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DYNAMIC SYSTEM MODELING AND FAULT DETECTION WITH PROBABILISTIC FINITE STATE AUTOMATA

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Dissertação submetida ao Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal de Pernambuco como parte dos requisitos para obtenção do grau de **Mestre em Engenharia Elétrica**.

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2017

To my grandmother Zélia

To my parents and sister for all the support they gave me.

To my advisers for all the guidance provided.

RESUMO

TODO

Palavras-chaves: Probabilistic finite state automata, dynamic systems, system modeling, fault detection, conditional entropy, Kullback-Leibler divergence.

ABSTRACT

TODO

Keywords: Probabilistic finite state automata, dynamic systems, system modeling, fault detection, conditional entropy, Kullback-Leibler divergence.

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CAPÍTULO 2

REVISION

THIS chapter revises the concepts needed to develop the algorithms and their application.

2.1 Sequences of Discrete Symbols

This section provides tools to describe sequences of discrete symbols. The length of a sequence u is denoted by $|u|$. The empty sequence ϵ is defined as the sequence with length 0. The set of all possible symbols for a sequence is called its alphabet, represented as Σ. The set of all possible sequences of n, $n\ in\mathbb{Z}$ symbols from Σ is Σ^n and the set of all sequences of symbols from Σ with all possible lengths, including the empty sequence sequence ϵ, is Σ^*.

Two sequences u and $v \in \Sigma^*$ can be concatenated to form a sequence uv. For example, using a binary alphabet $\Sigma = \{0, 1\}$ and u = 1010 and v = 111, they can be concatenated to form uv = 1010111.

Note that $|uv| = |u| + |v|$. Concatenation is associative, which means $u(vw) = (uv)w = uvw$, but it is not commutative, as uv is not necessarily equal to vu. This means that Σ^* with the operation of concatenation is a Monoid, as it is a set with an associative operation with an identity element (the empty string).

A sequence $v \in \Sigma^*$ is called a suffix of a sequence $w \in \Sigma^*$ (given $|w| > |v|$) if w can be written as a concatenation uv, where $u \in \Sigma^*$, that is $w = uv$. In this same sense, the sequence u is called a prefix of w.

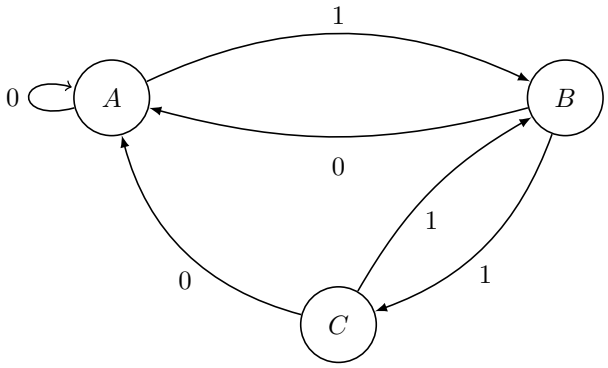

Figura 2.1: *Example of a graph with Q = {A, B, C} and* $\Sigma = \{0, 1\}$.

2.2 Graphs

Definition 2.1 – Graph

A graph G over the alphabet Σ *consists of a triple (Q,* Σ*,* δ*):*

- ▷ *Q is a finite set of states with cardinality* $|Q|$*;*
- ▷ Σ *is a finite alphabet with cardinality* $|\Sigma|$*;*
- ▷ δ *is the state transition function* $Q \times \Sigma \rightarrow Q$*;* □

Each state q from Q has at most $|\Sigma|$ outgoing edges. Each outgoing edge from a state is labeled with a unique symbol from Σ and it arrives at only one state $q' \in Q$. This behavior is described by the function $\delta : Q \times \Sigma \rightarrow Q$, which is the transition function. For example, leaving state q with the edge labeled with a and arriving at state q' is represented by $\delta(q, 0) = q'$. Figure 2.1 shows an example of a graph over a binary alphabet.

