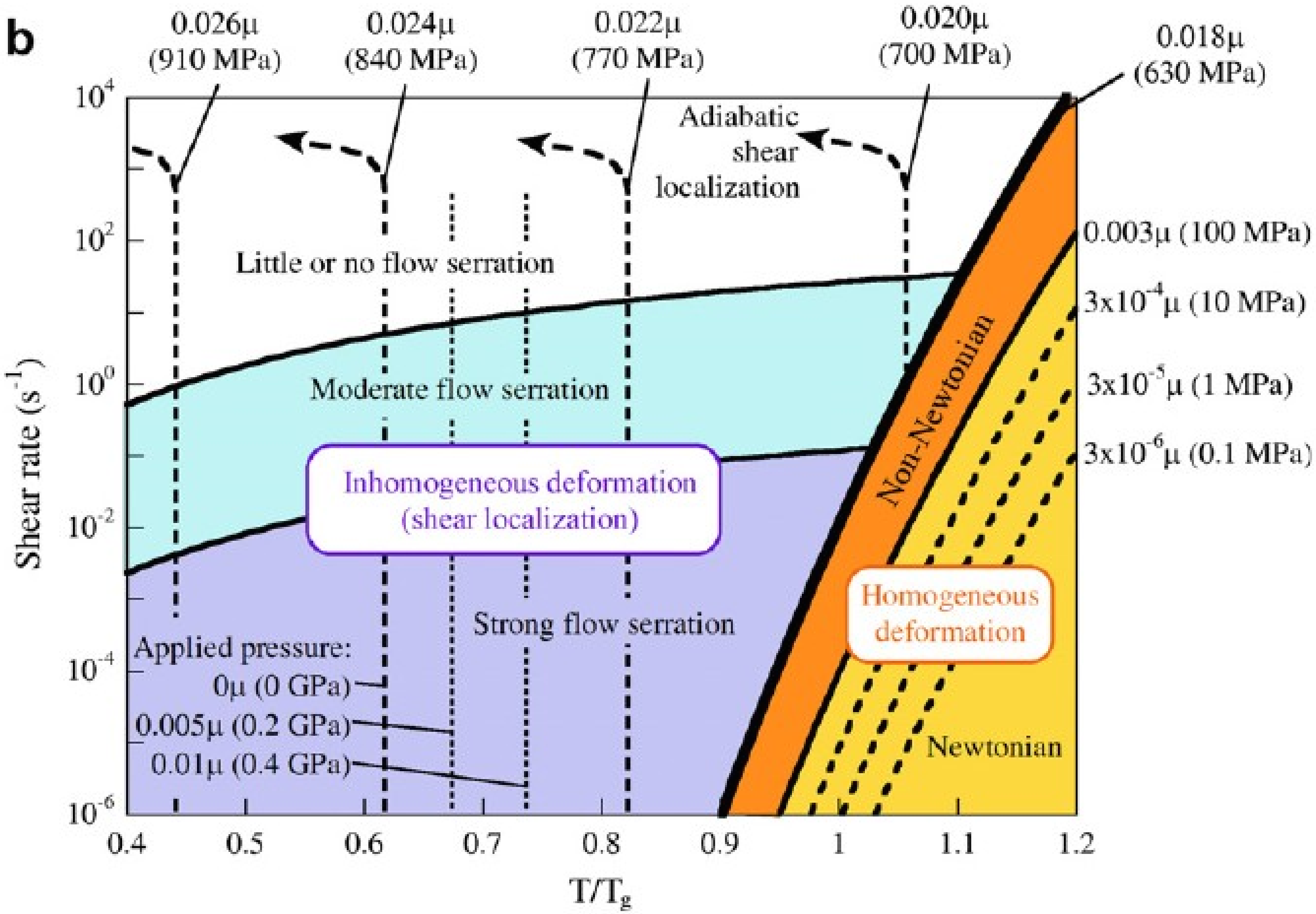
Noise and temperature effects on avalanches in strained amorphous solids

Anaël Lemaître

In collaboration with: C. Caroli, J. Chattoraj

Rhéophysique

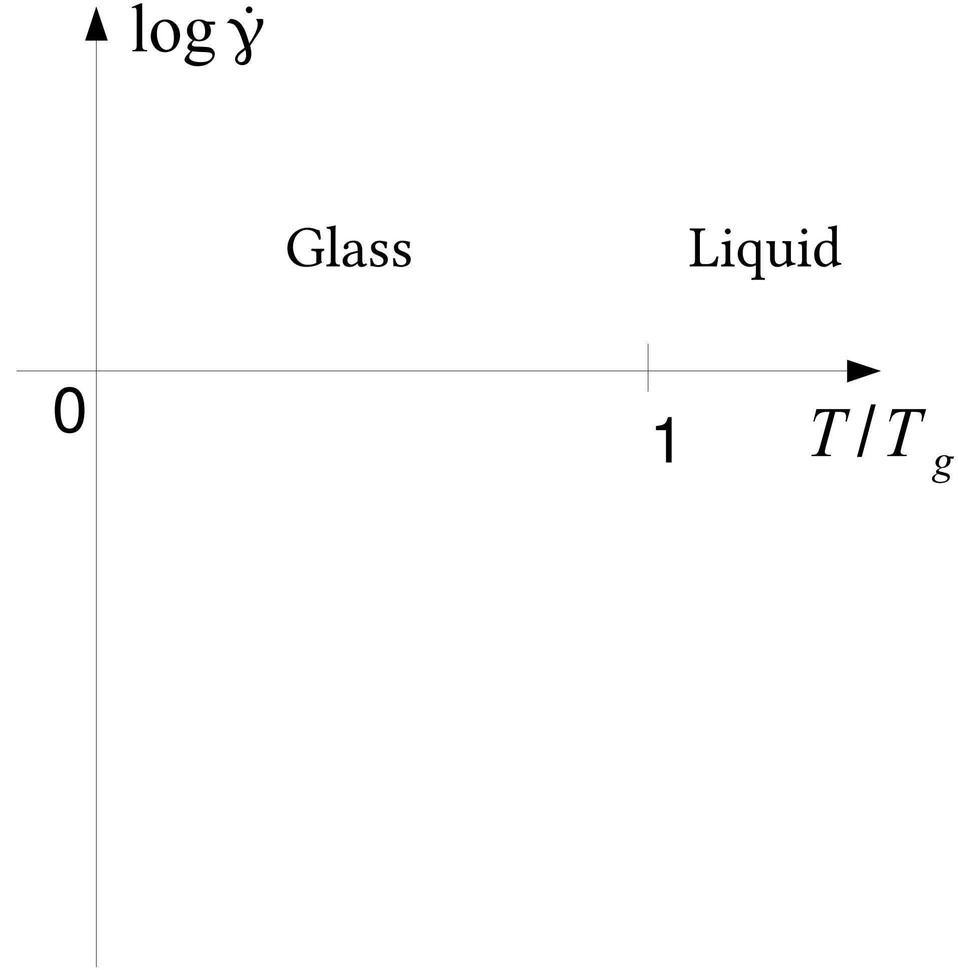
Deformation map for a metallic glass

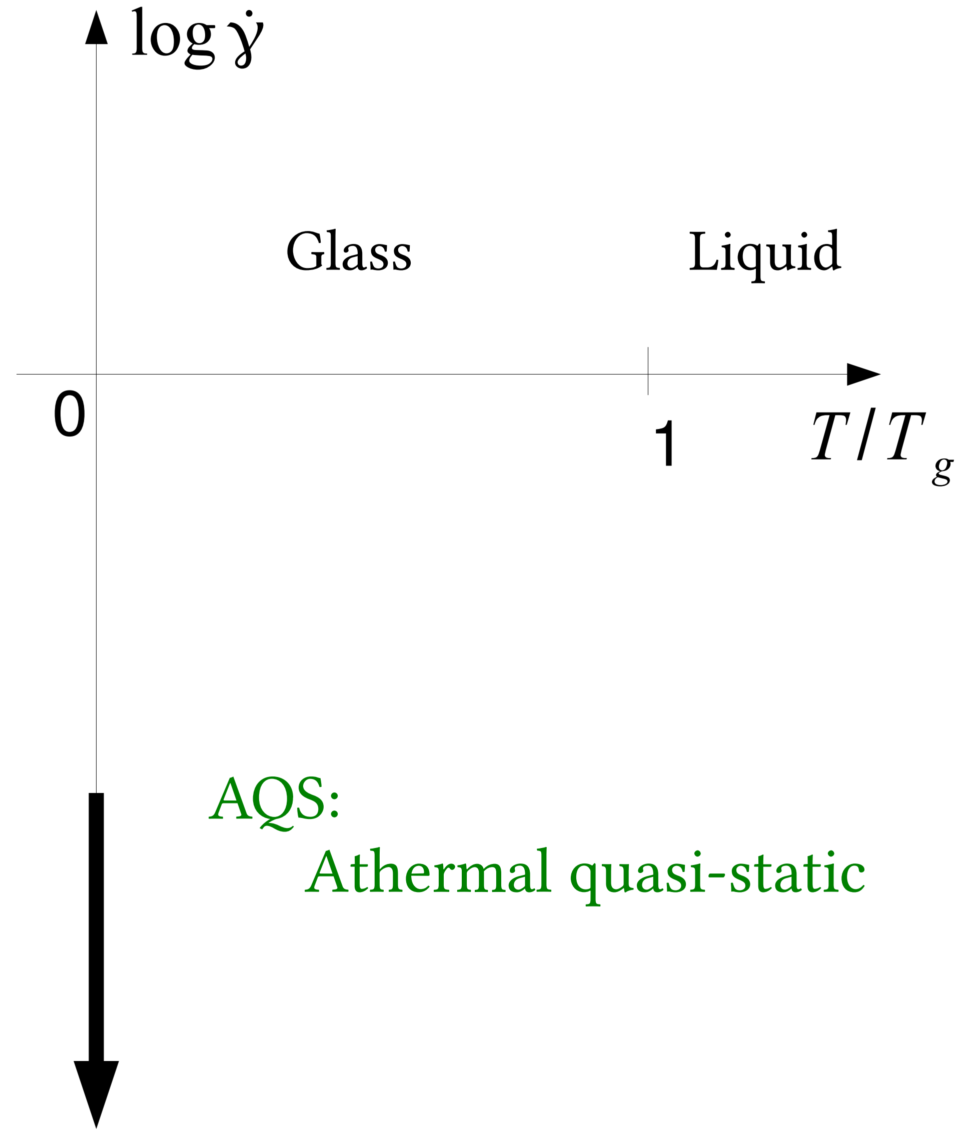


Schuh *et al*, Acta Mat. 55, 4067 (2007)

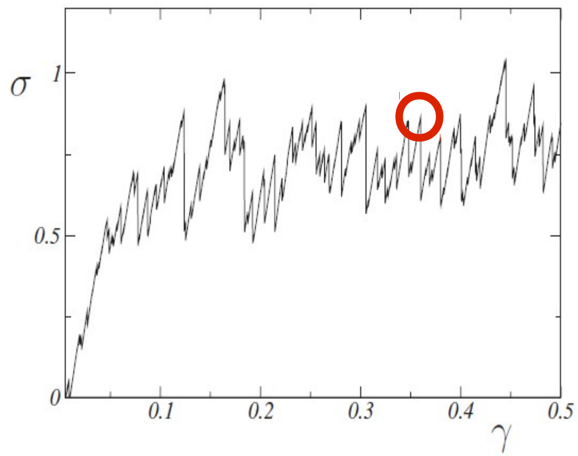
Identify the mechanisms that govern plasticity

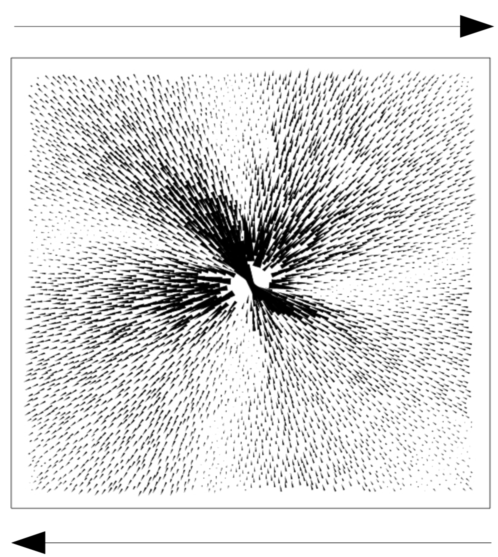
- nature of events?

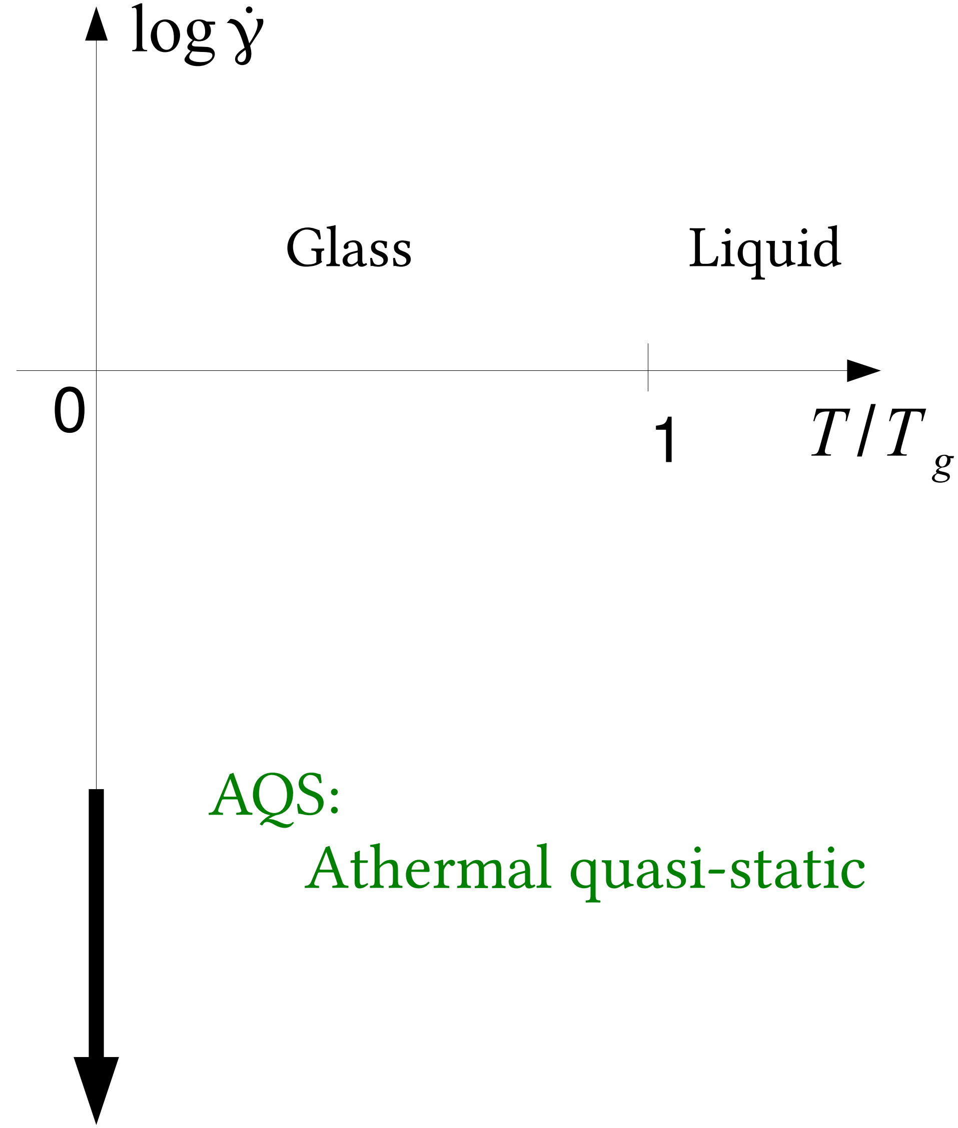

$\log \dot{\gamma}$
Glass
Liquid
0
1
T/T_g



$\log \dot{\gamma}$
Glass
Liquid
0
1
T/T_g
AQS:
Athermal quasi-static



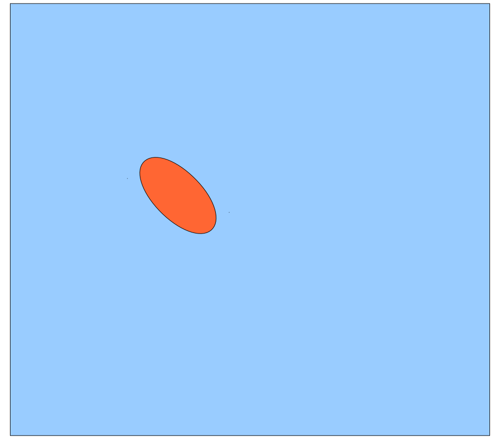


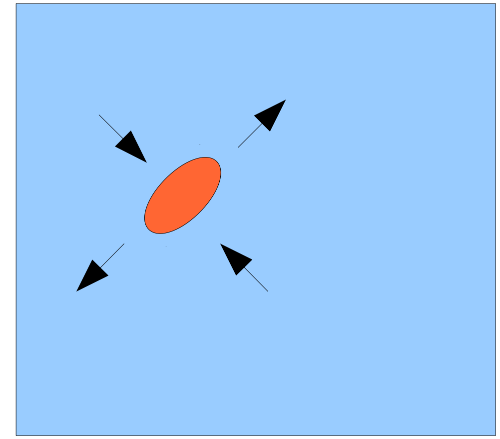


Maloney and AL
PRL 93, 016001 (2004)
PRE 74, 016118 (2006).

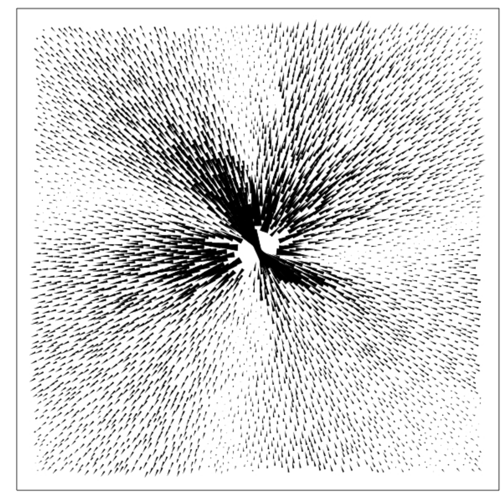
Question: What are plastic events?

