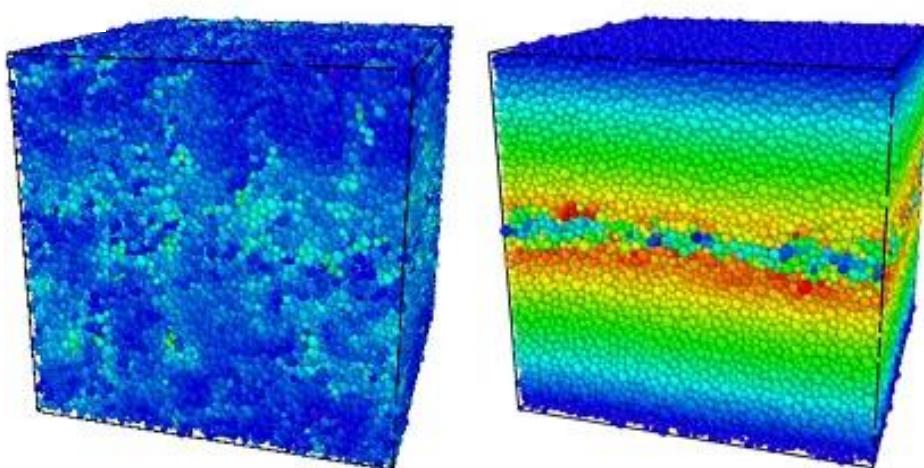


# Random critical point separates brittle and ductile yielding transitions in amorphous materials



Misaki Ozawa

Montpellier University



Collaborators

L. Berthier, G. Biroli, A. Rosso, and G. Tarjus

Ozawa, Berthier, Biroli, Rosso, and Tarjus, PNAS 2018

# Amorphous solids



METALLIC  
AND  
SILICATE  
GLASSES

POLYMER  
GLASSES

COLLOIDAL GLASSES

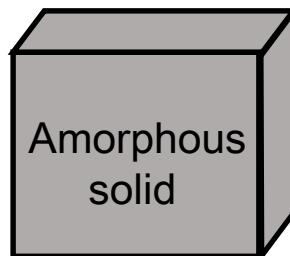
COLLOIDAL GELS

FOAMS

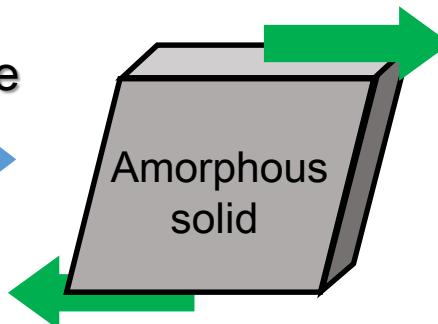
EMULSIONS

GRANULAR SOLIDS

DENSE GRANULAR SUSPENSIONS



Deformation,  
External force

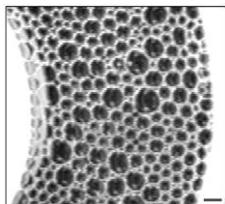


- Yielding
- Plastic flow
- Avalanche
- Shearband
- etc...

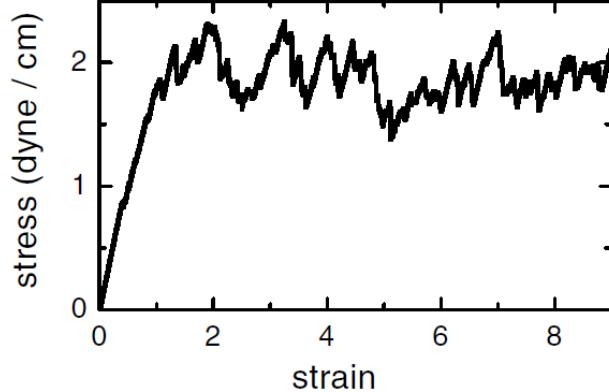
Universal!?

# Stress vs. strain curves

Foams

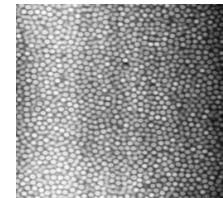


Lauridsen et al., PRL 2002

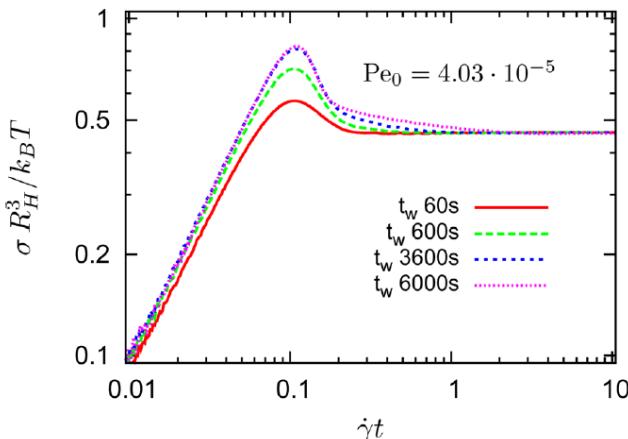


Monotonic crossover  
(Ductile)

Colloids

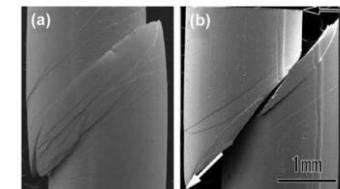


Amann et al., J. Rheol. 2013

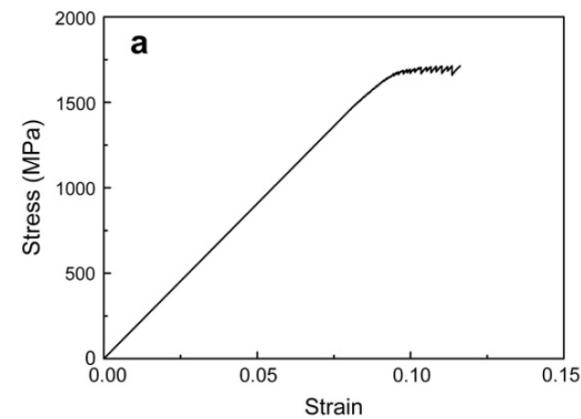


Mild stress overshoot  
(Ductile)

Metallic glasses



Song et al., INTERMETALLICS 2008

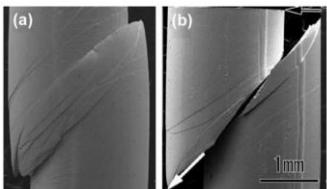


Sharp stress drop  
and shearband  
(Brittle)

What is the origin of different yielding behaviors? Unified picture?

