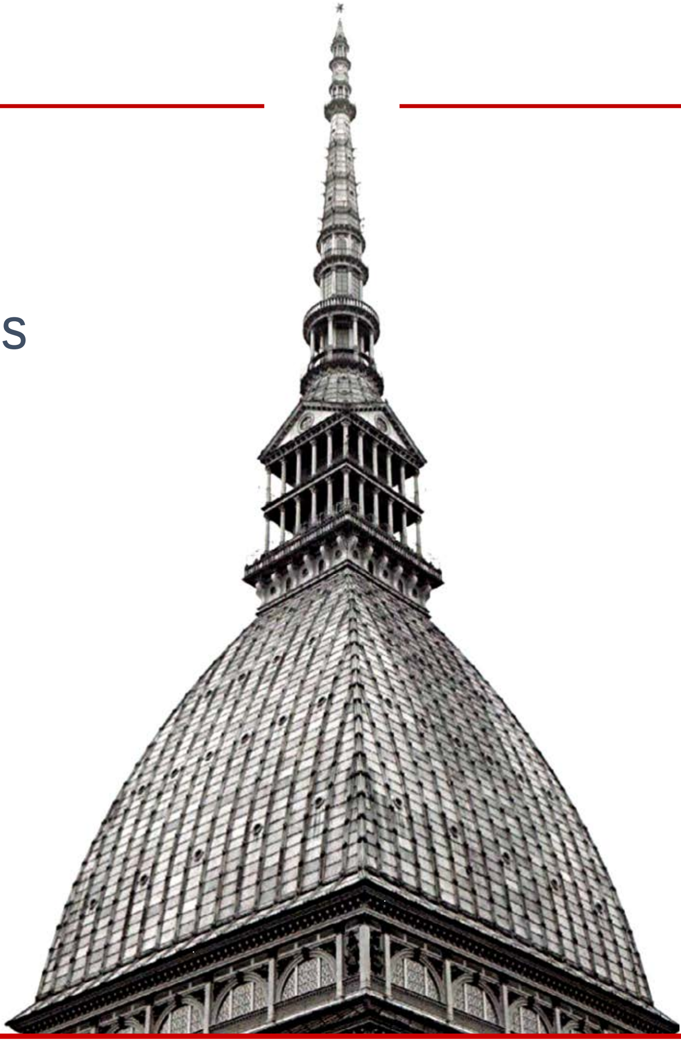
Politecnico di Torino

Chapters



Interference fitted connections are widely used in transmitting motion via two cylindrical parts. Examples of such applications are crank shaft-belt, shaft-bearing and others.

