Far Eastern University, Second Semester, A.Y 23-24
Theory of Interest Problem
set 1

Instructions: WORK INDEPENDENTLY. Answer *ALL* items. Show your complete proof and/or solutions to merit full points.

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1. A Filipino couple, Miguel and Maria, decide to purchase a residential property in Makati City for 8,000,000. They secure a 20-year loan from a local bank at an annual interest rate of 7.5%. The bank requires them to make monthly payments using the amortization method. Assuming there are no additional fees or charges, determine the following:
   a. The total interest paid over the life of the loan. **7694750.661**
   b. The monthly payment amounts. **784737.5331**
   c. If they decide to pay an additional 20,000 towards the principal every six months, how much time and interest would they save over the life of the loan?
   c. **Time Save:** 2
   c. **Interest Save:** 867523.198

TABLE #1 (Given Values)

Periods	Payment	Interest	Payments for the Principal	Balance
0				8000000
1	784737.5	600000	184737.5331	7815262.5
2	784737.5	586144.7	198592.848	7616669.6
3	784737.5	571250.2	213487.3116	7403182.3
4	784737.5	555238.7	229498.86	7173683.4
5	784737.5	538026.3	246711.2745	6926972.2
6	784737.5	519522.9	265214.6201	6661757.6
7	784737.5	499631.8	285105.7166	6376651.8
8	784737.5	478248.9	306488.6454	6070163.2
9	784737.5	455262.2	329475.2938	5740687.9
10	784737.5	430551.6	354185.9408	5386502
11	784737.5	403987.6	380749.8864	5005752.1
12	784737.5	375431.4	409306.1278	4596445.9
13	784737.5	344733.4	440004.0874	4156441.9
14	784737.5	311733.1	473004.394	3683437.5
15	784737.5	276257.8	508479.7235	3174957.7
16	784737.5	238121.8	546615.7028	2628342
17	784737.5	197125.7	587611.8805	2040730.2
18	784737.5	153054.8	631682.7715	1409047.4
19	784737.5	105678.6	679058.9794	729988.4
20	784737.5	54749.13	729988.4028	-8.38E-09

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TABLE #1.1 (DP:20,000/6months)

Period	Payment	Interest	Payments for the Principal	Balance
0				8000000
1	824737.53	600000	224737.533	7775262.467
2	824737.53	583144.685	241592.848	7533669.619
3	824737.53	565025.2214	259712.312	7273957.307
4	824737.53	545546.798	279190.735	6994766.572
5	824737.53	524607.4929	300130.04	6694636.532
6	824737.53	502097.7399	322639.793	6371996.739
7	824737.53	477899.7554	346837.778	6025158.961
8	824737.53	451886.9221	372850.611	5652308.35
9	824737.53	423923.1263	400814.407	5251493.944
10	824737.53	393862.0458	430875.487	4820618.456
11	824737.53	361546.3842	463191.149	4357427.307
12	824737.53	326807.0481	497930.485	3859496.822
13	824737.53	289462.2617	535275.271	3324221.551
14	824737.53	249316.6163	575420.917	2748800.634
15	824737.53	206160.0476	618577.485	2130223.149
16	824737.53	159766.7362	664970.797	1465252.352
17	824737.53	109893.9264	714843.607	750408.7453
18	824737.53	56280.6559	768456.877	-18048.13185

Such that, every periods is (6+6) months

SUMMARY

TABLE #1 (Given Values)

a. Interest Paid	7694751
b. Monthly Payment Amount	784737.5

TABLE #1.1 (DP:20,000/6months)

a. Interest Paid	**6827227.46**
b. Monthly Payment Amount	**824737.533**
c. Time Save	**2**
c. Interest Save	**867523.198**

2. A Filipino entrepreneur, Juan, wants to expand his business by purchasing a commercial property in Quezon City for 15,000,000. He secures a 15-year loan from a local bank at an annual interest rate of 6.8%. The bank offers two repayment options:

    a. monthly payments using the amortization method or

    b. an initial lump sum payment of 25% of the loan amount, followed by monthlypayments using the amortization method for the remaining balance. Determine the following:

    a. The total interest paid over the life of the loan for option (a).

    b. The total interest paid over the life of the loan for option (b).

    c. The difference in total interest paid between the two options.

    d. If Juan chooses option (b) and decides to pay an additional 50,000 towards the principal every quarter, how much time and interest would he save over the life of the loan?

3. The Philippine Government plans to issue 100 billion worth of 10-year Treasury bonds to finance its infrastructure projects. The bonds will pay semi-annual coupons (interest payments) and have a face value of 10,000 each. Assuming the bonds are issued at par (face value), determine the following:

   a. The number of bonds that will be issued.
   b. If the annual coupon rate is set at 5.25%, calculate the semi-annual coupon payment per bond.
   c. Suppose an investor purchases 1,000 bonds at the time of issuance. Calculatethe total amount the investor will receive at maturity, including the principal and all coupon payments.
   d. If the bonds are trading at a price of 9,800 per bond after 3 years, calculate the current yield and the yield to maturity for the investor.

