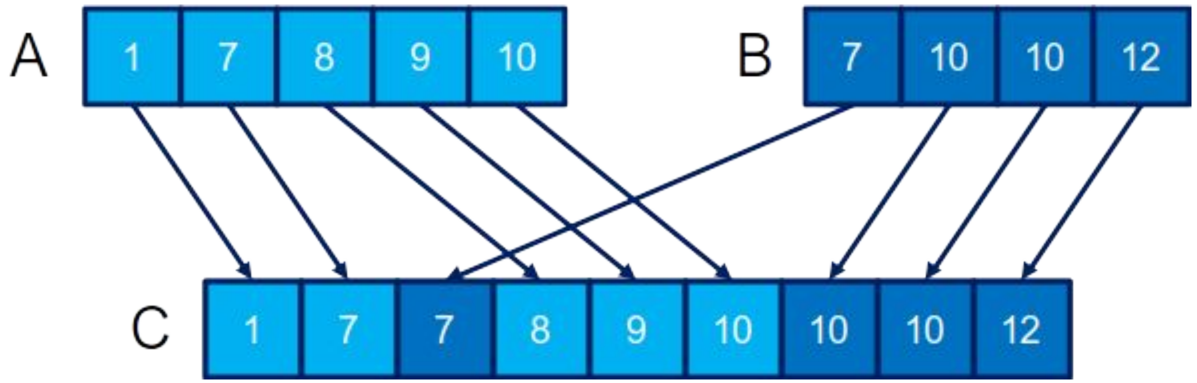
Merge

Dynamic input data identification

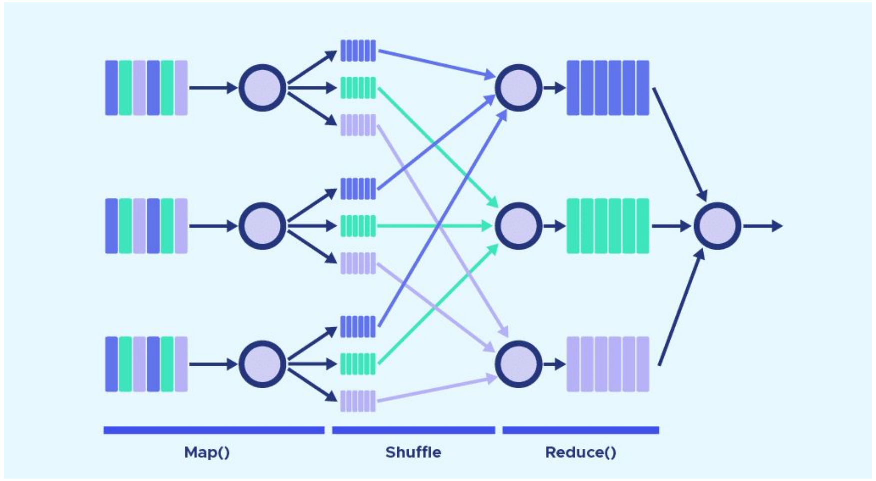
Merge Operation

- It takes two sorted lists and generates a combined sorted list
  - An ordered merge function takes two sorted lists, A and B, and merges them into a single sorted list, C
  - There is an order relation (e.g., less than or equal to) in the sorted lists and the merged list
- Focus on Stable Sort
  - Whenever the numerical values are equal between an element of A and an element of B, the element of A should appear first in the output list C
  - This requirement ensures the stability of the ordered merge operation.



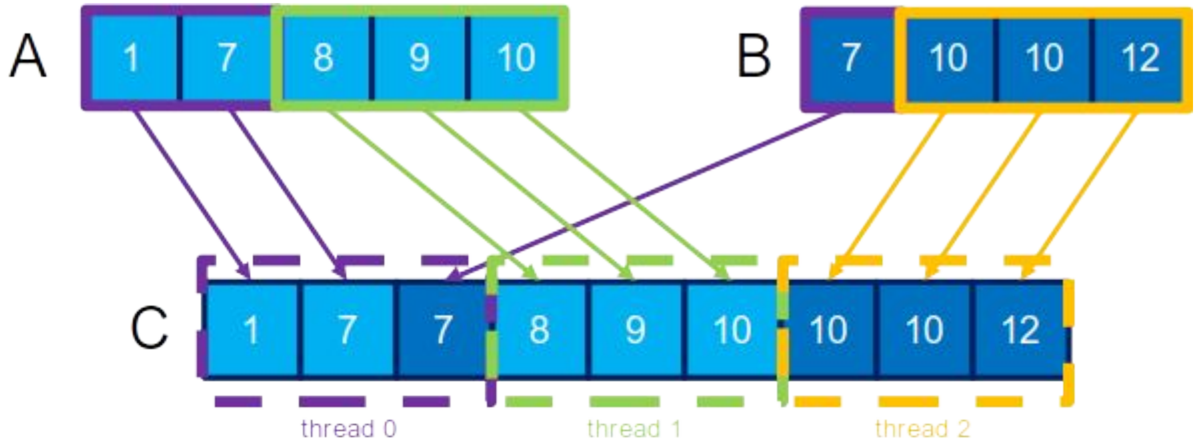
Why does it matter?

- It is a building block of modern mapreduce frameworks
  - For example, Hadoop distributes the computation over a large number of nodes
  - The reduce process assembles the result of these compute nodes into the final result
  - Results may be sorted according to an ordering relation.