$d_{a,i}$ inner diameter of the shaft

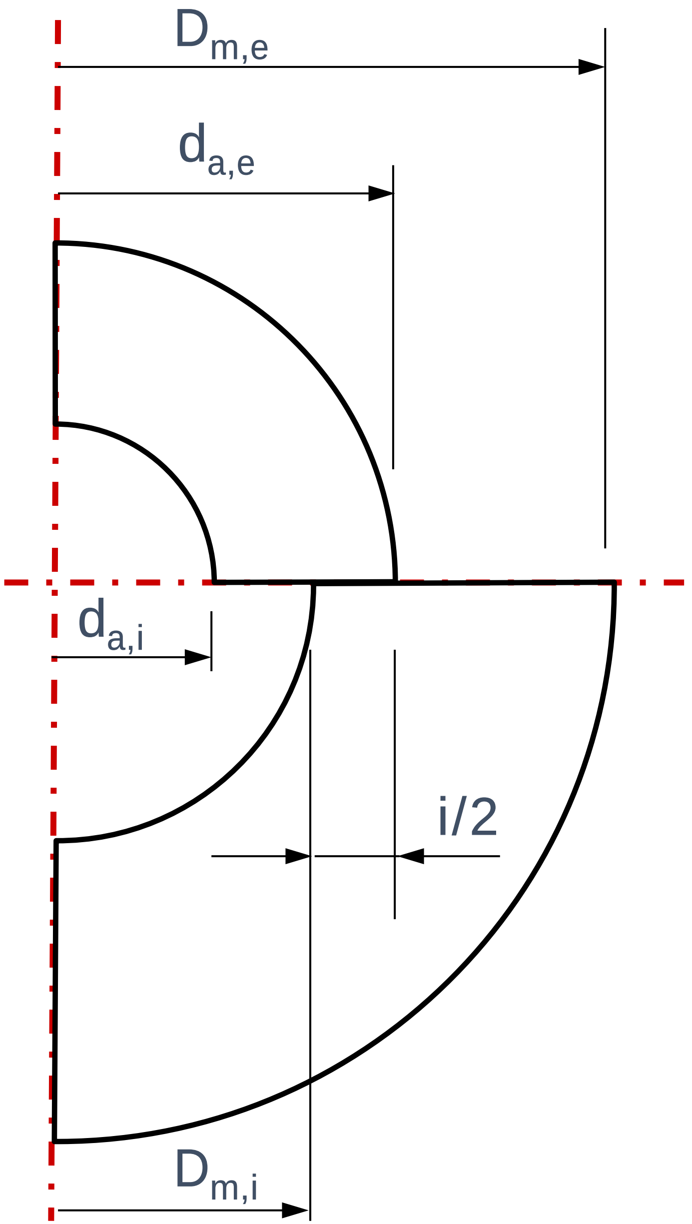
$d_{a,e}$ outer diameter of the shaft

$D_{m,i}$ inner diameter of the hub

$D_{m,e}$ outer diameter of the hub

The difference between the outer diameter of the shaft and the inner diameter of the hub is denoted to as interference

$$i = d_{a,e} - D_{m,i}$$



The displacements are known from the relations obtained on disks with constant thickness b.

Shaft: $p_i = 0$; $p_e = p$ Hub: $p_i = p$; $p_e = 0$

$$u = \frac{D}{2}\frac{p_i}{E}\left[\frac{\frac{D_i^2}{D^2}(1+\nu) + \frac{D_i^2}{D_e^2}(1-\nu)}{1 - \frac{D_i^2}{D_e^2}}\right] - \frac{D}{2}\frac{p_e}{E}\left[\frac{\frac{D_i^2}{D^2}(1+\nu) + (1-\nu)}{1 - \frac{D_i^2}{D_e^2}}\right]$$

In the given relationships E_a, ν_a and E_m, ν_m are the modulus of elasticity and the Poisson's coefficient for the shaft and the hub, respectively.

The displacements are known from the relations obtained on disks with constant thickness b.

Shaft

$$u_a = -\frac{d}{2E_a}\frac{(1+\nu_a)\dfrac{d_{a,i}^2}{d^2} + (1-\nu_a)}{1-\dfrac{d_{a,i}^2}{d_{a,e}^2}}\,p$$

Hub

$$u_m = \frac{D}{2E_m}\frac{(1+\nu_m)\dfrac{D_{m,i}^2}{D^2} + (1-\nu_m)\dfrac{D_{m,i}^2}{D_{m,e}^2}}{1-\dfrac{D_{m,i}^2}{D_{m,e}^2}}\,p$$

In the given relationships E_a, ν_a and E_m, ν_m are the modulus of elasticity and the Poisson’s coefficient for the shaft and the hub, respectively.

In a compact form the displacement u can be written

Shaft

$$u_a = -\frac{d}{2}\delta_a(d)\,p$$

Hub

$$u_m = \frac{D}{2}\delta_m(D)\,p$$

where δ can be defined as a deformability coefficient that relates displacements with pressure in a linear way. The deformability coefficient depends on the specific radius at which δ is needed.

$$\delta_a(d) = \frac{1}{E_a}\frac{(1+\nu_a)\dfrac{d_{a,i}^2}{d^2} + (1-\nu_a)}{1-\dfrac{d_{a,i}^2}{d_{a,e}^2}}$$

$$\delta_m(D) = \frac{1}{E_m}\frac{(1+\nu_m)\dfrac{D_{m,i}^2}{D^2} + (1-\nu_m)\dfrac{D_{m,i}^2}{D_{m,e}^2}}{1-\dfrac{D_{m,i}^2}{D_{m,e}^2}}$$

