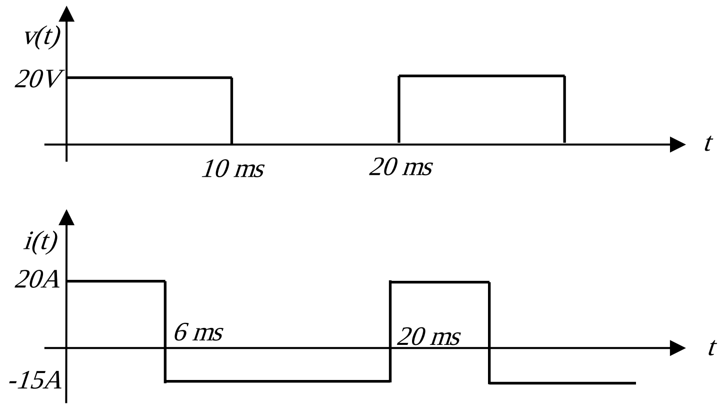
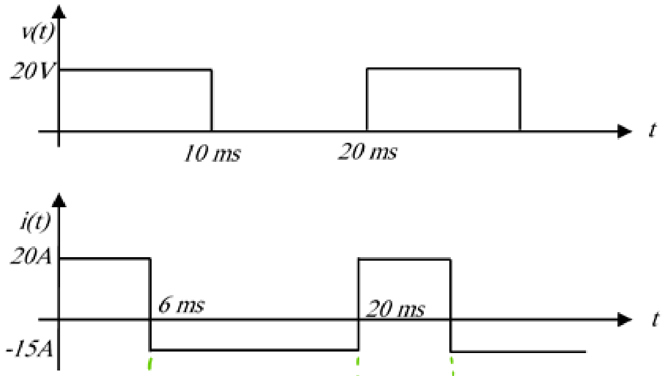
Tutorial 1 - Solutions

Problem 1:

Voltage and current for a device are shown in figures below.

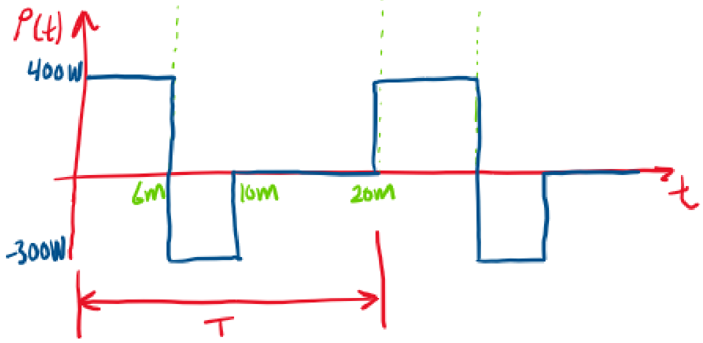
a. Determine the instantaneous power absorbed by the device
b. Determine the energy absorbed by the device in one period
c. Determine the average power absorbed by the device





Solution:

a)

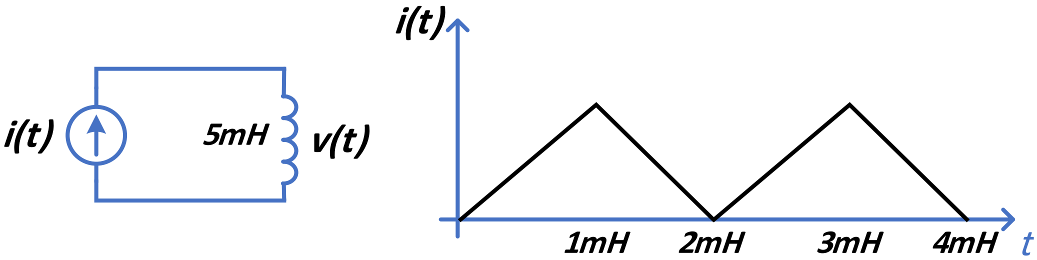


b)
$$E = \int_0^T p(t)\,dt$$
$$= \int_0^{6m} 400\,dt + \int_{6m}^{10m} -300\,dt$$
$$= 400\,t\Big]_0^{6m} - 300\,t\Big]_{6m}^{10m}$$
$$= 400 \times 6\times 10^{-3} - 300 \times (10-6)\times 10^{-3}$$
$$\therefore E = 2.4 - 1.2 = 1.2\,J$$

