00_Leukemia_in_NY

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This project fits 6 models (fixed effects, random effects (iid), SLM, ICAR, BYM and Leroux et al.), produces summary results and plots results.

Load packages

```
# Load packages
library(spdep)     # for spatial weights matrix objects
```

```
## Loading required package: sp

## Loading required package: spData

## Loading required package: sf

## Linking to GEOS 3.9.1, GDAL 3.4.0, PROJ 8.1.1; sf_use_s2() is TRUE
```

```
library(DClusterm) # data
```

```
## Loading required package: parallel

## Loading required package: spacetime

## Loading required package: DCluster

## Loading required package: boot

## Loading required package: MASS
```

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.8
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x dplyr::select() masks MASS::select()
```

```
library(pander)
library(ggplot2)
library(gridExtra)
```

```
##
## Attaching package: 'gridExtra'

## The following object is masked from 'package:dplyr':
##
##     combine
```

```
library(DClusterm)
data(NY8)
```

The `NY8` data

The `NY8` data set contains the number of leukemia cases in an eight-country region of upstate New York from 1978-1982.

```
# Load data
data(NY8)

# View data
#head(NY8)
NY8
```

```
## class       : SpatialPolygonsDataFrame
## features    : 281
## extent      : 358241.9, 480393.1, 4649755, 4808545  (xmin, xmax, ymin, ymax)
## crs         :  +proj=utm +zone=18 +ellps=WGS84 +units=m +no_defs
## variables   : 17
## names       :    AREANAME,     AREAKEY,        X,        Y,  POP8, TRACTCAS,  PROPCAS, PCTOWNHOME,
## min values  : Auburn city, 36007000100, -55.4823, -75.2907,     9,        0,        0, 0.00082237, 0
## max values  : Vestal town, 36109992300,  53.5086, 56.41013, 13015,     9.29, 0.006993,          1, 0
```

```
# Check class
class(NY8)
```

```
## [1] "SpatialPolygonsDataFrame"
## attr(,"package")
## [1] "sp"
```

```
# Convert it to a df?
# https://www.paulamoraga.com/book-geospatial/sec-spatialdataandCRS.html
NY8@data %>% head %>% pander
```

Table 1: Table continues below

	AREANAME	AREAKEY	X	Y	POP8	TRACTCAS
0	Binghamton city	36007000100	4.069	-67.35	3540	3.08
1	Binghamton city	36007000200	4.639	-66.86	3560	4.08
2	Binghamton city	36007000300	5.709	-66.98	3739	1.09
3	Binghamton city	36007000400	7.614	-66	2784	1.07
4	Binghamton city	36007000500	7.316	-67.32	2571	3.06
5	Binghamton city	36007000600	8.559	-66.93	2729	1.06

Table 2: Table continues below

	PROPCAS	PCTOWNHOME	PCTAGE65P	Z	AVGIDIST	PEXPOSURE
0	0.00087	0.3277	0.1466	0.142	0.2374	3.167
1	0.001146	0.4268	0.2351	0.3555	0.2087	3.039
2	0.000292	0.3377	0.138	-0.5817	0.1709	2.838
3	0.000384	0.4616	0.1189	-0.2963	0.1406	2.643
4	0.00119	0.1924	0.1416	0.4569	0.1578	2.759
5	0.000388	0.3652	0.1411	-0.2812	0.1726	2.848

	Cases	Xm	Ym	Xshift	Yshift
0	3.083	4069	-67353	423391	4661502
1	4.083	4639	-66862	423961	4661993
2	1.087	5709	-66978	425031	4661878
3	1.065	7614	-65996	426935	4662859
4	3.06	7316	-67318	426638	4661537
5	1.064	8559	-66934	427880	4661921