Eshelby (1957): the effect of the transformation can be modeled by forces lying at the boundary



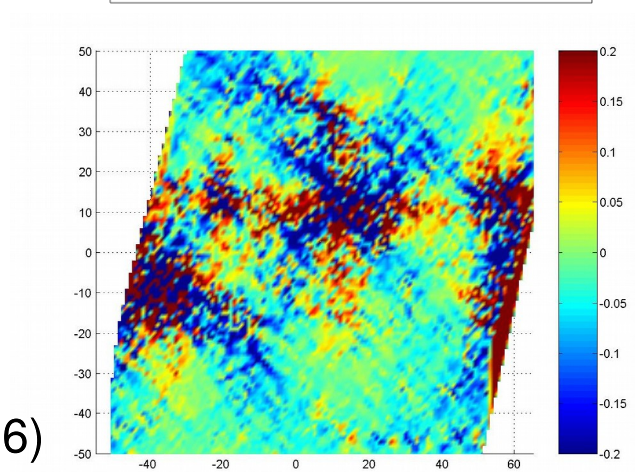


$$\vec{u} = \frac{2a^2 \Delta \epsilon_0}{\pi} \frac{xy}{r^4} \vec{r}$$



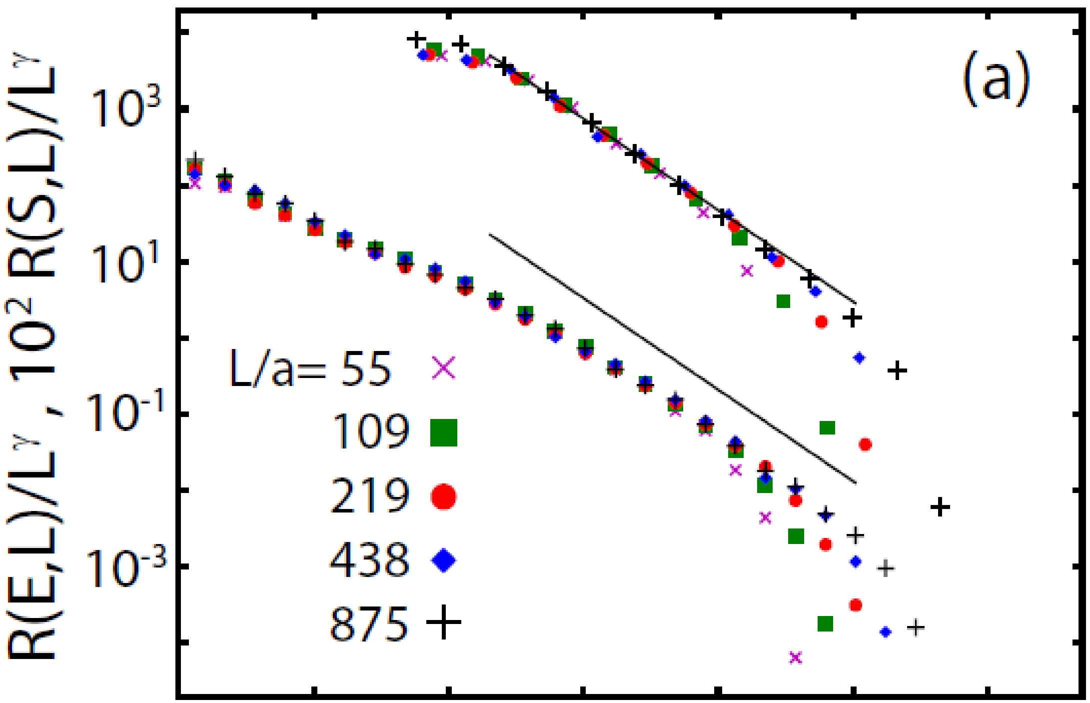
Maloney, AL (2004)

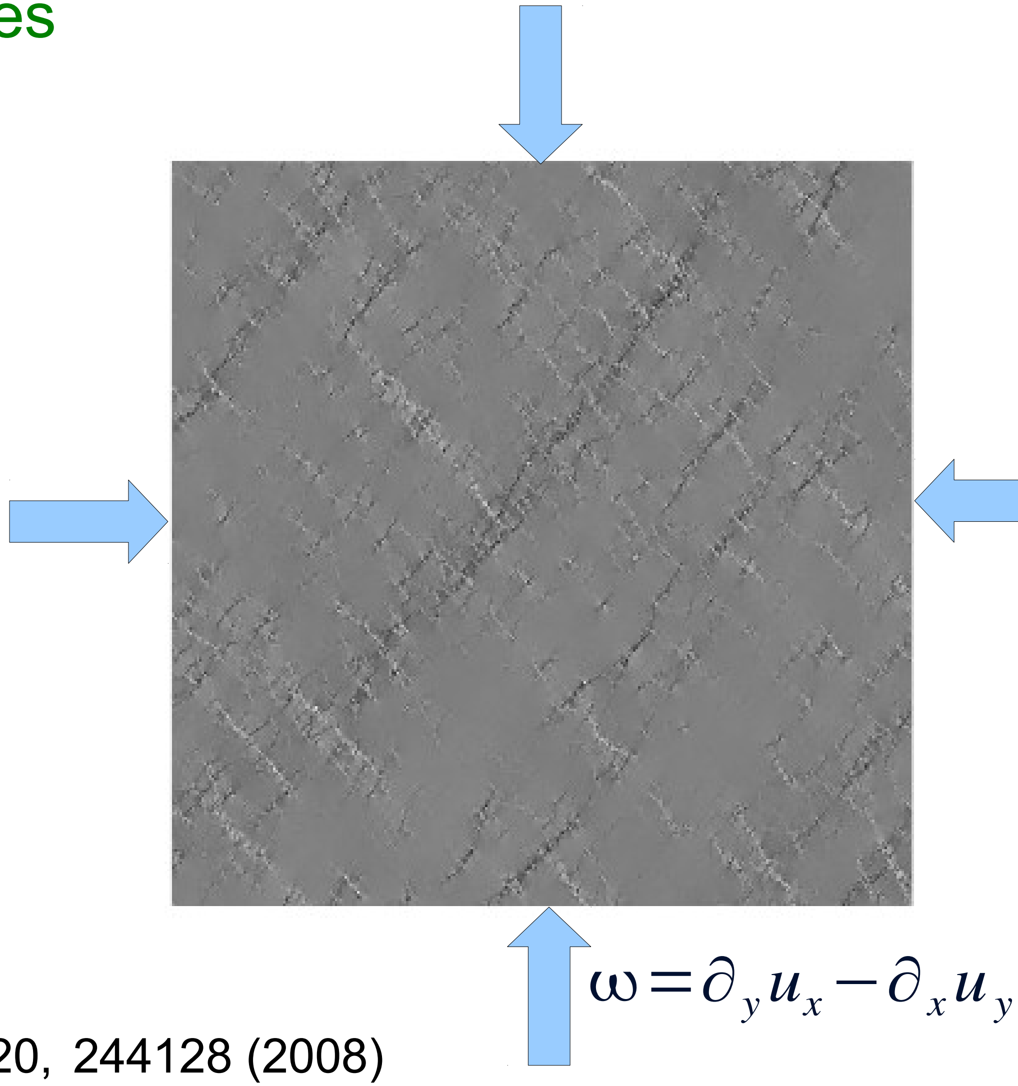
$$\Delta \sigma_{xy} = \frac{2 \mu a^2 \Delta \epsilon_0}{\pi} \frac{\cos(4\theta)}{r^2}$$



Tanguy et al (2006)

AQS: Plasticity results from avalanches





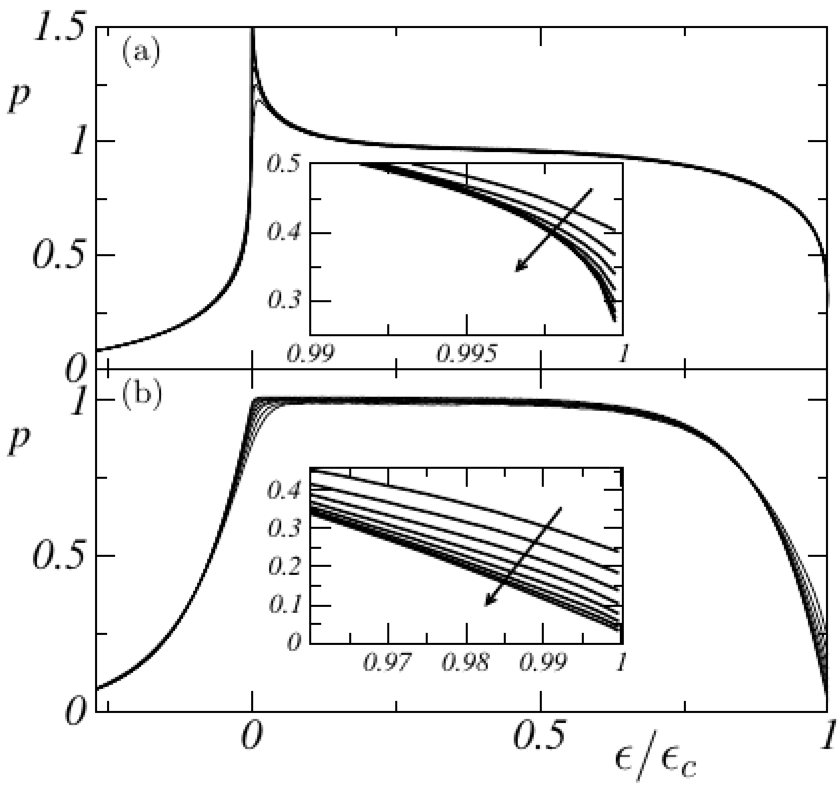
$$\omega = \partial_y u_x - \partial_x u_y$$

2D: Maloney and AL, PRL 93, 016001 (2004);
PRE 74, 016118 (2006).
Maloney & Robbins, J. Phys. Cond. Mat. 20, 244128 (2008)
Lerner and Procaccia, PRE 79, 066109 (2009)
Salerno, Maloney, Robbins (2012)

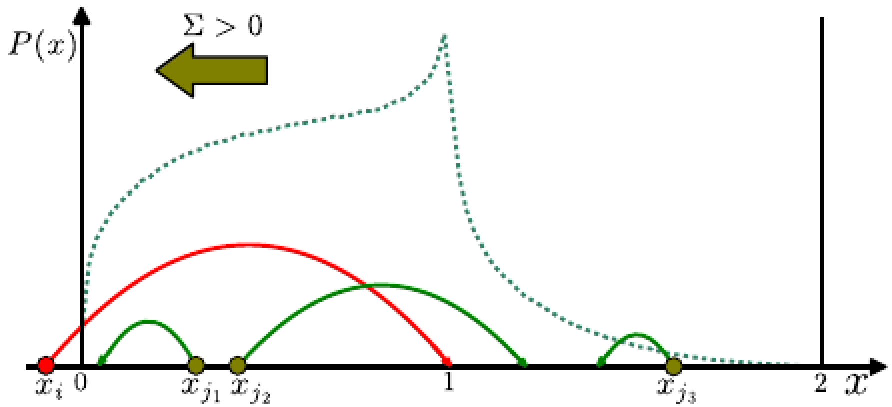
3D: Bailey et al PRL 98, 095501 (2007)
Salerno, Robbins (2013)

Marginal dynamics

Each zone is driven by loading + noise from previous events
Simplified model neglecting a priori spatial correlations



Lemaître and Caroli arXiv/0609689 (2006)
Lemaître and Caroli arXiv/0705.3122 (2007)



Lin and Wyart PRX, 6 011005 (2016)

Importance of noise kernel
=> correlations between events

Identify the mechanisms that govern plasticity

- nature of events: flips, avalanches (slip lines)

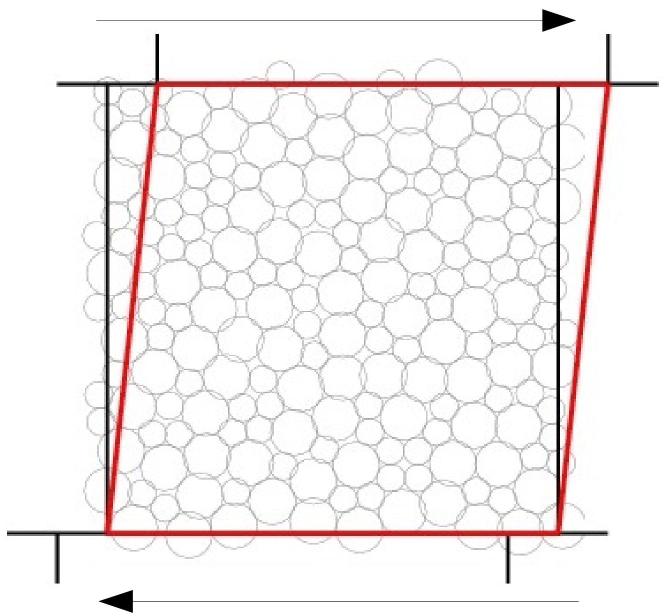
- observation of avalanches => correlations

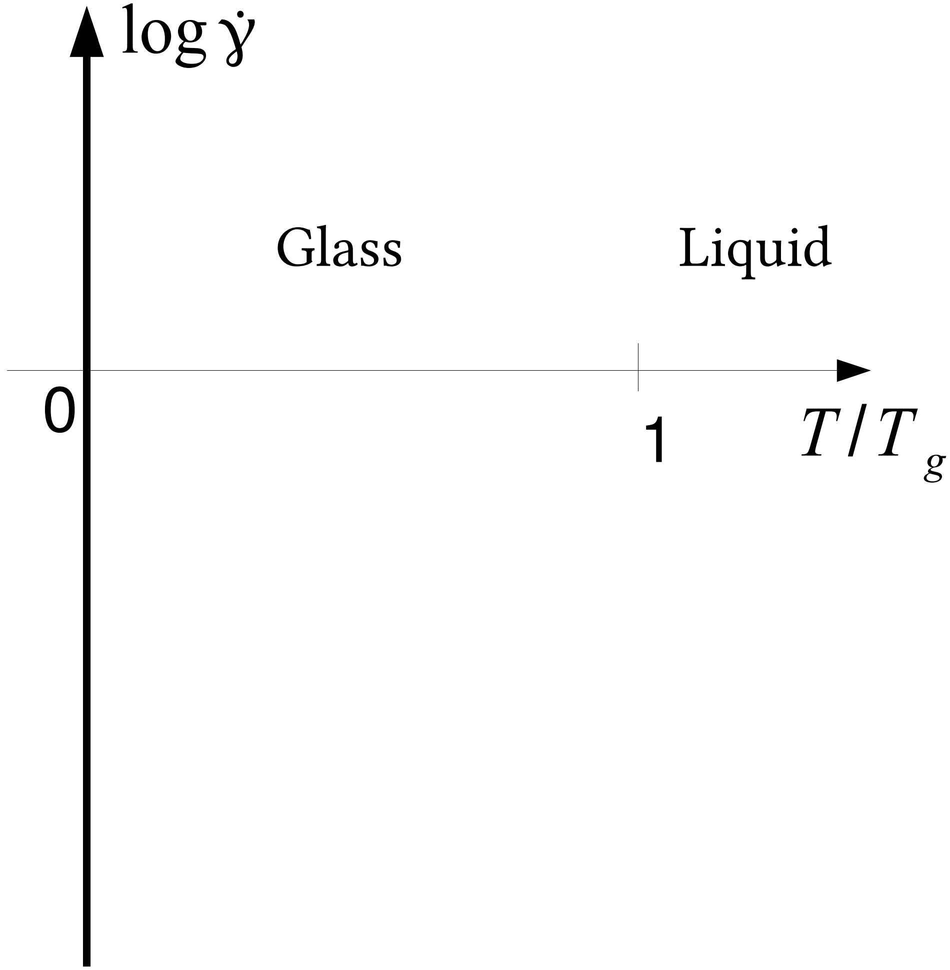
Validity at finite rates?

Athermal limit

Non-affine velocity

$\vec{v}_i - \dot{\gamma}\, y_i \vec{e}_x$





Athermal, finite strain-rate simulations: $T=0 \quad \dot{\gamma}\neq 0$

- Standard MD simulation
- Damping forces

$$f_{ij} = \frac{m}{\tau}\phi(r)(\vec{v}_j - \vec{v}_i)$$

AL and C. Caroli, PRL 103, 065501 (2009)

Athermal limit

Non-affine velocity

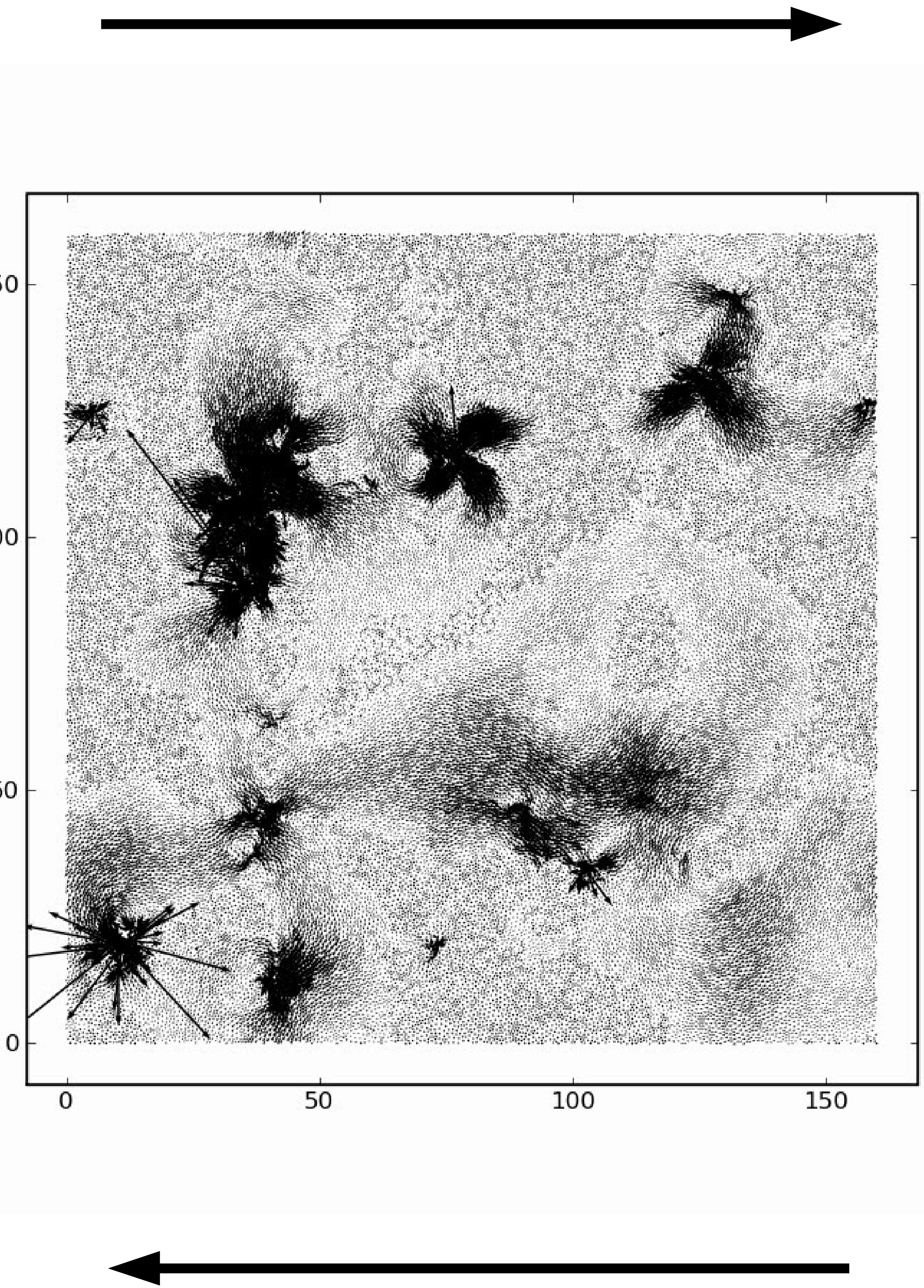
$$\vec{v}_i - \dot{\gamma}\, y_i \vec{e}_x$$

$$L = 160$$

$$\dot{\gamma} = 5.10^{-5}$$

PRL 103, 065501 (2009)

$$T < 10^{-4}$$

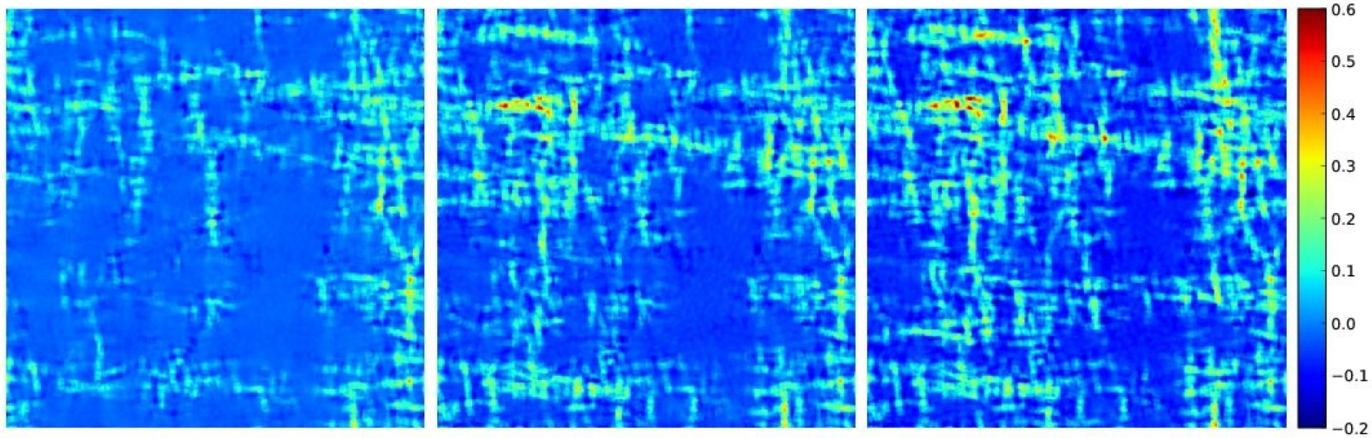


AL and C. Caroli, PRL 103, 065501 (2009)

Avalanches?