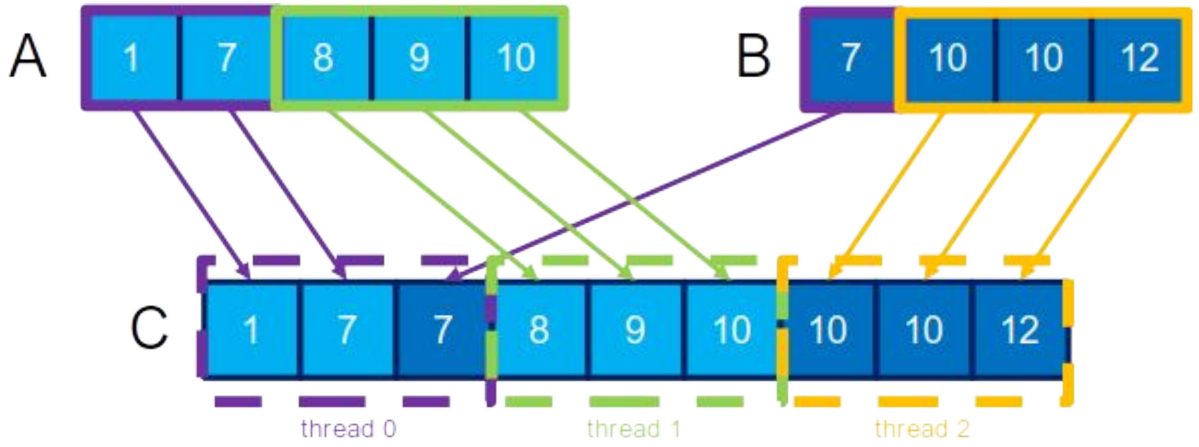
Naive Implementation - 1

- Sequential approach has $O(m+n)$ complexity
  - Where m and n are A and B dimensions
- <u>We can gather the merge computation among threads</u>
  - We partition the output list equally among threads
  - Each thread collects input elements for its section of the output
  - The range of input elements to be used by each thread is a function of the input elements



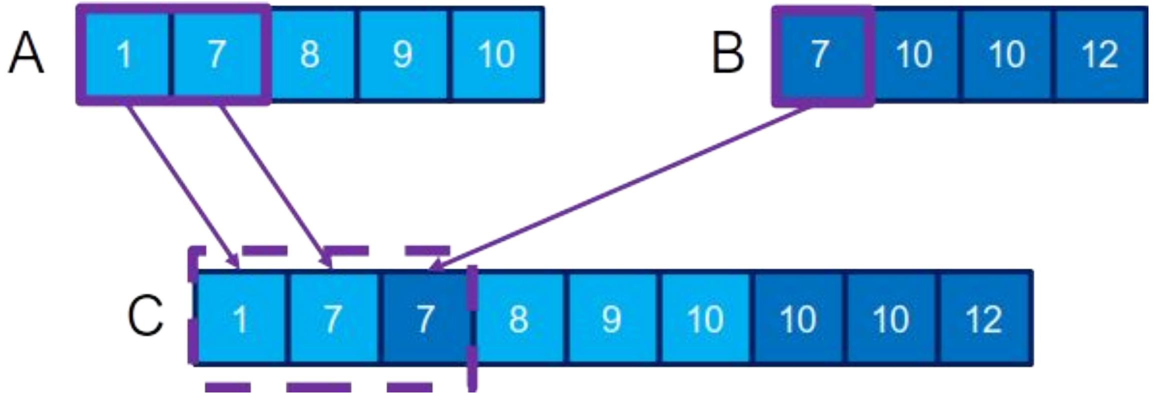
Naive Implementation - 2

- We can gather the merge computation among threads
  - Each output section receives its elements from a continuous section of A and B
- All threads identify the starting and ending locations of the continuous sections of the inputs (A and B) that they will use
  - All threads perform merge for their sections in parallel
  - Each thread executes a sequential merge for its own section

