ENGINE COMPANY

STUDENTS GROUP

Samuele Cucchi, Francesco Gemignani

LOGISTICS PROJECT

Data Science & Business Informatics

Academic Year 2020/2021

Engine Company

The Problem

An engine company manufactures small engines at three different plants. From the plants, the engines are transported to two different warehouses before being distributed to three wholesales distributors. Three plants make engines with respect of the following conditions. Each of them produce a certain number of engines, with minimum and maximum limits.

PLANT	COST	MINIMUM REQUIRED	CAPACITY
1	$ 13	150	400
2	$ 15	150	300
3	$ 12	150	600

Figure 1: Plants costs, minimum requirements and capacity.

Moreover, the unit cost of transporting engines from each plant to each warehouse is shown below:

PLANT	WAREHOUSE 1	WAREHOUSE 2
1	$ 4	$ 5
2	$ 6	$ 4
3	$ 3	$ 3

Figure 2: Unit transportation costs between warehouses and plants.

Two different warehouses receive the engines and send them to distributors. Each of them can process up to 500 engines. The unit cost from each warehouse to each distributor is shown below along with the daily demand for each distributor.

WAREHOUSE	DISTRIBUTOR 1	DISTRIBUTOR 2	DISTRIBUTOR 3
1	$ 6	$ 4	$ 3
2	$ 3	$ 5	$ 2
DEMAND	300	600	100

Figure 3: Unit transportation costs between warehouses to distributors.

The optimization problem consists on how to transport the engines from the plants to the warehouses, and from the warehouses to the distributors, by satisfying all the stated constraints and minimizing the total cost.

Model Classification

This is a production and transportation problem, whose goal is to determine how to transport all the engines from plants (supply nodes) to warehouses (transshipment nodes), and from those latter to distributors (demand nodes) satisfying all constraints and minimizing the total cost (objective function).

A Graphical Representation

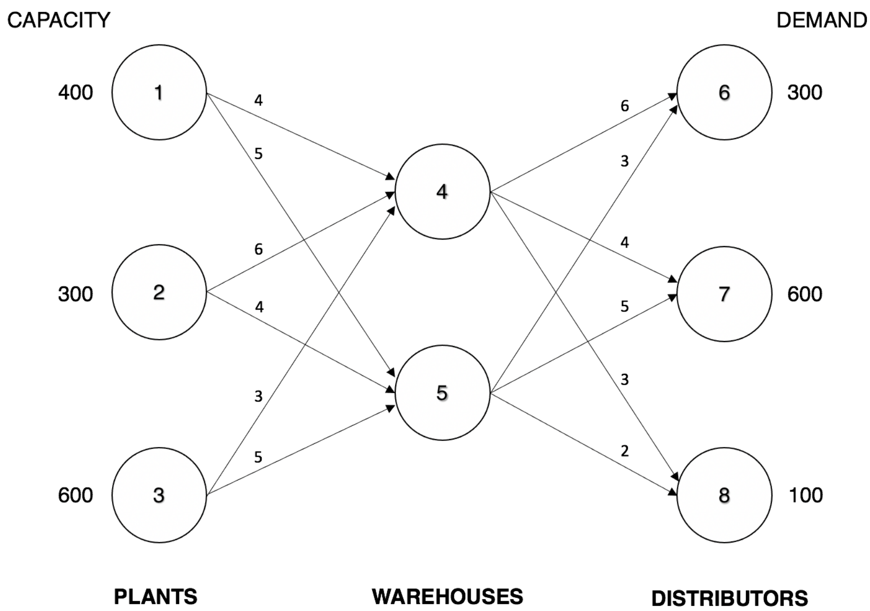

Figure 4: Transportation network.

