Network Sensitivity of Systemic Risk

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The recent stream of literature on systemic risk in financial markets emphasized the key importance of considering the complex interconnections among financial institutions. Much efforts has been put to model the contagion dynamics of financial shocks, and to assess the resilience of specific financial markets—either using real data, reconstruction techniques or simple toy networks. Here we address the more general problem of how the shock propagation dynamics depends on the topological details of the underlying network. To this end, we consider different network topologies, all consistent with balance sheets information obtained from real data on financial institutions. In particular, we consider networks with varying density and different mesoscale structures, and diversify as well in the details of the shock propagation dynamics. We show that the systemic risk properties of a financial network are extremely sensitive to its network features. Our results can thus aid in the design of regulatory policies to improve the robustness of financial markets.

I. INTRODUCTION

The several crises that happened in the last two decades lead scientists and regulators to rethink the approach used to assess market risk with a systemic perspective [1–7]. A common denominator that emerged from these work is the importance of considering the structure of financial dependencies [8–12], a thing that pushed the research in the direction of designing novel systemic risk mechanisms [13]—from the seminal approaches of Eisenberg & Noe [14] and Furfine [15] to the recently introduced DebtRank centrality [16]. These methods are nowadays implemented in stress tests performed by central banks [17], thus the current scientific challenge is no longer to quantify the systemic risk, but to suggest specific regulatory solutions to improve the structure of the system and reduce risk. To this end, it is essential to understand which features of a financial network make it more or less resilient to systemic risk. In this work we focus precisely on this challenge.

One of the first contributions in this direction is that of Gai & Kapadia [2], who showed that random Erdös-Rényi networks are "robust-yet-fragile": the probability of contagion is maximal for intermediate network densities, whereas, the amount of systemic losses monotonically increases with the network connectivity. Mastromatteo et al. [18] further showed that, under the Furfine dynamics, sparse Erdös-Rényi networks in general lead to more defaults than very dense networks.

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Roukny et al. [19] showed that no single topology always leads to lowest risk level (in particular, scale-free networks can be both more robust and more fragile than Erdös-Rényi architectures). Leon & Berndsen [20] argued that modular scale-free architectures can favor robustness, whereas, Montagna & Lux [21] show that the dependence of systemic risk on the density is changes if shocks are correlated. Hurd et al. [22] observed that, under the Gai & Kapadia dynamics, degree assortativity can strongly affect the course of contagion cascades [22], whereas, Bardoscia et al. [23] showed that, under the DebtRank dynamics, the stability of the system decreases monotonically with the density due to the presence of cycles which lead to instability.

In this work we set an exploratory route for the network sensitivity of systemic risk, which aims at encompassing these previous findings and extending them by considering modular, core-periphery and bipartite network structures.

II. METHODOLOGY

A. Data

To build financial (interbank) networks, we use the Bankscope dataset [24] containing the balance sheet of the 100 largest European banks. In particular, we have information about the interbank assets A_i, the interbank liabilities L_i and the equities E_i of each i of these banks, and we consider data for years 2008 and 2013 (i.e, before and after the global financial crisis) [25]. We recall that the equity of a bank is the difference between its positive positions and its obligations to creditors. When the equity is positive the bank is solvent, otherwise it goes bankrupt (defaults) because it would not be able to refund its debts. Since the chosen group of banks is not an isolated systems, interbank assets and liabilities do not sum up to the same value. In order to have a closed system, we rescale them to have $\sum_j A_j = \sum_j L_j$.

B. Network Generation

In the literature on financial networks, interbank markets are typically reconstructed from balance sheet data—before being tested for systemic risk [26]. Here we use and generalize the approach of Cimini et al. [27] to generate (rather than reconstruct) financial networks compatible with balance sheet information. The method is based on a combination of Exponential Random Graphs [28, 29] and the fitness model [30] (see further details in [27, 31]).

First, we generate an unweighted directed graph by drawing each edge $i \to j$ independently with probability:

$$p_{i\to j} = \frac{zA_iL_j}{1 + zA_iL_j}, \tag{1}$$

where $z \in (0, \infty)$ is a parameter controlling for the density of the network. Indeed, since the values of assets and liabilities are given, this probability is an increasing function of z, hence the link density of the network is proportional to the parameter z. We consider also the possibility of having self-loops in the graph because some of the top European banks do represent banking group with internal flow of money. The alternative possibility would be to use the RAS algorithm to get rid of self loops [32].

