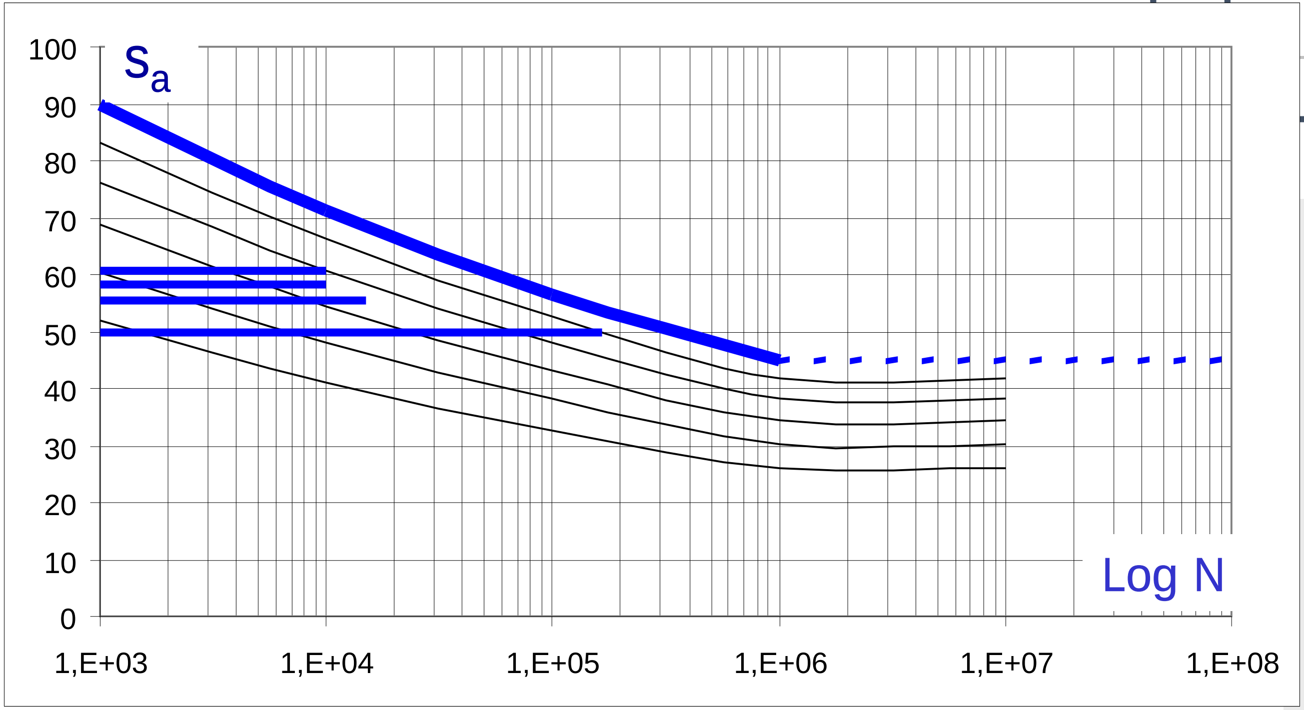
	$S_{a,i}$	n_i	N_i	n_i / N_i
1	61	1,0E+04	4,8E+04	0,21
2	58	1,0E+04	7,5E+04	0,13
3	55	1,5E+04	1,3E+05	0,12
4	50	1,7E+05	3,5E+05	0,47
				Σ=0,93



On the basis on the total number of valid cycles, n_{TOT}, the utilisation fraction α_i is now introduced:

$$\alpha_i = \frac{n_i}{n_{TOT}}$$

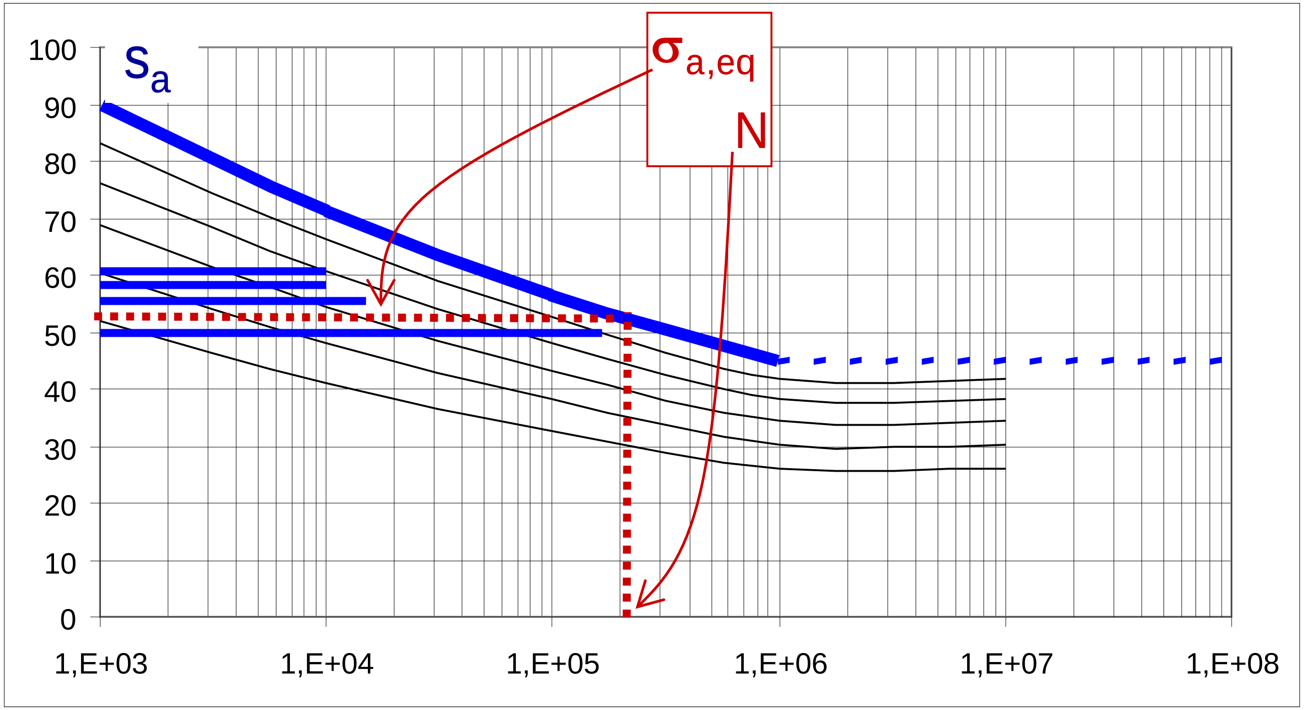
	$s_{a,i}$	n_i	α_i
1	61	1,0E+04	0,049
2	58	1,0E+04	0,049
3	55	1,5E+04	0,073
4	50	1,7E+05	0,829
	n_{TOT} = 2,05E+05		Σ=1,000



The aim is to obtain an "equivalent stress amplitude" acting for the total number of cycles n_{TOT}.

The "equivalent stress amplitude" $\sigma_{a,eq}$ when applied for the total number of n_{TOT} (valid) cycles would produce the same damage of all the single stress σ_a levels applied each for its own n cycle number.

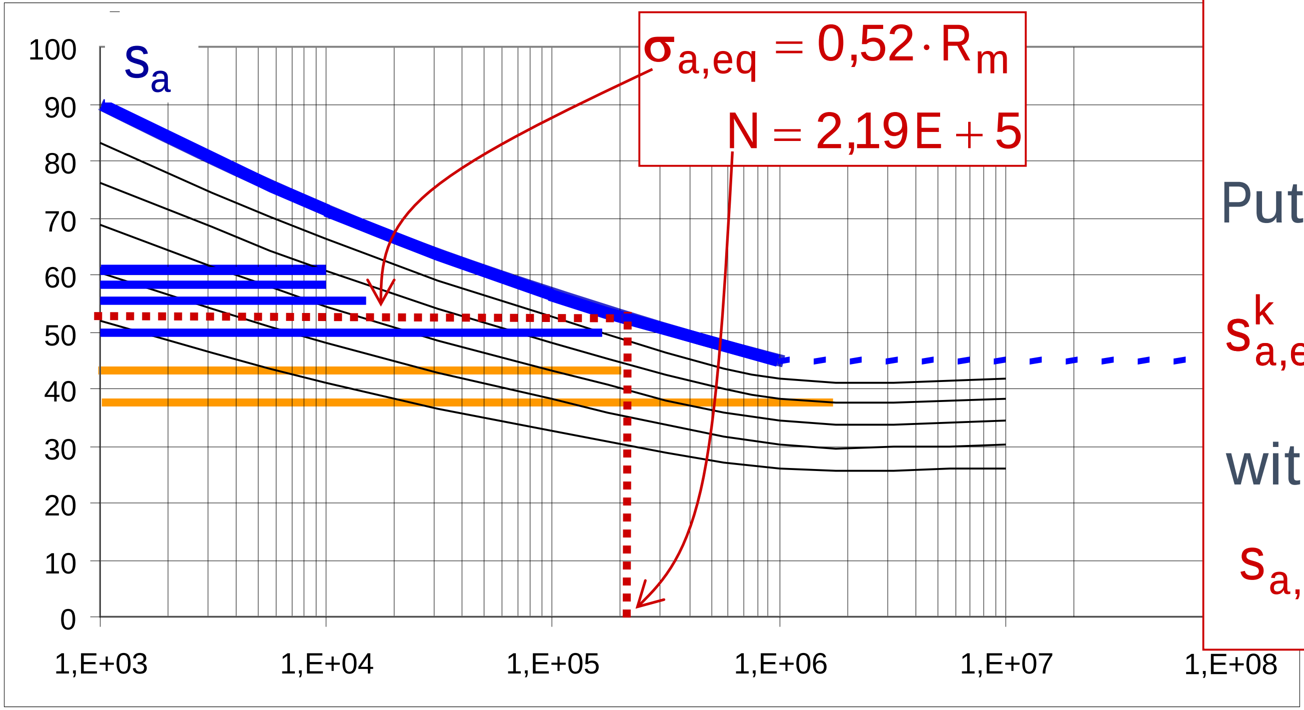
Wöhler equation $\sigma_a^k N \le \sigma_{D-1}^k 10^6$ is here written in the alternative form $s_a^k N \le s_{D-1}^k 10^6$ with: $s_a = \sigma_a / R_m * 100$.



The developments in the example of next slide are valid only for the simplest case, where σ_m is the same for all σ_a

(in the example σ_m=0; it could be σ_m=any but equal for all)

In cases when σ_m is the same for all $\sigma_{a,i}$, formula (6 for the "equivalent stress" $s_{a,eq}$ holds; it is the stress amplitude which would produce the same damage if applied for all n_{TOT} cycles, as stated in formula (5 written for the case σ_m=0



$$N_i = 10^6 \left(\frac{s_{D-1}}{s_{a,i}} \right)^k \quad (1$$

$$\sum_i \frac{n_i}{N_i} \equiv \sum_i w_i \leq 1 \quad (2$$

$$\sum_i n_i = n_{TOT} \quad (3$$

$$n_i = \alpha_i \, n_{TOT} \quad (4$$

Put 1) and 4) in 2):

$$s_{a,eq}^k \, n_{TOT} \leq s_{D-1}^k \, 10^6 \quad (5$$

with:

$$s_{a,eq} = \left(\sum_i \alpha_i \, s_{a,i}^k \right)^{1/k} \quad (6$$

When, instead, each $\sigma_{a,i}$ or $s_{a,i}$ has a different $\sigma_{m,i}$ or $s_{m,i}$, then:

$$\left[s_{a,i}\left(s_{m,i}\right)\right]^k N_i = \left[s_D\left(s_{m,i}\right)\right]^k 10^6 \quad (1$$

$$N_i = \frac{\left[s_D\left(s_{m,i}\right)\right]^k}{\left[s_{a,i}\left(s_{m,i}\right)\right]^k} 10^6 \quad (1'$$

$$\sum_i \frac{n_i}{N_i} \equiv \sum_i w_i \leq 1 \quad (2$$

$$\sum_i n_i = n_{TOT} \quad (3$$

$$n_i = \alpha_i \, n_{TOT} \quad (4$$

1') and 4) in 2):

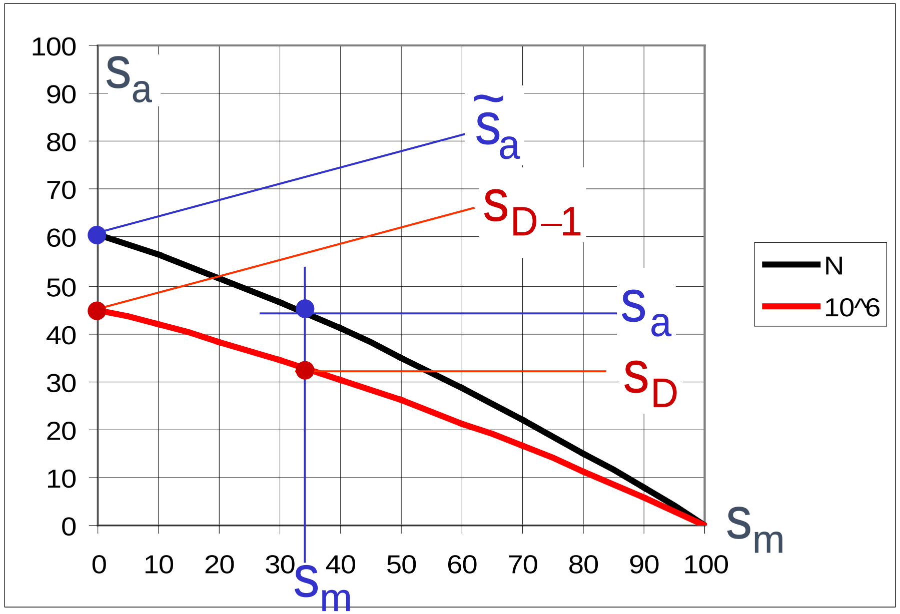
$$n_{TOT} \sum_i \alpha_i \frac{\left[s_{a,i}\left(s_{m,i}\right)\right]^k}{\left[s_D\left(s_{m,i}\right)\right]^k} \frac{1}{10^6} = 1 \quad (5$$

In this case the formulation of an equivalent stress amplitude acting for the total number of cycles n_{TOT} is not possible because a term $s_{a,eq}^k$ in:

$$s_{a,eq}^k \, n_{TOT} \leq const$$

cannot be isolated.