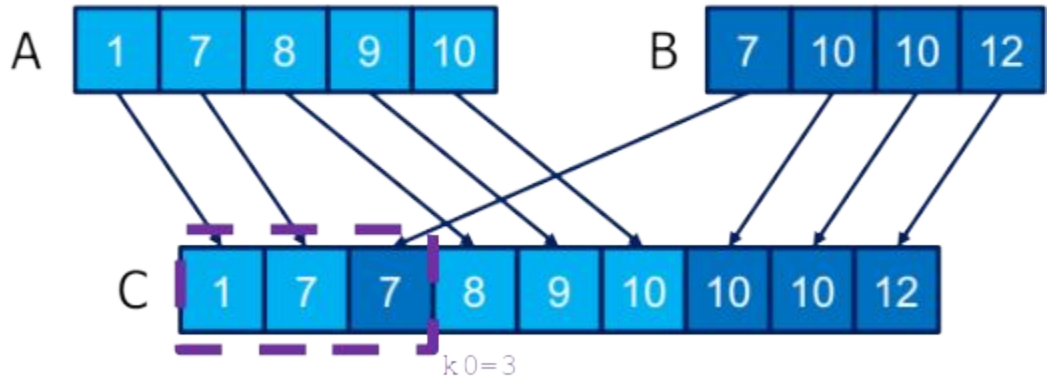


Co-Rank Calculation

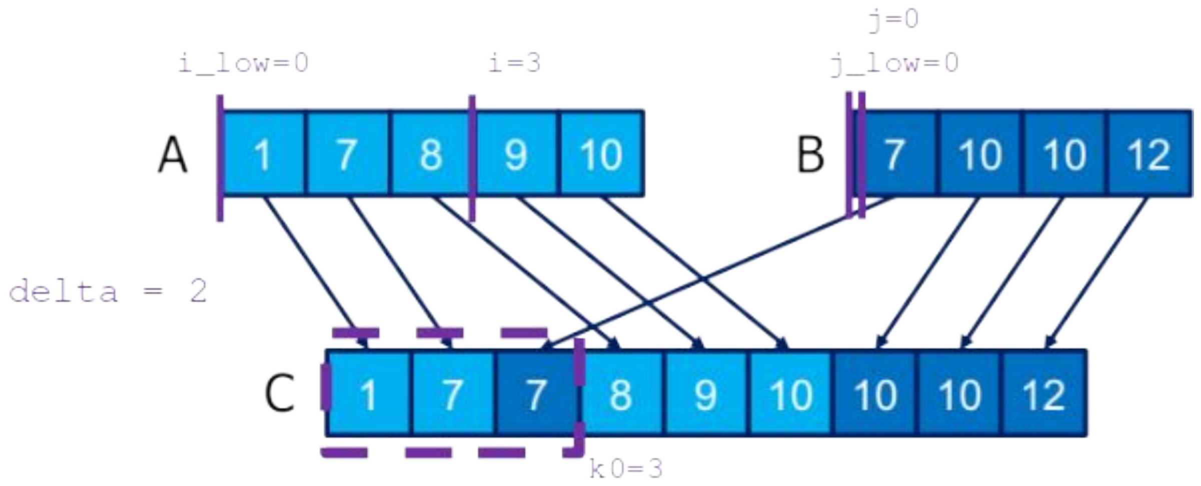
- For an element $C[k]$, k is referred to as its rank, and i and j are referred to as its co-ranks
  - For any k such that $0 \le k < m+n$, we can find i and j such that $k=i+j$, $0 \le i < m$ and $0 \le j < n$
    - $A[i-1] \le B[j]$
    - $B[j-1] < A[i]$
- Finding co-rank values for different threads is not balanced
- Use binary search for i and j values
  - Complexity of $O(\log N)$