We then assign a weight to each link, in accordance with the generated graph adjacency matrix $a_{i\to j}$, as follow:

$$w_{i\to j} = \frac{A_iL_j}{Wp_{i\to j}}\, a_{i\to j} \tag{2}$$

where $W = \sqrt{\left(\sum_i A_i\right)\left(\sum_j L_j\right)}$. The final result is a weighted directed network given by the corresponding adjacency matrix A whose entries are $A_{ij} = w_{i\to j}$. In the economic network literature this matrix is referred to as the asset matrix while its transpose is called the liability matrix.

The distribution of assets and liabilities across banks is heterogeneous, and with such an input our network construction method generates a core-periphery structure, independently on the network density. In order to get rid of this constraint, we introduce a generalization of Eq. (1):

$$p_{i\to j} = \frac{z\,(A_i L_j)^{\phi}}{1 + z\,(A_i L_j)^{\phi}}\,, \quad \phi \in [0,1]. \tag{3}$$

The new parameter ϕ allows to model a wide range of network topologies (for fixed z), including the fitness-induced configuration model and the Erdös-Rényi random graphs as the two limits $\phi = 1$ and $\phi = 0$, respectively.

C. Block Structure

The network generation method allows one to explore different network structures. To this end, we can further decompose the adjacency matrix A into blocks. Here, for the sake of simplicity, we shall restrict our consideration to the case in which there are only four blocks present in A as following

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

Each block A_{nm}, $n, m = \overline{1,2}$ represents a subgraph of the network in which the link density is characterized by z_{nm}, $n, m = \overline{1,2}$, and which thus is generated via Eqs (1)-(2) using that z_{nm}. Furthermore, among all possible topological configurations, there are three distinct ones that we shall focus on, namely the modular, the core-periphery and the bipartite-like structures.

Modular topology — In this case the network is clustered into two groups of nodes, with dense connections within the groups and sparse connections between them. This configuration corresponds to the choice of z_{11} and z_{22} both much larger than z_{12} and z_{21}. Without loss of generality, we implement the modular topology by setting $z_{11} = z_{22} = z$ and $z_{12} = z_{21} = \lambda z$, where $\lambda \in [0,1]$.

Bipartite-like topology — As an opposing configuration to the modular structure, one can consider the case in which the interconnections between the two communities dominate over the connections inside each community. The parameterization now is given by $z_{12} = z_{21} = z$ and $z_{11} = z_{22} = \beta z$, where $\beta \in [0,1]$.

Core-periphery topology — Of special interest in the investigation of financial networks is the core-periphery topology, in which there are two groups of banks (core and periphery) and a much higher link density in the first group than in the second group. As mentioned in the last section, the network generated by Eq. (1) inherently possesses a core-periphery structure. Therefore, any parameterization of the form $z_{12} = z_{21} = \gamma z$, $z_{22} = \gamma^2 z$ and $z_{11} = z$, where $\gamma \in [0,1]$, would show only small differences between the two extreme $\gamma = 0$ and $\gamma = 1$. This is why we need to use Eq. (3) to explore the transition between core-periphery and homogeneous topologies.

D. DebtRank

Once the network is constructed, we use the DebtRank to model the propagation of shocks in the network of banks [33, 34]. We consider the relative loss of equity $h_i(t)$ for bank i and the interbank leverage matrix $\Lambda_{ij}(t)$:

$$h_i(t) = \frac{E_i(0) - E_i(t)}{E_i(0)} \tag{4}$$

$$\Lambda_{ij}(t) = \begin{cases} \dfrac{A_{ij}(0)}{E_i(0)} & \text{if bank } j \text{ has not defaulted up to time } (t-1) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Assuming a loss given default of 100%, the dynamical equation for the relative equity $h_i(t)$ reads:

$$h_i(t+1) = \min\left[1, h_i(t) + \sum_{j=1}^{N} \Lambda_{ij}(t)[p_j^D(t+1) - p_j^D(t)]\right] \tag{6}$$

where $p_j^D(t) = h_j(t)e^{\alpha[h_j(t)-1]}$ is the probability of default of bank j at time t, and $\alpha \in (0, \infty)$ is a controlling parameter which allows to switch continuously from the linear DebtRank ($\alpha = 0$) [33] to the Furfine algorithm ($\alpha \to \infty$) [15].