However, an equivalent stress amplitude can be found under certain assumptions. If Haigh master curves are obtained according to an interpolation done as in Sect. 8 of Ch. 2, the formula of sl. 9 of that Section is now written:



$$\frac{\tilde{s}_a}{s_{D-1}} = \frac{s_a(s_m)}{s_D(s_m)}$$ which for the level i :

$$\frac{\tilde{s}_{a,i}}{s_{D-1}} = \frac{s_{a,i}(s_{m,i})}{s_D(s_{m,i})} \quad (6$$

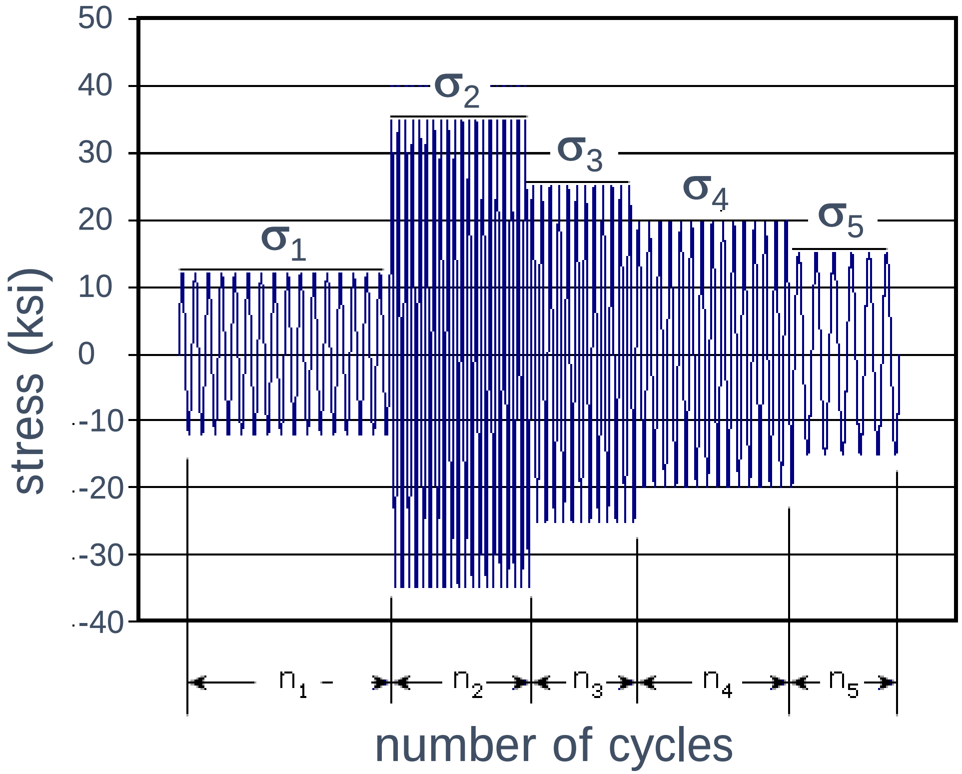
Eq. (6 put in eq. (5 gives: $$n_{TOT} \sum_i \alpha_i \frac{\tilde{s}_{a,i}^k}{s_{D-1}^k} \frac{1}{10^6} = 1 \quad (7$$

Now the denominator is no longer dependent on $s_{m,i}$ then:

$$n_{TOT} \underbrace{\sum_i \alpha_i \ \tilde{s}_{a,i}^k}_{\tilde{s}_{a,eq}^k} = s_{D-1}^k 10^6 \quad (8$$

2. Cycle counting (1/8)

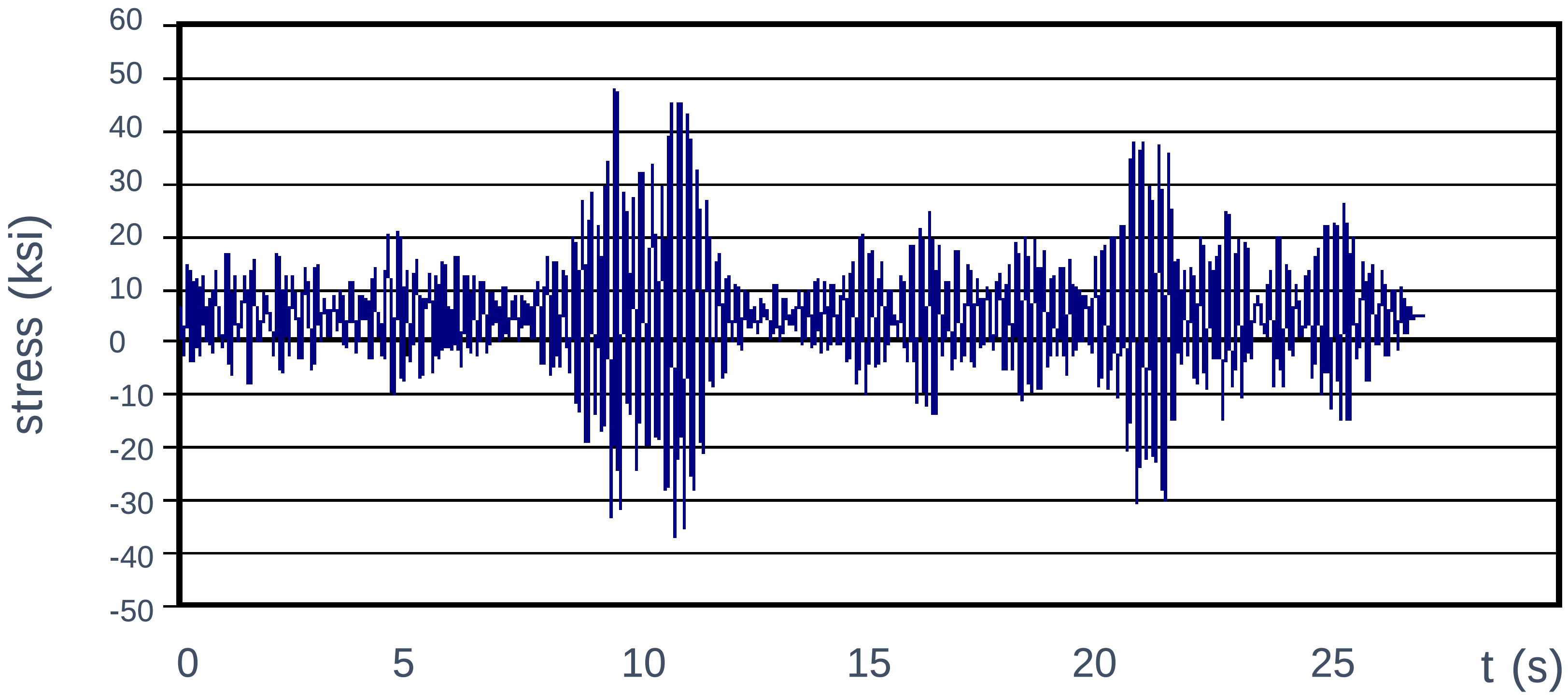
The number of cycles at each stress amplitude needs to be known, or "counted". The constant amplitude blocks used by whatever cumulative damage theory are extracted from a non-uniform time history, as it will be shown in this section.



from: http://www.engrasp.com/doc/etb/mod/fm1/miner/miner_help.html

The figure to the left illustrates a very simple case of a "block" spectrum, which is of course a gross simplification of reality for educational purposes.

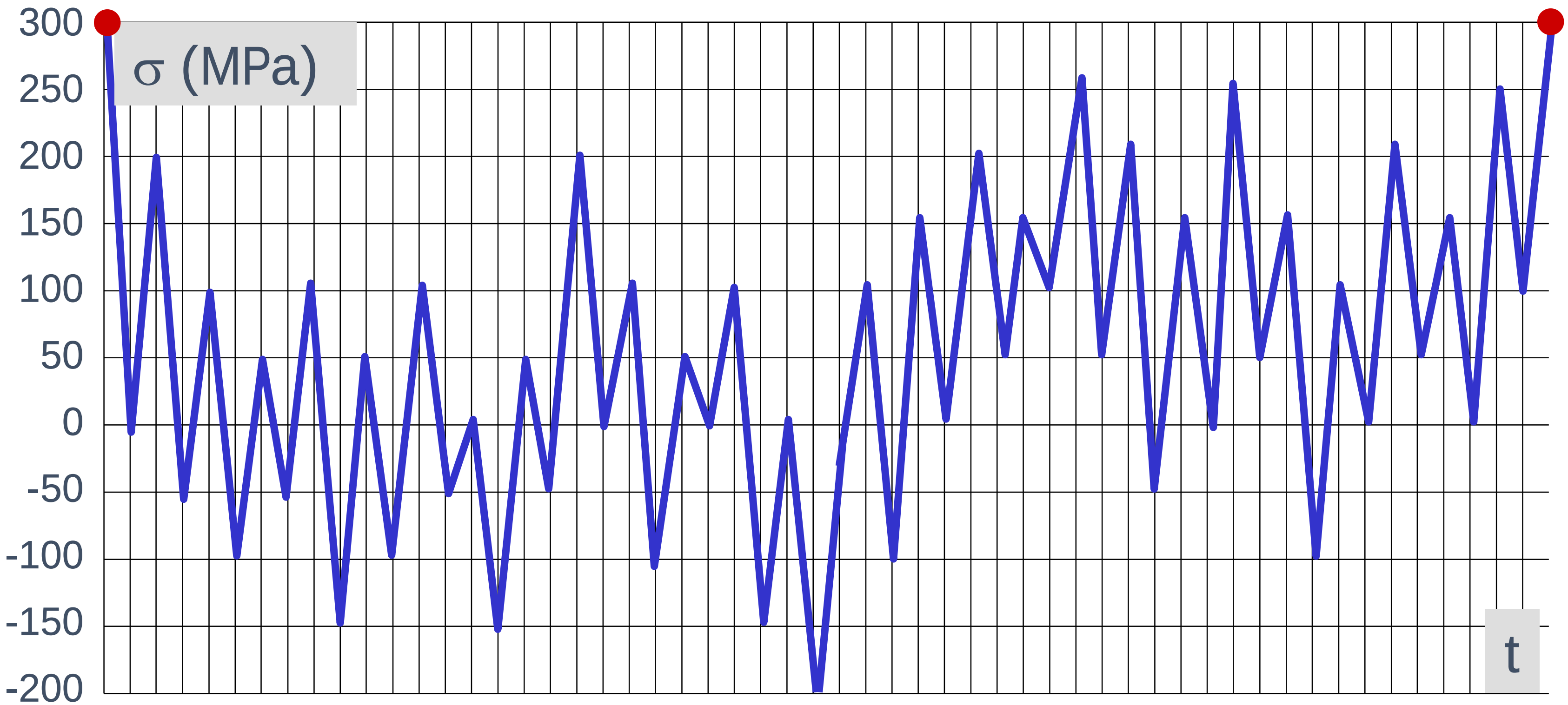
The figure below shows one possible case of (experimentally detected) time history of stresses. It is necessary to find a method to extract cycles which must be used for damage calculations.



from: http://www.engrasp.com/doc/etb/mod/fm1/miner/miner_help.html

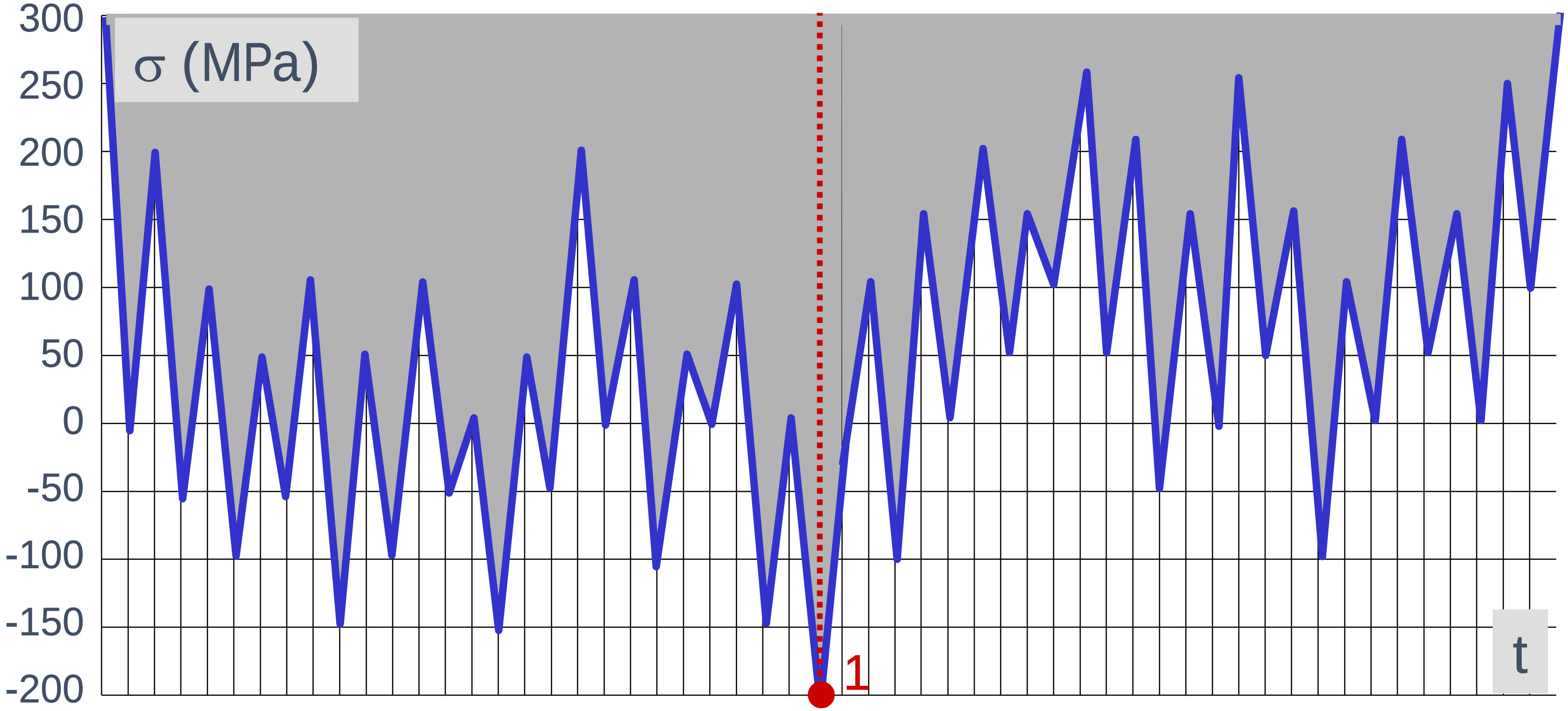
We describe here the “rainflow” counting method in the so called “bathtub” version. They extract constant amplitude cycles from a non-uniform time history.