```r
# # Plot it
# plot(NY8) # Just the map now.
```

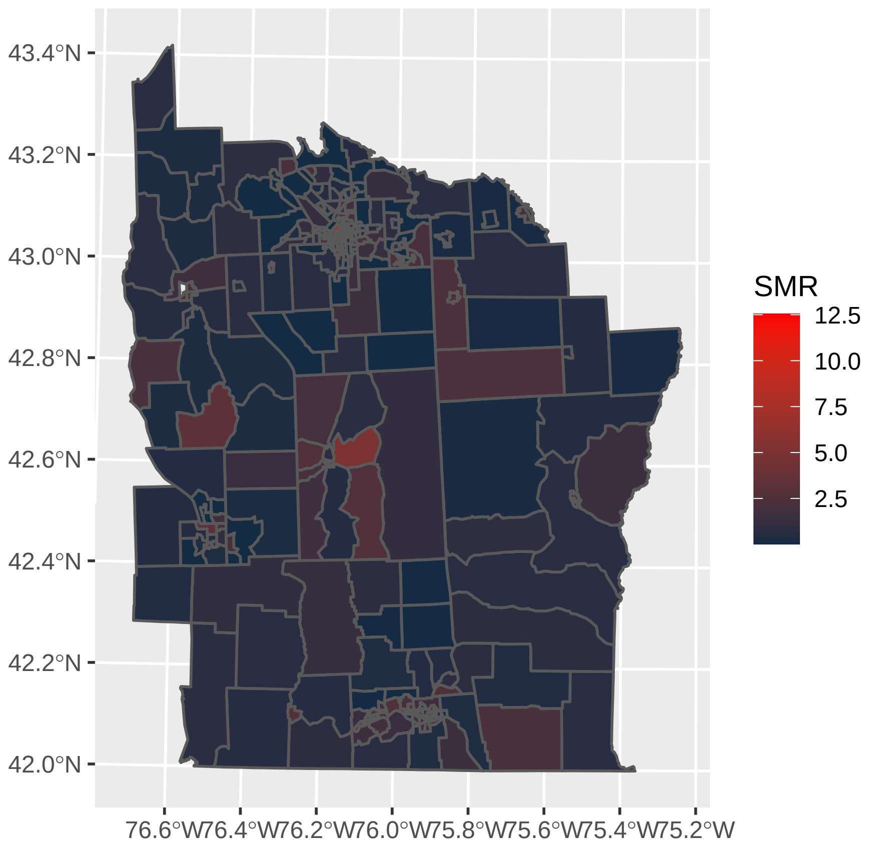
Plotting

```r
# Convert to sf
library(sf)
NY8_sf <- st_as_sf(NY8)
```

```r
# Create the standardized mortality ratio (SMR) variable
# https://www.r-bloggers.com/2019/11/spatial-data-analysis-with-inla/
rate <- sum(NY8_sf$Cases) / sum(NY8_sf$POP8)

NY8_sf <- NY8_sf %>% mutate(
  Expected = POP8 * rate,
  SMR = Cases / Expected
)
```

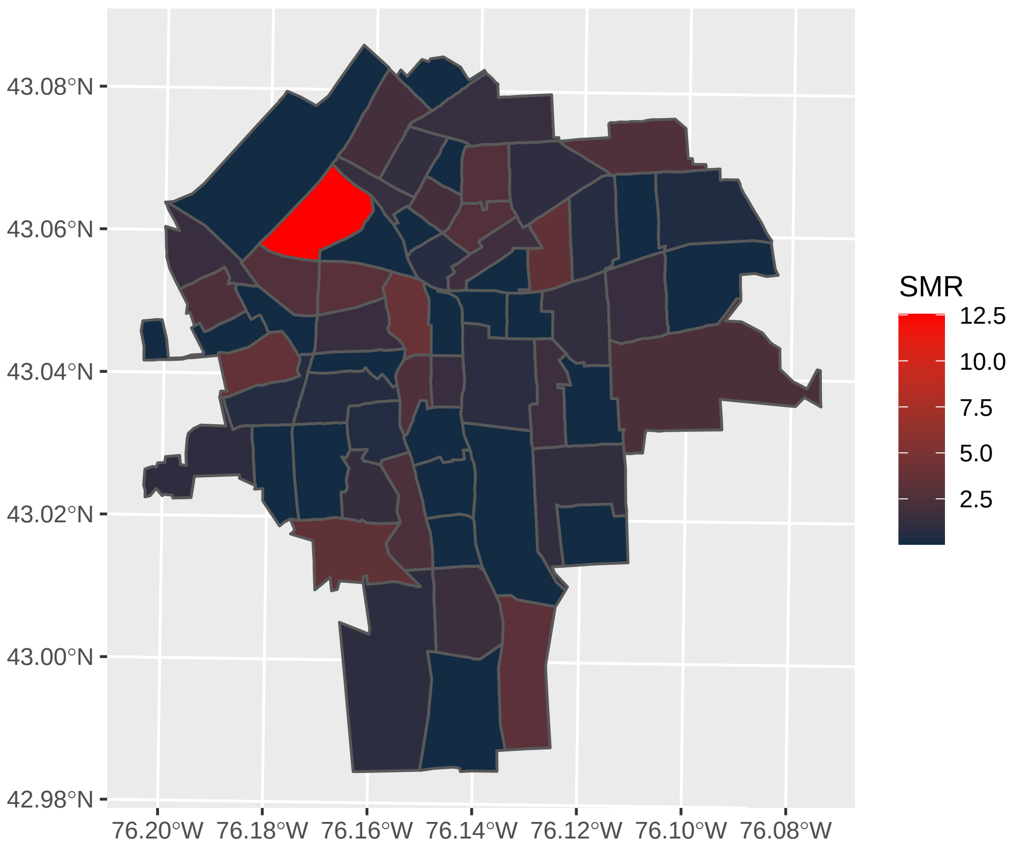
```
# Plot SMR
ggplot(NY8_sf) + geom_sf(aes(fill = SMR)) + # Look nice!
  scale_fill_gradient(high = "red")
```



Subsetting then plotting