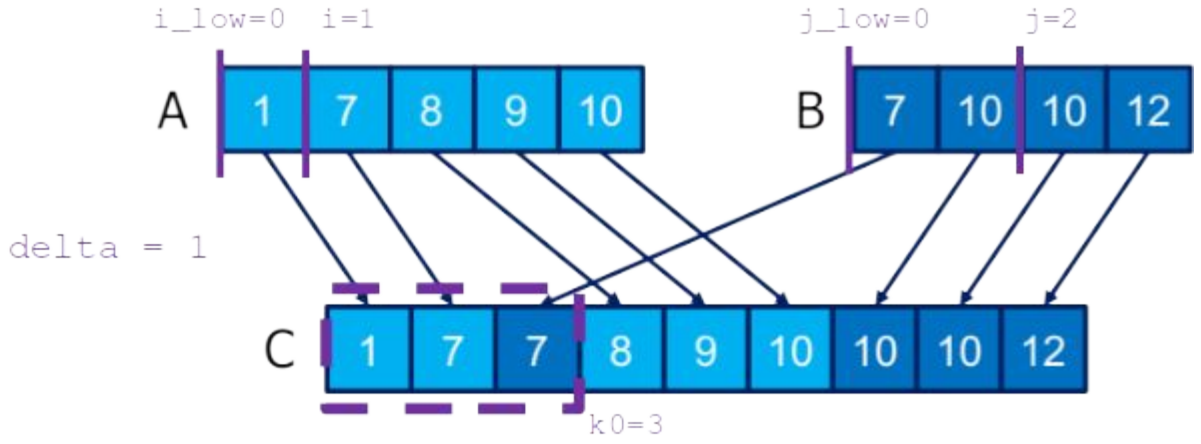
Co-Rank Example



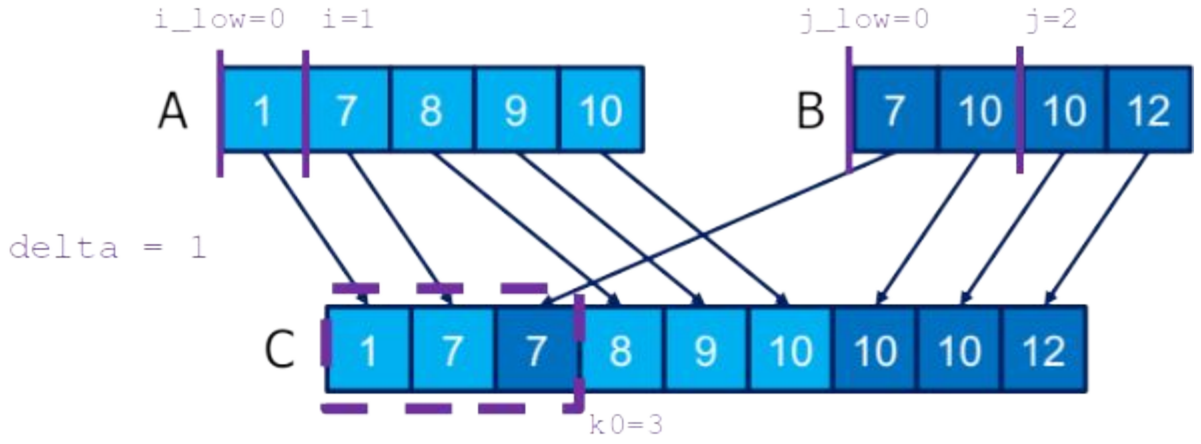
Co-Rank Example



Co-Rank Example

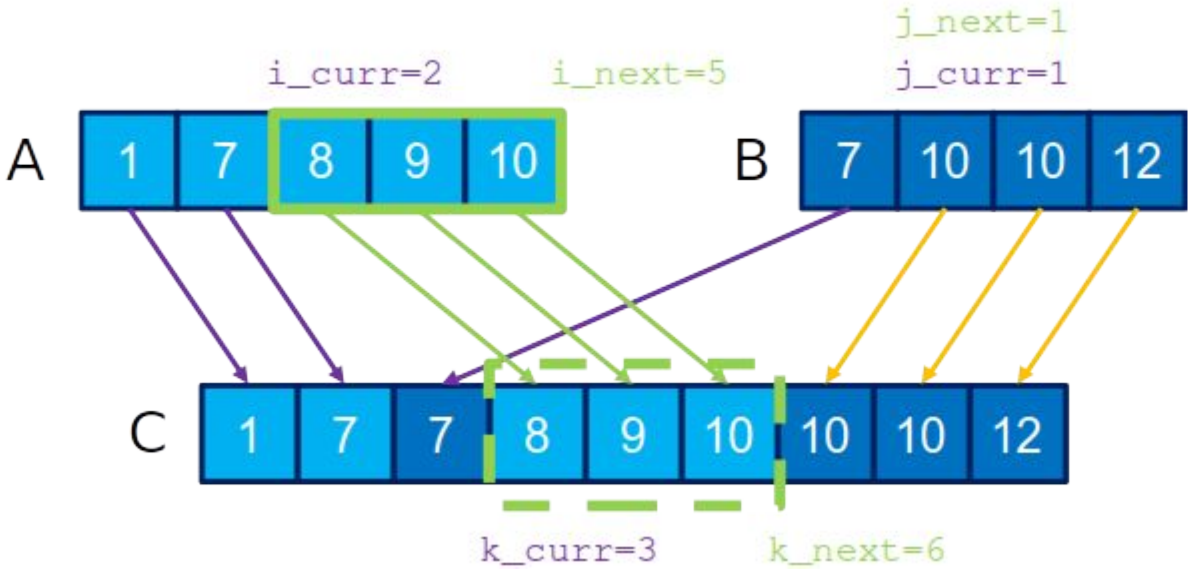


Co-Rank Example

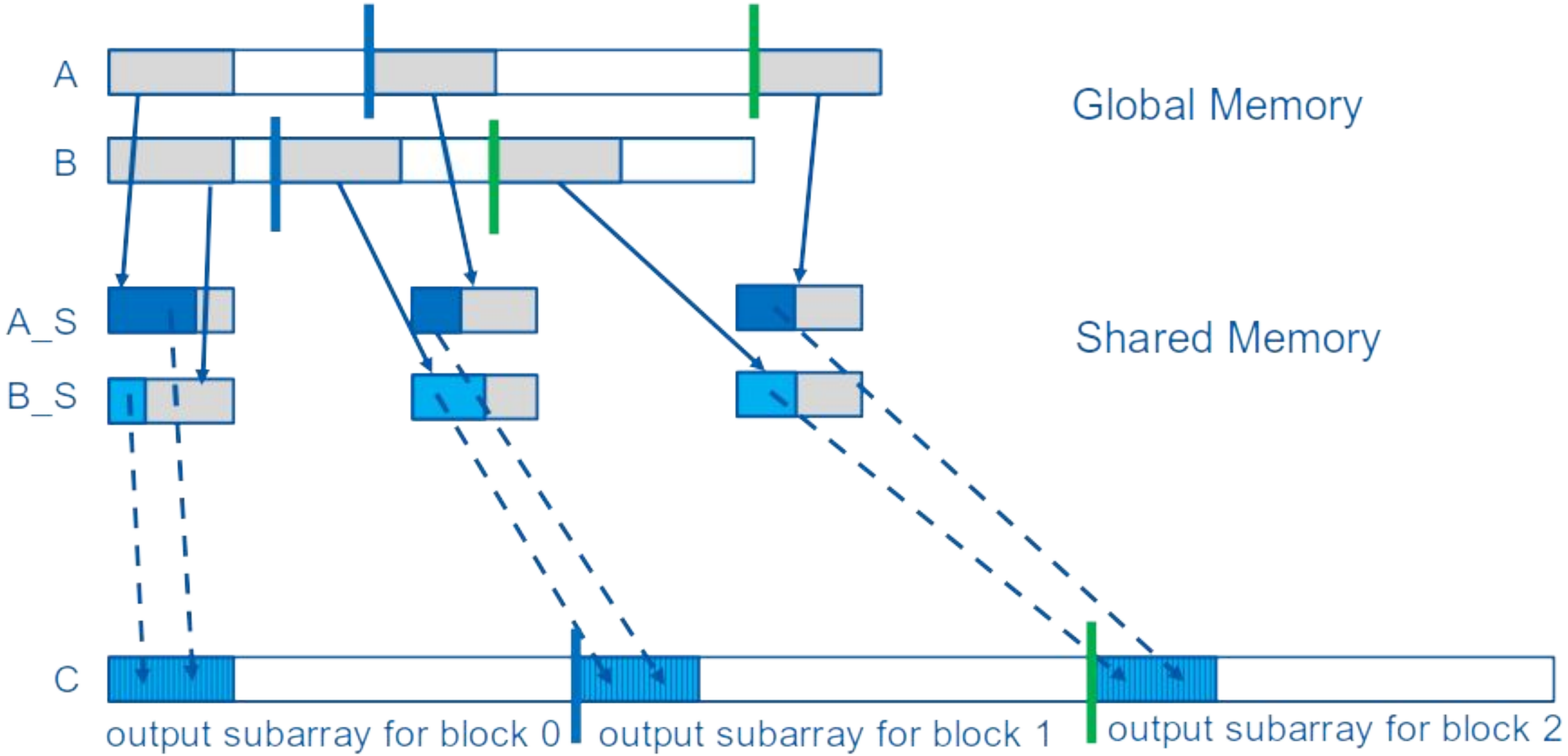


Parallel Merge

- Each thread is in charge of a continuous section of the output
- Problem
  - Uncoalesced memory accesses
  - We can improve the solution by collaborative loading sections of A and B into shared memory