The average relative equity loss at the end of the shock propagation dynamics is:

$$\overline{E}_{loss} = \sum_i \frac{[h_i(t) - h_i(1)]E_i(0)}{\sum_i E_i(0)} \tag{7}$$

where $E_i(0)$ is the initial equity of the bank i, and $h_i(1)$ is the initial shock on i. Hence $\overline{E}_{loss}$ does not account for the initial shock on the system, but only for the network effect on systemic losses.

Note that we use two kind of stopping conditions for simulations: either when the difference $\|[h(t) - h(t-1)]E(0)\|_2$ becomes smaller than a tolerance tol, or when the number of interactions is equal to max_{iter}. The value of the parameters tol and max_{iter} depends on the cases considered.

III. RESULTS

Our operative basic framework works as follows. We reconstruct the interbank network for years 2008 and 2013. We then run the stress test according to DebtRank shock propagation mechanism on the reconstructed network, and measure $\overline{E}_{loss}$. We do this for different network densities ρ. For each ρ, we sample 10 networks and calculate the average $\overline{E}_{loss}$ over these networks. In terms of initial shock, we consider an uniform shock by reducing the equity value of each bank by a fraction θ of its initial equity, which means $h_i(1) = \theta\ \forall i$. Figure 1 shows the result of this exercise for ρ ranges from 0 to 1, and θ ranges from 0 to 0 to 0.6.

First, we see from the figure that the $\overline{E}_{loss}$ increases monotonically with ρ. This implies that the network becomes more fragile when it becomes more dense. Second, in the 2008 data we find a very high value of $\overline{E}_{loss}$ also for very small θ: network amplification effects are so important that they can wipe out the whole system also when the initial perturbation is minimal. The decreasing of $\overline{E}_{loss}$ with θ is instead mainly due to the fact that initial shocks are not included in the computation of $\overline{E}_{loss}$, and indeed this quantity is bounded from above by $1 - \theta$. Finally, by comparing the 2008 data and the 2013 data, we find that the $\overline{E}_{loss}$ for every combination of ρ and θ has substantially changed from before to after the global financial crisis: the network is much more robust in 2013 than in 2008, especially for what concerns small shocks (and even in the high density regime).

Previously, we have looked at the case of linear DebtRank, which corresponds to the choice $\alpha = 0$ in Eq. (6). Here we do the analogous exercise, but looking at different values of α. In particular, we