At the shaft-hub interface the displacements are

Shaft

$$u_{a,e} = -\frac{D_c}{2}\delta_{a,e}\,p$$

Hub

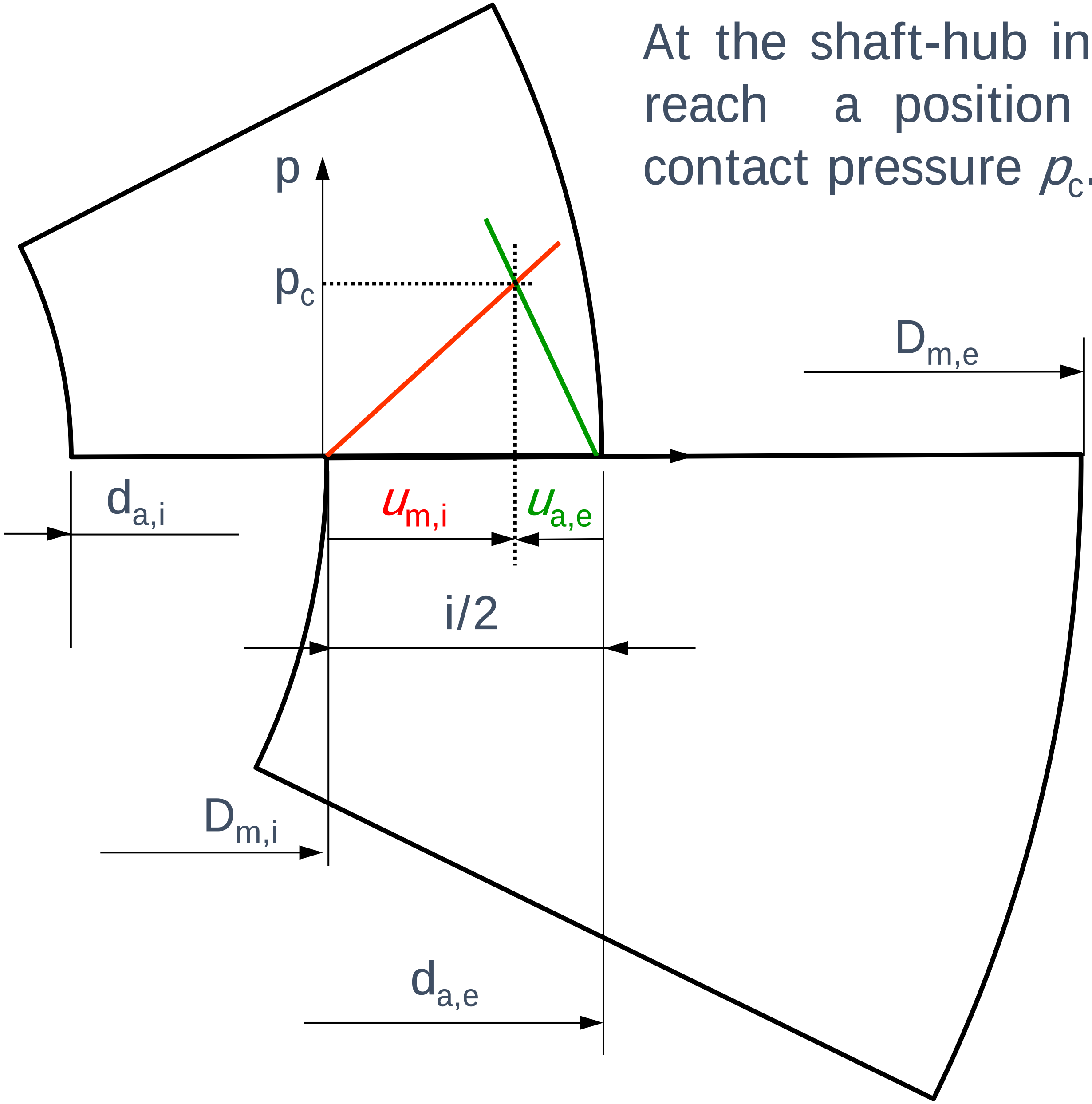
$$u_{m,i} = \frac{D_c}{2}\delta_{m,i}\,p$$

