01-SPDEtoy-INLA

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Explanations are a bit confusing but we will follow the example for the purpose of learning the **R** package `R-INLA`.

Load packages

```
library(INLA)
```

```
## Loading required package: Matrix
```

```
## Loading required package: foreach
```

```
## Loading required package: parallel
```

```
## Loading required package: sp
```

```
## This is INLA_22.03.16 built 2022-03-16 13:24:07 UTC.
##  - See www.r-inla.org/contact-us for how to get help.
```

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --
```

```
## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.8
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x purrr::accumulate() masks foreach::accumulate()
## x tidyr::expand()     masks Matrix::expand()
## x dplyr::filter()     masks stats::filter()
## x dplyr::lag()        masks stats::lag()
## x tidyr::pack()       masks Matrix::pack()
## x tidyr::unpack()     masks Matrix::unpack()
## x purrr::when()       masks foreach::when()
```

```
library(pander)
library(ggplot2)
library(gridExtra)
```

```
##
## Attaching package: 'gridExtra'

## The following object is masked from 'package:dplyr':
##
##     combine
```

Load data

```
class(SPDEtoy)
```

```
## [1] "data.frame"
```

```
SPDEtoy %>% head %>% pander
```

s1	s2	y
0.08266	0.05641	11.52
0.6123	0.9168	5.278
0.162	0.357	6.903
0.7526	0.2576	13.18
0.851	0.1541	14.6
0.001806	0.7353	9.78

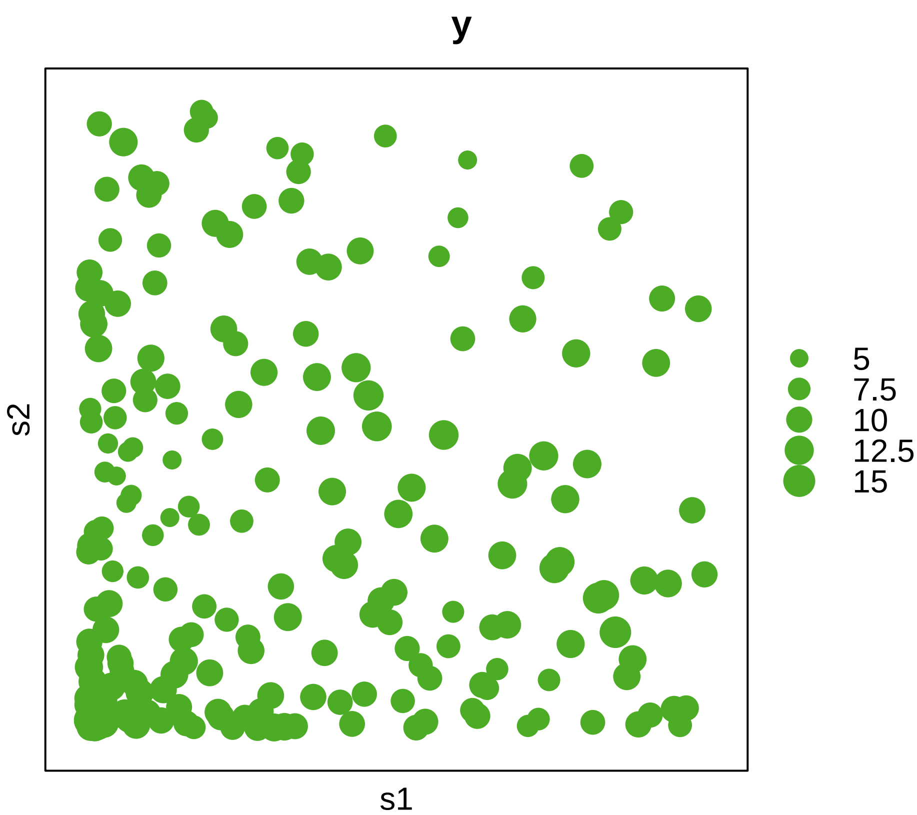
Convert to a SpatialPointsDataFrame

```
SPDEtoy.sp <- SPDEtoy
coordinates(SPDEtoy.sp) <- ~ s1 + s2 # Isn't this from the package sp? Yes.
```

Plot it

`s1` and `s2` are x- and y-coordinate and `y` is the simulated observations at the locations.