```
# Subset to include Syracuse city only
syracuse <- which(NY8$AREANAME == "Syracuse city")

# Plot it
ggplot(NY8_sf[syracuse, ]) + geom_sf(aes(fill = SMR)) +
  scale_fill_gradient(high = "red")
```



Poisson Models

Fitting a Poisson regression model

```
#install.packages("INLA") # run once
#not available for this R version...
#install.packages("INLA", repos=c(getOption("repos"), INLA="https://inla.r-inla-download.org/R/stable")
library(INLA) # Now it works.
```

```
## Loading required package: Matrix
```

```
##
## Attaching package: 'Matrix'
```

```
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
```

```
## Loading required package: foreach
```

```
##
## Attaching package: 'foreach'
## The following objects are masked from 'package:purrr':
##
##     accumulate, when
```

```
## This is INLA_22.03.16 built 2022-03-16 13:24:07 UTC.
##  - See www.r-inla.org/contact-us for how to get help.
```

Let's work on some toy examples first before coming to fix the issue. Toy examples work fine. Issues seem to related to `Cases`. Rounding `Cases` work but results differ a bit.

```
# Fit a Poisson regression model
m1 <- inla(round(Cases) ~ 1 + AVGIDIST,
           data = NY8_sf,
           family = "poisson",
           E = NY8_sf$Expected,
           control.predictor = list(compute = TRUE),
           control.compute = list(dic = TRUE, waic = TRUE))
```

```
# summary(m1) %>% pander # very bad!
```

```
summary(m1)
```

```
##
## Call:
##    c("inla.core(formula = formula, family = family, contrasts = contrasts,
##    ", " data = data, quantiles = quantiles, E = E, offset = offset, ", "
##    scale = scale, weights = weights, Ntrials = Ntrials, strata = strata,
##    ", " lp.scale = lp.scale, link.covariates = link.covariates, verbose =
##    verbose, ", " lincomb = lincomb, selection = selection, control.compute
##    = control.compute, ", " control.predictor = control.predictor,
##    control.family = control.family, ", " control.inla = control.inla,
##    control.fixed = control.fixed, ", " control.mode = control.mode,
##    control.expert = control.expert, ", " control.hazard = control.hazard,
##    control.lincomb = control.lincomb, ", " control.update =
##    control.update, control.lp.scale = control.lp.scale, ", "
##    control.pardiso = control.pardiso, only.hyperparam = only.hyperparam,
##    ", " inla.call = inla.call, inla.arg = inla.arg, num.threads =
##    num.threads, ", " blas.num.threads = blas.num.threads, keep = keep,
##    working.directory = working.directory, ", " silent = silent, inla.mode
##    = inla.mode, safe = FALSE, debug = debug, ", " .parent.frame =
##    .parent.frame)")
## Time used:
##     Pre = 2.57, Running = 0.415, Post = 0.0207, Total = 3
## Fixed effects:
##               mean    sd 0.025quant 0.5quant 0.975quant   mode kld
## (Intercept) -0.097 0.046     -0.188   -0.096     -0.008 -0.096   0
## AVGIDIST     0.324 0.078      0.163    0.327      0.471  0.332   0
##
## Deviance Information Criterion (DIC) ...............: 1016.44
## Deviance Information Criterion (DIC, saturated) ....: -649.28
## Effective number of parameters .....................: 2.00
##
```

```
## Watanabe-Akaike information criterion (WAIC) ...: 1017.37
## Effective number of parameters .................: 2.69
##
## Marginal log-Likelihood:  -514.42
##  is computed
## Posterior summaries for the linear predictor and the fitted values are computed
## (Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
```

Fitting a Poisson regression model with random effects

```r
# Fit a Poisson regression model with random effects
NY8_sf <- NY8_sf %>% mutate(
  ID = 1:nrow(NY8)) # Use ID as the random effect

m2 <- inla(round(Cases) ~ 1 + AVGIDIST + f(ID, model = "iid"),
  data = NY8_sf,
  family = "poisson",
  E = NY8_sf$Expected,
  control.predictor = list(compute = TRUE),
  control.compute = list(dic = TRUE, waic = TRUE))
```