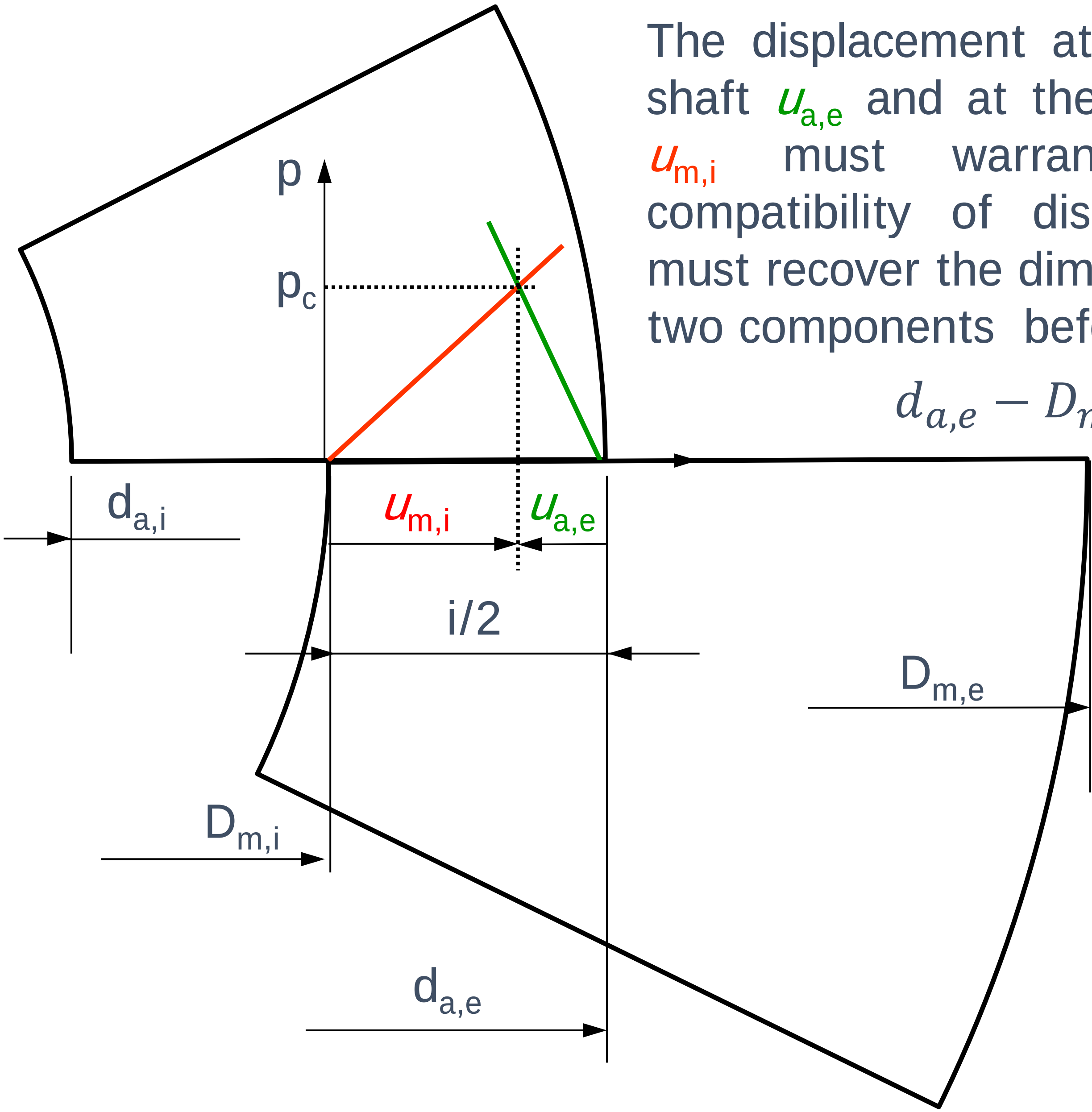
where the approximation $d_{a,e} \sim D_c$ and $D_{m,i} \sim D_c$ has been used as the diameters differ only for the dimensional tolerances, and

$$\delta_{a,e}(D_c) = \frac{1}{E_a}\frac{(1+\nu_a)\dfrac{d_{a,i}^2}{D_c^2} + (1-\nu_a)}{1-\dfrac{d_{a,i}^2}{D_c^2}}$$

$$\delta_m(D_c) = \frac{1}{E_m}\frac{(1+\nu_m) + (1-\nu_m)\dfrac{D_c^2}{D_{m,e}^2}}{1-\dfrac{D_c^2}{D_{m,e}^2}}$$

At the shaft-hub interface the two bodies reach a position of equilibrium with a contact pressure p_c.





The displacement at the outer radius of the shaft $u_{a,e}$ and at the inner radius of the hub $u_{m,i}$ must warranty the condition of compatibility of displacements. Then, they must recover the dimensional tolerances of the two components before the assembling

$$d_{a,e} - D_{m,i} = i = 2\left(\left|u_{a,e}\right| + \left|u_{m,i}\right|\right)$$

The contact pressure p_c is then directly related to the dimensional tolerances through the elastic properties $\delta_{a,e} = \delta_a(D_c)$ and $\delta_{m,i} = \delta_m(D_c)$ of shaft and hub

$$i/2 = |u_{a,e}| + |u_{m,i}| \qquad i = p_c\, D_c\left(\delta_{a,e} + \delta_{m,i}\right)$$

$$p_c = \frac{i}{D_c\left(\delta_{a,e} + \delta_{m,i}\right)}$$

where the deformability coefficients are

$$\delta_{a,e}(D_c) = \frac{1}{E_a}\frac{(1+\nu_a)\frac{d_{a,i}^2}{D_c^2} + (1-\nu_a)}{1 - \frac{d_{a,i}^2}{D_c^2}}$$

$$\delta_m(D_c) = \frac{1}{E_m}\frac{(1+\nu_m) + (1-\nu_m)\frac{D_c^2}{D_{m,e}^2}}{1 - \frac{D_c^2}{D_{m,e}^2}}$$

Local plastic deformation occurs at the shaft-hub interface during assembling. Plastic flattening of surface roughness at the bearing area is designated as “embedding”.