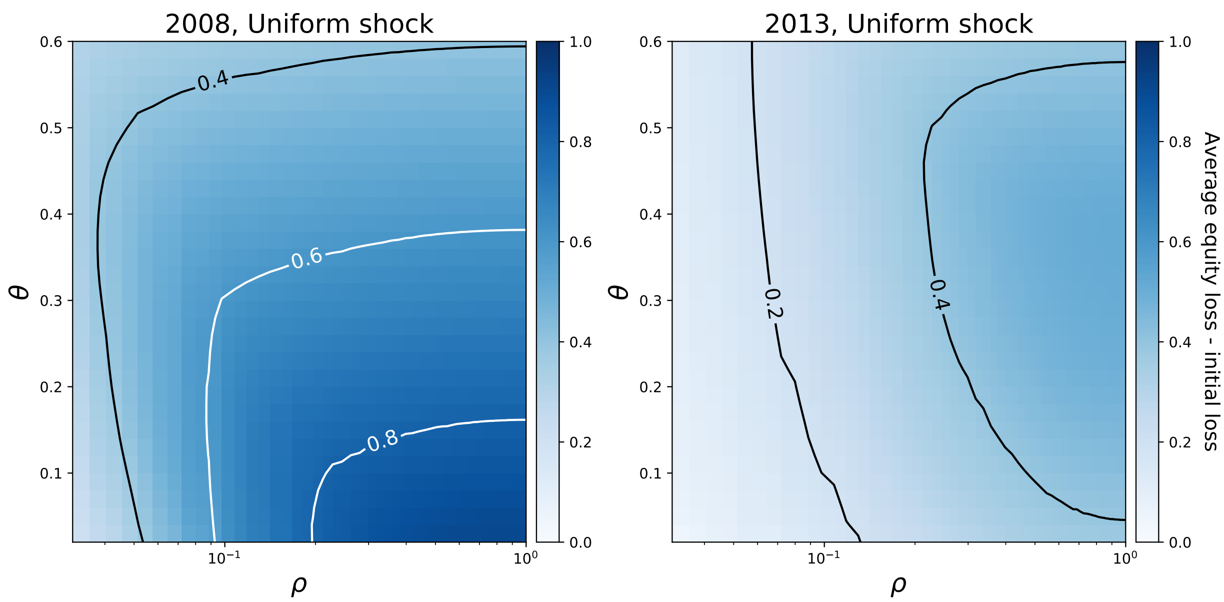

FIG. 1: $\overline{E}_{loss}$ as a function of the link density and the magnitude of the uniform shock for the 2008 (left) and the 2013 data (right). Darker (brighter) color refers to the higher (smaller) DebtRank, which corresponds to a more fragile (resilient) financial network.

are interested in the cases of DebtRank, Furfine, and the non-linear default probability in between. To this end, we look at the value of $\alpha = 0, 2, 3, 4, 5, 6, 7, \infty$. Figure 2 shows the results for this exercise. First, we look at the 2008 networks. In general, for small α we see that $\overline{E}_{loss}$ increases with ρ, and converges towards the highest value of $\overline{E}_{loss} = 1 - \theta$. In contrast, for large α equity losses due to the network remain very small also for very dense networks. The case $\alpha = 0.5$ marks a kind of transition between these two regimes: while for small ρ the behavior is similar to the case of large α, for large densities losses converge to the highest possible values. The increasing behavior of $\overline{E}_{loss}$ with ρ is generally observed also for 2013 networks. However, in this case the $\overline{E}_{loss}$ for completely connected networks does depend on the value of α, and $\alpha = 0.5$ still seems to mark the transition between systemic and localized losses.

Up to this point, we have looked at the case $\phi = 1$ where we use the fitness configuration model to reconstruct the interbank network. We now consider other values of ϕ. Figure 3 shows that network losses $\overline{E}^{*}_{loss}$ always increase with ϕ. This means that a network with a marked core-periphery structure is more fragile than a homogeneous network. Interestingly, even the initial default of a peripheral node propagates faster in the core-periphery structure compared to homogeneous case. This behavior is consistent for different network densities, and is similar in both 2008 and 2013, the only difference being the curve slope—which is lower in 2013, confirming the increased post-crisis stability.

We finally consider the modular and bipartite topologies. Figure 4 shows the results of an exercise in which we uniformly shock banks of the first community and measure $\overline{E}^{*}_{loss}$ for the second community. Here we see that the modular structure can be quite more resilient if the different blacks are scarcely connected. When we move to the bipartite case the behavior of systemic risk is similar, however it decreases when we move away from a pure bipartite structure. For a constant density, this is an effect of the increasing probability of a connection between two big institutions from different groups.