# Unified descriptions

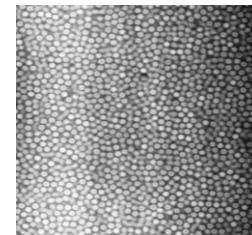
Metallic glasses



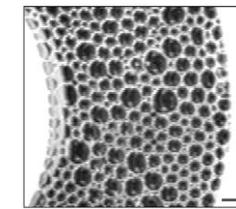
Oxide glasses



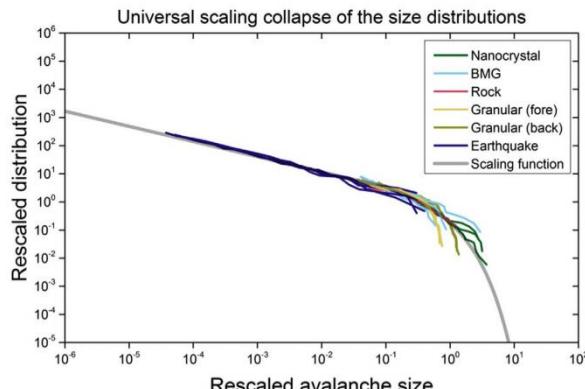
Colloids



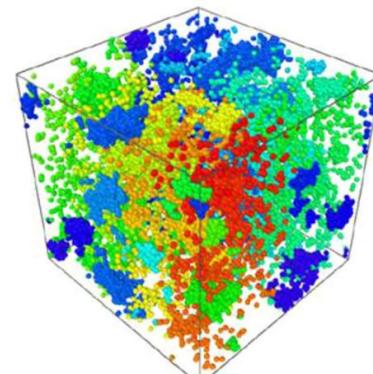
Foams



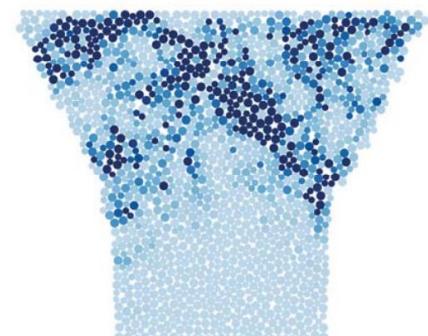
Avalanche scaling



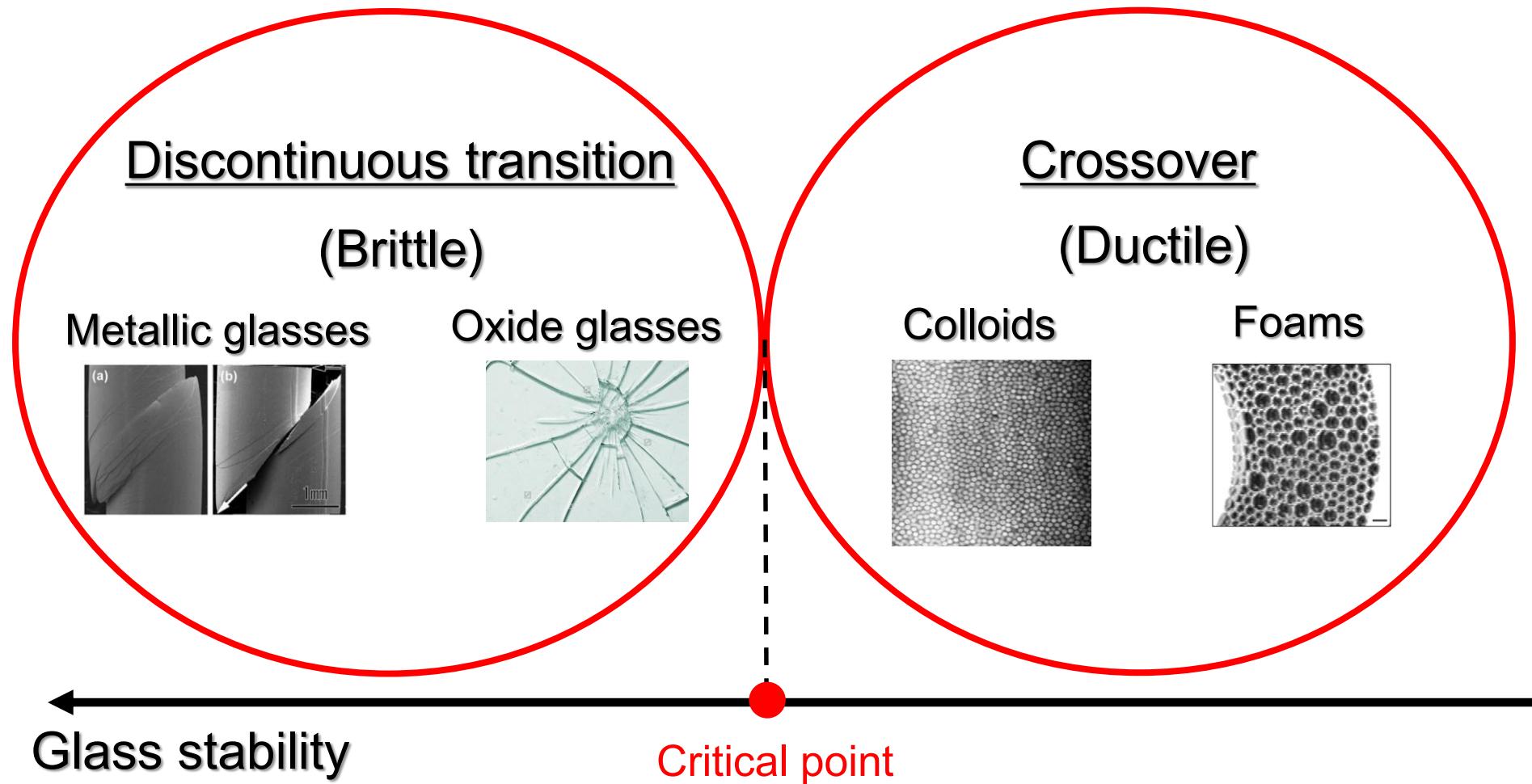
Growing avalanche



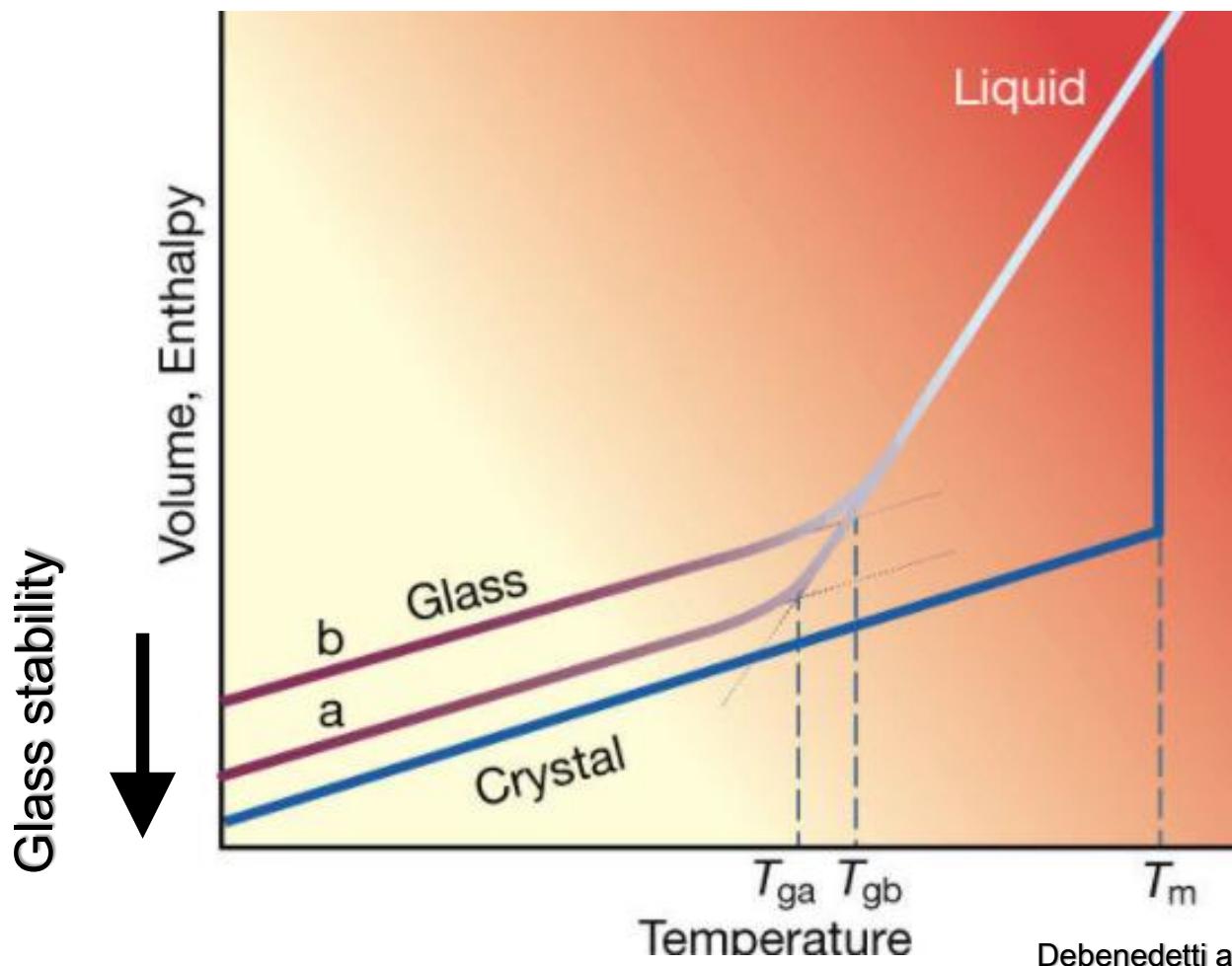
Soft spots



# Our unified picture of yielding transition



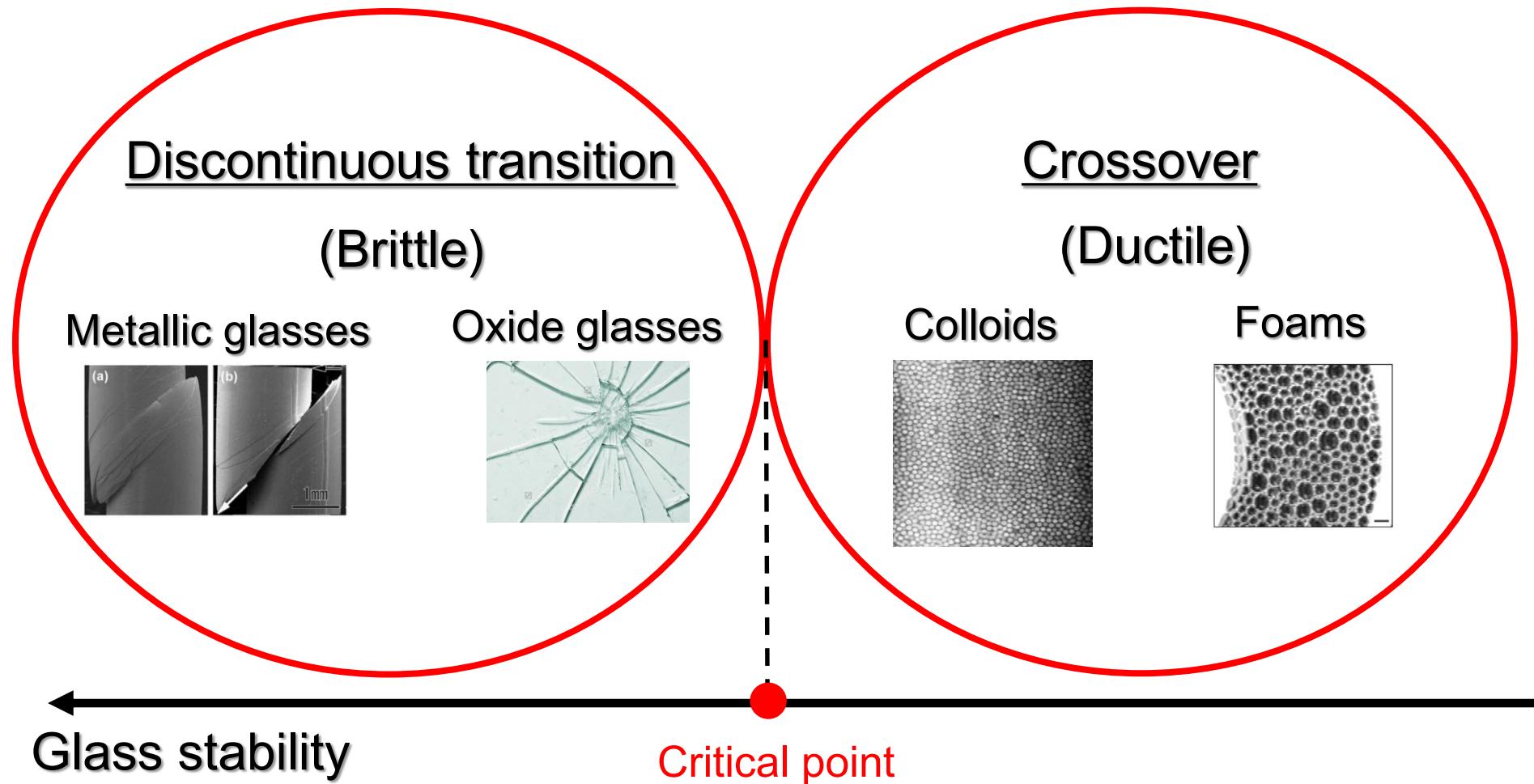
# Glass stability



Debenedetti and Stillinger, Nature 2001

Glass stability increases with decreasing cooling rate

# Our unified picture of yielding transition



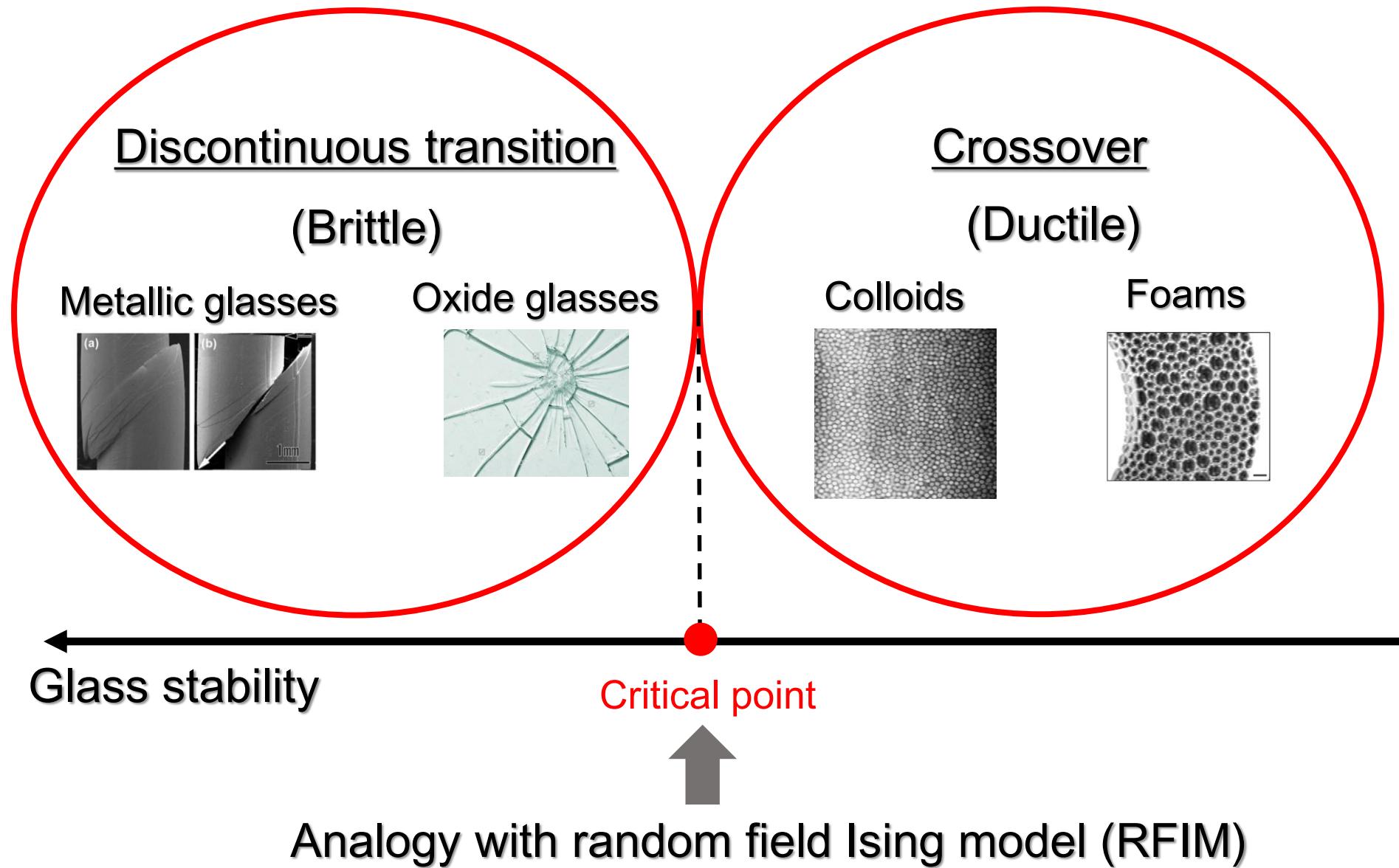
Many papers about preparation protocols, aging, stability of glasses

Kumar, Neibecker, Liu, and Schroes, Nat. Commun. 2013

Jin, Urbani, Zamponi, and Yoshino, arXiv 2018

Vasoya, Rycroft, and Bouchbinder, PRAp 2016, etc...

