

# Pure-Past Linear Temporal and Dynamic Logic on Finite Traces

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Survey Track



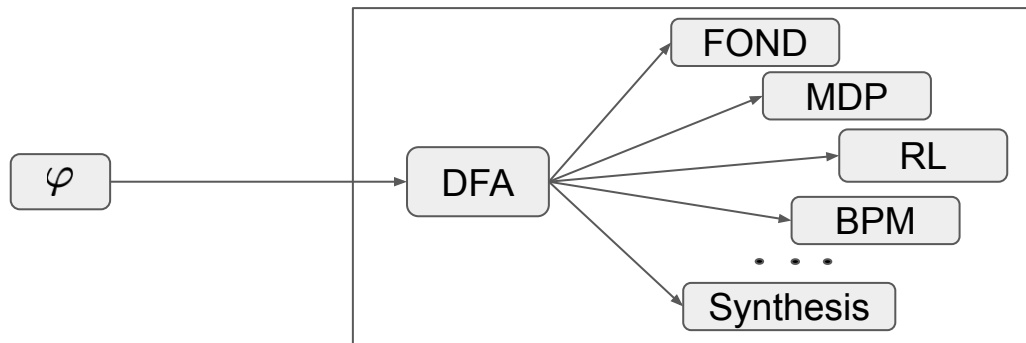
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# Context and Motivation

- Linear Temporal and Dynamic Logic on *finite* traces ( $LTL_f$  /  $LDL_f$ ) studied as compelling formal languages to express temporal specifications
  - *Ease of use* and *intuitiveness*
  - Possibility of using DFAs in the translation



# PLTL<sub>f</sub>/PLDL<sub>f</sub>

- Most of work focused on the pure-future LTL<sub>f</sub>/LDL<sub>f</sub>
- Sometimes specifications are *easier* and *more natural* to express referring to the past (PLTL<sub>f</sub>/PLDL<sub>f</sub>) [Lichtenstein et al. 1985]
  - non-Markovian models [Gabaldon2011]
  - non-Markovian rewards in MDPs [Bacchus et al. 1996]
  - normative properties in multi-agent systems [FisherWooldridge2005;Knobbout et al. 2016;Alechina et al. 2018]
- Computational advantage wrt LTL<sub>f</sub>/LDL<sub>f</sub> [Chandra et al., 1981]

# Examples

**Example 1:** “every time you took the bus, you bought a new ticket beforehand”

$$\text{PLTL}_{\text{f}}: \quad \Box(\text{take}B \Rightarrow \Theta(\neg \text{take}B \mathcal{S} \text{buy}T))$$

$$\text{LTL}_{\text{f}}: \quad (\text{buy}T \mathcal{R} \text{take}B) \wedge \Box(\text{take}B \Rightarrow (\text{buy}T \vee \bigcirc(\text{buy}T \mathcal{R} \neg \text{take}B)))$$

**Example 2:** “every time, if the cargo-ship departed (*cs*), then there was an alternation of *grab* and unload (*unl*) of containers before”

$$\text{PLDL}_{\text{f}}: \quad \llbracket \text{true}^* \rrbracket (\langle\langle \text{cs} \rangle\rangle tt \Rightarrow \langle\langle \text{unl}; \text{grab} \rangle^*; (\text{unl}; \text{grab}) \rangle\rangle \text{start})$$

$$\text{LDL}_{\text{f}}: \quad \langle (\neg \text{cs} + (\text{grab} \wedge \neg \text{cs}); (\text{unl}; (\text{grab} \wedge \neg \text{cs}))^*; (\text{cs} \wedge \text{unl})) ; \neg \text{cs}^* \rangle \text{end}$$

# From $\varphi$ to Automata

- $LTL_f/LDL_f$ :

