QLS 2021/2022

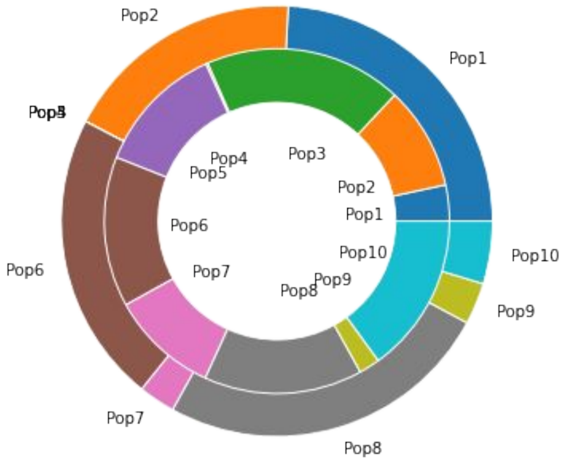
Spatial ecology for territorial populations

Benjamin G. Weiner, Anna Posfai, and Ned S. Wingreen

THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

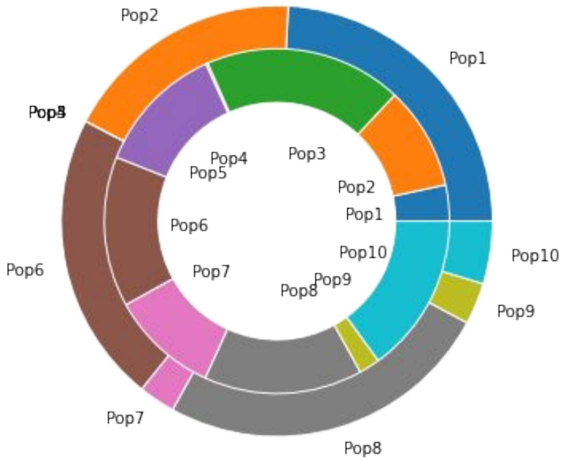
1. *m* **species** competing for *p* **nutrients**



THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

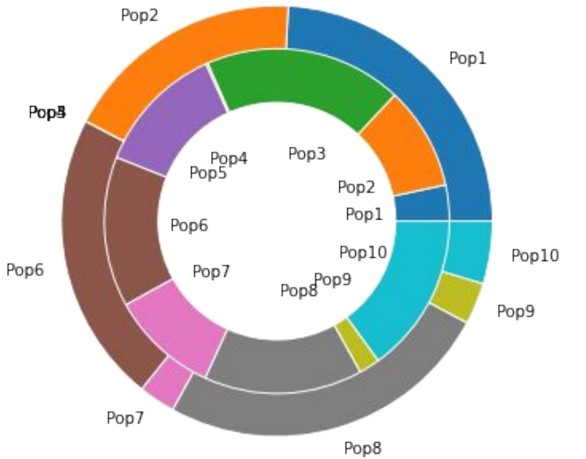
1. *m* species competing for *p* nutrients
2. They live in a **1-D space** of size L with **periodic boundary** conditions (a ring)



THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

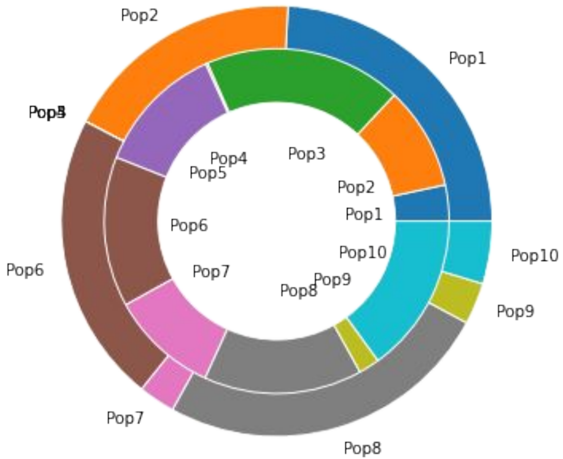
1. *m* species competing for *p* nutrients
2. They live in a 1-D space of size L with periodic boundary conditions (a ring)
3. Each species occupies a **segment of the ring** whose **size** is given by its **population** n_σ



THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

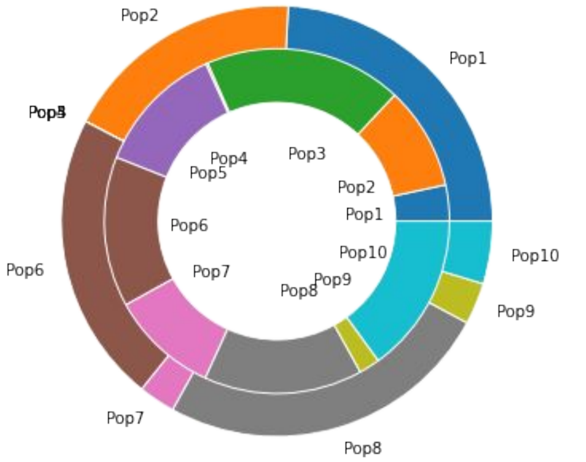
1. *m* species competing for *p* nutrients
2. They live in a 1-D space of size L with periodic boundary conditions (a ring)
3. Each species occupies a segment of the ring whose size is given by its population n_σ
4. Populations **never overlap**



THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

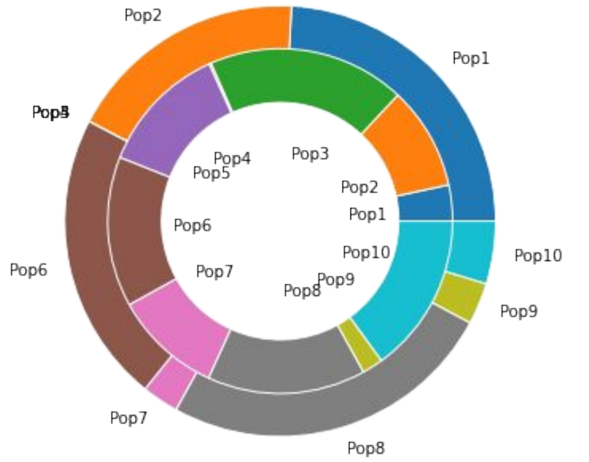
1. *m* species competing for *p* nutrients
2. They live in a 1-D space of size L with periodic boundary conditions (a ring)
3. Each species occupies a segment of the ring whose size is given by its population n_σ
4. Populations never overlap
5. Presence of a **metabolic tradeoff** that determines some constraints in the definition of the metabolic strategies



THE MODEL: GENERAL OVERVIEW

In this work it’s presented an ecological model that allows to describe simple spatial structures. The main features are:

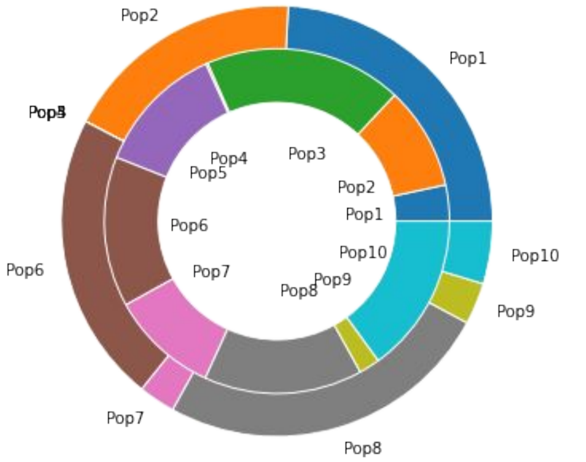
1. *m* species competing for *p* nutrients
2. They live in a 1-D space of size L with periodic boundary conditions (a ring)
3. Each species occupies a segment of the ring whose size is given by its population n_σ
4. Populations never overlap
5. Presence of a metabolic tradeoff that determines some constraints in the definition of the metabolic strategies
6. The **nutrients diffuse in space** with time according to a differential equation



THE MODEL: GENERAL OVERVIEW

In this work it's presented an ecological model that allows to describe simple spatial structures. The main features are:

1. *m* species competing for *p* nutrients
2. They live in a 1-D space of size L with periodic boundary conditions (a ring)
3. Each species occupies a segment of the ring whose size is given by its population n_σ
4. Populations never overlap
5. Presence of a metabolic tradeoff that determines some constraints in the definition of the metabolic strategies
6. The nutrients diffuse in space with time according to a differential equation
7. The space dimension L and consequently the **total population remains constant in time**



THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

This equation tells how the nutrients' abundance change in time

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

$c_{\sigma i}$ indicates the abundance of the nutrient i consumed by the population σ

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

Si indicates the supply rate of the *i-th* nutrient, i.e. the rate at which the resource is injected into the environment

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

This term quantifies the speed at which the resource i resource is consumed by the population sigma. $\alpha_{\sigma i}$ represents the metabolic strategy of a particular pair of resource-population (a deeper explanation will be given later)

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

D is the constant diffusion rate at which the resources spread in the territory

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

The nutrients spread is influenced by the spatial distribution of them at the current time, which is quantified by the second derivative of the abundance w.r.t. the space

THE MODEL: RESOURCES

$$\frac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \frac{\partial^2 c_{\sigma i}}{\partial x^2}$$

- $c_{\sigma i}$: nutrient *i* abundance distribution in the territory occupied by the population σ
- S_i : supply rate of the i–th resource
- $\alpha_{\sigma i}$: metabolic strategy of the pair σi
- D : diffusion rate

THE MODEL: STATIONARITY

The differential equations for the dynamics of the nutrients are too complex to simulate numerically

↓

Given the fact that nutrients processing is generally much faster than species growth, we can assume **timescale separation**

↓

$$\frac{\partial c}{\partial t} = 0$$

QUASI-STATIONARY APPROXIMATION FOR NUTRIENTS

THE MODEL: RESOURCES

What remains is a second order differential equation in space, which can be solved analytically, returning:

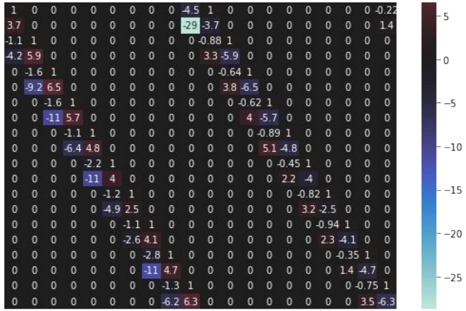
$$c_{\sigma i} = \frac{S_i}{\alpha_{\sigma i}} + A_{\sigma i} \exp\left(x\sqrt{\frac{\alpha_{\sigma i}}{D}} \right) + B_{\sigma i} \exp\left(- x\sqrt{\frac{\alpha_{\sigma i}}{D}} \right)$$

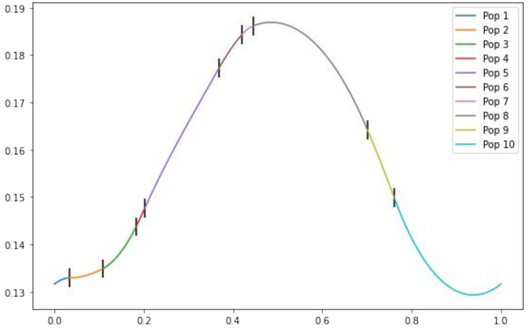
The expression above returns the spatial function that describes the **distribution of the nutrient i in the territory occupied by the population σ**

THE MODEL: RESOURCES

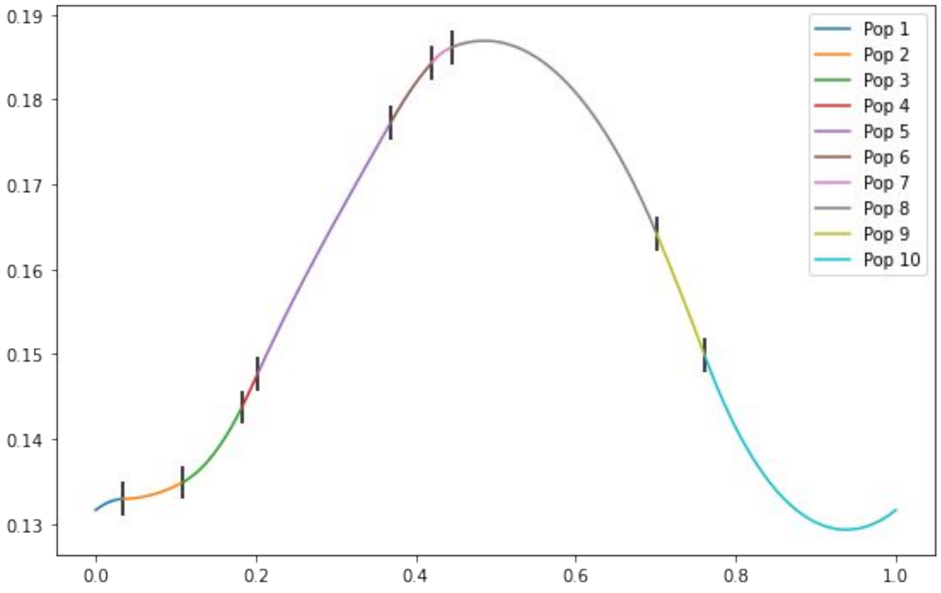
$$c_{\sigma i} = \frac{S_i}{\alpha_{\sigma i}} + A_{\sigma i} \exp\left(x\sqrt{\frac{\alpha_{\sigma i}}{D}} \right) + B_{\sigma i} \exp\left(-x\sqrt{\frac{\alpha_{\sigma i}}{D}} \right)$$

The integration constants $A_{\sigma i}$ $B_{\sigma i}$ are fixed by the physical constraints of the system. In particular the nutrients distributions have to be **continuous and differentiable in any point,** also in the conjunction between the areas occupied by different populations





THE MODEL: RESOURCES



THE MODEL: POPULATIONS

$$\frac{dn_\sigma}{dt} = \sum_i \alpha_{\sigma i} \left(v \int_0^{n_\sigma} c_i(x) dx \right) - \delta n_\sigma$$