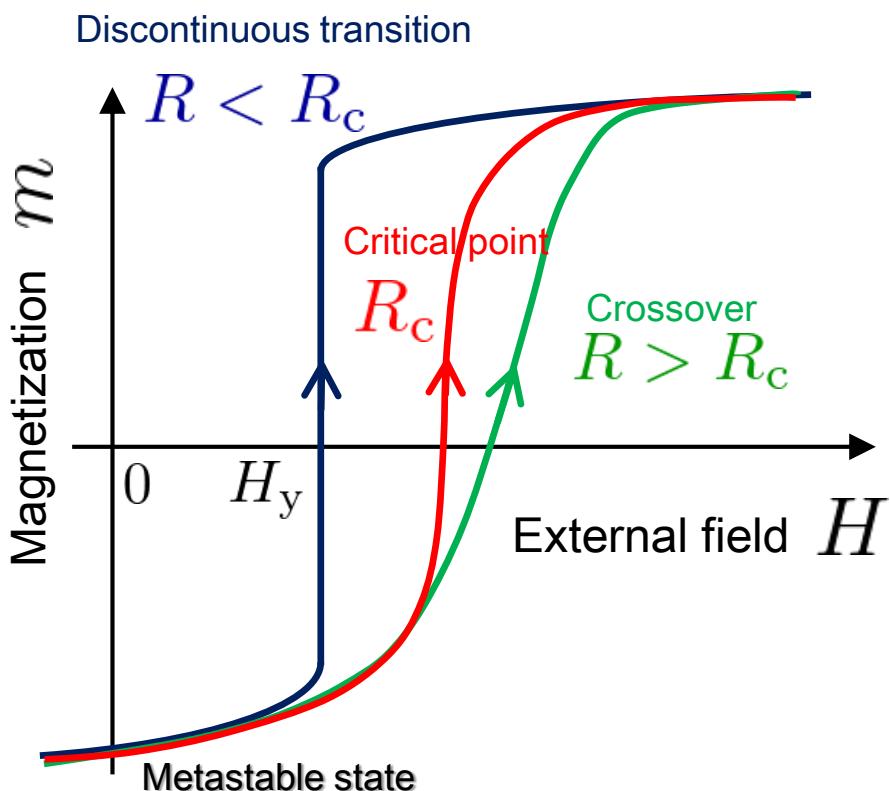
# Our unified picture of yielding transition



# Athermally ( $T=0$ ) driven random field Ising model (RFIM)

Model

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i (h_i + H(t)) S_i$$



Sethna, Dahmen, and Perkovic,  
The science of Hysteresis, 2005

Gaussian random fields

$$\overline{h_i} = 0$$

$$\overline{h_i^2} = R^2 : \text{Strength of disorder}$$

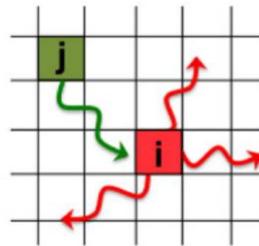
Analogy

RFIM	Yielding
Magnetization	Stress
External field	Strain
$R < R_c$	Brittle yielding
$R > R_c$	Ductile yielding
Rare droplet	Soft spot

Nandi, Biroli, and Tarjus, PRL 2016

# Our strategy

## ■ A mean-field elastoplastic model (Talk by Alberto Rosso)

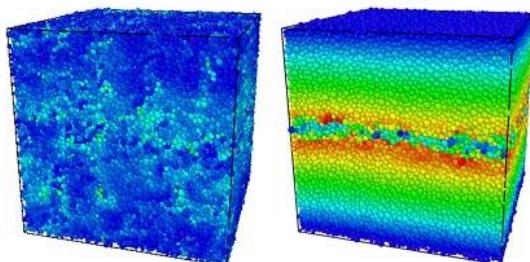


Inspired by a depinning model Jagla, Landes, and Rosso, PRL 2014

Universality class of the athermally driven RFIM

Bocquet, Colin, Ajdari, PRL 2009

## ■ Molecular simulations (This talk!)



Our Model: Polydisperse soft spheres  $N = 1000 - 96000$

Glass preparation: Swap Monte-Carlo

Ninarello, Berthier, and Coslovich, PRX 2017

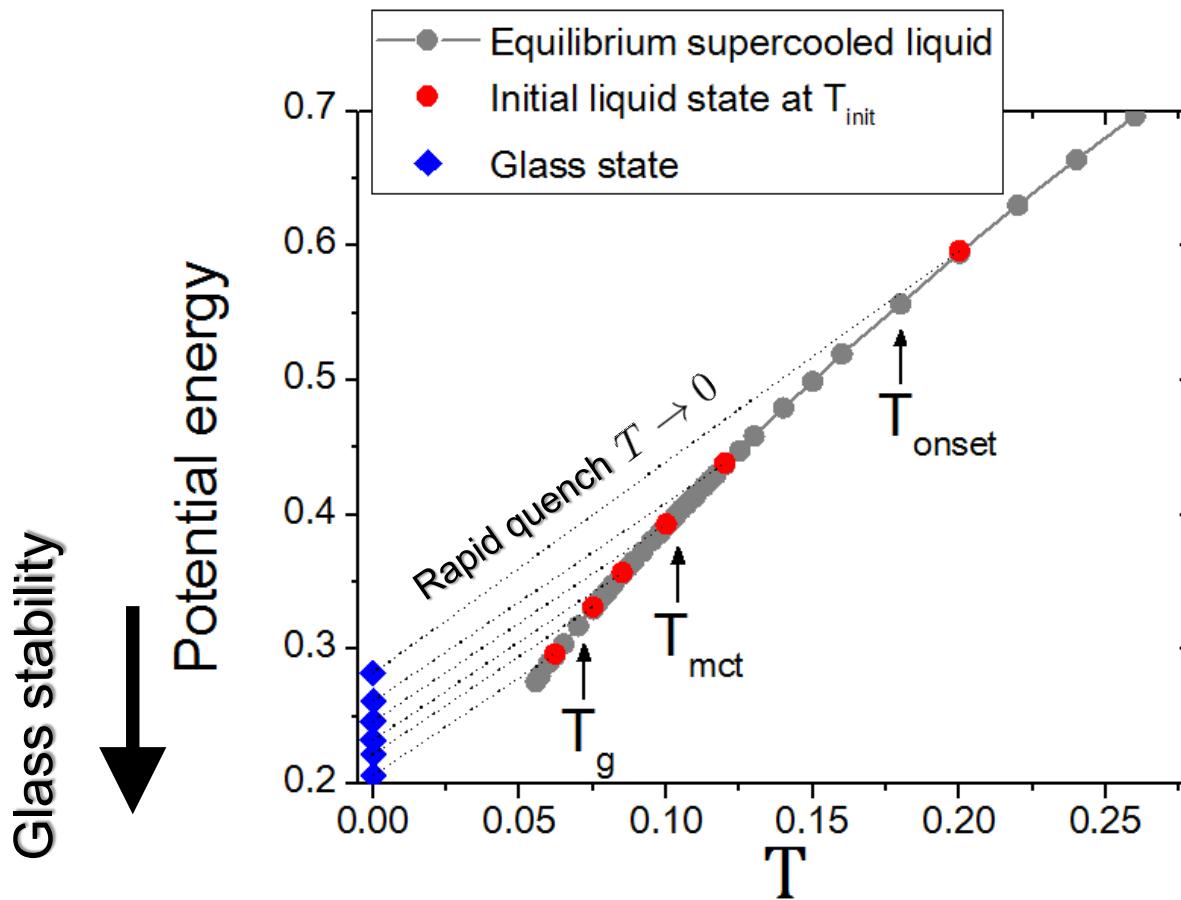
Shear protocol: Athermal quasistatic shear

Maloney and Lemaître, PRE 2006

We will show

Critical point, rare droplet (soft spots), dimensional dependence

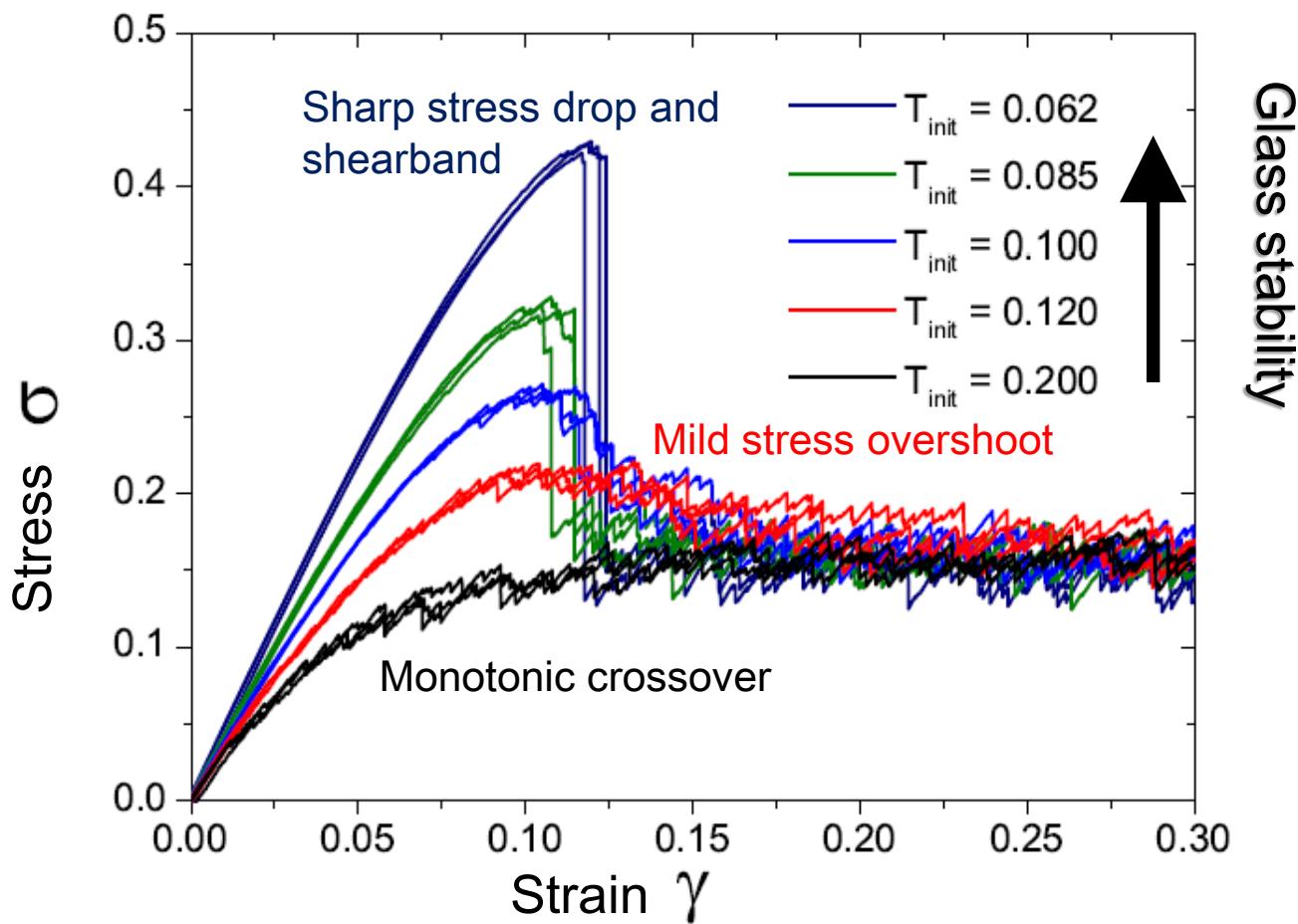
# How we prepare glass samples



Glasses are generated by rapid quench of liquids at  $T_{\text{init}}$

$T_{\text{init}}$  characterizes glass stability ( $R$  in RFIM)

# Stress vs. strain curve



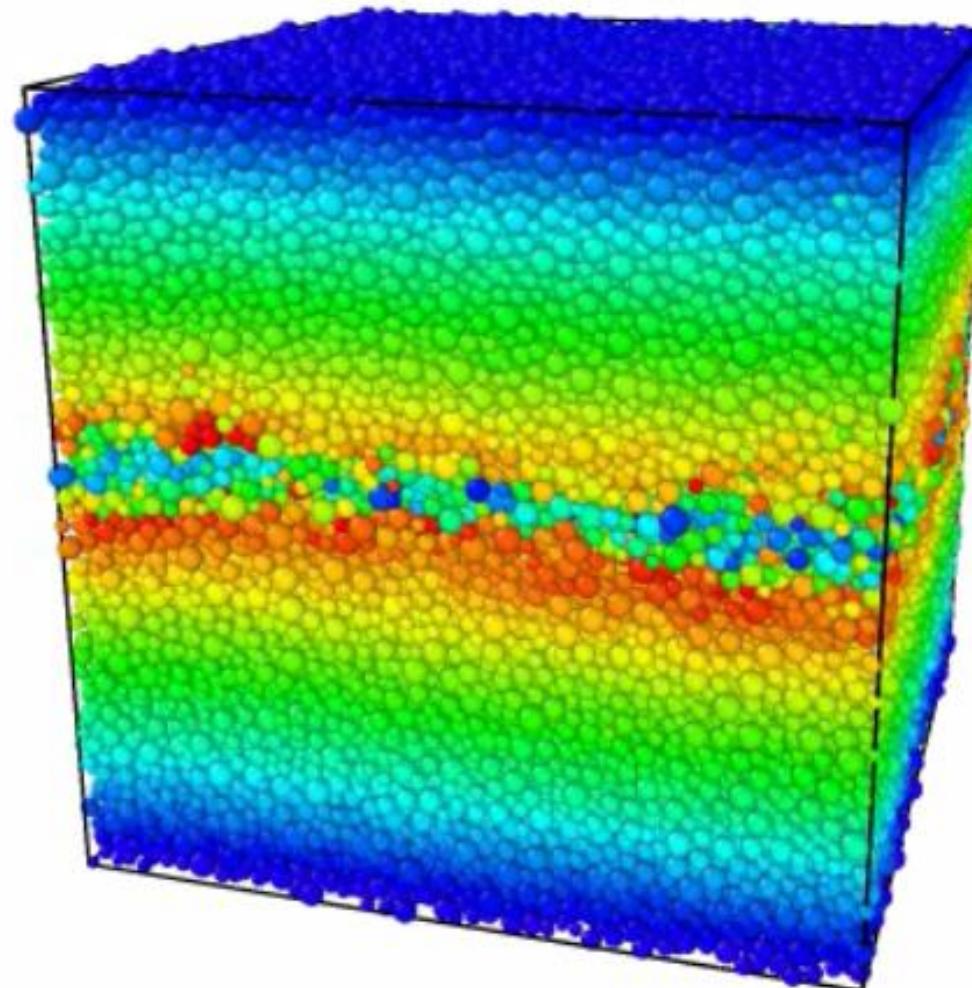
Athermal quasi-static shear simulation at  $T = 0$