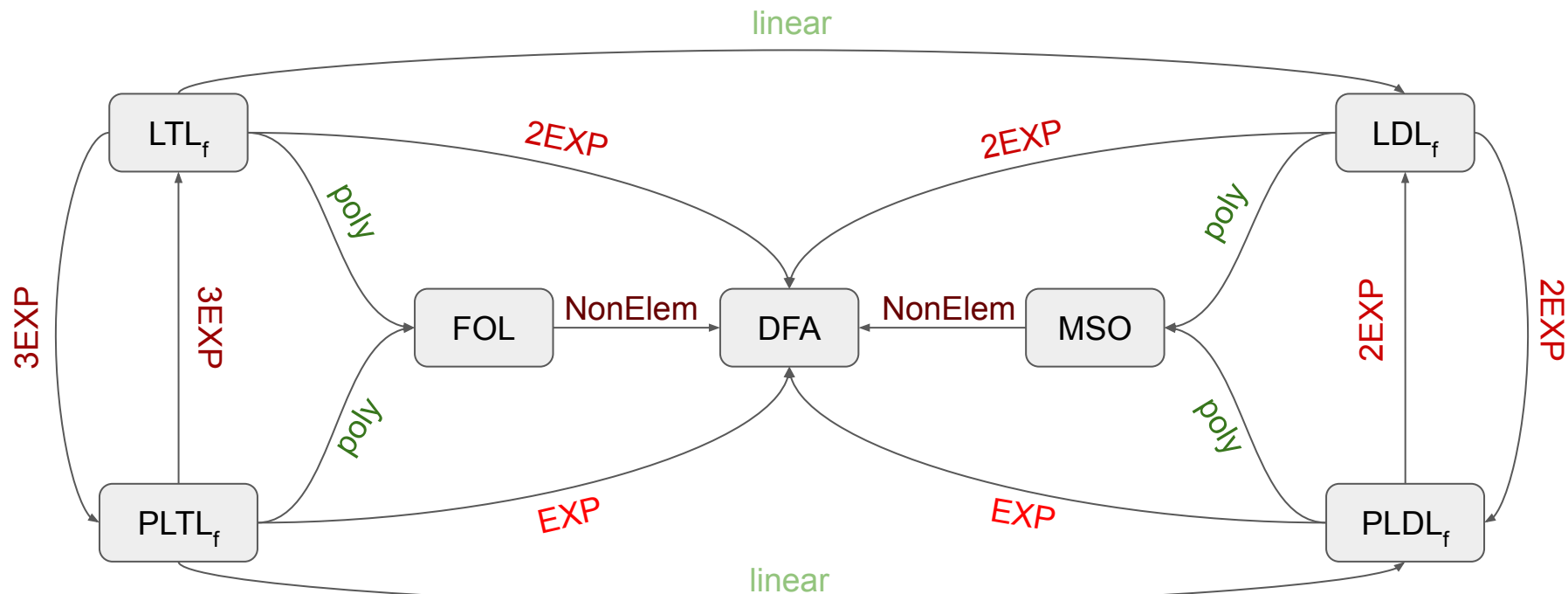


- $PLTL_f/PLDL_f$ :



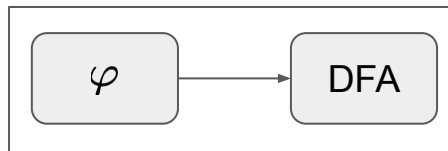
- Given an AFA of  $k$  states for language  $L$ , there exists a DFA of at most  $2^k$  states for language  $L^{reverse}$  [Chandra et al. 1981]

# Transformations

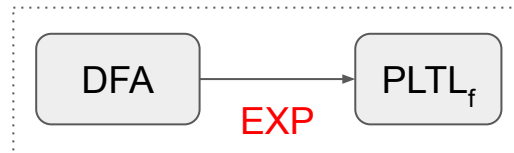


# Useful Results

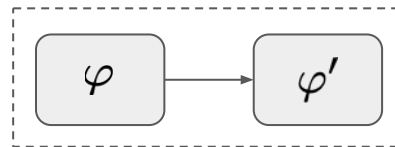
1. Transformations seen before



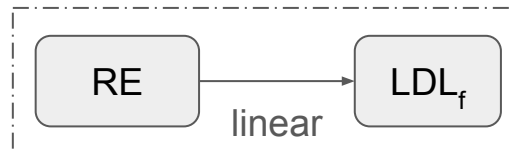
2. From DFA (star-free) to  $\text{PLTL}_f$  [Maler&Pnueli 1990]