```r
summary(m2)
```

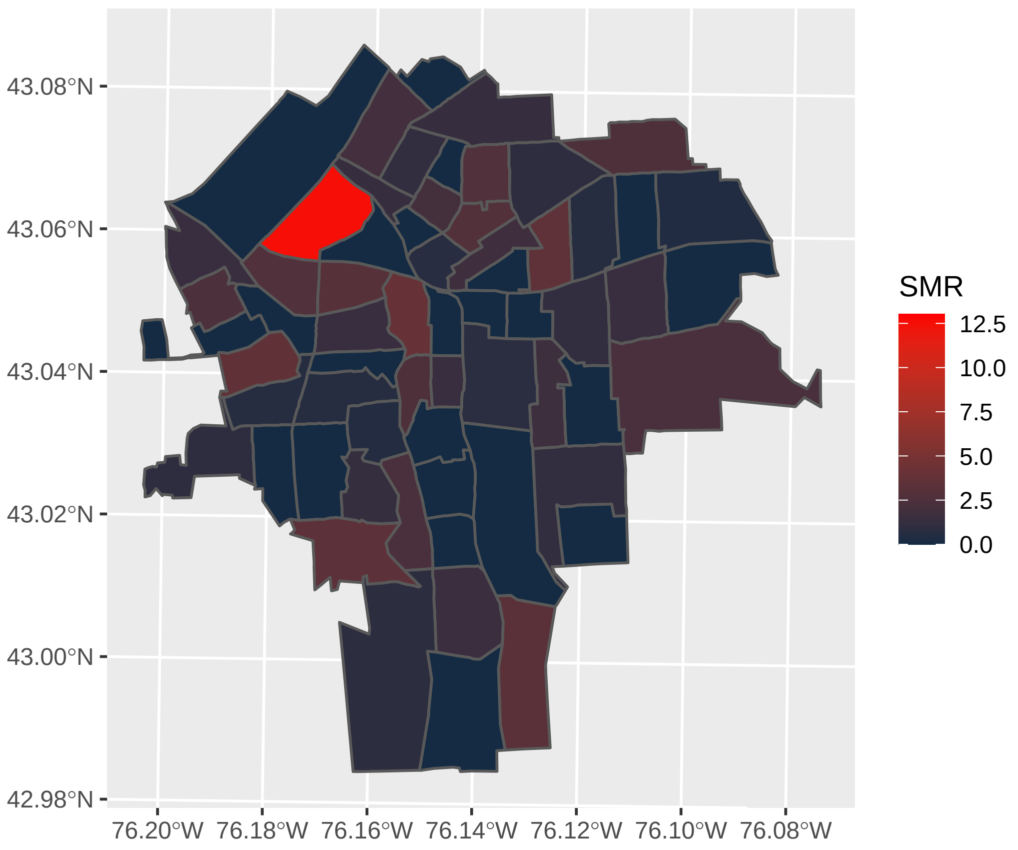
```
##
## Call:
##    c("inla.core(formula = formula, family = family, contrasts = contrasts,
##    ", " data = data, quantiles = quantiles, E = E, offset = offset, ", "
##    scale = scale, weights = weights, Ntrials = Ntrials, strata = strata,
##    ", " lp.scale = lp.scale, link.covariates = link.covariates, verbose =
##    verbose, ", " lincomb = lincomb, selection = selection, control.compute
##    = control.compute, ", " control.predictor = control.predictor,
##    control.family = control.family, ", " control.inla = control.inla,
##    control.fixed = control.fixed, ", " control.mode = control.mode,
##    control.expert = control.expert, ", " control.hazard = control.hazard,
##    control.lincomb = control.lincomb, ", " control.update =
##    control.update, control.lp.scale = control.lp.scale, ", "
##    control.pardiso = control.pardiso, only.hyperparam = only.hyperparam,
##    ", " inla.call = inla.call, inla.arg = inla.arg, num.threads =
##    num.threads, ", " blas.num.threads = blas.num.threads, keep = keep,
##    working.directory = working.directory, ", " silent = silent, inla.mode
##    = inla.mode, safe = FALSE, debug = debug, ", " .parent.frame =
##    .parent.frame)")
## Time used:
##     Pre = 2.44, Running = 0.283, Post = 0.0213, Total = 2.74
## Fixed effects:
##               mean    sd 0.025quant 0.5quant 0.975quant   mode kld
## (Intercept) -0.184 0.062     -0.311   -0.182     -0.066 -0.179   0
## AVGIDIST     0.363 0.114      0.137    0.363      0.586  0.365   0
##
## Random effects:
##   Name     Model
##     ID IID model
```