TABLE #3 (Given Values)

Periods	Payment
1	-9800
2	262.50
3	262.50
4	262.50
5	262.50
6	262.50
7	262.50
8	262.50
9	262.50
10	262.50
11	262.50
12	262.50
13	262.50
14	10,262.50

Bond Price	100,000,000,000.00
T	10
M	2
Purchase Bond	1000

Trading	
Price/Bond	9800
Bond Years Duration	3

SUMMARY

Face Value	10,000.00
Annua Coupon Rate	0.0525
Total coupon payments	5,250,000.00
Total principal	10,000,000.00
Current yield	5.358
Semi-Annual YTM	2.81%
Annual YTM	0.056216
Annual YTM Rate	5.622
Annua Coupon Rate	0.0525

TABLE #3 (Given Values)

a. The number of bonds that will be issued.	10,000,000.00
b. If the annual coupon rate is set at 5.25%, calculate the semi-annual coupon payment per bond	262.50
c. Suppose an investor purchases 1,000 bonds at the time of issuance. Calculate the total amount the investor will receive at maturity, including the principal and all coupon payments.	15,250,000.00
d. If the bonds are trading at a price of ₱9,800 per bond after 3 years, calculate the current yield and the yield to maturity for the investor.	Current Yield: 5.358 YTM: 5.622

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4. A Filipino couple, Juan and Maria, are planning for their retirement. They decide toinvest in an annuity that will provide them with a retirement income starting at age 65. The annuity has the following terms:

   · Initial investment:2,000,000 **(PV)**

   · Annual interest rate: 7% **(r)**

   · The first payment at age 65 will be 80,000 **(P)**

   · Each subsequent payment will increase by 4% annually **(g)**

   Determine the following:

   a. The number of years the annuity will make payments.

In the excel I have shown you how to approach the problem of finding out how many years the annuity would make payments. To get the number of years, we need to first calculate how much were the payments made on that year and calculate the present value of each payment using the PV formula for compound interest to help us determine the value of future payments in terms of today's money. We then track the cumulative present value of payments until it reached or exceeded the initial investment amount, in other words it calculates the number of payments per year until the initial investment amount is depleted. By doing this process we would be able to determine the number of years the annuity would make payments, which in the excel we have found out to be **49 Years**.

I checked my results by using a formula of the Growing Annuity (a type of annuity) to simply calculate how many years the annuity will make payments. The Growing Annuity fits the problem since the payments increases by a fixed percentage each year. Using the formula can help accurately determine the number of years the annuity will make payments since the formula takes into account both the initial payment and the growth rate of subsequent payments.

The Growing Annuity Formula: **PV =** $\frac{P}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^{n}\right]$

From the calculations we get:

$$2{,}000{,}000 = \frac{80{,}000}{0.07-0.04}\left[1-\left(\frac{1.04}{1.07}\right)^{n}\right]$$

$$2{,}000{,}000 = \frac{80{,}000}{0.03}\left[1-\left(\frac{1.04}{1.07}\right)^{n}\right]$$

$$2{,}000{,}000 = 2{,}666{,}666.667\left[1-\left(\frac{1.04}{1.07}\right)^{n}\right]$$

$$\frac{2{,}000{,}000}{2{,}666{,}666.667} = \left[1-\left(\frac{1.04}{1.07}\right)^{n}\right]$$

$$0.75 = 1 - (\frac{1.04}{1.07})^n$$

$$0.25 = (\frac{1.04}{1.07})^n$$

$$\ln(0.25) = n \ln\left(\frac{1.04}{1.07}\right)$$

$$n = \frac{\ln(0.25)}{\ln(\frac{1.04}{1.07})}$$

$$\boldsymbol{n = 48.74806646 \; or \; 49 \; Years}$$

b. The total amount Juan and Maria will receive from the annuity.

Based on the table made in the Excel for letter (a), we can calculate the total amount received by the couple from the annuity by adding all the payments from each year until the annuity is depleted. Thus, the couple have approximately received **Php 11,666,699** from the annuity.

c. If they decide to withdraw an additional lump sum of 500,000 at age 75, how would this affect the remaining annuity payments?

The problem stated that Juan and Maria decided to withdraw an additional lump sum of Php 500,000 at age 75. It would mean that it is an extra withdrawal on top of the regular annuity payment for that specific year. After calculating the remaining investment from the age 75, we update/adjust the other calculations of the following year back to the normal previous payment +4% (without the 500,000 lump sum).

Thus, based on the table if the couple decided to withdraw an additional lump sum of Php 500,000 on top of the annuity payment then it would affect the remaining annuity. Specifically, the Total Amount received from the annuity **(Approximately Php 7,732,732)** and the number of years of payment until the annuity is depleted **(Approximately 39 Years)**.

