**Far Eastern University, Second Semester, A.Y 23-24**

**Theory of Interest Problem set 1**

**Instructions: WORK INDEPENDENTLY**. Answer *ALL* items. Show your complete proof and/or solutions to merit full points.

1. A Filipino couple, Miguel and Maria, decide to purchase a residential property in Makati City for 8,000,000. They secure a 20-year loan from a local bank at an annual interest rate of 7.5%. The bank requires them to make monthly payments using the amortization method. Assuming there are no additional fees or charges, determine the following:
   1. The total interest paid over the life of the loan.

7694750.661

* 1. The monthly payment amounts.

784737.5331

* 1. If they decide to pay an additional 20,000 towards the principal every six months, how much time and interest would they save over the life of the loan?

c. Time Save:2

c. Interest Save:867523.198

1. A Filipino entrepreneur, Juan, wants to expand his business by purchasing a commercial property in Quezon City for 15,000,000. He secures a 15-year loan from a local bank at an annual interest rate of 6.8%. The bank offers two repayment options:
   1. monthly payments using the amortization method or
   2. an initial lump sum payment of 25% of the loan amount, followed by monthlypayments using the amortization method for the remaining balance. Determine the following:
   3. The total interest paid over the life of the loan for option (a).
   4. The total interest paid over the life of the loan for option (b).
   5. The difference in total interest paid between the two options.
   6. If Juan chooses option (b) and decides to pay an additional 50,000 towards the principal every quarter, how much time and interest would he save over the life of the loan?
2. The Philippine Government plans to issue 100 billion worth of 10-year Treasury bonds to finance its infrastructure projects. The bonds will pay semi-annual coupons (interest payments) and have a face value of 10,000 each. Assuming the bonds are issued at par (face value), determine the following:
   1. The number of bonds that will be issued.

* 10,000,000.00
  1. If the annual coupon rate is set at 5.25%, calculate the semi-annual coupon payment per bond.
* 262.50
  1. Suppose an investor purchases 1,000 bonds at the time of issuance. Calculatethe total amount the investor will receive at maturity, including the principal and all coupon payments.
* 15,250,000.00
  1. If the bonds are trading at a price of 9,800 per bond after 3 years, calculate the current yield and the yield to maturity for the investor.
* Current Yield: 5.358 and YTM:5.622

1. A Filipino couple, Juan and Maria, are planning for their retirement. They decide toinvest in an annuity that will provide them with a retirement income starting at age

65. The annuity has the following terms:

· Initial investment: 2,000,000 **(PV)**

· Annual interest rate: 7% **(r)**

· The first payment at age 65 will be 80,000 **(P)**

· Each subsequent payment will increase by 4% annually **(g)**

Determine the following:

1. The number of years the annuity will make payments.

In the excel I have shown you how to approach the problem of finding out how many years the annuity would make payments. To get the number of years, we need to first calculate how much were the payments made on that year and calculate the present value of each payment using the PV formula for compound interest to help us determine the value of future payments in terms of today’s money. We then track the cumulative present value of payments until it reached or exceeded the initial investment amount, in other words it calculates the number of payments per year until the initial investment amount is depleted. By doing this process we would be able to determine the number of years the annuity would make payments, which in the excel we have found out to be **49 Years**.

I checked my results by using a formula of the Growing Annuity (a type of annuity) to simply calculate how many years the annuity will make payments. The Growing Annuity fits the problem since the payments increases by a fixed percentage each year. Using the formula can help accurately determine the number of years the annuity will make payments since the formula takes into account both the initial payment and the growth rate of subsequent payments.

A number of numbers and symbols

Description automatically generated with medium confidence

The Growing Annuity Formula: **PV =**  
  
From the calculations we get:

1. The total amount Juan and Maria will receive from the annuity.

Based on the table made in the Excel for letter (a), we can calculate the total amount received by the couple from the annuity by adding all the payments from each year until the annuity is depleted. Thus, the couple have approximately received **Php 11,666,699** from the annuity.

1. If they decide to withdraw an additional lump sum of 500,000 at age 75, how would this affect the remaining annuity payments?

The problem stated that Juan and Maria decided to withdraw an additional lump sum of Php 500,000 at age 75. It would mean that it is an extra withdrawal on top of the regular annuity payment for that specific year. After calculating the remaining investment from the age 75, we update/adjust the other calculations of the following year back to the normal previous payment +4% (without the 500,000 lump sum).