```
##
## Model hyperparameters:
##                  mean   sd 0.025quant 0.5quant 0.975quant mode
## Precision for ID 6.10 2.92       3.11     5.42      13.32 4.63
##
## Deviance Information Criterion (DIC) ...............: 979.25
## Deviance Information Criterion (DIC, saturated) ....: -686.48
## Effective number of parameters .....................: 73.40
##
## Watanabe-Akaike information criterion (WAIC) ...: 983.62
## Effective number of parameters .................: 64.16
##
## Marginal log-Likelihood:  -512.10
##  is computed
## Posterior summaries for the linear predictor and the fitted values are computed
## (Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
```

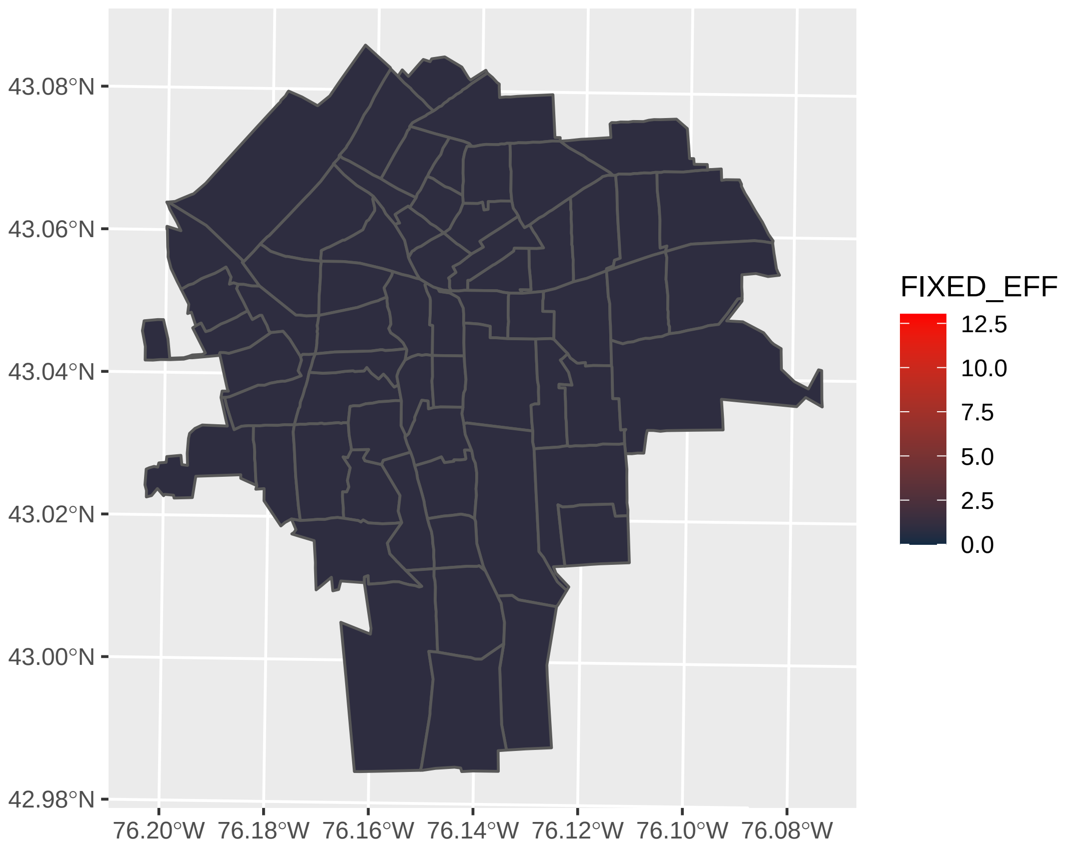
Plotting