This equation tells how the species populations change in time

THE MODEL: POPULATIONS

$$\frac{dn_\sigma}{dt} = \sum_i \alpha_{\sigma i}\left(v\int_0^{n_\sigma} c_i(x)dx\right) - \delta n_\sigma$$

n_σ is the population of the species σ , and also corresponds to the fraction of the territory occupied by such a species

THE MODEL: POPULATIONS

$$\frac{dn_\sigma}{dt} = \sum_i \alpha_{\sigma i} \left(v \int_0^{n_\sigma} c_i(x)dx \right) - \delta n_\sigma$$

Conversion rate between nutrients consumption and population /occupied territory increase

THE MODEL: POPULATIONS

$$\frac{dn_\sigma}{dt} = \sum_i \alpha_{\sigma i} \left(v \int_0^{n_\sigma} c_i(x) dx \right) - \delta n_\sigma$$

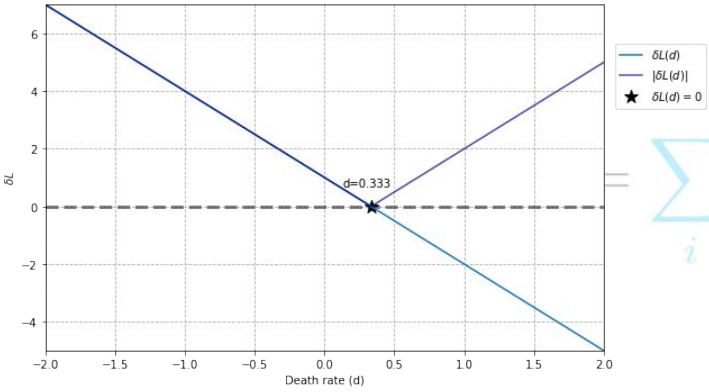
The integral return the quantity of nutrient i contained in the territory of species σ. By multiplying for the metabolic strategies and summing over all the nutrients, we have the growth rate of the species

THE MODEL: POPULATIONS

$$\frac{dn_\sigma}{dt} = \sum_i \alpha_{\sigma i} \left(v \int_0^{n_\sigma} c_i(x)dx \right) - \delta n_\sigma$$

It is the term related to the decreasing of the species abundance due to the death of some individuals, i.e. the negative growth

THE MODEL: POPULATIONS



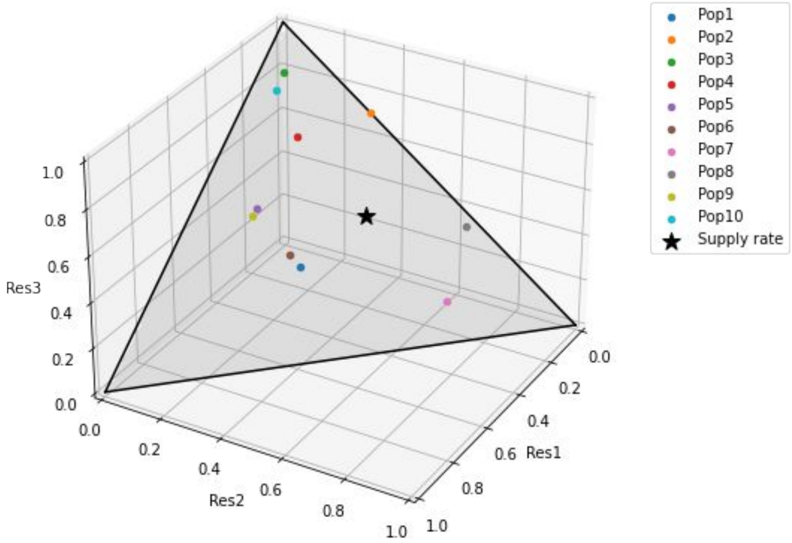
The **death rate** is a fundamental parameter in the simulation. It's possible to see that it has to be tweaked perfectly in order to have a constant total population in the territory.

Find δ such that $\sum_{\sigma} \frac{dn_\sigma}{dt} = 0$

It can be shown that it has to be equal to $\frac{1}{p}$ with p the number of nutrients.

Even small variations make the system diverge (or vanish).

THE MODEL: METABOLIC TRADEOFF

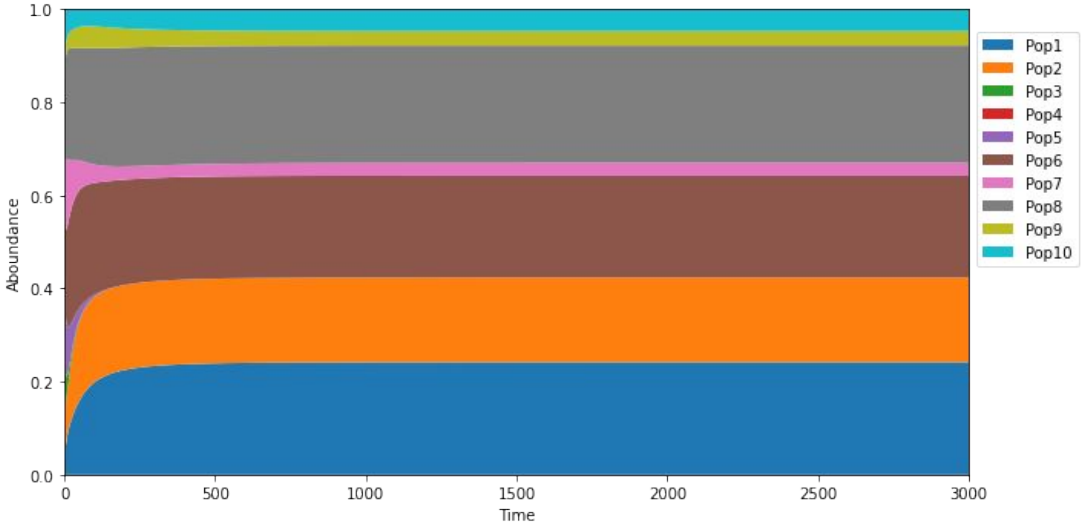


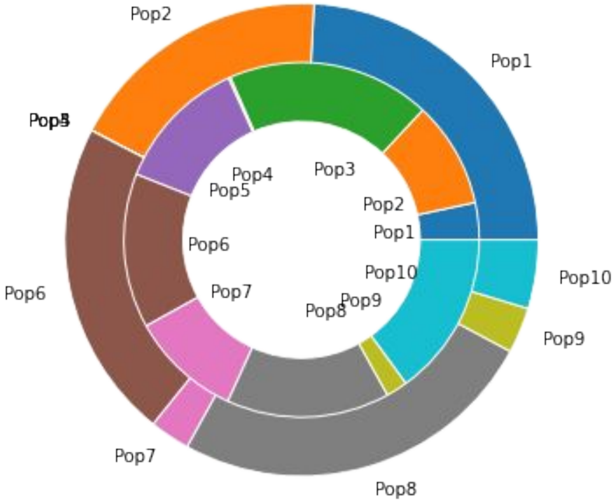
$$\sum_i \alpha_{\sigma i} = E$$

The metabolic constraint forces the alphas to lie in *p-1* dimensional simplex.

It derives from the observation that every living organism has a fixed quantity of energy to invest into metabolic dynamics.

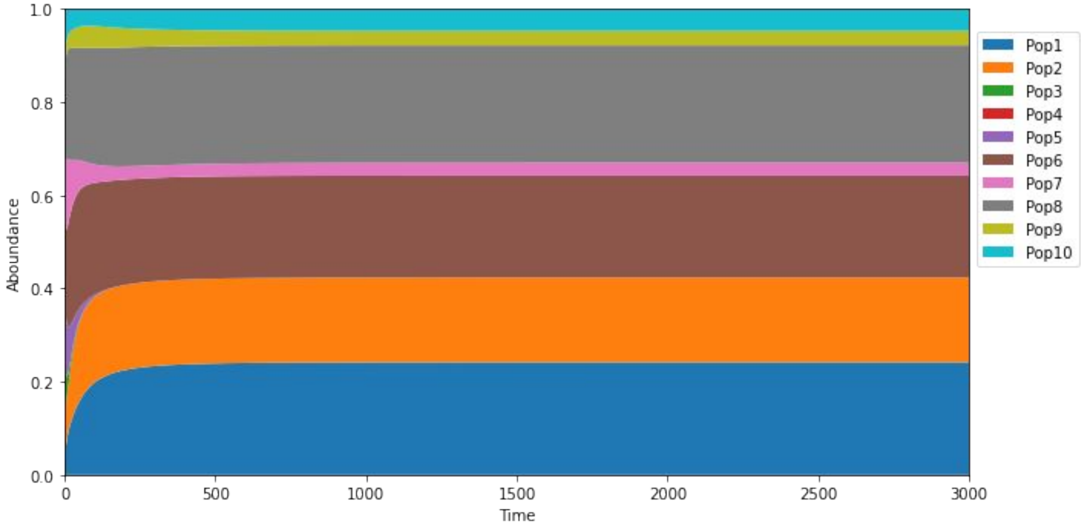
COMPLETE SIMULATION

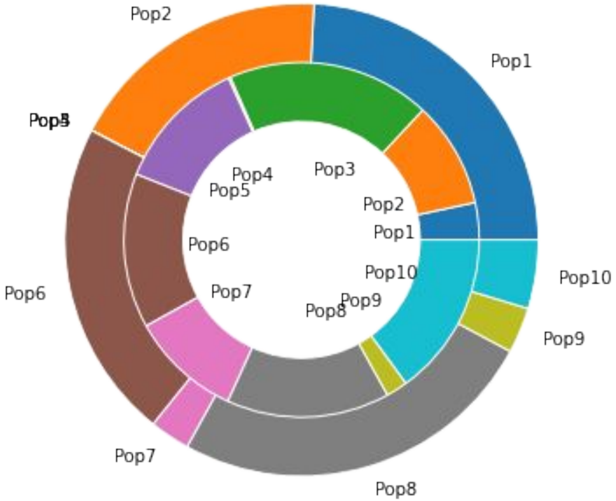




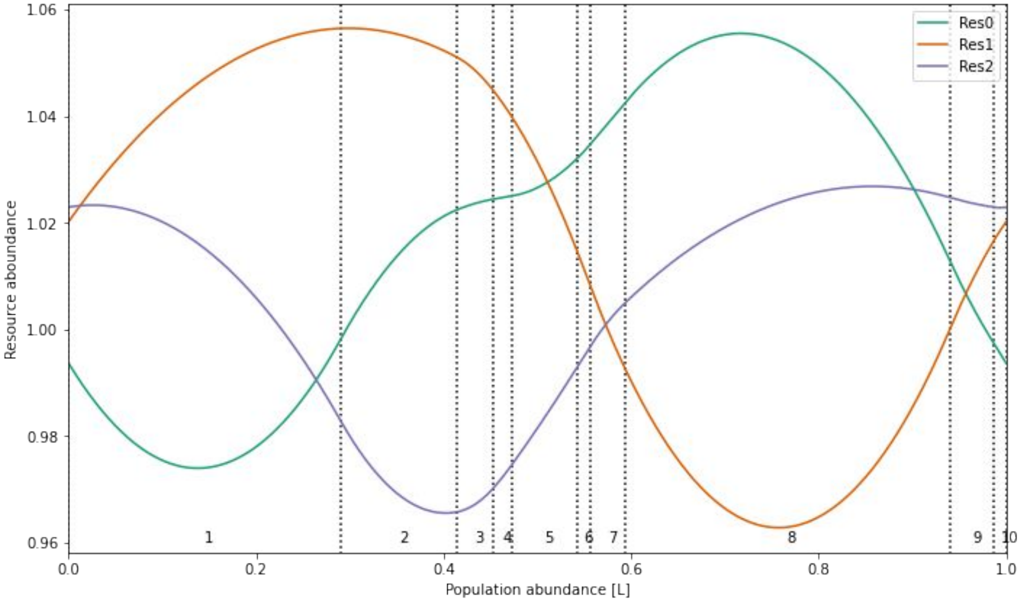
COMPLETE SIMULATION

CEP violated!!!