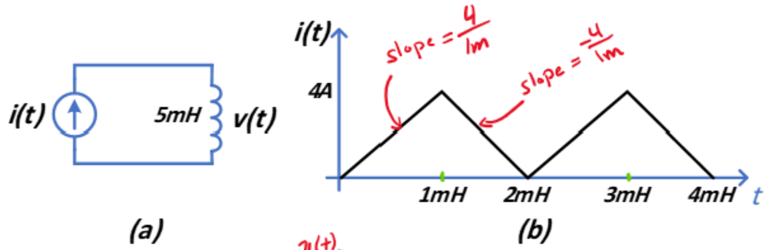
c)
$$P = \frac{1}{T}\underbrace{\int_0^T p(t)\,dt}_{E}$$
$$P = \frac{1}{T}\times E = \frac{1}{0.02}\times 1.2 = 60W$$

<u>Problem 2:</u>

The current in a 5 mH inductor in Figure (a) is the periodic triangular wave shown in Fig (b). Determine the voltage, instantaneous power and average power for the inductor.



(a) (b)



Solution:

The voltage across the inductor

$v_L = L \frac{di}{dt}$

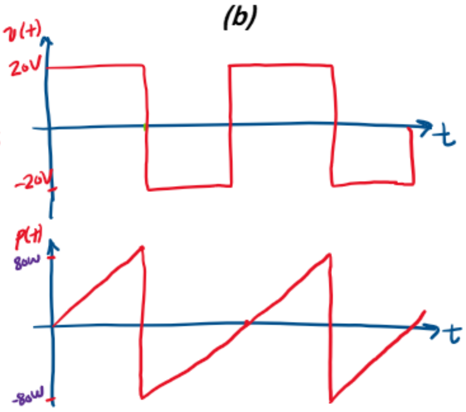
5mH

slope from i(t)

$\therefore v_L = 5m \times \frac{4}{1m}$

$= 20V$

$p(t) = v(t) \times i(t)$



$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = 0$ because of symmetry on the +ve and -ve sides.

$P_{avg} = \frac{1}{T}\int_0^T v(t) \times i(t)\,dt = 0$ because of symmetry on the +ve and -ve sides.

Conclusion / The ideal inductor is a lossless component

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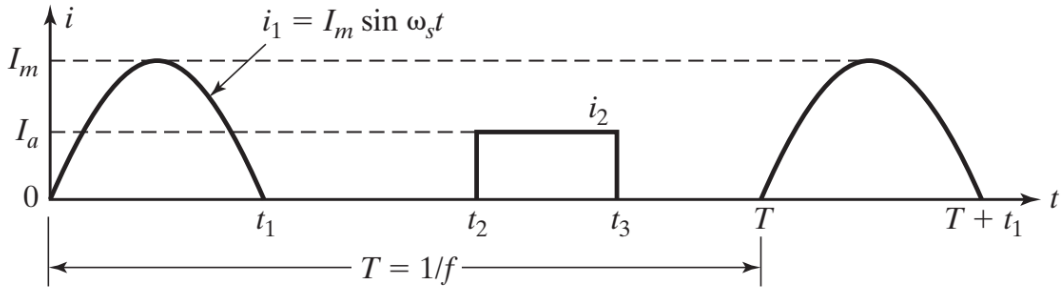
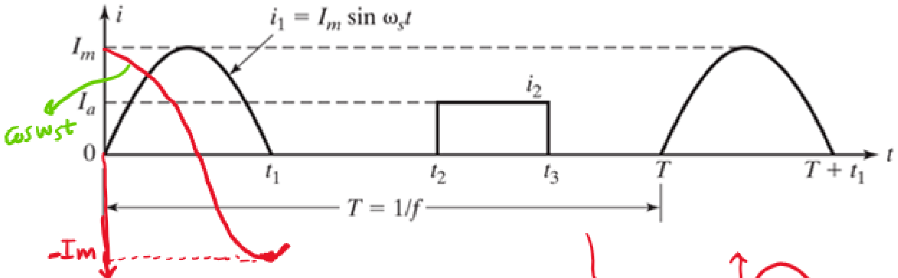
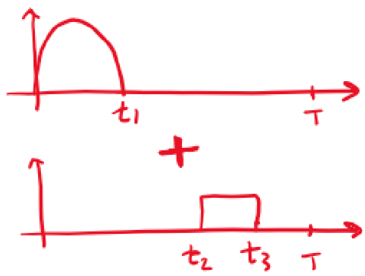
Problem 3:

The current through a diode is shown in the Figure below. Determine (a) the average current and (b) the rms diode current if t_1 = 100 μs, t_2 = 350 μs, t_3 = 500 μs, f = 250 Hz, f_s = 5 kHz, I_m = 450 A, and I_a = 150 A.