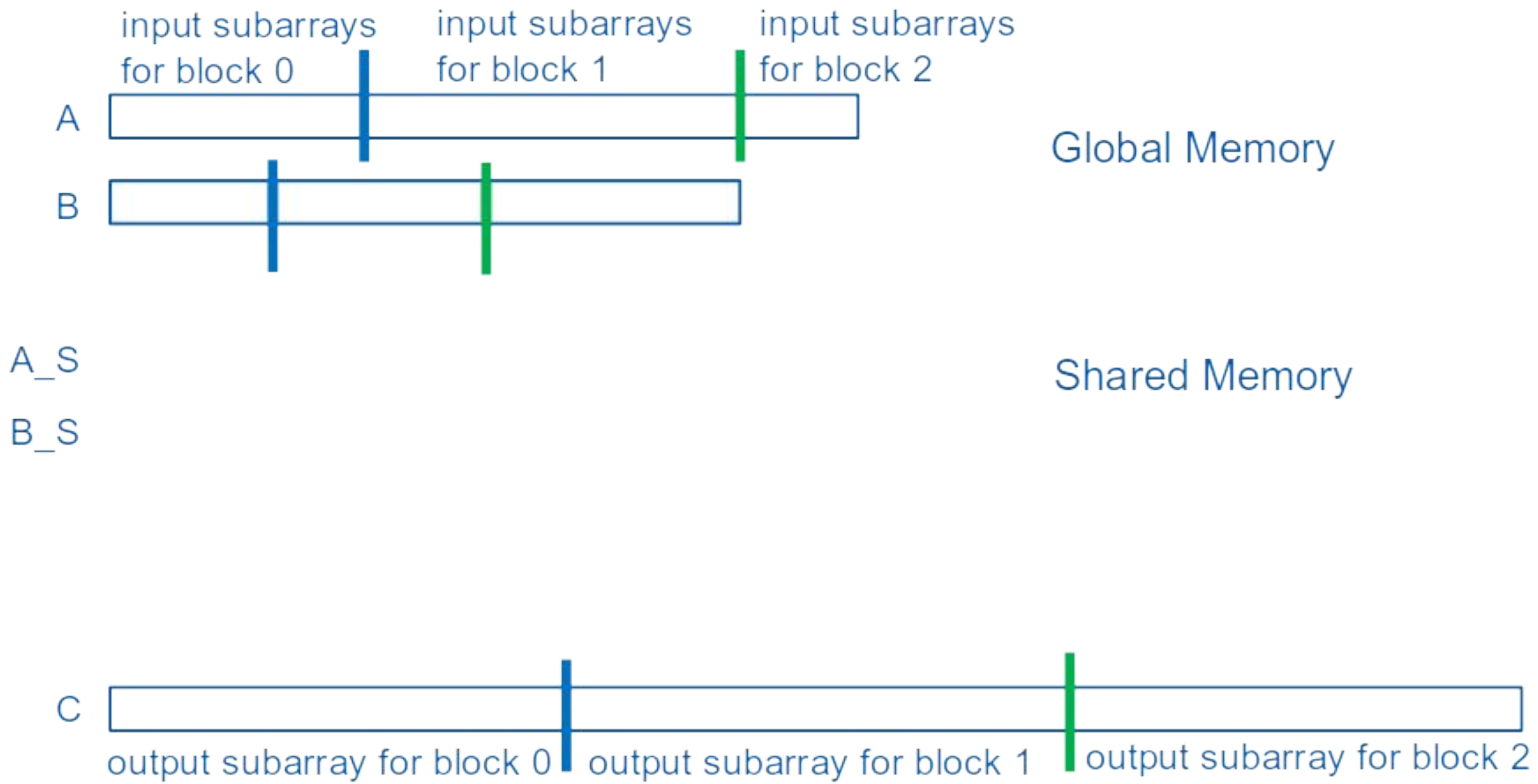
Tiled Merge


A
B
A_S
B_S
C
Global Memory
Shared Memory
output subarray for block 0
output subarray for block 1
output subarray for block 2

