3_proc_Hawkes

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```
library(tidyverse)
library(ggplot2)
```

```
# Simuluate HPP
set.seed(1) # for reproducibility

t_max <- 10
t <- 0

lmbda <- 10

t_vec <- numeric(0) # vector of t # consider change it to t_vec

while(t <= t_max){
  u       <- runif(1)
  t       <-  t - log(u)/lmbda                # t ~ exp(1/lambda)
  if(t < t_max) {
    t_vec <- c(t_vec,t)
  }
}
```

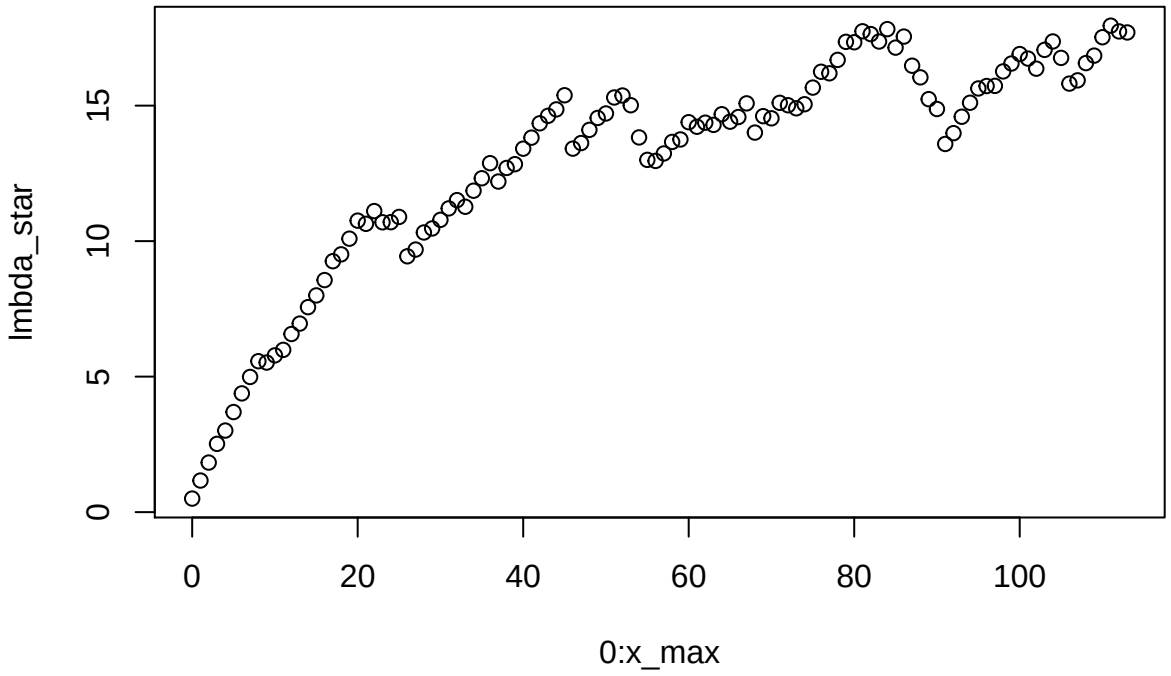
```
# Create lambda(t) function

# Initialize
# Note that mu > 0 and 0 < alpha < beta ??
mu = 0.5
alpha = 0.7
beta = 0.5

lmbda_fun <- function(time){
  diff = time - t_vec
  diff = diff[diff > 0]
  a = sum(alpha * exp(-beta * diff))
  out = mu + a
  return(out)
}
```

```
lmbda_star <- sapply(X = t_vec, FUN = lmbda_fun)
lmbda_star
```

```
##   [1]  0.500000  1.166243  1.828709  2.518966  3.009789  3.692638  4.381577
##   [8]  4.987644  5.568817  5.519164  5.784735  5.987685  6.572629  6.956240
##  [15]  7.563257  7.997083  8.562208  9.258648  9.511981 10.090501 10.755817
##  [22] 10.638557 11.108964 10.694468 10.698810 10.892377  9.440549  9.688126
##  [29] 10.319339 10.467464 10.785304 11.207891 11.512135 11.268051 11.859905
##  [36] 12.319467 12.876089 12.198703 12.699850 12.839321 13.411322 13.818262
```