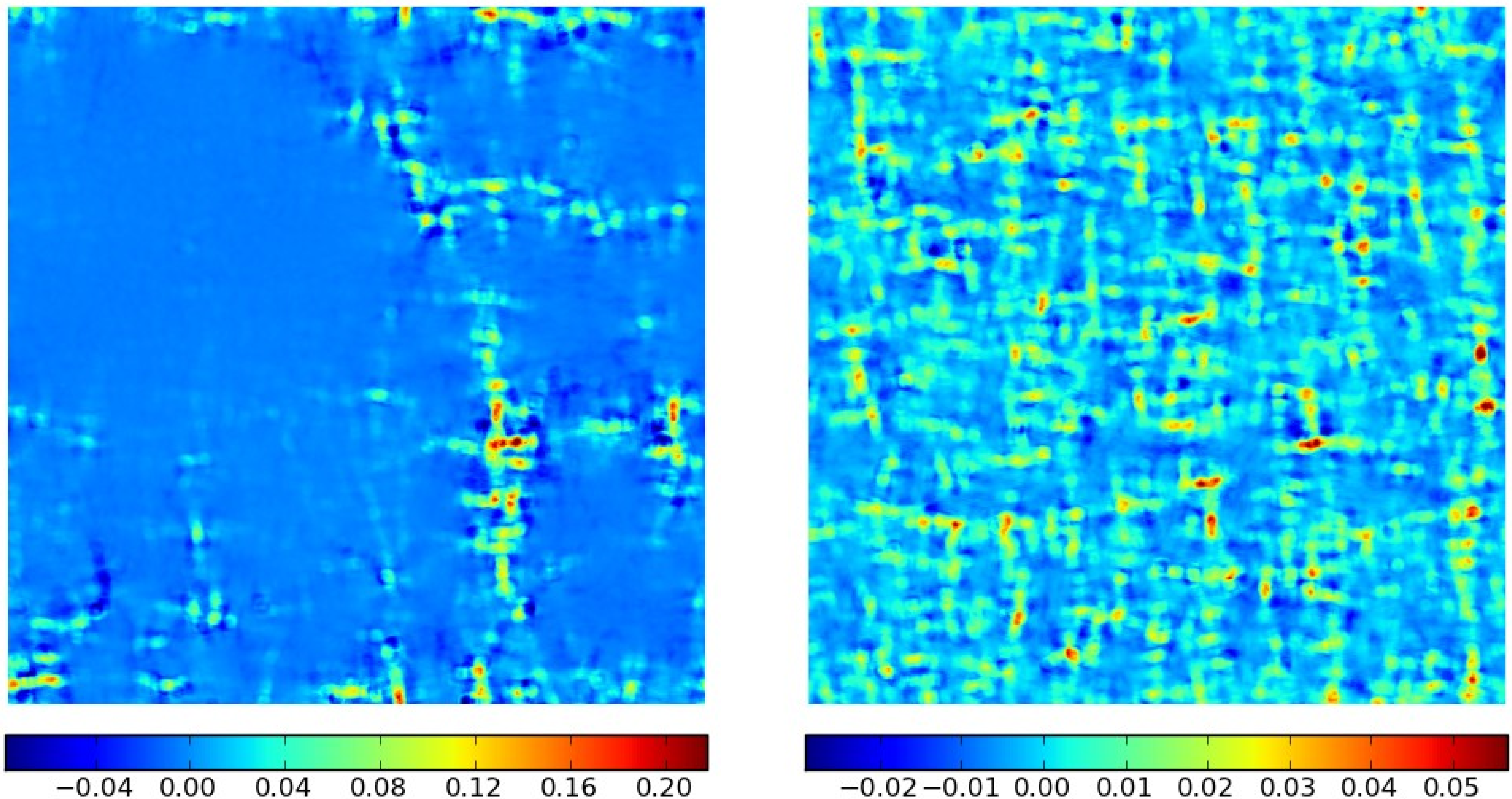
Deformation maps

$\epsilon_{xy}(\vec{r})$



$\gamma = 1\%$ $\gamma = 5\%$ $\gamma = 20\%$

$\dot{\gamma} = 10^{-4}$ $\dot{\gamma} = 10^{-2}$

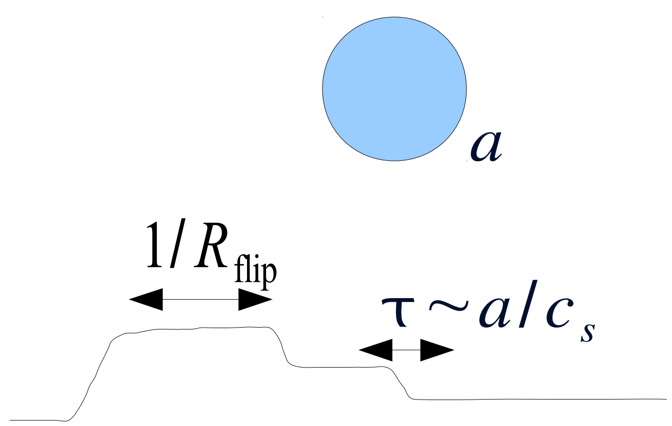


$\Delta\gamma = 1\,\%$

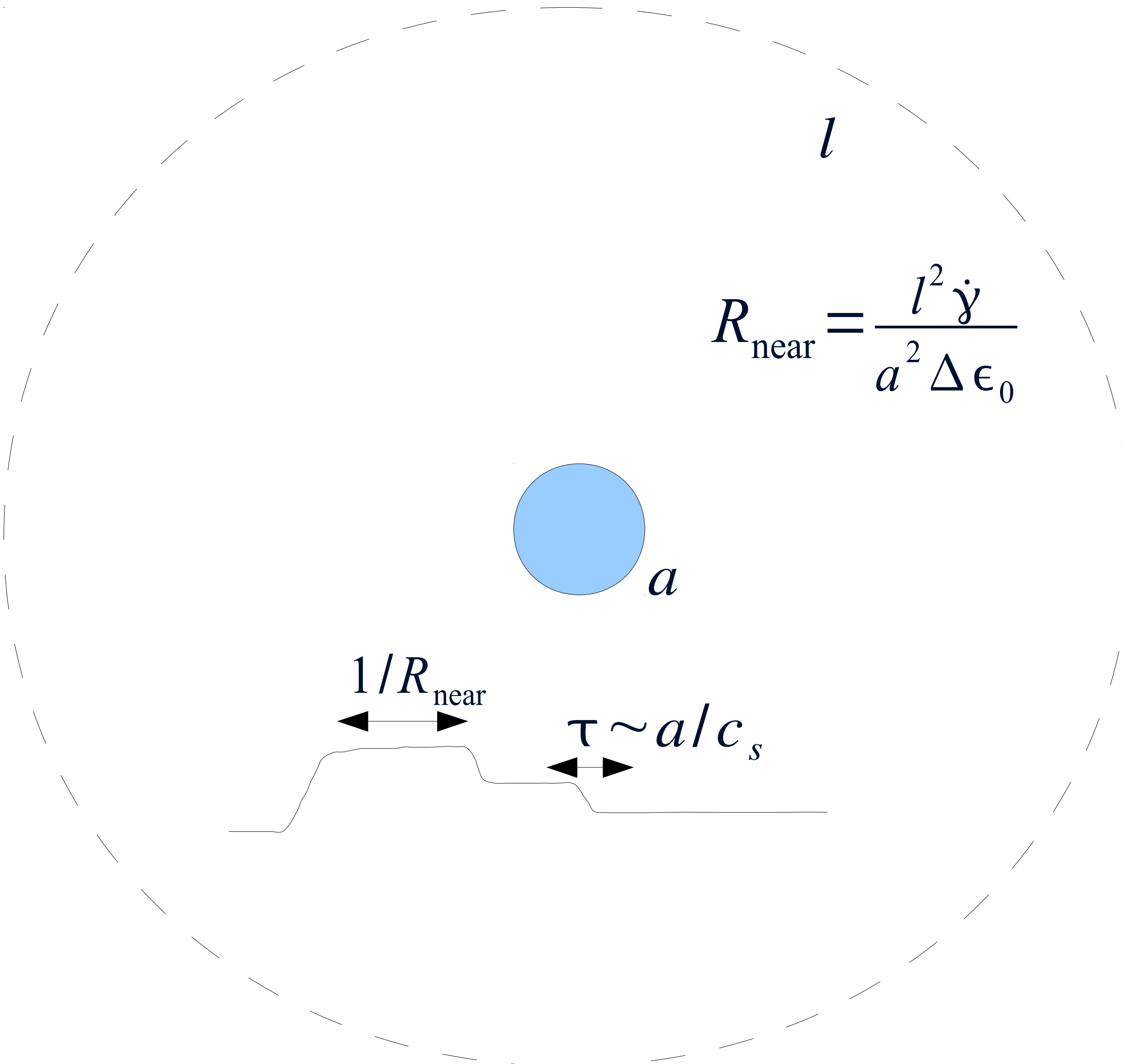
What is the noise received by a weak (marginal) zone?

System size: L

Total flip rate: $R_{\text{flip}} = \frac{L^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$



What is the noise received by a weak (marginal) zone?



System size: L

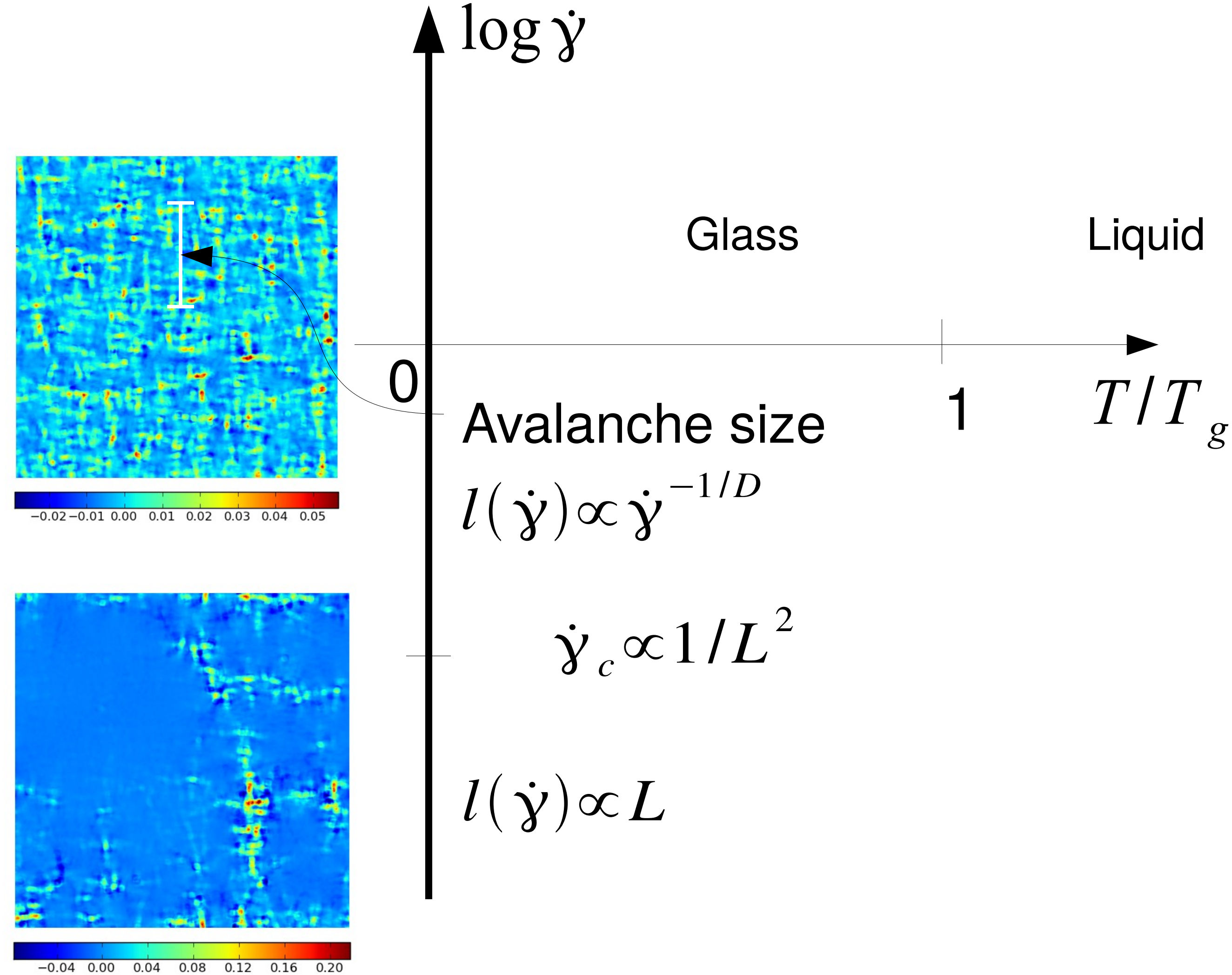
Total flip rate: $R_{\text{flip}} = \frac{L^2 \dot{\gamma}}{a^2 \Delta \epsilon_0}$

Signals originating from the sphere of radius

$$l \sim \sqrt{a^2 \Delta \epsilon_0 / \dot{\gamma} \tau_{\text{flip}}}$$

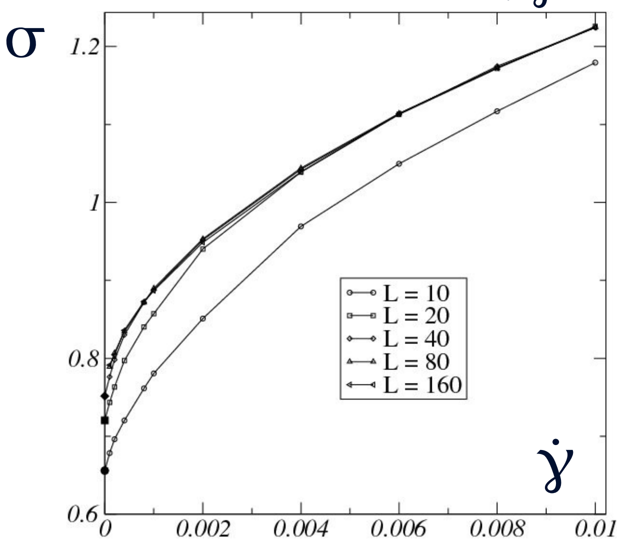
are non-overlapping

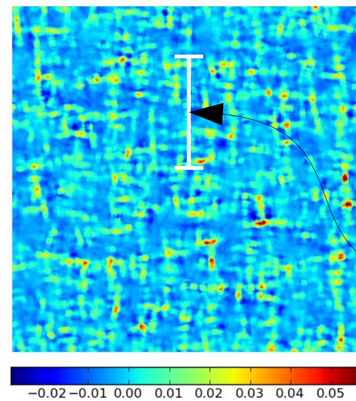
Athermal, finite strain rate

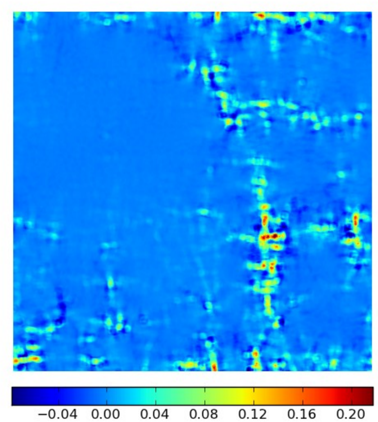


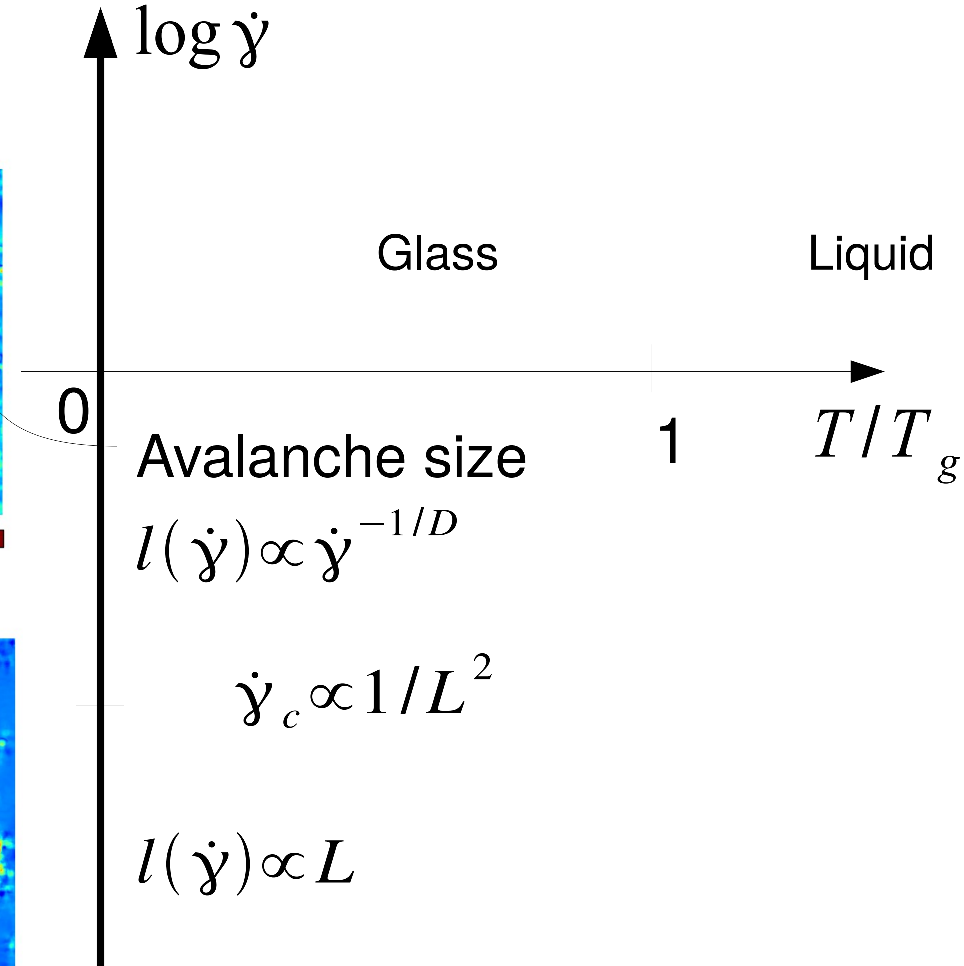
Athermal, finite strain rate

$\sigma = 0.74 + 4.87\sqrt{\dot{\gamma}}$









guess: $\sigma - \sigma_y \approx \mu\dot{\gamma}\tau_{av}/2$

event duration: $\tau_{av} \sim l/c_s$

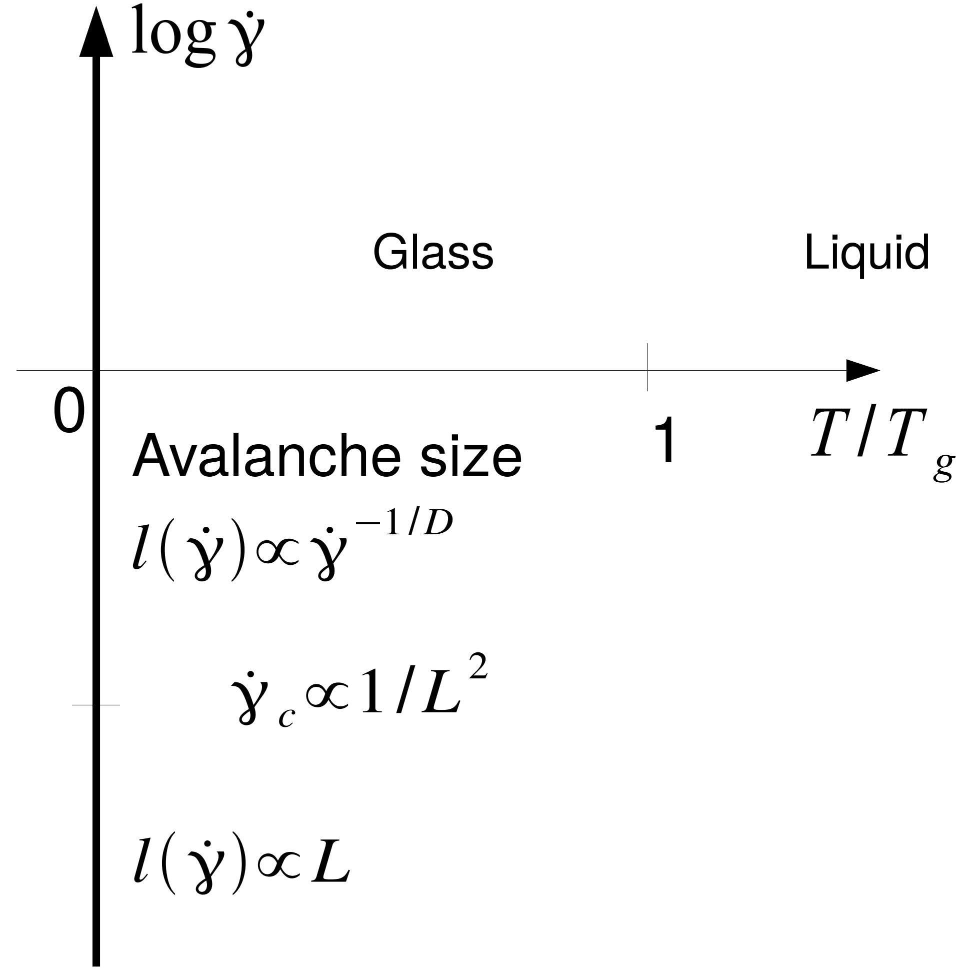
$$l \sim \sqrt{a^2 \Delta\epsilon_0 / \dot{\gamma}\tau_{flip}}$$

$$\Rightarrow \boxed{\sigma = \sigma_y + C\sqrt{\dot{\gamma}}}$$

$$C = \frac{\mu}{2c_s} a \sqrt{\frac{\Delta\epsilon_0}{\tau}} \approx 7.5$$

AL and C. Caroli, PRL 103, 065501 (2009)

At finite T


$\log\dot\gamma$
Glass
Liquid
0
1
T/T_g
Avalanche size
$l(\dot\gamma)\propto\dot\gamma^{-1/D}$
$\dot\gamma_c\propto 1/L^2$
$l(\dot\gamma)\propto L$

Identify the mechanisms that govern plasticity

- nature of events: flips, avalanches (slip lines)

