IOT Homework Exercise #3

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1 Introduction

We were assigned an RFID system based on Dynamic Frame ALOHA, composed of N=4 tags. We were given the following tasks:

1. Find the overall collision resolution efficiency η with different initial frame sizes r1=1,2,3,4,5,6. We were given two assumptions:
   - After the first frame, the frame size is correctly set to the current backlog size.
   - The duration of the arbitration period with N=2,3, when r=N is given ($L_2 = 4, L_3 = \frac{51}{8}$).
2. After the computation of the efficiency values, plot the values of η over r1.
3. Comment for which values of r1 we have maximum η.

2 Definitions

- **# resolved tags combinations**: number of all possible combinations of 'i' tags taken from a set of N tags.
- **# resolved tags positions combinations**: number of possible arrangements without repetitions of the resolved tags in the r slots.
- **# colliding tags positions combinations**: number of possible arrangements without repetitions, of groups of at least 2 tags, in the set of r-'i' slots, where 'i' is the number of resolved tags which is equal to the number of slots occupied by them.

3 Probabilities

$P(S=i) = (\frac{1}{r})^N * \#_{\text{resolved tags combinations}} * \#_{\text{resolved tags position combinations}} * \#_{\text{colliding tags position combinations}}$

	N = 4, i = 0
r = 1	$P(S=0) = \frac{1}{1^4} * 1 * 1 * 1 = 1$
r = 2	$P(S=0) = \frac{1}{2^4} * 1 * 1 * 8 = \frac{1}{2}$
r = 3	$P(S=0) = \frac{1}{3^4} * 1 * 1 * 21 = \frac{7}{27}$
r = 4	$P(S=0) = \frac{1}{4^4} * 1 * 1 * 40 = \frac{5}{32}$
r = 5	$P(S=0) = \frac{1}{5^4} * 1 * 1 * 65 = \frac{13}{125}$
r = 6	$P(S=0) = \frac{1}{6^4} * 1 * 1 * 96 = \frac{2}{27}$

	N = 4, i = 1
r = 1	$P(S=1) = \frac{1}{1^4} * 4 * 1 * 0 = 0$
r = 2	$P(S=1) = \frac{1}{2^4} * 4 * 2 * 1 = \frac{1}{2}$
r = 3	$P(S=1) = \frac{1}{3^4} * 4 * 3 * 2 = \frac{8}{27}$
r = 4	$P(S=1) = \frac{1}{4^4} * 4 * 4 * 3 = \frac{3}{16}$
r = 5	$P(S=1) = \frac{1}{5^4} * 4 * 5 * 4 = \frac{16}{125}$
r = 6	$P(S=1) = \frac{1}{6^4} * 4 * 6 * 5 = \frac{5}{54}$

	N = 4, i = 2
r = 1	$P(S=2) = \frac{1}{1^4} * 6 * 0 * 0 = 0$
r = 2	$P(S=2) = \frac{1}{2^4} * 6 * 2 * 0 = 0$
r = 3	$P(S=2) = \frac{1}{3^4} * 6 * 6 * 1 = \frac{4}{9}$
r = 4	$P(S=2) = \frac{1}{4^4} * 6 * 12 * 2 = \frac{9}{16}$
r = 5	$P(S=2) = \frac{1}{5^4} * 6 * 20 * 3 = \frac{72}{125}$
r = 6	$P(S=2) = \frac{1}{6^4} * 6 * 30 * 4 = \frac{5}{9}$

	N = 4, i = 3
r = 1	$P(S=3) = 0$
r = 2	$P(S=3) = 0$
r = 3	$P(S=3) = 0$
r = 4	$P(S=3) = 0$
r = 5	$P(S=3) = 0$
r = 6	$P(S=3) = 0$

It is impossible, given 4 tags, to solve 3 of them and not solve the last one. This is due to the fact that each tag, in order to be solved, needs to be alone in a slot. By having a single remaining tag after solving the first three, the last will also be solved.

4 Arbitration periods

Recursive steps:

- $L_0 = 0$
- $L_1 = 1$
- $L_2 = 4$
- $L_3 = \frac{51}{8}$
- $L_4 = 4 + P(S=4)L_0 + P(S=3)L_1 + P(S=2)L_2 + P(S=1)L_3 + P(S=0)L_4 =$
  $\frac{4+0+P(S=3)L_1+P(S=2)L_2+P(S=1)L_3}{1-P(S=0)} = \frac{4+0+\frac{9}{16}*4+\frac{3}{16}*\frac{51}{8}}{1-\frac{5}{32}} = \frac{(512+288+153)\div(128)}{27\div32} = \frac{953}{128} * \frac{32}{27} = \frac{953}{108} = 8.8240$

4.1 N = 4, r1 = 1

$L_4^* = 1 + P(S=4)*L_0 + P(S=3)*L_1 + P(S=2)*L_2 + P(S=1)*L_3 + P(S=0)*L_4 =$
$1 + 0 + 0 + 0 + 0 + 1 * 8.8240 = 9.8240$

4.2 N = 4, r1 = 2

$L_4^* = 2 + P(S=4)*L_0 + P(S=3)*L_1 + P(S=2)*L_2 + P(S=1)*L_3 + P(S=0)*L_4 =$
$2 + 0 + 0 + 0 + \frac{1}{2} * \frac{51}{8} + \frac{1}{2} * 8.8240 = 9.5995$