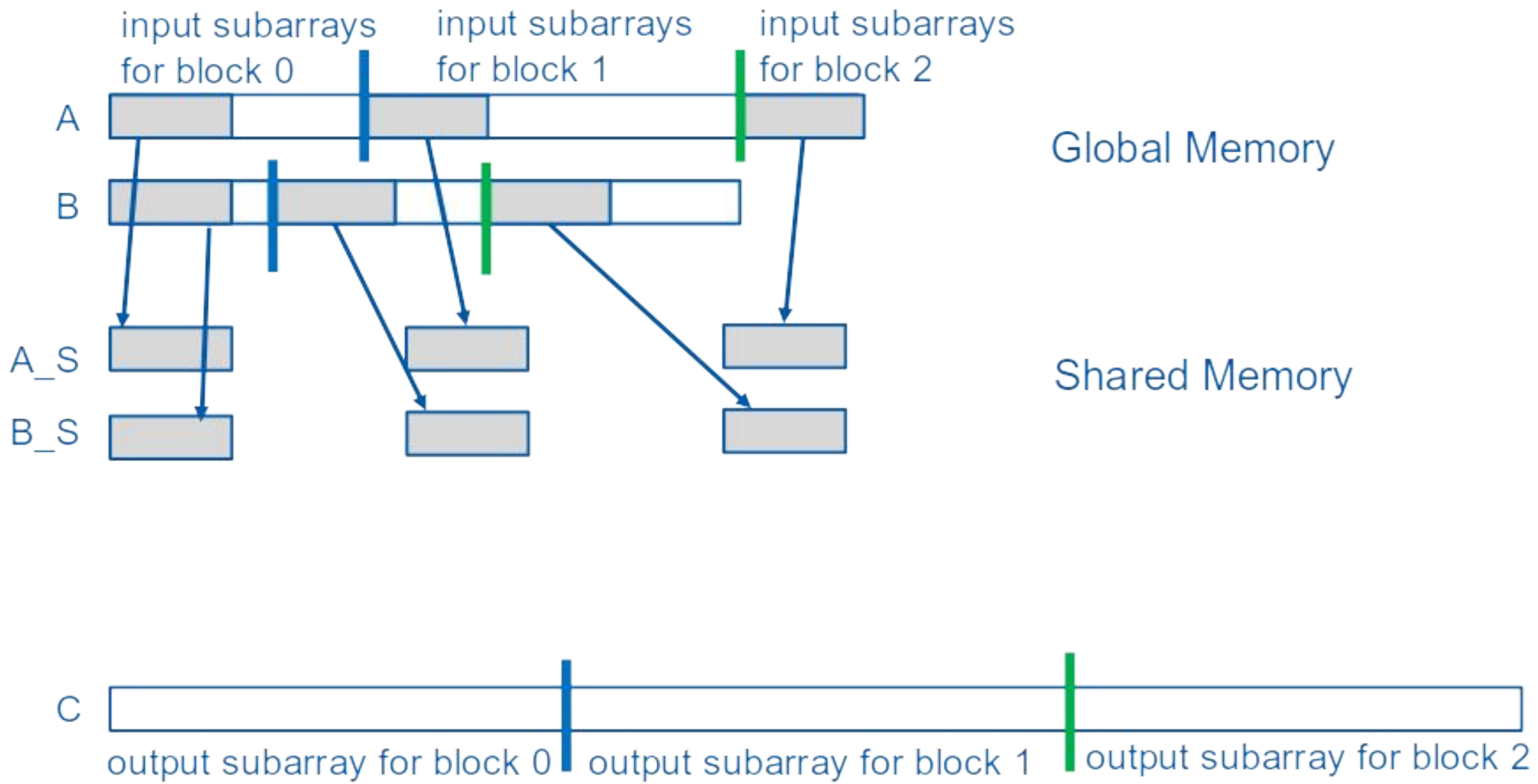
Tiled Merge

- Each thread block computes an output sections
  - It should be a multiple of the *TILE_SIZE* in the exercise
- The leader thread performs the binary search to identify the input sections
- Each block iteratively generates its output section; at each iteration
  - Threads of a block collaboratively load a tile of A and a tile of B into shared memory
  - Divide the output tile into subsections and assign them to threads
  - All threads perform parallel merge on input and output tiles

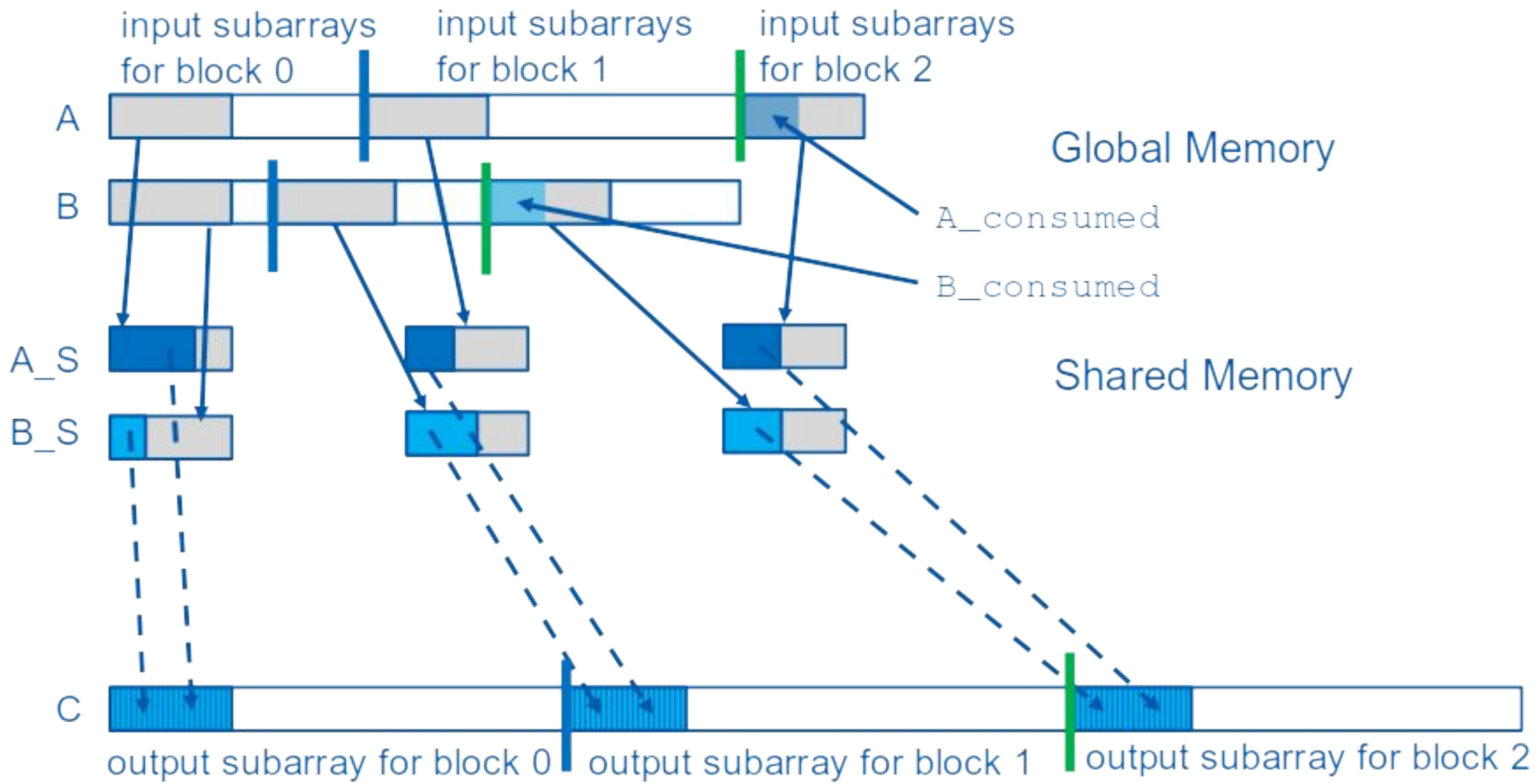
Tiled Merge Example (Iteration 0)


input subarrays for block 0
input subarrays for block 1
input subarrays for block 2
A
B
Global Memory
A_S
B_S
Shared Memory
C
output subarray for block 0
output subarray for block 1
output subarray for block 2