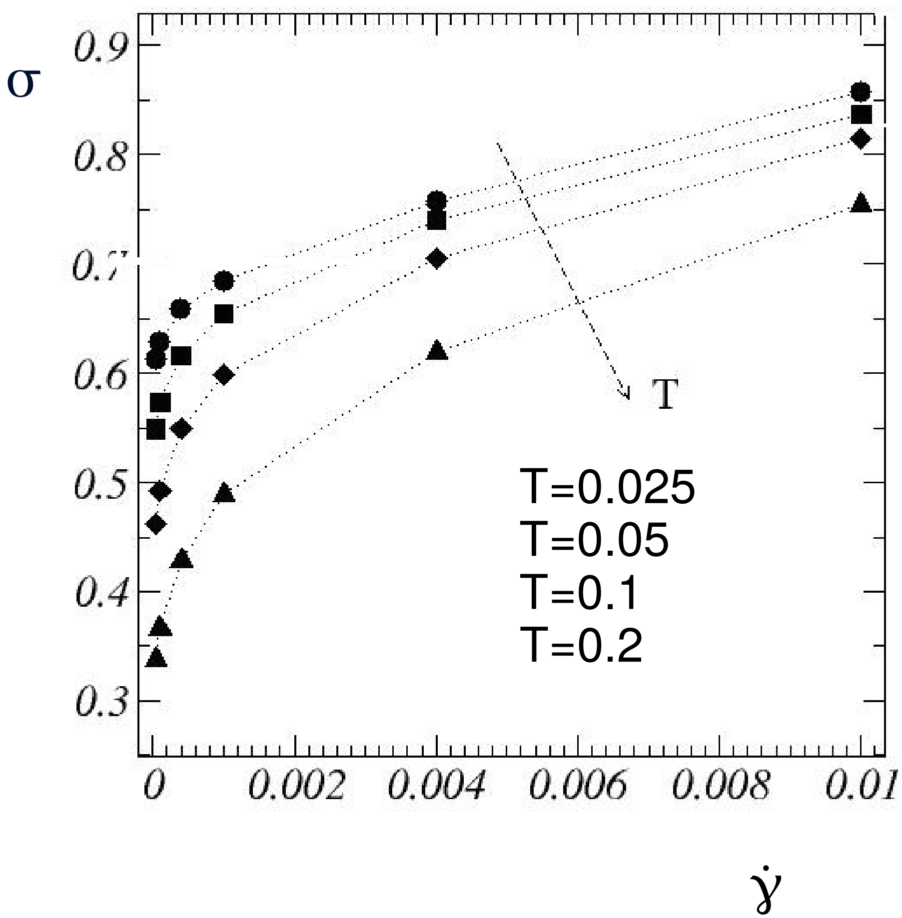
- observation of avalanches => correlations

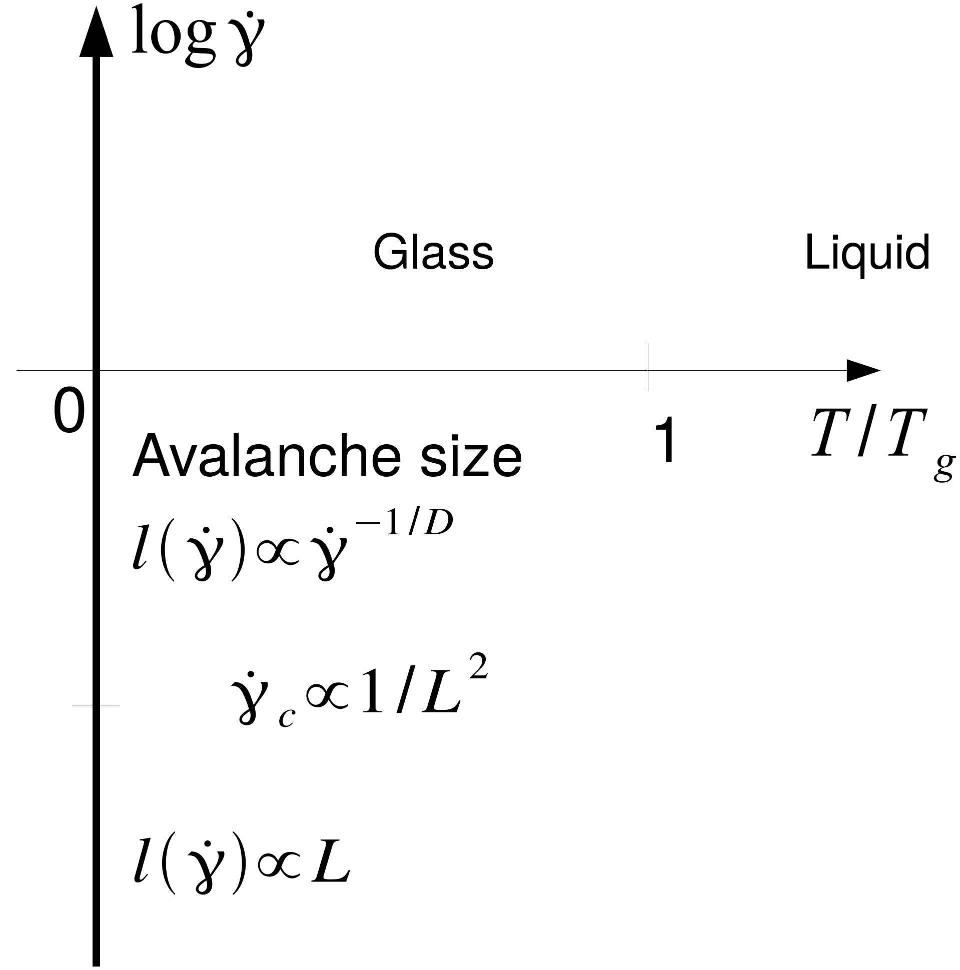
Validity at finite rates? Yes.

- avalanches related to correlations and rheology

Relevance at finite temperatures?

Finite T

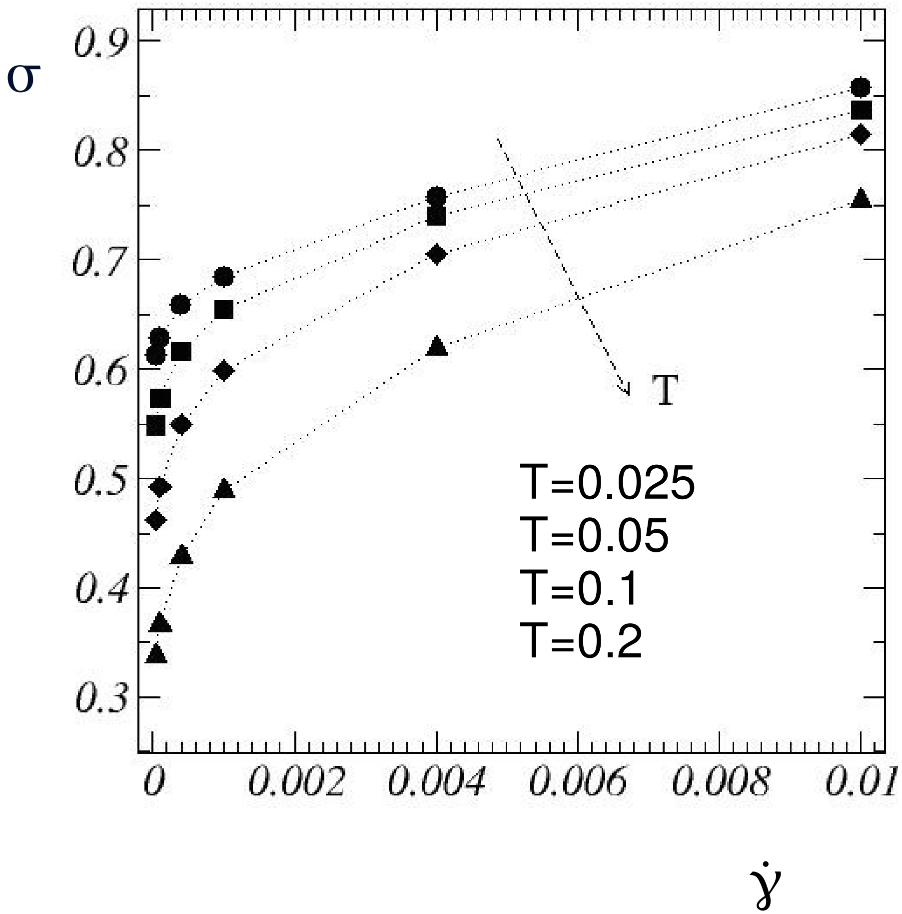


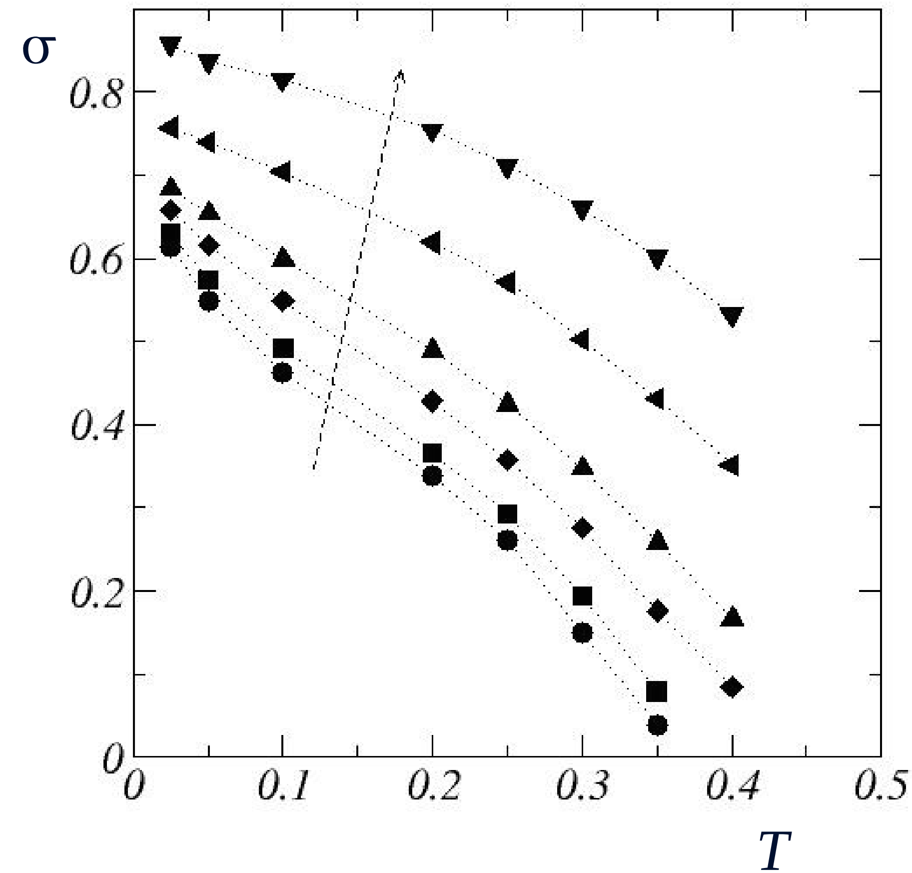


$\sigma(\dot{\gamma})$

- Decreases strongly with T
- No longer fits Hershel Bulkley law

Finite T

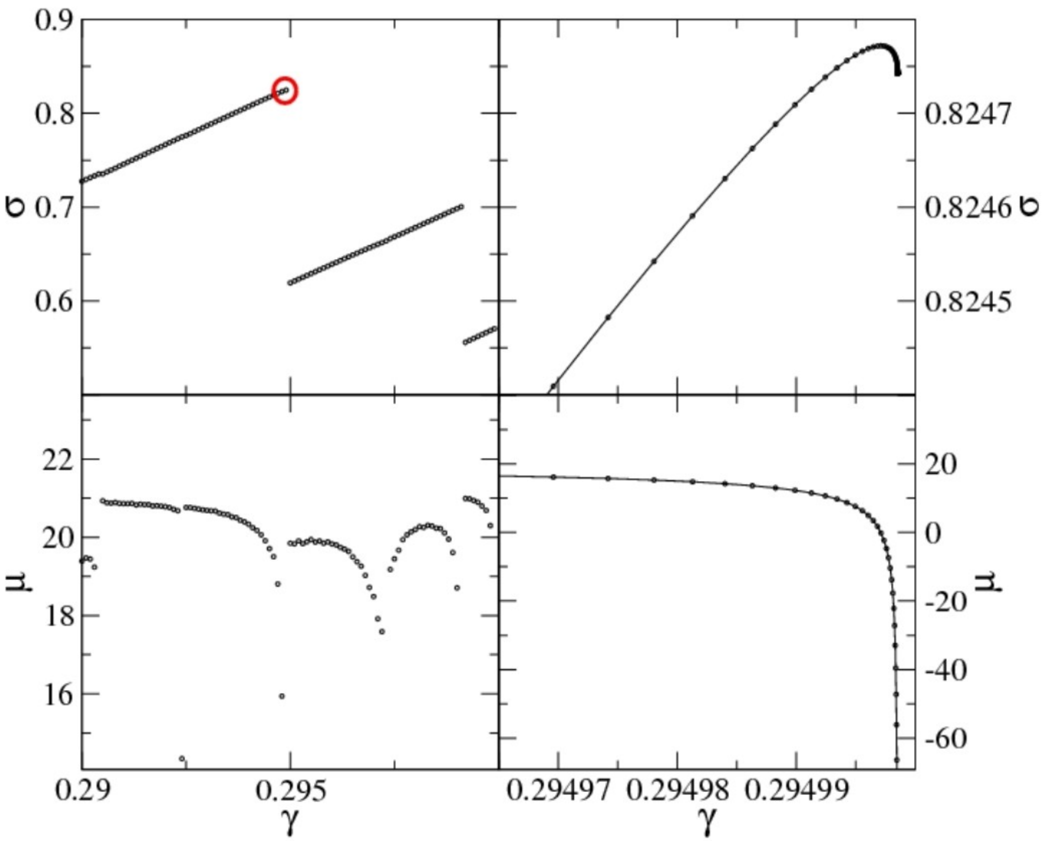


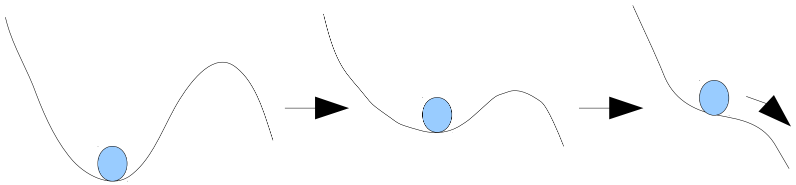


$\sigma(\dot{\gamma})$ - Decreases strongly with T

- No longer fits Hershel Bulkley law

AQS: Events = saddle node crossings





$$\sigma \sim -A\sqrt{\gamma_c - \gamma}$$

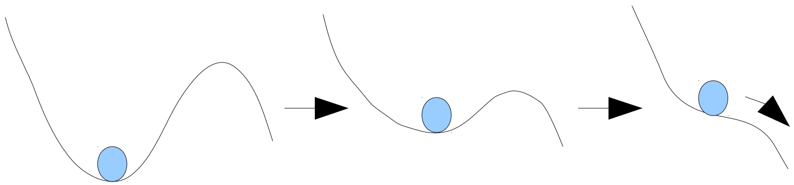
$$\Delta E \sim (\gamma_c - \gamma)^{3/2}$$

Maloney, AL, 195501 PRL (2004)

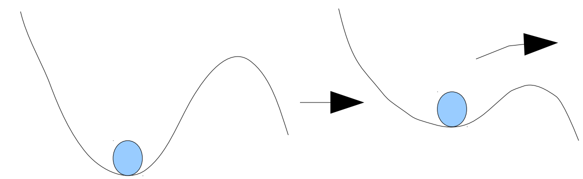
Activation over driven barriers

Chattoraj et al, PRL 105, 26601 (2010)

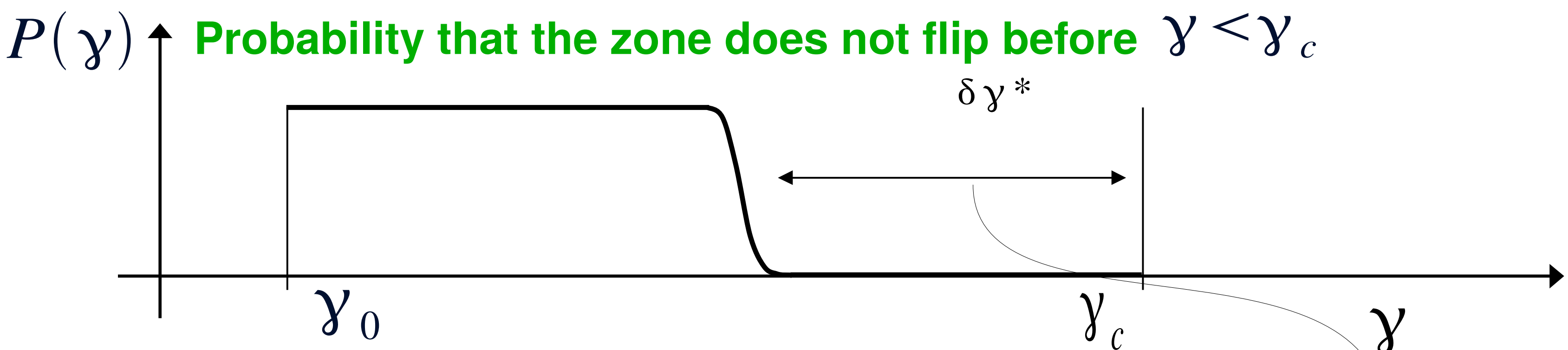
$T=0$



$T \neq 0$



Probability that the zone does not flip before $\gamma < \gamma_c$



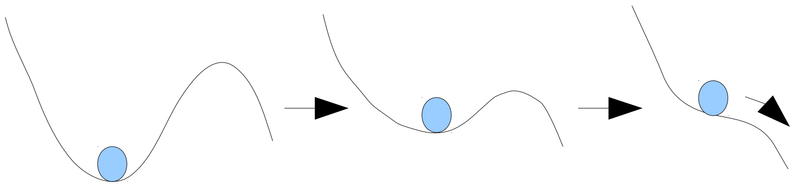
$$\delta\gamma^* \sim \left[\frac{T}{B} \ln\left(\frac{2}{3} \frac{\nu}{\dot{\gamma}} \left(\frac{T}{B} \right)^{5/6} \right) \right]^{2/3}$$

$$\sigma(\dot{\gamma};T) = \sigma(\dot{\gamma};T=0) - 2\mu\,\overline{\delta\gamma^*}$$

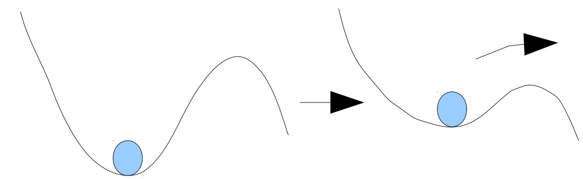
Activation over driven barriers

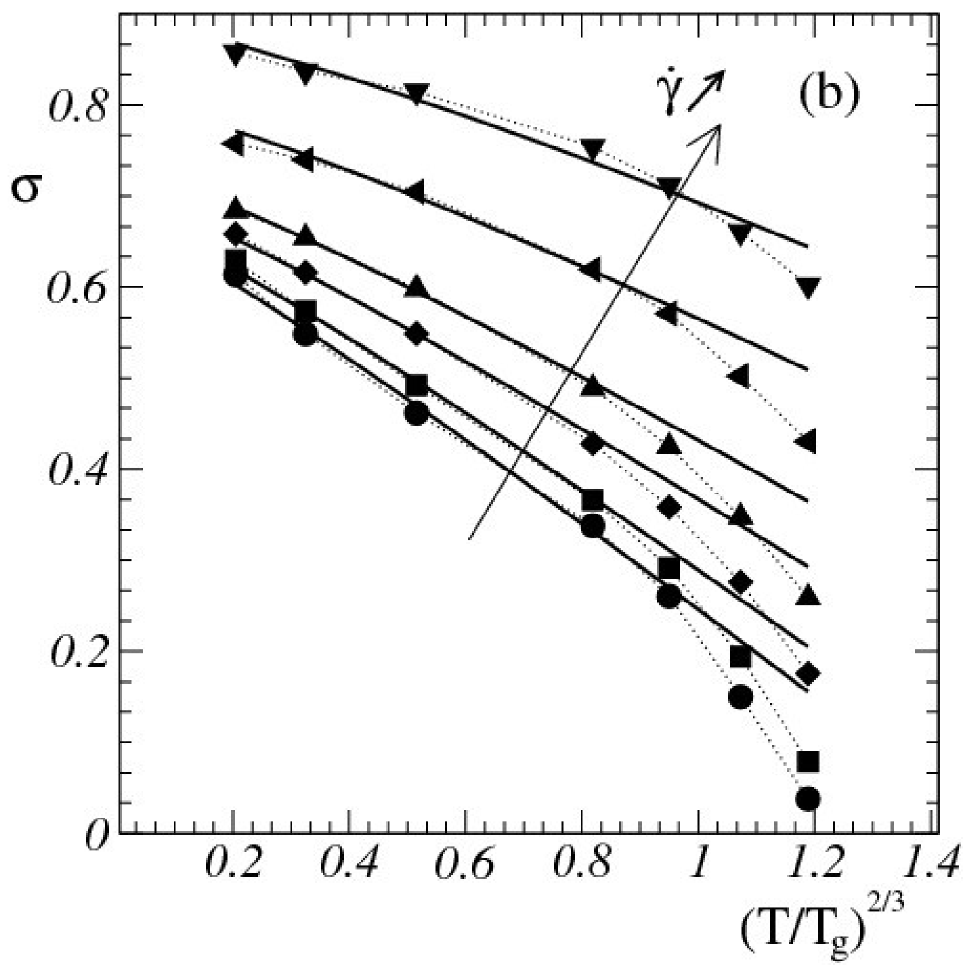
Chattoraj et al, PRL 105, 26601 (2010)

$T=0$



$T\neq 0$





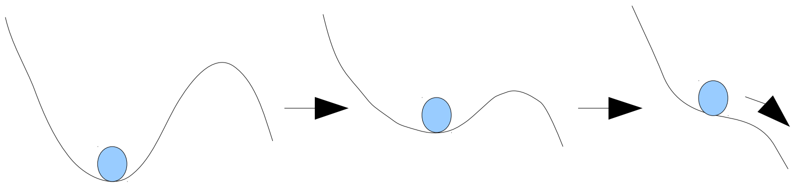
$$\delta\gamma^{*} \sim \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot{\gamma}}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

$$\sigma(\dot{\gamma};T)=\sigma(\dot{\gamma};T=0)-2\mu\,\overline{\delta\gamma^{*}}$$

