The Timetabling Examination problem

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The problem

Aim: schedule a set of exams (E), in ordered time-slots (T), for a set of students (S).

- n_{e_1,e_2}: the number of students enrolled both in e_1 and in e_2
- $e_1,\ e_2 \in$ E are considered ***conflicting*** if $n_{e_1,e_2} > 0$

Given $e_1, e_2 \in$ E, schedulated at distance of i time-slots, with $1 \leq i \leq 5$, a penalty is assigned and defined in this way:

$$2^{5-i}\,\frac{n_{e_1,e_2}}{|S|}$$

The problem

The goal in the Examination Timetabling problem is scheduling the exams into the available time-slots, by considering the following:

- Each item is scheduled exactly once during the examination eriod
- Two conflicting exams can not be scheduled in the same time-slot
- The total penalty resulting from the created timetable is minimized

Model: inputs

- Set of students: $S = s_1, ..., s_{|S|}$
- Set of exams: $E = e_1, ..., e_{|E|}$
- Set of the available time-slots: $T = t_1, ..., t_{|T|}$

Inputs

- Enrol matrix A, $(|S| \times |E|)$, defined as follows
  $$a_{s,e} = \begin{cases} 1 & \text{if the student s is enrolled in exam e} \\ 0 & \text{otherwise} \end{cases}$$
- Conflict matrix C, $(|E| \times |E|)$, where
  - $c_{i,j}$ is the number of students enrolled in both the exams
  - $c_{i,j} > 0 \iff$ exam i and exam j are conflicting

  In this implementation, C is an upper triangular matrix.

Variables

Aim: to assign exams to time-slots (binary variables) Each variable is associated to a couple e, t, where e is an exam and t is a time-slot, selected from sets E and T, respectively. The total number of variables is $|E| \times |T|$.

$$x_{e,t} = \begin{cases} 1 & \text{if exam } e \text{ is scheduled in time-slot } t \\ 0 & \text{otherwise} \end{cases}$$

Constraints

- Each exam must be scheduled exactly once.
  For each exam e, the sum of the variables $x_{e,t}$, for all time-slots t, must be equal to 1:

$$\sum_{t \in T} x_{e,t} = 1, \quad \forall e \in E$$

- Conflicting exams can not be scheduled in the same time-slot.
  If two exams are conflicting, they must be placed in different time-slots:

$$x_{e_i,t} + x_{e_j,t} \leq 1, \quad \forall t \in T, \forall e_i, e_j \in E \text{ s.t. } c_{e_i,e_j} > 0$$

Objective function

Aim: minimizing the total penalty of the created timetable.

$$\text{obj} = \sum_{e_i, e_j \in E} \sum_{t_m, t_n \in T} 2^{5-|t_m - t_n|} \frac{c_{e_i, e_j}}{|S|} x_{e_i, t_m} x_{e_j, t_n}$$

where $1 \leq |t_m - t_n| \leq 5$

- Only conflicting exams ($c_{e_i, e_j} > 0$), scheduled in time-slots with a distance between 1 and 5, contribute to the total penalty.
- Only the penalty of pairs of exams that are effectively present in the solution of the problem ($x_{e_i, t_m}, x_{e_j, t_n} > 0$) is considered.

Equity measure

There are many other different ways of measuring the goodness of a timetable: the equity measures.

Example: the total number of times students have back-to-back exams.

- Back-to-back: when a student is enrolled in exams scheduled in two consecutive time-slots.
- The lower the number of back-to-back situations, the more equitable is the timetable for the students.

