Tools and Frameworks for Causal Inference

ECON 31720 Applied Microeconometrics

Francesco Ruggieri

The University of Chicago

September 30, 2020

Monte Carlo Simulations

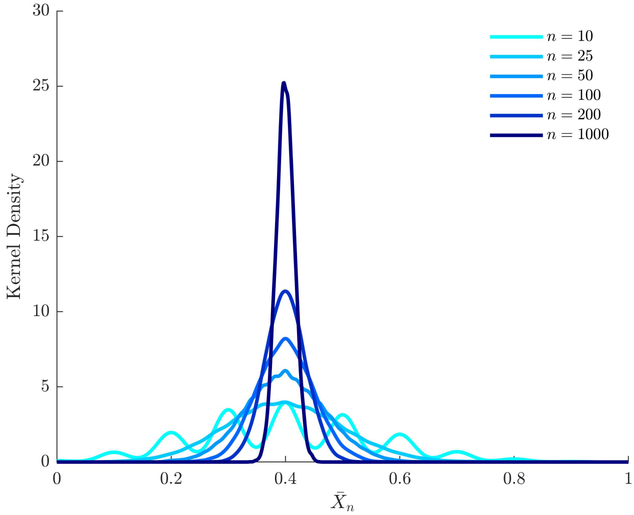
Monte Carlo experiments hinge on **repeated random sampling** to fulfill various goals:

- Provide a **numerical approximation to integrals** using empirical means
  - Applications: Method of Simulated Moments (MSM) and Maximum Simulated Likelihood (MSL)
  - The “quality” of the approximation increases with the number of simulations (via LLN)
- Investigate the **properties** of any statistical procedure via simulation

Monte Carlo Simulations: Implementation

1. Specify the number of Monte Carlo samples, $\boldsymbol{m}$, and the size of each sample, $\boldsymbol{n}$
2. Assume **knowledge of the joint distribution** of all random variables in the experiment
   - E.g., $Y = 5 + 1.6 \cdot \sin(X) \cdot \log(X) + R$, with $X \sim \mathcal{U}\,[0, 10]$, $R \sim \mathcal{N}\,(0, 9)$, and $X \perp\!\!\!\perp R$
3. For each one of the m Monte Carlo iterations:
   - **Simulate a $\boldsymbol{n}$-dimensional sample** from the joint distribution of random variables
     - E.g. $\{(Y_i, X_i, R_i)\}_{i=1}^{n}$ is an i.i.d. sample from the joint probability distribution of (Y, X, R)
   - Perform a **deterministic computation** on this sample
     - E.g. Perform local linear regression of Y on X with bandwidth $h = 0.5$, and store fitted values
4. Compute one or more **statistics** by averaging quantities across Monte Carlo samples
   - E.g. compute the Mean Squared Error of local linear regression fitted values

Monte Carlo Simulations: Example



Nonparametric Bootstrap

- A variant of Monte Carlo simulation that can be implemented with
  1. **Fewer parametric assumptions** on the Data Generating Process
  2. **Little additional code** beyond the one required to estimate the model in the first place
- Nonparametric bootstrap is typically used to compute analytically complex statistics
  - E.g. **standard errors**, when working with Continuous Mapping and Delta Method is hard
- It hinges on **sampling with replacement** from the empirical distribution of the data
  - Let $P = \{1, 2, 3, 4, 5\}$ be the population. Examples of 3-dimensional samples with replacement are $S_1 = \{4, 1, 2\}$, $S_2 = \{5, 2, 5\}$, and $S_3 = \{4, 4, 4\}$
- **Smoothness** conditions must be satisfied for nonparametric bootstrap to be valid
  - E.g. Bootstrap is invalid if applied to one-to-one propensity score matching (more later)

Nonparametric Bootstrap

- $X \in \mathbb{R}^{d_x}$ is a random vector distributed according to some **population cdf** F
- $\{X_i\}_{i=1}^n$ is a n-dimensional collection of draws from F (i.e., a **sample**)
- $\widehat{F}$ is the **empirical cdf** of X
    - This distribution places equal probability mass to each of the n draws, $\{X_i\}_{i=1}^n$
- θ is a parameter of interest that can be estimated by $\widehat{T} = T\left(\widehat{F}\left(X_1, \ldots, X_n\right)\right)$
- A **bootstrap sample** of size n, $\left\{X_i^b\right\}_{i=1}^n$, is a collection of **i.i.d.** draws from $\widehat{F}$
    - Sampling **with replacement** is necessary for $X_1^b, \ldots, X_n^b$ to be i.i.d.