Thus, based on the table if the couple decided to withdraw an additional lump sum of Php 500,000 on top of the annuity payment then it would affect the remaining annuity. Specifically, the Total Amount received from the annuity **(Approximately Php 7,732,732)** and the number of years of payment until the annuity is depleted **(Approximately 39 Years)**.

Excel Reference:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| **# of Years** | **Age** | **Payments** | **Present Value (PV) of the Current Payment** | **The Investment Left from the Annuity** |  |
| 1 | 65 | 80000 | 74766.35514 | 1,925,234 |  |
| 2 | 66 | 83200 | 72670.10219 | 1852563.543 |  |
| 3 | 67 | 86528 | 70632.62269 | 1781930.92 |  |
| 4 | 68 | 89989.12 | 68652.26878 | 1713278.651 |  |
| 5 | 69 | 93588.6848 | 66727.43882 | 1646551.212 |  |
| 6 | 70 | 97332.23219 | 64856.57605 | 1581694.636 |  |
| 7 | 71 | 101225.5215 | 63038.16737 | 1518656.469 |  |
| 8 | 72 | 105274.5423 | 61270.74212 | 1457385.727 |  |
| 9 | 73 | 109485.524 | 59552.87085 | 1397832.856 |  |
| 10 | 74 | 113864.945 | 57883.16419 | 1339949.692 |  |
| 11 | 75 | 118419.5428 | 56260.27173 | 1283689.42 |  |
| 12 | 76 | 123156.3245 | 54682.88094 | 1229006.539 |  |
| 13 | 77 | 128082.5775 | 53149.71605 | 1175856.823 |  |
| 14 | 78 | 133205.8806 | 51659.5371 | 1124197.286 |  |
| 15 | 79 | 138534.1158 | 50211.13886 | 1073986.147 |  |
| 16 | 80 | 144075.4804 | 48803.34992 | 1025182.797 |  |
| 17 | 81 | 149838.4997 | 47435.0317 | 977747.7655 |  |
| 18 | 82 | 155832.0396 | 46105.07754 | 931642.688 |  |
| 19 | 83 | 162065.3212 | 44812.41181 | 886830.2762 |  |
| 20 | 84 | 168547.9341 | 43555.98905 | 843274.2871 |  |
| 21 | 85 | 175289.8514 | 42334.7931 | 800939.494 |  |
| 22 | 86 | 182301.4455 | 41147.83628 | 759791.6577 |  |
| 23 | 87 | 189593.5033 | 39994.15863 | 719797.4991 |  |
| 24 | 88 | 197177.2435 | 38872.82708 | 680924.672 |  |
| 25 | 89 | 205064.3332 | 37782.93473 | 643141.7373 |  |
| 26 | 90 | 213266.9065 | 36723.60011 | 606418.1372 |  |
| 27 | 91 | 221797.5828 | 35693.96646 | 570724.1707 |  |
| 28 | 92 | 230669.4861 | 34693.20105 | 536030.9697 |  |
| 29 | 93 | 239896.2655 | 33720.49448 | 502310.4752 |  |
| 30 | 94 | 249492.1162 | 32775.06005 | 469535.4151 |  |
| 31 | 95 | 259471.8008 | 31856.13313 | 437679.282 |  |
| 32 | 96 | 269850.6728 | 30962.97052 | 406716.3115 |  |
| 33 | 97 | 280644.6997 | 30094.84985 | 376621.4616 |  |
| 34 | 98 | 291870.4877 | 29251.06902 | 347370.3926 |  |
| 35 | 99 | 303545.3072 | 28430.94559 | 318939.447 |  |
| 36 | 100 | 315687.1195 | 27633.81627 | 291305.6308 |  |
| 37 | 101 | 328314.6043 | 26859.03638 | 264446.5944 |  |
| 38 | 102 | 341447.1885 | 26105.97928 | 238340.6151 |  |
| 39 | 103 | 355105.076 | 25374.03594 | 212966.5792 |  |
| 40 | 104 | 369309.2791 | 24662.61437 | 188303.9648 |  |
| 41 | 105 | 384081.6502 | 23971.1392 | 164332.8256 |  |
| 42 | 106 | 399444.9162 | 23299.05118 | 141033.7744 |  |
| 43 | 107 | 415422.7129 | 22645.80676 | 118387.9676 |  |
| 44 | 108 | 432039.6214 | 22010.8776 | 96377.09005 |  |
| 45 | 109 | 449321.2063 | 21393.75019 | 74983.33986 |  |