Essential phenomenology of yielding are covered by changing glass stability

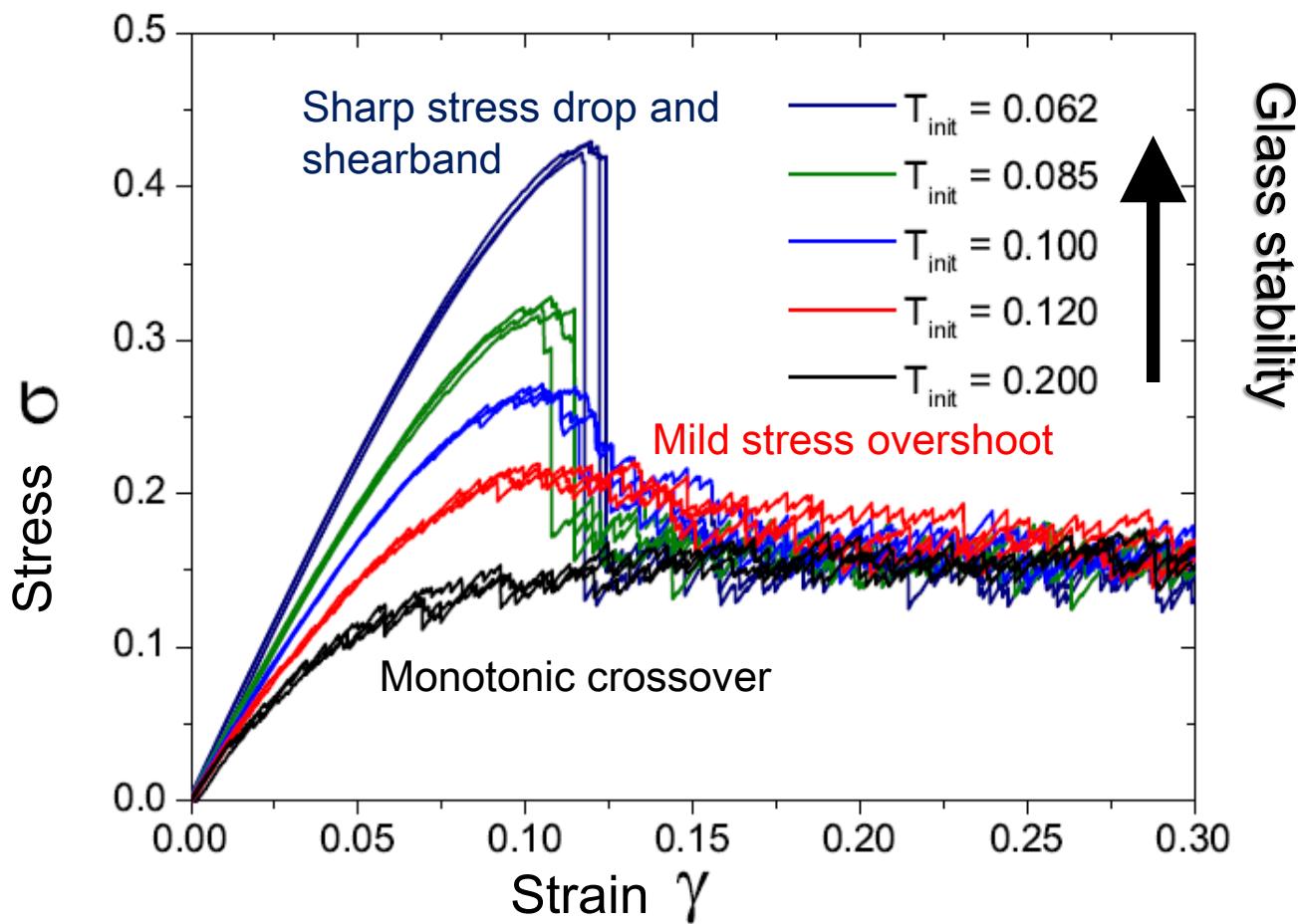
# Shearband movie

$\gamma = 122\text{e-}3$

Non-affine displacement  
0 2



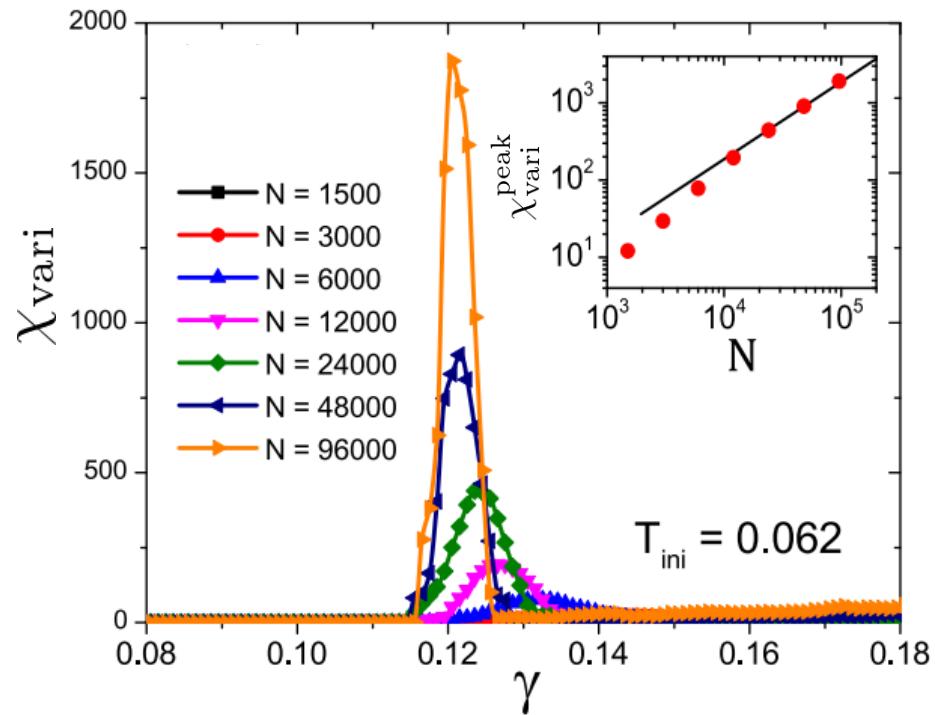
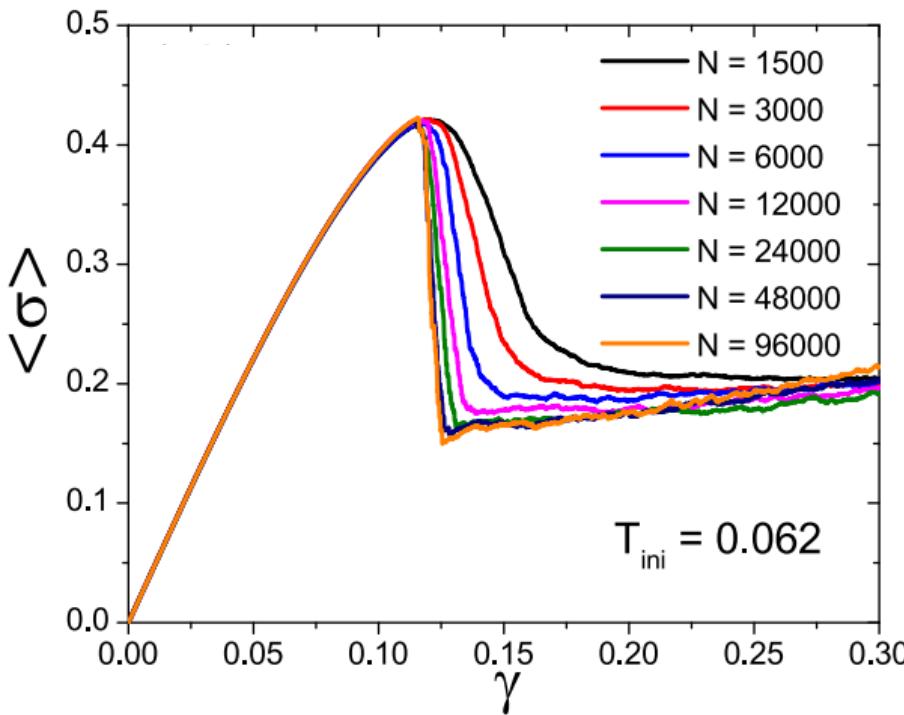
# Stress vs. strain curve



Athermal quasi-static shear simulation at  $T = 0$

Essential phenomenology of yielding are covered by changing glass stability

# Yielding transition (Brittle)



$\langle (\dots) \rangle$  : Averaging over samples

$$\chi_{vari} = N(\langle \sigma^2 \rangle - \langle \sigma \rangle^2)$$

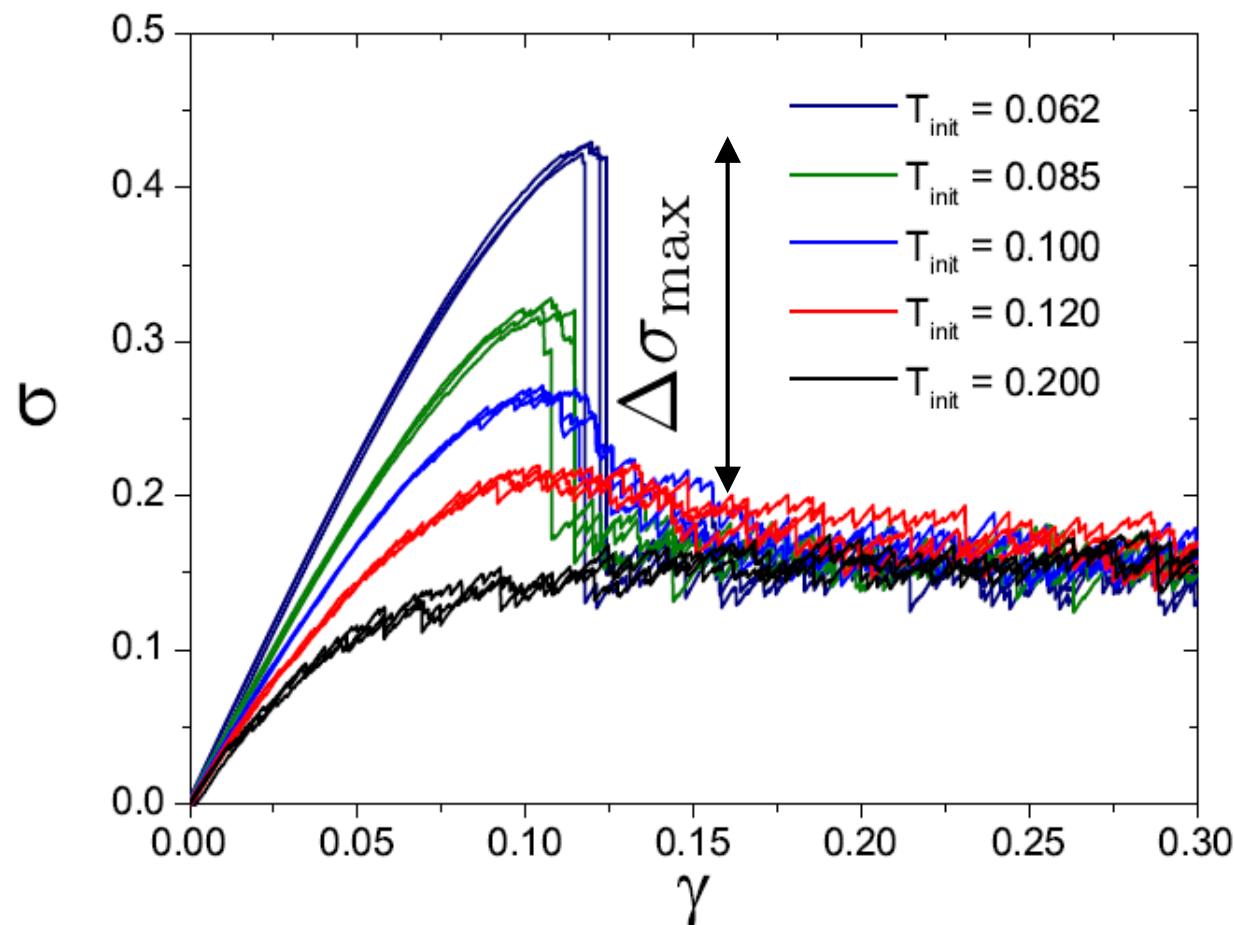
Brittle yielding is a discontinuous “transition” that survives at  $N \rightarrow \infty$  !

