Politecnico Di Torino

Master Degree in Physics of Complex Systems

Image Reconstruction via Expectation Propagation with auxiliary informative variables

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Abstract

Tomography is an imaging technique that allows one to reconstruct object sections by analyzing particular penetrating waves; this method finds applications in different areas of science. The mathematical procedure used to reconstruct images is generally called tomographic reconstruction; when X-rays are exploited, the overall procedure is called Computed Tomography scan (CT scan). Historically, this technique was first idealized by Johann Radon in 1917, when he introduced the so-called Radon transform: he showed that a cross-section of an image can be reconstructed in a single step through an infinite set of its projections. The first practical application in CT scan dates back to 1971, more than fifty years later. Nowadays, image reconstruction algorithms use, conversely, a finite number of projections, and iterative reconstruction methods. However, the original problem is mapped into an ill-posed linear problem, so one has to add more information, such as image regularizations, to find a feasible solution. In recent years, it has been possible to formulate the problem of image reconstruction in a Bayesian context, that is, in a probabilistic framework. Here, one has to find the pixel assignment that maximizes a posterior probability distribution. The huge advantage carried by this framework is that of encompassing the ability to introduce non-convex priors which reflect the typical characteristics of the images. However, this makes the problem intractable for general optimization methods, i.e. linear programming. Thanks to the Expectation Propagation method, it is possible to deal with these posteriors using approximations. Moreover, the ability to deal with non-convex functions allows us to introduce designed priors according to the empirical behavior of some pixel functions. This can be done for the so-called pixel-difference variables or for some other auxiliary variables obtained through a linear operator. Therefore, the idea behind the present work is to create a dataset of images somewhat similar to the tomographic ones and to apply a linear transform on the pixel variables: by studying the empirical behavior of the auxiliary variables, we try to deduce the functional form of the priors describing these auxiliary variables. In this thesis, we apply the Haar transform to the images of the dataset and we study the behavior of these auxiliary variables to define priors describing the Haar coefficients. Then, we compare the performances obtained by some implementations of the EP algorithm (differing in the priors used) in several measurement regimes. The results shown in the simulations confirm the validity of using auxiliary variables in the EP algorithm, in particular the pixel-difference variables.