Solution:

a) $AVG = \frac{1}{T}\int_0^T i(t)\, dt$

$$I_{AVG} = \frac{1}{T}\left[\int_0^{t_1} I_m \sin w_s t\, dt + \int_{t_2}^{t_3} I_a\, dt\right]$$

$$= \frac{1}{T}\left[\left[-I_m \frac{\cos w_s t}{w_s}\right]_0^{t_1} + \left[I_a t\right]_{t_2}^{t_3}\right]$$

$$= \frac{1}{T}\left[-I_m\left(\frac{\overset{-1}{\cancel{\cos w_s t_1}} - 1}{w_s}\right) + I_a(t_3 - t_2)\right]$$

$$= \frac{1}{T}\left[\frac{2 I_m}{w_s} + I_a(t_3 - t_2)\right]$$

$2\pi f_s$

$$\therefore I_{AVG} = 250\left[\frac{2 \times 450}{2\pi \times 5k} + 150(500\mu - 350\mu)\right]$$

$$\boxed{I_{AVG} = 12.79\ A}$$

b) $I_{rms} = \sqrt{I_{rms_1}^2 + I_{rms_2}^2}$

$I_{rms_1} = I_{pk}\sqrt{\frac{\delta}{2}}$ (from slides)

$$= I_m \sqrt{\frac{t_1}{2T}}$$

$$= 450\sqrt{\frac{100\mu}{2 \times \frac{1}{250}}}$$

$$= 450\sqrt{\frac{100\mu \times 250}{2}}$$

$$= 50.31\,A$$

$$I_{rms2} = I_p\sqrt{\delta} \quad \left(\text{from slides}\right)$$

$$= I_a\sqrt{\frac{t_{on}}{T}}$$

$$= I_a\sqrt{\frac{(t_3 - t_2)}{T}}$$

$$= 150\sqrt{\frac{(500\mu - 350\mu)}{1/250}}$$

$$= 29.05\,A$$

$$\therefore\ I_{rms} = \sqrt{(50.31)^2 + (29.05)^2}$$

$$\boxed{I_{rms} = 58.09\,A}$$

Problem 4:

A 1kW heater load in a food processing system is supplied by an 86% efficiency converter which costs a £100. A higher efficiency (96.8%) converter is offered by you to the system technical team with a cost of £200. The team claims that 10% more efficiency needs 10 years to be paid back …. If the energy cost is £0.1/kWh, discuss the correctness of their claim with cost feasibility of your offer.

Solution:

$\eta = 86\%$, $P_o = 1000W$

$\therefore P_{in} = \frac{P_o}{\eta} = \frac{1000}{0.86} = 1162.8\,W$

and $P_{loss} = P_{in} - P_o = 162.8\,W$

Energy loss [kWh] $= 162.8\,W \times 1h$
$= 162.8\,Wh$
$= 0.1628\,kWh$

cost of Loss/Year $= 0.1628\,kWh \times 24 \times 365 \times £0.1$
$= 1426.13 \times £0.1$
$= £142.6/year$

Then in two year times, the loss costs £285.2 which is more than the new converter cost.

For the new converter $\eta = 96.8\%$

$\therefore P_L = 33.06\,W$

Energy loss $= 33.06\,Wh = 0.033\,kWh$

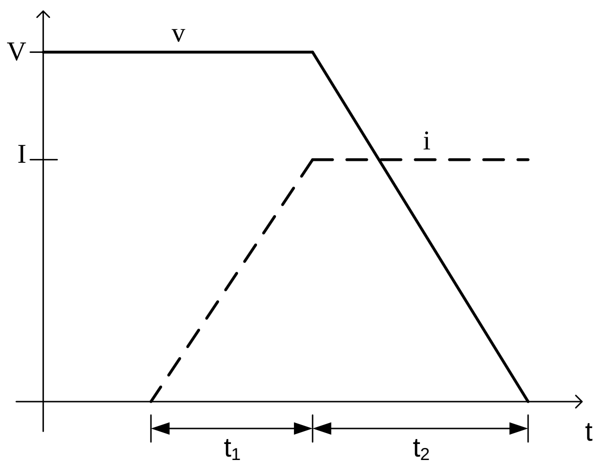
Loss cost/year $= 0.033\,kWh \times 24 \times 365 \times £0.1$
$= £28.9$

In two years times: Total costs $= 200£\,[\text{new conv.}] + 28.9 \times 2\,[\text{loss}]$
$= £257.8$

$\therefore$ The Pay back will be within two years and not 10 years.