4.3 N = 4, r1 = 3

$L_4^* = 3 + P(S=4)*L_0 + P(S=3)*L_1 + P(S=2)*L_2 + P(S=1)*L_3 + P(S=0)*L_4 =$
$3 + 0 + 0 + \frac{4}{9} * 4 + \frac{8}{27} * \frac{51}{8} + \frac{7}{27} * 8.8240 = 8.9543$

4.4 N = 4, r1 = 4

$L_4^* = L_4 = 8.8240$

4.5 N = 4, r1 = 5

$L_4^* = 5 + P(S=4)*L_0 + P(S=3)*L_1 + P(S=2)*L_2 + P(S=1)*L_3 + P(S=0)*L_4 =$
$5 + 0 + 0 + \frac{72}{125} * 4 + \frac{16}{125} * \frac{51}{8} + \frac{13}{125} * 8.8240 = 9.0377$

4.6 N = 4, r1 = 6

$L_4^* = 6 + P(S=4)*L_0 + P(S=3)*L_1 + P(S=2)*L_2 + P(S=1)*L_3 + P(S=0)*L_4 =$
$6 + 0 + 0 + \frac{5}{9} * 4 + \frac{5}{54} * \frac{51}{8} + \frac{2}{27} * 8.8240 = 9.4661$

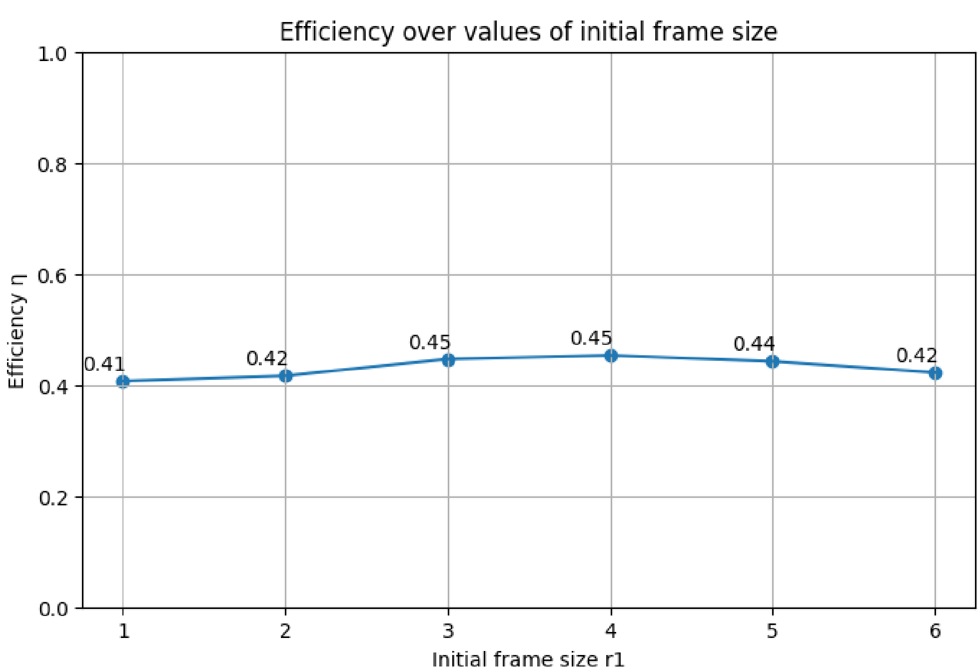

Figure 1: Plot of η over r1

5 Comments

The analysis shows that, for a system with N=4 tags, the maximum efficiency is achieved when the initial frame size (r1) is set to 4. This suggests that an initial frame size of 4 is the most effective setting to minimize arbitration periods and maximize overall efficiency. It allows definition of a frame having size equal to the number of tags that needs to be resolved. The frame size is also equal to the slot number. Due to the backlog's assumption, the frame size is set to the number of yet to be solved tags from the second step of the process. By trying all initial frame sizes, we noticed that the highest efficiency value was obtained when the initial frame size was set to the number of unsolved tags. Thus, we set the frame size to the number of yet to solve tags from the first step. By not approximating the value of η's, we can see that a value of r1 $= 4$ is the best.

6 Summary

r1	$P(S=0)$	$P(S=1)$	$P(S=2)$	$P(S=3)$	L_4^*	$\eta = \frac{N}{L_4^*}$	
1	1	0	0	0	9.8240	0.4072	
2	$\frac{1}{2}$	$\frac{1}{2}$	0	0	9.5995	0.4167	
3	$\frac{7}{27}$	$\frac{8}{27}$	$\frac{4}{9}$	0	8.9543	0.4467	
4	$\frac{5}{32}$	$\frac{3}{16}$	$\frac{9}{16}$	0	8.8240	0.4533	✓
5	$\frac{13}{125}$	$\frac{16}{125}$	$\frac{72}{125}$	0	9.0377	0.4426	
6	$\frac{2}{27}$	$\frac{5}{54}$	$\frac{5}{9}$	0	9.4661	0.4226	