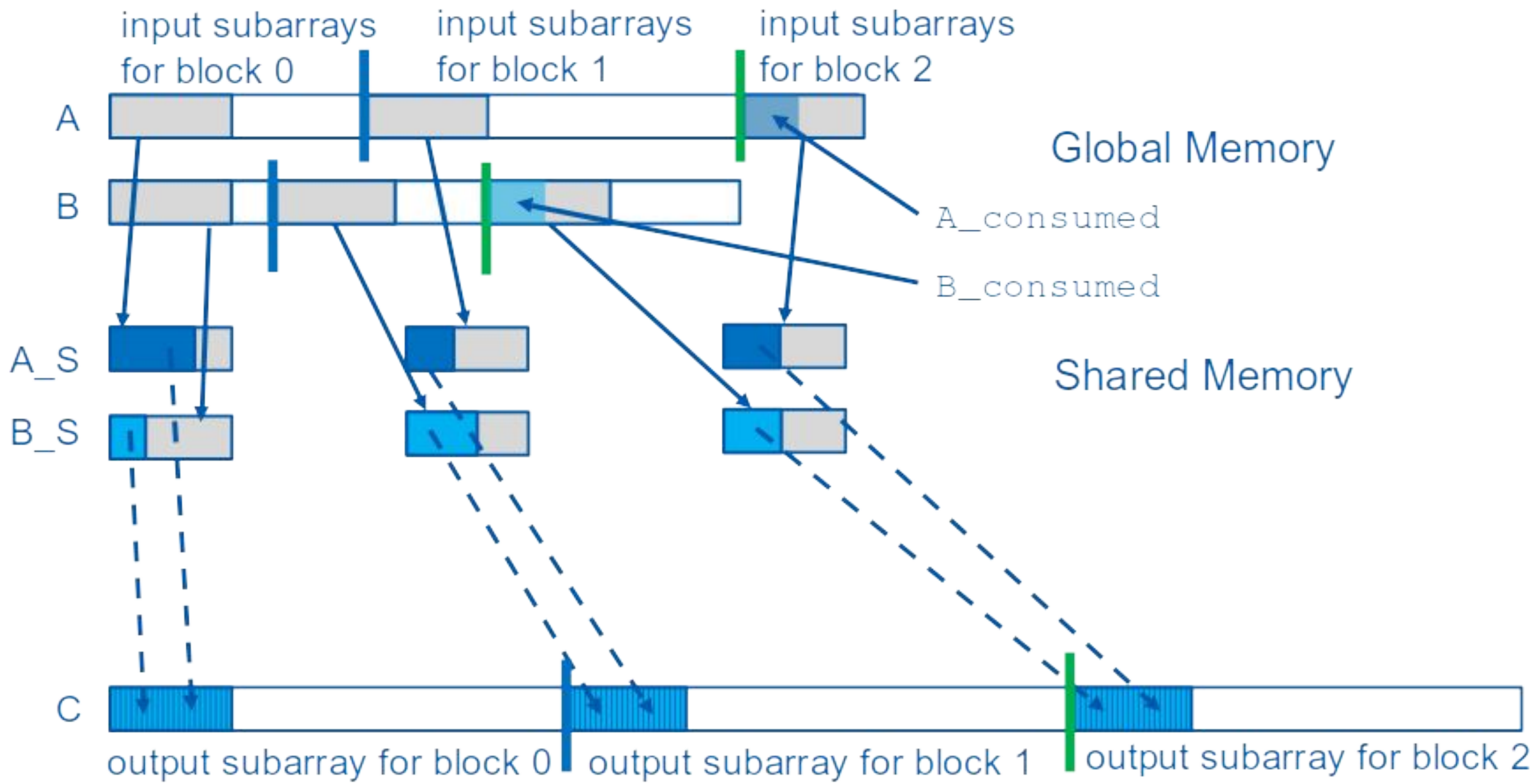
Tiled Merge Example (Iteration 0)


input subarrays for block 0
input subarrays for block 1
input subarrays for block 2
A
B
Global Memory
A_S
B_S
Shared Memory
C
output subarray for block 0
output subarray for block 1
output subarray for block 2

Tiled Merge Example (Iteration 0)


input subarrays for block 0
input subarrays for block 1
input subarrays for block 2
A
B
Global Memory
A_consumed
B_consumed
A_S
B_S
Shared Memory
C
output subarray for block 0
output subarray for block 1
output subarray for block 2