A graph G can also be represented as a triple (Q, E, L) where Q is the set of states, E is the set of edges connecting the states and L is the set of edges' labels. A walk in a graph G is the sequence of labels $l \in L$ formed by starting at a state $q \in Q$ and a string s that starts as the empty string and going to a next state connected to it by a vertex and appending the vertex's label to s. This process can be repeated and when it stops, s defines a walk over G starting at q. Calling the graph of Figure 2.1 as G, the following is an example of a walk: start at state A and go to A, A, B, C, B, A. This forms the string $s = 001110$.

Definition 2.2 – Follower Set

The follower set of the graph G rooted at state $q \in Q$ *is defined as the set of all possible walks that can be formed by starting at state q. That is:* □

$$F(q) = \{\omega \in \Sigma^* | q \cdot \omega \in Q\},$$

where $q \cdot \omega$ denotes the state arrived after starting at q and following the path determined by the word ω.

Definition 2.3 – Language of a Graph

The language $\mathcal{L}$ of a graph G is the the set of follower sets for each state $q \in Q$: □

$$\mathcal{L} = \{F(q), \forall q \in Q\}.$$

A word *w* is called a synchronizing word of *G* if all walks in *G* that generate *w* terminate at the same state and *G* is called *synchronizing* if there exists a synchronization word for every state $q \in Q$.

2.3 Graph Minimization

This section explains the two most widely used algorithms for automata minimization: Moore and Hopcroft.

Definition 2.4 – Partitions and Equivalence Relations

Given a set E, a partition of E is a family $\mathcal{P}$ of nonempty, pairwise disjoint subsets of E such that $\bigcup_{P \in \mathcal{P}} P = E$. The index of the partition is its number of elements. The partition defines an equivalence relation on E and the set of all equivalence classes [x], $x \in E$, of an equivalence relation in E defines a partition of the set. □

When a subset *F* of *E* is the union of classes of $\mathcal{P}$ it said that *F* is saturated by $\mathcal{P}$. Given $\mathcal{Q}$, another partition of *E*, it said to be a *refinement* of $\mathcal{P}$ (or that $\mathcal{P}$ is coarser than $\mathcal{Q}$) if every class of $\mathcal{Q}$ is contained by some class of $\mathcal{P}$ and it is written as $\mathcal{Q} \leq \mathcal{P}$. The index of $\mathcal{Q}$ is greater than the index of $\mathcal{P}$.

Given partitions $\mathcal{P}$ and $\mathcal{Q}$ of $E, \mathcal{U} = \mathcal{P} \wedge \mathcal{Q}$ denotes the coarsest partition which refines $\mathcal{P}$ and $\mathcal{Q}$. The elements of $\mathcal{U}$ are non-empty sets $P \cap Q$, $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$. The notation is extended for multiple sets as $\mathcal{U} = \mathcal{P}_1 \wedge \mathcal{P}_2 \wedge \ldots \wedge \mathcal{P}_n$. When $n = 0$, $\mathcal{P}$ is the universal partition comprised of just *E* and it is the neutral element for the $\wedge$-operation.

Given $F \subseteq E$, a partition $\mathcal{P}$ of *E* induces a partition $\mathcal{P}'$ of *F* by intersection. $\mathcal{P}'$ is composed by the sets $P \cap F$ with $P \subseteq \mathcal{P}$. If $\mathcal{P}$ and $\mathcal{Q}$ are partitions of *E* and $\mathcal{Q} \leq \mathcal{P}$, the restrictions $\mathcal{P}'$ and $\mathcal{Q}'$ to *F* maintain $\mathcal{Q}' \leq \mathcal{P}'$.

Given partitions $\mathcal{P}$ and $\mathcal{P}'$ of disjoint sets *E* and *E'*, the partition of set $E \cup E'$ whose restriction to *E* and *E'* are $\mathcal{P}$ and $\mathcal{P}'$ is denoted by $\mathcal{P} \vee \mathcal{P}'$. It is possible to write $\mathcal{P} = \vee_{P \vee \mathcal{P}} \{P\}$.

Definition 2.5 – Irreducible Graph

A graph G is said to be irreducible if all its states have distinct follower sets (from Definition 2.2), that is F(p) ≠ F(q) for each pair of distinct states p, q ∈ Q. □

From Definition 2.5 it is possible to define an equivalence relation called the Nerode equivalence:

$$p, q \in Q, p \equiv q \Leftrightarrow F(p) = F(q).$$

A graph is considered minimal if and only if its Nerode equivalence is the identity. The problem of minimizing a graph is that of computing the Nerode equivalence. The quotient graph *G/*$\equiv$ obtained by taking for *Q* the set of Nerode equivalence classes. The minimal graph is unique and it accepts the same language as the original graph.