A section of the time history must be isolated so that it begins and ends at the maximum value, points ●.



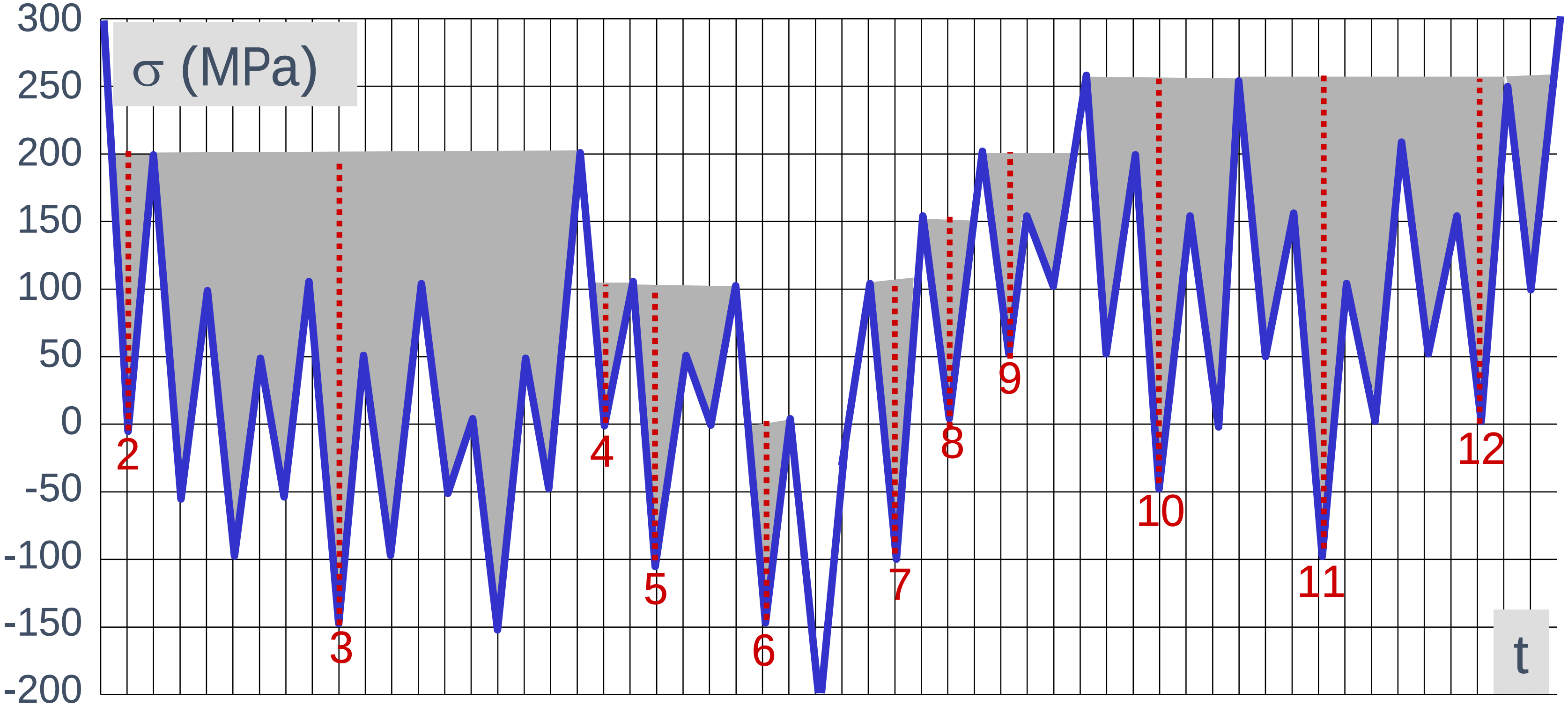
The diagram is interpreted as if it were a basin full of liquid.

The bottom point 1 is where liquid is spilled, and the basin partially emptied...

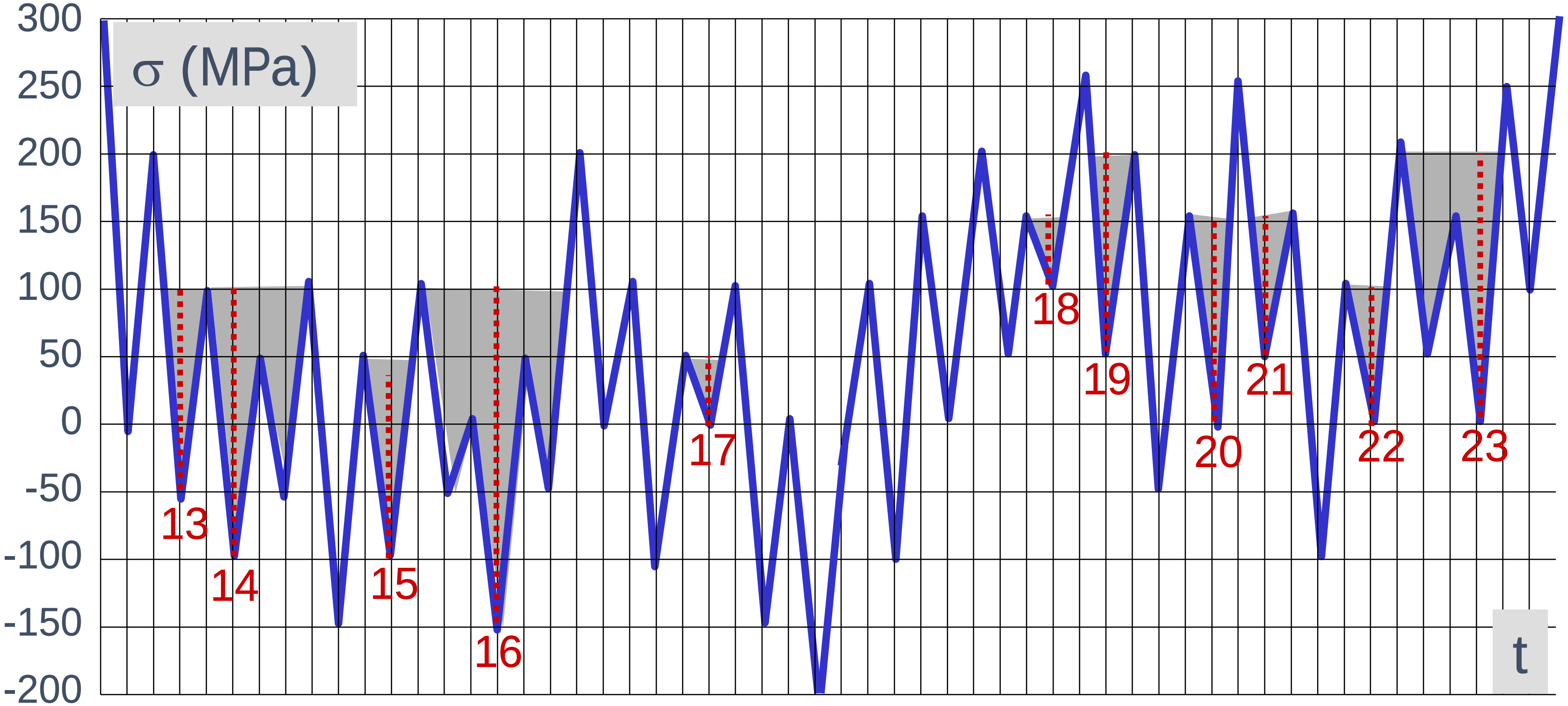


... the total level descent at point 1 (from +300 to –200 MPa , i.e. σ_a=250, σ_m=50 MPa) is the cycle n. 1 detected.

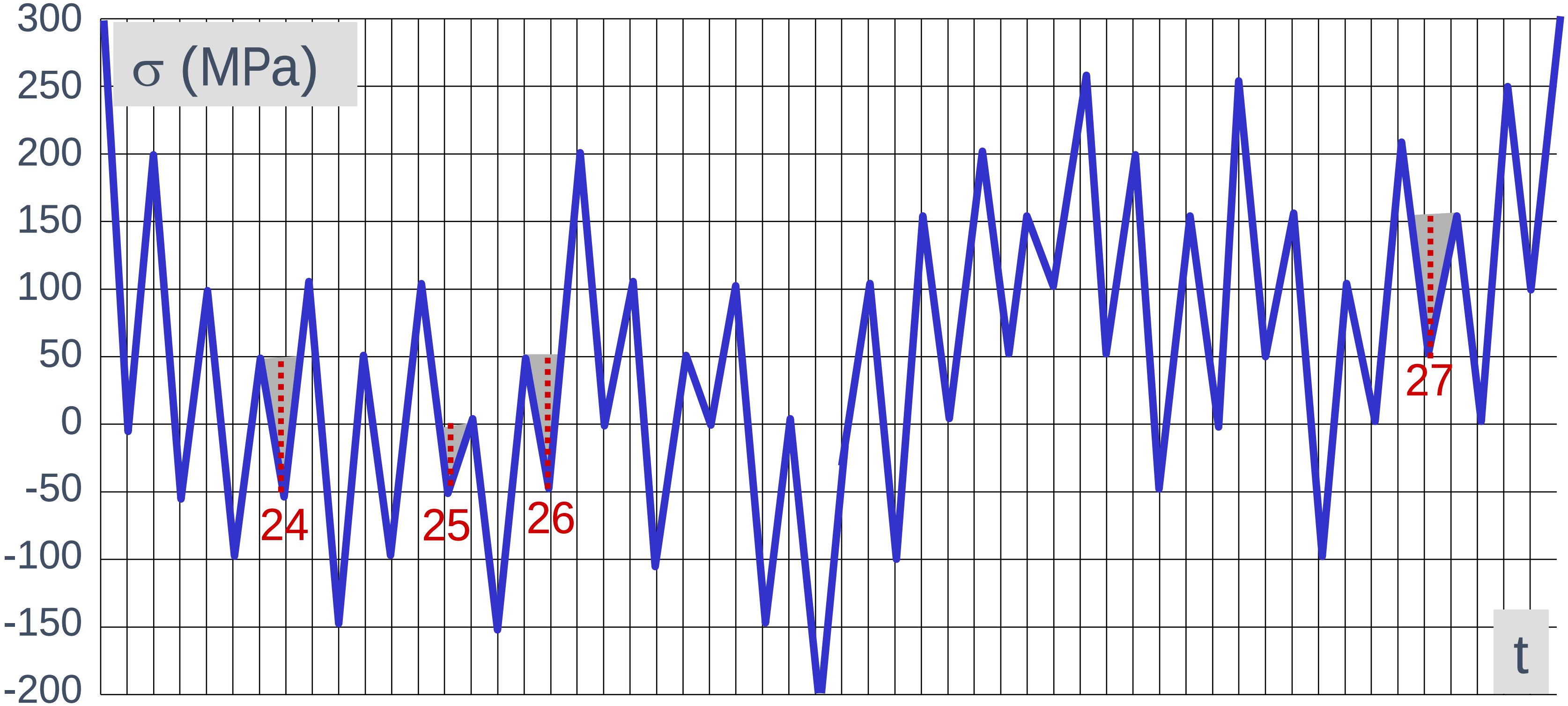
This leaves a number of secondary basins to empty by spilling at the bottom point of each: 2 through 12 .



... further levels are detected, 13 through 23 ...



... and the last cycles 24 to 27.



2. Cycle counting (8/8)

The table below summarises the results:

stresses (MPa)					
n	σ_{min}	σ_{max}	$\Delta\sigma$	σ_a	σ_m
1	-200	300	500	250	50
2	0	200	200	100	100
3	-150	200	350	175	25
4	0	100	100	50	50
5	-100	100	200	100	0
6	-150	100	250	125	-25
7	-100	100	200	100	0
8	0	150	150	75	75
9	50	200	150	75	125
10	-50	250	300	150	100
11	-100	250	350	175	75
12	100	250	150	75	175
13	-50	100	150	75	25
14	-100	100	200	100	0
15	-100	50	150	75	-25
16	-150	100	250	125	-25
17	0	50	50	25	25
18	100	150	50	25	125
19	50	200	150	75	125
20	0	150	150	75	75
21	50	150	100	50	100

n	σ_{min}	σ_{max}	$\Delta\sigma$	σ_a	σ_m
22	0	100	100	50	50
23	0	200	200	100	100
24	-50	50	100	50	0
25	-50	0	50	25	-25
26	-50	50	100	50	0
27	50	150	100	50	100

These cycle counts can be grouped in blocks. See colours: in this example they are not numerous, but in the real cases and with digit rounding they may number up to thousands. One group is missing: which one?

To each block a “total occurrence” number will be associated:

α=2/27	α=2/27
α=3/27	α=2/27
α=2/27	α=2/27

others α=1/27

Sections 3, 4 – Applications of finite life concepts

Also these sections refer to one-dimensional tensile (mainly tensile, but compressive can be included) stresses,

Section 3 shows how a stress spectrum can be used to assess finite life at fatigue, deepening the tool presented in Section 1.

Section 4 introduces advanced matters on the exceedance diagram.