Jaiswal, Procaccia, Rainone, and Singh, PRL 2016

Urbani and Zamponi, PRL 2017

Kapteijns, Ji, Brito, Wyart, and Lerner, arXiv 2018

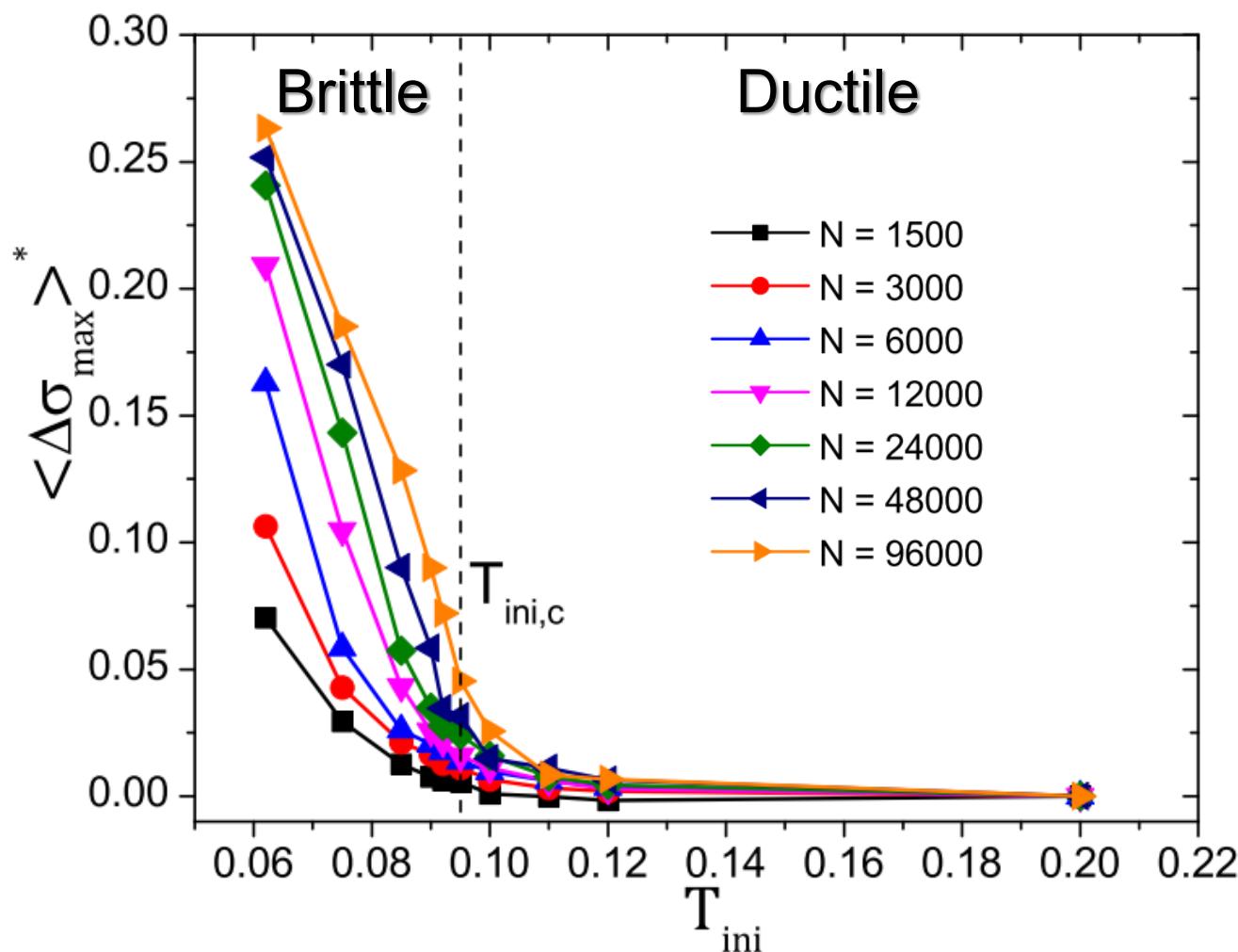
# Order parameter



Control parameter:  $T_{\text{init}}$

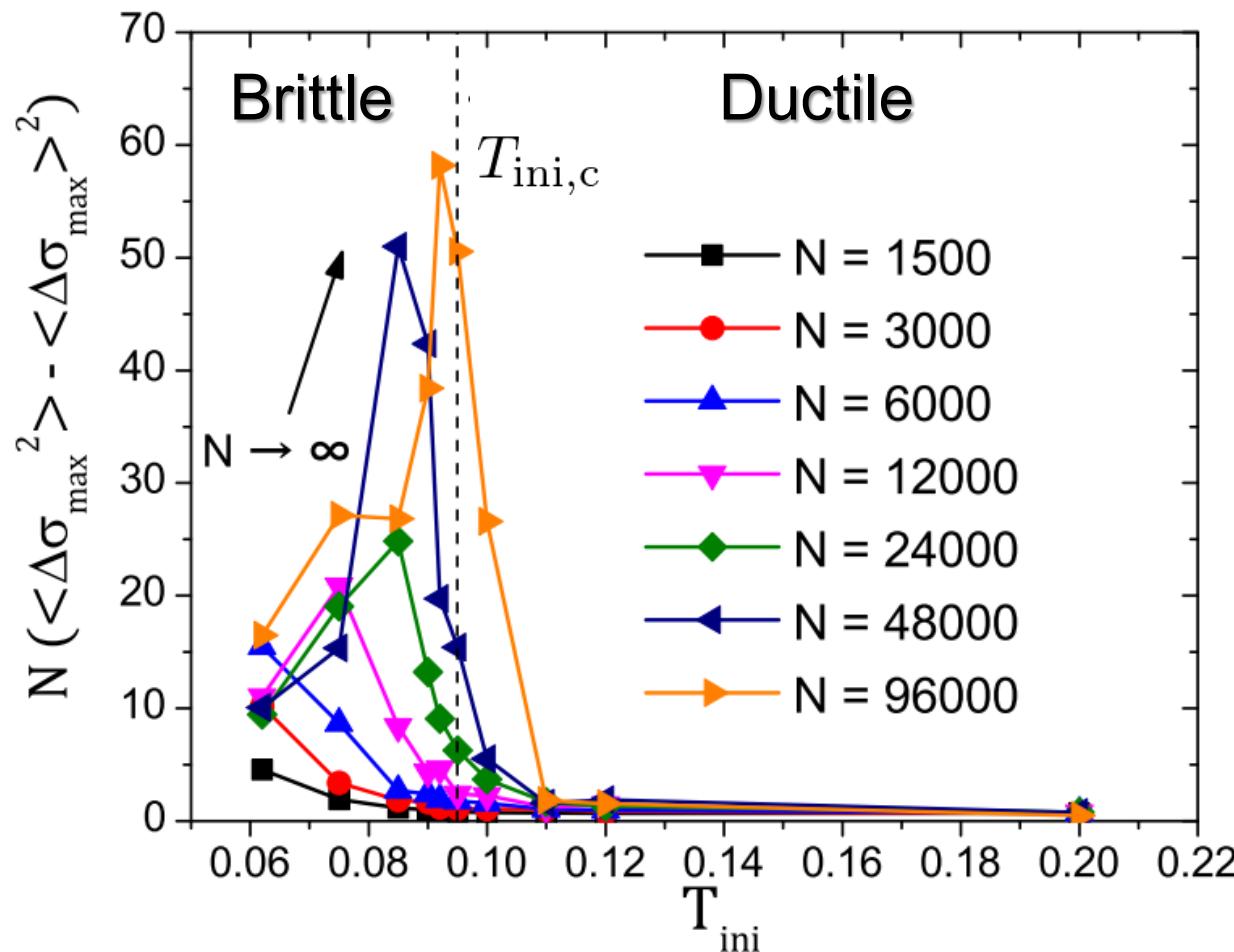
Order parameter for brittle-to-ductile transition :  $\Delta\sigma_{\text{max}}$

# Growth of order parameter



$\langle \Delta\sigma_{\max} \rangle$  develops below  $T_{\text{ini},c}$ , suggesting a critical point

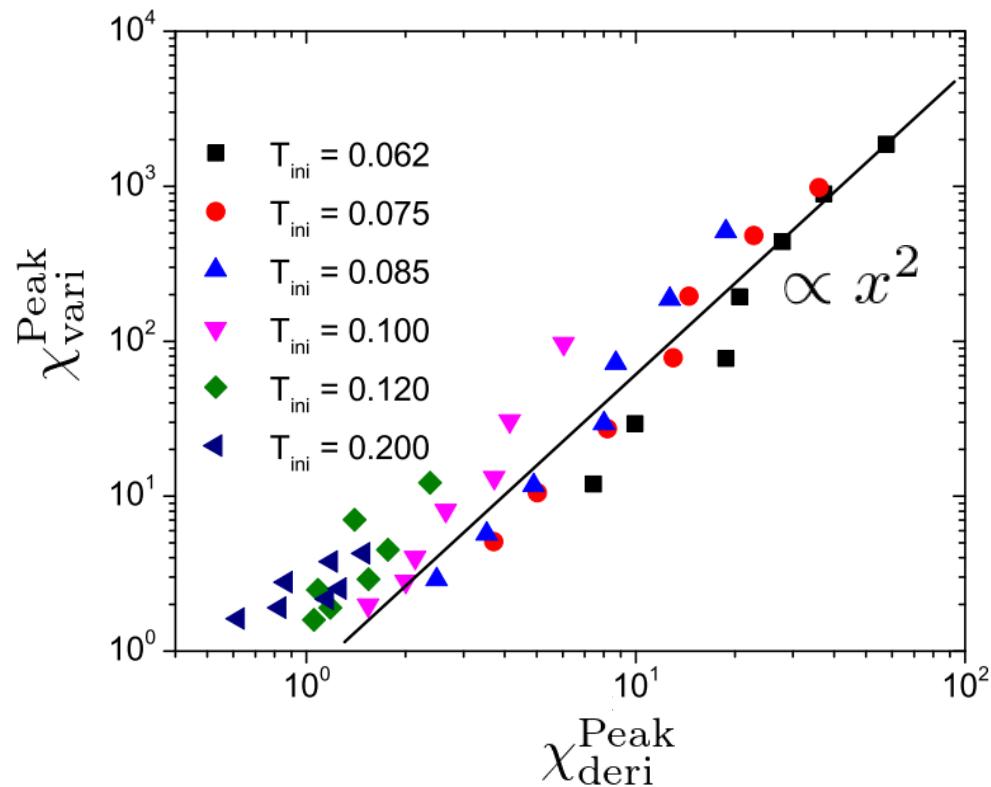
# Divergence of variance



Rapid growth of the variance further supports the existence of the critical point