Given a set of states $P \subset Q$ and a symbol $\sigma \in \Sigma$, let $\sigma^{-1}P$ denote the set of states q such that $\delta(q, \sigma) \in P$. Consider *P*, *R* $\subset$ *Q* and $\sigma \in \Sigma$, the partition of R

$$(P, \sigma)|R$$

the partition composed of two non-empty subsets:

$$R \cap \sigma^{-1}P = \{r \in R | \delta(r, \sigma) \in P\}$$

and

$$R \backslash \sigma^{-1}P = \{r \in R | \delta(r, \sigma) \notin P\}.$$

The pair (*P*, σ) is called a splitter. Observe that (*P*, σ)|R = R if either $\delta(R, \sigma) \subset P$ or $\delta(R, \sigma) \cap P = \emptyset$ and (*P*, σ)|R is composed of two classes if both $\delta(R, \sigma) \cap P \neq \emptyset$ and $\delta(R, \sigma) \cap P^c \neq \emptyset$ or equivalently if $\delta(R, \sigma) \not\subset P$ and $\delta(R, \sigma) \not\subset P^c$. If (*P*, σ)|R contains two classes, then we say that (*P*, σ) splits R. This notation can also be extended to sequences, using a sequence $\omega \in \Sigma^*$ instead of the symbol $\sigma \in \Sigma$.

Proposition 2.1

The partition corresponding to the Nerode equivalence is the coarsest partition $\mathcal{P}$ such that no splitter (P,σ), with P ∈ $\mathcal{P}$ and σ ∈ Σ, splits a class in $\mathcal{P}$, that is such that (P,σ)|R = R for all P, R ∈ $\mathcal{P}$ and σ ∈ Σ. □

Lemma 2.1

Let P be a set of states and $\mathcal{P} = P_1, P_2$ *a partition of P. For any symbol* σ *and for any set of states R, one has:* □

$$(P,\sigma)|R \wedge (P_1,\sigma)|R = (P,\sigma)|R \wedge (P_2,\sigma)|R = (P_1,\sigma)|R \wedge (P_2,\sigma)|R,$$

and consequently

$$(P,\sigma)|R \geqslant (P_1,\sigma)|R \wedge (P_2,\sigma)|R,$$

$$(P_1,\sigma)|R \geqslant (P,\sigma)|R \wedge (P_2,\sigma)|R.$$

2.3.1 Moore's Algorithm

Algorithm 1 Moore(G)

```
1: P ← InitialPartition(G)
2: repeat
3:     P' ← P
4:     for all σ ∈ Σ do
5:         P_σ ← ⋀_{P∈P} (P,σ)|Q
6:     P ← P ∧ ⋀_{σ∈Σ} P_σ
7: until P = P'
```

2.3.2 Hopcroft's Algorithm

Algorithm 2 Hopcroft(G)

```
1: P ← InitialPartition(G)
2: W ← ∅
3: for all σ ∈ Σ do
4:     Append((min(F, F^c,σ),W)
5:     while W ≠ ∅ do
6:         (W,σ) ← TakeSome(W)
7:         for each P∈ P which is split by (W,σ) do
8:             P', P'' ← (W,σ)|P Replace P by P' and P'' in P
9:             for all τ ∈ Σ do
10:                if (P,τ) ∈ W then
11:                    Replace (P,τ) by (P',τ) and (P",τ) in W
12:                else
13:                    Append((min(P', P",τ),W)
```

