# Homework 3

$$Z(x,L;00) = \int_{x(0)=0}^{x(L)=x} \mathcal{D}[x(t)]\, e^{-\beta\sigma L - \frac{\beta\sigma}{2}\int_0^L dt \left(\frac{dx}{dt}\right)^2}$$

Consider the Langevin process $\frac{dx}{dt} = \xi(t)$ as in the text

1 $$P[\xi(t)] = \mathcal{N} \exp\left[-\frac{1}{2D}\int_0^L \xi^2(t)\,dt\right]$$

$\xi(t)$ defined ~~between~~ $t \in [0,L]$

2 $$P[x(t)] = \left\langle \delta\left[\frac{dx}{dt} - \xi(t)\right] \left|\det\left(\frac{d}{dt}\right)\right| \right\rangle_{\text{over } \xi(t)}$$

The Jacobian $\left|\det\left(\frac{d}{dt}\right)\right|$ is a constant, thus

$$P[x(t)] = \mathcal{N} \int P[\xi(t)]\, \delta\left[\frac{dx}{dt} - \xi(t)\right] \mathcal{D}[\xi(t)]$$

$$= \mathcal{N}' \exp\left[-\frac{1}{2D}\int_0^L \left(\frac{dx}{dt}\right)^2 dt\right]$$

$$\boxed{D = \frac{1}{\beta\sigma}}$$

3 $$\int_{x(0)=0}^{x(t)=x} P[x(t)]\,\mathcal{D}[x(t)] = P(xL;00) = \frac{e^{-\frac{x^2}{2DL}}}{\sqrt{2\pi DL}}$$

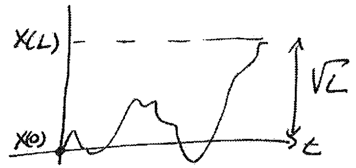


$$Z(xL;00) = \mathcal{N} e^{-\beta\sigma L} P(xL;00) = \mathcal{N}' \exp\left[-\beta\sigma L - \frac{\beta\sigma x^2}{L}\right]$$

4 The probability that the interface ends in $x(L) = x$ is

$$\frac{Z(xL;00)}{\int d\bar{x}\, Z(\bar{x},L;00)} = P(xL;00)$$

$$\Rightarrow \left\langle \left(x(L) - x(0)\right)^2 \right\rangle = \frac{L}{\beta\sigma}$$




Energy of the interface:

$$+\sigma L + \int_0^L dt \left[\left(\frac{dx}{dt}\right)^2 + V(x(t),t)\right]$$

1 if $V=0$, the energy is minimized by $x(t) = \text{constant}$. Fixing ~~the~~ $x(0)=0$ $\Rightarrow x_{GS}(t) = 0$

2 For small disorder the energy reads

$$E = \sigma L + \int_0^L dt \left(\frac{dx}{dt}\right)^2 + \int_0^L h(t)\,x(t)\,dt$$

the equation for $x_{GS}(t)$:

$$\frac{\delta E}{\delta x(t)} = 0 \Rightarrow -\sigma \frac{d^2 x_{GS}}{dt^2} + h(t) = 0$$

In Fourier space $K^2 \hat{x}_{GS}(k) + \hat{h}(k) = 0$

$$\Rightarrow \hat{x}_{GS}(k) = \frac{-\hat{h}(k)}{\sigma k^2}$$

3
$$\overline{\hat{h}(k)\hat{h}(k')} = \int dt\,dt'\, e^{-ikt} e^{-ik't'}\, \overline{h(t)h(t')}$$
$$= \Delta^2 \int dt\,dt'\, e^{-ik(t)-ik't'}\, \delta(t-t')$$
$$= \Delta^2 \int dt\, e^{-i(k+k')t} = \Delta^2 (2\pi)\, \delta(k+k')$$

4
$$\overline{(x_{GS}(t) - x_{GS}(t'))^2} =$$
$$\overline{\left(\int \frac{dk}{2\pi} e^{+ikt}\, \hat{x}_{GS}(k) - \int \frac{dk'}{2\pi} e^{ik't'}\, \hat{x}_{GS}(k')\right)^2} =$$
$$= \int \frac{dk}{2\pi}\frac{dk'}{2\pi}\, \overline{\hat{x}_{GS}(k)\hat{x}_{GS}(k')}\left[e^{i(k+k')t} + e^{i(k+k')t'} - 2e^{ikt + ik't'}\right]$$

$$\overline{\hat{x}_{GS}(k)\hat{x}_{GS}(k')} = \frac{\overline{\hat{h}(k)\hat{h}(k')}}{\sigma^2 k^2 (k')^2} = \frac{2\pi\,\delta(k+k')\,\Delta^2}{\sigma^2 k^4}$$

4

$$\overline{\left(x_{GS}(t) - x_{GS}(t')\right)^2} = \int \frac{dk}{2\pi} \frac{\Delta^2}{\sigma^2} \frac{1}{k^4}\left[2 - 2e^{ik[t-t']}\right]$$

By changing variable $k \to k(t-t')$

$$= C\,\frac{\Delta^2}{\sigma^2}\, |t-t'|^3$$

$\Rightarrow$ Even for $\Delta \ll 1$ if

$$|t-t'| > \left(\frac{\sigma^2}{\Delta^2 C}\right)^{\frac{1}{3}} \sim \Delta^{-\frac{2}{3}}$$

The interface fluctuates of $O(1)$

$\Rightarrow \Delta$ is NEVER a small perturbation.

The length $\ell = \frac{1}{\Delta^{\frac{2}{3}}}$ is called Larkin length.

(RK $C$ is dependent on the total length of the interface and diverges with it)