Remember that warehouses can process up to 500 engines for each day and each plant must produce at least 150 engines

Decision Variable

x_{ij}: number of engines sent from node i to node j

Flow Conservation Constraints

Some example of flow conservation constraints that we use to define the linear problem:

- plant 1 (node 1), which is a supply node: $x_{14} + x_{15} \leq 400$
- warehouse 2 (node 5), which is a transshipment node with demand equal to 0: $x_{15} + x_{25} + x_{35} - x_{56} - x_{57} - x_{58} = 0$
- distributor 2 (node 7), which is a demand node: $x_{47} + x_{57} = 600$

Observations

The objective function not only needs to minimize the transportation costs from plants to warehouses and to distributors but also needs to minimize the engines production costs. Furthermore, we don't care to verify if integrity property is satisfied because for each question, as we can see below, the solver returns an integer optimal solution to formulate the problem described.

A Mathematical Model Solution

Question 1

Propose an ILP model to formulate the problem, implement it by means of the modeling language AMPL and solve it using the optimization solver CPLEX.

ILP Model

$$\begin{aligned}
\min \quad & [13(x_{14}+x_{15})+15(x_{24}+x_{25})+12(x_{34}+x_{35})]+ && \text{(production cost)}\\
& [4x_{14}+5x_{15}+6x_{24}+4x_{25}+3x_{34}+5x_{35}]+ && \text{(plants-warehouses costs)}\\
& [6x_{46}+4x_{47}+3x_{48}+3x_{56}+5x_{57}+2x_{58}] && \text{(warehouses-distributors costs)}
\end{aligned}$$

$$\begin{aligned}
& -x_{14}-x_{15} \geq -400 \\
& -x_{24}-x_{25} \geq -300 && \text{(plant capacities constraints)}\\
& -x_{34}-x_{35} \geq -600
\end{aligned}$$

$$\begin{aligned}
& x_{14}+x_{24}+x_{34}-x_{46}-x_{47}-x_{48}=0 \\
& x_{15}+x_{25}+x_{35}-x_{56}-x_{57}-x_{58}=0 && \text{(warehouses flow constraints)}
\end{aligned}$$

$$\begin{aligned}
& x_{46}+x_{56}=300 \\
& x_{47}+x_{57}=600 && \text{(distributors demands)}\\
& x_{48}+x_{58}=100
\end{aligned}$$

$$\begin{aligned}
& x_{14}+x_{15} \geq 150 \\
& x_{24}+x_{25} \geq 150 && \text{(minimum plants production)}\\
& x_{34}+x_{35} \geq 150
\end{aligned}$$

$$\begin{aligned}
& x_{14}+x_{24}+x_{34} \leq 500 \\
& x_{15}+x_{25}+x_{35} \leq 500 && \text{(warehouses process constraints)}
\end{aligned}$$

$$x_{14}, x_{24}, x_{34}, x_{15}, x_{25}, x_{35}, x_{46}, x_{56}, x_{47}, x_{57}, x_{48}, x_{58} \geq 0 \qquad \text{(no negativity)}$$

The Optimal Solution

The minimum total cost that solve the problem is \$ 20.150. It includes minimum production and trasportation costs. The graph in figure 5 shows how many engines to send over each link.
How we can see, the optimal integer values of decision variables x_{ij} are:

$$\begin{array}{llllll}
x^*_{14}=0 & x^*_{15}=250 & x^*_{24}=0 & x^*_{25}=150 & x^*_{34}=500 & x^*_{35}=100 \\
x^*_{46}=0 & x^*_{47}=500 & x^*_{48}=0 & x^*_{56}=300 & x^*_{57}=100 & x^*_{58}=100
\end{array}$$