Physical meaning: Strong sample-to-sample fluctuations near  $T_{\text{ini},c}$

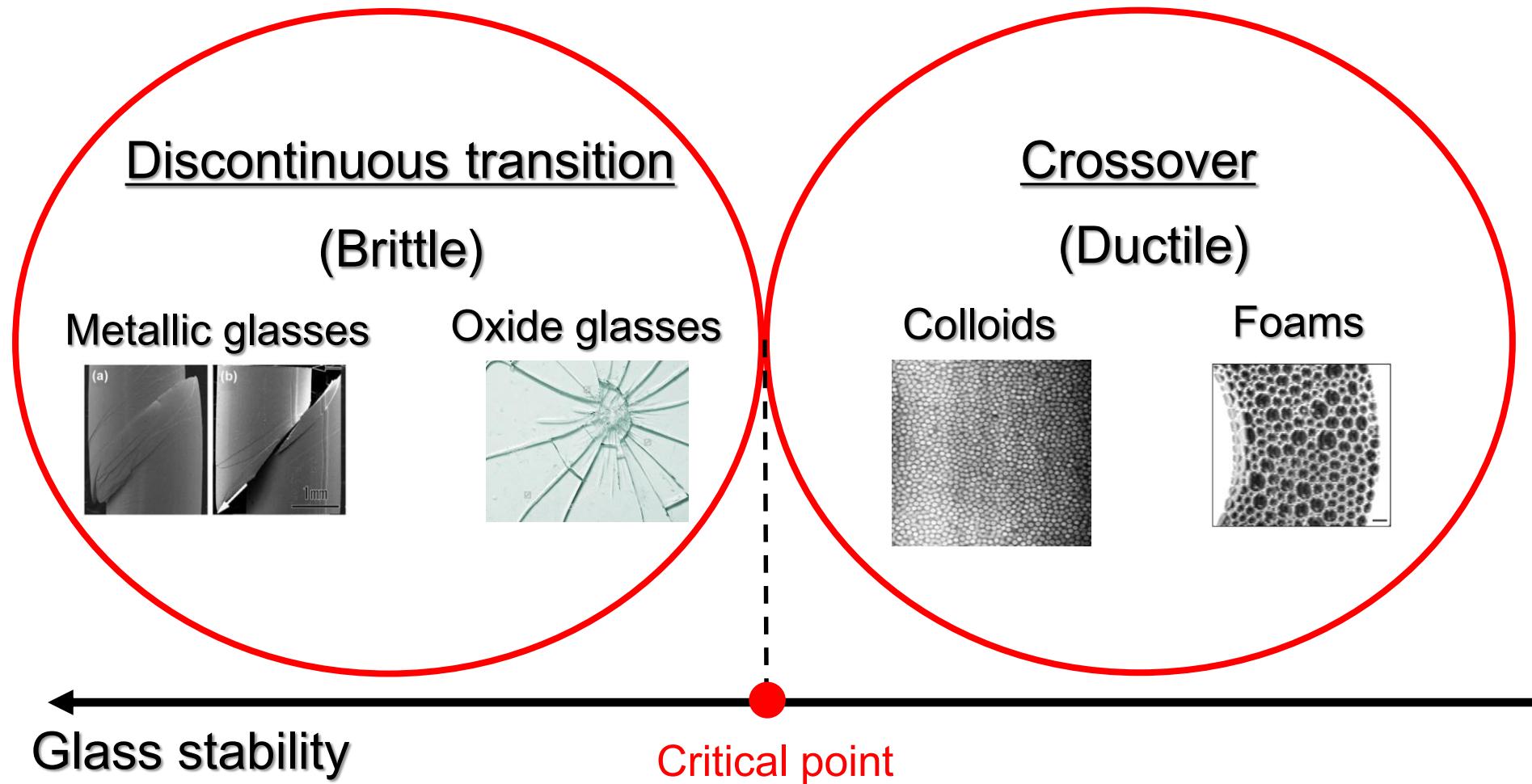
# Comparison with RFIM



$$\chi_{\text{vari}} = N(\langle \sigma^2 \rangle - \langle \sigma \rangle^2) \quad \chi_{\text{deri}} = -\frac{\partial \langle \sigma \rangle}{\partial \gamma}$$

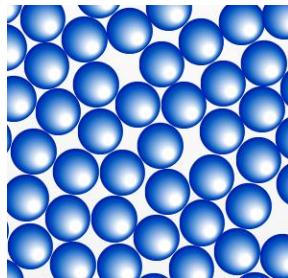
Below  $T_{\text{ini},c} \approx 0.095$  we find  $\chi_{\text{vari}} \propto (\chi_{\text{deri}})^2$  : the same scaling as RFIM  
(Strong sample-to-sample fluctuation)

# Our unified picture of yielding transition



# Dimensional dependence of disordered systems

## Jamming transition

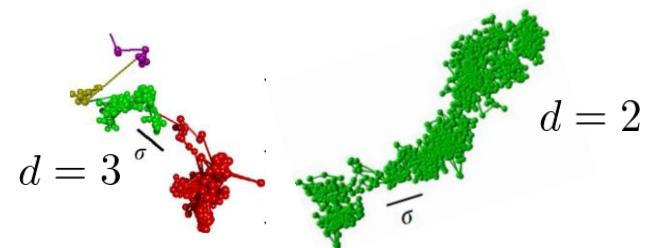


-The upper critical dimensions  $d_{uc} = 2$

Goodrich, Liu, and Nagel, PRL 2012

Charbonneau, Kurchan, Parisi, Urbani, and Zamponi,  
Annu. Rev. Condens. Matter Phys. 2017

## Supercooled liquids and glasses



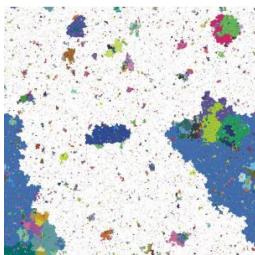
-Fundamental differences between dynamics in  $d = 2$  and  $d = 3$

Flenner and Szamel, Nat. Commun. 2015

-The lower critical dimensions  $d_{lc} = 2$ ?

Berthier, Charbonneau, Ninarello, Ozawa, and Yaida, arXiv 2018

## Random field Ising model

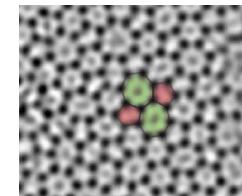


-Equilibrium  $d_{lc} = 2$

-Athermally driven  $d_{lc} = 2???$

Balog, Tarjus, and Tissier, PRB 2018

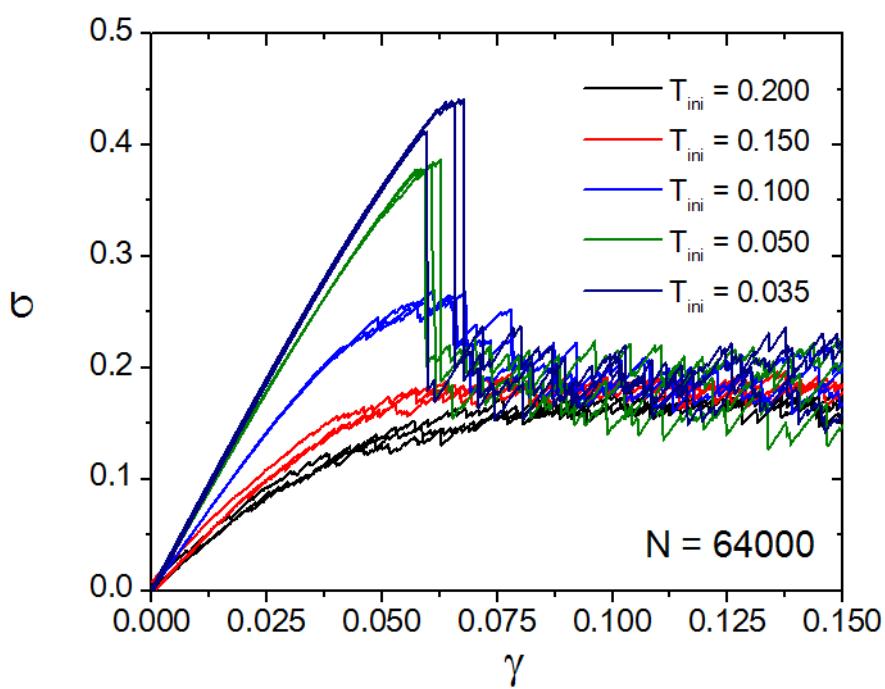
Many real 2D amorphous materials  
e.g., foams, colloids, silica



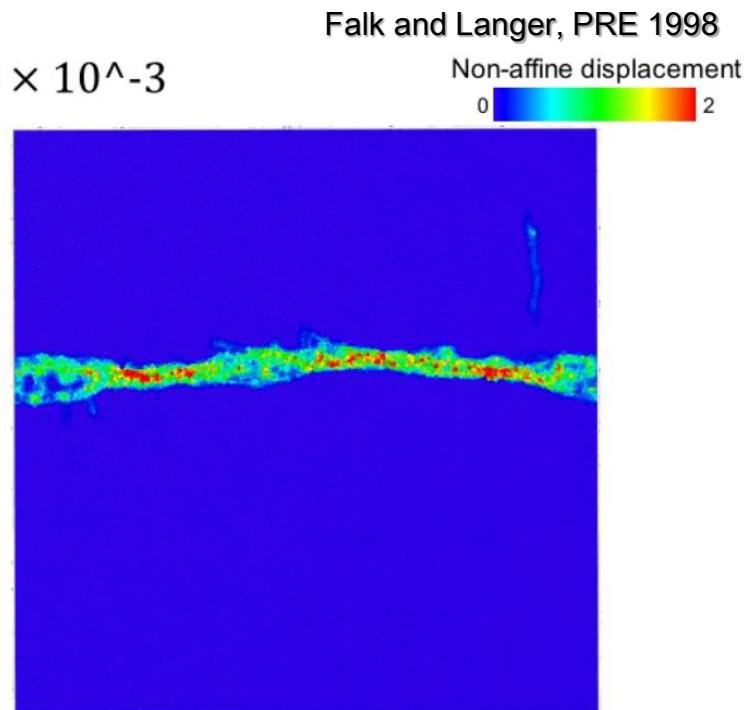
Huang, et al., Science 2013

How about yielding in  $d = 2$  ?

# Yielding transition in $d = 2$



$\gamma = 66 \times 10^{-3}$

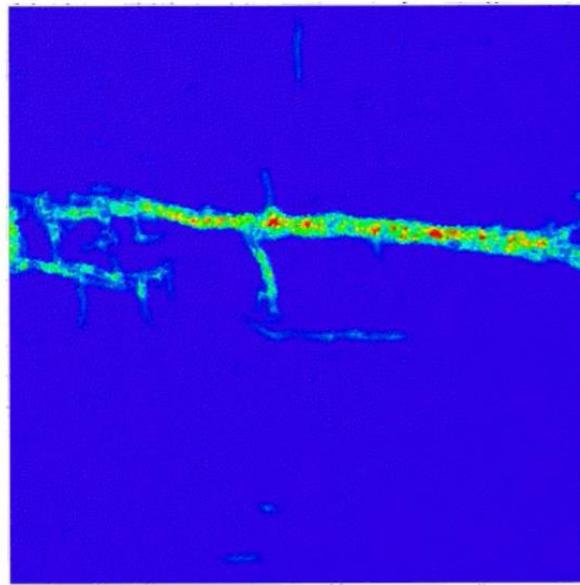


Sharp brittle yielding can be also observed in  $d = 2$

# Yielding transition in $d = 2$

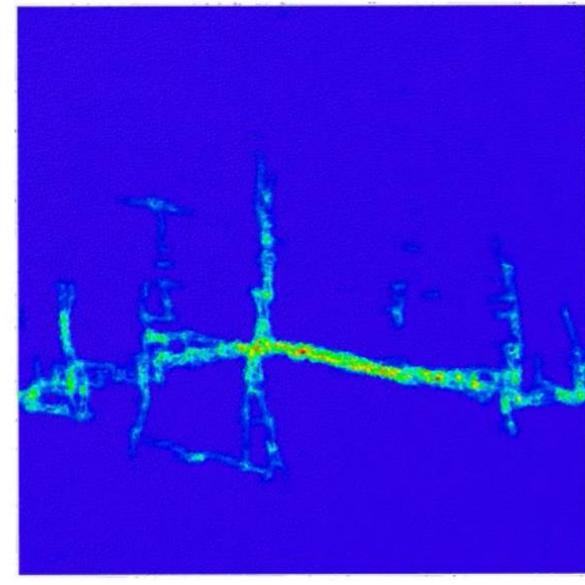
$\gamma = 66 \times 10^{-3}$

Non-affine displacement  
0 2



$\gamma = 71 \times 10^{-3}$

Non-affine displacement  
0 2



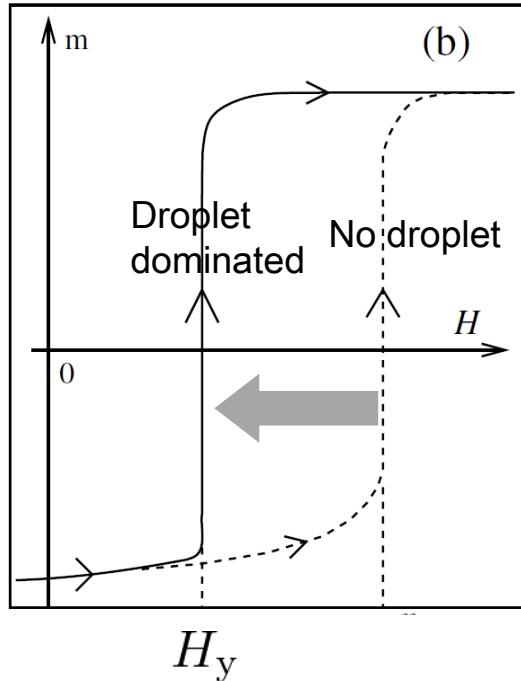
Very occasionally dirty shearbands are observed in stable glasses

Dimensional effect?