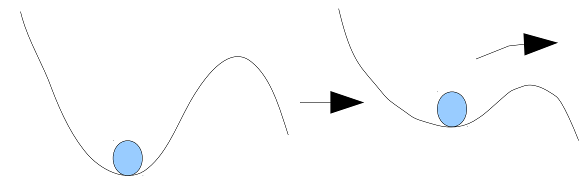
Activation over driven barriers

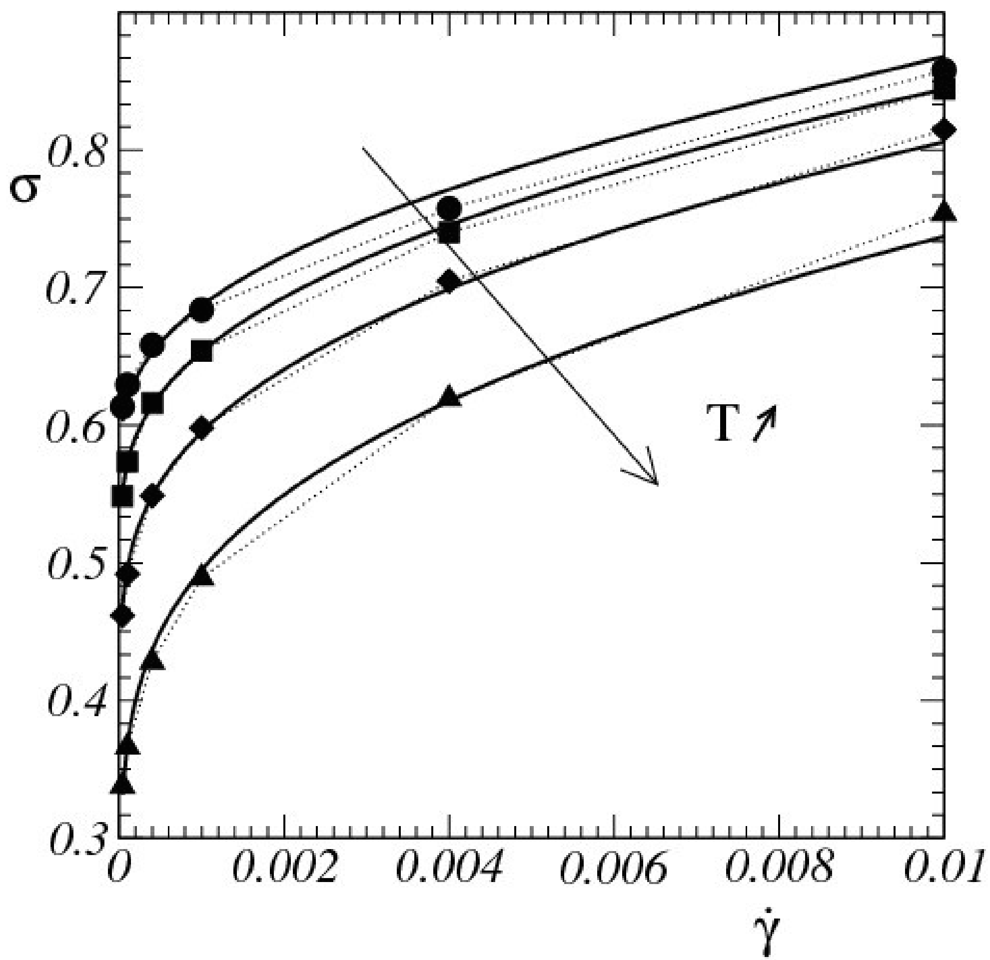
Chattoraj et al, PRL 105, 26601 (2010)

$T=0$



$T\neq 0$



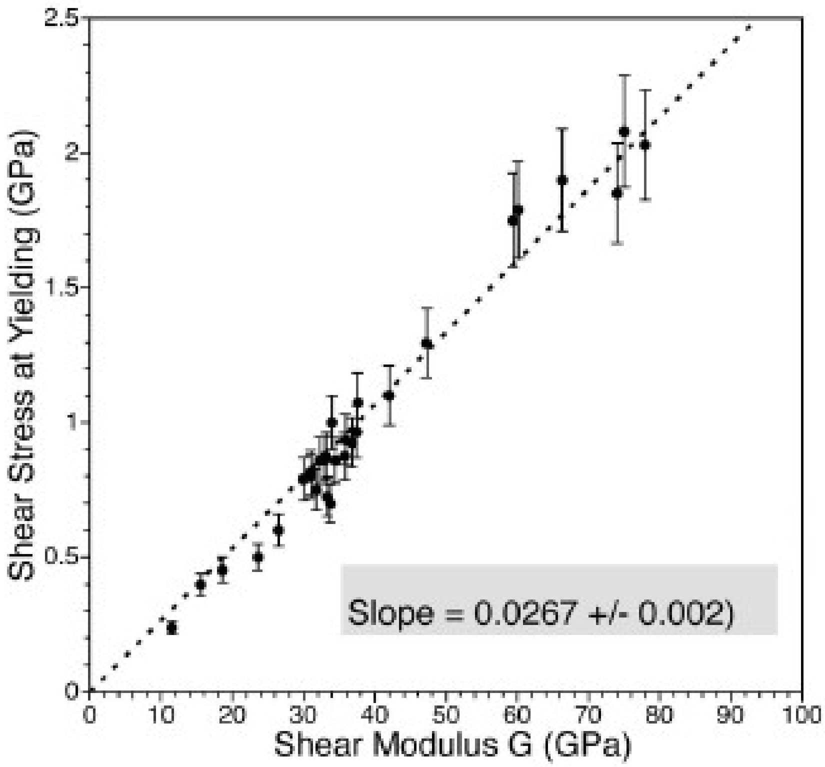


$$\delta\gamma^* \sim \left[\frac{T}{B}\ln\left(\frac{2}{3}\frac{\nu}{\dot\gamma}\left(\frac{T}{B}\right)^{5/6}\right)\right]^{2/3}$$

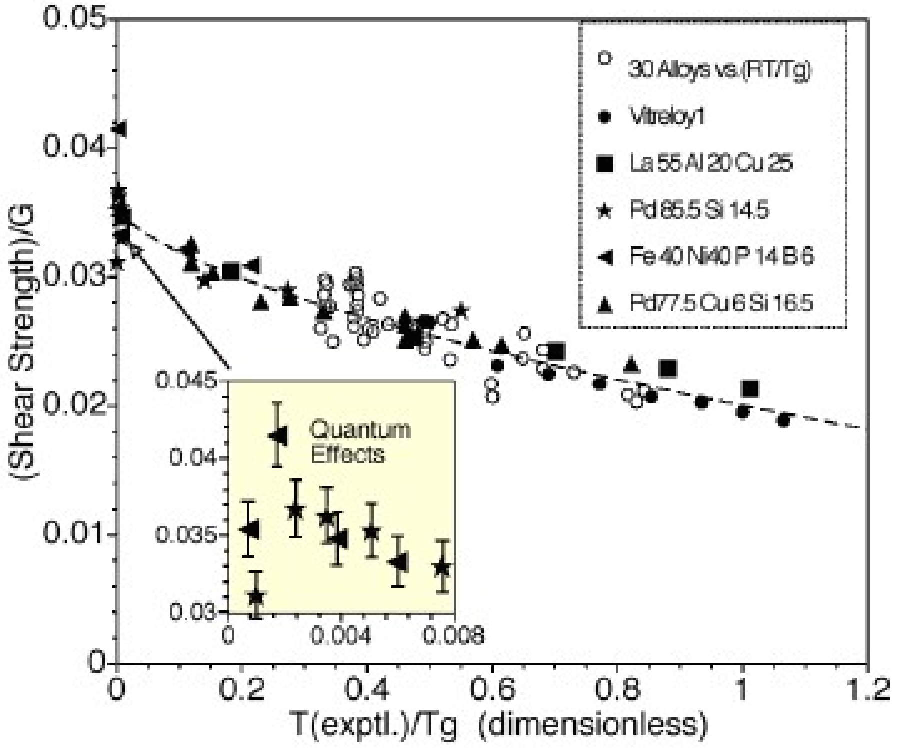
$$\sigma(\dot\gamma;T)=\sigma(\dot\gamma;T=0)-2\mu\overline{\delta\gamma^*}$$

Metallic glass yield stress

Johnson & Samwer 95, 195501 (2005)



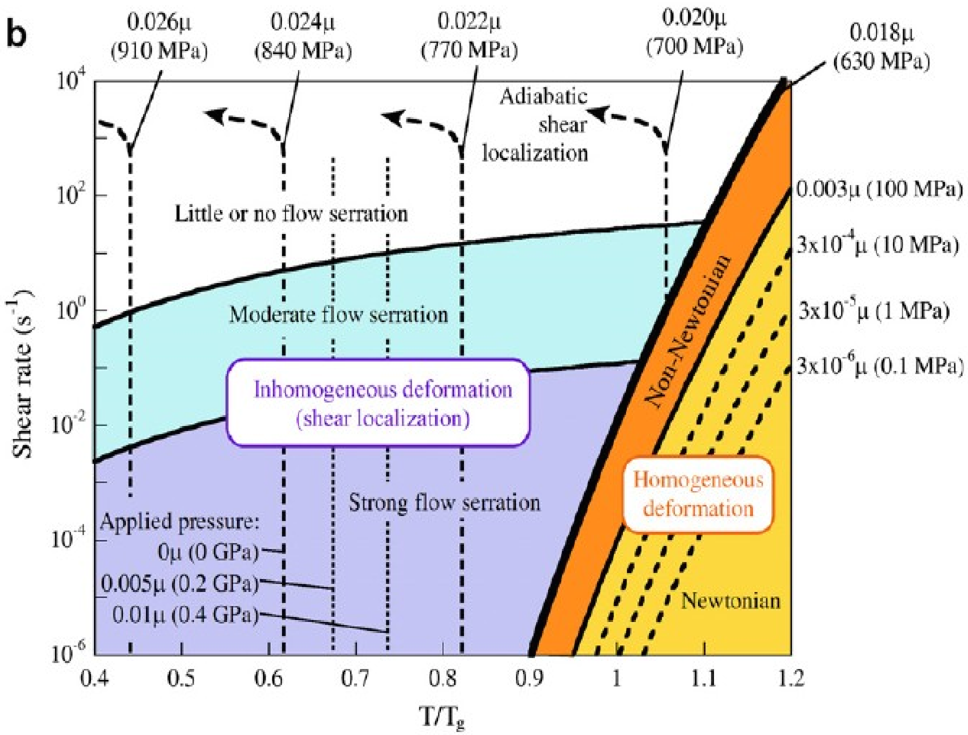
$$\sigma - \sigma_Y \propto T^{2/3}$$

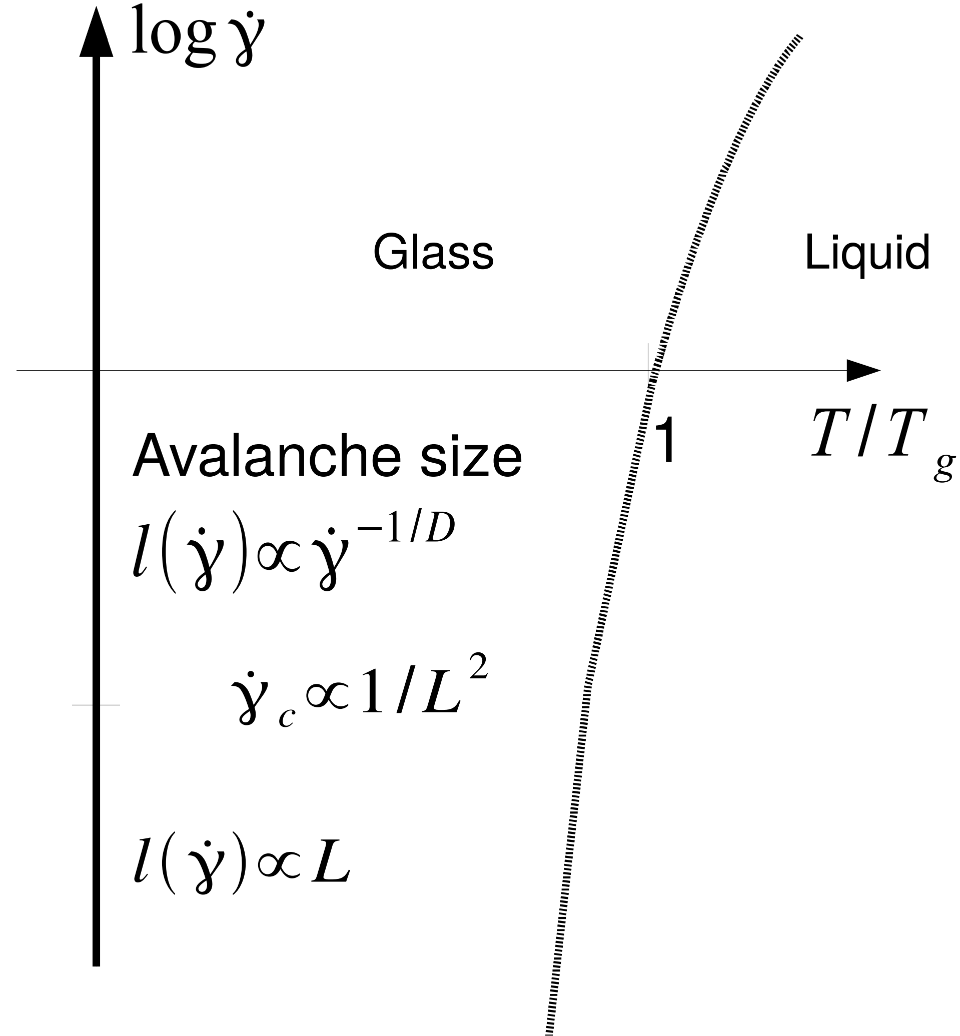


At finite T

Schuh *et al*,

Acta Mat. 55, 4067 (2007)

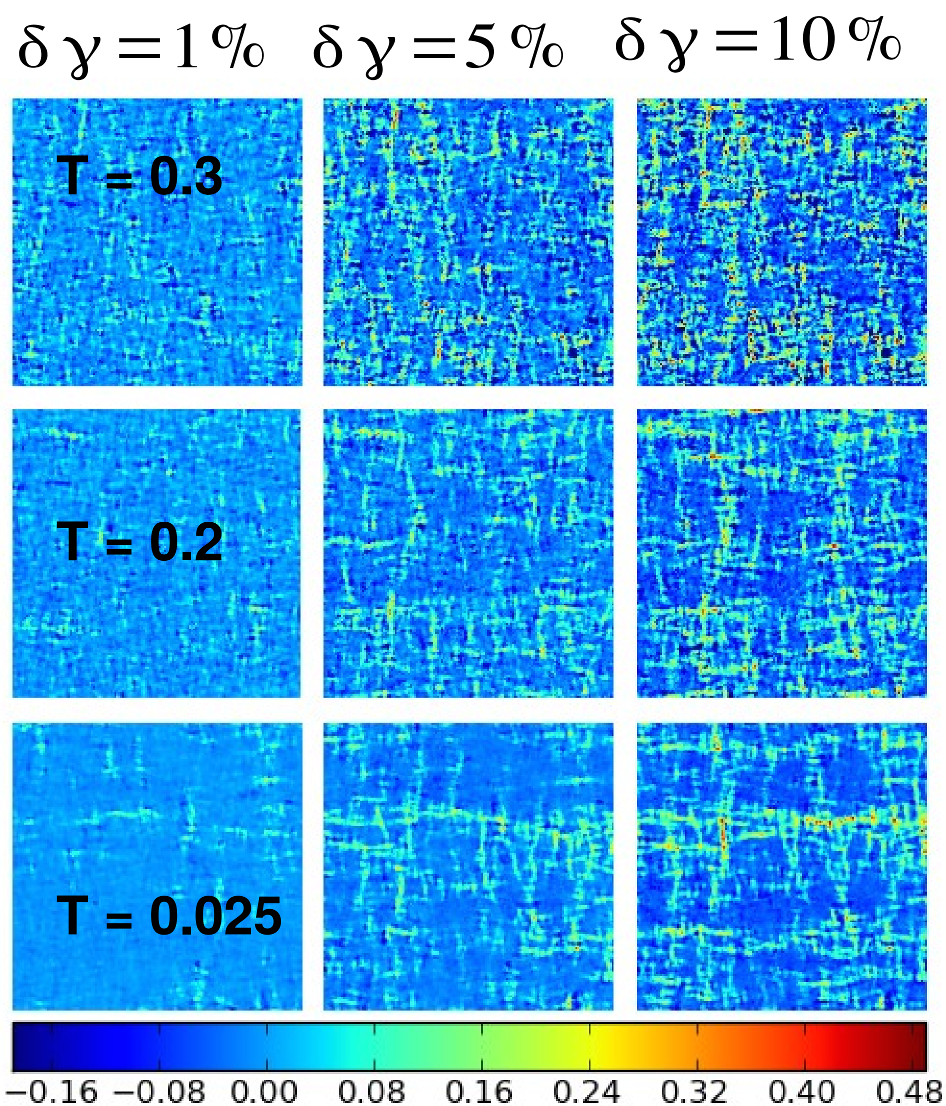


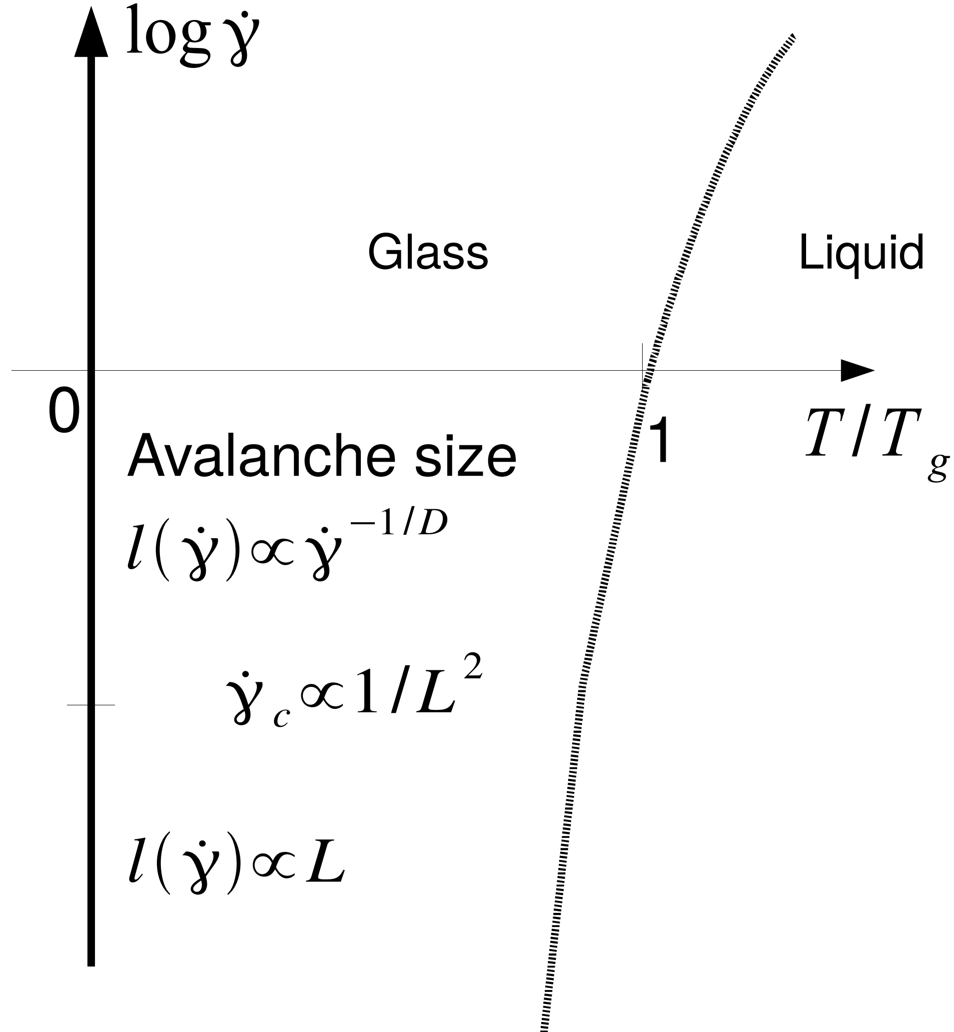


Chattoraj *et al* PRL 105, 266001 (2010)

Chattoraj, et al, PRE 011501 (2011)

At finite T





Chattoraj *et al* PRL 105, 266001 (2010)
Chattoraj, et al, PRE 011501 (2011)

Identify the mechanisms that govern plasticity

- nature of events: flips, avalanches (slip lines)

- observation of avalanches => correlations

Validity at finite rates? Yes.