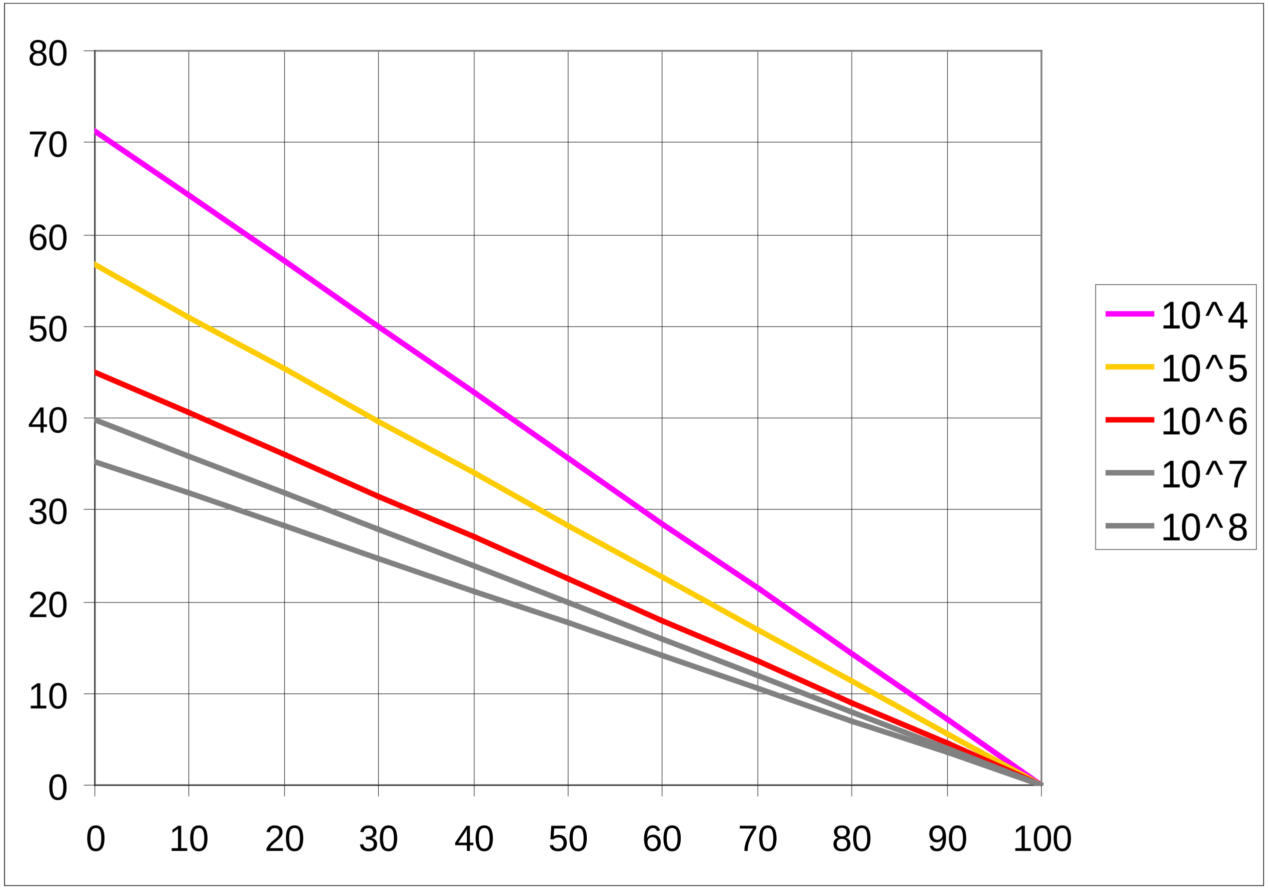
As an application of ideas developed so far, the one shown on the right is a very simplified example of spectrum of stresses (expressed as a percent of R_m) applied at a design point.

We shall not use the results from the cycle counting of the preceding section, in order to have a smaller and more treatable, yet representative of the concept, number of points.

We shall also make use of the Wöhler curves in the extended Haibach form explained in Sect. 5, Appendix to this chapter, so to become familiar with it.

s_m	s_a	n_i
0	50	1,1E+05
15	47	1,5E+04
18	50	6,7E+03
40	35	6,7E+03
30	30	2,3E+05
20	32	1,2E+06
0	25	1,3E+07
30	20	5,3E+07

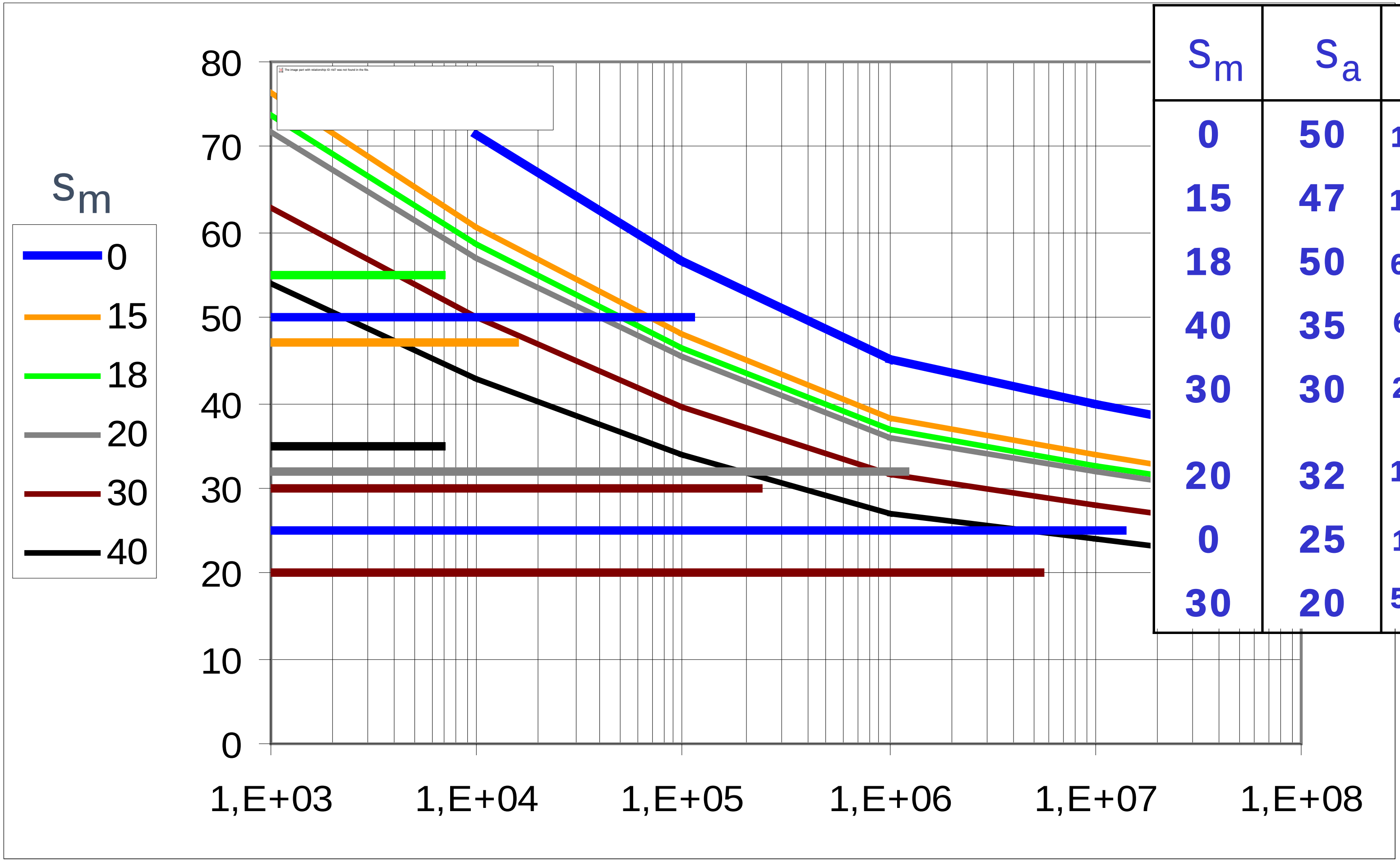
$$\boxed{s_x = \sigma_x / R_m}$$



s_m	s_a	n_i
0	50	1,1E+05
15	47	1,5E+04
18	50	6,7E+03
40	35	6,7E+03
30	30	2,3E+05
20	32	1,2E+06
0	25	1,3E+07
30	20	5,3E+07

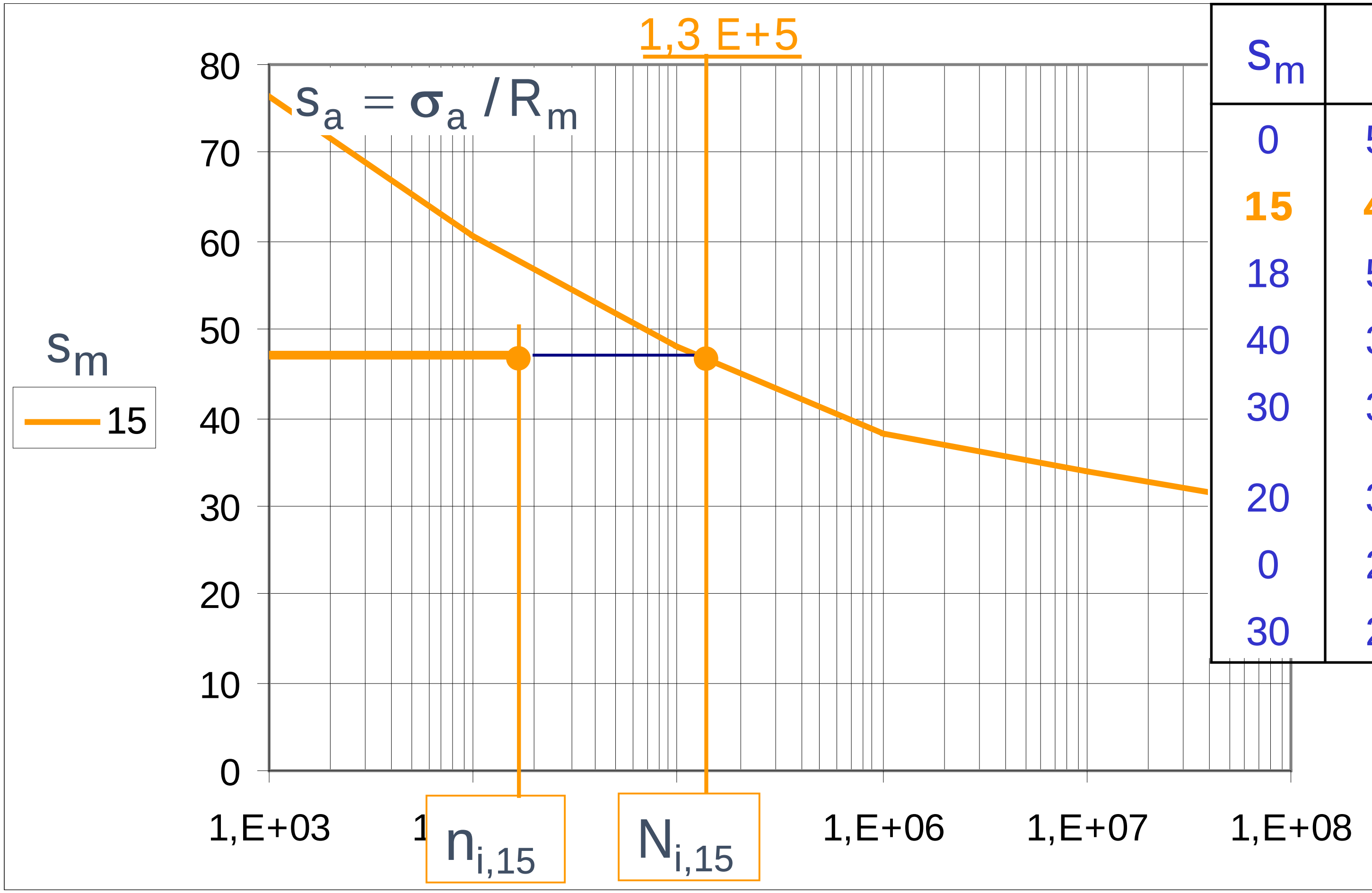
$$\boxed{s_x = \sigma_x / R_m}$$

Each (s_m ; s_a) point can be represented on the Haigh diagram.



s_m	s_a	n_i
0	50	1,1E+05
15	47	1,5E+04
18	50	6,7E+03
40	35	6,7E+03
30	30	2,3E+05
20	32	1,2E+06
0	25	1,3E+07
30	20	5,3E+07

Wöhler curves are traced for each s_m, and n_i's represented for each s_a . Mind the colour code coupling each s_a to its s_m



s_m	s_a	n_i
0	50	1,1E+05
15	**47**	**1,5E+04**
18	50	6,7E+03
40	35	6,7E+03
30	30	2,3E+05
20	32	1,2E+06
0	25	1,3E+07
30	20	5,3E+07

Then the "damage fraction" w_i for each level can be calculated (here shown for $w_{i,15}$, i.e. $w_{i,15}=n_{i,15}/N_{i,15}=$ **1,5E+4** / **1,29E+5**)