COMPLETE SIMULATION

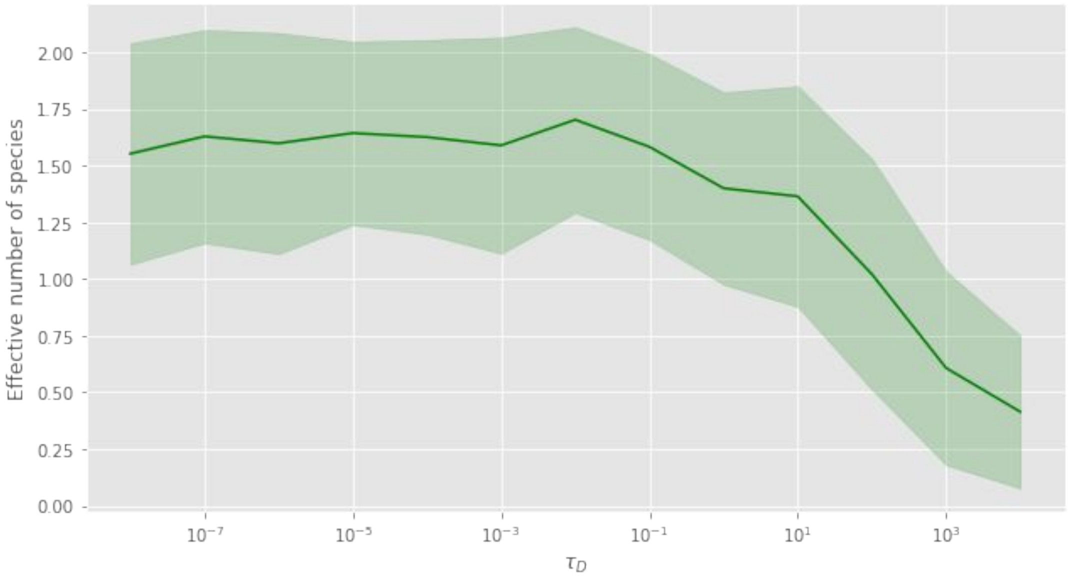


VARIATION OF THE DIFFUSION RATE

Effective number of species: it is defined as the Shannon entropy of the population densities

$$H = -\sum_{\sigma} p_\sigma \log p_\sigma$$

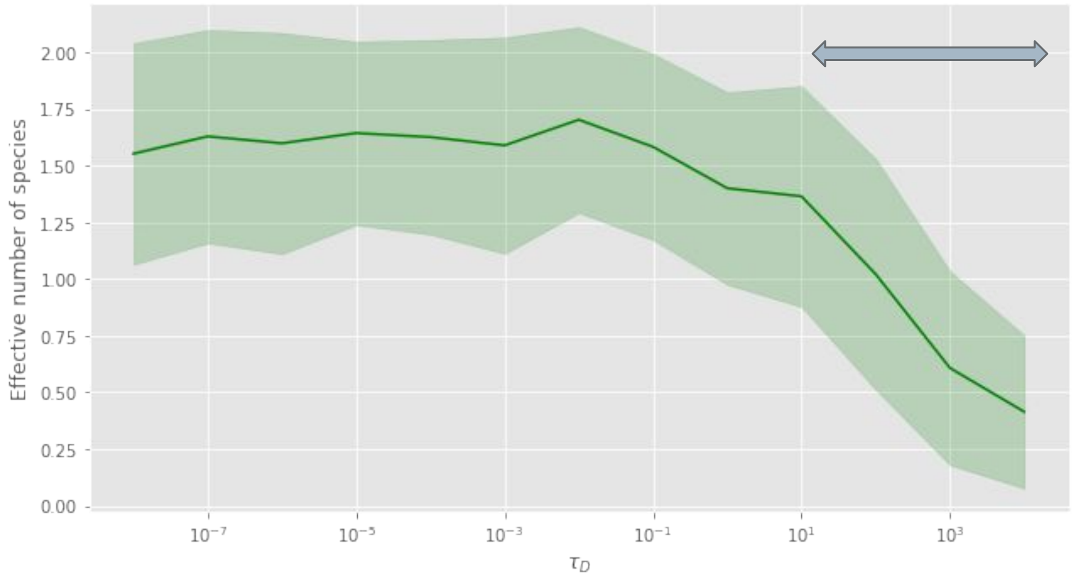
It allows to quantify the biodiversity of the system without setting an arbitrary threshold to state if a species is extinct or not.



$$\tau_D = \frac{LE}{D}$$

Diffusion time, the time required for nutrients to diffuse a distance L relative to the uptake time. It is inversely proportional to the Diffusion Rate

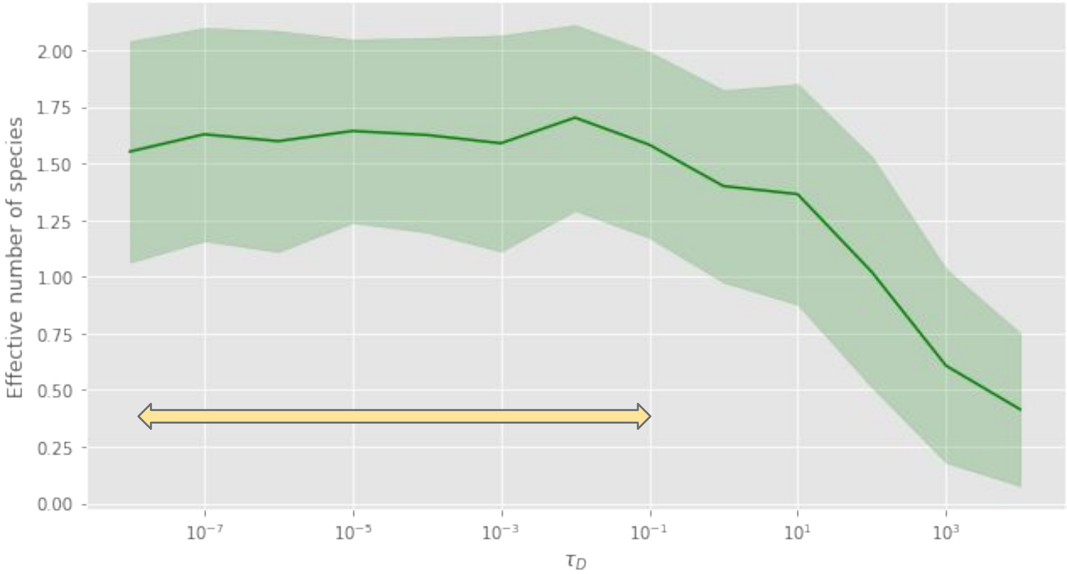
VARIATION OF THE DIFFUSION RATE



The biodiversity of the system decreases as the diffusion rate increase

For small values of D, the nutrients spread in the territory with a timescale comparable to the one of the species growth, and so the spatial features of the system are relevant.

VARIATION OF THE DIFFUSION RATE



The biodiversity of the system decreases as the diffusion rate increase

For large values of D, the nutrients spread very fast, and the concentration of each one is almost constant in each point of the space in every instant.

In the limit of , $D \longrightarrow \infty$ we talk about **WELL MIXED CASE**

NULL MODEL: WELL-MIXED CASE

$$\boxed{\text{Spatial} \xrightarrow{D \to \infty} \text{Well-Mixed}}$$

$$\begin{cases} \dfrac{\partial c_{\sigma i}}{\partial t} = S_i - \alpha_{\sigma i} c_{\sigma i} + D \dfrac{\partial^2 c_{\sigma i}}{\partial x^2} \\ \dfrac{dn_\sigma}{dt} = \displaystyle\sum_i \alpha_{\sigma i} \left(v \int_0^{n_\sigma} c_i(x) dx \right) - \delta n_\sigma \end{cases} \xrightarrow{D \to \infty}$$

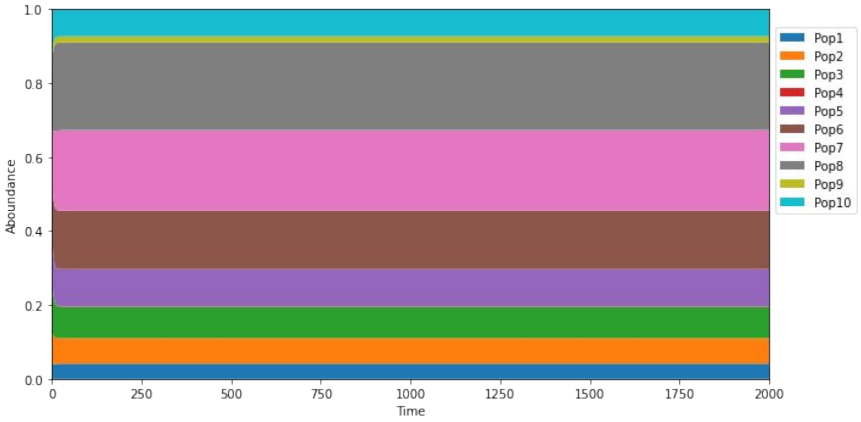
Applying expansions and approximations

$$\frac{\partial c_i}{\partial t} = S_i \left(\sum_\sigma n_\sigma \alpha_{\sigma i} \right) r_i(c_i)$$

QUASI-STATIONARY APPROXIMATION

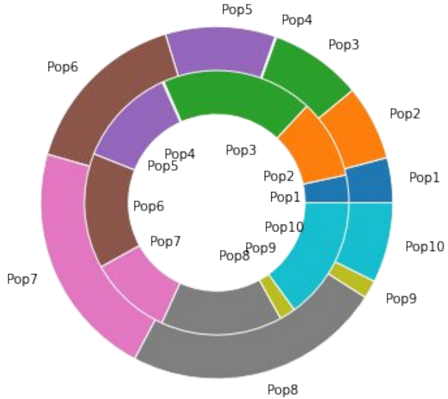
$$\boxed{\frac{dn_\sigma}{dt} = \left(\sum_i \alpha_{\sigma i} \frac{S_i}{\sum_\gamma \alpha_{\gamma i} n_\gamma} - \delta \right) n_\sigma}$$

NULL MODEL: WELL-MIXED CASE



In general the **typical timescale** for the population dynamics in the *well-mixed case* in **much faster** than the one of the *spatial case*

If the *metabolic tradeoff condition holds*, typically **all** or almost all **the species survive** asymptotically, and this leads to an high biodiversity degree



Before the comparison, one more ingredient ...

OLIGOTROPHS

High **variability** in the results w.r.t. the choice of alpha

Something is happening!

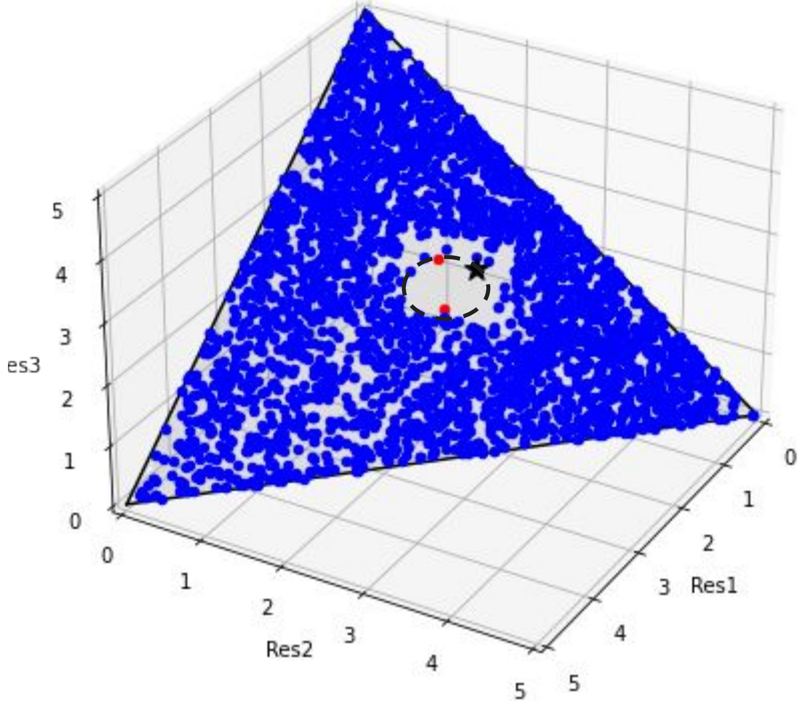
Generation of many **different combinations of alpha** and computed effective number of species

Experiment

Take alpha combinations with corresponding **e.n.s.** belonging to the **85% highest values** and study the distribution

Analysis

OLIGOTROPHS

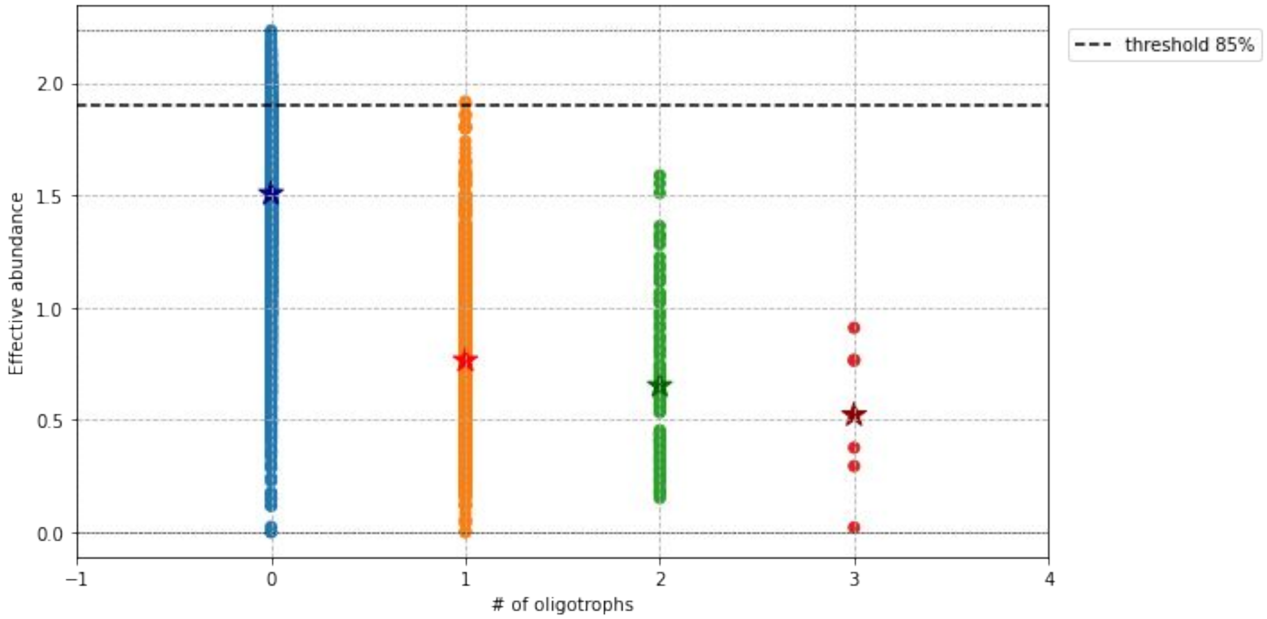


Inside the region delimited by the dashed line, there are almost no points

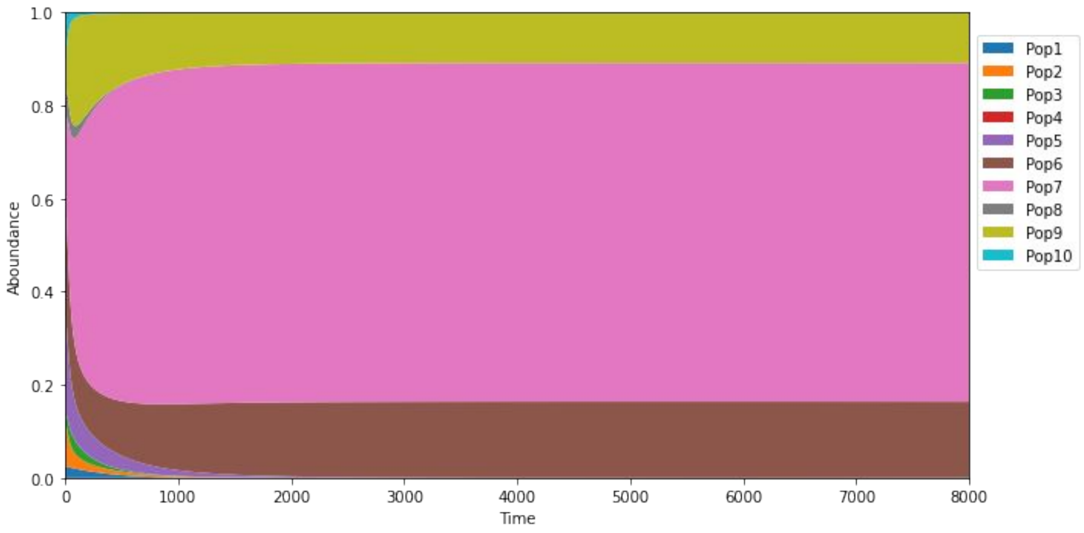
Oligotrophs: they are species able to survive also at minimum total nutrient level, and in such a way they drive other competitors to extinction. This property is mathematically given by the formula below

