Lin_Masters

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I. Abstract

II. Introduction

Motivation

Applications

Applications of Hawkes Processes can be found in a wide variety of fields such as seismology, criminology, insurance, finance, social media, and neuroscience.

In seismology, an event can be an earthquake occurrence that causes aftershocks. In criminology, an event can be a gang rivalry that triggers retaliations following the gang crime. In insurance, an event can be a standard claim that increases claims. In finance, an event can be a transcation that influences future prices or volumes of transcations or a news that leads to movements in stock prices or trading behaviors. In social media, an event can be a tweet about an event on Twitter that follows a cascade of retweets from other users on the same social networking platform. In neuroscience, an event can be firing of a neuron that triggers spikes (or action potentials) of other neurons.

Objectives

III. Definitions, Properties and Graphs

Counting Process

(Point Process) Let $\{T_i, i \in N\}$ be a sequence of non-negative random variables such that $T_i < T_{i+1}\ \forall i \in N$, a point process on R^+ is defined as

$$\{T_i, i \in N\}$$

(Stochastic Process) A stochastic process is a family of random variables and is defined as

$$\{X(t), t \in T\}$$

(Counting Process) Let *N(t)* be the total number of events up to some time *t*, a stocastic process is said to be a counting process and is defined as

$$\{N(t), t \geq 0\}$$

(Counting Process) Let $\{T_i, i \in N\}$ be a point process, a counting process associated with $\{T_i, i \in N\}$ is defined as

$$N(t) = \sum_{i \in N} I_{\{T_i \leq t\}}$$

A counting process has to satisfy

1. $N(t) \geq 0$
2. $N(t)$ is an integer
3. If $s \leq t$, then $N(s) \leq N(t)$
4. If $s < t$, then $N(t) - N(s)$ is the number of events occur in the interval (s, t]

Three properties of counting process are

1. Independence
2. Stationarity
3. Homogeneity

Poisson Process

(Poisson Process) If the following conditions hold, a counting process $\{N(t), t \geq 0\}$ is said to be a Poisson Process with constant rate (or intensity) $\lambda > 0$

1. $N(0) = 0$
2. $N(t)$ has independent increments
3. $P(N(t+h)) - N(t) = 1)) = \lambda h + o(h)$
4. $P(N(t+h)) - N(t) \geq 2)) = o(h)$

where

1. the process starts at $t = 0$
2. the increments are independent
3. λ is the rate (or intensity)
4. no 2 or more events can occur at the same location

Properties of Poisson process are

1. the number of events in any interval t $N(t) \sim Pos(\lambda t)$

for all $s, t \geq 0$

$$P(N(t+s) - N(s) = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

$n = 0, 1, ...$

2. interarrival times $w \sim exp(\frac{1}{\lambda})$

for rate $\lambda > 0$, interarrival times W_i $i = 1, 2, ...$

$$P(W_1 > t) = P(N(t) = 0) = e^{-\lambda t}$$

This is because p(1^{st} arrivial arrives after time t) is the same as p(no arrivial in the interval [0,t]).

In addition,

$$F(t) = P(W_1 \leq t) = 1 - e^{-\lambda t}$$

and

$$f(t) = \lambda e^{-\lambda t}$$

Poisson process has stationary increments since the distribution of the number of events that occur in any interval of time depends only on the length of the time interval but not on the location of the interval. In contrast, nonhomogeneous Poisson process does not have stationary increments since the distribution can change when shifted in time.

Nonhomogeneous Poisson Process

(Nonhomogeneous Poisson Process) If the following conditions hold, a counting process $\{N(t), t \geq 0\}$ is said to be a nonhomogeneous Poisson Process with intensity function of time $\lambda(t), t > 0$

1. $N(0) = 0$
2. $N(t)$ has independent increments
3. $P(N(t+h)) - N(t) = 1)) = \lambda(t)h + o(h)$
4. $P(N(t+h)) - N(t) \geq 2)) = o(h)$

Properties of nonhomogeneous Poisson process are

1. the number of events in any interval t $N(t) \sim Pos(\int_0^t \lambda(s)ds)$

for all $s, t \geq 0$

$$P(N(t) = n) = \frac{(\int_0^t \lambda(s)ds)^n e^{\int_0^t \lambda(s)ds}}{n!}$$

or

$$P(N(b) - N(a) = n) = \frac{(\int_a^b \lambda(s)ds)^n e^{\int_a^b \lambda(s)ds}}{n!}$$

$n = 0, 1, ...$

Hawkes Process

In previous cases, the events either arrive independently, arrive at a constant rate (the Poisson process), or depend on an intensity function (the nonhomogeneous Poisson Process). In the case of Hawkes process, the arrivial rate of the events depends on past events (i.e. the arrivial of an event increases the occurrence of future events).

(Hawkes Process) A counting process $\{N(t), t \geq 0\}$ associated with past events $\{\mathcal{H}_t, t > 0\}$ is said to be a Hawkes process with conditional intensity function $\lambda(t|\mathcal{H}_t), t > 0$ and takes the form

$$\lambda(t|\mathcal{H}_t) = \lambda_0(t) + \sum_{i:T_i<t} \phi(t - T_i)$$

where

- $\lambda_0(t)$ is the base intensity function
- $T_i < t$ are the events time occur before current time t
- $\phi(\cdot)$ is the kernel function through which intensity function depends on past events

A few choices of $\phi(\cdot)$ are exponentially decaying function and power-law kernel, and they take the form of

$$\phi(x) = \alpha e^{-\beta x}$$

$$\phi(x) = \frac{\alpha}{(x+\beta)^{\eta+1}}$$

There are two types of Hawkes processes

1. Intensity-based Hawkes Process
2. Cluster-based Hawkes Process

IV. Algorithms

V. Conclusions and Discussion

Acknowledgments

Reference

Obral, K. (2016). Simulation, estimation and applications of hawkes processes (Master's thesis, University of Minnesota, Twin Cities, United States).

Rizoiu, M. A., Lee, Y., Mishra, S., & Xie, L. (2017). A tutorial on hawkes processes for events in social media. arXiv preprint arXiv:1708.06401.