| 46 | 110 | 467294.0545 | 20793.92542 | 54189.41444 |  |
| 47 | 111 | 485985.8167 | 20210.91816 | 33978.49628 |  |
| 48 | 112 | 505425.2494 | 19644.25691 | 14334.23938 |  |
| 49 | 113 | 525642.2593 | 14334.48335 | 0 |  |
|  |  |  |  |  |  |
| The Number of Years the annuity will make payments is **49 years** | | |  |  |  |
|  |  |  |  |  |  |
| **For B** |  |  |  |  |  |
| Total Amount received from the annuity | | **11666698.74** | or 11,666,699 **(Approximate Value)** |  |  |
|  |  |  |  |  |  |
| **For C** |  |  |  |  |  |
| **# of Years** | **Age** | **Payments** | **Present Value (PV) of the Current Payment** | **The Investment Left from the Annuity** |  |
| 1 | 65 | 80000 | 74766.35514 | 1,925,234 |  |
| 2 | 66 | 83200 | 72670.10219 | 1,852,564 |  |
| 3 | 67 | 86528 | 70632.62269 | 1,781,931 |  |
| 4 | 68 | 89989.12 | 68652.26878 | 1,713,279 |  |
| 5 | 69 | 93588.6848 | 66727.43882 | 1,646,551 |  |
| 6 | 70 | 97332.23219 | 64856.57605 | 1,581,695 |  |
| 7 | 71 | 101225.5215 | 63038.16737 | 1,518,656 |  |
| 8 | 72 | 105274.5423 | 61270.74212 | 1,457,386 |  |
| 9 | 73 | 109485.524 | 59552.87085 | 1,397,833 |  |
| 10 | 74 | 113864.945 | 57883.16419 | 1,339,950 |  |
| 11 | 75 | 618419.5428 | 293806.6699 | 1,046,143 |  |
| 12 | 76 | 123156.3245 | 54682.88094 | 991,460 |  |
| 13 | 77 | 128082.5775 | 53149.71605 | 938,310 |  |
| 14 | 78 | 133205.8806 | 51659.5371 | 886,651 |  |
| 15 | 79 | 138534.1158 | 50211.13886 | 836,440 |  |
| 16 | 80 | 144075.4804 | 48803.34992 | 787,636 |  |
| 17 | 81 | 149838.4997 | 47435.0317 | 740,201 |  |
| 18 | 82 | 155832.0396 | 46105.07754 | 694,096 |  |
| 19 | 83 | 162065.3212 | 44812.41181 | 649,284 |  |
| 20 | 84 | 168547.9341 | 43555.98905 | 605,728 |  |
| 21 | 85 | 175289.8514 | 42334.7931 | 563,393 |  |
| 22 | 86 | 182301.4455 | 41147.83628 | 522,245 |  |
| 23 | 87 | 189593.5033 | 39994.15863 | 482,251 |  |
| 24 | 88 | 197177.2435 | 38872.82708 | 443,378 |  |
| 25 | 89 | 205064.3332 | 37782.93473 | 405,595 |  |
| 26 | 90 | 213266.9065 | 36723.60011 | 368,872 |  |
| 27 | 91 | 221797.5828 | 35693.96646 | 333,178 |  |
| 28 | 92 | 230669.4861 | 34693.20105 | 298,485 |  |
| 29 | 93 | 239896.2655 | 33720.49448 | 264,764 |  |
| 30 | 94 | 249492.1162 | 32775.06005 | 231,989 |  |
| 31 | 95 | 259471.8008 | 31856.13313 | 200,133 |  |
| 32 | 96 | 269850.6728 | 30962.97052 | 169,170 |  |
| 33 | 97 | 280644.6997 | 30094.84985 | 139,075 |  |
| 34 | 98 | 291870.4877 | 29251.06902 | 109,824 |  |
| 35 | 99 | 303545.3072 | 28430.94559 | 81,393 |  |
| 36 | 100 | 315687.1195 | 27633.81627 | 53,759 |  |
| 37 | 101 | 328314.6043 | 26859.03638 | 26,900 |  |
| 38 | 102 | 341447.1885 | 26105.97928 | 794 |  |
| 39 | 103 | 355105.076 | 794.0359373 | 0 |  |
|  |  |  |  |  |  |
| Total Amount received from the annuity | | **7732731.977** | or 7,732,732 **(Approximate Value)** |  |  |
|  |  |  |  |  |  |

|  |  |  |
| --- | --- | --- |
| **Given:**  Investment (IN) | 2,000,000 |  |
| First Payment (p1) | 80000 |  |
| i | 0.07 |  |
| r | 0.07 | 7% |
| Growth Rate (g) | 0.04 | 4% |

*”hiraya manawari - sana matupad puhon - balang araw”*