The total number of back-to-backs is given by:

$$b2b = \sum_{t=1}^{|T|-1} \sum_{e_i, e_j \in E} c_{e_i, e_j} \left(x_{e_i, t} x_{e_j, t+1} + x_{e_j, t} x_{e_i, t+1} \right)$$

Additional restrictions - 1

Change the constraints that impose that no conflicting exams can be scheduled in the same time slot. Instead, impose that at most 3 conflicting pairs can be scheduled in the same time slot.

$$\sum_{e_i, e_j \in E} x_{e_i,t} x_{e_j,t} \leq 3, \quad \forall t \in T$$

where $c_{e_i,e_j} > 0$, $\forall e_i, e_j \in E$

Additional restrictions - 2

At most 3 consecutive time slots can have conflicting exams. It is necessary to add additional binary variables z_t, defined through a set of constraints:

$$z_t = \begin{cases} 1 & \text{if } t \text{ contains at least a pair of conflicting exams} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{e_i, e_j \in E} x_{e_i,t} x_{e_j,t} \leq M z_t$$

$$z_t \leq \sum_{e_i, e_j \in E} x_{e_i,t} x_{e_j,t}$$

where $e_i, e_j > 0$, $\forall t \in T$ and M is a large enough number.

Additional restrictions - 2

Whenever 3 consecutive time-slots have conflicting exams, two constraints are imposed to limit the presence of conflicting exams in the following and in the previous time-slots:

$$5 - \sum_{i=0}^{2} z_{t+i} \geq 3z_{t+3}, \quad \forall t \in \{1, \dots, |T| - 3\}$$

$$5 - \sum_{i=0}^{2} z_{t+i} \geq 3z_{t-1}, \quad \forall t \in \{1, \dots, |T| - 2\}$$

Additional restrictions - 3

If two consecutive time slots contain conflicting exams, then no conflicting exam can be scheduled in the next 3 time slots.
Another binary variable is added:

$$y_{e_i,e_j,t} = \begin{cases} 1 & \text{if } e_i \text{ and } e_j \text{ are scheduled in } t \text{ and } t+1 \\ 0 & \text{otherwise} \end{cases}$$

The values of y_t are imposed with this constraint:

$$y_{e_i,e_j,t} = x_{e_i,t}x_{j,t+1} + x_{e_j,t}x_{i,t+1}$$

which guarantees that $y_{e_i,e_j,t}$ is 0 if the right-hand side sum is 0, and 1 if the sum is equal to 1.

Additional restrictions - 3

The following expression represents the restriction on the three consecutive time-slots:

$$\sum_{e \in E^*} \sum_{k=t+2}^{t+5} x_{e,k} y_{e_i,e_j,t} = 0$$

where:
$E^* = \{e \in E : e \neq e_i \text{ and } e \neq e_j \text{ and } (c_{e,e_i} > 0 \text{ or } c_{e,e_j} > 0)\}$ and $t \in \{1, \ldots, |T| - 5\}$, $\forall e_i, e_j \in E$ s.t. $c_{i,j} > 0$.

E^* allows selecting all the exams that are different from e_i and e_j, and that are in conflict with at least one exam between e_i and e_j.

Bonus profit

Include a bonus profit each time no conflicting exams are scheduled for 6 consecutive time slots.

$$6 - \sum_{i=0}^{5} z_{t+i} \geq 6 b_t \quad \forall t \in \{1, \ldots, |T| - 5\}$$

$$b_t \geq 1 - \sum_{i=0}^{5} z_{t+i} \quad \forall t \in \{1, \ldots, |T| - 5\}$$

Where b_t is a variable defined as follows:

$$b_t = \begin{cases} 1 & \text{if the bonus is assigned to time-slot } t \\ 0 & \text{otherwise} \end{cases}$$

After defining b_t, it should be integrated into the objective function.

Results: basic model

Name	Result	Benchmark
test	3.375	3.375
instance01	157.736	157.033
instance02	44.233	34.709
instance03	51.429	32.627
instance04	12.565	7.717
instance05	19.342	12.901
instance07	12.967	10.050
instance08	30.202	24.769
instance09	14.477	9.818

Results: equity measure

Name	Back-to-backs	Penalty
test	0	3.375
instance01	3021	165.347
instance02	1315	49.357
instance03	1208	47.531

Results: additional constraints

Name	Additional Restrictions	Basic
test	3.375	3.375
instance01	194.088	157.736