3. Finite life and stress spectra (5/9)

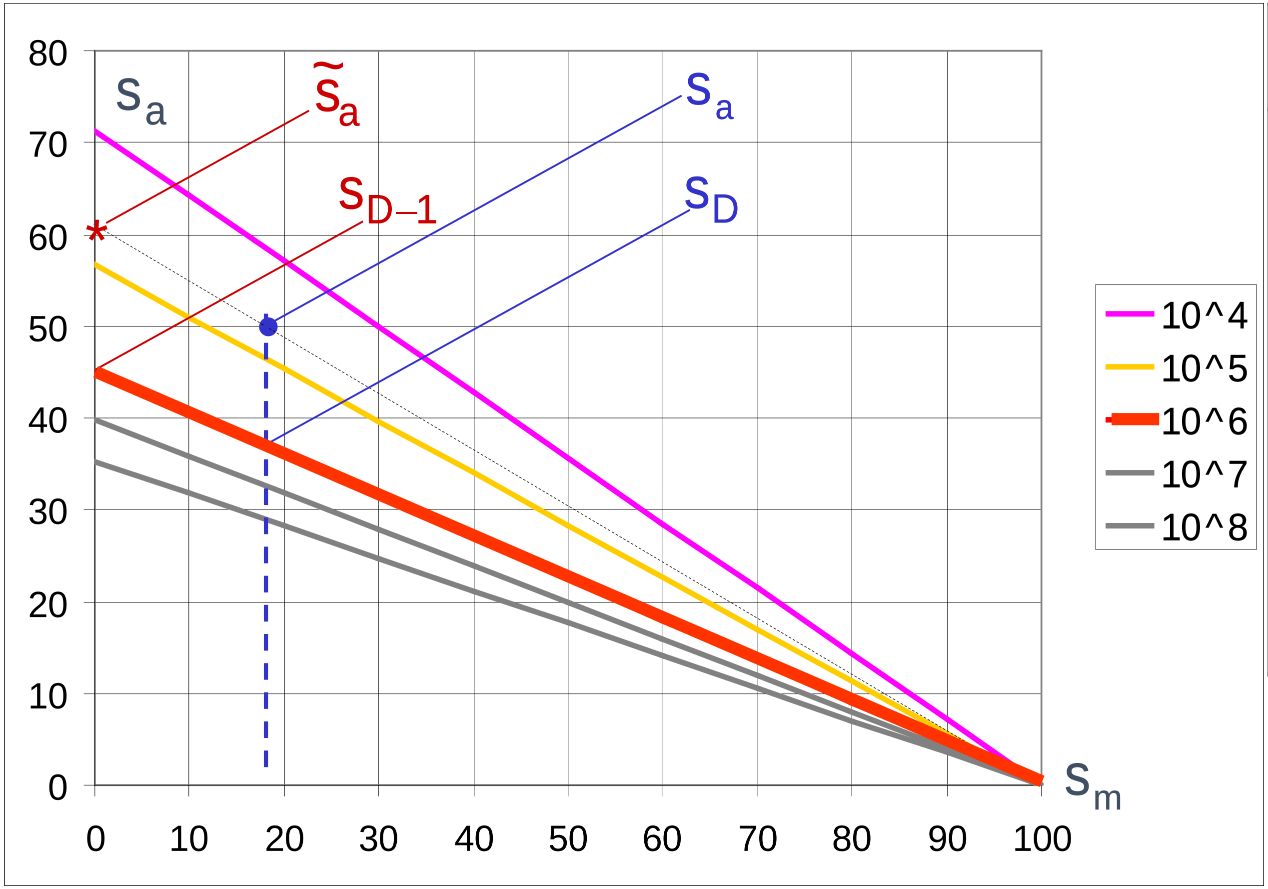
The foregoing procedure is quite clear in principle, but has the disadvantage that Wöhler curves for each s_m must be calculated. Then, stress levels s_a must be coupled with the corresponding curve having the same s_m.

This is not terribly inconvenient, but there is a better way, in that it simplifies the graphics, and makes use of the only Wöhler curve for the life 10^6 cycles.

s_m	s_a	n_i
0	50	1,1E+05
15	47	1,5E+04
18	50	6,7E+03
40	35	6,7E+03
30	30	2,3E+05
20	32	1,2E+06
0	25	1,3E+07
30	20	5,3E+07

Let us take for instance point s_m=18, s_a=50 ... ...

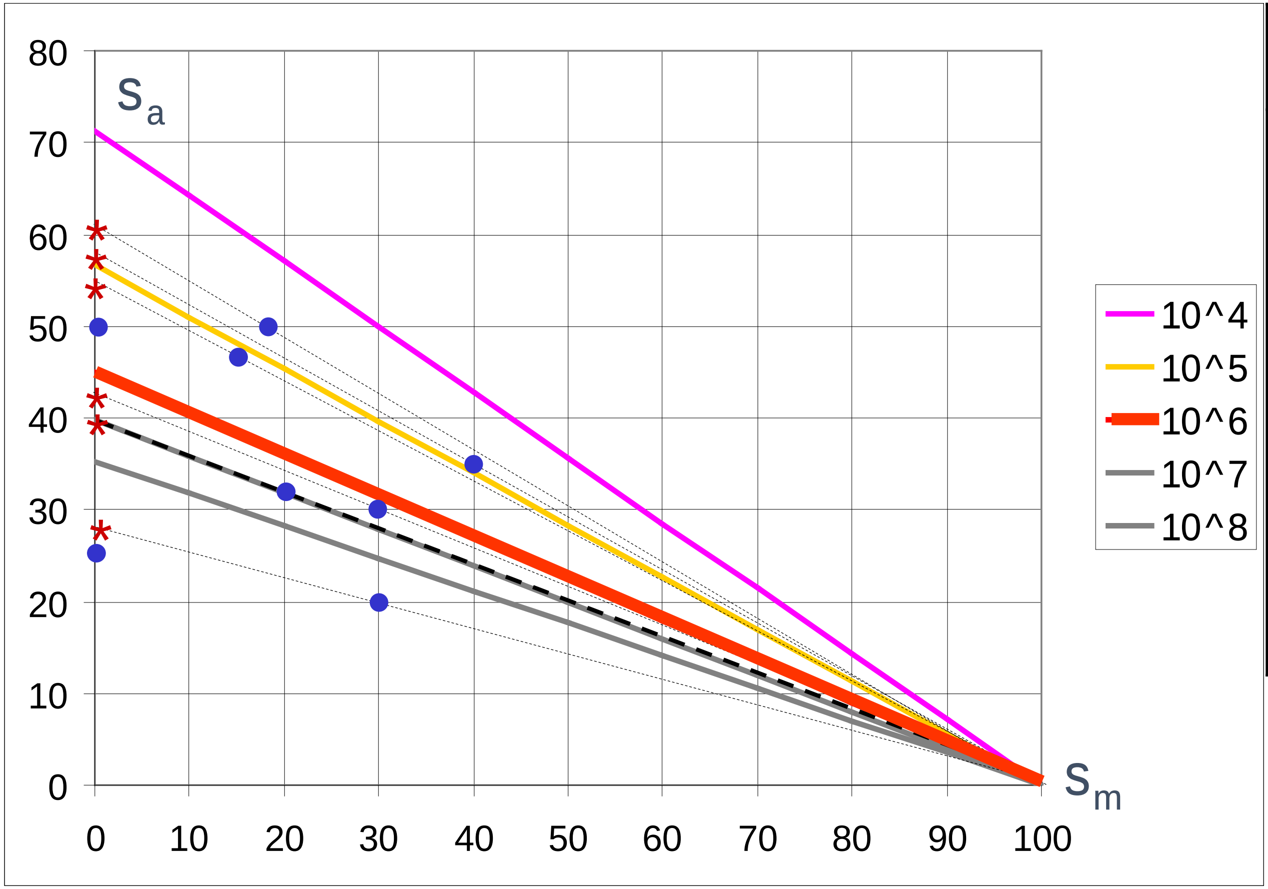
3. Finite life and stress spectra (6/9)



s_m	s_a	$\tilde{s}_a$
18	**50**	**61**

$$s_x = \sigma_x / R_m$$

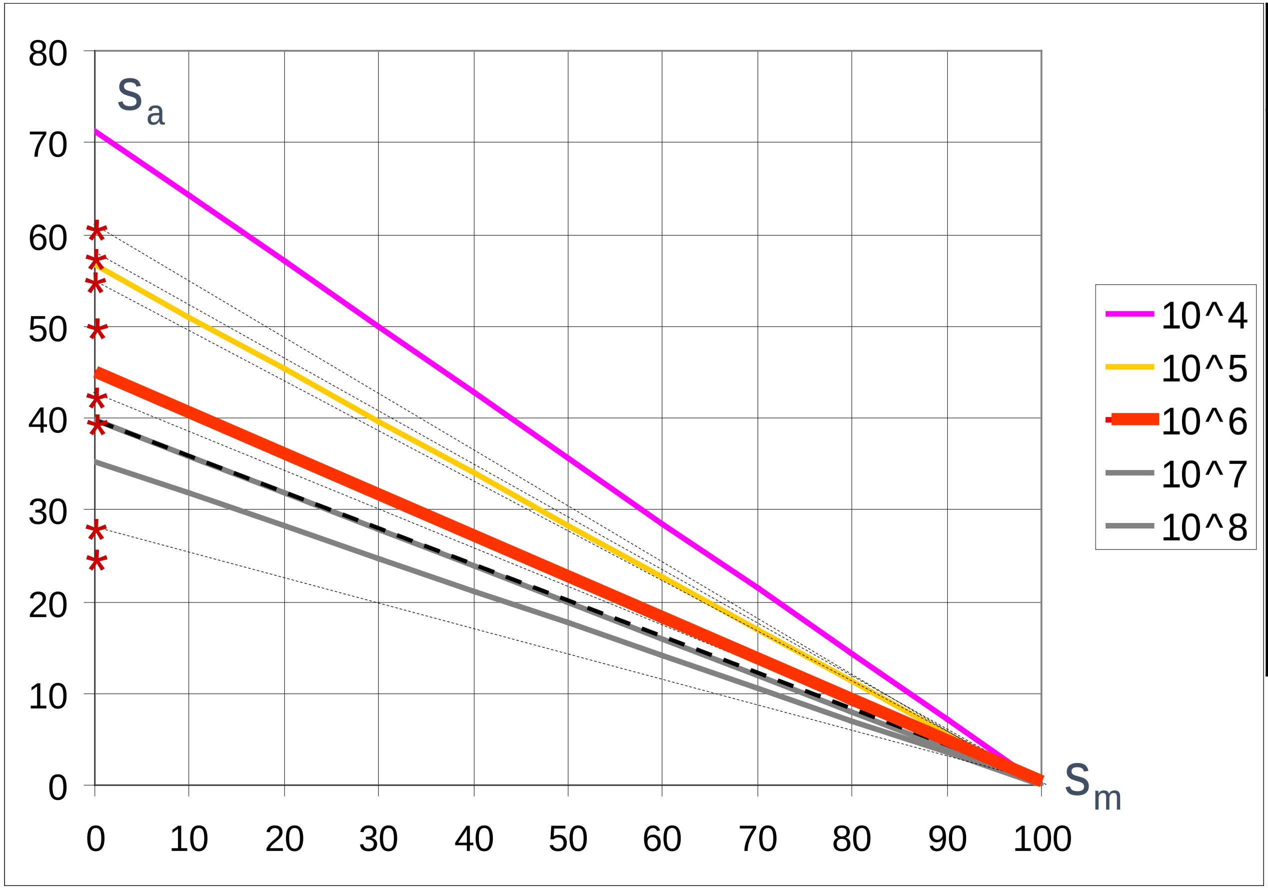
As shown in sl.7-Sect.8-Ch.2, $s_a / s_D = \tilde{s}_a / s_{D-1} = \left(10^6 / N\right)^{1/k}$
so for the same life N we can use $\tilde{s}_a$ at s_m=0 instead of s_a .



s_m •	s_a •	$\tilde{s}_a$ *
0	**50**	50
15	**47**	55
18	**50**	61
40	**35**	58
30	**30**	43
20	**32**	40
0	**25**	25
30	**20**	29

$$s_x = \sigma_x / R_m$$

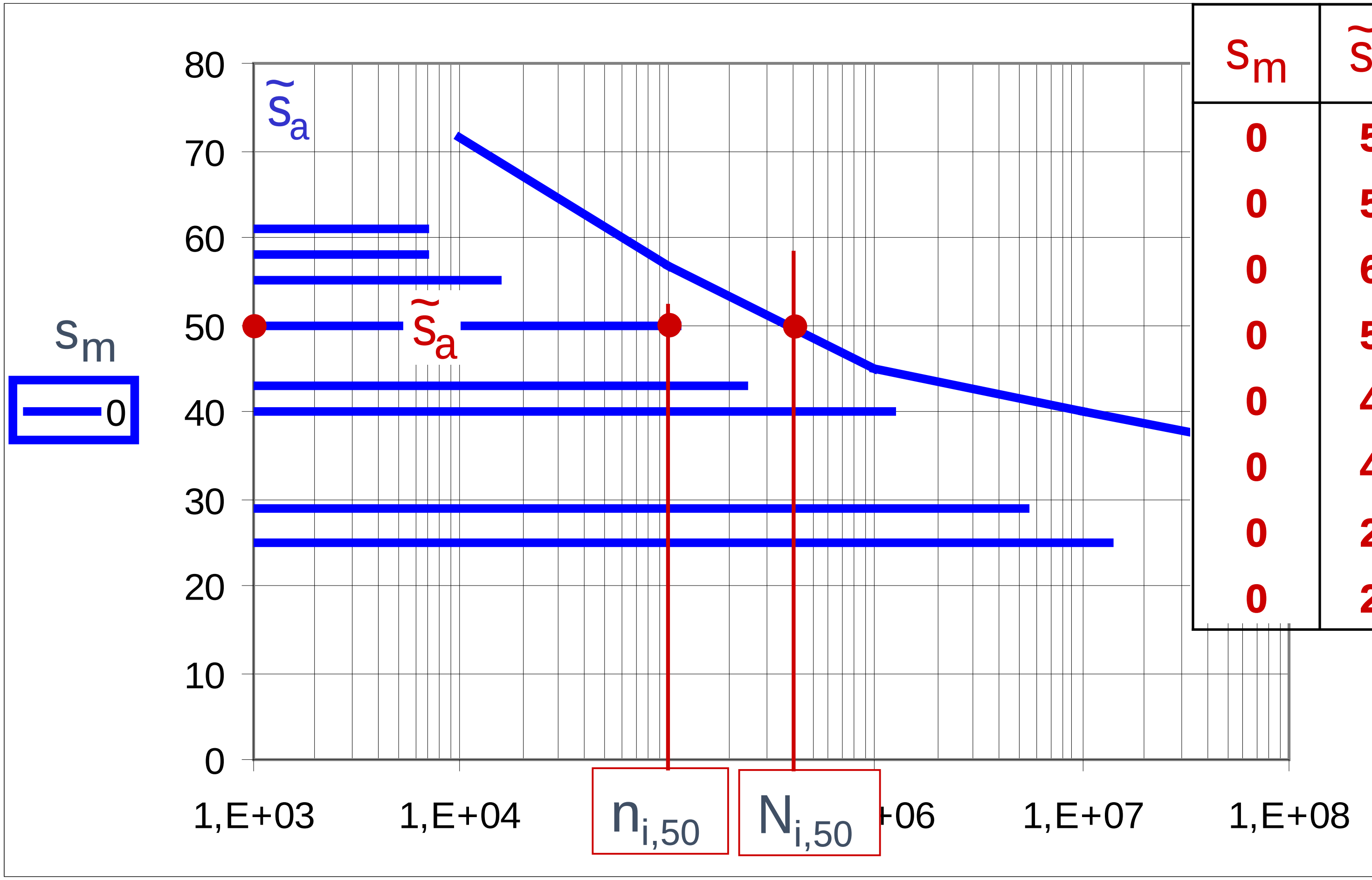
So all points • can be transformed into equal life * points at s_m=0. Let us now remark that for life calculation $s_a / s_D \equiv \sigma_a / \sigma_D$, then Haigh and Wöhler diagrams in non-dimensional form are useful.