```
# Add fitted values for both m1 & m2
NY8_sf <- NY8_sf %>% mutate(
  FIXED_EFF = m1$summary.fitted[, "mean"],
  IID_EFF = m2$summary.fitted[, "mean"]
  )
```

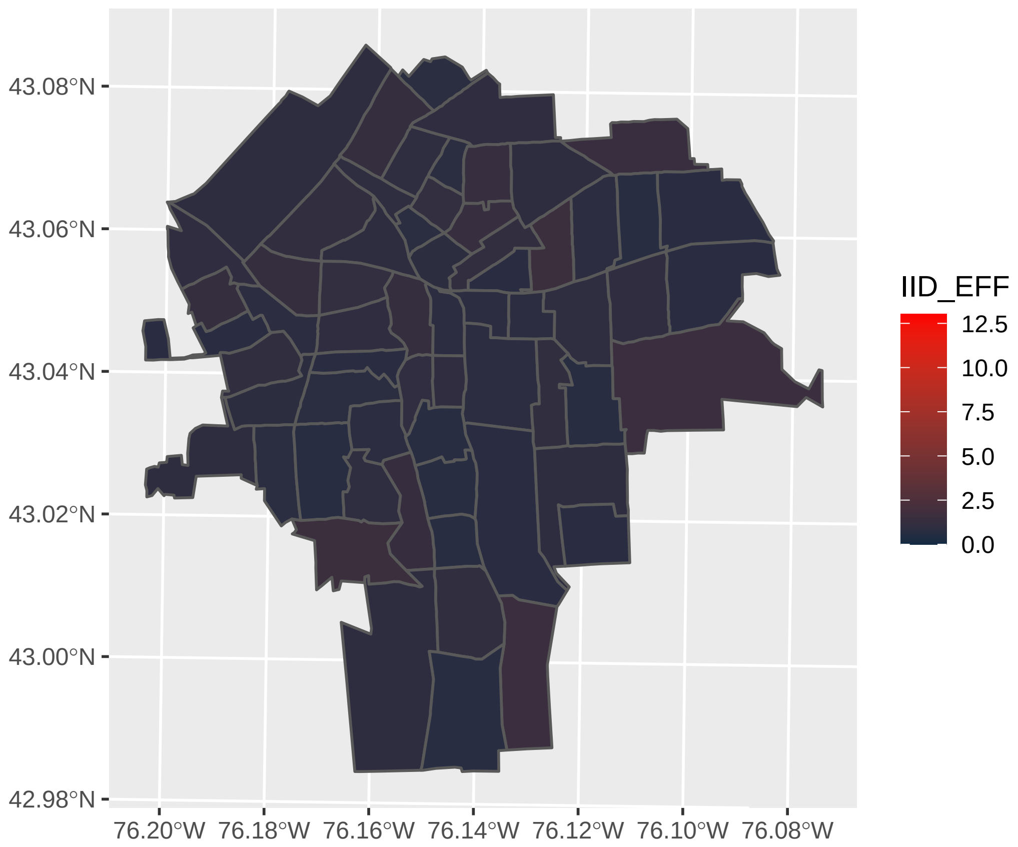
```
# Plot them but for Syracuse city only
ggplot(NY8_sf[syracuse, ]) + geom_sf(aes(fill = SMR)) +
  scale_fill_gradient(high = "red", limits = c(0, 13)) -> p_m0
p_m0
```



```
ggplot(NY8_sf[syracuse, ]) + geom_sf(aes(fill = FIXED_EFF)) + #, show.legend = FALSE) +
  scale_fill_gradient(high = "red", limits = c(0, 13)) -> p_m1
p_m1
```



```
ggplot(NY8_sf[syracuse, ]) + geom_sf(aes(fill = IID_EFF)) + # , show.legend = FALSE) +
  scale_fill_gradient(high = "red", limits = c(0, 13)) -> p_m2
p_m2
```



We might want them plotted with the same scale.

```
#grid.arrange(p_m0, p_m1, p_m2, nrow = 3, ncol = 1)
```

Spatial Models for Areal (or Lattice) Data

Plot spatial neighbors

An adjacency (or neighbour) matrix W is often used to describe spatial proximity in areal (lattice) data. Element W_{ij} is non-zero, if area i and j are neighbors. Element W_{ij} is zero, otherwise.

```
# Compute adjacency matrix
NY8.nb <- poly2nb(NY8) # construct the neighbours list / neighbour matrix
NY8.nb
```

```
## Neighbour list object:
## Number of regions: 281
## Number of nonzero links: 1624
## Percentage nonzero weights: 2.056712
## Average number of links: 5.779359
```

```
class(NY8.nb)
```

```
## [1] "nb"
```

Plot spatial neighbors using ggplot2

```r
# Plot spatial neighbors using ggplot2
# https://mbjoseph.github.io/posts/2018-12-27-plotting-spatial-neighbors-in-ggplot2/
NY8_sp <- as(NY8_sf, 'Spatial')  # NY8_sf is a "sf" "data.frame"
class(NY8_sp) # Now is a "SpatialPolygonsDataFrame"
```

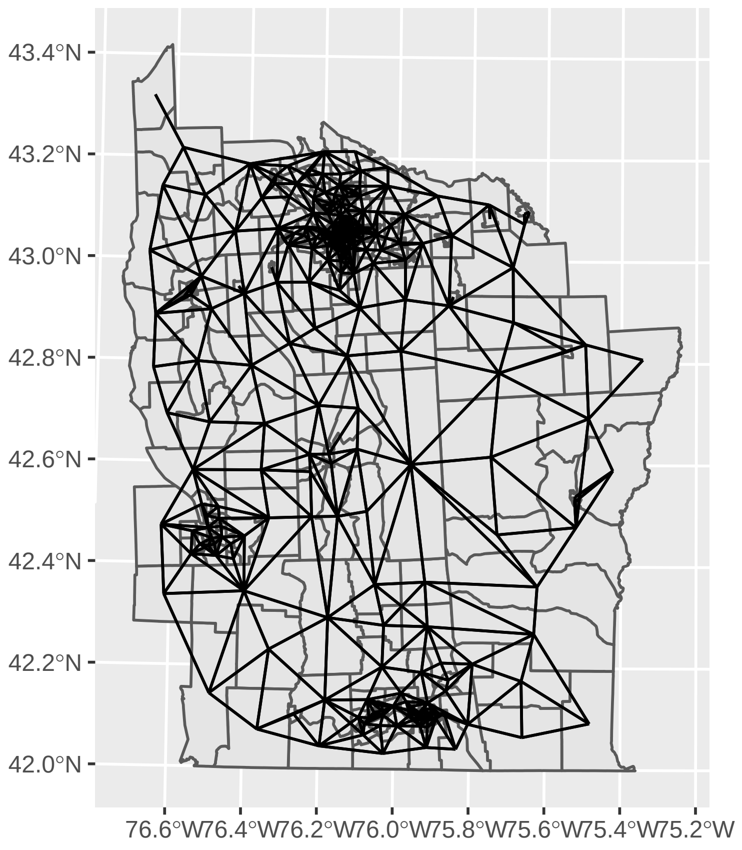
```
## [1] "SpatialPolygonsDataFrame"
## attr(,"package")
## [1] "sp"
```

