

# notes

## Files

Filename	What it does
sim1.R	fit variogram through WOLS on <b>meuse</b> data
sim2.R	fit variogram through REML on <b>meuse</b> data
sim3.R	fit variogram through WOLS on simulated random field
sim4.R	fit variogram through REML on simulated random field

## Issues

1. Dimension of grid seems to lead to computational problem for (i) taking pps sample, (ii) estimating variogram through REML with all the data.

## Some questions

1. Variogram/Covariogram

- Q1: The formulas in the paper are for the general case  $U \in \mathbb{R}^d$  or for the case  $U \in \mathbb{R}^2$ ?

R1: For the definition of  $G$  and  $C$ , this is the general case:  $U = \mathbb{R}^d$

### 2. Equation (5)

$Y : \Omega \rightarrow (U \rightarrow \mathcal{Y})$  Gaussian random process

$X : \Omega \rightarrow U$

$n, m \in \mathcal{N}, x \in U^n, x' \in U^m$

- Q2:  $n \neq m$ ?

R2: not necessarily.

Expected value of signal  $\mu : U \rightarrow \mathcal{Y}, x \rightarrow E[Y[x]]$  and covariance matrix between points  $\mathbf{x}_1, \dots, \mathbf{x}_n$  and  $\mathbf{x}'_1, \dots, \mathbf{x}'_m$   $\Sigma_{\mathbf{x}, \mathbf{x}', \theta} = Cov[Y[\mathbf{x}], [\mathbf{x}']]$ .

In the paper we have

$$f_{Y[\mathbf{x}]}(\mathbf{y}) = \frac{1}{2\pi^{n/2}} \exp \frac{-(\mathbf{y} - \mu(\mathbf{x}))\Sigma_{\mathbf{x}, \mathbf{x}}^{-1}(\mathbf{y} - \mu(\mathbf{x}))}{2}.$$

- Q3: Most likely I'm mistaken, but I was wondering if (i) in the first fraction the denominator has also the term  $|\Sigma|^{n/2}$  where  $|\Sigma|$  is the determinant of  $\Sigma$ , and (ii) the first  $(\mathbf{y} - \mu(\mathbf{x}))$  is transposed, so that we have

$$f_{Y[\mathbf{x}]}(\mathbf{y}) = \frac{1}{2\pi^{n/2}|\Sigma|^{n/2}} \exp \frac{-(\mathbf{y} - \mu(\mathbf{x}))^T \Sigma_{\mathbf{x}, \mathbf{x}}^{-1}(\mathbf{y} - \mu(\mathbf{x}))}{2}.$$

R3: You are right.

### 3. Design/sample

$D$  is the random process for the design.

- Q4:  $D$  has domain equal to the power set of  $U^n$ , i.e.  $P(U^n)$ ?

R4: we have made the assumption of a fixed size design, so in this case,  $D(\omega)$  has domain equal to the power set of  $U^n$ , i.e.  $P(U^n)$ .  $D$  is a random variable:  $D : \Omega \rightarrow$  the set of all probabilities on  $U^n$ ,  $\omega \mapsto D(\omega)$

- Q5: the codomain is the set of probability distributions on  $U^n$ , right? Can we indicate that with  $\mathcal{P}_{U^n}$ ?  
R5: the codomain of  $D$  is the set of all probabilities distributions on  $U^n$  the codomain of  $D(\omega)$  is  $[0, 1]$
- Q6: If Q4 and Q5 are right, then we should have  $D : P(U^n) \rightarrow \mathcal{P}_{U^n}$

R6:  $D(\omega) : \mathcal{P}_{U^n} \rightarrow [0, 1]$

### 4. Exchangeability condition

- Q7: Why do we use it? (I guess to have property 2.3) R7: this is a way to ignore the order in which elements are selected. Adaptive sampling is a counter example
- Q8: It is verified in our case? I mean, it's just for SRS or not? R8: Yes, we limit ourselves to these cases. The selection proportional to size, this is also the case.