s_m*	$\tilde{s}_a$ *	n_i
0	50	1,1E+05
0	55	1,5E+04
0	61	6,7E+03
0	58	6,7E+03
0	43	2,3E+05
0	40	1,2E+06
0	25	1,3E+07
0	29	5,3E+07

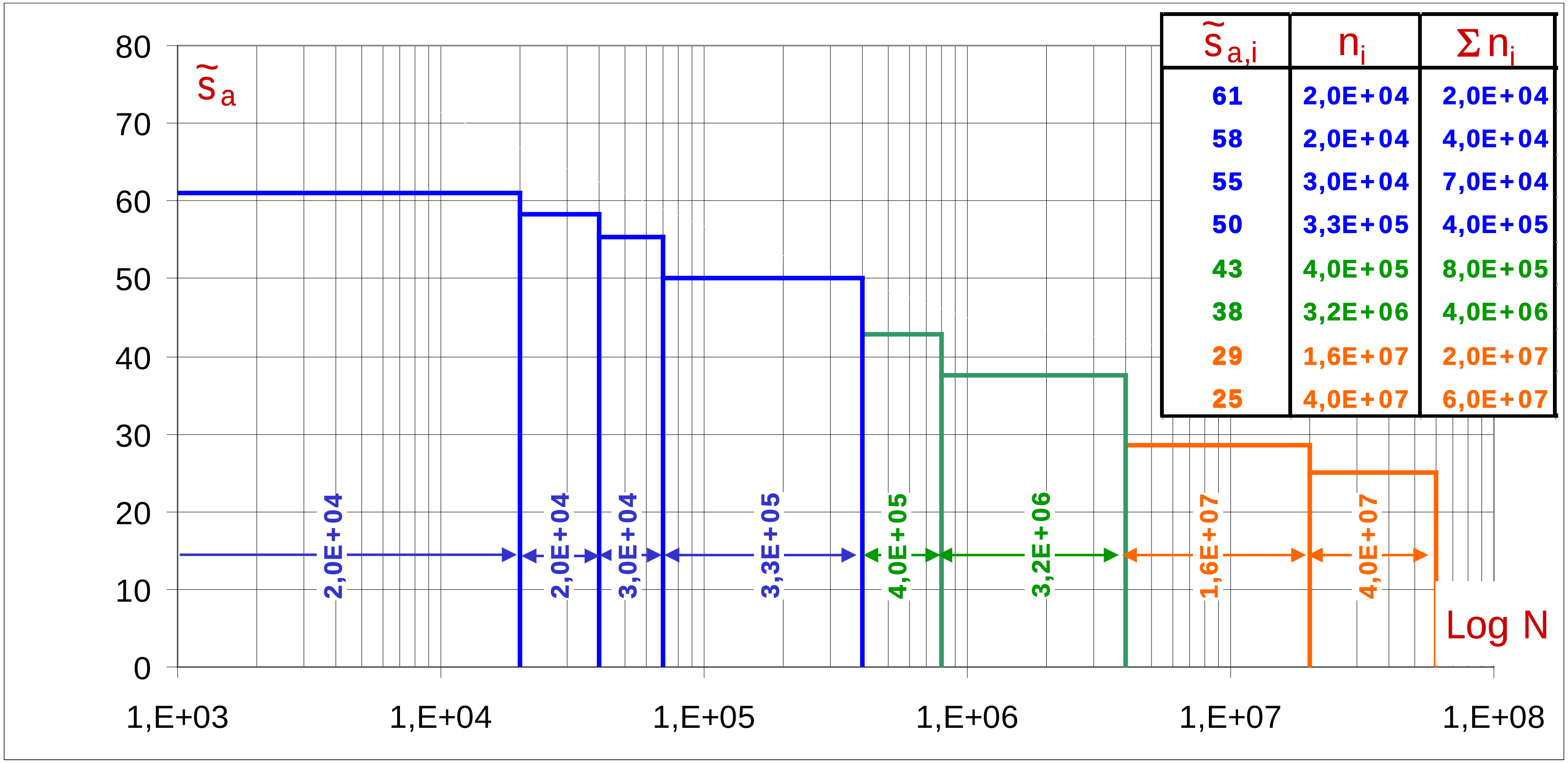
$$s_x = \sigma_x / R_m$$

All points to consider are now those listed and marked with *



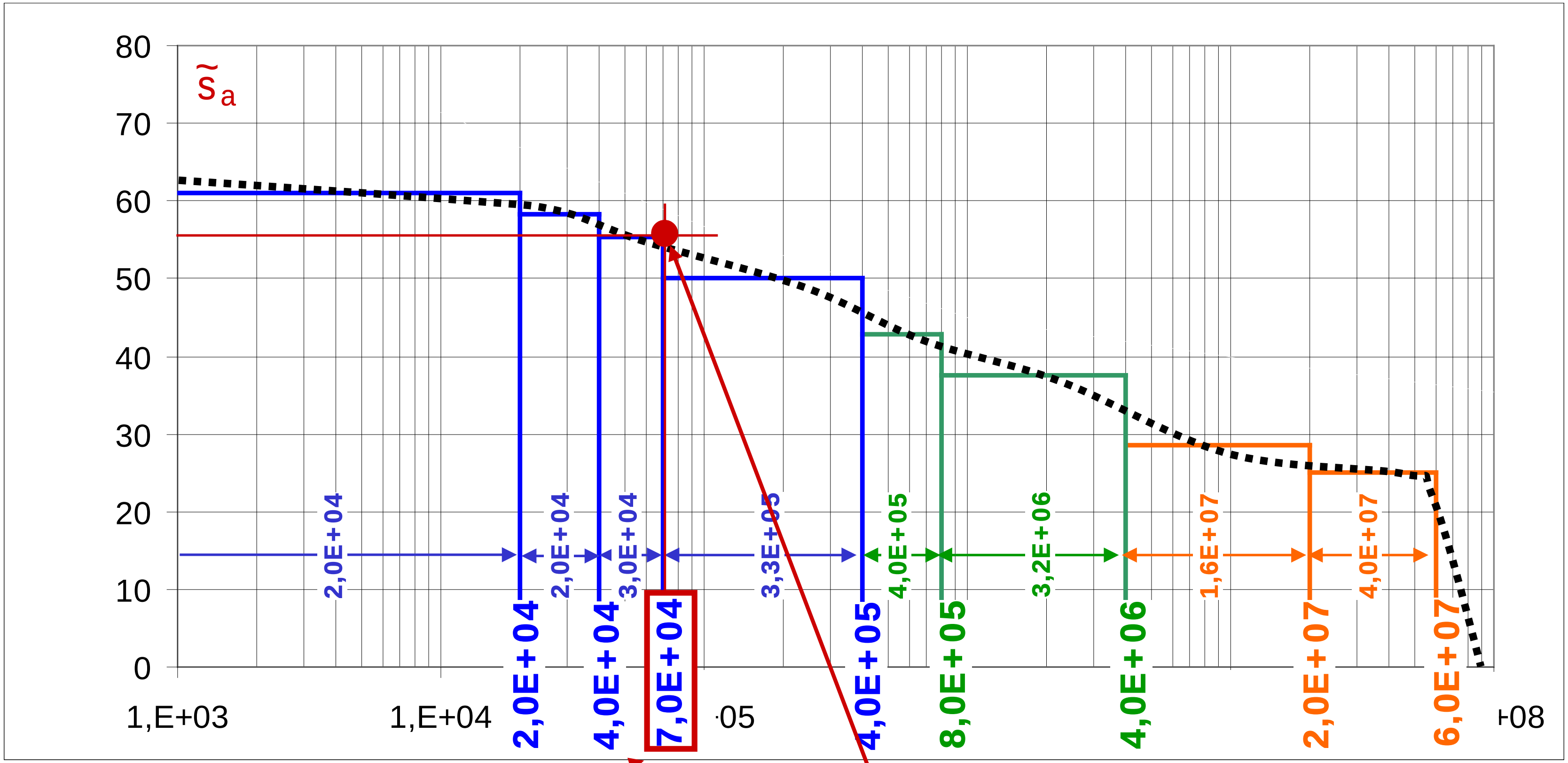
s_m	$\tilde{s}_a$	n_i
0	50	1,1E+05
0	55	1,5E+04
0	61	6,7E+03
0	58	6,7E+03
0	43	2,3E+05
0	40	1,2E+06
0	25	1,3E+07
0	29	5,3E+07

Now all "damage fractions" w_i will be calculated as n_i/N_i , where the only Woehler curve used is the one for σ_m=0 .



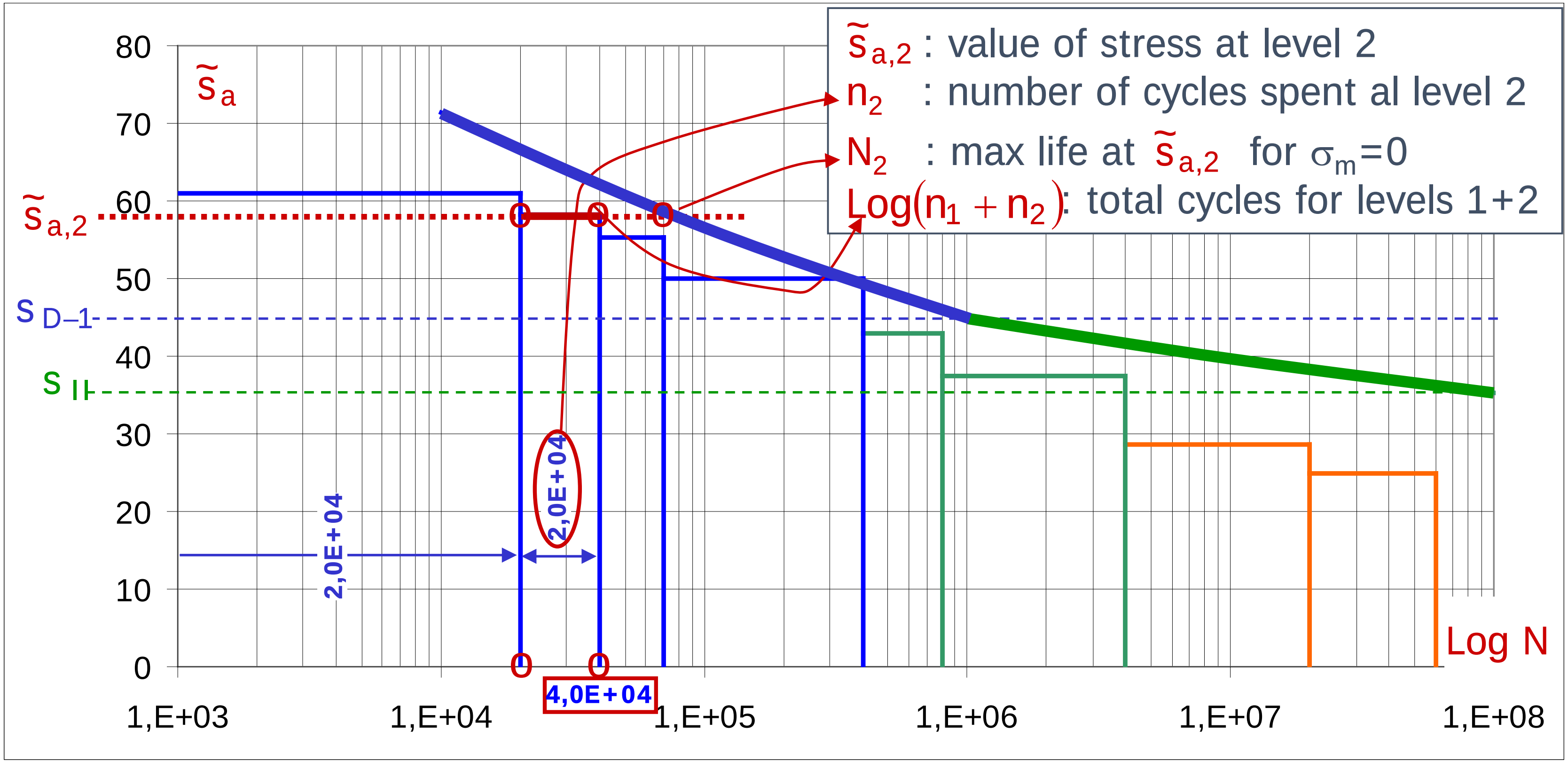
$\tilde{s}_{a,i}$	n_i	Σn_i
61	2,0E+04	2,0E+04
58	2,0E+04	4,0E+04
55	3,0E+04	7,0E+04
50	3,3E+05	4,0E+05
43	4,0E+05	8,0E+05
38	3,2E+06	4,0E+06
29	1,6E+07	2,0E+07
25	4,0E+07	6,0E+07

Each value $\tilde{s}_{a,i}$ is represented with its life n_i in the “exceedance diagram”, which comes from the cycle count of a typical time history of stresses. Stresses are re-ordered in descending order, and are represented against the total cycle number $\mathrm{Log}\left(\sum n_i\right)$.