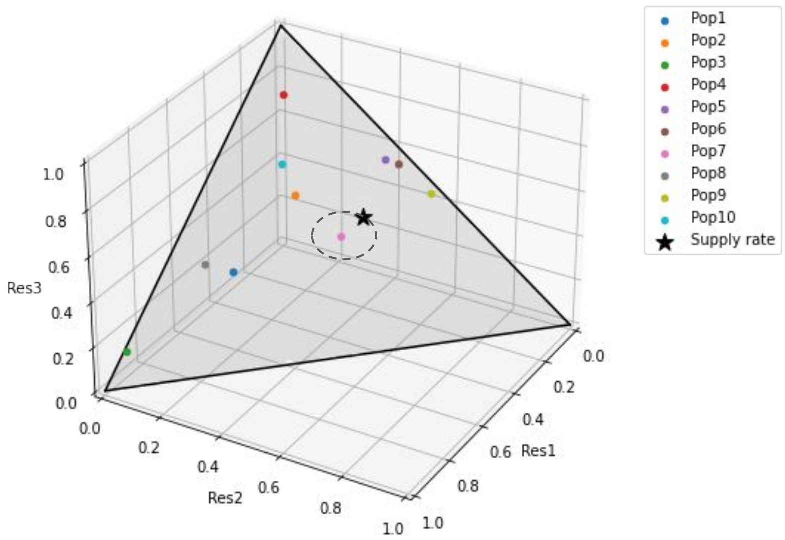
$$\sum_i \frac{S_i}{\alpha_{\sigma i}} < \frac{p}{\mathrm{E}}$$

OLIGOTROPHS

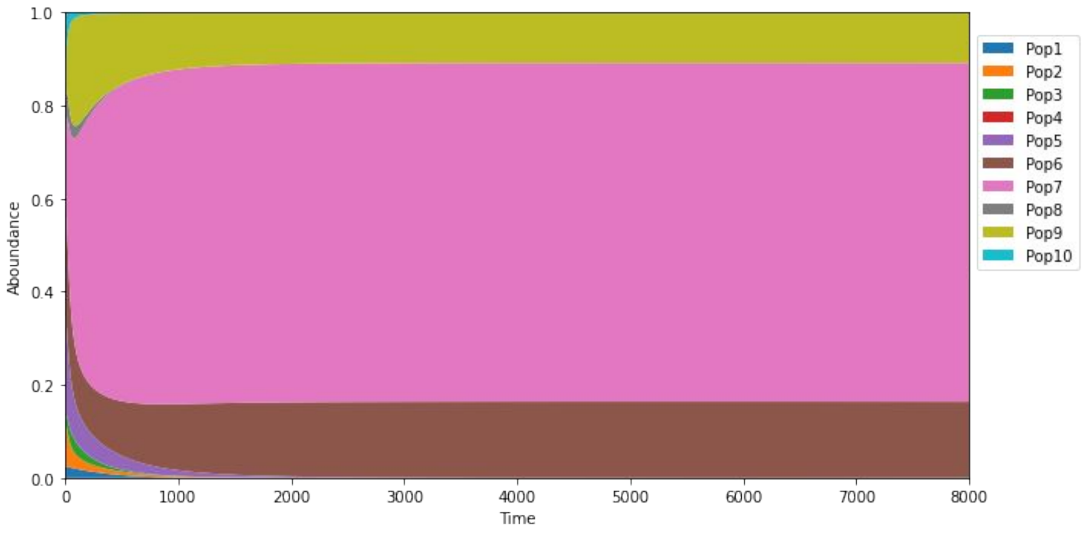


OLIGOTROPHS

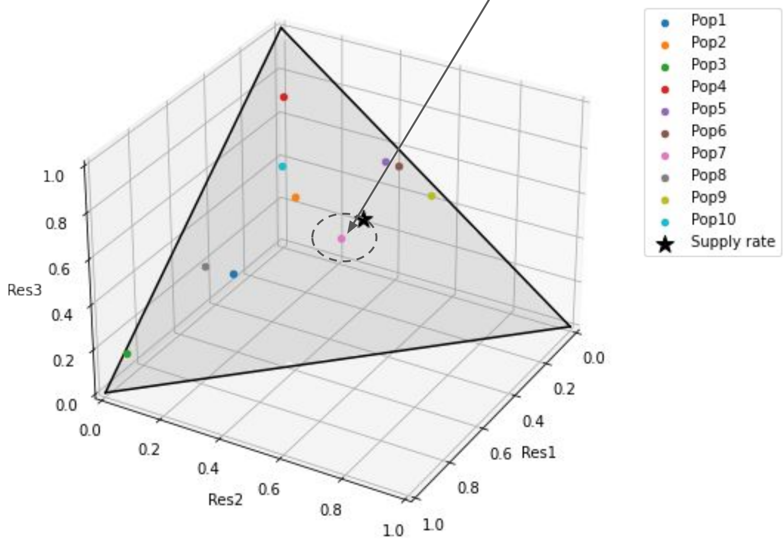




OLIGOTROPHS

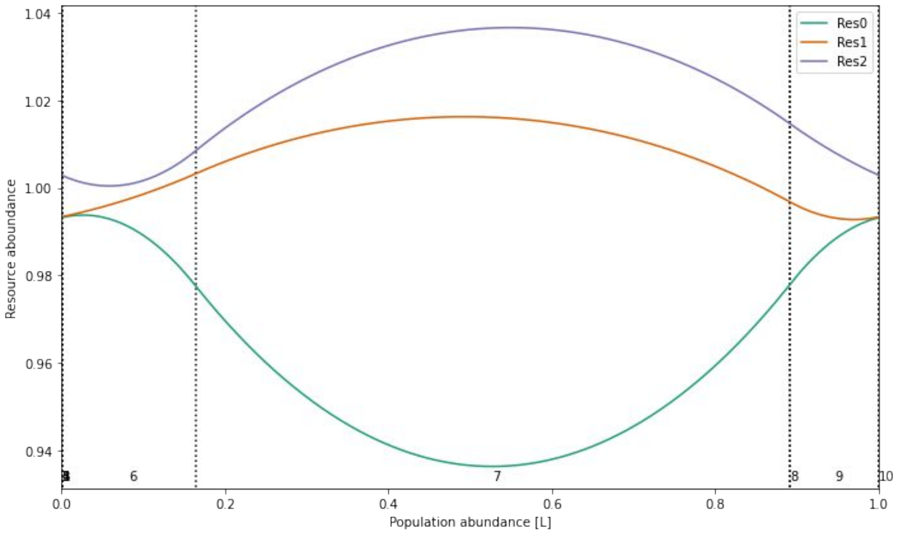


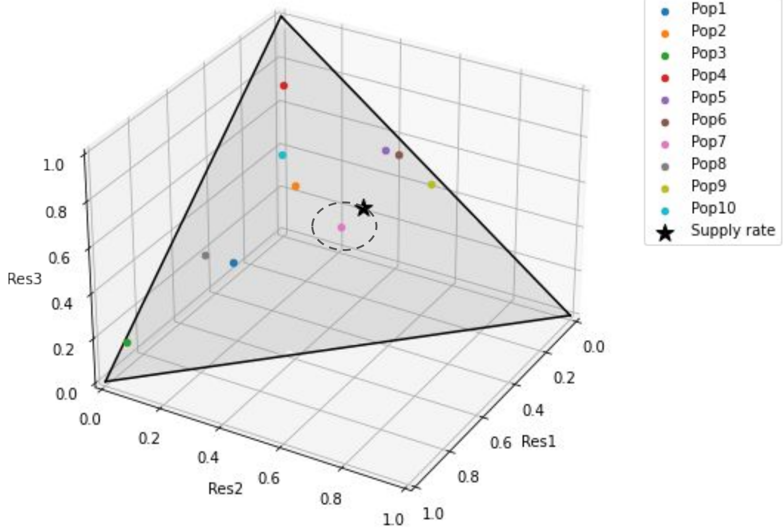
Species 7 is an oligotroph



OLIGOTROPHS

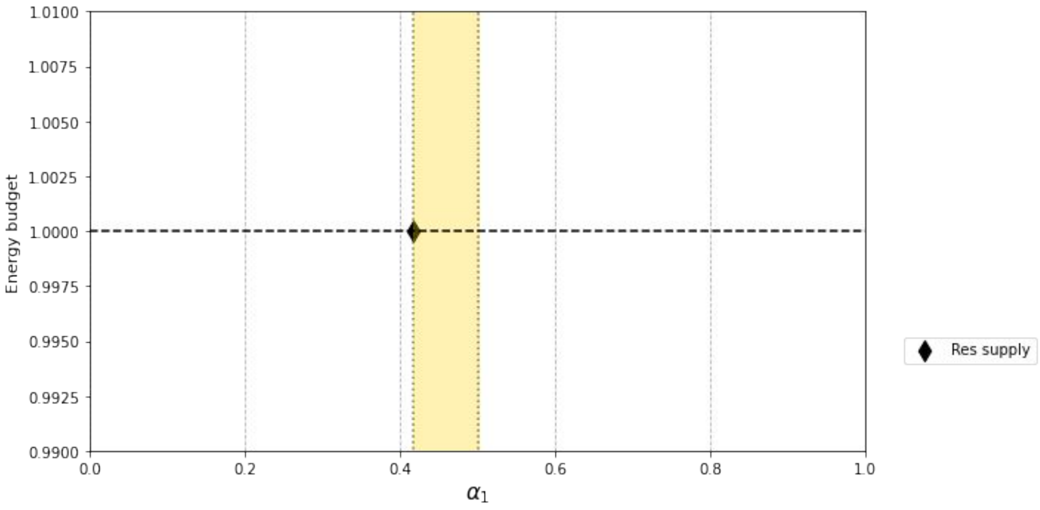
The majority of the resources have flowed in the area occupied by species 7 !

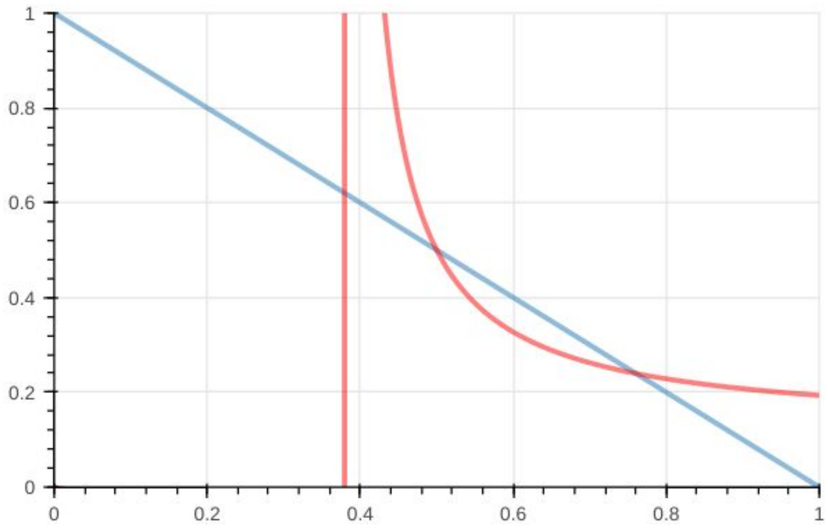




OLIGOTROPHS with 2 nutrients

In the case of 2 nutrients is easier to determine the position of the oligotrophs region because it's possible to solve analytically the system of equations.