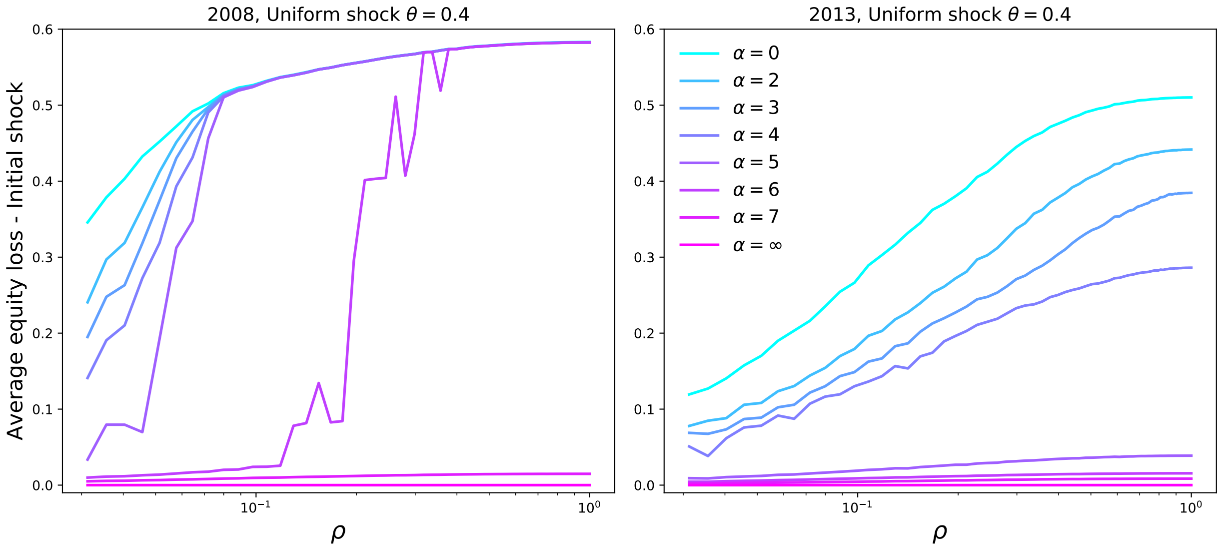

FIG. 2: $\overline{E}_{loss}$ as a function of the link density for a fixed value $\theta = 0.4$ of uniform initial shock, in the 2008 (left) and 2013 data (right). Different curves correspond to different magnitudes of non-linearity in the default probabilities entering in Eq. (6).

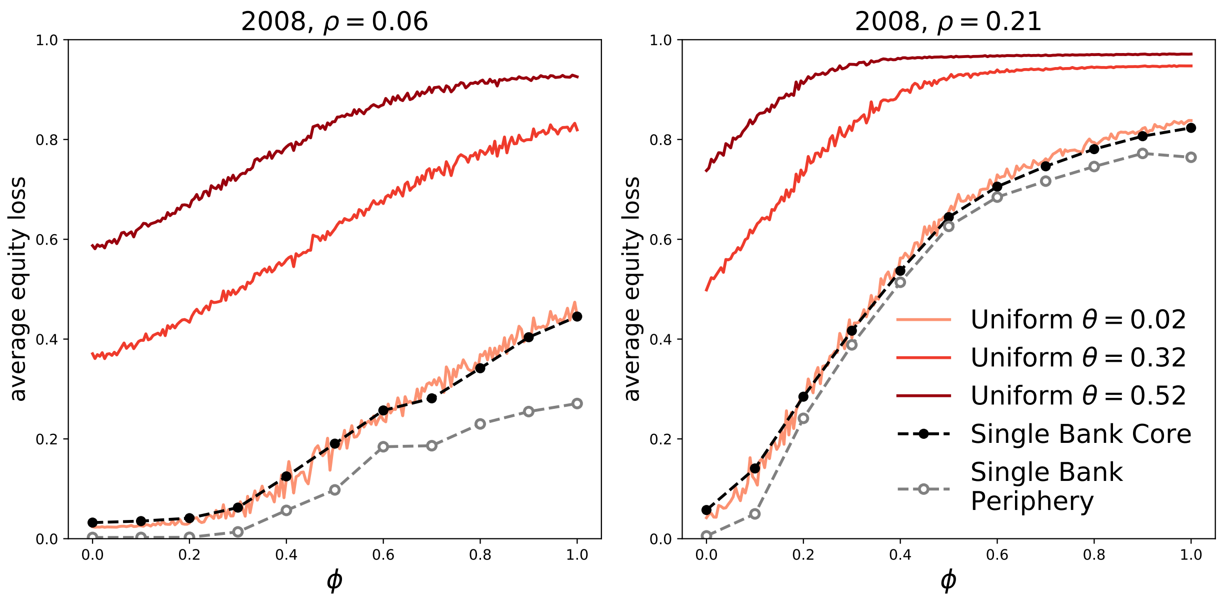

FIG. 3: $\overline{E}^*_{loss}$ as a function of the parameter ϕ tuning the strength of the core-periphery structure, for 2008 interbank networks. We consider fixed values of the density $\rho = 0.06$ (left) and $\rho = 0.21$ (right), and different initial shocks: either uniform for all banks, or consisting of a single initial default (in the core or in the periphery).