```
##  [43] 14.347787 14.623249 14.859711 15.382651 13.413329 13.619000 14.105405
##  [50] 14.545215 14.710365 15.299386 15.372810 15.014675 13.826841 12.998089
##  [57] 12.959886 13.234890 13.660645 13.751166 14.387725 14.220679 14.370096
##  [64] 14.289392 14.681589 14.406944 14.578472 15.083097 14.004828 14.610564
##  [71] 14.530923 15.102575 15.013133 14.900957 15.051268 15.664532 16.249309
##  [78] 16.192795 16.687621 17.353729 17.337470 17.742754 17.639249 17.365307
##  [85] 17.822599 17.140228 17.547158 16.473199 16.042486 15.238914 14.874533
##  [92] 13.584656 13.982873 14.589516 15.105905 15.633533 15.722693 15.728607
##  [99] 16.262510 16.553932 16.902863 16.735650 16.363206 17.057126 17.367689
## [106] 16.761283 15.812665 15.932645 16.569077 16.844507 17.523966 17.949392
## [113] 17.737683 17.699430
```

```r
# Plot lmbda_star
# Not thinned yet
x_max = length(lmbda_star) - 1
plot(x = 0:x_max, y = lmbda_star)
```




```r
# while(t <= t_max){
#   if(runif(1) < min(lmbda/lmbda_star, 1)){
#     X_keep <- c(X, t)
#   }
#   return(X_keep)
# }
```

```r
# length(X_keep)
```

```r
# Try James's way
lmbda <- 10
p_keep <- lmbda / lmbda_star
t_keep <- t_vec[runif(length(p_keep)) < min(p_keep, 1)]
t_keep
```

```
##  [1] 0.1326108 0.2871762 0.2968044 0.4733173 0.5147480 0.5610923 0.9974999
##  [8] 1.1709112 1.2084500 1.3041343 1.4332490 1.4340617 1.7606083 2.1000774
## [15] 2.7226938 2.7366555 2.9173984 2.9685534 3.2072522 3.5121305 3.5444669
## [22] 3.6333163 3.6530461 3.6965777 3.7210485 3.7802817 3.8438224 3.8674762
## [29] 4.3172482 4.4590082 4.4739499 4.5564814 4.9621745 5.4149875 5.5049234
## [36] 6.0030084 6.0767089 6.3507260 6.4721964 6.7795215 6.8055072 6.8996707
## [43] 6.9288788 7.0460935 7.2500087 7.4374430 7.4715343 7.6821605 7.8226110
## [50] 8.0168895 8.1597556 8.4871605 8.5003687 8.6266909 8.8294128 9.1107075
## [57] 9.2652562 9.4697622 9.5435520 9.5514483 9.6051493 9.6363752 9.7394536
## [64] 9.8235085
```

```
# Plot using ggplot2
# Create a df
df_Hawkes = tibble(
  x = t_keep,
  y = 0:(length(t_keep) - 1)
)

p_Hawkes <- ggplot(data=df_Hawkes, mapping=aes(x=x, y=y)) +
  geom_step() +
  labs(title = "Hawkes Process",
       x = "t",
       y = "N(t)")
#p_Hawkes
```