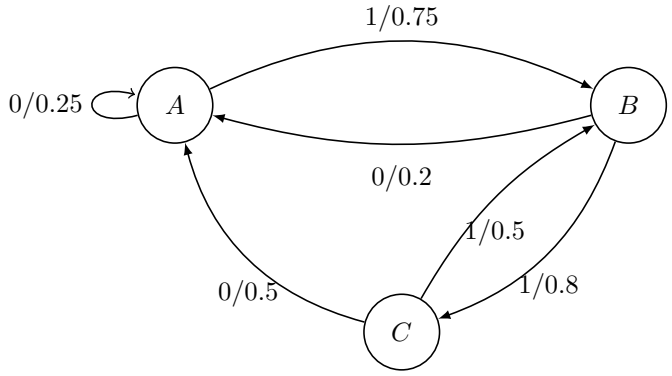

Figura 2.2: *A probabilistic version of the graph of Figure 2.1.*

2.4 Probabilistic Finite State Automata

Definition 2.6 – Probabilistic Finite State Automata

A Probabilistic Finite State Automaton (PFSA) P is defined as a quadruple $(Q, \Sigma, \delta, \mathcal{V})$. *The first three items are the same as of a graph as defined in Definition 2.1, while* $\mathcal{V}$ *is a probability function,* $\mathcal{V}$*:* $\delta \rightarrow [0, 1)$ *which associates a probability to each edge.* □

The function $\mathcal{V}$ gives the probabilistic factor to the PFSA. It is the associated probability distribution for the outgoing edges of each state. This means that for each state $q \in Q$, there will be a probability $\mathcal{V}(\delta(q, \sigma)), \forall\sigma \in \Sigma$ associated to each edge in such a way that $\Sigma_\sigma \mathcal{V}(\delta(q, \sigma)) = 1$ and $0 \leq \mathcal{V}(\delta(q, \sigma)) \leq 1$. The probability associated to an edge is the probability of taking this path once the system is in the state q.

Definition 2.7 – Morph

The probability distribution $\mathcal{V}(q) = \{\mathcal{V}(\delta(q, \sigma)); \forall\sigma \in \Sigma\}$ *of a state is called the state morph.* □

A PFSA can be represented with a graph, in which each $q \in Q$ is represented as a node. The edges are given by function $\delta(q, a) = q'$, indicating there is an edge going from q to q' labeled with the symbol a. The probability associated with an edge, $\mathcal{V}(\delta(q, a))$ is also in the edge label.

Definition 2.8 – Deterministic PFSA

A PFSA is called deterministic if the outgoing edges of each state are labeled with distinct symbols. □

An example of a graph of a PFSA is shown in Figure 2.2, for which $Q = \{A, B, C\}$, $\Sigma = \{0, 1\}$ and the functions δ and $\mathcal{V}$ are represented in the edges of the graph.

A sequence can be generated by a PFSA as the sequence formed by starting at a given state $q \in Q$ then following a path with its edges and concatenating the labels for each edge. Its probability is given by multiplying the probability of each edge that was taken.

From Figure 2.2, starting at the state *A*, it is possible to form the sequence *u = 1011001* by taking a path going to states *B,A,B,C,A,A* and *B* and concatenating the labels of the path from each of these transitions. By multiplying the probabilities of these edges, it is seen that p(*u*) = $0.75 \times 0.2 \times 0.75 \times 0.8 \times 0.5 \times 0.25 \times 0.75 = 0.0084375$.

It is useful do adapt the concept of synchronization word to the context of PFSA:

Definition 2.9 – PFSA Synchronization Word

For a state $q \in Q$, w is a synchronization word if, $\forall u \in \Sigma^*$ and $\forall v \in \Sigma^*$:

$$\Pr(u|w) = \Pr(u|vw). \tag{2.1}$$

□

Definition 2.9 means that the probability of obtaining any sequence after the synchronization word does not depend on whatever came before *w*. The main problem with this definition is the fact that is not possible to check (2.1) for all $u \in \Sigma^*$ and for all $v \in \Sigma^*$ as there are an infinite number of sequences.

A solution uses the *d*-th order derived frequency, which is the probability using *u* and *v* from Σ^d, $d \in \mathbb{Z}$, instead of taking them from Σ^*. Calling $\Pr_d(\omega)$ the d-th order derived frequency of ω, a statistical test (such as the Chi-Squared or Kolmogorov-Smirnov) with significance level α has to be performed with the following null hypothesis for *w* being a synchronization word:

$$\Pr_d(w) = \Pr_d(uw), \forall u \in \cup_{i=1}^{L_1} \Sigma^i, \forall d = 1, 2, \ldots, L_2, \tag{2.2}$$

where L_1 and L_2 are precision parameters. This means that the statistical test compares the probabilities of words *w* with length from 0 to L_2 with the probabilities of words *uw*, where *u* is a prefix of *w* with lengths from 0 to L_1. This limits the number of tests to be realized.