# Effect of soft spot seed on yielding

## Athermally driven RFIM

Rare droplet triggers transition

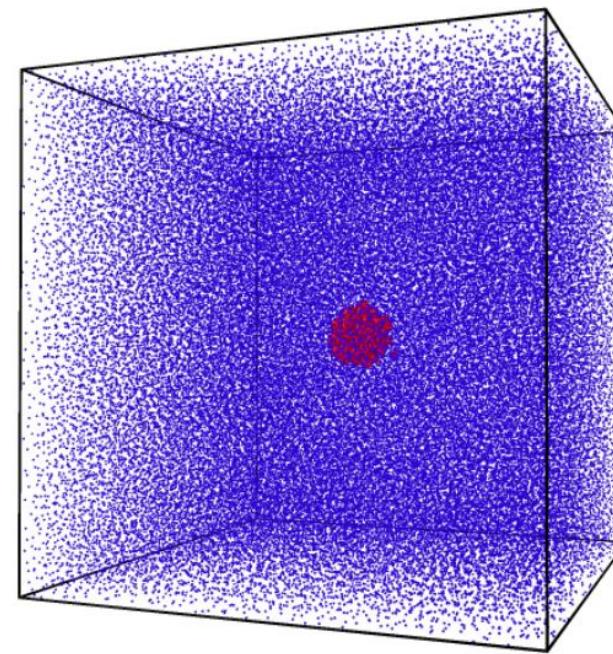


$H_y$  is shifted by rare droplet

Nandi, Biroli, and Tarjus, PRL 2016

## Yielding

Soft spot seed is rare droplet?



Inserting a soft spot seed “by hand”

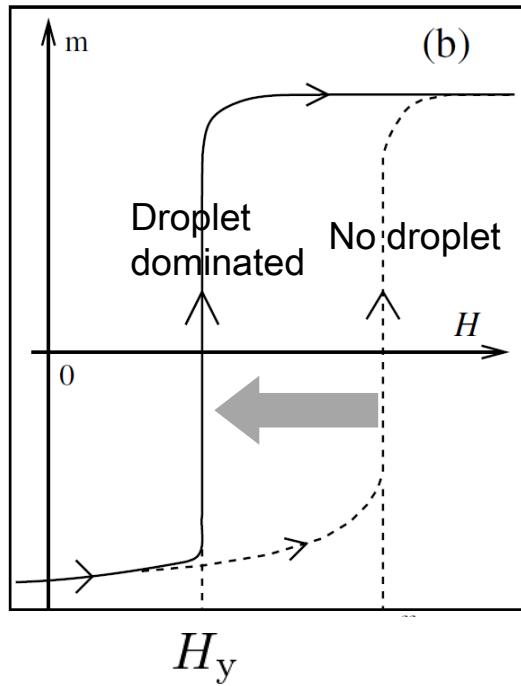
Manning and Liu, PRL 2011

Ding, Patinet, Falk, Cheng, and Ma, PNAS 2014

# Effect of soft spot seed on yielding

## Athermally driven RFIM

Rare droplet triggers transition



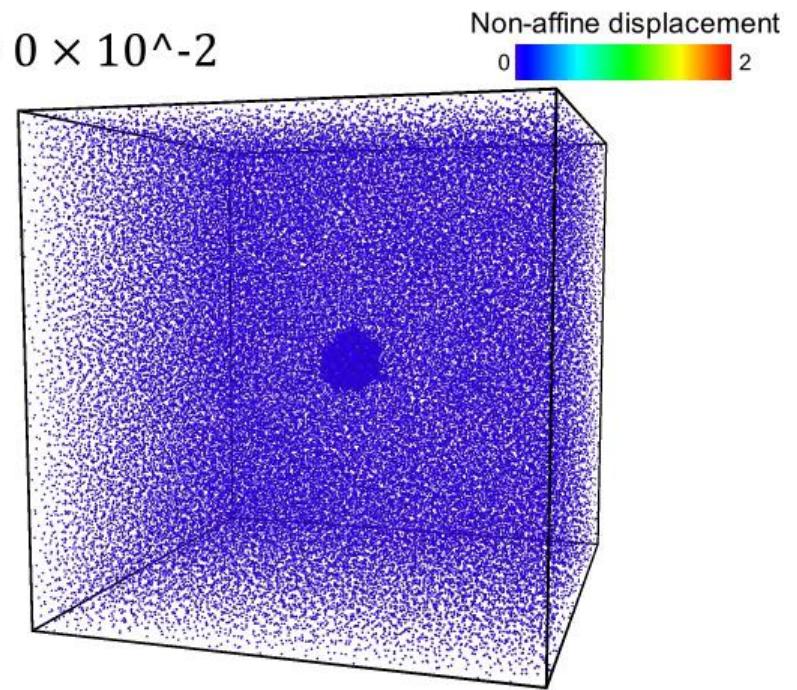
$H_y$  is shifted by rare droplet

Nandi, Biroli, and Tarjus, PRL 2016

## Yielding

Soft spot seed is rare droplet?

$$\gamma = 0 \times 10^{-2}$$



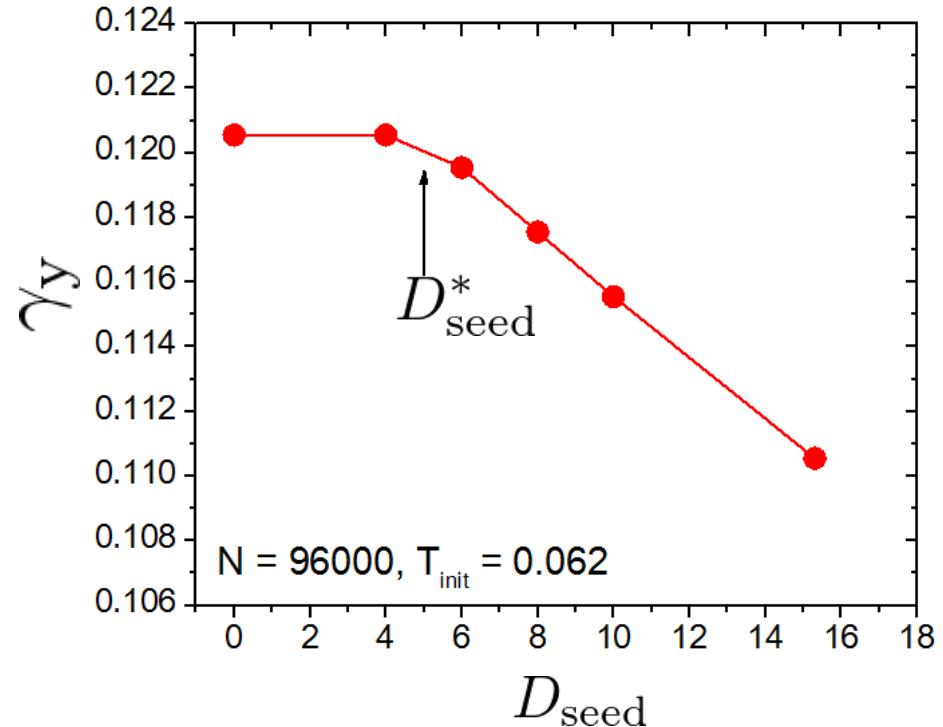
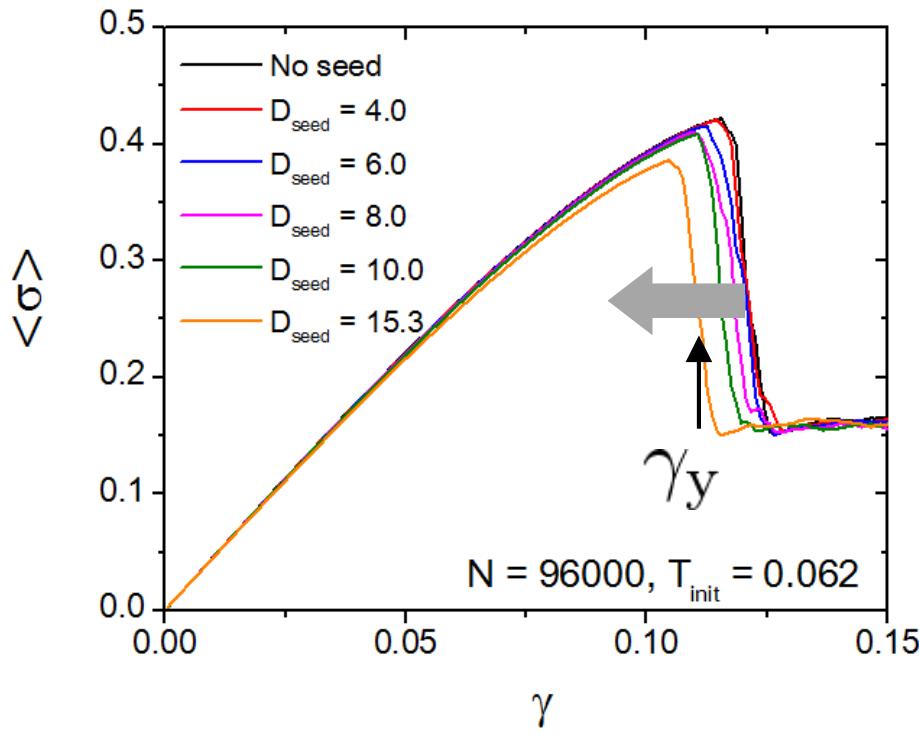
Inserting a soft spot seed “by hand”

Manning and Liu, PRL 2011

Ding, Patinet, Falk, Cheng, and Ma, PNAS 2014

# Effect of soft spot seed on yielding

$D_{\text{seed}}$  : Diameter of artificially inserted soft spot seed



Yielding point is decreased with increasing  $D_{\text{seed}}$ , but only above  $D_{\text{seed}}^* \approx 5$

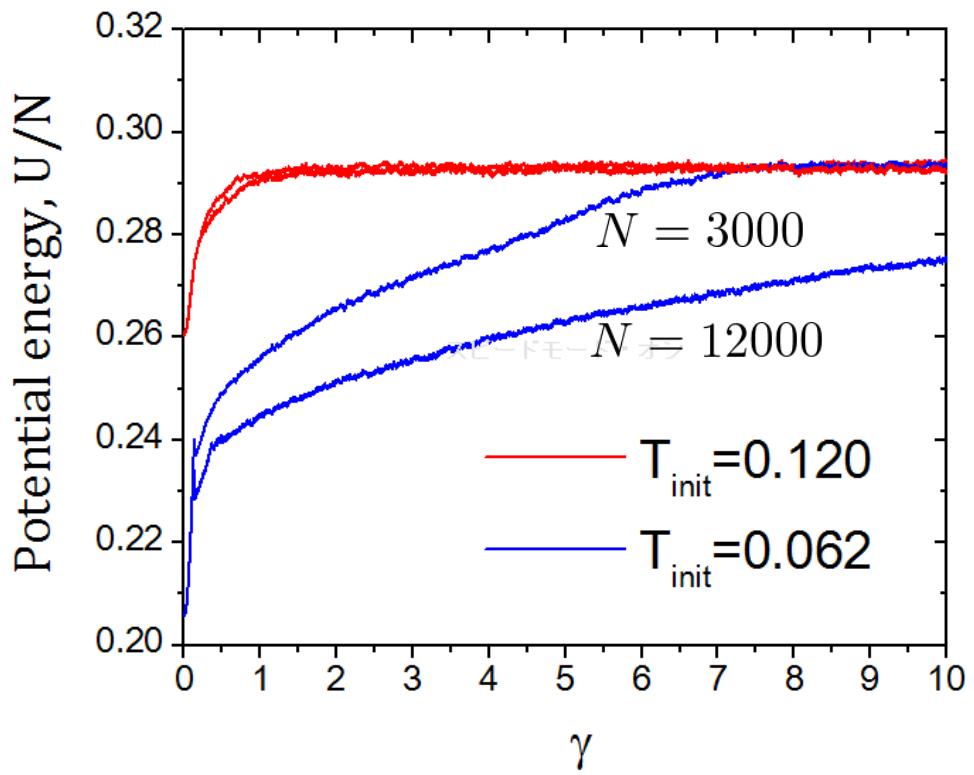
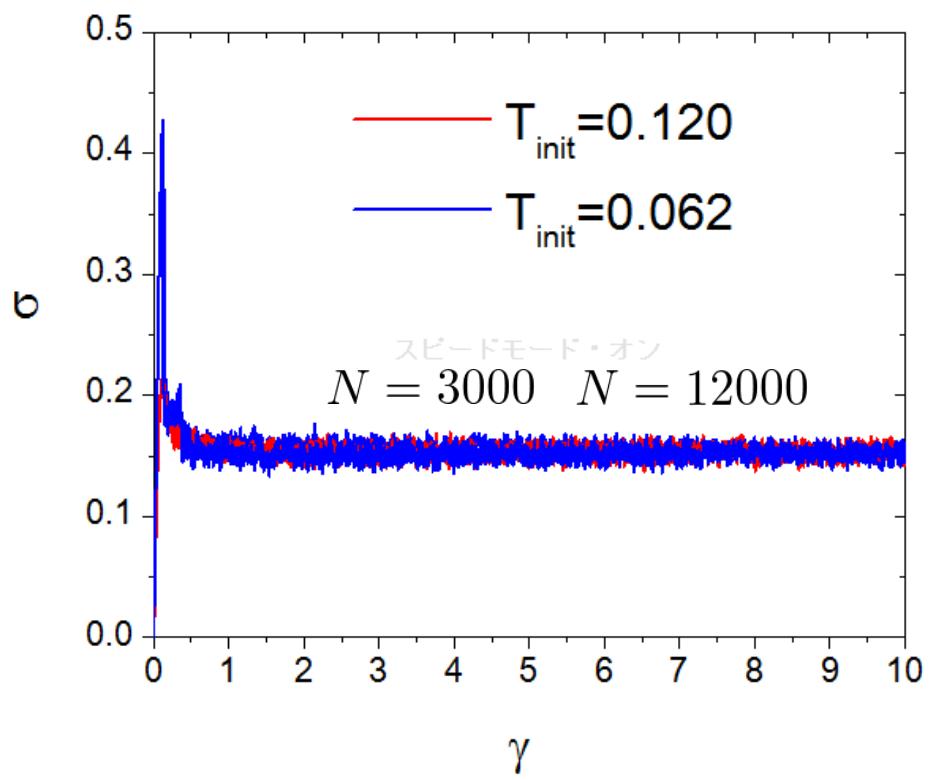
Threshold size?