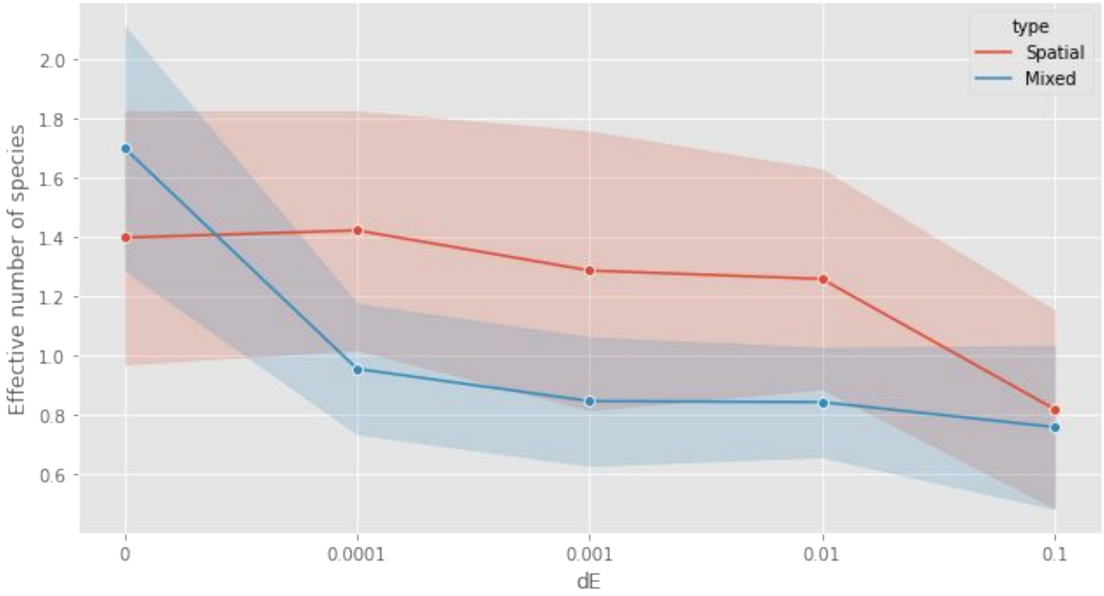
UNEQUAL ENERGY BUDGET

The condition for which the sum of the metabolic strategy coefficients should be equal to a constant E allows the violation of the CEP and the resulting increase of the system's biodiversity. Now we want to explore what will happen if this condition is relaxed and each population is characterized by a specific value of E. In particular we want:

$$\sum_i \alpha_{\sigma i} = E_\sigma \qquad\qquad E_\sigma \sim \mathcal{N}(E, dE)$$

We want now to study the different behaviour of systems following the **spatial model** and the **well-mixed one**

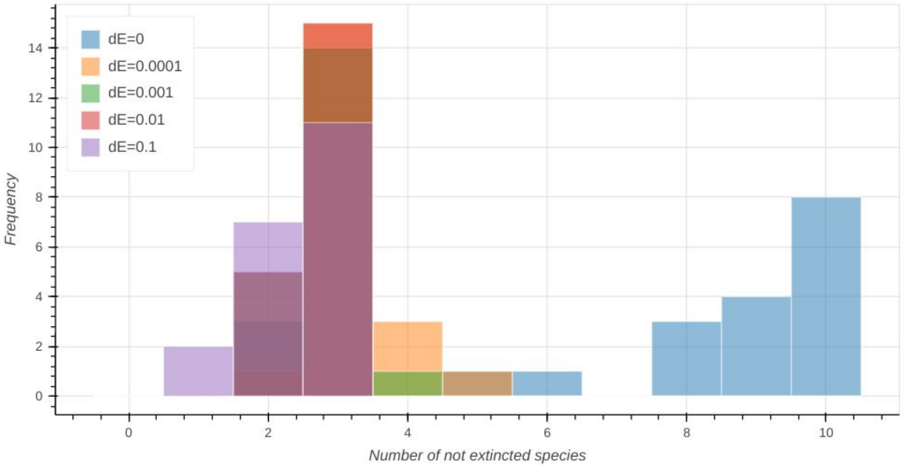
UNEQUAL ENERGY BUDGET



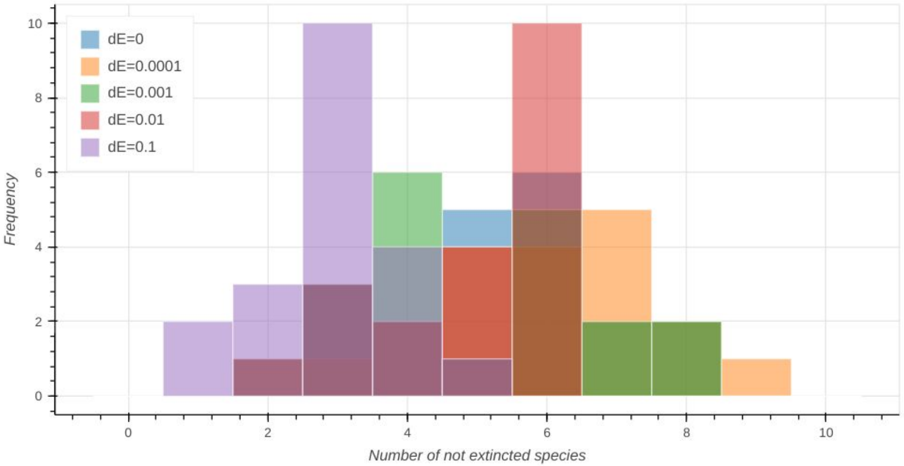
- For **dE=0** the **well-mixed** case leads to a **greater biodiversity**
- As soon as **dE becomes different from 0** the **e.n.s.** relative the the **well-mixed** case drastically **drops**.
- The spatial case biodiversity is almost constant for a wide range of dE
- For too large values of dE also the spatial system starts to fail
- The **variability of the spatial case** is significantly **greater** than the one of the well-mixed case, probably due to phenomenon of the **oligotrophs**

UNEQUAL ENERGY BUDGET

Well-mixed



Spatial



UNEQUAL ENERGY BUDGET
2 nutrients and presence of oligotrophs

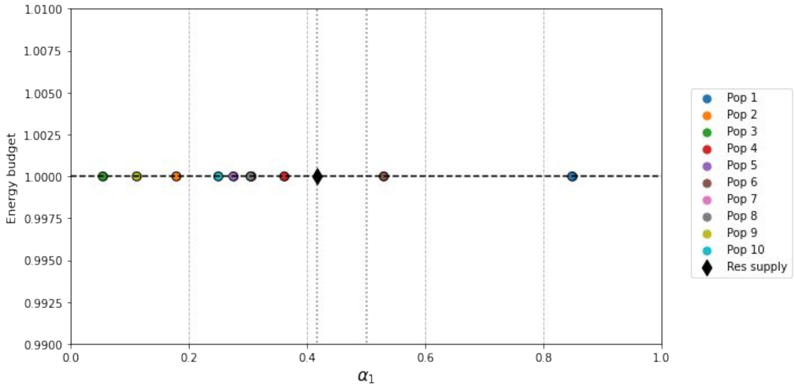
Fix $dE = 10^{-3}$

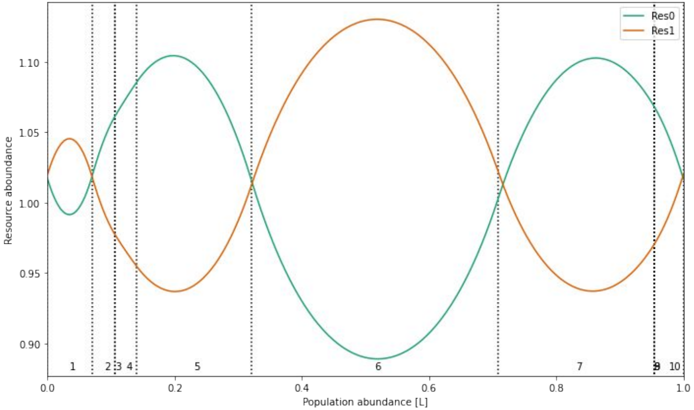
Study behaviour of both spatial and w–m cases **both with perfect and not perfect metabolic tradeoff**

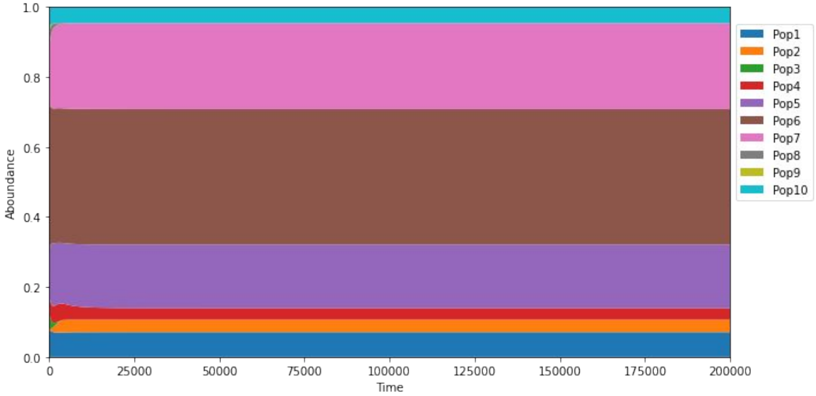
Repeat the previous point **introducing** some species with metabolic strategy coefficients in the **oligotroph**'s region