A synchronization words is a good starting point to model a system from its output sequence because the probability of its occurrence does not depend on what came before it. Therefore its prefix can be regarded as a transient.

2.5 Consolidated Algorithms

2.5.1 D-Markov Machines

2.5.2 CRiSSiS

CAPÍTULO 3

ALGORITHMS DESCRIPTIONS

NESTE capítulo propomos técnicas para o projeto de RNG a partir de mapas caóticos e introduzimos técnicas de discretização codificada variante no tempo. Apesar dos métodos considerado serem independente do mapa caótico, empregamos para o estudo de caos o mapa tanh. Para testar a aleatoriedade das sequências geradas empregamos a entropia condicional e o teste NIST.

3.1 A New Algorithm for Finding Synchronization Words

Given a sequence X of length L over an alphabet Σ which is the output of a dynamical system, the proposed algorithm is an alternative to find possible synchronization words in X. The typical method would be using Equation 2.2 and testing for all possible values. The proposed algorithm will use data structures in order to speed up the process.

The algorithm uses two rooted tree with probabilities to search for synchronization words. The main tree $\mathcal{T}$ represents the statistics of sub-sequences of the original sequence X and the auxiliary tree $\mathcal{S}$ is used to search the suffixes of those sub-sequences and keep track of their status as valid candidates for synchronization words. The auxiliary tree regulates how the main tree is explored while the search is performed. Once the expansion reaches its end, a list of the most likely synchronization words is returned.

A Rooted Tree with Probabilities $\mathcal{T}$ over $\Sigma = \{0, 1\}$ is presented via an example in figure 3.1. $\mathcal{T}$ consists of a set of branching nodes $\mathcal{B}$ and a set of leaf nodes $\mathcal{L}$. All nodes have exactly one predecessor (with the exception of the root node, which has no predecessors). Leaf nodes have no successors, while each branching node has $|\Sigma|$ successors as each element of Σ labels one of the

outgoing branches. Those branches are also labeled with the probability of leaving the node with that symbol. Each node is labeled with the string formed from concatenating the symbols in the branches in the path from the root to the current node. The root node is labeled with the empty string ϵ. The probability of reaching a node is given by multiplying the probabilities labeling the branches in the path from the root node to the current node.

For example, the leaf node *10* is labeled as so because to reach it, the path taken from the root node ϵ is first *1* and then *0*. The probability of reaching this node is $P(1) \times P(0|1)$, that is the probability of leaving the root node with 1 (which is $P(1)$) multiplied by the probability of leaving the node 1 with 0 (that is, $P(0|1)$.

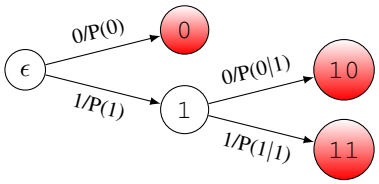

Figura 3.1: *Example of a rooted tree with probabilities.*

The two rooted tree with probabilities used in the algorithm are $\mathcal{T}$ and $\mathcal{S}$. The primary tree $\mathcal{T}$ has branch probabilities taken from the conditional probabilities from the sequence. The auxiliary tree $\mathcal{S}$ is constructed from $\mathcal{T}$. Each node has a label that is the inverse from the node label in $\mathcal{T}$, but keeping the same branch probabilities. This is done in order to use the nodes in $\mathcal{S}$ to verify the suffixes of nodes in $\mathcal{T}$ as it is explained in an example.

From these two trees, two dynamic lists are created. The list Δ is initialized with the root node from $\mathcal{T}$ and all of its children nodes. The elements in the second list Γ are triples *(s, candidacy, tested)*. *s* is a state from $\mathcal{S}$ and the other two elements are binary flags. The *candidacy flag* is checked true if the state *s* is a valid candidate for synchronization word. The *test flag* is checked true if *s* have been through all the statistical tests to check its candidacy for synchronization word status.