- avalanches related to correlations and rheology

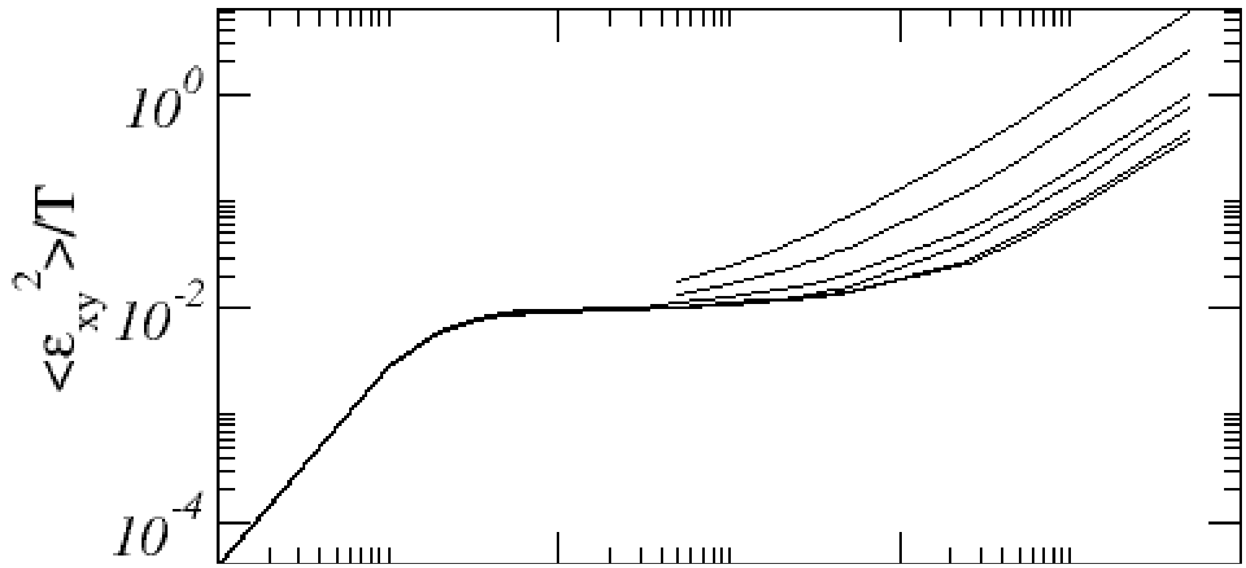
Relevance at finite temperatures?

- avalanches ~ unchanged

- shifts in strain / time ==> rheology

What information is contained in the shear strain autocorrelation?

$$C_{xy} = \langle \epsilon_{xy}(\underline{r}; t, t+\Delta t)\, \epsilon_{xy}(\underline{r} + \underline{\Delta r}; t, t+\Delta t) \rangle$$



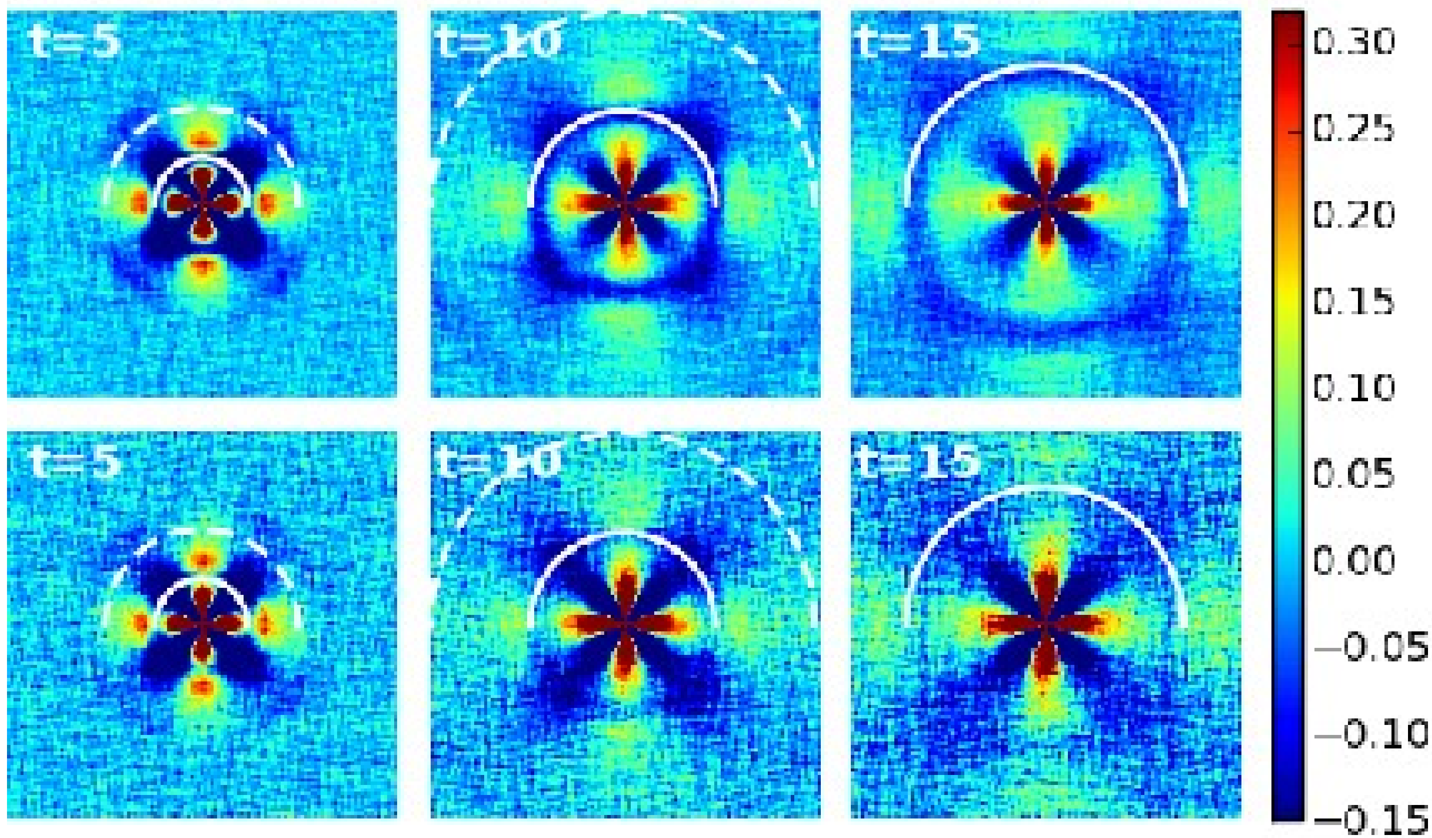
Short time: vibrations around inherent states

Long time: elasticity + accumulation of plastic events

Chattoraj, AL, PRL (2013)

$$C_{xy} = \langle \epsilon_{xy}(\underline{r}; t, t+\Delta t)\, \epsilon_{xy}(\underline{r} + \underline{\Delta r}; t, t+\Delta t) \rangle$$

$$C_{xy}/T$$



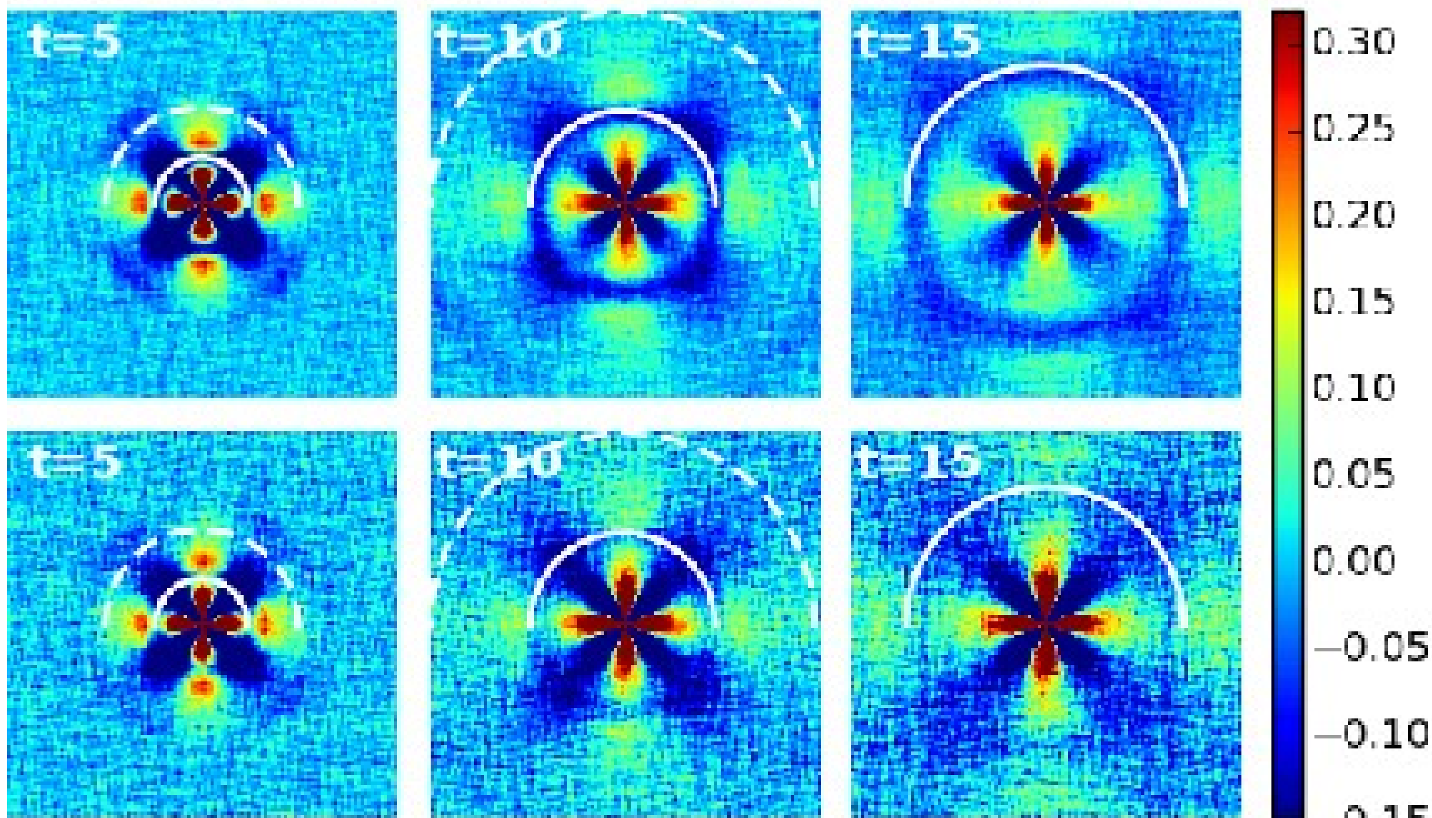
Chattoraj, AL, PRL (2013)

$$C_{xy} = \langle \epsilon_{xy}(\underline{r}; t, t+\Delta t)\, \epsilon_{xy}(\underline{r} + \underline{\Delta r}; t, t+\Delta t) \rangle$$

C_{xy}/T

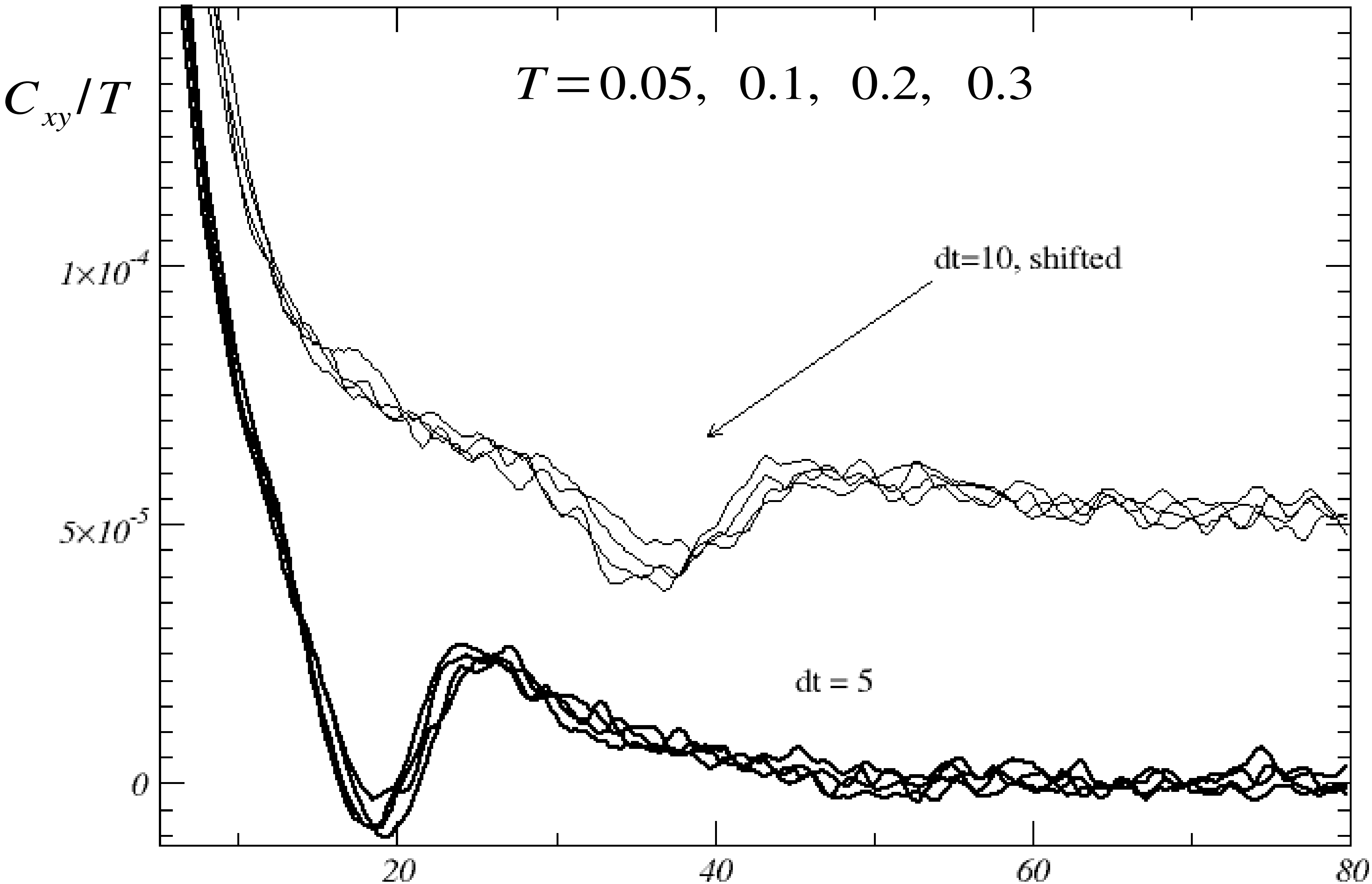
$T = 0.1$

$T = 0.3$



Chattoraj, AL, PRL (2013)

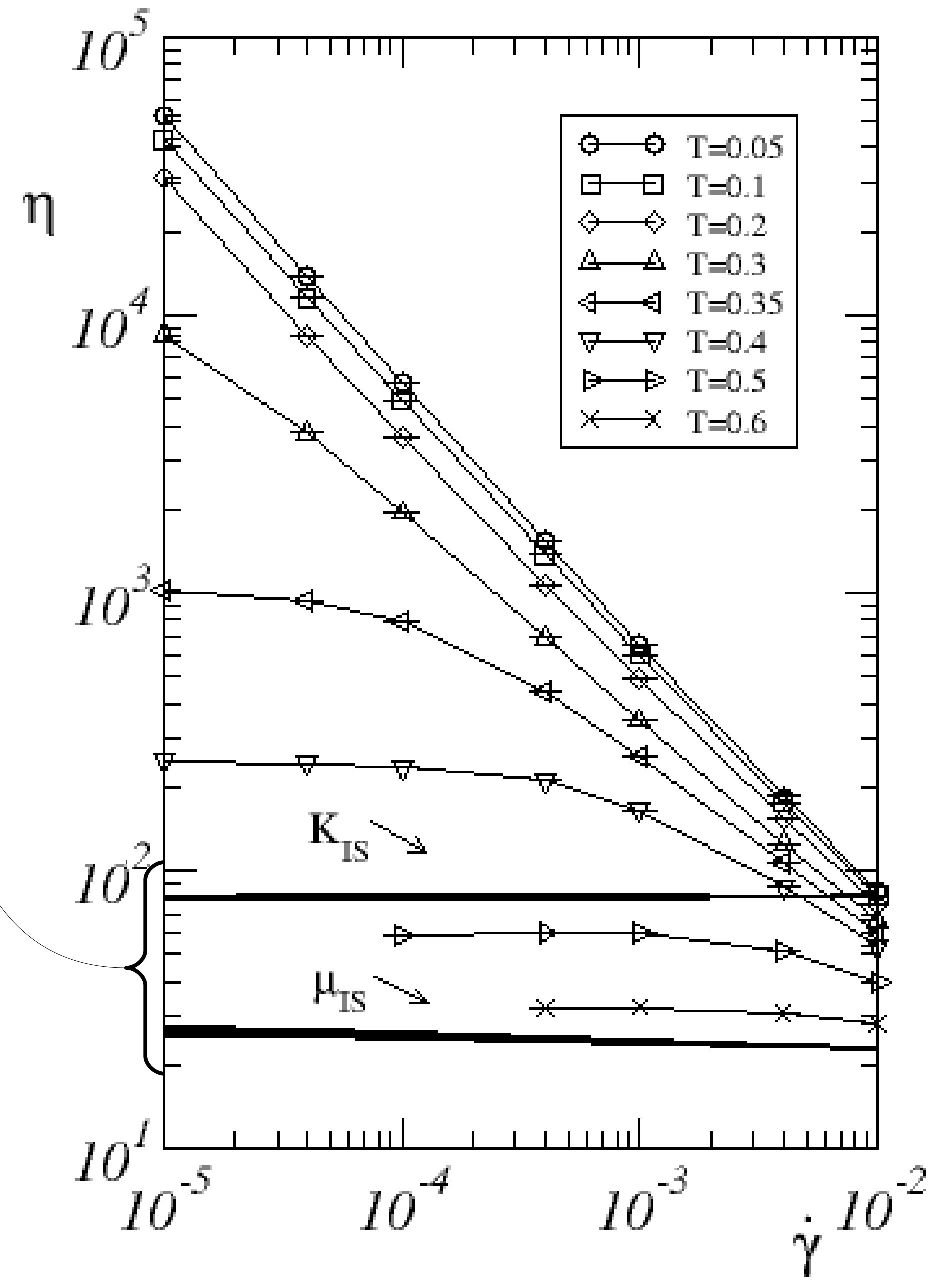
Fronts



Chattoraj, AL, PRL (2013)

Elastic moduli of inherent states

~ T-independent



Chattoraj, AL, PRL (2013)

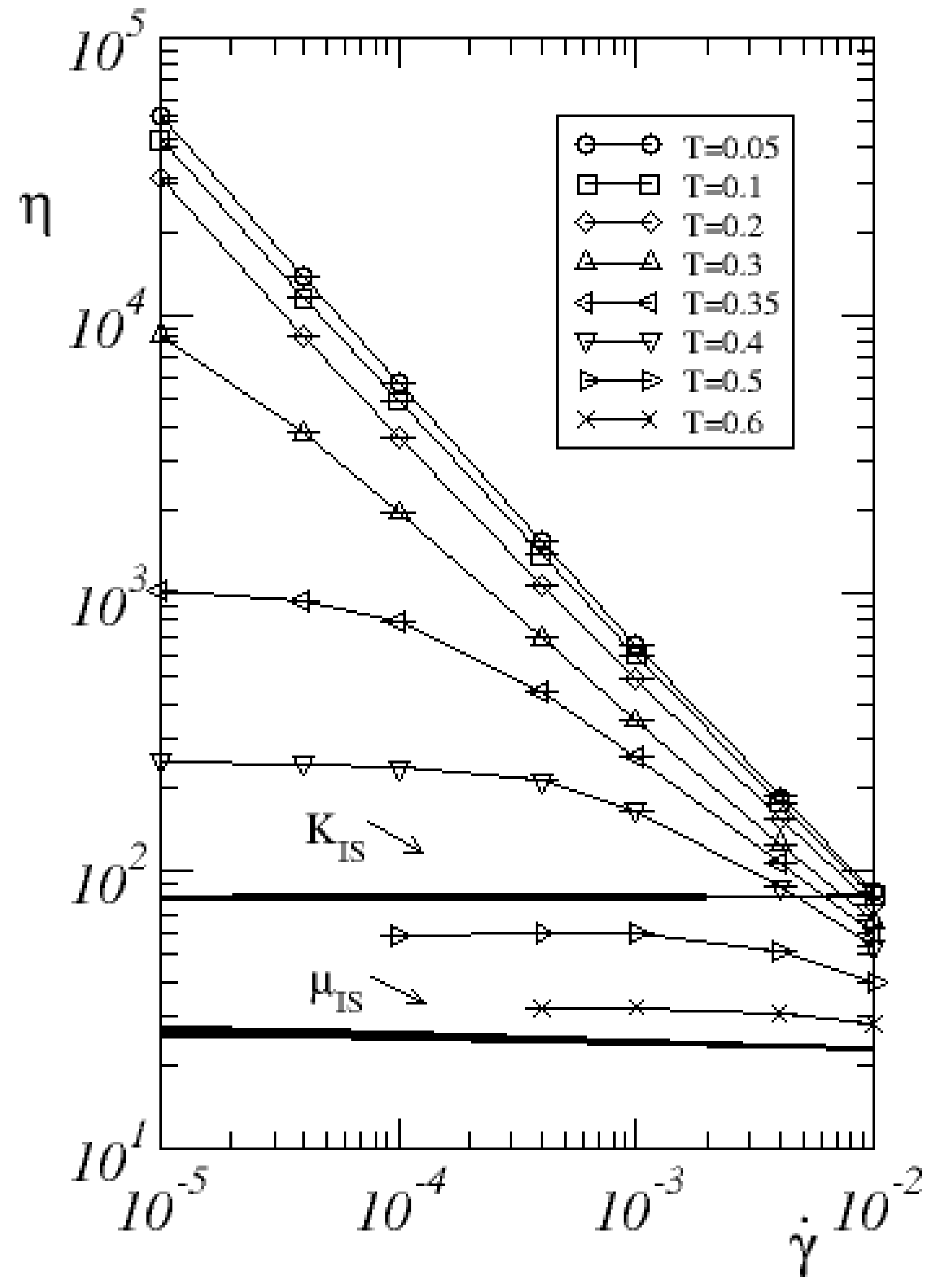
Denote $C_{xy} = \langle \underline{\underline{\epsilon}}(\underline{r}; t, t+\Delta t)\, \underline{\underline{\epsilon}}(\underline{r}+\underline{R}; t, t+\Delta t) \rangle$