IV. DISCUSSION

We have examined different topological properties and shown how they affect the systemic risk. In addition, we also used a variety of shock types and changed the way they propagate across the network. These results provide additional evidence of how complex the interbank system is and

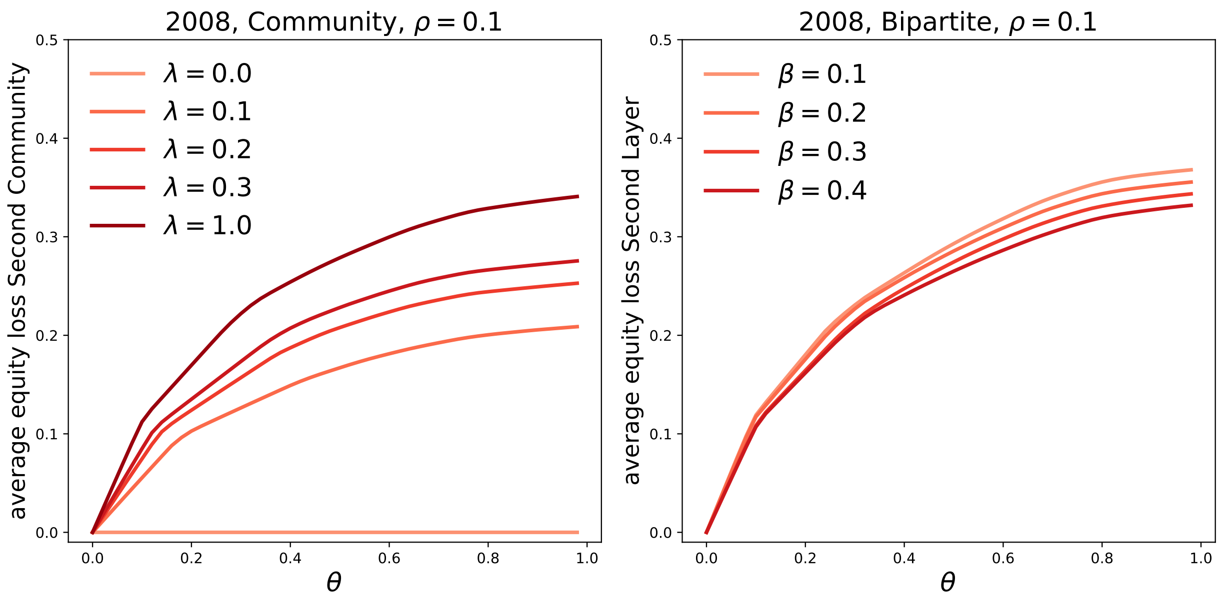

FIG. 4: $\overline{E}^*_{loss}$ as a function of the uniform initial shock θ, for 2008 interbank networks and the modular (left) and the bipartite (right) topological structure, with $\rho = 0.1$.

how many variables are involved in shaping its resilience.

In the simplest situation of a single-block network, systemic risk monotonically increases with the density, but there is a qualitative difference between the behavior observed in 2008 and 2013: the crisis shaped the interbank market in a way that afterwards it became much more robust to small shocks even in the high density regime.

Analysis of different shapes of the bank default probability further shows the differences between pre and post crisis. In 2008 there are basically two regimes of the shocks propagation, depending on the network density and the parameter α—which can be seen as the amount of confidence market participants have in the ability of counterparts to recover from equity losses. Indeed if the confidence in the system is not high enough, systemic losses become widespread, otherwise they remain small. On the other hand in 2013, even for low confidence levels, increasing the density does not lead to overall losses equal to what is observed for the linear case (that corresponding to the lowest level of trust in the counterpart). Note that these results were obtained with uniform shock in all equities. As shown in [18], if we consider the default of single banks as initial shocks, we expect that increasing the density can help the system to withstand the shock.

The method of network reconstruction described by Eq. (1) imposes, as a result of fat tail distribution of assets and liabilities, a core-periphery structure. In this scenario, the only topological parameter that can be used is the network density, which allows to switch from networks with a few contracts of large amounts to networks with many contracts of small amounts. However, by introducing the parameter ϕ in Eq. (3) we were also able to continuously change the network structure from a random one ($\phi = 0$) to a core-periphery one ($\phi = 1$). We then found that the core periphery structure is less resilient, even in the case of a single initial default. This confirms the well known observation that strongly connected nodes enhance the shock propagation.

In the last step of our analysis we looked at a block structure of a network. In particular, a modular structure can well represent a market of several countries, in which home and foreign connectivities are different. On the other hand, the bipartite case is a simple approximation of a bow-tie structure, which is often observed among financial networks. In both cases we were interested to see how a shock in one block propagates to the other.

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