An appreciable loss of interference i and, consequently, of contact pressure is expected.

As a rule of thumb the asperities on each contact surface decrease by around 40% of the surface roughness, $R_{A,a}$ and $R_{A,m}$ of the shaft and of the hub respectively, and the effective interference i_{eff} is less of the nominal (expected by design) interference i_{nom}

$$i_{eff} = i_{nom} - 2 \cdot 0.40\left(R_{A,a} + R_{A,m}\right)$$

$$p_c = \frac{i_{eff}}{D_c\left(\delta_{a,e} + \delta_{m,i}\right)}$$

According to the international system of tolerances (UNI EN ISO 286), it is possible to define two limits of interference, i_{min} and i_{MAX} which represent the minimum and maximum interference, respectively.

During the design phase, the minimum interference i_{min} must be selected to ensure the minimum pressure at the contact.

$$i_{eff,min} = i_{nom,min} - 2 \cdot 0.40\left(R_{A,a} + R_{A,m}\right)$$

$$\boxed{p_{c,min} = \frac{i_{eff,min}}{D_c\left(\delta_{a,e} + \delta_{m,i}\right)}}$$

The state of stress in the shaft and hub can be calculated with the known equation derived from disks with the proper loading conditions in terms of boundary pressures.

Shaft: $p_i = 0$; $p_e = p$ Hub: $p_i = p$; $p_e = 0$

$$\sigma_r = A + \frac{B}{r^2} = -p_i \frac{\frac{D_i^2}{D^2} - \frac{D_i^2}{D_e^2}}{1 - \frac{D_i^2}{D_e^2}} - p_e \frac{1 - \frac{D_i^2}{D^2}}{1 - \frac{D_i^2}{D_e^2}}$$

$$\sigma_c = A - \frac{B}{r^2} = p_i \frac{\frac{D_i^2}{D^2} + \frac{D_i^2}{D_e^2}}{1 - \frac{D_i^2}{D_e^2}} - p_e \frac{1 + \frac{D_i^2}{D^2}}{1 - \frac{D_i^2}{D_e^2}}$$

During the assessment phase, it is necessary to select the maximum interference i_{MAX} to assess the strength of components with the worst possible condition.

$$i_{eff,min} = i_{nom,MAX} - 2 \cdot 0.40\left(R_{A,a} + R_{A,m}\right)$$

$$\boxed{p_{c,MAX} = \frac{i_{eff,MAX}}{D_c\left(\delta_{a,e} + \delta_{m,i}\right)}}$$

The state of stress in the shaft and hub can be calculated with the known equation derived from disks with the proper loading conditions in terms of boundary pressures.

Shaft: $p_i = 0; \qquad p_e = p_{c,MAX}$

$$\sigma_r = -p_{c,MAX}\frac{1-\frac{d_{a,i}^2}{d^2}}{1-\frac{d_{a,i}^2}{d_{a,e}^2}}$$

$$\sigma_c = -p_{c,MAX}\frac{1+\frac{d_{a,i}^2}{d^2}}{1-\frac{d_{a,i}^2}{d_{a,e}^2}}$$

Hub: $p_i = p_{c,MAX}; \qquad p_e = 0$

$$\sigma_r = -p_{c,MAX}\frac{\frac{D_{m,i}^2}{D^2}-\frac{D_{m,i}^2}{D_{m,e}^2}}{1-\frac{D_{m,i}^2}{D_{m,e}^2}}$$

$$\sigma_c = p_{c,MAX}\frac{\frac{D_{m,i}^2}{D^2}+\frac{D_{m,i}^2}{D_{m,e}^2}}{1-\frac{D_{m,i}^2}{D_{m,e}^2}}$$

A proper verification point (diameter d or D for shaft or hub, respectively) must be chosen according to the most stressed region.

Shaft: $p_i = 0;\qquad p_e = p_{c,MAX}$

$$\sigma_r(d_{a,i}) = p_{c,MAX}\frac{1-\dfrac{d_{a,i}^2}{d_{a,i}^2}}{1-\dfrac{d_{a,i}^2}{d_{a,e}^2}} = 0$$