Focus on
$$\left\{ \begin{array}{l} \dot{\gamma} = 10^{-4} \\ \Delta \gamma = 20\% \\ \Delta t = 2000 \end{array} \right.$$

Transition between:

$$T = 0.35$$

$$T = 0.4$$



Chattoraj, AL, PRL (2013)

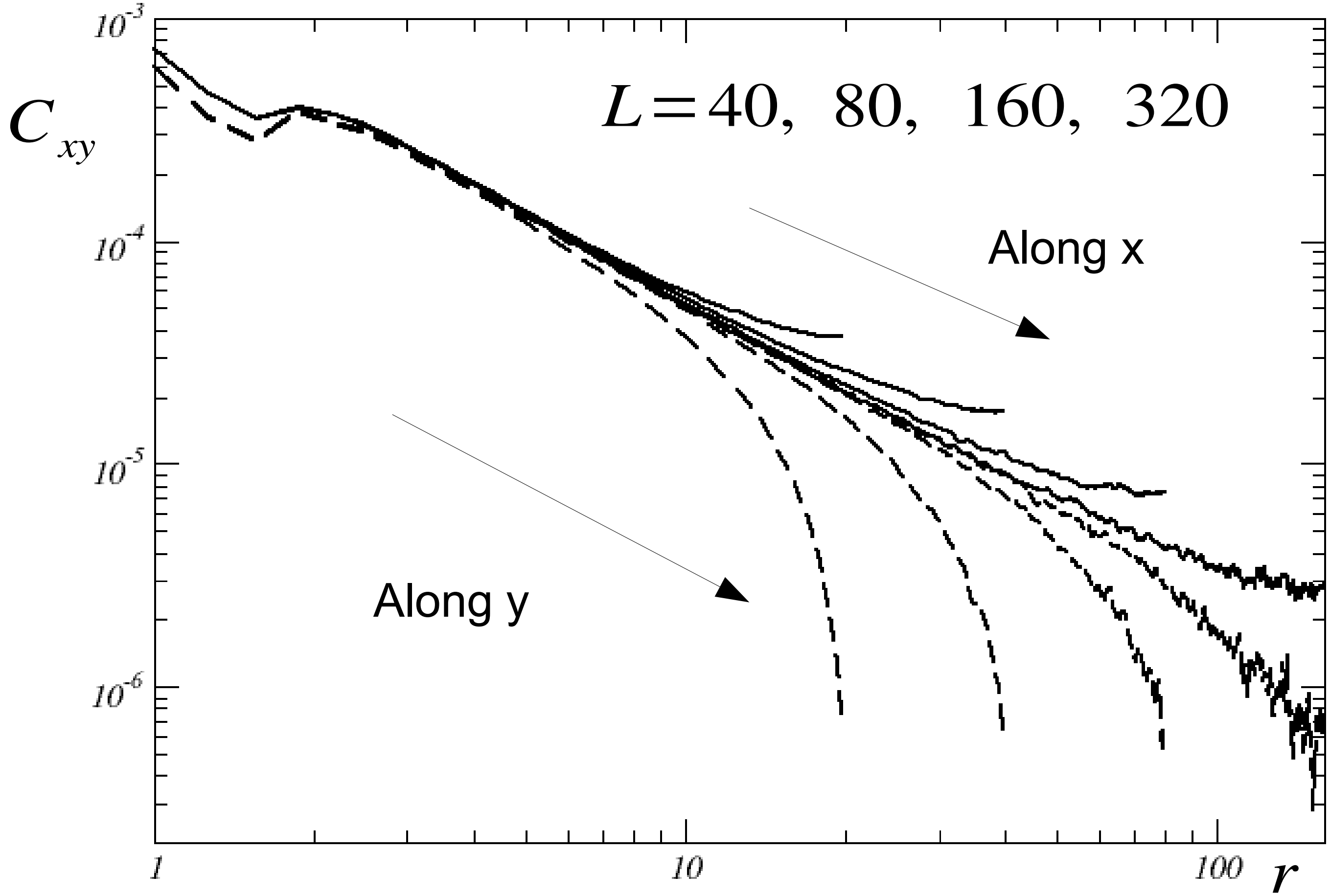
Effect of system size

$T = 0.1$

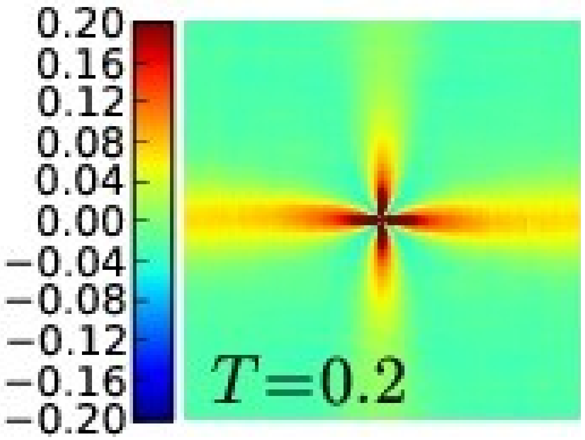
$\dot{\gamma} = 10^{-4}$

$\Delta\gamma = 20\%$

$\Delta t = 2000$

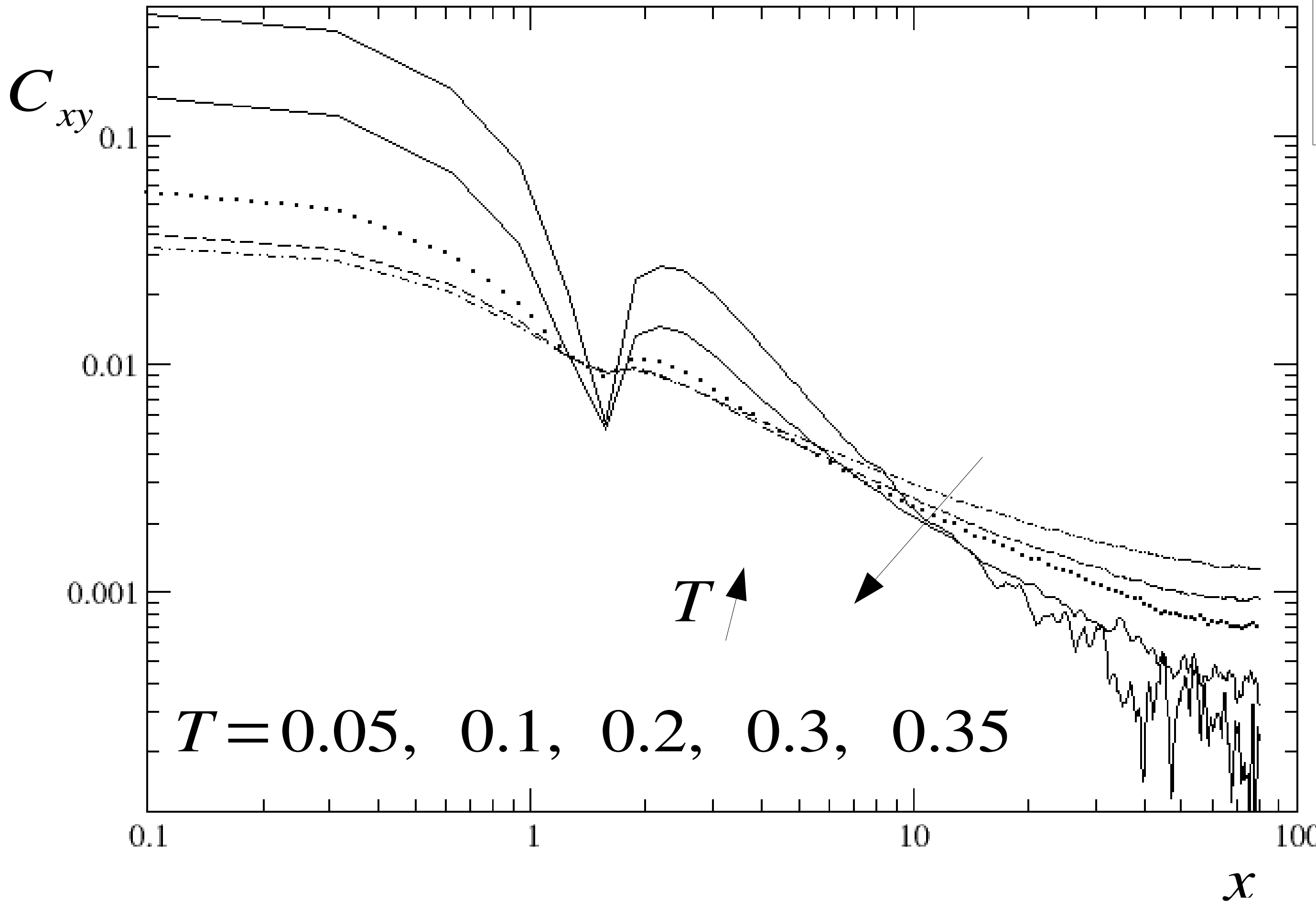


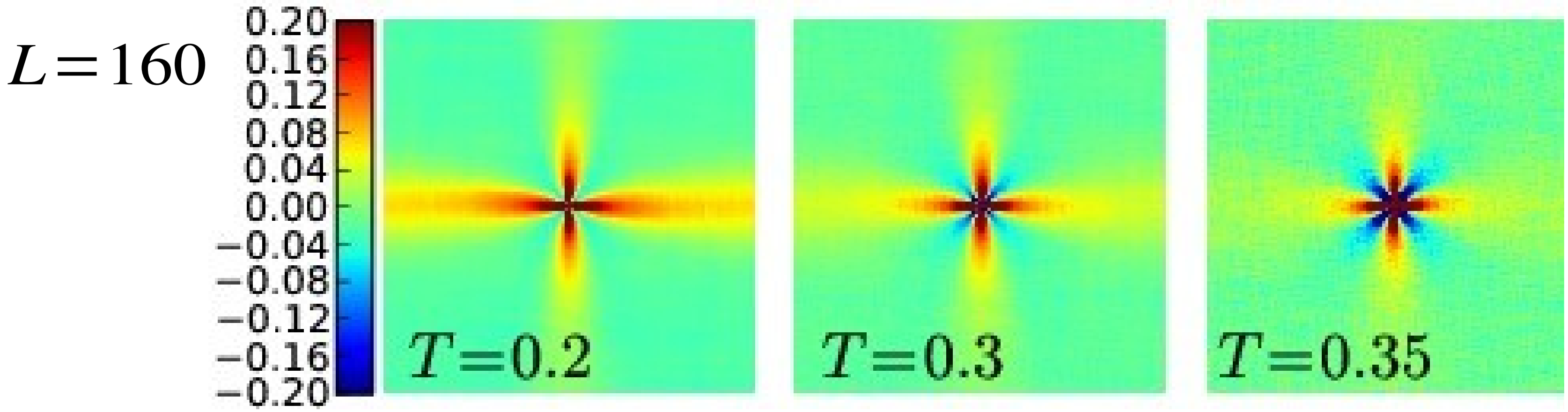
$L = 160$



Effect of T

$$\dot{\gamma} = 10^{-4}$$
$$\Delta\gamma = 20\%$$
$$\Delta t = 2000$$



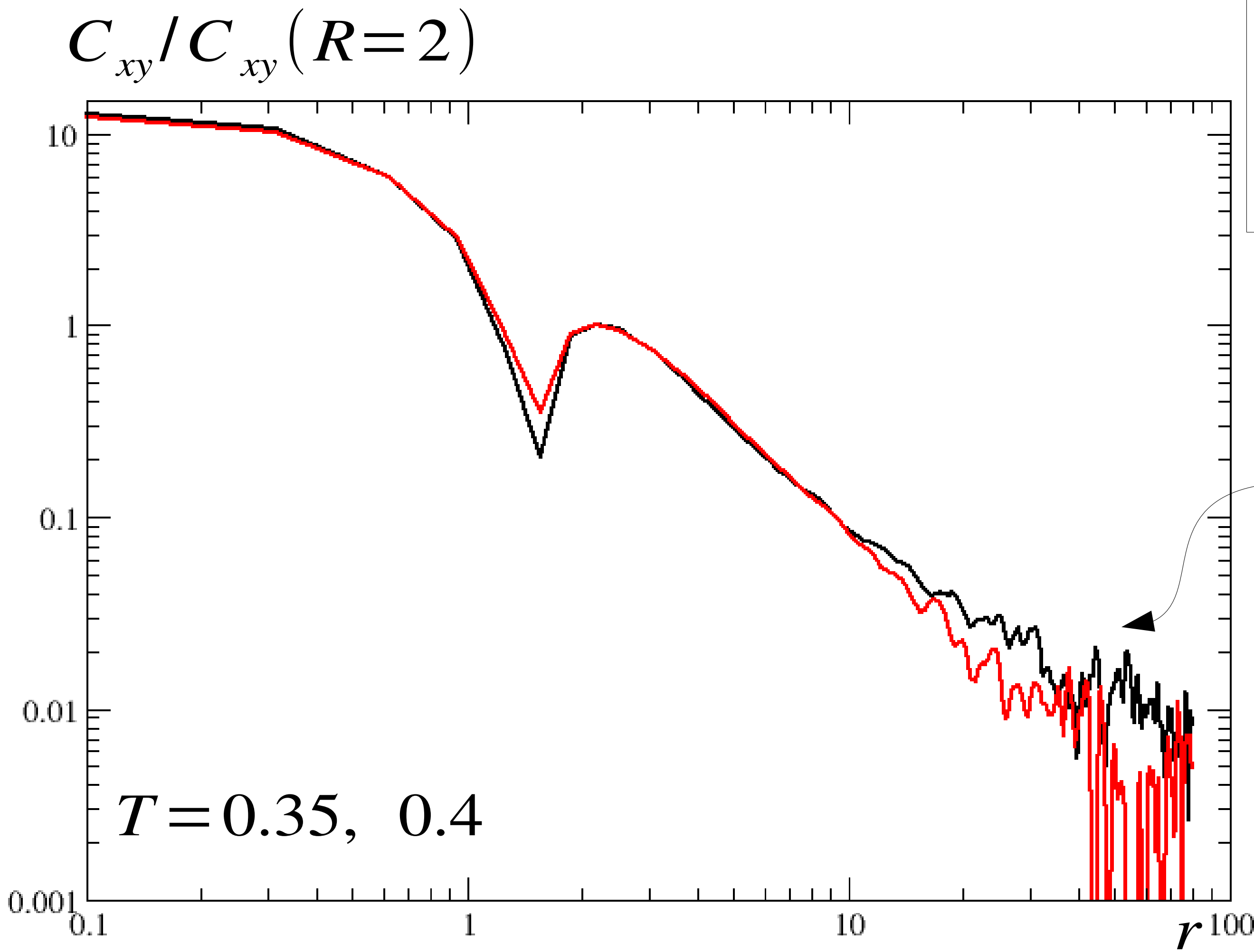


Effect of T

$\dot{\gamma} = 10^{-4}$

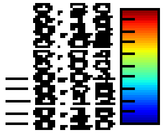
$\Delta\, \gamma = 20\,\%$

$\Delta\, t = 2000$



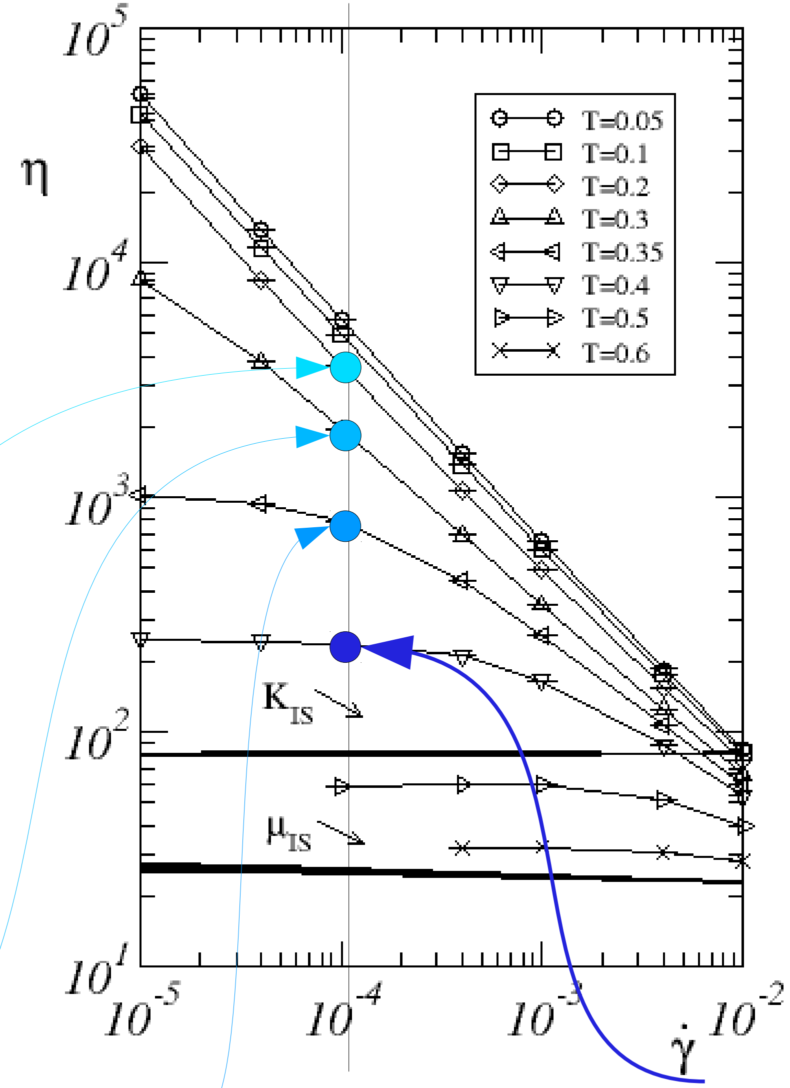
Excess of correlation

$L = 160$

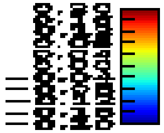


$T = 0.2$ $T = 0.3$ $T = 0.35$ $T = 0.4$

Interpreting the excess correlation



$L = 160$



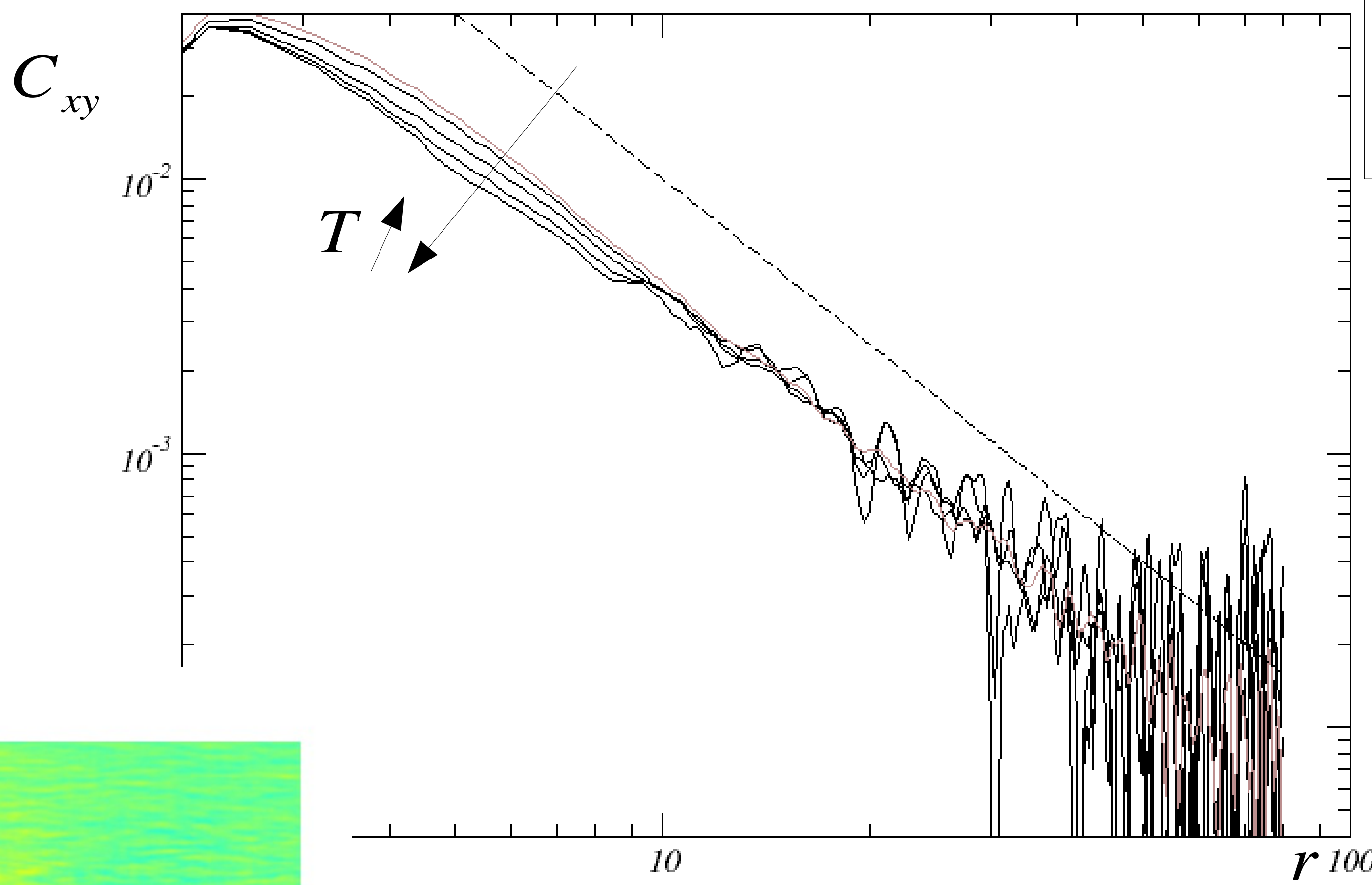
$T=0.2$ $T=0.3$ $T=0.35$ $T=0.4$

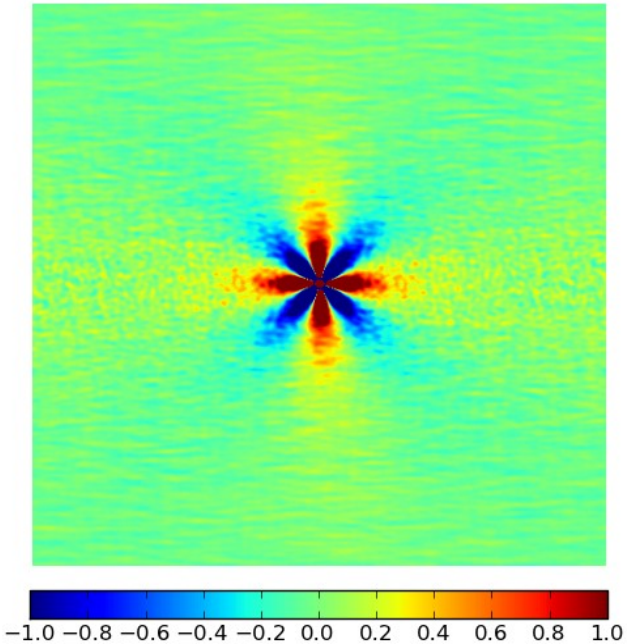
$T = 0.35, \quad \tau_\alpha = 100$

$\dot{\gamma} = 10^{-5}$

$\Delta\gamma \leq 20\,\%$

$\Delta t \leq 2.10^4$

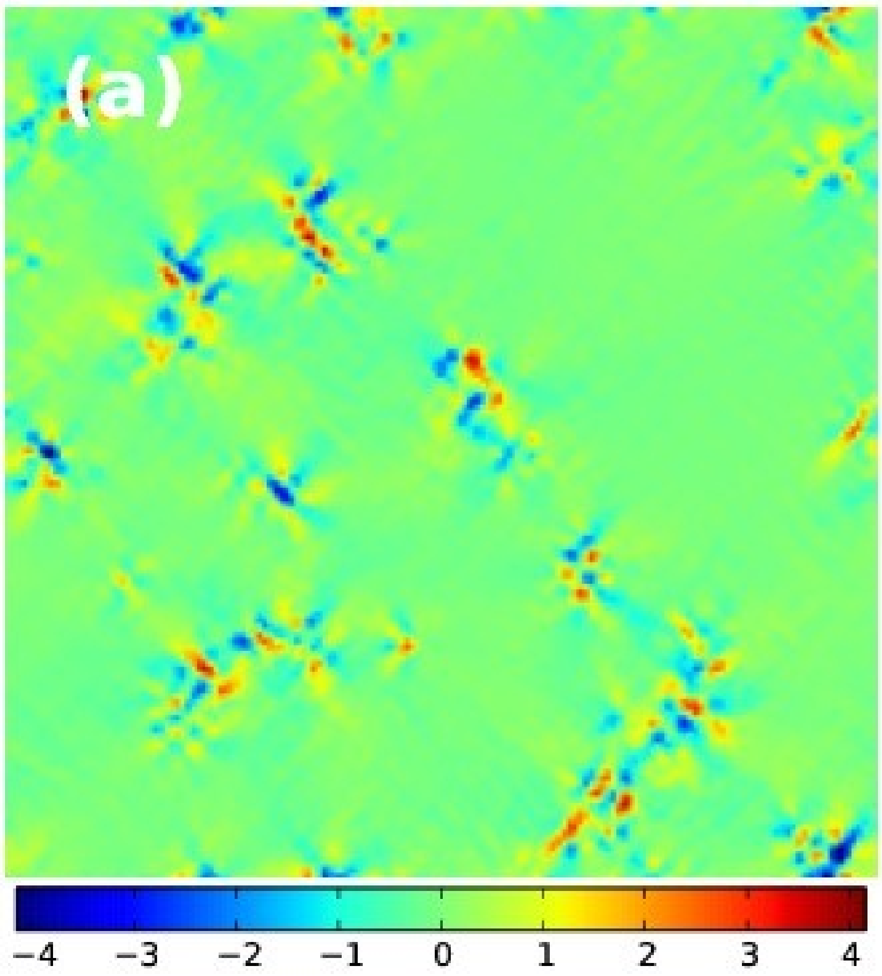




Signature of Eshelby fields

Stress increments in the relaxing liquid

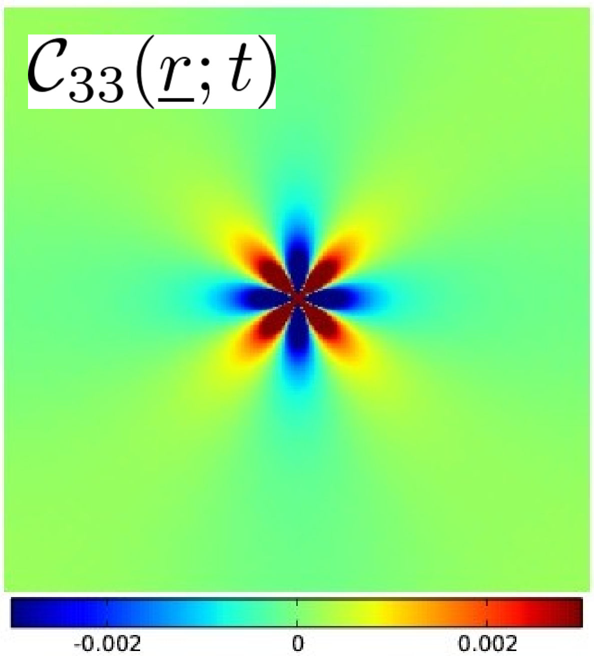
$\delta\sigma_3$

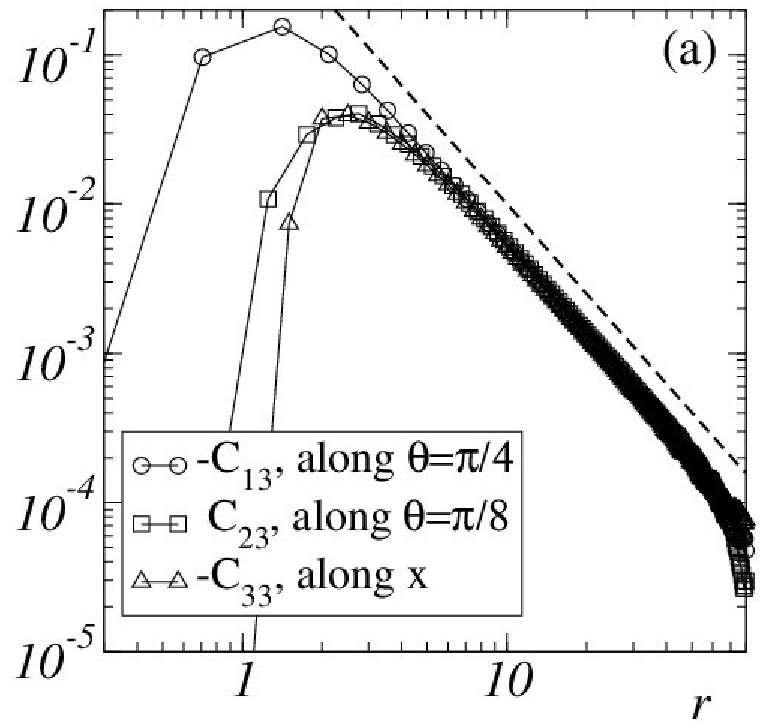