Compute the effective number of species in each case and compare the results

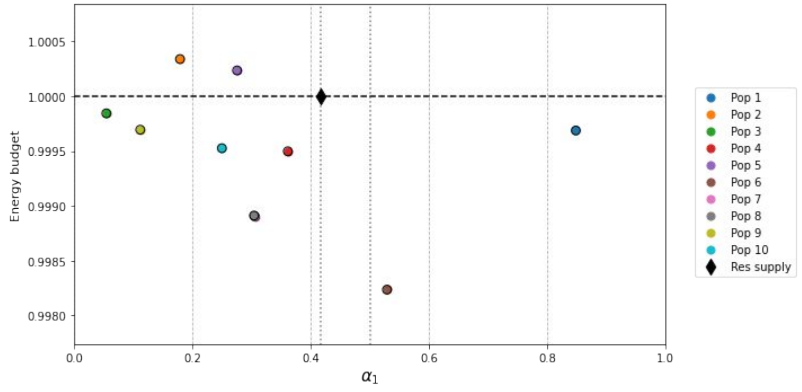
SPATIAL CASE: PERFECT METABOLIC TRADEOFF

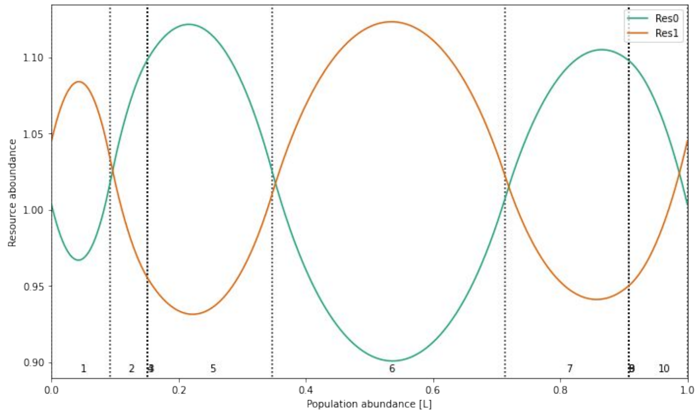


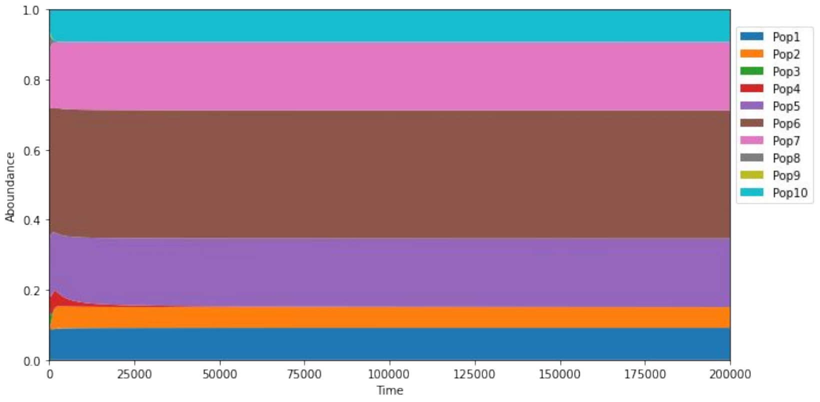




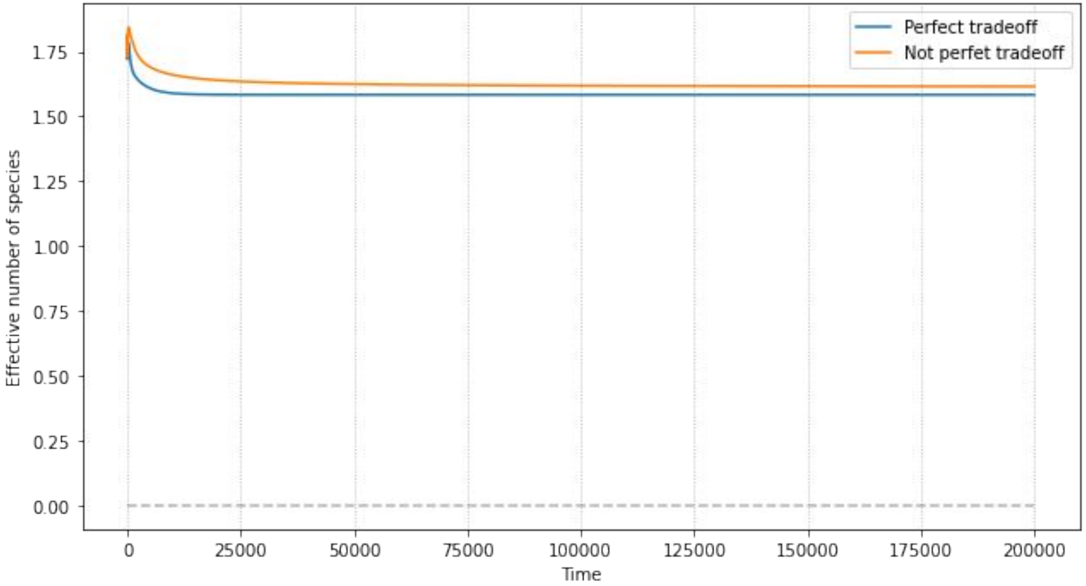
SPATIAL CASE: NOT PERFECT METABOLIC TRADEOFF





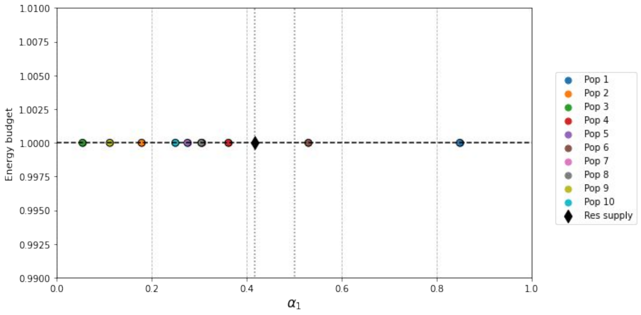


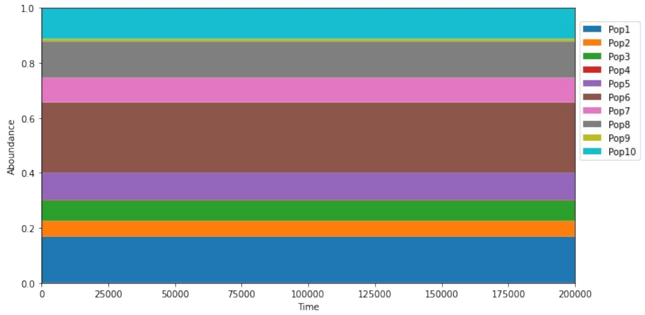
SPATIAL CASE: COMPARISON



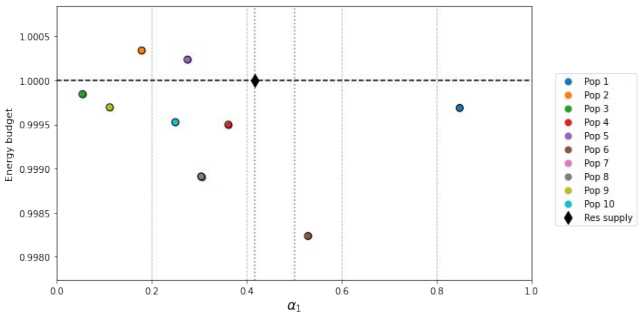
WELL-MIXED CASE

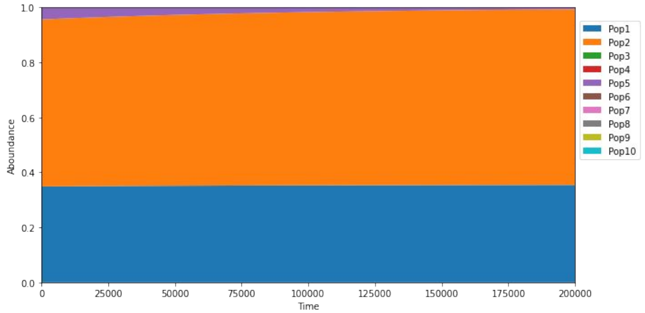
Perfect metabolic tradeoff



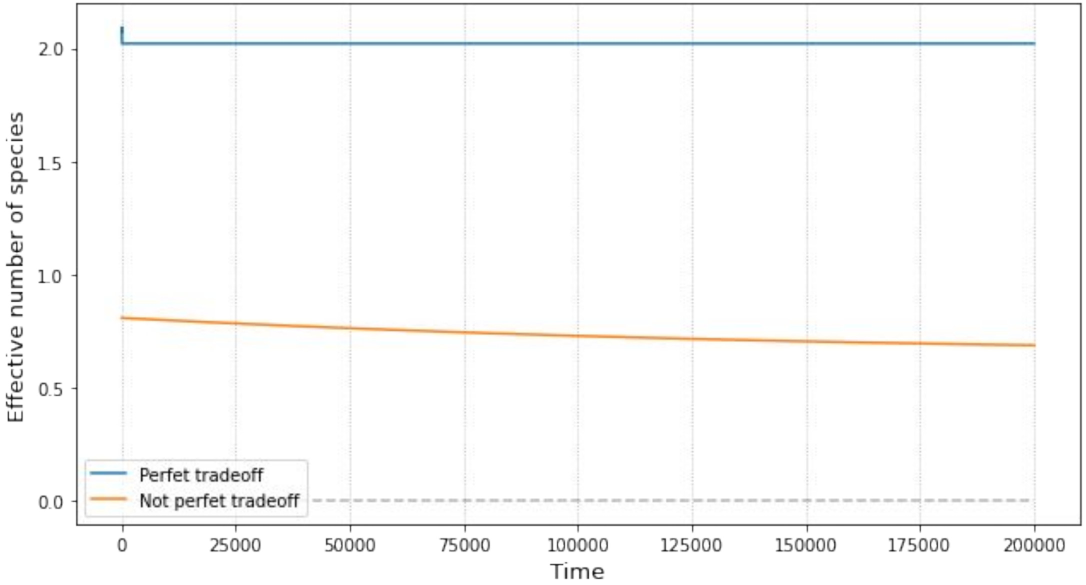


Not perfect metabolic tradeoff



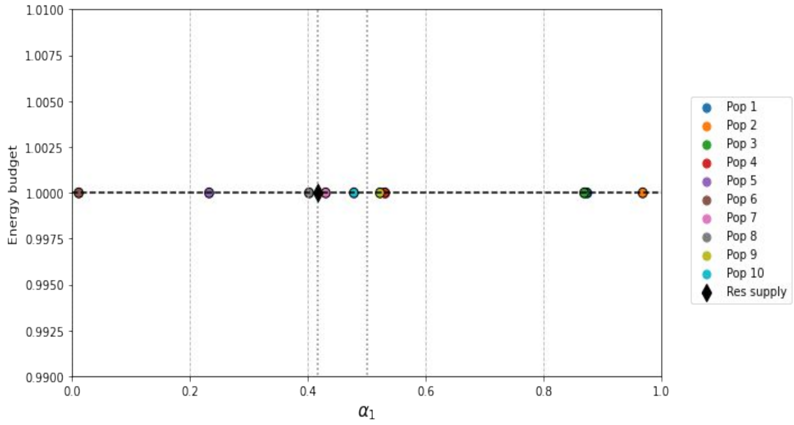


WELL-MIXED CASE: COMPARISON

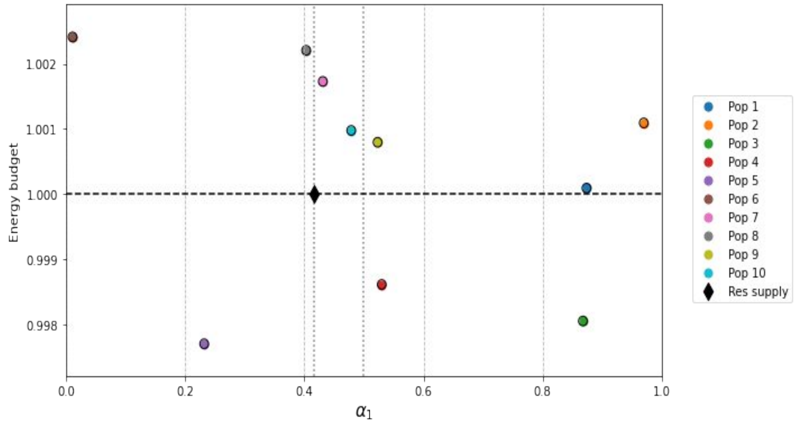


INTRODUCTION OF OLIGOTROPHS

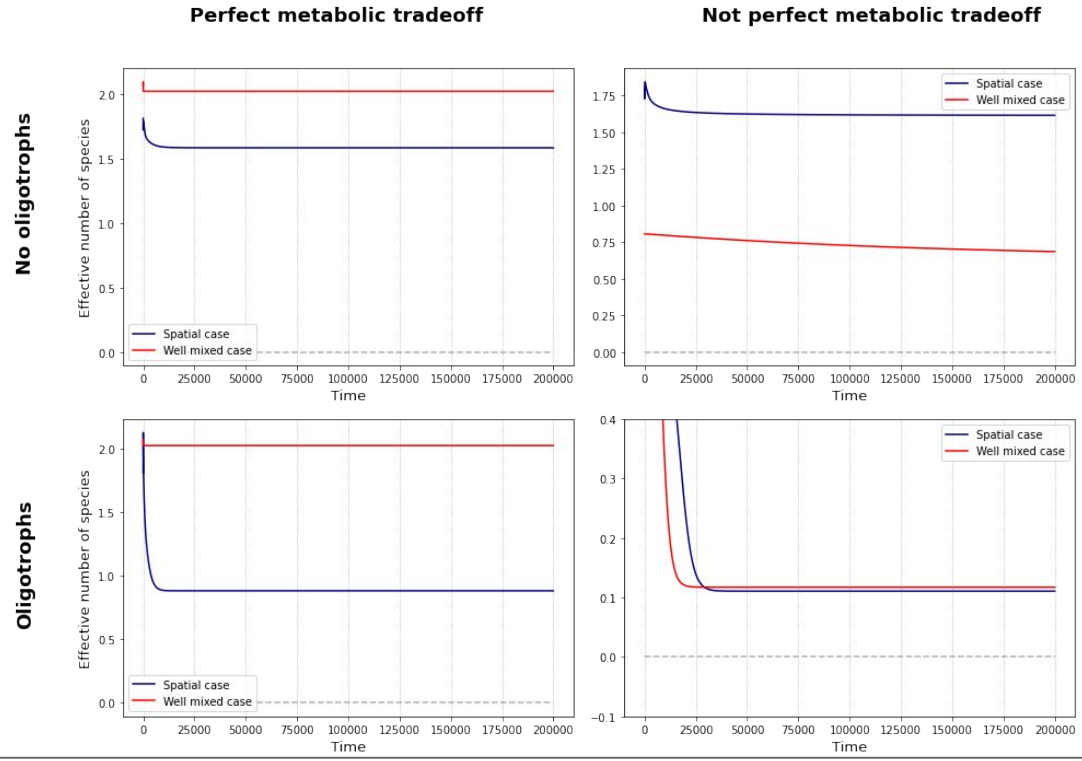
Perfect metabolic tradoff



Not perfect metabolic tradoff



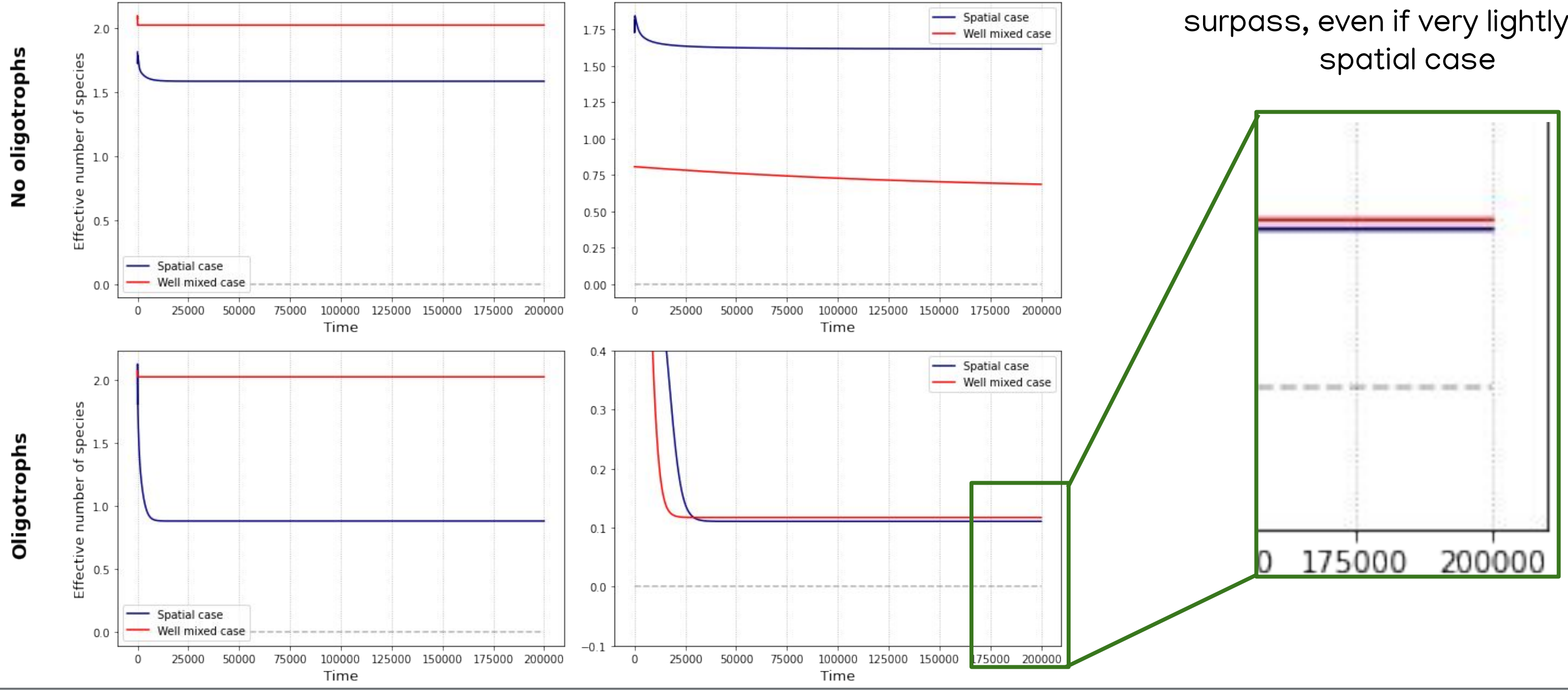
COMPARISON



Perfect metabolic tradeoff
Not perfect metabolic tradeoff
No oligotrophs
Oligotrophs
Effective number of species
Time
Spatial case
Well mixed case

COMPARISON

Perfect metabolic tradeoff | **Not perfect metabolic tradeoff**



In this case the w-m model surpass, even if very lightly, the spatial case

VARIATION OF NUMBER OF NUTRIENTS

Presence of oligotrophs deeply influences the biodiversity as stationarity

How does the **number of nutrients influence** the **mean biodiversity?**

In spatial model grate biodiversity is possible even with a small number of nutrients

VARIATION OF NUMBER OF NUTRIENTS

ANSATZ: the oligotroph's region in the metabolic strategies plane can be approximated as an **hypersfere** with diameter equal to the distance between the center of the corresponding hyper pyramid and the point corresponding to the supply rate *d*