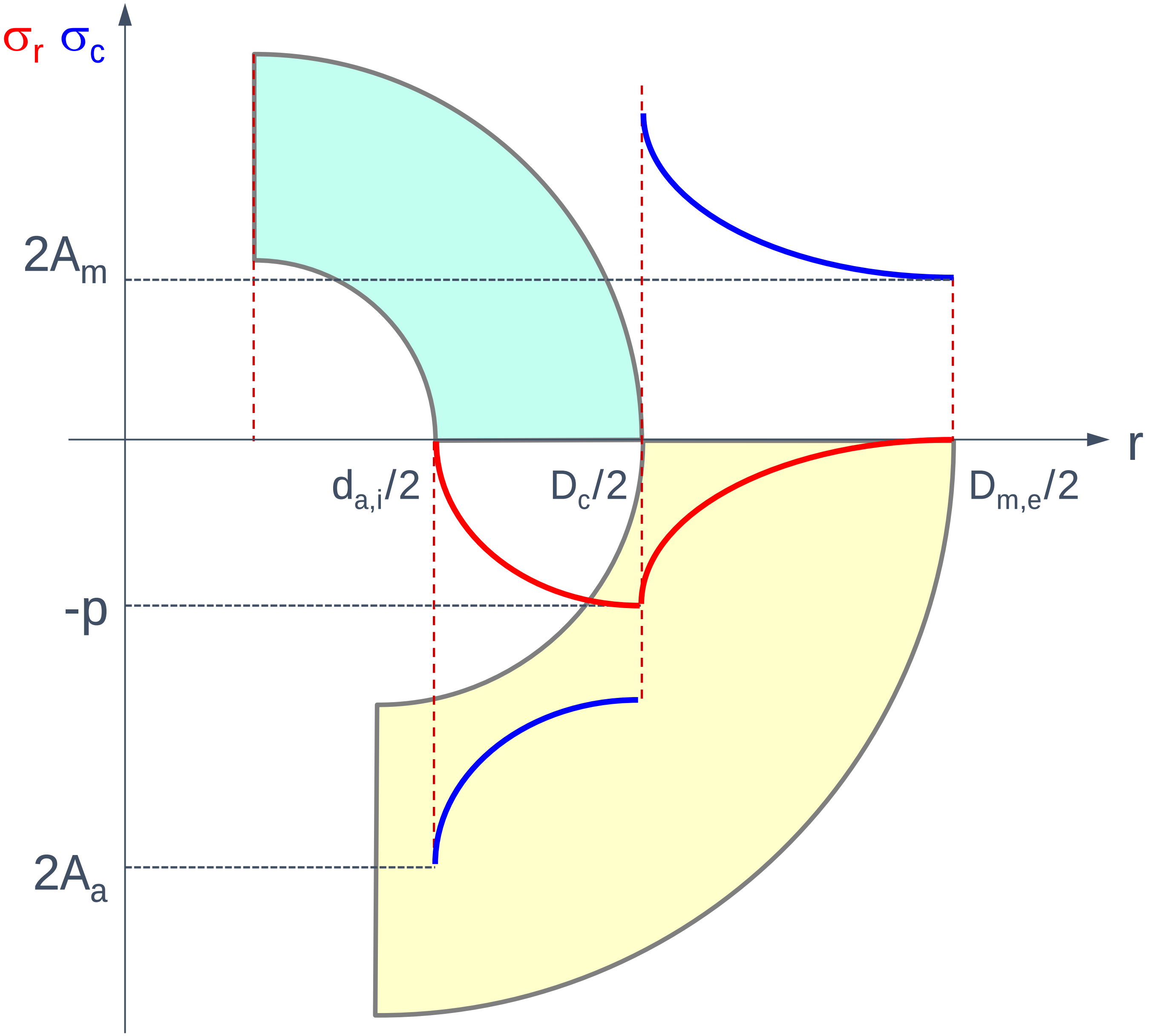
$$\sigma_c(d_{a,i}) = -p_{c,MAX}\frac{1+\dfrac{d_{a,i}^2}{d_{a,i}^2}}{1-\dfrac{d_{a,i}^2}{d_{a,e}^2}} = -p_{c,MAX}\frac{2}{1-\dfrac{d_{a,i}^2}{d_{a,e}^2}}$$

A proper verification point (diameter d or D for shaft or hub, respectively) must be chosen according to the most stressed region.

Hub: $p_i = p_{c,MAX}$; p_e =0

$$\sigma_r\left(D_{m,i}\right) = -p_{c,MAX}\frac{\dfrac{D_{m,i}^2}{D_{m,1}^2} - \dfrac{D_{m,i}^2}{D_{m,e}^2}}{1 - \dfrac{D_{m,i}^2}{D_{m,e}^2}} = -p_{c,MAX}$$

$$\sigma_c\left(D_{m,i}\right) = p_{c,MAX}\frac{\dfrac{D_{m,i}^2}{D_{m,i}^2} + \dfrac{D_{m,i}^2}{D_{m,e}^2}}{1 - \dfrac{D_{m,i}^2}{D_{m,e}^2}} = p_{c,MAX}\frac{1 + \dfrac{D_{m,i}^2}{D_{m,e}^2}}{1 - \dfrac{D_{m,i}^2}{D_{m,e}^2}}$$