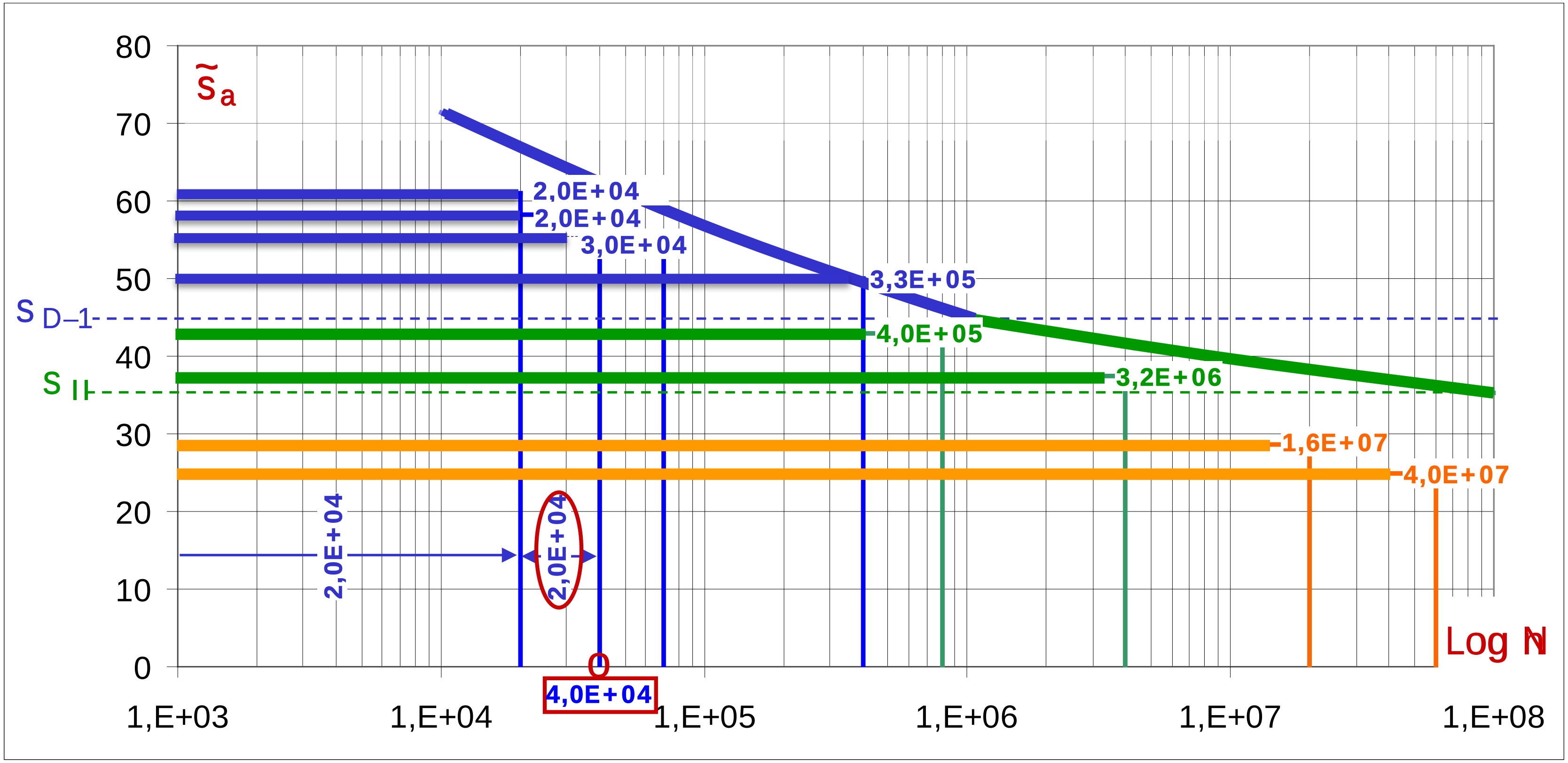


The exceedance diagram is called so because it shows for how many cycles n the stress amplitude $\tilde{s}_a$ is exceeded.

The block-diagram version is a simplification of a continuos curve, typical of a service condition (dotted grey line).



This semi-log representation of the exceedance diagram can be superimposed to the σ_m=0 Wöhler curve, here in the Haibach form (see Appendix). Remark the meaning of points for the example of level i=2, which can easily extended to any level i.



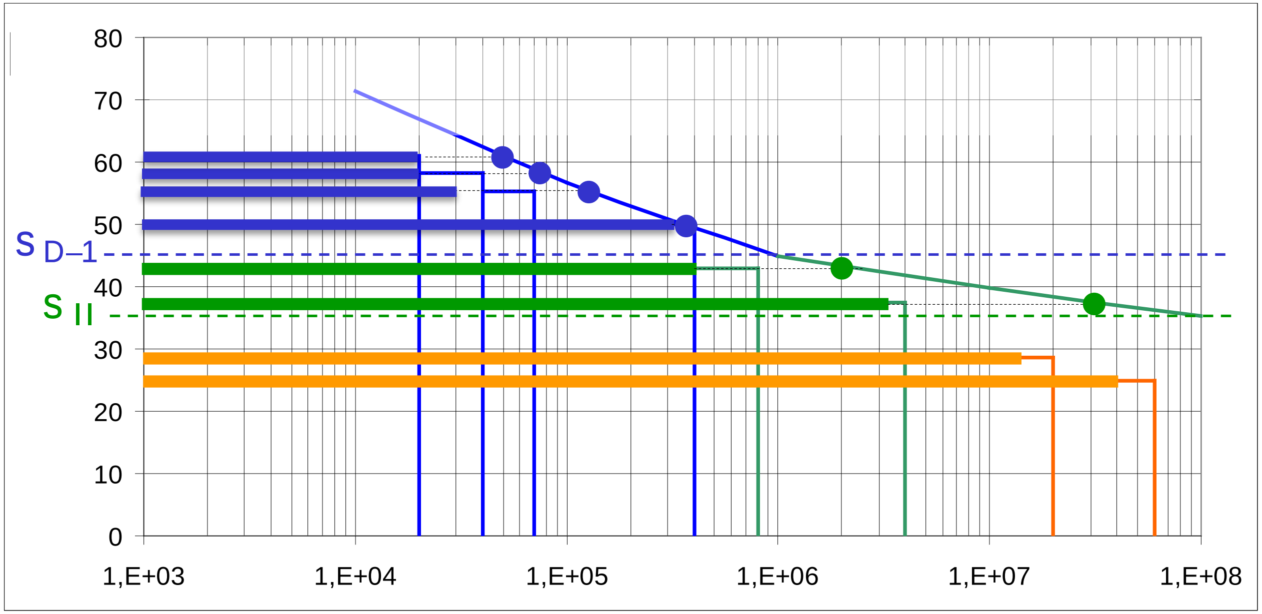
It is perhaps a better and clearer idea to extract from the exceedance diagram the bars representing the n_i , i.e. number of applied cycles – at each level.

4. Finite life and the exceedance diagram (5/12)

From:

$$N_i = 10^6 \left(\frac{s_{D-1}}{\tilde{s}_{a,i}} \right)^k$$

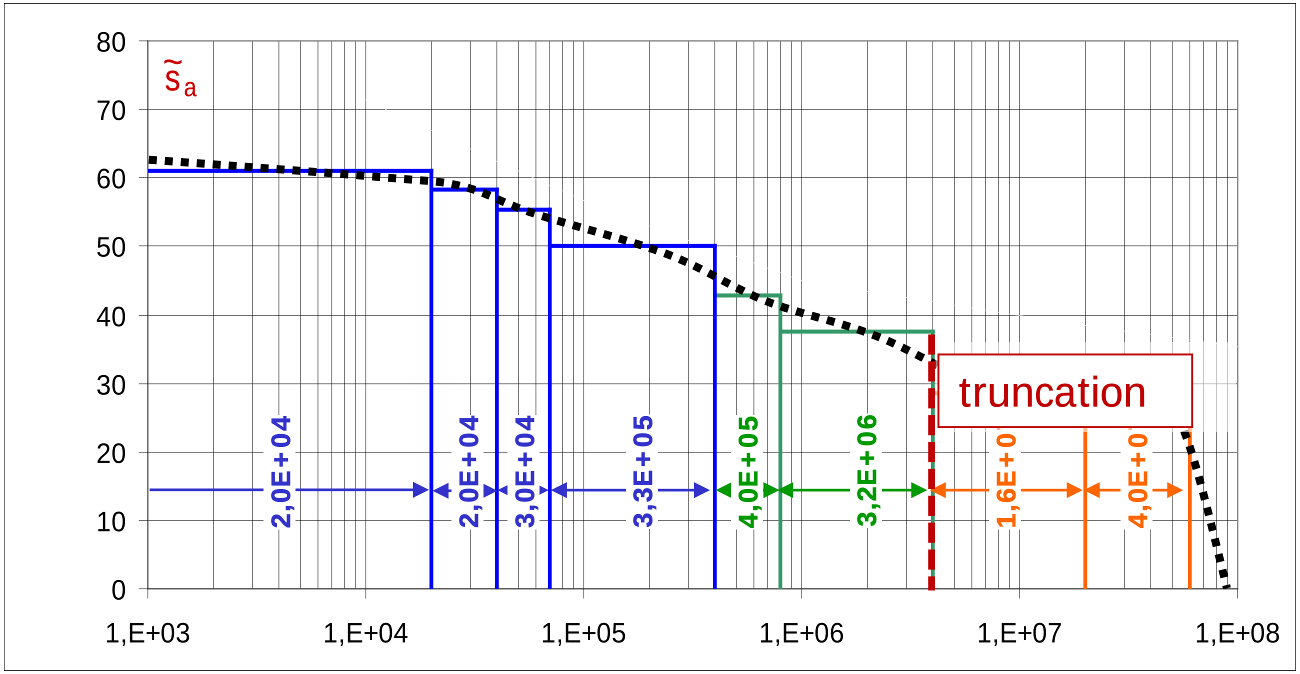
we can calculate N_i and then the ratio n_i/N_i i.e. the damage index.



It is easily seen that it is absolutely worthless to include cycles with $\tilde{s}_a$ lower than s_{II} (orange values); it is then customary to truncate the exceedance diagram excluding all values $\tilde{s}_a < s_{II}$.

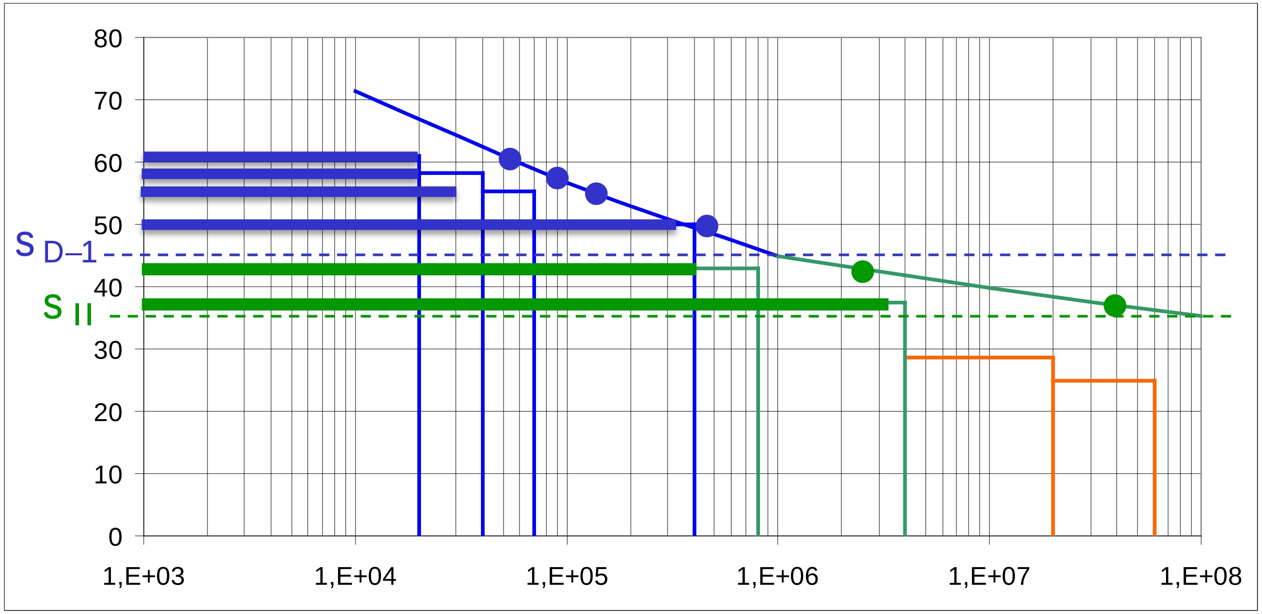
$\tilde{s}_{a,i}$	n_i	Σn_i	N_i	n_i/N_i
61	2,0E+04	2,0E+04	4,8E+04	0,4173
58	2,0E+04	4,0E+04	7,5E+04	0,2680
55	3,0E+04	7,0E+04	1,3E+05	0,2354
50	3,3E+05	4,0E+05	3,5E+05	0,9464
43	4,0E+05	8,0E+05	2,5E+06	0,1583
38	3,2E+06	4,0E+06	3,2E+07	0,1002
29	1,6E+07	2,0E+07	5,6E+09	0,0029
25	4,0E+07	6,0E+07	7,1E+10	0,0006

Both in numerical simulations and in laboratory tests the exceedance diagram is truncated by eliminating the large number of small cycles which do not contribute to damage.



It takes as much computer time (and testing time) to deal with one small cycle as with one large cycle. Thus the small cycles at the lower end of the exceedance diagram consume most of the time (cost) while their effect may be very small. Therefore, it would be advantageous if their number could be reduced. This is called truncation.

Quoted from: David Broek, The practical use of fracture mechanics, Kluwer Academic Publishers, 1989 (a warmly suggested reading!)



The total number of effective cycles of this load history is then: $n_{TOT}=4\times10^6$.

