

Prequel to Hawkes Processes: An Overview of Temporal and Spatio-Temporal Point Processes and Some Simulations

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Background: Hawkes Process

Events such as earthquake epicenters, crime patterns, forest wildfires, financial transactions, etc. often exhibit triggering and clustering behavior.

Hawkes process are also known as self-exciting point processes (SEPP). The original Hawkes processes are temporal, whereas the more recently developed SEPP have been extended to account for both the spatial and temporal aspects of the data.

Background: Characteristics

The defining characteristic of SEPP is that it 'self-excites', i.e., the occurrence of an event increases the occurrence of future events nearby in space and/or time, although the events don't self-excite in perpetuity.

In seismology, an event can be an earthquake that causes aftershocks. In criminology, an event can be a gang rivalry that triggers retaliations following the gang crime. In both cases, the initial event can continue to spawn 'offspring' events and the 'offspring' events can spawn 'offspring' events of their own, but the spawns fade out eventually.

Background: Applications

The ability to capture events with such behavior gives SEPP the potential to become a powerful predictive tool for a wide variety of applications. To date, the models have been used to model

- ▶ earthquake epicenters (Ogata, 1988, 1998)
- ▶ crime patterns (Mohler et al., 2011; Reinhart & Greenhouse, 2018),
- ▶ forest wildfires (Peng, et al., 2005),
- ▶ insurance claims (Stabile et al., 2010),
- ▶ financial transactions (Bauwens and Hautsch, 2009; Embrechts et al., 2011; Bacry et al., 2015),
- ▶ social network events (Zhao et al., 2015; Rizoïu et al., 2017),
- ▶ neuron activities (Johnson, 1996; Gerhard et al., 2017), and
- ▶ disease spread or transmission (Meyer et al., 2012; Meyer & Held, 2014)

Background and Objectives

It is surprising that SEPP have not gained enough attention from the machine learning communities, which would find their predictive capabilities beneficial. Some of the relevant point processes (e.g. nonhomogeneous Poisson, Cox and cluster processes), which are often left out from graduate-level, introductory spatial statistics and stochastic processes courses.

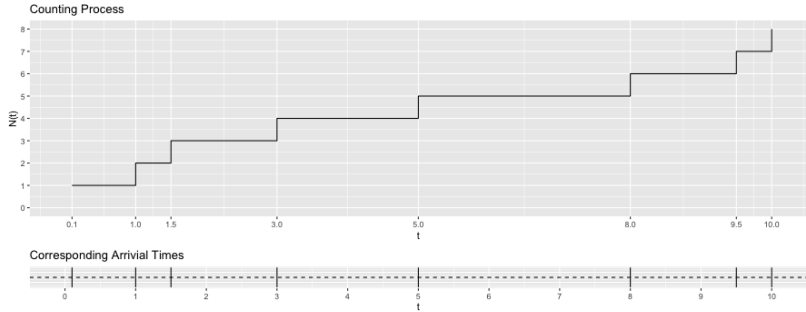
The objective of this project is then to give an overview of various types of point processes so that readers of interest have the background knowledge to understand to read and comprehend existing SEPP literature as well as explore the field further.

Introductions, Definitions, Properties, and Applications

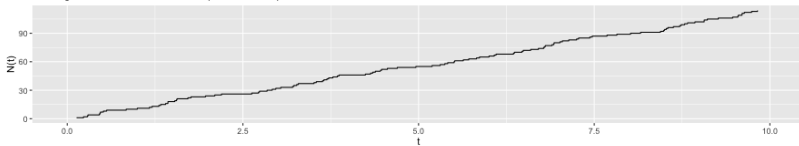
1. Counting Process
2. HPP (Homogeneous Poisson Process)
3. NPP (Nonhomogeneous Poisson Process)
4. Cox and Cluster Processes
5. Hawkes Process
6. Spatio-Temporal SEPP (Self-Exciting Point Process)

Counting Process

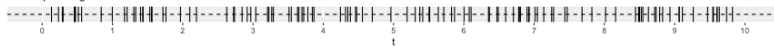
Counting Process



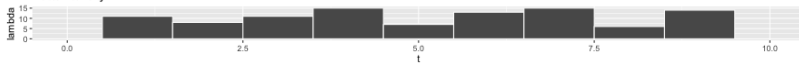
HPP (Homogeneous Poisson Process)

Homogeneous Poisson Process ($\lambda = 10$)

Corresponding Inter-Arrival Times



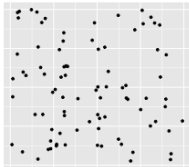
Rate/Intensity



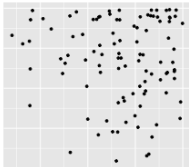
NPP (Nonhomogeneous Poisson Process)

HPP vs. NPP in 2D

HPP (rate = 100)



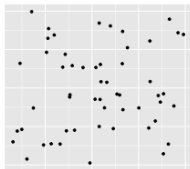
NPP



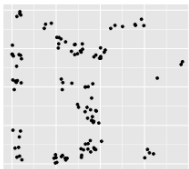
Cox and Cluter Processes

Cox vs. Matern Cluster in 2D

Cox Process

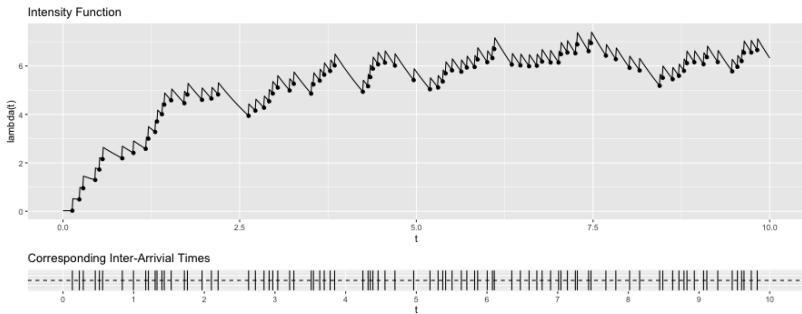


Matern Cluster Process



(Temporal) Hawkes Process

Hawkes Process



Thinning Algorithm

Thinning algorithm is also called acceptance-rejection method.

Algorithm 2: Simulations of a Hawkes Process via Thinning Algorithm

Input $\mu, \alpha, \beta, \lambda, t_{max}$

1. Simulate a HPP using Algorithm 1
2. Create a $\lambda(t)$ function where the function
$$= \mu + \sum_{i: T_i < t} \alpha e^{-\beta x}$$
3. Set $\lambda^* =$ apply the $\lambda(t)$ function to the HPP
4. Generate $u \sim U(0, 1)$
5. **if** $(u < \min(\frac{\lambda^*}{\lambda}, 1))$ where the accepting probability
 $= \min(\lambda^*/\lambda, 1)$
6. | Keep the points
7. **else**
8. | “Thin” or reject the points and **return** $\{t_k\}_{k=0,1,\dots}$

Spatio-temporal SEPP (Self-Exciting Point Process)

Recent Advancement and Future Work of SEPP

Recent work has extended the use of SEPP to novel applications such as

- ▶ mass shootings (Boyd & Molyneux, 2021),
- ▶ COVID-19 transmission (Chiang et al., 2020), and
- ▶ gang violence (Park et al., 2021).

However, there is still much work to be done which include computational advances to ease the burden of applying the models to bigger data sets (Holbrook et al., 2021), residual and model diagnostics, methods that make the models more flexible and applicable, etc.

Thank you!

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Report, code, etc. are available at

GitHub: franceslinyc, Hawkes-Process-2021

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