σr σc
2Am
-p
2Aa
r
da,i/2
Dc/2
Dm,e/2

Basic Terminology

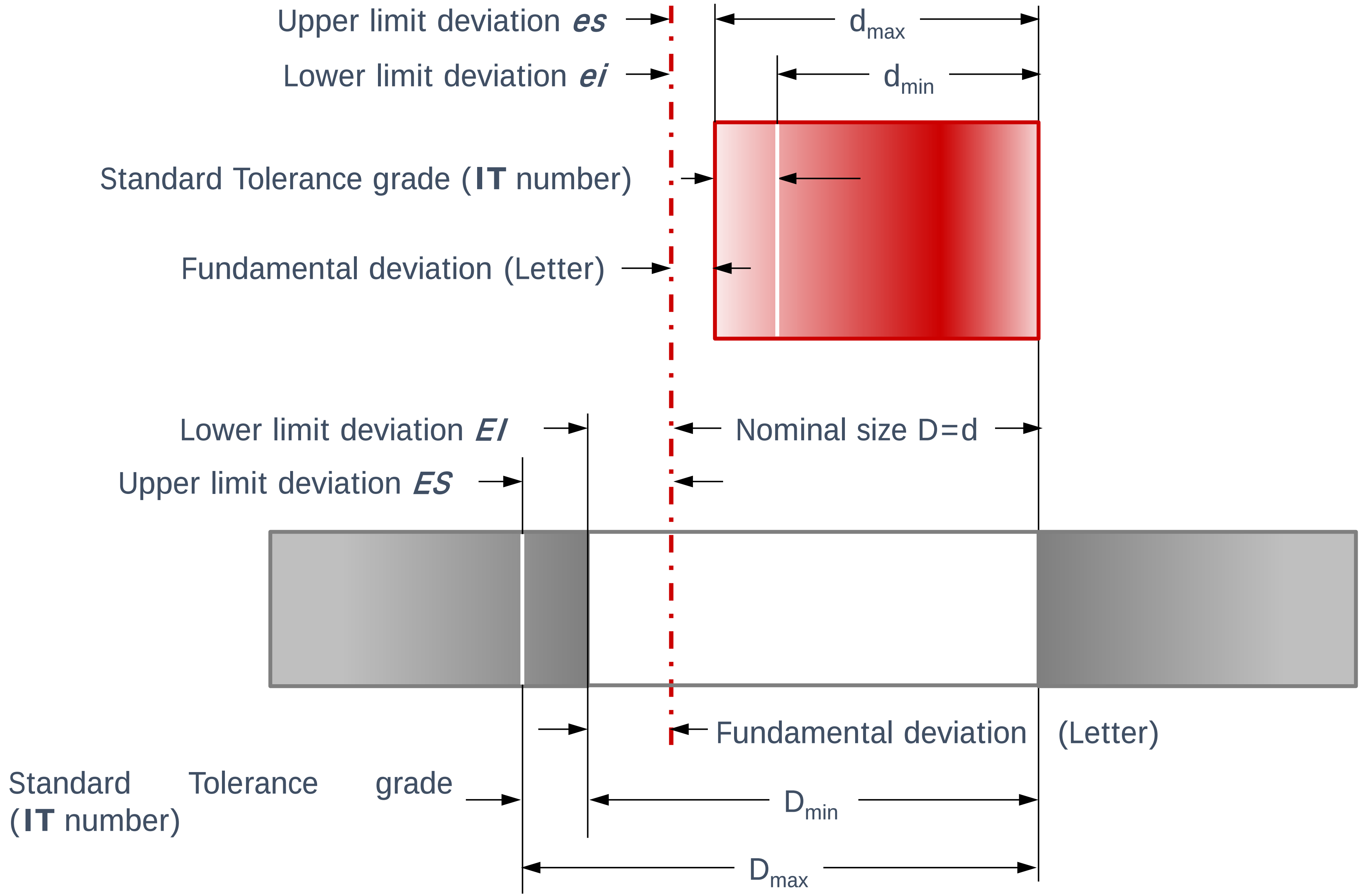
Nominal size	The size of a feature of perfect form as defined by the drawing specification.
Deviation	The algebraic difference between a size its nominal size.
Upper limit deviation	The algebraic difference between the maximum deviation and its corresponding nominal size. ***Es*** and ***es*** for internal and external feature, respectively.
Lower limit deviation	The algebraic difference between the minimum deviation and its corresponding nominal size. ***EI*** and ***ei*** for internal and external feature, respectively.