Definition 3.1

A state s from a rooted tree with probabilities $\mathcal{S}$ is called a valid state if the triple (s, candidacy, tested) in list Γ has the candidacy flag set to true and the tested flag set to false. □

From the this definition, a state is called valid when it is a valid candidate for a synchronization word, but its status is yet to be tested. Whenever a new element is added to Γ it is added as a valid state. Γ is initialized with the triple *(ϵ, True, False)*, where ϵ is the root of $\mathcal{S}$ and, as stated before, it is initialized as a valid state. The function *nextValidState* returns the valid state in Γ with the shortest

label.

The algorithm receives the parameter *W* which indicates the maximum length of subsequences it will take into consideration, i.e. the depth of the tree $\mathcal{T}$. The depth of $\mathcal{S}$ is $W - 1$.

Figures 3.2 and 3.3 are used as examples. They are both taken over binary alphabet and with branch probabilities taken from a computer generated sequence. *W* is taken to be equal to 3. $\mathcal{S}$ is constructed from $\mathcal{T}$ by taking the labels from each state and reversing them, while keeping the branch probabilities. To show how this is useful to find suffixes, take the node *110* from $\mathcal{T}$, reverse it (obtaining *011*) and walk through $\mathcal{S}$. Starting at ϵ, the walk goes through *0* and then *01*. Reverting each visited state, it is seen that ϵ, *0* and *10* are suffixes of *110*, as it was expected.

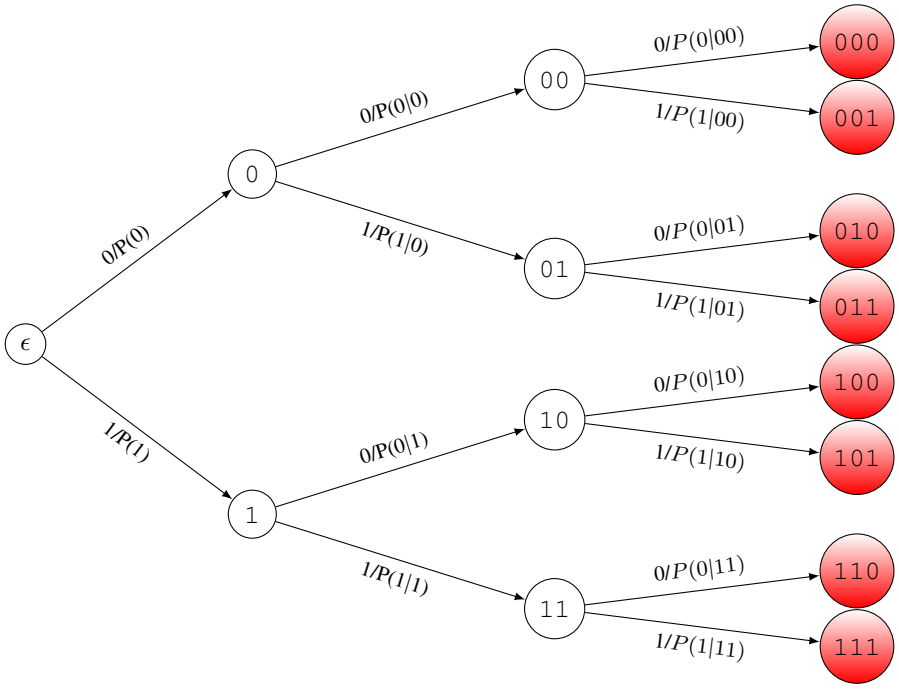

Figura 3.2: *Example of binary* $\mathcal{T}$

When two morphs are compared as in $\mathcal{V}(q) = \mathcal{V}(p)$ it means that both of these distributions are being compared via an appropriate statistical test and this operation returns a true or false whether the test was successful or not for a predetermined confidence level α.