```r
neighbors <- poly2nb(NY8) # construct the neighbours list
neighbors_sf <- as(nb2lines(neighbors, coords = coordinates(NY8_sp)), 'sf')
```

```
## Warning in CRS(proj4string): CRS: projargs should not be NULL; set to NA
```

```r
neighbors_sf <- st_set_crs(neighbors_sf, st_crs(NY8_sf))
```

```r
ggplot(NY8_sf) +
  geom_sf() + # remove aes(fill = SMR)
  geom_sf(data = neighbors_sf)
```



```
#plot(NY8)
```

```
# plot(NY8)
# plot(NY8.nb, coordinates(NY8), add = TRUE, pch = ".", col = "gray")
```

```
# Create sparse adjacency matrix
# Or use the function nb2INLA to generate spatial neighbours for INLA
NY8.mat <- as(nb2mat(NY8.nb, style = "B"), "Matrix") # generate a weights matrix for a neighbours list
# Use this (NY8.mat) for the graph argument in the function inla
#NY8.mat
class(NY8.mat)
```

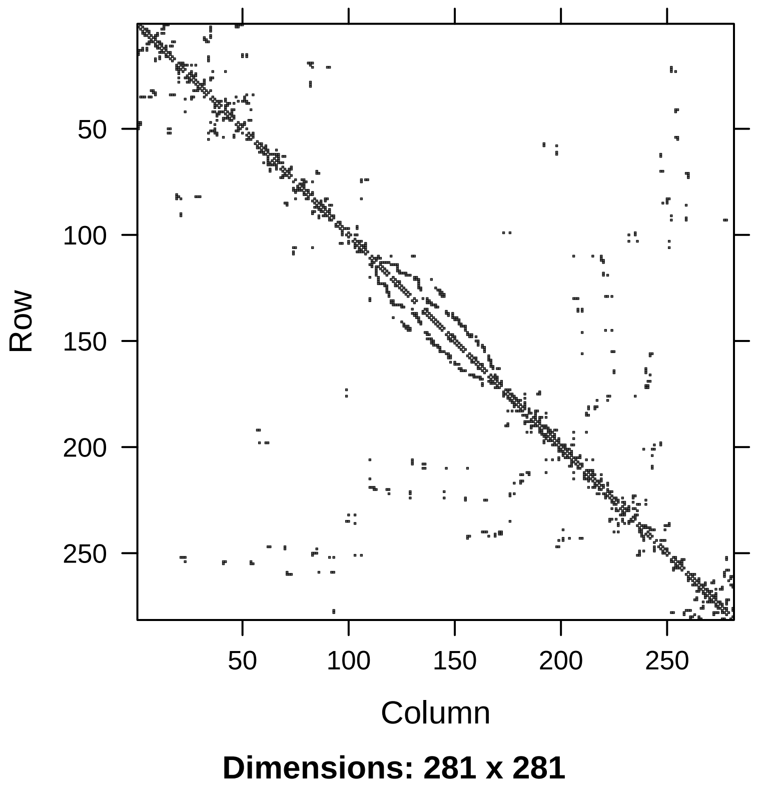
```
## [1] "dgCMatrix"
## attr(,"package")
## [1] "Matrix"
```

Here is a post that discusses the function `poly2nb` vs. `nd2INLA`. We might also need to check this tutorial to do more.

Plot the adjacency matrix

Is the adjacency matrix the same as spatial neighbor?