Basic Terminology

Fundamental deviation The limit deviation, which defines that limit of size which is closer to the nominal size. The fundamental deviation is identified by a letter. Capital letter, e.g. **H**, and small letter, e.g. **h**, for internal and external feature, respectively.

Basic Terminology

Tolerance	The difference between the upper and lower limit of size.
Standard tolerance grade	Any tolerance belonging to the code system tolerances. Designate groups of tolerances such that the tolerances for a particular IT number have the same relative level of accuracy but vary depending on the basic size. IT stands for International Tolerance.
Basic hole	hole chosen as a basis for a hole-basis fit system. The fundamental deviation is **H**.
Basic shaft	Shaft chosen as a basis for a shaft-basis fit system. The fundamental deviation is **h**.


Upper limit deviation *es*
Lower limit deviation *ei*
d_{max}
d_{min}
Standard Tolerance grade (**IT** number)
Fundamental deviation (Letter)
Lower limit deviation *EI*
Upper limit deviation *ES*
Nominal size D=d
Fundamental deviation (Letter)
Standard Tolerance grade (**IT** number)
D_{min}
D_{max}

2. Interference fitted shaft-hub system (5/7)

Excerpt from **UNI EN ISO 286-1:2010**

Geometrical product specifications (GPS) — ISO code system for tolerances on linear sizes — Part 1: Basis of tolerances, deviations and fits

Limit deviation for holes - Fundamental deviation H

Upper limit deviation *Es*

Lower limit deviation Ei

Nominal size mm		H																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Above	Up to and including	Deviations µm											Deviations mm						
10	18						+11 0	+18 0	+27 0	+43 0	+70 0	+110 0							
18	30						+13 0	+21 0	+33 0	+52 0	+84 0	+130 0							
30	50	+1.5 0	+2.5 0	+4 0	+7 0	+11 0	+16 0	+25 0	+39 0	+62 0	+100 0	+160 0	+0.25 0	+0.39 0	+0.62 0	+1 0	+1.6 0	+2.5 0	+3.9 0
50	80						+19 0	+30 0	+46 0	+74 0	+120 0	+190 0							
80	120						+22 0	+35 0	+54 0	+87 0	+140 0	+220 0							

Limit deviation for shaft

Upper limit deviation es
Lower limit deviation ei

Nominal size mm		h																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14[a]	15[a]	16[a]	17	18
Above	Up to and including	Deviations µm											Deviations mm						
30	50	0 -1.5	0 -2.5	0 -4	0 -7	0 -11	0 -16	0 -25	0 -39	0 -62	0 -100	0 -150	0 -0.25	0 -.0.39	0 -0.62	0 -1	0 -1.6	0 -2.5	0 -3.9

Nominal size mm		js																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14[a]	15[a]	16[a]	17	18
Above	Up to and including	Deviations µm											Deviations mm						
30	50	±0.75	±1.25	±2	±3.5	±5.5	±8	±12.5	±19.5	±31	±50	±80	±0.125	±0.195	±0.31	±0.5	±0.8	±1.25	±0.195

Nominal size mm		j				k										
Above	Up to and including	5	6	7	8	3	4	5	6	7	8	9	10	11	12	13
		µm														
30	50	+6 +5	+11 +5	+15 +10		+4 0	+9 +2	+13 +2	+18 +2	+27 +2	+39 0	+62 0	+100 0	+160 0	+250 0	+390 0

Limit deviation for shaft

Upper limit deviation es

Lower limit deviation ei

Nominal size mm		m							n						
Above	Up to and including	3	4	5	6	7	8	9	3	4	5	6	7	8	9
		µm													
30	50	+13 +9	+16 +9	+20 +9	+25 +9	+34 +9	+48 +9	+71 +9	+21 +17	+24 +17	+28 +17	+33 +17	+42 +17	+56 +17	+79 +17

Nominal size mm		p								r								s		
Above	Up to and including	3	4	5	6	7	8	9	10	3	4	5	6	7	8	9	10	5	6	7
		µm																		
30	50	+30 +26	+33 +26	+37 +26	+42 +26	+51 +26	+65 +26	+88 +26	+125 +26	+38 +34	+41 +34	+45 +34	+50 +34	+59 +34	+73 +34	+96 +34	+134 +34	+54 +43	+59 +43	+68 +43

Nominal size mm		t*			u		x
Above	Up to and including	5	6	7	6	7	7
		µm					
30	40	+59 +48	+64 +48	+73 +48	+76 +60	+85 +60	+105 +80
40	50	+65 +54	+70 +55	+79 +56	+86 +70	+95 +70	+122 +97