Nonparametric Bootstrap

1. Specify the number of bootstrap samples $\overline{\boldsymbol{b}}$
2. Let $b \in \{1, 2, \ldots, \overline{b}\}$. For each bootstrap iteration:
   - Extract a **bootstrap sample** of size n, $\{X_i^b\}_{i=1}^n$
   - Perform a **deterministic computation** on this bootstrap sample
     - E.g. Regress X_1 on $X_2, \ldots, X_k$ and store the coefficient associated with X_2, i.e., $\widehat{\beta_2^b}$
3. Compute one or more **statistics** by averaging quantities across bootstrap samples
   - E.g. compute the standard error associated with $\widehat{\beta}_2$ as $\sqrt{\frac{1}{\overline{b}} \sum_{b=1}^{\overline{b}} \left(\widehat{\beta_2^b} - \frac{1}{\overline{b}} \sum_{j=1}^{\overline{b}} \widehat{\beta_2^j} \right)^2}$
   - As above, approximation quality increases with the number of bootstrap samples $\overline{b}$

Nonparametric Bootstrap

Object	Data	Bootstrap
Population Distribution	F	$\widehat{F}$
Sample	$\{X_i\}_{i=1}^n$	$\{X_i^b\}_{i=1}^n$ i.i.d.
Empirical Distribution	$\widehat{F}$	$\widehat{F}^b$
Parameter	$\theta = T(F)$	$\widehat{T} = T\left(\widehat{F}\right)$
Estimator	$\widehat{T} = T\left(\widehat{F}\right)$	$\widehat{T}^b = T\left(\widehat{F}^b\right)$

Source: Lecture notes by Charles J. Geyer

Frameworks for Causal Inference

Two main frameworks for causal inference:

1. **Latent variables**, or **all-causes**, model:

$$Y = g\left(D, U\right)$$

   where D and U denote the **observed** and **unobserved** determinants of Y, respectively. Together, D and U **exhaustively** cause the outcome.

2. **Potential outcomes** model:

$$Y = \sum_{d \in \mathcal{D}} Y(d) \mathbb{I}\left[D = d\right]$$

   where $Y(d)$ is the counterfactual Y associated with treatment state d.

Example: from Potential Outcomes to All-Causes

- Consider a binary treatment $D \in \{0, 1\}$
- Assume that D and Y are linked by **potential outcomes** $Y(0)$ and $Y(1)$
- Let us derive a **linear all-causes model** from the potential outcomes model:

$$
\begin{aligned}
Y &= DY(1) + (1 - D)Y(0) \\
&= \underbrace{\mathbb{E}\left[Y(0)\right]}_{\equiv \alpha} + \underbrace{\left(Y(1) - Y(0)\right)}_{\equiv \beta} D + \underbrace{Y(0) - \mathbb{E}\left[Y(0)\right]}_{\equiv U} \\
&= \alpha + \beta D + U \equiv g\left(D, U\right)
\end{aligned}
$$

- β could be assumed to be a **deterministic constant** (homogeneous treatment effects) or a **nondegenerate random variable** (heterogeneous treatment effects)

Example: from All-Causes to Potential Outcomes

- Maintain the assumption that the treatment is binary
- Assume a **linear all-causes model** of the observed and unobserved determinants of Y,

$$Y = \alpha + \beta D + U$$

- To derive a **potential outcomes model**, it is sufficient to define

$$Y(0) \equiv g(0, U) = \alpha + U$$
$$Y(1) \equiv g(1, U) = \alpha + \beta + U$$

- Both $Y(0)$ and $Y(1)$ are random variables because U is a random variable
- In addition, β may be a random variable if the effect of D is heterogeneous

Unobserved Determinants of the Outcome vs. Linear Regression Residual

- In general, the conditional expectation function is **nonlinear**:

$$\mathbb{E}[Y|D] = h(D, \theta)$$

where $h(\cdot)$ is some function of a random vector D and a parameter vector θ

- $h(D, \theta)$ is typically unknown but can be approximated:
  - A convenient approximation choice is the "best" **linear approximation**

$$\beta^* \in \arg\min_{\beta \in \mathbb{R}^{d_d}} \mathbb{E}\left[\left(\mathbb{E}[Y|D] - D'\beta\right)^2\right] \tag{BLA}$$

  - This minimization problem is equivalent to the **linear prediction** problem

$$\beta^* \in \arg\min_{\beta \in \mathbb{R}^{d_d}} \mathbb{E}\left[\left(Y - D'\beta\right)^2\right] \tag{BLP}$$

Unobserved Determinants of the Outcome vs. Linear Regression Residual

- The first-order necessary conditions associated with both minimization problems are

$$\mathbb{E}[D\,(Y - D'\beta^*)] = \mathbb{E}[DU] = 0$$

where $U \equiv Y - D'\beta^*$ is a **statistical residual**

- U captures the **"quality"** of the linear approximation to $\mathbb{E}[Y|D] = h\,(D, \theta)$

- Being a statistical residual, U has **no causal interpretation**

- Analogously, β^* is the solution to a Mean Squared Error minimization problem

Unobserved Determinants of the Outcome vs. Linear Regression Residual

- Consider the linear causal model

$$Y = D'\beta + U$$

- If U were interpreted as encompassing the **unobserved determinants** of Y, then
  - $\mathbb{E}[DU] = 0$ would imply that observed and unobserved determinants of Y are **linearly unrelated**
  - This is not a statistical property, but a causal one, and its credibility is **assessed subjectively**

- The first part of this course will be devoted to studying cases in which $\boldsymbol{\beta^* = \beta}$
  - Under selection on observables, β^* identifies only some components of β