Lin_Masters_Written

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2.1 Counting Process

Counting processes deal with the occurrences (or numbers) of events over time.

For example, if we were to count the numbers of events $N(t)$ such as the numbers of customers arriving at a supermarket or the numbers of phone calls receiving at the help line up to some time t, we can consider using counting processes.

2.2 Poisson Process

Poisson processes is one of the simplest and most-widely used point processes. Here, we assume that the numbers of events $N(t)$ follows a Poisson distribution with a constant rate λ and the interarrival times between events W are exponentially distributed.

For example, if we were to model the numbers of bus arrivals at a bus stop, the numbers of car accidents at a site or the requests for documents on a web server, we can consider modelling using Poisson processes.

2.3 Nonhomogeneous Poisson Process

Nonhomogeneous Poisson processes is a generalization of homogeneous Poisson processes that allow the rate λ to vary with function of time t. Previously, we assume that the rate is constant. If we have reasons to believe that the rate is not constant, we should consider modelling using nonhomogeneous Poisson processes.

For example, if we were to model the number of customers arriving at a supermarket and we have reasons to believe that the arrivial rate of customers is higher during lunch time as compared to 2pm, we should consider modelling using nonhomogeneous Poisson processes.

2.4 Cox and Cluster Process

In previous cases, we assume independence between events. That is, whether events occur at a constant rate λ (e.g. Poisson process) or depend on an intensity function $\lambda(t)$ (e.g. nonhomogeneous Poisson process), they occur independently. Here, we discuss models that allow dependence between events.

Examples that can be modelled using Cox and cluster processes include seedlings and saplings of California redwood, locations of emergent plants, and locations of trees. In these examples, the patterns appear to be clustered.

(?) We can think of Cox process as a hierarchical model with two levels and cluster process such as Neyman-Scott process a hierarchical model with three levels.

In Cox processes (or doubly stochastic Poisson process), the intensity function $\Lambda(u)$ is treated as random. Examples of Cox processes include mixed Poisson process, log Gaussian Cox process, and shot noise Cox process.

In cluster processes, the randomness arises from two steps: First, 'parent' points $\mathbf{Y}$ is generated. Then, each 'parent' point $y_i \in \mathbf{Y}$ gives to 'offspring' points z_{ij}. All the 'offspring' points Z_{ij} form a Cox process $\mathbf{X}$ and only $\mathbf{X}$ is observed.

Specific models of cluster processes depend on the choices of assumptions. Examples of cluster processes include Neyman-Scott process, Matern cluster process, and Thomas cluster process.

2.5 Hawkes Process

Hawkes process is also known as a self-exciting point process. (?) Hawkes processes can be temporal or spatio-temporal. When we retain the spatial components of Hawkes's, it becomes closely related to Cox processes.

In Hawkes processes, the events also do not occur independently. The occurrence rate of the events depends not only on time t but also past events $\mathcal{H}_t^N$ up to some time t.

Examples that can be modelled using Hawkes processes include locations of earthquake epicenters, locations of crimes, and locations of patients with a disease. In these examples, the occurrence of an event increases the occurrence of subsequent events.