Diameter

$$d = |\vec{S} - \vec{c}|$$

$$c_i = \frac{E}{p} \quad i = 1, ..., p$$

$$\Longrightarrow$$

Volume

$$V = \int_{\mathcal{C}} dV$$

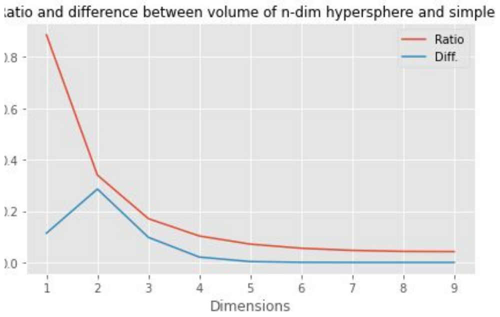
$$\mathcal{C} = \{x_i, i = 1, ..., p-1 : \sum_i x_i^2 < d/2\}$$

VARIATION OF NUMBER OF NUTRIENTS

From math, the ration between the volume of the hyperpyramid and the hypersphere decreases as the number of dimension in which they leave increase

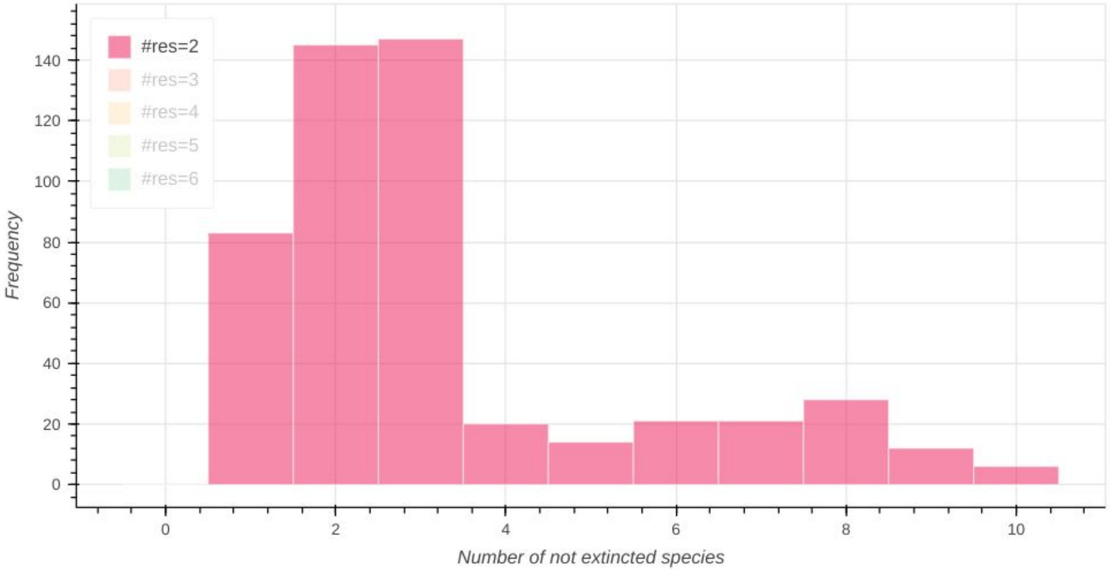
In our case the number of dimensions correspond to *p-1*

The probability that a random metabolic strategy lying in the *p-1* dimensional simplex falls in the metabolic region is proportional to the ratio between the hypersphere and the simplex

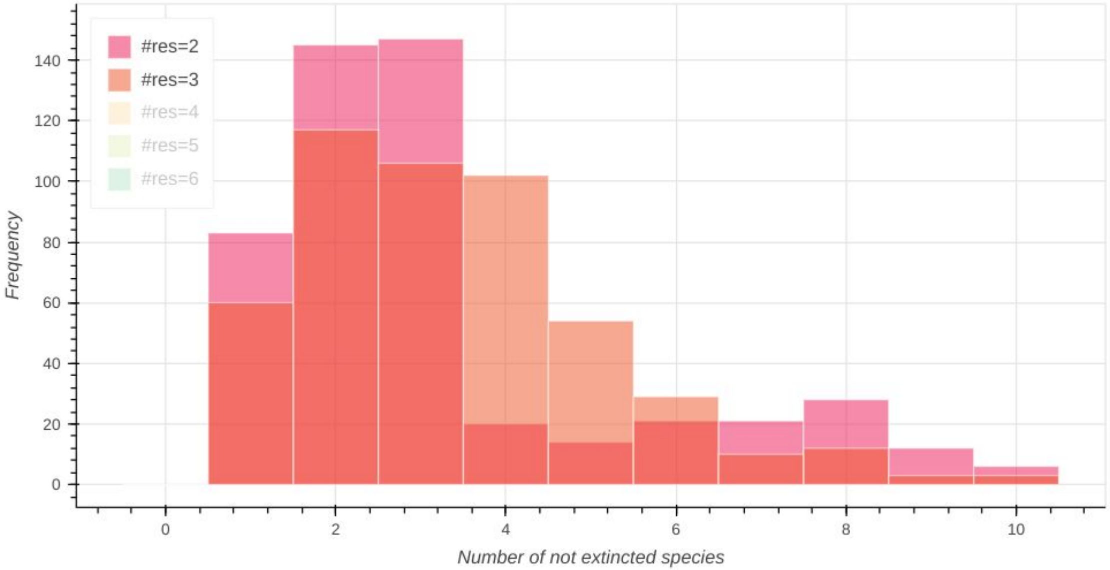


The occurrence of an oligotroph becomes rarer as the number of nutrients increases, and so the <u>biodiversity should increase</u>

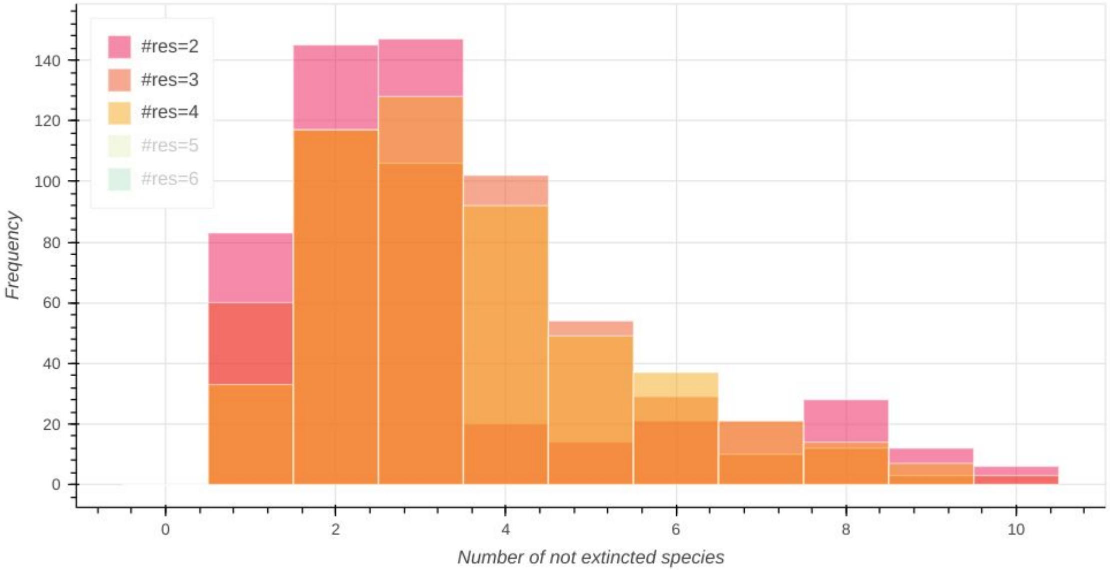
VARIATION OF NUMBER OF NUTRIENTS



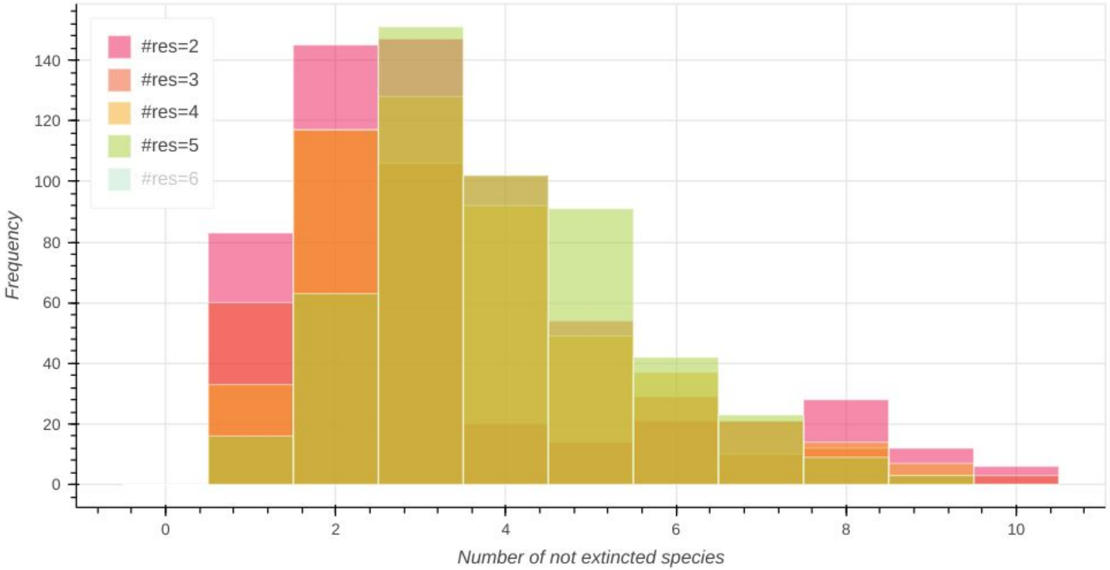
VARIATION OF NUMBER OF NUTRIENTS



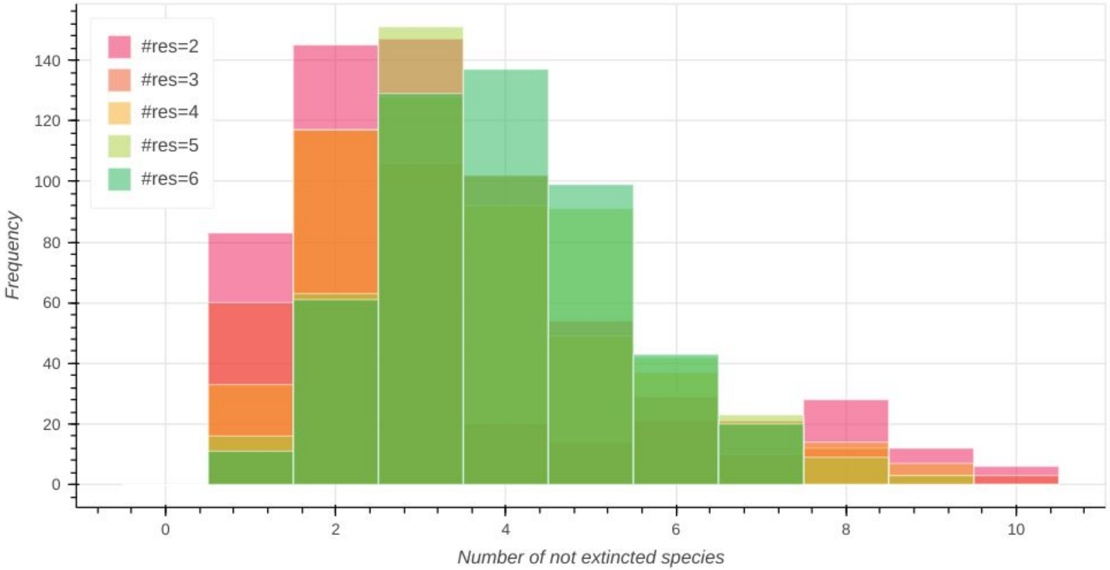
VARIATION OF NUMBER OF NUTRIENTS



VARIATION OF NUMBER OF NUTRIENTS

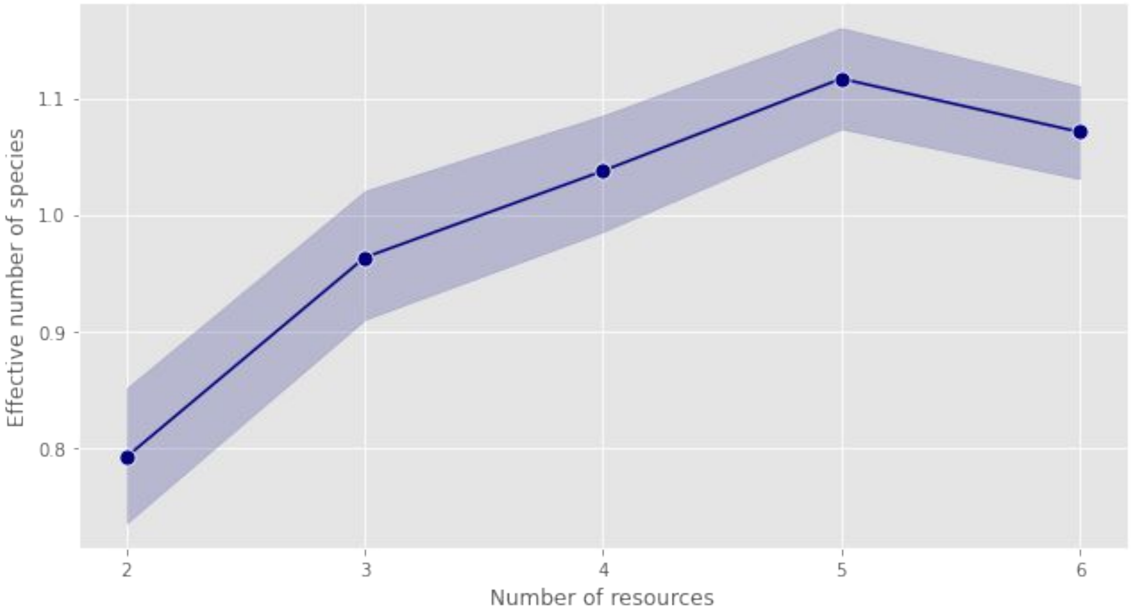


VARIATION OF NUMBER OF NUTRIENTS



VARIATION OF NUMBER OF NUTRIENTS

As expected as the number of nutrients increase, also the biodiversity increase



CONCLUSIONS

In general **reduced mean biodiversity** w.r.t. well mixed case

Small diffusion rates reduce mean biodiversity

Robustness to small **variations** in the metabolic **energy budget**

Presence of **oligotrophs,** species that thanks to their high degree of fitness with the environment, leads the other populations to extinction

THANKS!