```
# Plot the adjacency matrix
image(NY8.mat)
```



```r
# summ <- summary(NY8.mat)
# #summ
# NY8.mat.df <- data.frame(
#   Origin = rownames(NY8.mat)[summ$i],
#   Destination = colnames(NY8.mat)[summ$j],
#   Weight = NY8.mat$x)
```

Generalized Linear Models With Spatial Random Effects

The GLMs have the following form:

$$Y = X\beta + Zu + \varepsilon,$$

where β is a vector of fixed effects, u is a vector of random effects,……

The vector of random effects u is modeled as MVN:

$$u \sim N(0, \sigma_u^2 \Sigma),$$

where Σ is defined s.t. it induces higher correlation with adjacent areas.

There are a few ways to include spatial dependence in Σ:

1. SAR (Simultaneous autoregressive):

$$Q = \Sigma^{-1} = ((I - \rho W)^T((I - \rho W))),$$

where I is the identity matrix, ρ is a spatial autocorrelation parameter, and W is the adjacency matrix.

2. CAR (Conditional autoregressive):

$$Q = \Sigma^{-1} = (I - \rho W)$$

3. ICAR (Intrinsic CAR):

$$\Sigma^{-1} = diag(n_i) - W,$$

where n_i is the number of neighbors of area i.

4. Mixture of matrices (Leroux et al.'s model):

$$\Sigma^{-1} = ((1 - \lambda)I_n + \lambda M), \lambda \in (0, 1)$$

where M is precision of intrinsic CAR specification.

Fit a SLM (spatial lag model)

Fit a ICAR (Intrinsic CAR) model

Fit a BYM (Besag-York-Mollié) model

The BYM (Besag-York-Mollié) model is a convolution model of an ICAR (intrinsic CAR) effect and an iid Gaussian latent effect.

```r
m.bym = inla(round(Cases) ~ 1 + AVGIDIST +
               f(ID, model = "bym", graph = NY8.mat),
             data = NY8_sf,
             E = NY8_sf$Expected,
             family ="poisson",
             control.predictor = list(compute = TRUE),
             control.compute = list(dic = TRUE, waic = TRUE))
```

Results differ a bit.

```r
summary(m.bym)
```

```
##
## Call:
##    c("inla.core(formula = formula, family = family, contrasts = contrasts,
##    ", " data = data, quantiles = quantiles, E = E, offset = offset, ", "
##    scale = scale, weights = weights, Ntrials = Ntrials, strata = strata,
##    ", " lp.scale = lp.scale, link.covariates = link.covariates, verbose =
##    verbose, ", " lincomb = lincomb, selection = selection, control.compute
##    = control.compute, ", " control.predictor = control.predictor,
##    control.family = control.family, ", " control.inla = control.inla,
##    control.fixed = control.fixed, ", " control.mode = control.mode,
##    control.expert = control.expert, ", " control.hazard = control.hazard,
##    control.lincomb = control.lincomb, ", " control.update =
##    control.update, control.lp.scale = control.lp.scale, ", "
##    control.pardiso = control.pardiso, only.hyperparam = only.hyperparam,
##    ", " inla.call = inla.call, inla.arg = inla.arg, num.threads =
##    num.threads, ", " blas.num.threads = blas.num.threads, keep = keep,
##    working.directory = working.directory, ", " silent = silent, inla.mode
##    = inla.mode, safe = FALSE, debug = debug, ", " .parent.frame =
##    .parent.frame)")
## Time used:
##     Pre = 2.52, Running = 1, Post = 0.0249, Total = 3.55
## Fixed effects:
##               mean    sd 0.025quant 0.5quant 0.975quant   mode kld
## (Intercept) -0.163 0.054     -0.271   -0.163     -0.060 -0.161   0
## AVGIDIST     0.322 0.125      0.070    0.324      0.563  0.327   0
##
## Random effects:
##   Name     Model
##     ID BYM model
##
## Model hyperparameters:
##                                          mean      sd 0.025quant 0.5quant
## Precision for ID (iid component)      1772.87 1759.70     113.36  1249.70
## Precision for ID (spatial component)     2.70    1.14       1.21     2.46
##                                      0.975quant   mode
## Precision for ID (iid component)        6482.92 304.49
## Precision for ID (spatial component)       5.58   2.06
##
## Deviance Information Criterion (DIC) ...............: 967.44
## Deviance Information Criterion (DIC, saturated) ....: -698.29
## Effective number of parameters .....................: 51.67
```

```
##
## Watanabe-Akaike information criterion (WAIC) ...: 971.93
## Effective number of parameters .................: 48.34
##
## Marginal log-Likelihood:  -458.85
##  is computed
## Posterior summaries for the linear predictor and the fitted values are computed
## (Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
```

Fit a mixture (Leroux et al.) model

Reference

Gómez-Rubio, V. (2019). R-bloggers. Spatial Data Analysis with INLA. https://www.r-bloggers.com/2019/11/spatial-data-analysis-with-inla/.