To find the synchronization words, algorithm 3 is used. It receives as parameters both trees and *W*. The lists Γ and Δ are then created as described above. Besides that, the list Ω_{syn}, which will receive the results from the algorithm is initialized as an empty list. Another list Θ is created to store triples *(t, s, result)*, where *t* is a state from $\mathcal{T}$, *s* is a state from $\mathcal{S}$ and *result* is the result from the statistical test of comparing the morphs of *t* and *s*. Whenever a statistical test needs to be performed,

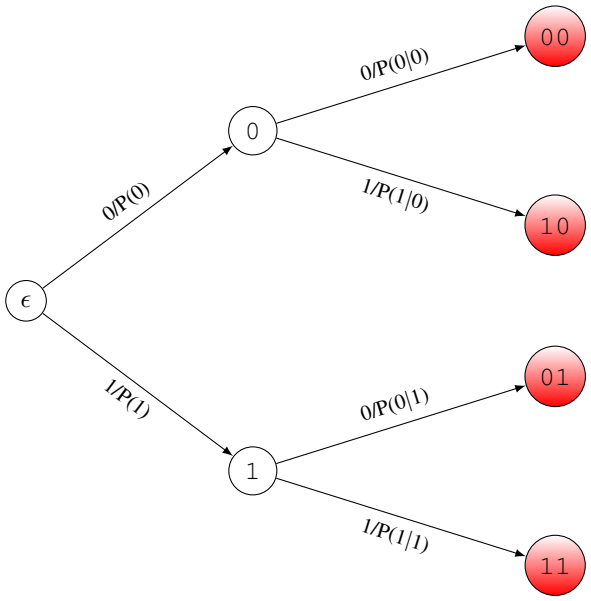

Figura 3.3: *Example of binary $\mathcal{S}$*

the result is first checked in this list. If the test has been performed in a given pair of states before, it saves time not to do it again. Θ is initialized empty.

At the start of each iteration, the function *nextValidState* is applied to Γ and returns the shortest valid candidate for synchronization word c. If no valid states are found, Ω_{syn} will receive the labels of all elements in Γ for which the candidacy flags are set to true. Given the tuple $\omega = (x_1, x_2, \ldots, x_t)$, the notation $\omega.x_i$ is used to denote the element x_i from ω. In other words, Ω_{syn} receives all elements γ from Γ for which $\gamma.candidacy$ is true. This list is then returned. As there are no valid states, this means that all possible states have been tested and the states which still have the candidacy flag set to true are the synchronization words to be returned.

If there is a valid state c, a new list Ψ is created empty. It is used to store states from $\mathcal{T}$ for which the suffix was already found. This is done in order to save time. If the suffix was found the algorithm will not search for it again. If the label of c is shorter than W, another list Λ is created with all the elements from Δ whose lengths are less than the length of c. They also cannot be present in the Ψ list nor the pair *(c.label, s)* can be present in the list Θ. These restrictions prevent the algorithm from doing redundant work of finding suffixes which were already found or repeating tests for which the results are known.

Each element λ of Λ is then compared to c. First, it is checked if *c.label* is a suffix of λ. In the affirmative case, λ is added to Ψ. Then the label of c and λ are compared via a statistical test.

If the test result is positive, the current candidate keeps its candidacy status and the algorithm

keeps iterating. After c is tested against all states from Λ for which it is a suffix, the candidate is marked as tested.

On the other hand, if the test fails, Γ and Δ are expanded via algorithm 4 and all states from Γ are marked as untested as they will have to go through a new phase of testing with the new states that were added to the lists Γ and Δ from the expansion.

This is a way of implementing Equation 2.2. Each word is tested against all its possible suffixes in each iteration and if it still keeps the candidacy flag set to true, it is a synchronization word. The tree structure improves on the brute force method as no unnecessary tests will be performed for longer words if a shorter one is still valid.

Algorithm 4 will properly expand Γ and Δ after a statistical test fails. First, the candidate for which the test failed is expanded in Δ: its children are stored in a list called Φ and Δ is updated appending these new states. As the nodes in $\mathcal{S}$ have inverted labels in regard to $\mathcal{T}$, the components of Φ have their names inverted and stored in Υ. For each element υ in Υ, its shortest valid suffix in $\mathcal{S}$ is found and stored in η. To find the shortest valid suffix, the procedure described before to find a suffix is used and it stops when a state in $\mathcal{S}$ is found for which its *candidacy* flag in Γ is true. If the label of υ and η are the same state, the list Π receives all of the children of υ. After repeating this process for all elements in Υ, the updated Π is appended to the list Δ.