KPI OVERVIEW

PEOPLE

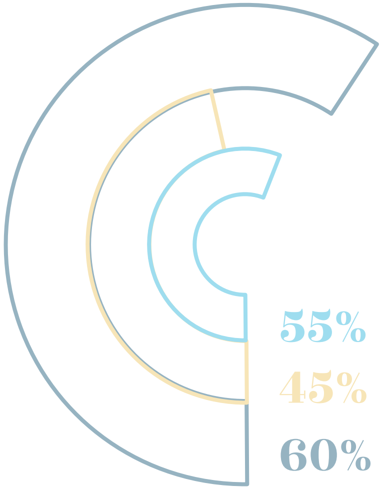
Is the closest planet of all

SALES

Mars is a cold place

MARKET

Jupiter is a gas giant



458K

Neptune is the farthest

7,520,000$

Has a beautiful name

86%

Mars is a cold place

OUR TEAM

AMANDA RICHARDS

You can replace the image on the screen with your own

JENNIFER ADAMS

You can replace the image on the screen with your own

A PICTURE IS WORTH MORE THAN A THOUSAND WORDS

AWESOME WORDS

ICON PACK

RESOURCES

PHOTOS

- Cute girl with her hairstyle
- Front view of business people talking
- Girl
- Front view of business woman talking at phone

VECTORS

- Colorful weekly schedule template with flat design

ICON PACK

- Business Icon Pack

ALTERNATIVE RESOURCES

PHOTOS

- Men shaking hands
- Thoughtful young man sitting near laptop against wall with notes
- Cheerful colleagues looking at project on laptop in office
- Smiling couple sitting at desk with laptops against wall with notes
- Colleagues happy to work together
- Adult women working on new project together
- Businesswoman posing outdoors with tablet ad notepad
- Front view of businesswoman working on laptop in the office
- Group of businesspeople making plans on energy saving at workplace

Instructions for use

In order to use this template, you must credit **Slidesgo** by keeping the **Thanks** slide.

You are allowed to:

- Modify this template.
- Use it for both personal and commercial projects.

You are not allowed to:

- Sublicense, sell or rent any of Slidesgo Content (or a modified version of Slidesgo Content).
- Distribute Slidesgo Content unless it has been expressly authorized by Slidesgo.
- Include Slidesgo Content in an online or offline database or file.
- Offer Slidesgo templates (or modified versions of Slidesgo templates) for download.
- Acquire the copyright of Slidesgo Content.

For more information about editing slides, please read our FAQs or visit Slidesgo School:
https://slidesgo.com/faqs and https://slidesgo.com/slidesgo-school

Fonts & colors used

This presentation has been made using the following fonts:

Barlow
(https://fonts.google.com/specimen/Barlow)

Barlow Condensed
(https://fonts.google.com/specimen/Barlow+Condensed)

#c1e1d6 **#f8c2b2** **#d9d9d9** **#c1cfe1** **#1d262d**

Storyset

Create your Story with our illustrated concepts. Choose the style you like the most, edit its colors, pick the background and layers you want to show and bring them to life with the animator panel! It will boost your presentation. Check out How it Works.

Pana

Amico

Bro

Rafiki

Cuate

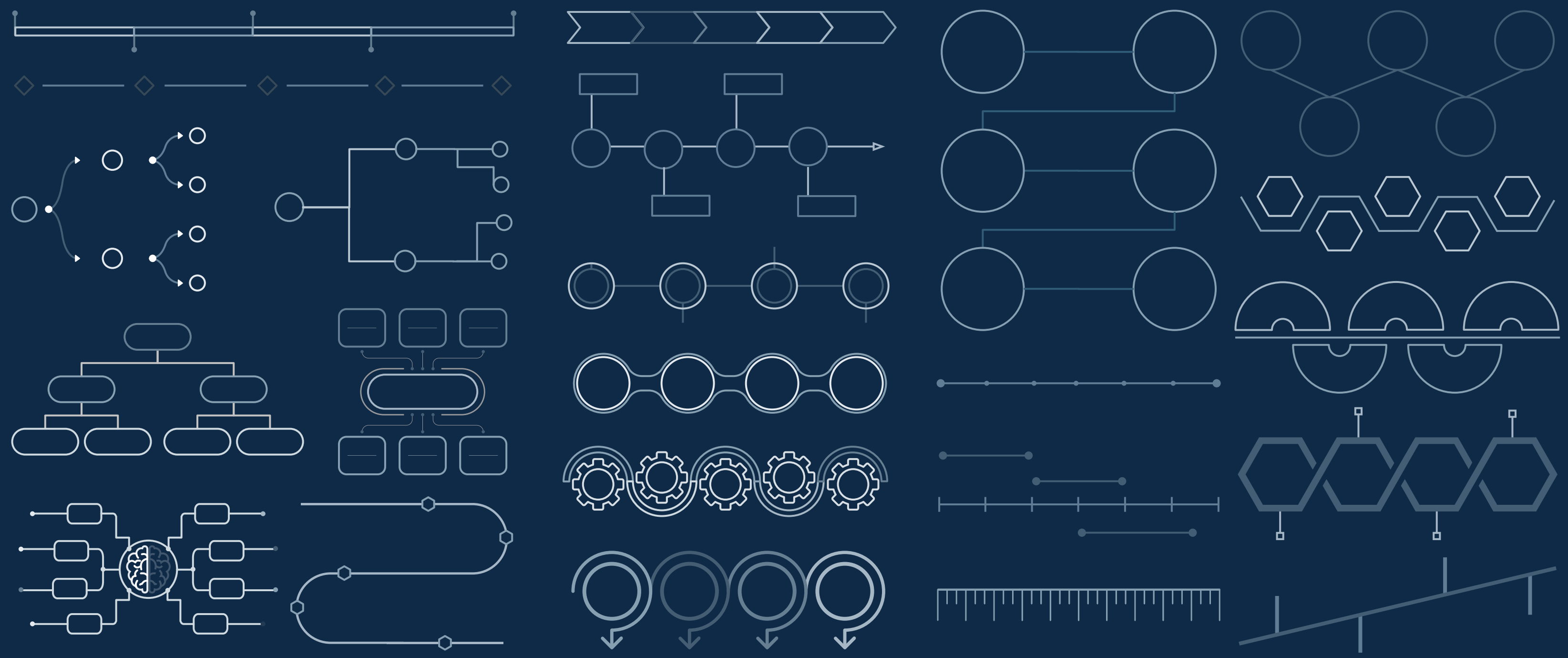
Use our editable graphic resources...

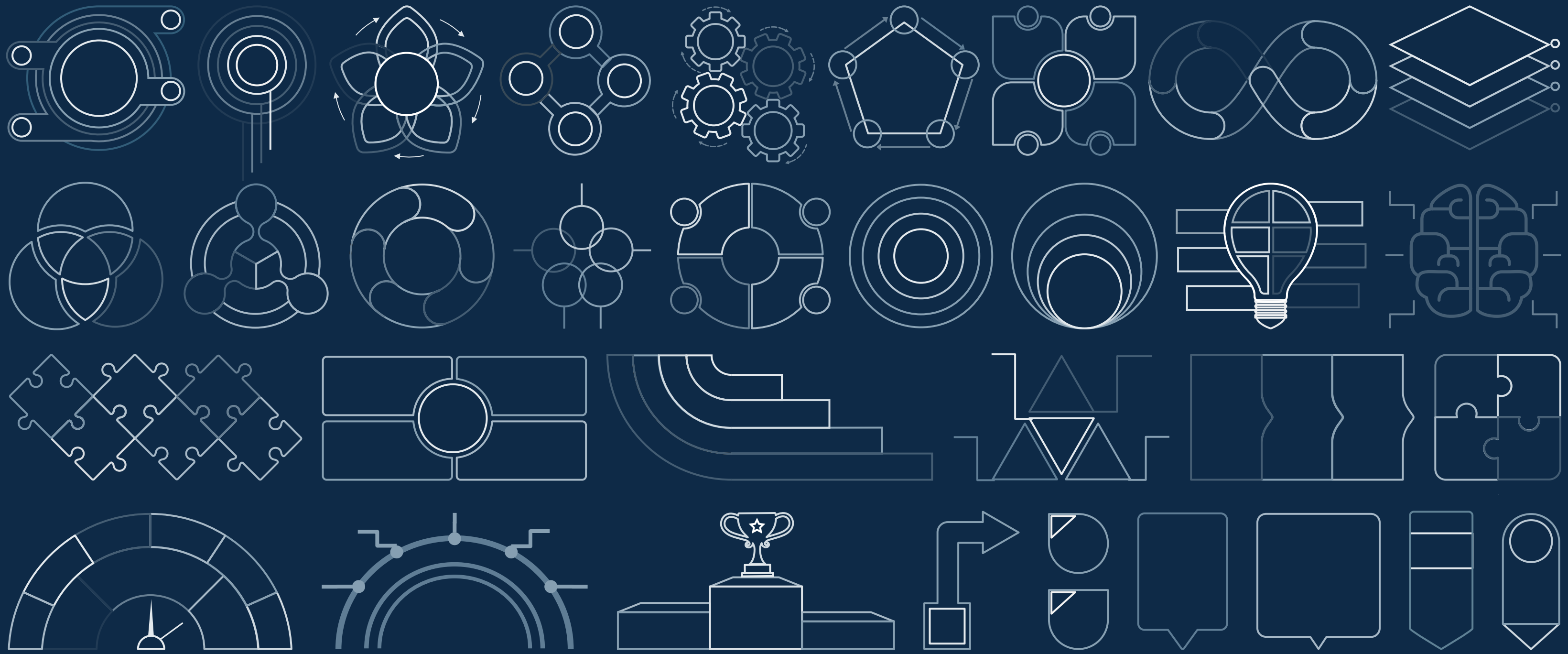
You can easily resize these resources without losing quality. To change the color, just ungroup the resource and click on the object you want to change. Then, click on the paint bucket and select the color you want. Group the resource again when you're done. You can also look for more infographics on Slidesgo.

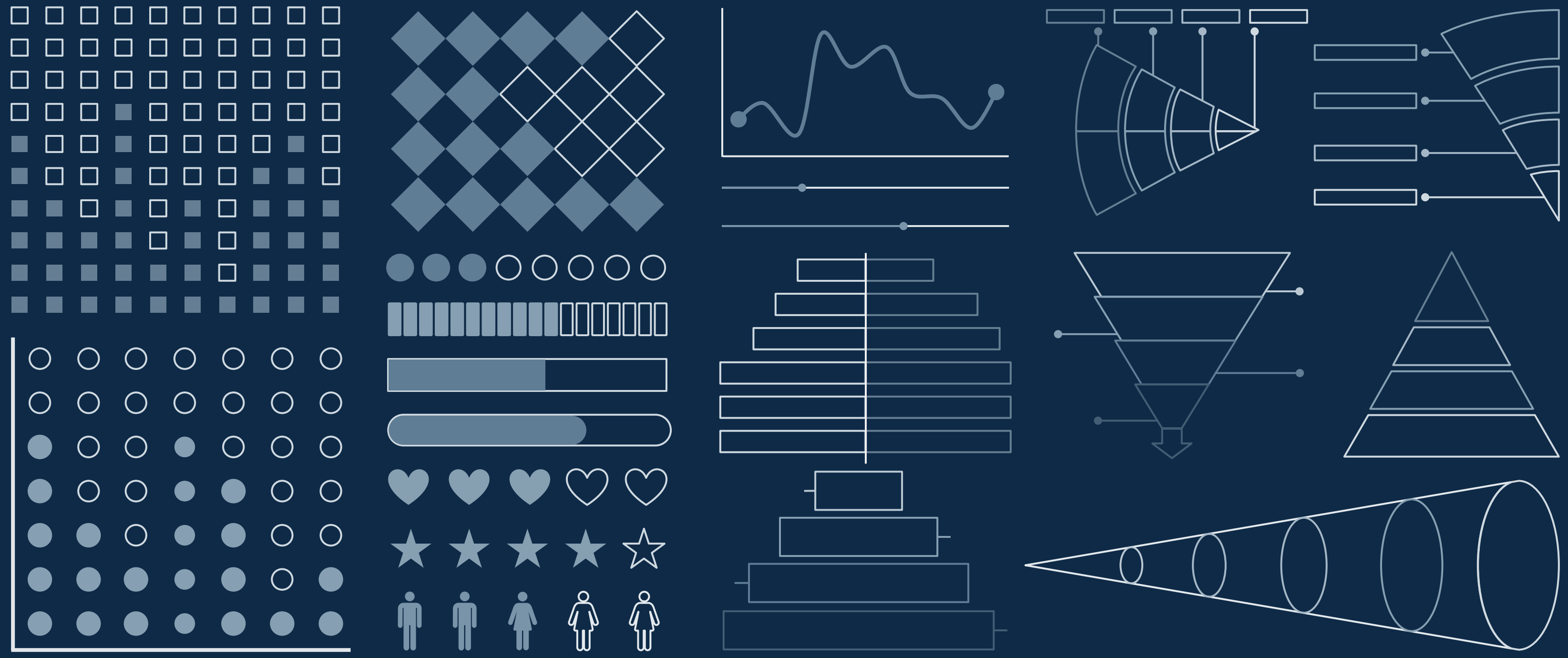












...and our sets of editable icons

You can resize these icons without losing quality.
You can change the stroke and fill color; just select the icon and click on the paint bucket/pen.
In Google Slides, you can also use Flaticon’s extension, allowing you to customize and add even more icons.

Educational Icons

Medical Icons

Business Icons

Teamwork Icons

Help & Support Icons

Avatar Icons

Creative Process Icons

Performing Arts Icons

Nature Icons

SEO & Marketing Icons

slidesgo