Lin_Masters_Written

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2.1 Counting Process

Counting processes count the occurrences (or numbers) of events over time. For example, if we were to count the numbers of events $N(t)$ such as the numbers of customers arriving at a supermarket or the numbers of phone calls receiving at the help line up to some time t, we can use counting processes.

Counting processes is independent, stationary, and homogeneous. In addition,......

2.2 Poisson Process

Homogeneous Poisson process (HPP) is one of the simplest and most-widely used point processes. For example, if we were to model the numbers of events such as the numbers of bus arrivals at a bus stop, the numbers of car accidents at a site or the requests for documents on a web server, we can use Poisson processes.

HPP is independent, stationary, and homogeneous. In addition, we assume that the numbers of events $N(t)$ follows a Poisson distribution with a constant rate λ and the interarrival times between events W are exponentially distributed.

2.3 Nonhomogeneous Poisson Process

Assuming that the rate is constant is not that realistic in practice. We may want a model that allows for more flexibility. Nonhomogeneous Poisson processes (NPP) is a generalization of homogeneous Poisson processes that allow the rate (or intensity) λ to vary with function of time t.

Previously, we assume that the intensity λ is constant. If we have reasons to believe that the intensity is not constant, we should model using nonhomogeneous Poisson processes. For example, if we were to model the number of customers arriving at a supermarket and we have reasons to believe that the arrivial rate of customers is higher during lunch time as compared to say, 2pm, we should model using nonhomogeneous Poisson processes.

In contrast to HPP, NPP is independent but not stationary nor homogeneous. In addition, for NPP, we assume that $N(t)$ follows a Poisson distribution with an intensity function $\lambda(t)$.

2.4 Cox and Cluster Process

Even more flexible models than NPP are Cox and cluster processes that allow depdence between events. Previously, we assume independence between events. That is, whether events occur at a constant rate λ (e.g. HPP) or depend on an intensity function $\lambda(t)$ (e.g. NPP), they occur independently. Here, we discuss models that allow dependence between events. Examples that can be modelled using Cox and cluster processes include seedlings and saplings of California redwood, locations of emergent plants, and locations of trees. In these examples, the patterns appear to be clustered.

We can think of Cox process as a hierarchical model with two levels and cluster process such as Neyman-Scott process a hierarchical model with three levels. **ELABORATE MORE**.

In Cox processes (or doubly stochastic Poisson process), the randomness arises from two parts. Not only the randomness occurs at different location of the time interval as in the case of a NPP, but the governing function $\Lambda(u)$ is also random, instead of governing by one single function $\lambda(t)$ also as in the case of a NPP. In other words, the intensity function $\Lambda(u)$ is also treated as random. **PICK ONE TO PLOT THEN TALK IN DETAILS**. Other examples of Cox processes include mixed Poisson process, log Gaussian Cox process, and shot noise Cox process.

In cluster processes, the randomness also arises from two steps: First, 'parent' points $\mathbf{Y}$ is generated. Then, each 'parent' point $y_i \in \mathbf{Y}$ gives to 'offspring' points z_{ij}. All the 'offspring' points Z_{ij} form a cluster process $\mathbf{X}$ and only $\mathbf{X}$ is observed.

Specific models of cluster processes depend on the choices of assumptions. Matern cluster process, for example, involves generating homogeneous Poisson parents and each parent gives rise to Poisson number of offspring uniformmly distributed in a disc of radius r centered around the parent. Other examples of cluster processes include Neyman-Scott process and Thomas cluster process.

In addition, the differences between Cox process and cluster process are such that Cox process conditions on (?) random field, whereas cluster process conditions on...... (?) in bounded region.

2.5 Hawkes Process

Hawkes process is also known as a self-exciting point process.

In Hawkes processes, the events also do not occur independently. The occurrence rate of the events depends not only on time t but also past events $\mathcal{H}_t^N$ up to some time t.

Examples that can be modelled using Hawkes processes include locations of earthquake epicenters, locations of crimes, and locations of patients with a disease. In these examples, the occurrence of an event increases the occurrence of subsequent events.