Problem 5:

The figure below shows the voltage across a power semiconductor device and current through the device during a switching transition (turn on transition or turn off transition). What is the energy lost during the transition?



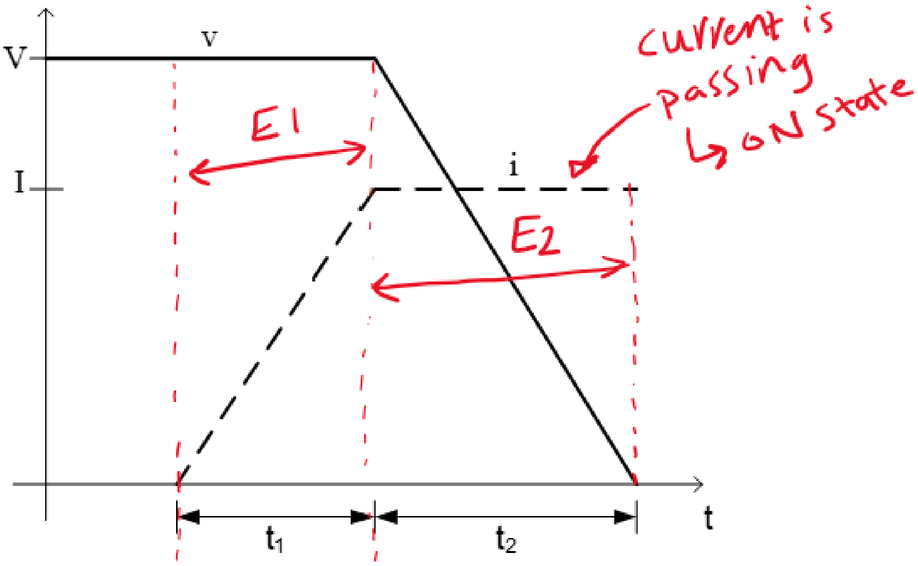
(a) Turn on, $\frac{VI}{2}(t_1 + t_2)$
(b) Turn off, $VI(t_1 + t_2)$
(c) Turn on, $VI(t_1 + t_2)$
(d) Turn off, $\frac{VI}{2}(t_1 + t_2)$

a. Turn on, $\frac{VI}{2}(t_1+t_2)$ ✓

b. Turn off, $VI(t_1+t_2)$ ✗

c. Turn on, $VI(t_1+t_2)$ ✗

d. Turn off, $\frac{VI}{2}(t_1+t_2)$ ✗



Solution:

$$E_1 = \int_0^{t_1} v(t)\, i(t)\, dt$$

$$v(t) = V$$

$$i(t) = \frac{I}{t_1} t$$

$$\therefore E_1 = \int_0^{t_1} \frac{VI\,t}{t_1} dt$$

$$= \frac{VI}{t_1} \left.\frac{t^2}{2}\right]_0^{t_1}$$

$$= \frac{VI}{t_1} \frac{t_1^2}{2}$$

$$\boxed{E_1 = \frac{VI\,t_1}{2}}$$

$$E_2 = \int_0^{t_2} v(t)\, i(t)\, dt$$

$$v(t) = -\frac{V}{t_2} t + V$$

$$i(t) = I$$

$$\therefore E_2 = \int_0^{t_2} I V\left(-\frac{t}{t_2}+1\right) dt$$

$$= VI\left[-\frac{t^2}{2t_2} + t\right]_0^{t_2}$$

$$= VI\left[-\frac{t_2^{\cancel{2}}}{2\cancel{t_2}} + t_2\right]$$

$$= \frac{VI\,t_2}{2}$$

$$\therefore \boxed{E_2 = \frac{VI\,t_2}{2}}$$

$$\therefore \text{Total } E = E_1 + E_2 = \frac{VI\,t_1}{2} + \frac{VI\,t_2}{2}$$

$$\boxed{E = \frac{VI\,(t_1+t_2)}{2}}$$