PROVA 11

$$\frac{d}{dr}(\sigma_r r b) - \sigma_c b = -\rho \omega^2 r^2 b$$

$b(r=0) = b_0$

$? = b_{max}$ @ r_{max}

avere un disco a $\sigma_r = \sigma_c = \sigma = cost$ implica la seguente semplificazione:

$$\frac{d}{dr}(\sigma r b) - \sigma b = -\rho \omega^2 r^2 b$$

$$\cancel{\sigma b} + r b \underbrace{\frac{d\sigma}{dr}}_{=0} + \sigma \cancel{r} \frac{db}{dr} - \cancel{\sigma b} = -\rho \omega^2 r^{\cancel{2}} b$$

$$\sigma \frac{db}{dr} = -\rho \omega^2 r b \rightarrow \frac{1}{b}\frac{db}{dr} = -\frac{\rho \omega^2}{\sigma} r$$

$$\frac{d}{dr}\left(\frac{db}{b}\right) = -\frac{\rho \omega^2}{\sigma} r \rightarrow \frac{d}{dr}(\ln b) = -\frac{\rho \omega^2}{\sigma} r$$

$$\ln b = -\frac{\rho \omega^2}{2\sigma} r^2 + A$$

siccome a $r=0$, $b=b_0$ $\rightarrow$ $b = b_0 \, e^{-\frac{\rho \omega^2}{2\sigma} r^2}$

posto $r_{max} = 300$ mm :

$$b_{r_{max}} = 0{,}08 \cdot e^{-\frac{7800 \cdot 1046{,}7^2}{2 \cdot 500 \cdot 10^6} \cdot 0{,}3^2} = 0{,}037 \text{ m} = 37 \text{ mm}$$

$$\omega = 10000 \text{ rpm} = \frac{10000}{60} \text{ rps} = \frac{10000}{\cancel{60}_{30}} \cdot 2\pi \frac{\text{rad}}{\text{s}} = 1046{,}7 \frac{\text{rad}}{\text{s}}$$

$$1 \text{ g} = 2\pi \text{ rad}$$

Massa da aggiungere:

la massa deve essere tale da generare la stessa forza centrifuga del disco:

$$F_c = \int_{r_{max}}^{\infty} \rho\, b\, 2\pi\, r\, dr\, r\omega^2 = \int_{r_{max}}^{\infty} \rho\, b_0\, e^{-\frac{\rho\omega^2 r^2}{2\sigma}} \cdot 2\pi r^2 \omega^2 dr$$

$$F_c = \underbrace{\rho\, b_0\, 2\pi\, \omega^2}_{= 4'295'452'357} \int_{r_{max}}^{\infty} e^{-\frac{\rho\omega^2 r^2}{2\sigma}} r^2\, dr$$

$$\int_{r_{max}}^{\infty} e^{-Ax^2} x^2\, dx = \frac{\sqrt{\pi}\, \mathrm{erf}(\sqrt{A}\, x) - 2\sqrt{A}\, x e^{-Ax^2}}{4 A^{3/2}} + C$$

$$F_c = 4{,}3 \cdot 10^9 \; \frac{\sqrt{\pi}\ \mathrm{erf}\left(\sqrt{A}\, r\right) - 2\sqrt{A}\ r e^{\cdot A r^2}}{4\ A^{3/2}} + C$$

$$r = r_{max} = 0{,}3\ m$$

$$A = -\frac{\rho \omega^2}{2\sigma} = -8{,}5455$$

$$\mathrm{erf}\left(\sqrt{A}\ r\right) = \mathrm{erf}\left(0{,}877\right) = 0{,}75$$

$$F_c = 4{,}3 \cdot 10^9 \; \frac{\left(\sqrt{3{,}14} \cdot 0{,}75\right) - \left(2\sqrt{8{,}5455} \cdot 0{,}3 \cdot e^{8{,}5455 \cdot 0{,}3^2}\right)}{-4 \cdot 8{,}5455^{3/2}}$$

$$= 4{,}3 \cdot 10^9 \; \frac{-2{,}455}{-99{,}9} = 105\ MN$$

$$m = \frac{F_c}{r_{max} \cdot \omega^2} = 321\ kg$$