Reduced-Rank Regression Model: A Review

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March 2022

Background and Introduction

A classical multivariate linear model, which is given as

$$Y_k = CX_k + \varepsilon_k, \ k = 1, ..., T$$

where $Y_i = (y_{1k}, ...y_{mk})^T$ is a $mx1$ response vector, C is a mxn regression coefficient matrix, $X_k = (x_{1k}, ...x_{mk})^T$ is a $nx1$ predictor vector, and $\varepsilon_k = (\varepsilon_{1k}, ...\varepsilon_{mk})^T$ is a $mx1$ error vector with $E(\varepsilon_k) = 0$ and $cov(\varepsilon_k) = \Sigma_{\varepsilon\varepsilon}$, does not make use of the fact that the response variables are likely correlated.

In many practical situations, there is also often a need reduce the number of parameters in the model since it can be too large.

Introduction

We further assume that

$$rank(C) = r \leq min(m, n),$$

which leads to two implications.

1. The linear combination, $l^T Y_k$, $i = 1, ..., (m - r)$, can be modeled through the distribution of the error term ε_k.
2. C can be expressed as $C = AB$, where A is of dimension mxr and B is of dimension rxn. Then, the above multivariate linear model can be rewritten as

$$Y_k = A(BX_k) + \varepsilon_k, \ \ k = 1, ..., T,$$

where BX_k is of reduced dimension $rx1$, and as a result, there is a gain in simplicity and interpretation.

Introduction

The first application of reduced-rank regression model appeared in an initial work of Anderson (1951) in the field of economics. The model and its statistical properties were further examined by a few other authors.

Subsequent but separate work that were studied using related concepts were

- principle components (Rao, 1964),
- simultaneous linear prediction modeling (Fortier, 1966),
- redundancy analysis, an alternative to canonical correlation analysis (van den Wollenberg, 1977), etc.

More complex models have also been developed ever since.

Applications

Applications of the reduced-rank regression model include

(1) the experimental properties of hydrocarbon fuel mixtures in relating response to composition (Davies and Tso, 1982),

(2) an econometric model of the United Kingdom from 1948 to 1956 (Gudmundsson, 1977), which consists of 37 time series of response variables and 32 time series of predictors,

(3) the relationship between measurements on solar radiation taken over various sites in Scotland and the physical characteristics of the sites (Glasbey, 1992),

(4) the joint effects of toxic compounds on the growth of larval fathead minnows (Ryan et al., 1992), and

(5) testing the efficiency of portfolios (Zhou, 1991, 1995).

Estimation