IOT Homework exercise #2

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1 Introduction

We were assigned the following c code, implementing an ESP32-based IoT monitoring system:

```
// Global Timer Handle
declare timer_handle as esp_timer_handle_t


// Called every 10 seconds
function process_frame(arg):
    image = capture_camera_frame()
    person_count = estimate_number_of_people(image)
    if person_count == 0:
        payload = create_message(size=1KB)
    else if person_count == 1:
        payload = create_message(size=3KB)
    else:
        payload = create_message(size=6KB)

function app_main():
    call setup_camera()
    call setup_timer()
    loop forever:
        delay(100 ms)

function setup_timer():
    declare timer_config as esp_timer_create_args_t
    set timer_config.callback to process_frame
    set timer_config.name to "10_sec_timer"
    call esp_timer_create(&timer_config,&timer_handle)
    call esp_timer_start_periodic(timer_handle, 10_000_000) // 10s

// Initialization
function setup_camera():
    initialize_camera(QVGA)
```

We were tasked with the following:

1. Compute the Probability Mass Function of the output rate of the ESP32 $P(r = r_0)$, $P(r = r_1)$, $P(r = r_2)$, where r_0, r_1, and r_2 are the output rates when there are 0, 1 or 2 or more people in the captured frame, respectively.
2. Based on the output rate PMF, compute a consistent slot assignment for the CFP in a monitoring system composed of 1 PAN coordinator and 3 camera nodes. Assume nominal bit rate R=250kbps, packets of L=128bytes, 1 packet fits exactly in one slot. Compute:
   - Ts (slot time)
   - Number of slots in the CFP
   - T_{active}
   - $T_{inactive}$
   - The system's duty cycle
3. How many additional cameras can be added to keep the duty cycle below 10%?

We also assumed that the system runs with IEEE 802.15.4 in beacon-enabled mode (CFP only), and that the number of people present in the camera frame at any moment follows a Poisson distribution with an average rate of lambda = 0.15 people per frame.

2 Solution 1

The problem tells us that we have a poisson distribution with $\lambda = 0.15\frac{people}{frame}$, therefore its Probability Mass Function formula is: $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$
The problem asks us to compute the Probability Mass Function of the output rate; however, since we have a Poisson parameter λ expressed in $\frac{people}{frame}$, we have to associate the different output rates with the number of people in the frame (since the output rate changes if the number of people found in the frame changes). In particular, we have the following:

1. When there are 0 people in the frame, the output rate is:
$$\frac{output\_message\_size}{period} = \frac{1KB}{10s} = \frac{1000*8\ bits}{10s} = 800\frac{bits}{s}$$
2. When there is one person in the frame, the output rate is:
$$\frac{output\_message\_size}{period} = \frac{3KB}{10s} = \frac{3000*8\ bits}{10s} = 2400\frac{bits}{s}$$
3. When there are 2 or more people in the frame, the output rate is:
$$\frac{output\_message\_size}{period} = \frac{6KB}{10s} = \frac{6000*8\ bits}{10s} = 4800\frac{bits}{s}$$

We can then compute the different Probability Mass Functions:

$$P(X=0) = \frac{e^{-0.15}0.15^0}{0!} = e^{-0.15} = 0.8607 = 86.07\%$$
$$P(X=1) = \frac{e^{-0.15}0.15^1}{1!} = e^{-0.15}0.15 = 0.1291 = 12.91\%$$
$$P(X>1) = 1 - P(X=0) - P(X=1) = 1 - 0.8607 - 0.1291 = 0.0102 = 1.02\%$$

Combining the two results, we obtain:

$$r_0 = 800\frac{bits}{s} \Rightarrow P(X=r_0) = 86.07\%$$
$$r_1 = 2400\frac{bits}{s} \Rightarrow P(X=r_1) = 12.91\%$$
$$r_2 = 4800\frac{bits}{s} \Rightarrow P(X=r_2) = 1.02\%$$

3 Solution 2

The slot time T_s must be at least equal to the time required to transmit a packet on the channel. Since the problem tells us that a packet fits exactly in a time slot, it means that the slot's time will be equal to the time required to send a packet. We have the nominal bit rate R and the packet size L:

$$R = 250\ kbps$$
$$L = 128\ bytes$$

We know that cameras send packets of 128 bytes and that the minimum output rate required by all cameras is $800\frac{bits}{s}$. Based on this, we can compute the beacon interval (BI) as:

$$BI = \frac{L}{R_{out,min}} = \frac{128*8 \text{ bits}}{800\frac{bits}{s}} = 1.28s$$

In the worst case, the output data rate for a single camera in a 10s interval is:

$$R_{out,worst} = 4800\frac{bits}{s}$$

This means that a camera to send an entire frame, in the worst case, will require:

$$\frac{R_{out,worst}}{L} = \frac{4800\frac{\text{bits}}{\text{s}}}{128*8 \text{ bits}} = 4.6875\frac{\text{packets}}{\text{s}}$$

Therefore, in a beacon interval (BI), it will send $4.6875\frac{\text{packets}}{\text{s}} * 1.28s = 6\ packets$.

- $T_s = \frac{L}{R} = \frac{128*8\ bits}{250\ kbps} = 0.004096 = 4.096\ ms$
- We have 3 cameras, each of them transmitting every beacon interval a total of 6 packets (in the worst case). Each packet fits perfectly in a time slot; we need a slot for each camera in the CFP for a total of 18 slots.

$$\#_{slots,CFP} = 6*3 = 18$$

- The total time spent transmitting packets is given only by the CFP size (since from the problem we know that the system is in beacon-enabled mode and it uses only the CFP and not also the CAP), which must contain 6 time slots for each camera or 18 in total. In addition, we have to sum the time needed to send the first beacon that we can assume to be equal to T_s. Therefore, we find that T_{active} time is:

$$T_{active} = T_s * 18 + T_s(Beacon) = 77.824\ ms$$

- Based on the beacon interval and T_{active}, we can compute the inactive time as:

$$T_{inactive} = BeaconInterval - T_{active} = 1.28s - 77.824ms = 1.202s$$

- The duty cycle of the system is the fraction of time in which the system is working in the beacon interval:

$$D = \frac{T_{active}}{T_{active}+T_{inactive}} = \frac{77.824\ ms}{1.28\ s} = 0.0608 = 6.08\%$$

4 Solution 3

To keep the duty cycle below 10%, we can increase the active time up to:

$$D = \frac{T_{active}}{T_{active}+T_{inactive}} < 0.1$$

$$T_{active} < 0.1 * (T_{active} + T_{inactive})$$

$$T_{active} < 0.1 * 1.28s = 0.128s$$

If we take the limit case ($T_{active} = 0.128s$), we can have up to:

$$T_{active} = T_s * X$$

$$X = \frac{T_{active}}{T_s} = \frac{0.128\ s}{4.096\ ms} = 31.25 \simeq 31 = \#_{time\ slots}$$

X time slots inside the CFP. Since each camera requires up to, in the worst case, 6 time slots, we can increase the number of cameras up to:

$$\#_{cameras} = \frac{\#_{time\ slots}}{6} = 5.16 \simeq 5\ cameras$$

Therefore, we can add two additional cameras to the other 3 and still maintain the duty cycle below 10%.