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TABLE #4

# of Years	Age	Payments	Present Value (PV) of the Current Payment	The Investment Left from the Annuity
1	65	80000	74766.35514	1,925,234
2	66	83200	72670.10219	1852563.543
3	67	86528	70632.62269	1781930.92
4	68	89989.12	68652.26878	1713278.651
5	69	93588.6848	66727.43882	1646551.212
6	70	97332.23219	64856.57605	1581694.636
7	71	101225.5215	63038.16737	1518656.469
8	72	105274.5423	61270.74212	1457385.727
9	73	109485.524	59552.87085	1397832.856
10	74	113864.945	57883.16419	1339949.692
11	75	118419.5428	56260.27173	1283689.42
12	76	123156.3245	54682.88094	1229006.539
13	77	128082.5775	53149.71605	1175856.823
14	78	133205.8806	51659.5371	1124197.286
15	79	138534.1158	50211.13886	1073986.147
16	80	144075.4804	48803.34992	1025182.797
17	81	149838.4997	47435.0317	977747.7655
18	82	155832.0396	46105.07754	931642.688
19	83	162065.3212	44812.41181	886830.2762
20	84	168547.9341	43555.98905	843274.2871
21	85	175289.8514	42334.7931	800939.494
22	86	182301.4455	41147.83628	759791.6577
23	87	189593.5033	39994.15863	719797.4991
24	88	197177.2435	38872.82708	680924.672
25	89	205064.3332	37782.93473	643141.7373
26	90	213266.9065	36723.60011	606418.1372
27	91	221797.5828	35693.96646	570724.1707
28	92	230669.4861	34693.20105	536030.9697
29	93	239896.2655	33720.49448	502310.4752
30	94	249492.1162	32775.06005	469535.4151
31	95	259471.8008	31856.13313	437679.282
32	96	269850.6728	30962.97052	406716.3115
33	97	280644.6997	30094.84985	376621.4616
34	98	291870.4877	29251.06902	347370.3926
35	99	303545.3072	28430.94559	318939.447
36	100	315687.1195	27633.81627	291305.6308
37	101	328314.6043	26859.03638	264446.5944
38	102	341447.1885	26105.97928	238340.6151
39	103	355105.076	25374.03594	212966.5792
40	104	369309.2791	24662.61437	188303.9648

41	105	384081.6502	23971.1392	164332.8256
42	106	399444.9162	23299.05118	141033.7744
43	107	415422.7129	22645.80676	118387.9676
44	108	432039.6214	22010.8776	96377.09005
45	109	449321.2063	21393.75019	74983.33986
46	110	467294.0545	20793.92542	54189.41444
47	111	485985.8167	20210.91816	33978.49628
48	112	505425.2494	19644.25691	14334.23938
49	113	525642.2593	14334.48335	0

The Number of Years the annuity will make payments is **49 years**

For B

Total Amount received from the annuity **11666698.74** or 11,666,699 **(Approximate Value)**

For C

# of Years	Age	Payments	Present Value (PV) of the Current Payment	The Investment Left from the Annuity
1	65	80000	74766.35514	1,925,234
2	66	83200	72670.10219	1,852,564
3	67	86528	70632.62269	1,781,931
4	68	89989.12	68652.26878	1,713,279
5	69	93588.6848	66727.43882	1,646,551
6	70	97332.23219	64856.57605	1,581,695
7	71	101225.5215	63038.16737	1,518,656
8	72	105274.5423	61270.74212	1,457,386
9	73	109485.524	59552.87085	1,397,833
10	74	113864.945	57883.16419	1,339,950
11	75	618419.5428	293806.6699	1,046,143
12	76	123156.3245	54682.88094	991,460
13	77	128082.5775	53149.71605	938,310
14	78	133205.8806	51659.5371	886,651
15	79	138534.1158	50211.13886	836,440
16	80	144075.4804	48803.34992	787,636
17	81	149838.4997	47435.0317	740,201
18	82	155832.0396	46105.07754	694,096
19	83	162065.3212	44812.41181	649,284
20	84	168547.9341	43555.98905	605,728
21	85	175289.8514	42334.7931	563,393
22	86	182301.4455	41147.83628	522,245

23	87	189593.5033	39994.15863	482,251
24	88	197177.2435	38872.82708	443,378
25	89	205064.3332	37782.93473	405,595
26	90	213266.9065	36723.60011	368,872
27	91	221797.5828	35693.96646	333,178
28	92	230669.4861	34693.20105	298,485
29	93	239896.2655	33720.49448	264,764
30	94	249492.1162	32775.06005	231,989
31	95	259471.8008	31856.13313	200,133
32	96	269850.6728	30962.97052	169,170
33	97	280644.6997	30094.84985	139,075
34	98	291870.4877	29251.06902	109,824
35	99	303545.3072	28430.94559	81,393
36	100	315687.1195	27633.81627	53,759
37	101	328314.6043	26859.03638	26,900
38	102	341447.1885	26105.97928	794
39	103	355105.076	794.0359373	0

Total Amount received from the annuity **7732731.977** or 7,732,732 **(Approximate Value)**

Given:		
Investment (IN)	2,000,000	
First Payment (p1)	80000	
i	0.07	
r	0.07	7%
Growth Rate (g)	0.04	4%

"hiraya manawari - sana matupad puhon - balang araw"