```
bubble(SPDEtoy.sp, "y", key.entries = c(5, 7.5, 10, 12.5, 15),
       maxsize = 2, xlab = "s1", ylab = "s2")
```



Fit a MLR model

By default, the prior on the intercept α is a uniform; the prior on the coefficients is a Gaussian with mean 0 and precision ($variance^{-1}$) 0.001, and the prior on the precision τ is a Gamma with parameters 1 and 0.000005. These priors can be adjusted.

The model is as follows:

$$y_i \sim N(\mu_i, \tau^{-1}), i = 1, ..., 200$$
$$\mu_i = \alpha + \beta_1 s_{1i} + \beta_2 s_{2i}$$
$$\alpha \sim Uniform$$
$$\beta_j \sim N(0, 0.001^{-1}), j = 1, 2$$
$$\tau \sim Gamma(1, 0.00005).$$

Produce result summaries

```
m0 <- inla(y ~ s1 + s2, data = SPDEtoy)
summary(m0)
```

```
##
## Call:
##    c("inla.core(formula = formula, family = family, contrasts = contrasts,
##    ", " data = data, quantiles = quantiles, E = E, offset = offset, ", "
```

```
##    scale = scale, weights = weights, Ntrials = Ntrials, strata = strata,
##    ", " lp.scale = lp.scale, link.covariates = link.covariates, verbose =
##    verbose, ", " lincomb = lincomb, selection = selection, control.compute
##    = control.compute, ", " control.predictor = control.predictor,
##    control.family = control.family, ", " control.inla = control.inla,
##    control.fixed = control.fixed, ", " control.mode = control.mode,
##    control.expert = control.expert, ", " control.hazard = control.hazard,
##    control.lincomb = control.lincomb, ", " control.update =
##    control.update, control.lp.scale = control.lp.scale, ", "
##    control.pardiso = control.pardiso, only.hyperparam = only.hyperparam,
##    ", " inla.call = inla.call, inla.arg = inla.arg, num.threads =
##    num.threads, ", " blas.num.threads = blas.num.threads, keep = keep,
##    working.directory = working.directory, ", " silent = silent, inla.mode
##    = inla.mode, safe = FALSE, debug = debug, ", " .parent.frame =
##    .parent.frame)")
## Time used:
##     Pre = 2.58, Running = 0.241, Post = 0.0255, Total = 2.85
## Fixed effects:
##               mean    sd 0.025quant 0.5quant 0.975quant   mode kld
## (Intercept) 10.132 0.242      9.656   10.132     10.608 10.132   0
## s1           0.762 0.429     -0.081    0.762      1.605  0.762   0
## s2          -1.584 0.429     -2.427   -1.584     -0.741 -1.584   0
##
## Model hyperparameters:
##                                          mean   sd 0.025quant 0.5quant
## Precision for the Gaussian observations 0.308 0.03      0.252    0.307
##                                         0.975quant  mode
## Precision for the Gaussian observations      0.371 0.305
##
## Marginal log-Likelihood:  -423.18
##  is computed
## Posterior summaries for the linear predictor and the fitted values are computed
## (Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
```

Return parts of the result summaries only

```
# Get estimate of marginal likelihood
m0$mlik
```

```
##                                             [,1]
## log marginal-likelihood (integration) -423.1822
## log marginal-likelihood (Gaussian)    -423.1824
```

```
# Get summary statistics of fixed effects
m0$summary.fixed
```

```
##                   mean        sd  0.025quant  0.5quant 0.975quant       mode
## (Intercept) 10.1321487 0.2421748  9.65618747 10.132142  10.607695 10.1321497
## s1           0.7624297 0.4293100 -0.08132075  0.762418   1.605443  0.7624316
## s2          -1.5836768 0.4293100 -2.42742131 -1.583691  -0.740658 -1.5836810
##                   kld
```

```
## (Intercept) 4.789212e-07
## s1          4.788751e-07
## s2          4.788754e-07
```

```
# Get summary statistics of random effects
m0$summary.random # There is no random effect in this model.
```

```
## list()
```

```
# Get summary statistics of hyperparameters
m0$summary.hyperpar
```

```
##                                              mean        sd 0.025quant
## Precision for the Gaussian observations 0.3083786 0.0304855  0.2516781
##                                          0.5quant 0.975quant      mode
## Precision for the Gaussian observations 0.3072898  0.3713048 0.3051805
```

Extract predicted values

```
# Get fitted values
m0$summary.fitted.values %>% head(3) %>% pander
```

Table 2: Table continues below

	mean	sd	0.025quant	0.5quant	0.975quant
fitted.Predictor.001	10.11	0.2071	9.699	10.11	10.51
fitted.Predictor.002	9.147	0.3089	8.541	9.147	9.753
fitted.Predictor.003	9.69	0.1478	9.4	9.69	9.98

	mode
fitted.Predictor.001	10.11
fitted.Predictor.002	9.147
fitted.Predictor.003	9.69

```
# # Get marginals of fitted values
# m0$marginals.fitted.values %>% head(3) %>% pander # Why is it empty?
```

Plot the marginal density for the MLR model

```
# Get posterior marginals of fixed effects
marginal_fixed <- m0$marginals.fixed
#m0$marginals.fixed$`(Intercept)`
```