When Algorithm 4 is called, the lists of tree elements are updated with the new nodes that will be needed for synchronization worda analysis.

Algorithm 3 findSynchWords($W, \mathcal{S}, \mathcal{T}$)

procedure INITIALIZATION
  $\Gamma \leftarrow \{(\epsilon, True, False)\}$
  $\Delta \leftarrow \{\epsilon, \delta(\epsilon_T, \sigma)\ \forall \sigma \in \Sigma\}$
  $\Omega_{syn} \leftarrow \{\emptyset\}$
  $\Theta \leftarrow \{\emptyset\}$
  $\Psi \leftarrow \{\emptyset\}$
procedure MAINLOOP
  $c \leftarrow nextValidState(\Gamma)$
  if $\nexists$ valid states in Γ **then**
    $\Omega_{syn} \leftarrow \{\gamma \in \Gamma : \gamma.candidacy = True\}$
    return Ω_{syn}
  else
    $l \leftarrow length(c.label)$
    if $l < W$ **then**
      $\Lambda \leftarrow \{s \in \Delta : length(s) > l, s \notin \Psi, (c.label, s) \notin \Theta\}$
      for each $\lambda \in \Lambda$ **do**
        if *c.label* is a suffix of λ **then**
          $\Psi \leftarrow \Psi \cup \lambda$
          $p \leftarrow \mathcal{V}(c(label)) = \mathcal{V}(h)$
          $\Theta \leftarrow \Theta \cup (c.label, \lambda, p)$
          if $p = False$ **then**
            c.candidacy = False
            $expandTrees(c.label, \mathcal{S}, \mathcal{T}, \Gamma, \Delta, \Sigma)$
            $\gamma.tested = False\ \forall \gamma \in \Gamma$
            $\Psi \leftarrow \{\emptyset\}$
            break
          else
            if all elements in Λ were tested **then**
              $c.tested = True$
    goto *MAINLOOP*.

Algorithm 4 expandTrees($c, \mathcal{S}, \mathcal{T}, \Gamma, \Delta, \Sigma$)

procedure EXPAND TREES
  $\Phi \leftarrow \{(\delta(c, \sigma), True, False),\ \forall \sigma \in \Sigma\}$
  $\Gamma \leftarrow \Gamma \cup \Phi$
  $\Upsilon \leftarrow \{invert(\phi.label), \forall \phi \in \Phi\}$
  $\Pi \leftarrow \{\emptyset\}$
  for each υ in Υ **do**
    $\eta \leftarrow$ shortest valid suffix of υ in $\mathcal{S}$
    if $\eta = \upsilon$ **then**
      $\Pi \leftarrow \Pi \cup \{\delta(\upsilon, \sigma)\ \forall \sigma \in \Sigma\}$
  $\Delta \leftarrow \Delta \cup \Pi$
  return

Capítulo 4

TODO

TODO

Capítulo 5

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Capítulo 6

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Apêndice A

TODO

TODO

Apêndice B

TODO

SOBRE O AUTOR

The author was born in Brasília, Brasil, on the 6^{th} of August of 1991. He graduated in Electronic Engineering in the Federal University of Technology of Paraná (UTFPR) in Curitiba, Brazil, in 2014. His research interests include Information Theory, Error Correcting Codes, Data Science, Cryptography, Digital Communications and Digital Signal Processing.

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Esta dissertação foi diagramada usando LaTeX 2_ε[1] pelo autor.

[1] LaTeX 2_ε é uma extensão do LaTeX. LaTeX é uma coleção de macros criadas por Leslie Lamport para o sistema TeX, que foi desenvolvido por Donald E. Knuth. TeX é uma marca registrada da Sociedade Americana de Matemática ($\mathcal{AMS}$). O estilo usado na formatação desta dissertação foi escrito por Dinesh Das, Universidade do Texas. Modificado por Renato José de Sobral Cintra (2001) e por Andrei Leite Wanderley (2005), ambos da Universidade Federal de Pernambuco. Sua úlltima modificação ocorreu em 2010 realizada por José Sampaio de Lemos Neto, também da Universidade Federal de Pernambuco.

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