3. Syntactic swap of temporal operators

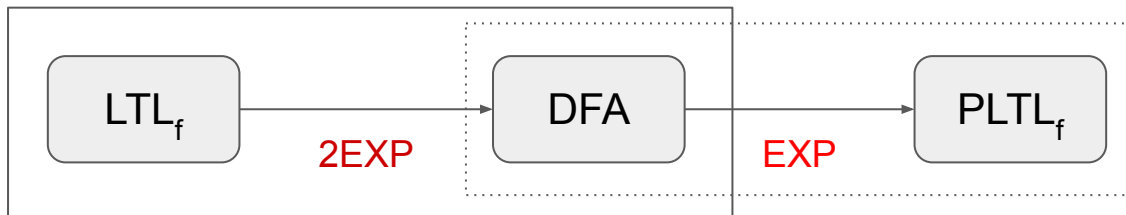


4. From RE to  $\text{LDL}_f$  [DeGiacomo&Vardi 2013]

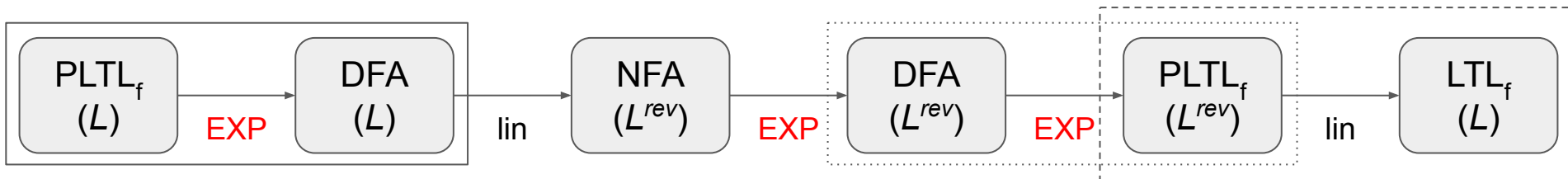


# $LTL_f$ to $PLTL_f$ (and vice versa)

- From  $LTL_f$  to  $PLTL_f$



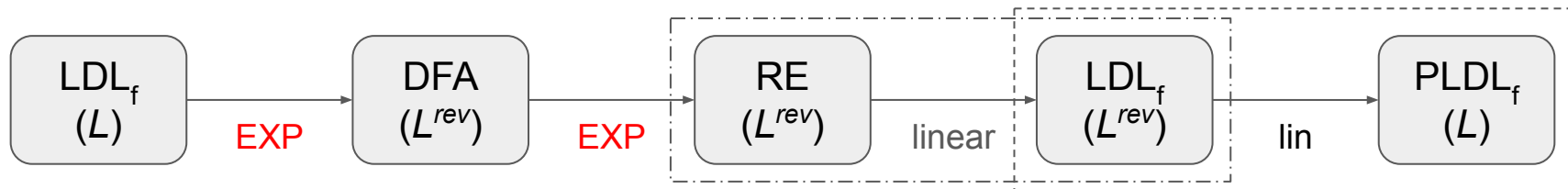
- From  $PLTL_f$  to  $LTL_f$



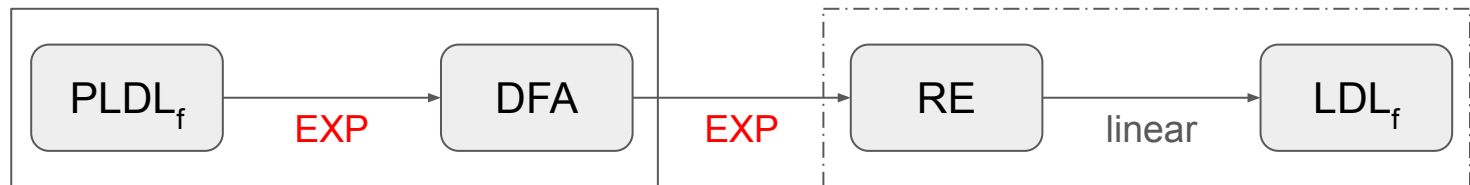


# $LDL_f$ to $PLDL_f$ (and vice versa)

- From  $LDL_f$  to  $PLDL_f$



- From  $PLDL_f$  to  $LDL_f$



# Impact of adopting $\text{PLTL}_f/\text{PLDL}_f$

- Exponential gain reflected in an exponential gain in solving different forms of sequential decision making involving temporal specifications:
  - FOND Planning for temporally extended goals
  - MDPs with non-Markovian rewards
  - Reinforcement Learning with temporally extended rewards
  - Planning in non-Markovian domains
  - non-Markovian decision processes
- Problems are EXPTIME-complete in the goal/reward natively expressed in  $\text{PLTL}_f/\text{PLDL}_f$  (vs. 2EXPTIME-complete in the  $\text{LTL}_f/\text{LDL}_f$  goal/reward)

# Takeaways

- If you can *naturally* express the specification in  $\text{PLTL}_f/\text{PLDL}_f$ , then do it to get the computational advantage.
- Converting  $\text{LTL}_f/\text{LDL}_f$  into  $\text{PLTL}_f/\text{PLDL}_f$  to get the exponential advantage is not computationally sensible
- Complexities are just worst-case, in many applications the size of the resulting DFA is actually manageable