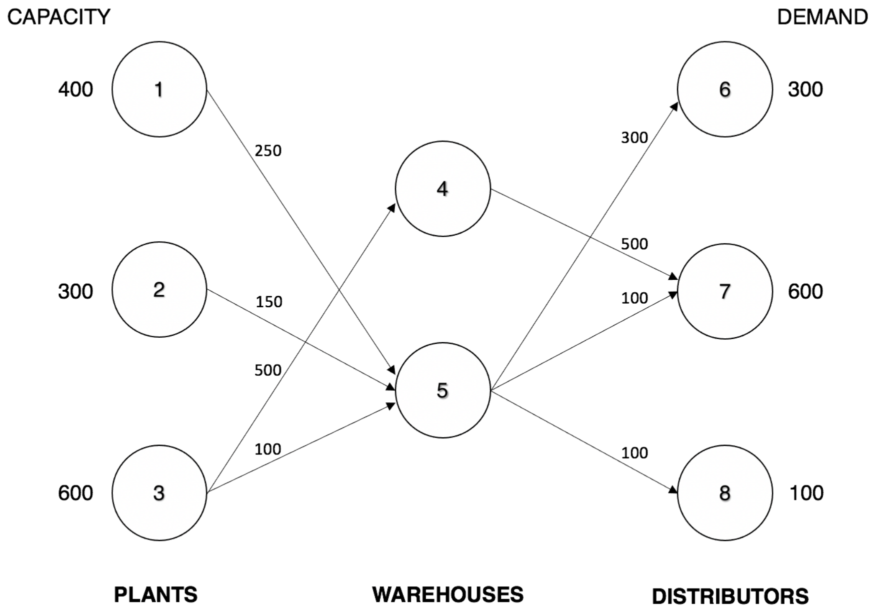

Figure 5: The graph optimal solution for question 1

Question 2

Propose an ILP model for the case in which, in addition to the stated constraints, Warehouse 2 can send at most 250 engines to Distributor 1. Then, implement and solve the model proposed and compare its optimal solution to the one found at the previous point.

ILP Model

If warehouse 2 (node 5) can send at most 250 engines to distributor 1 (node 6), we also add at previous ILP model the following constraint. All others constraints and objective function remain the same.

$x_{56} \leq 250$ (new link capacity constraint)

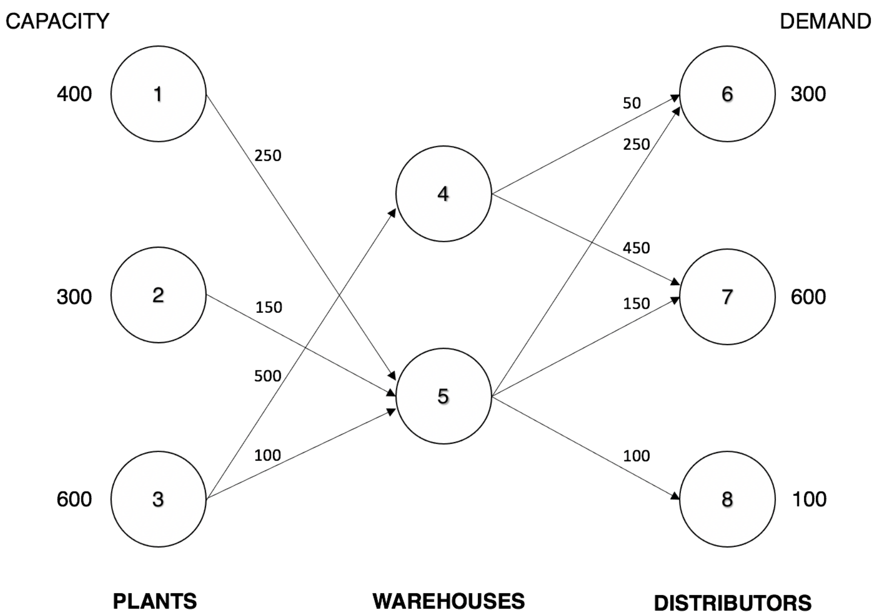

Figure 6: The graph optimal solution for question 2

The Optimal Solution

The minimum total cost that solve this problem is \$ 20.350. It includes minimum production and transportation costs. The graph in figure 6 shows how many engines send over each link.

In particular, we can see how this link capacity constrain lead to split the flow over other links. Therefore, the optimal integer values of decision variables x_{ij} are:

$x^*_{14} = 0$ $x^*_{15} = 250$ $x^*_{24} = 0$ $x^*_{25} = 150$ $x^*_{34} = 500$ $x^*_{35} = 100$
$x^*_{46} = 50$ $x^*_{47} = 450$ $x^*_{48} = 0$ $x^*_{56} = 250$ $x^*_{57} = 150$ $x^*_{58} = 100$