```r
# Plot the intercept using ggplot2
# https://www.paulamoraga.com/book-geospatial/sec-inla.html
#m0$marginals.fixed
alpha <- m0$marginals.fixed$`(Intercept)`
ggplot(data.frame(inla.smarginal(alpha)), aes(x, y)) +
  geom_line() +
  labs(x = expression(alpha), y = "density") -> p_alpha
```

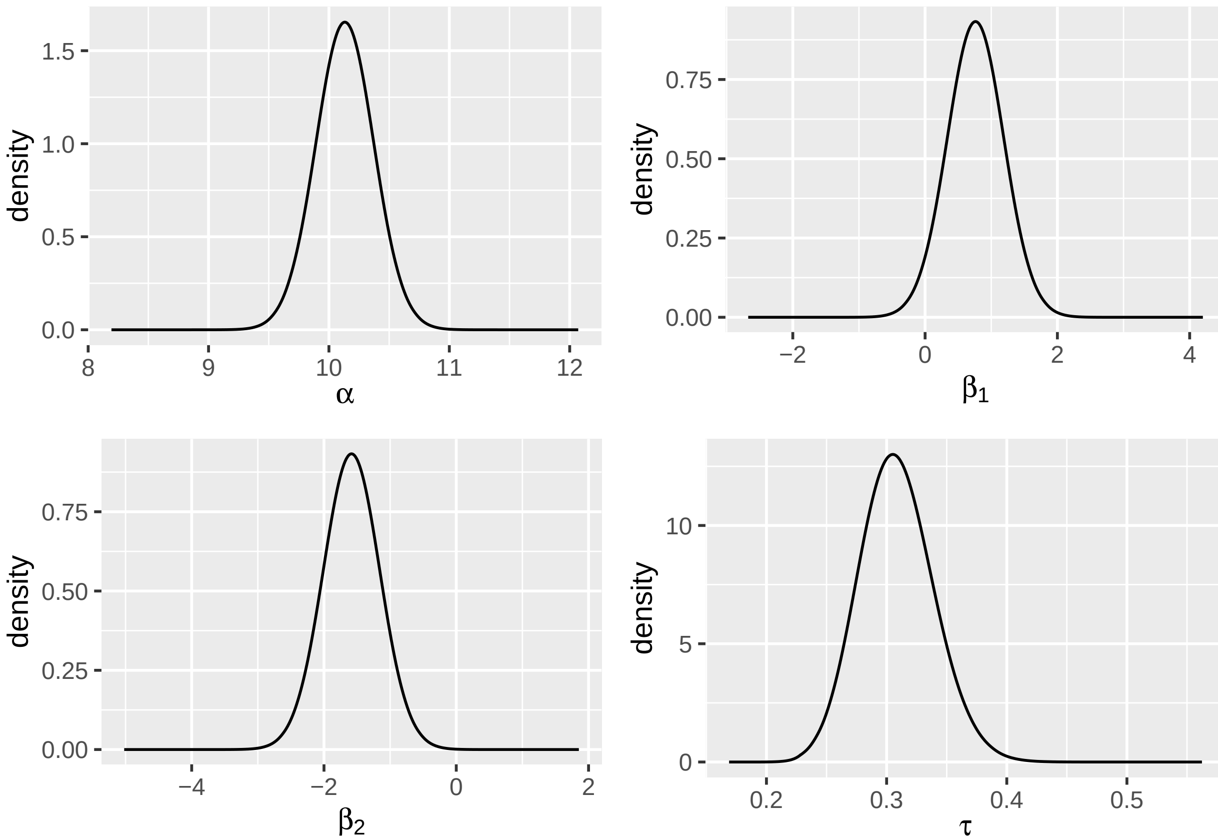
```r
# Plot b1
#m0$marginals.fixed
s1 <- m0$marginals.fixed$s1
ggplot(data.frame(inla.smarginal(s1)), aes(x, y)) +
  geom_line() +
  labs(x = expression(beta[1]), y = "density") -> p_beta_1
```

```r
# Plot b2
#m0$marginals.fixed
s2 <- m0$marginals.fixed$s2
ggplot(data.frame(inla.smarginal(s2)), aes(x, y)) +
  geom_line() +
  labs(x = expression(beta[2]), y = "density") -> p_beta_2
```

```r
#marginal_random <- m0$marginals.random # no random effects here
```

```r
# Plot tau
#m0$marginals.hyperpar
tau <- m0$marginals.hyperpar$`Precision for the Gaussian observations`
ggplot(data.frame(inla.smarginal(tau)), aes(x, y)) +
  geom_line() +
  labs(x = expression(tau), y = "density") -> p_tau
```

```r
# Combine plots
grid.arrange(p_alpha, p_beta_1, p_beta_2, p_tau, ncol = 2)
```



Fit another model

Reference

1.3 A simple example.