```
# Plot time plot
df_Hawkes <- df_Hawkes %>% add_column(
  t_fix = rep(2, length(df_Hawkes$x))
)

p_Hawkes_time <- ggplot(data=df_Hawkes, mapping=aes(x=x, y=t_fix)) +
  geom_point(size=0.5) +
  ylim(c(1,3)) +
  labs(title = "Corresponding Inter-Arrivial Times",
       x = "t",
       y = "y-axis has no meaning")
#p_Hawkes_time
```

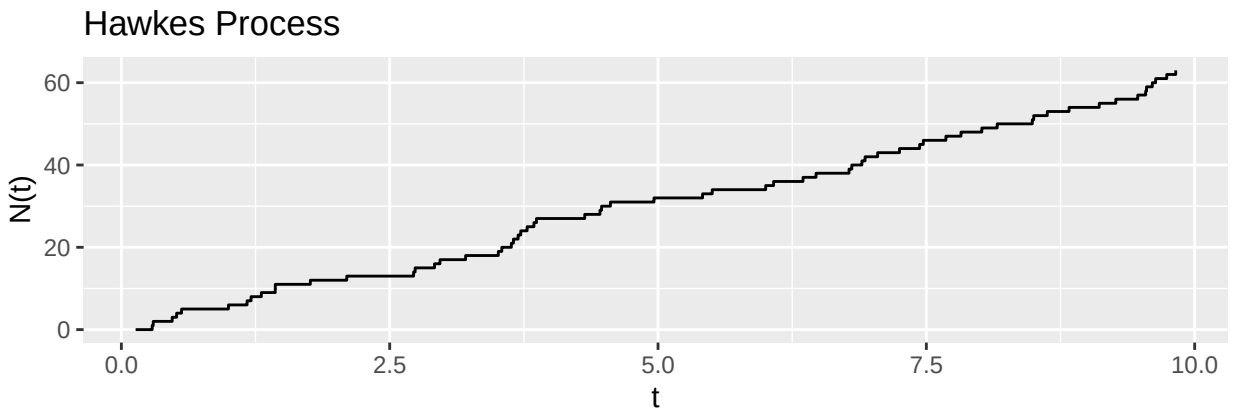
```
require(gridExtra)
```

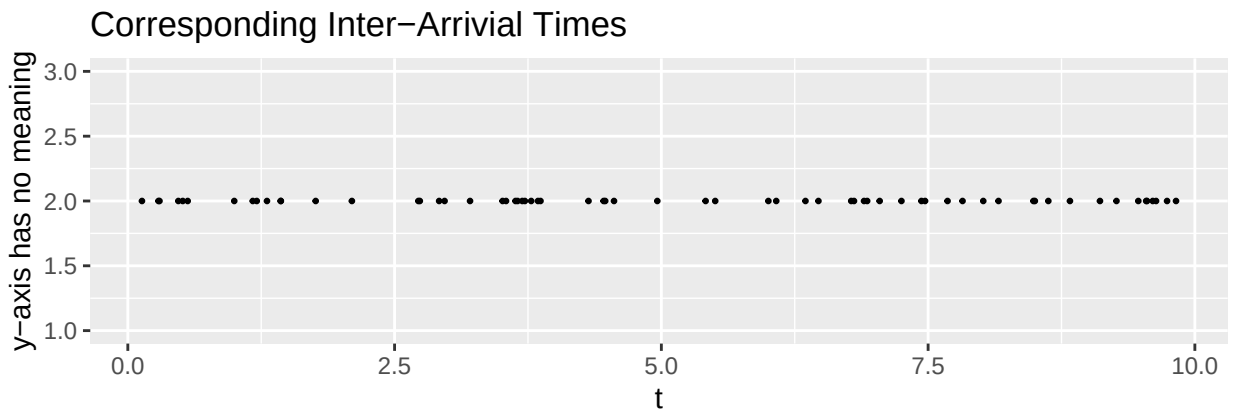
```
## Loading required package: gridExtra
```

```
##
## Attaching package: 'gridExtra'
```

```
## The following object is masked from 'package:dplyr':
##
##     combine
```

```
grid.arrange(p_Hawkes, p_Hawkes_time)
```





Plot lambda_star vs t next

```
# Plot lambda_star vs t
```