Barbot et al., PRE 2018

Popovic, de Geus, and Wyart, PRE 2018

# Summary and Future

- Random critical point separates brittle and ductile yielding transitions:  
Numerically tested Ozawa, Berthier, Biroli, Rosso, and Tarjus, PNAS 2018
- Dimensional dependence: Shape transition in both  $d = 3$  and  $d = 2$
- Effect of soft spot seed on yielding: Soft spot seed shifts the yielding point  
There is a threshold size
- Future: Finite temperature and shear rate
- On going big collaboration organized by L. Manning: Detailed structural analysis



-The stress is miss reading

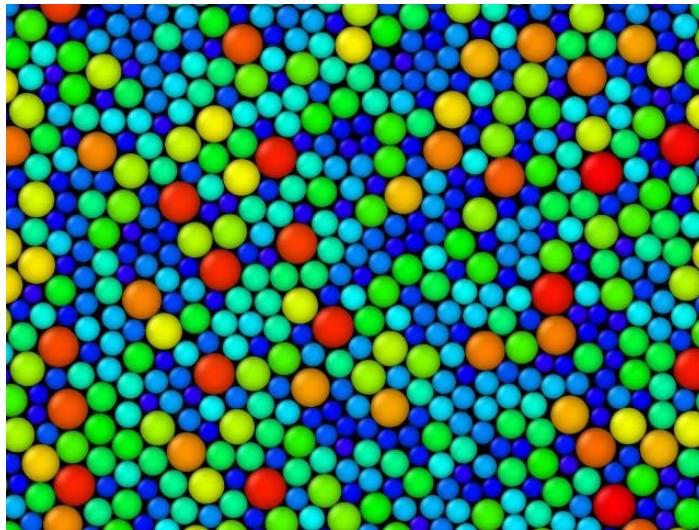
-Larger the system size, longer the time for the steady state



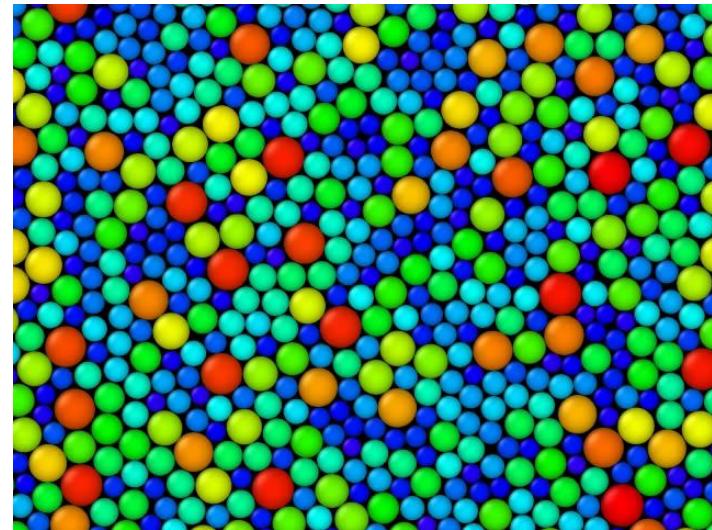
# Swap Monte-Carlo simulations

Equilibrium hard disks beyond jamming:  $\phi = 0.85 > \phi_J$

Normal Monte-Carlo



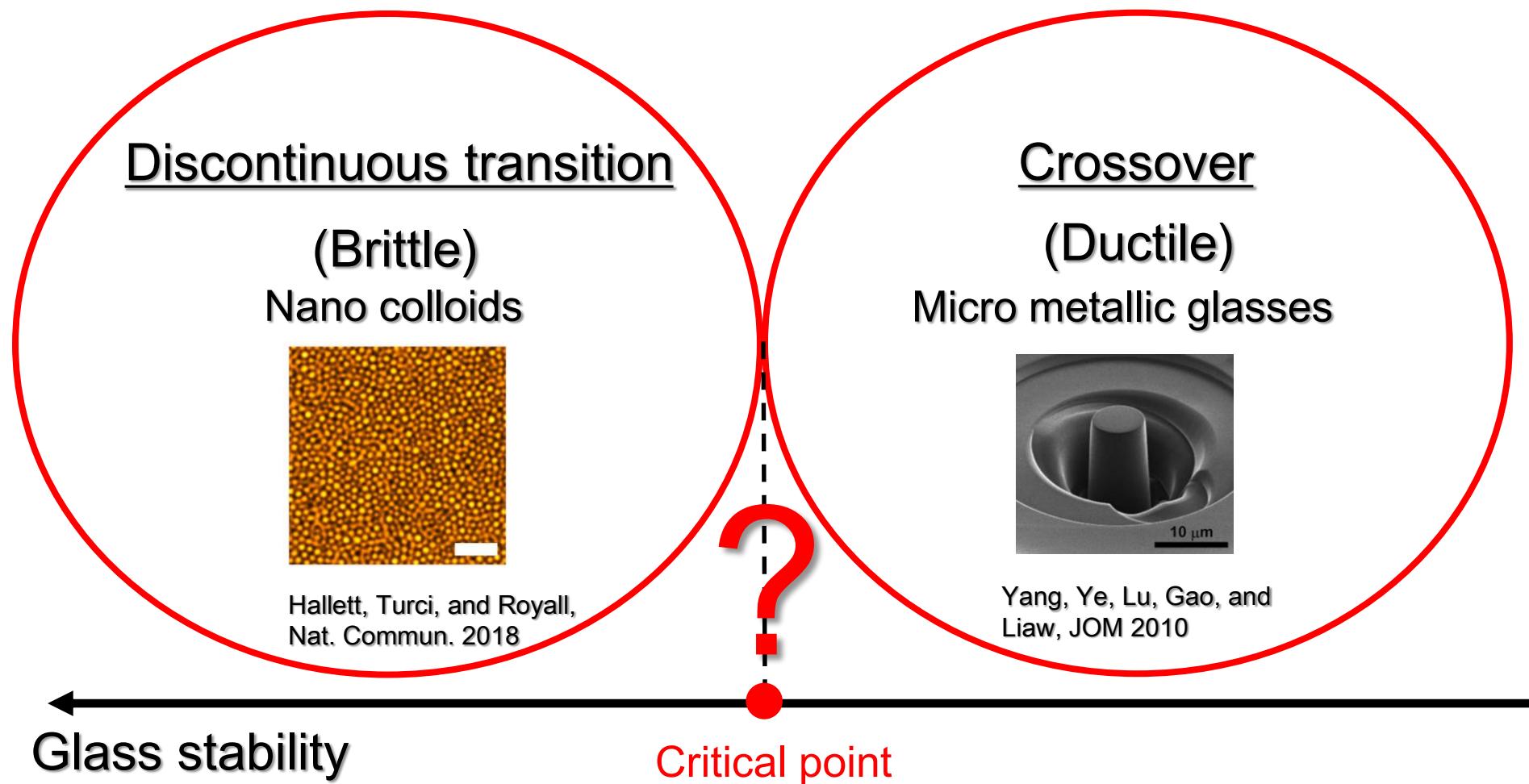
Swap Monte-Carlo



Berthier, Coslovich, Ninarello, and Ozawa, PRL 2016  
Ninarello, Berthier, and Coslovich, PRX 2017

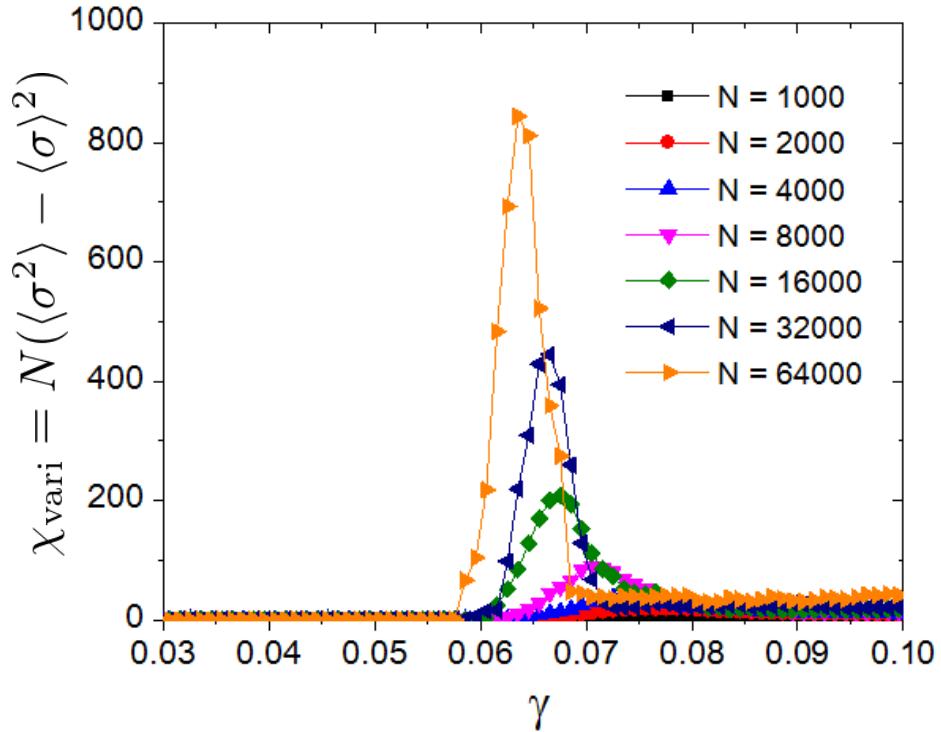
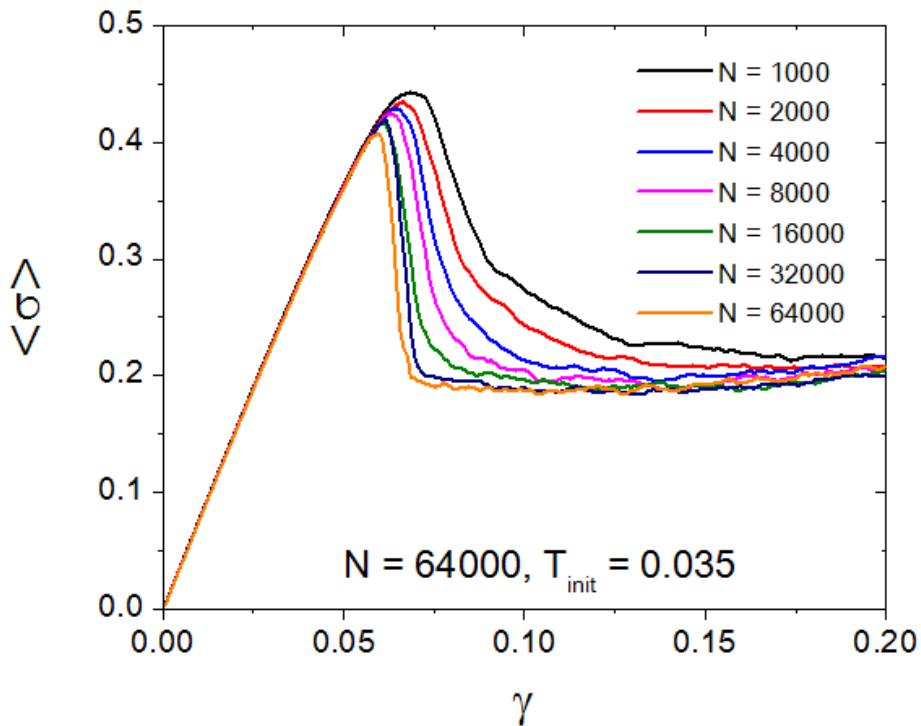
Swap MC is a very efficient thermalization technique,  
producing very stable glasses

# Discussion: Experimental realization



The critical point might be detected by some experiments

# Yielding transition in $d = 2$



Yielding becomes sharper with  $N \rightarrow \infty$