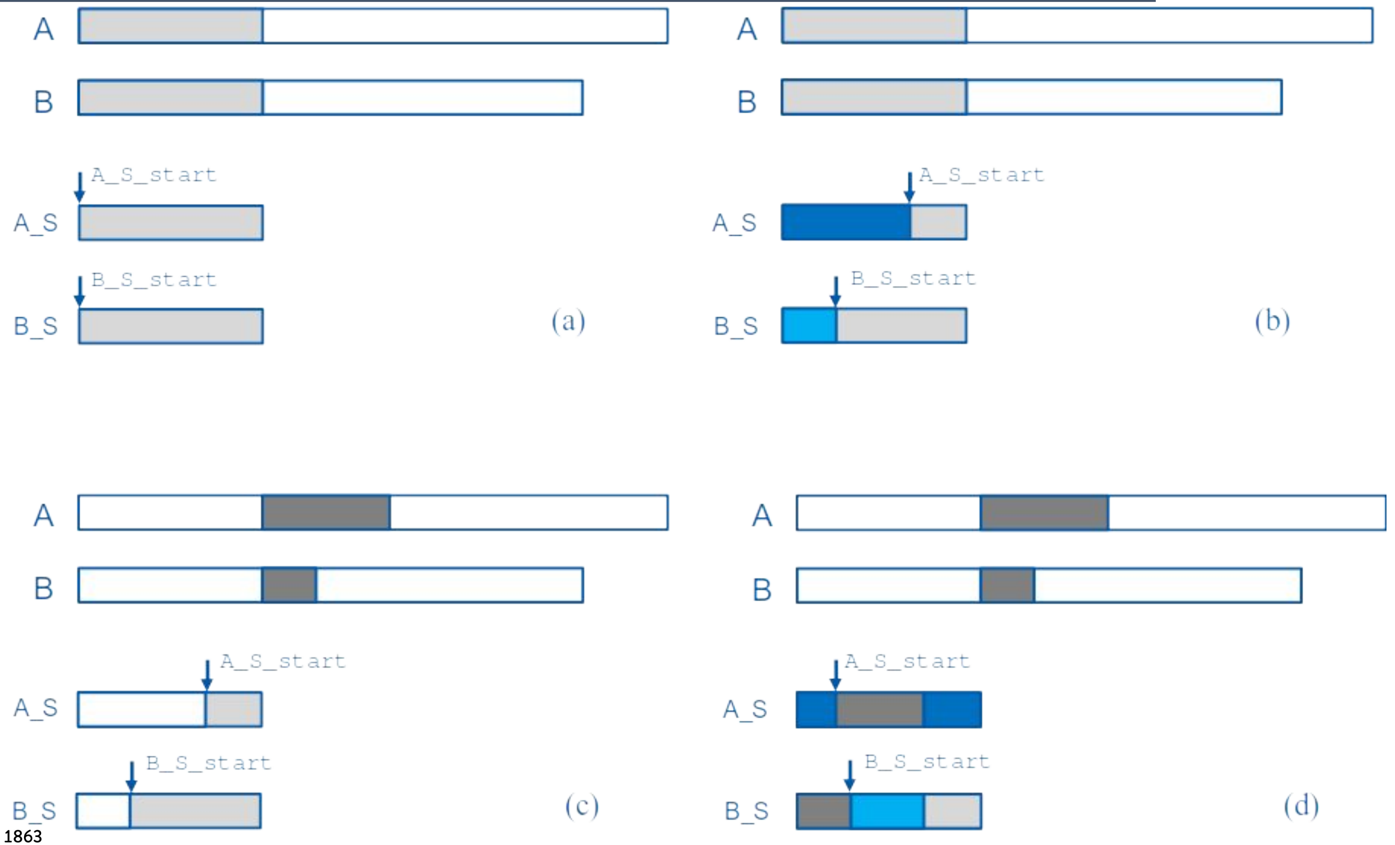
Tiled Merge Example (Iteration 1)


input subarrays for block 0
input subarrays for block 1
input subarrays for block 2
A
B
Global Memory
A_consumed
B_consumed
A_S
B_S
Shared Memory
C
output subarray for block 0
output subarray for block 1
output subarray for block 2

Tile Merge

- Coalesced Access
- Reduced global memory access on co-rank functions
- Improvement
  - Coalesced stores if results are written to shared memory
  - Call the *co_rank* twice during initialization to get this, and the next output section dimensions
    - Thus, being more accurate in loading data
  - Only half of the input elements loaded into shared memory are used (in the worst case)

Circular Buffering



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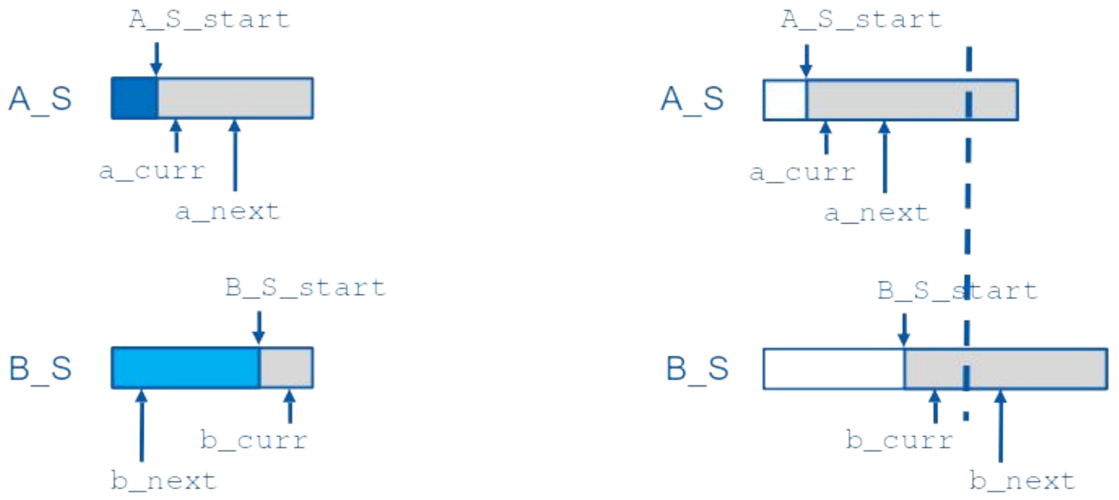
Circular Buffering: Co-Rank Values

- Handling the co-rank computation is now more complex



(a) Reality (b) Simplified

Credits

- PUMPS+AI 2022 Lecture 2: Input Regularization Techniques for Gather Parallelization, from *Wen-mei Hwu, Mateo Valero, Toni Pena, Juan Gomez-Luna, Marc Jorda, Leonidas Kosmidis, Bernat Font*

Thank you for your attention!

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