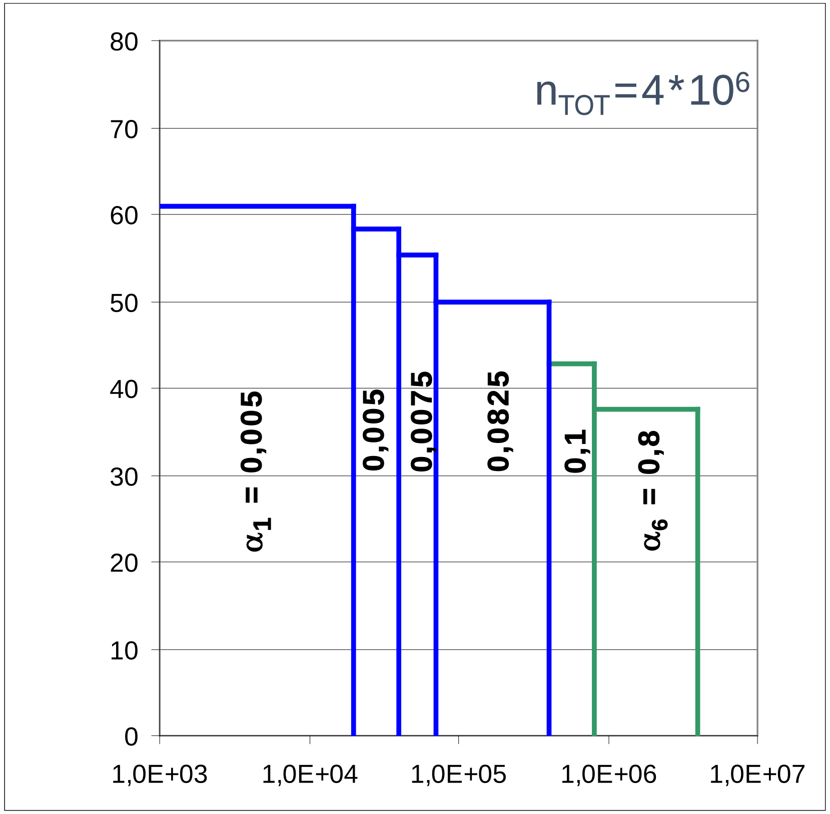
The utilisation factor at level i is α_i :

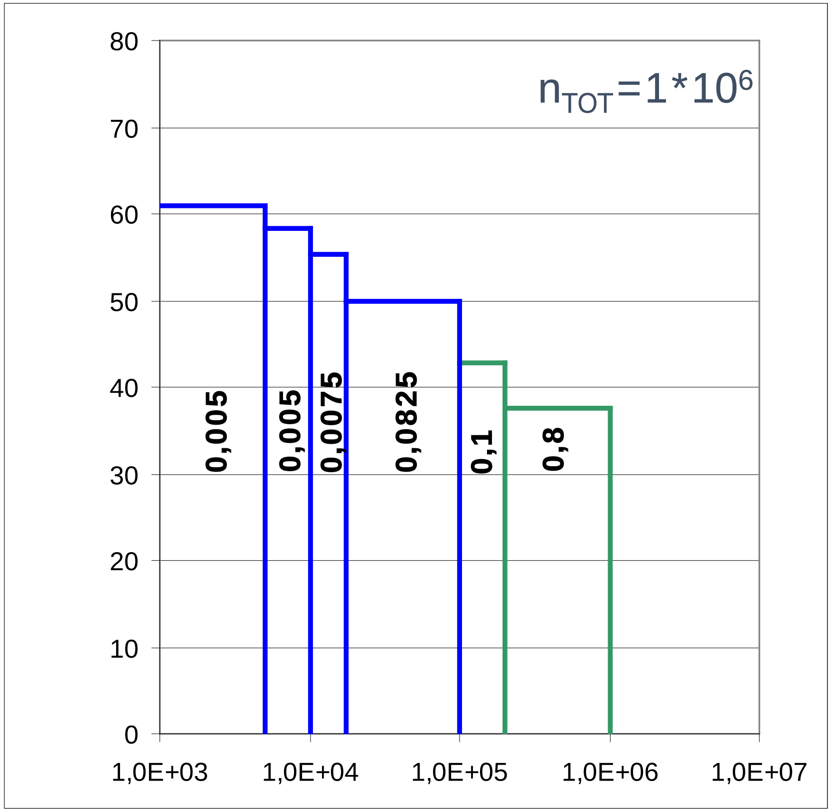
$$\alpha_i = \frac{n_i}{n_{TOT}}$$

i.e. the ratio between applied cycles n_i and n_{TOT}.

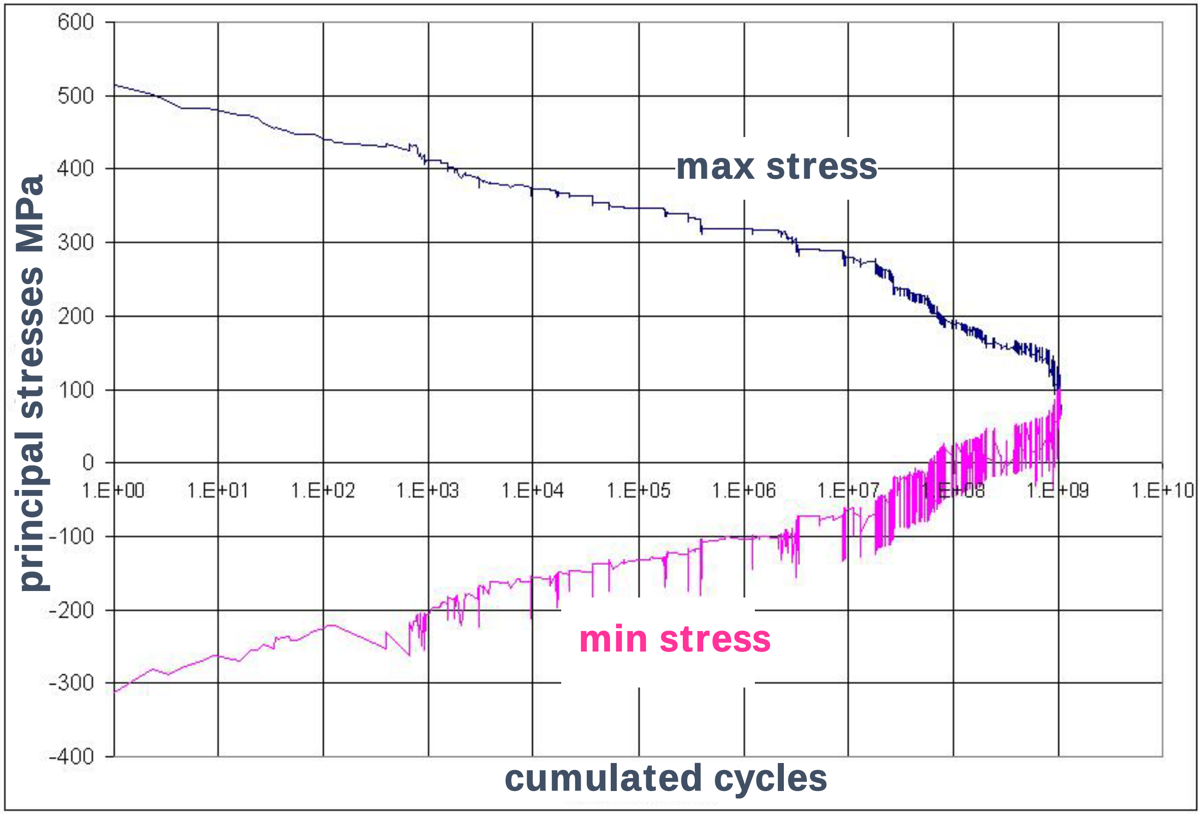
$\tilde{s}_{a,i}$	n_i	Σn_i	N_i	n_i/N_i	α_i
61	2,0E+04	2,0E+04	4,8E+04	0,4173	0,005
58	2,0E+04	4,0E+04	7,5E+04	0,2680	0,005
55	3,0E+04	7,0E+04	1,3E+05	0,2354	0,0075
50	3,3E+05	4,0E+05	3,5E+05	0,9464	0,0825
43	4,0E+05	8,0E+05	2,5E+06	0,1583	0,1
38	3,2E+06	4,0E+06	3,2E+07	0,1002	0,8

n_{TOT} = total cycle number





The concept of utilisation factor α_i is useful because the total cycle number n_{TOT} may change but, as the type of service of the mechanical part repeats in time over and over in the same way, the distribution of the values of α_i remains the same.



A practical example: the diagram above shows the load spectra for stresses at the inner radius of a propeller shaft due to an hydraulic device for the variation of propeller pitch-angle during flight.

	$\tilde{s}_{a,i}$	n_i	$\Sigma\, n_i$	N_i	n_i/N_i	α_i
1	61	2,0E+04	2,0E+04	4,8E+04	0,417	0,005
2	58	2,0E+04	4,0E+04	7,5E+04	0,268	0,005
3	55	3,0E+04	7,0E+04	1,3E+05	0,235	0,0075
4	50	3,3E+05	4,0E+05	3,5E+05	0,946	0,0825
5	43	4,0E+05	8,0E+05	2,5E+06	0,158	0,1
6	38	3,2E+06	4,0E+06	3,2E+07	0,100	0,8

In the case shown here the sum $\sum_i n_i / N_i \rightarrow$ **2,13** is much larger than 1.

Since all lives n_i in the sum of damage fractions: $\sum_i \frac{n_i}{N_i}$ appear in linear form, to get a desired damage sum W it will be sufficient to multiply all of them by the factor $\phi = W/2{,}13$.

The direct calculation of total cycles $\hat{n}_{TOT}$ can be done as follows:

$$N_i = 10^6 \left(\frac{s_{D-1}}{\tilde{s}_{a,i}} \right)^k \quad (1a$$

Wöhler curve: i=1,j

$$N_i = 10^6 \left(\frac{s_{D-1}}{\tilde{s}_{a,i}} \right)^{2k-1} \quad (1b$$

Haibach extension: i=j,n

The sum of damage fractions: $\sum_i \frac{n_i}{N_i} \le W \quad (2$

with the "damage criterion" W usually equal to 1.

Moreover is: $\sum_i n_i = n_{TOT} \quad (3$ and: $n_i = \alpha_i \, n_{TOT} \quad (4$

With (1a and (1b into (2 , and then (3 and (4 , we get:

$$\hat{n}_{TOT} \le \frac{W}{\sum_i \frac{\alpha_i}{N_i}} \equiv W \frac{10^6}{\sum_{i=1,j} \alpha_i \left(\frac{\tilde{s}_{a,i}}{s_{D-1}} \right)^k + \sum_{i=j,n} \alpha_i \left(\frac{\tilde{s}_{a,i}}{s_{D-1}} \right)^{2k-1}}$$

	$\tilde{S}_{a,i}$	n_i	Σn_i	N_i	n_i/N_i	α_i
1	61	9,4E+03	9,4E+03	4,8E+04	0,196	0,005
2	58	9,4E+03	1,9E+04	7,5E+04	0,126	0,005
3	55	1,4E+04	3,3E+04	1,3E+05	0,111	0,0075
4	50	1,6E+05	1,9E+05	3,5E+05	0,445	0,0825
5	43	1,9E+05	3,8E+05	2,5E+06	0,074	0,1
6	38	1,5E+06	1,9E+06	3,2E+07	0,047	0,8

$$\sum_i n_i / N_i \rightarrow \boxed{1,00}$$

In the example shown "total damage" 1 is found with the value $\phi=0,47$. The table above shows the modified values.

5. Appendix: the Haibach extension (1/2)

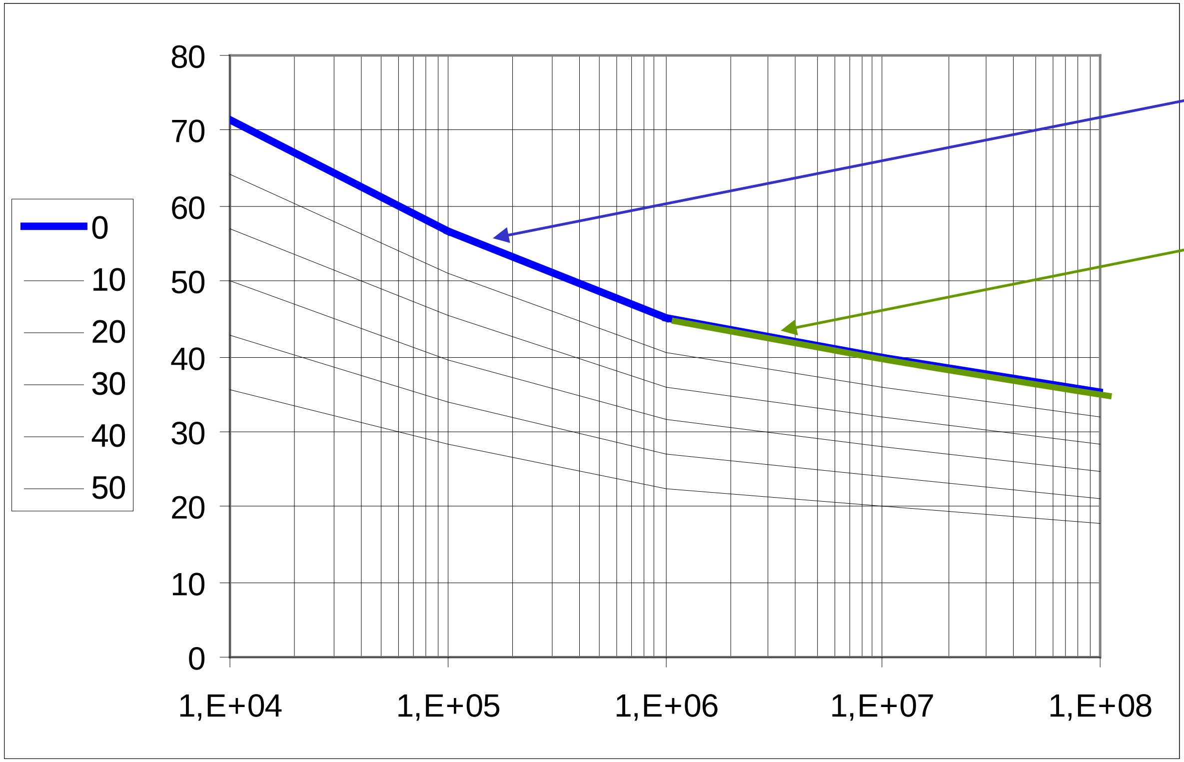
Experimental evidence has shown that even stresses below the fatigue limit (defined through experiments with single amplitude) can contribute to damage provided there are also some stress amplitudes which exceed the fatigue limit.

It is believed that the periodic overloads help overcome the grain barrier which does not let crack propagate at stresses below the fatigue limit, and cause them propagate the crack.

In order to allow for this, the original Miner's rule was improved by extending the fatigue curve beyond the fatigue limit life; following Haibach, the fatigue curve of exponent k is extended below the fatigue limit and towards zero stress with an exponent $2k-1$, as in the example shown in next slide, …

5. Appendix: the Haibach extension (2/2)

... the fatigue curve of exponent k is extended below the fatigue limit and towards zero stress with an exponent 2k-1 , as in the example shown here below (case k=10).



$$s_a^k N_i = 10^6 s_{D-1}^k$$

$$s_a^{2k-1} N_i = 10^6 s_{D-1}^{2k-1}$$