AL, PRL (2014)

Stress increments in the relaxing liquid

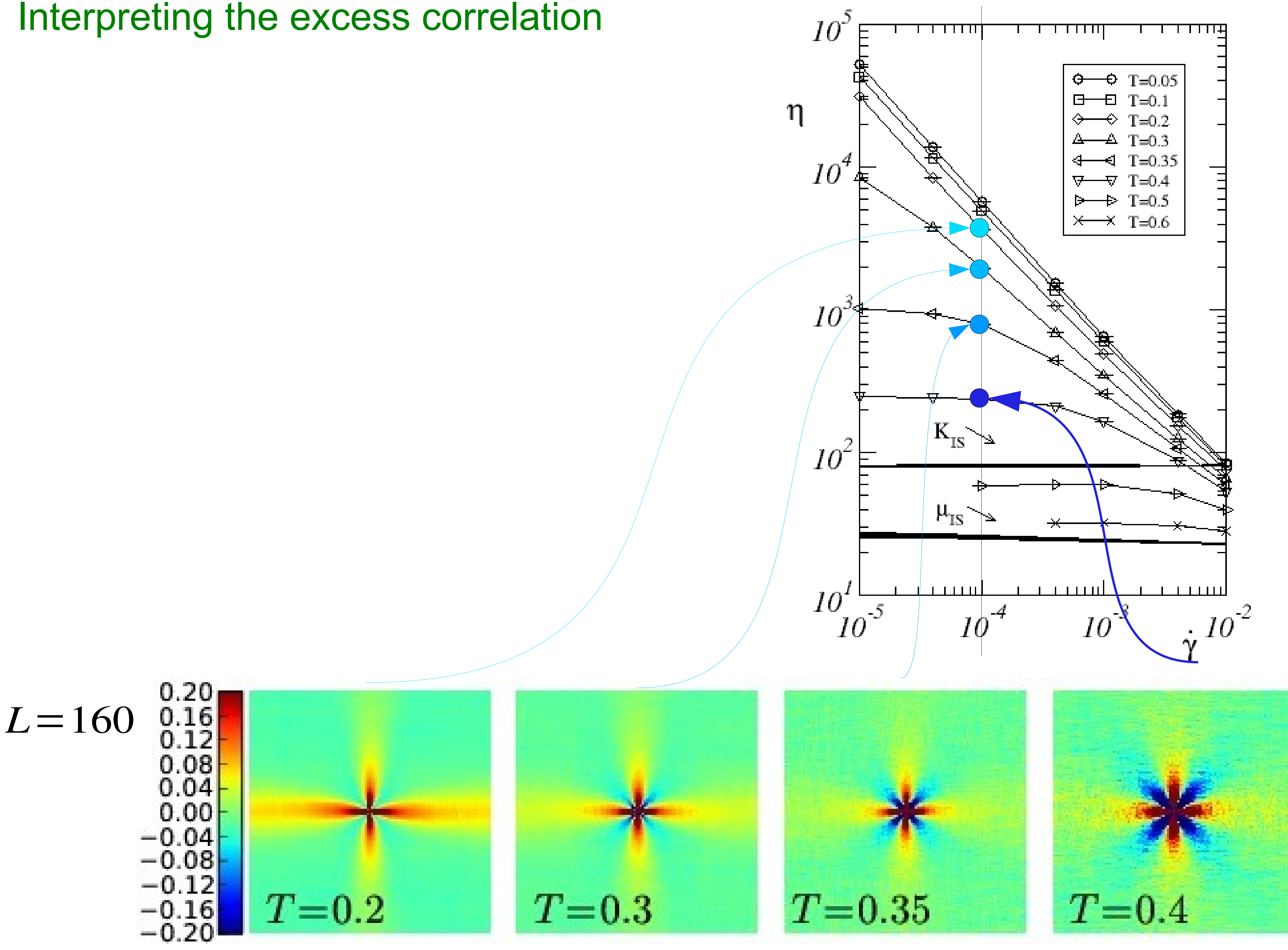
$$\mathcal{C}_{ij}(\underline{r};t) \equiv \langle \delta\sigma_i(\underline{r}_0;\, t_0, t_0+t)\delta\sigma_j(\underline{r}_0+\underline{r};\, t_0, t_0+t)\rangle$$

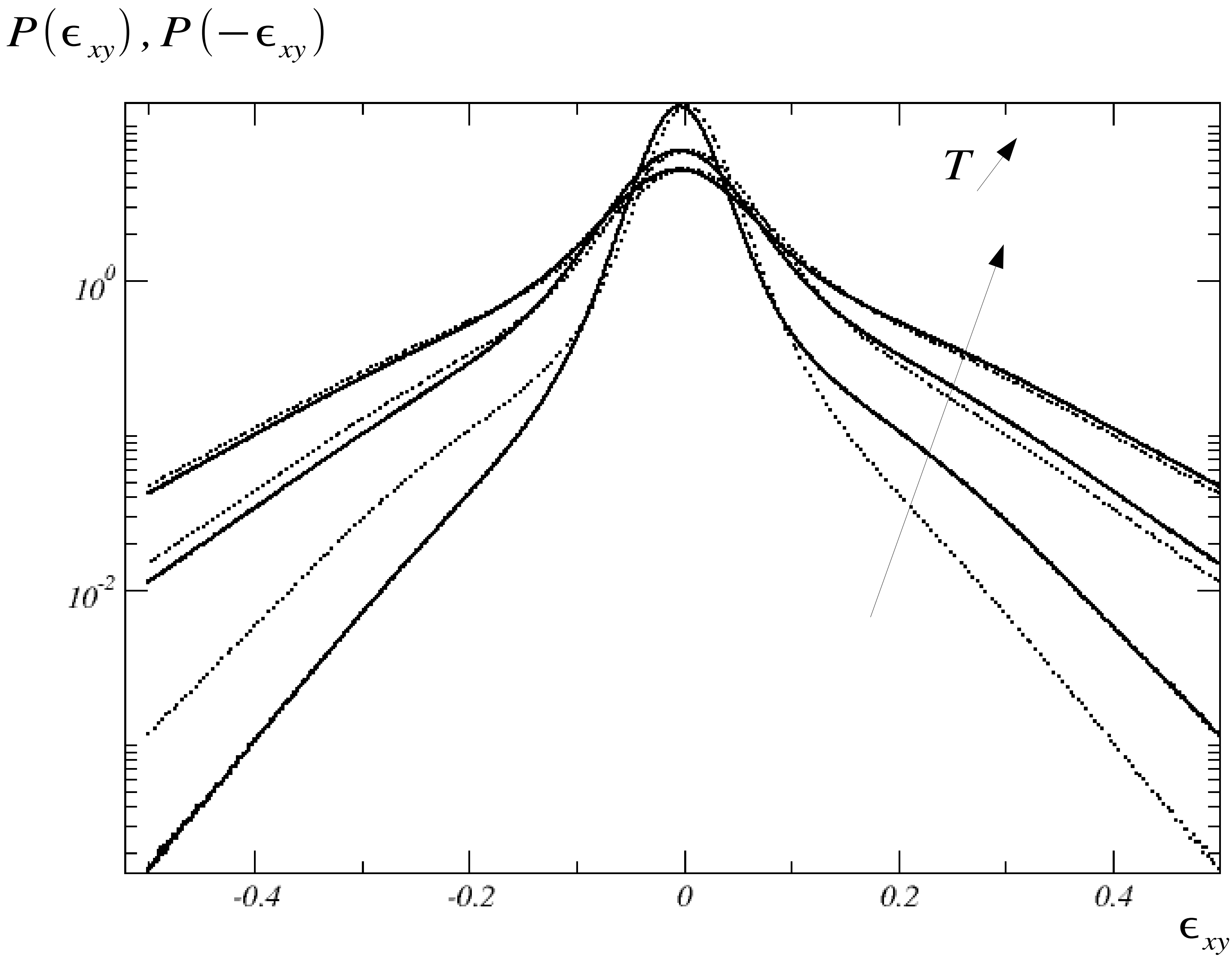




AL, PRL (2014)

Interpreting the excess correlation





$P(\epsilon_{xy}), P(-\epsilon_{xy})$
T
10^{0}
10^{-2}
-0.4
-0.2
0
0.2
0.4
ϵ_{xy}

Identify the mechanisms that govern plasticity

- nature of events: flips, avalanches (slip lines)

- observation of avalanches => correlations

Validity at finite rates? Yes.

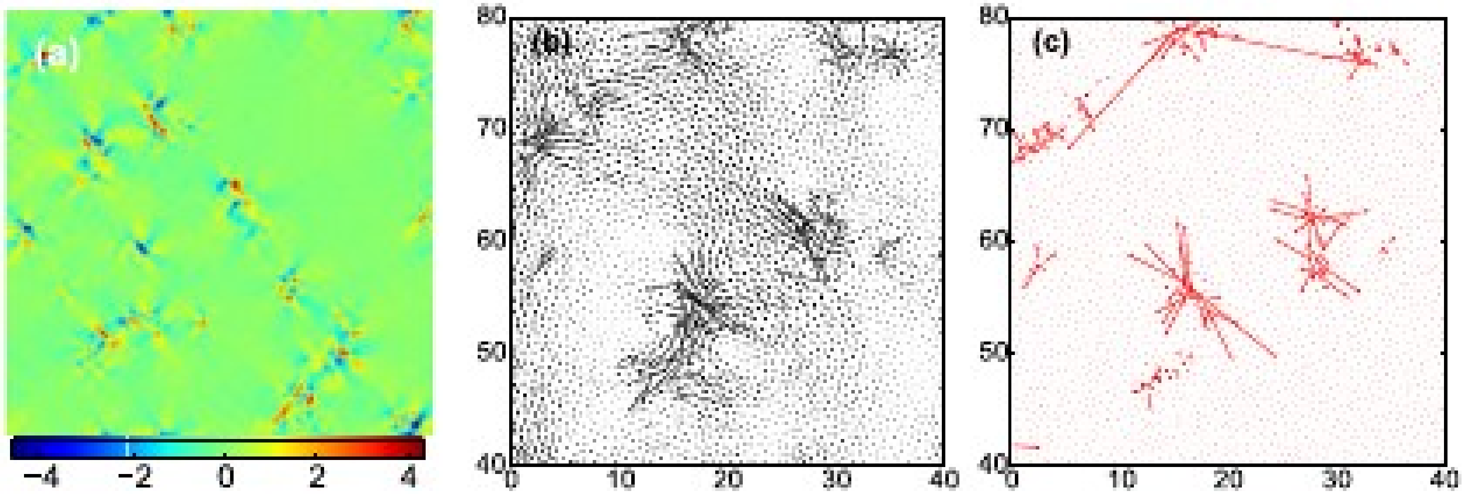
- avalanches related to correlations and rheology

Relevance at finite temperatures?

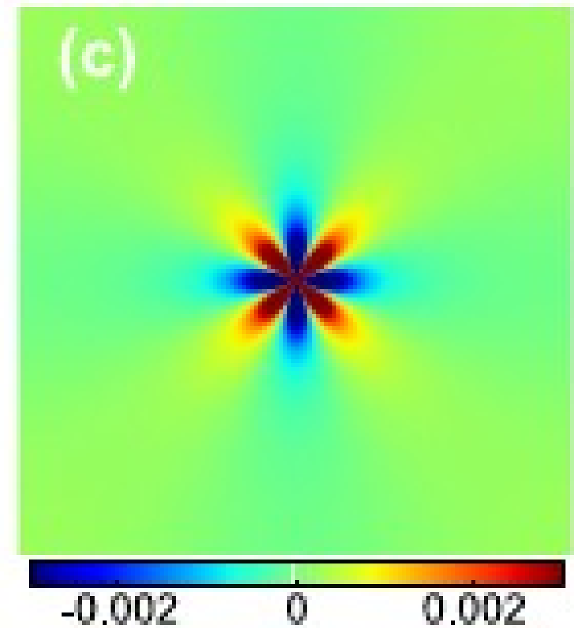
- avalanches ~ unchanged

- shifts in strain / time ==> rheology

- emergence of correlations in non-Newtonian crossover



Can we define a force field $\vec{f}$ such that $\vec{u}_i(t_0, t_0 + t)$ is exactly the elastic response to $\vec{f}$?



Around any IS, the elastic problem reads:

$$\mathcal{H} \cdot \underline{u} = \underline{f}$$

$$-\underline{u}_i(t_0, t_0 + t)$$

Conclusion

Plasticity results from avalanches

Avalanche size is rate-dependent

Correlations extend up to the Netownian regime

Stress is strongly T-dependent at low T

Rheology consequent of correlations and delays