1.4. a) $T(n) = \begin{cases} 1, & n \le a,\ a > 0 \\ T(n-a)+1 & n > a \end{cases}$

$$T(n) = T(n-a) + 1 = T(n-2a) + 2 = T\left(n - \frac{na}{a}\right) + \frac{n}{a} = \frac{n}{a} + 1 = \mathrm{ceil}\left(\frac{n}{a} + 1\right)$$

b) $T(n) = \begin{cases} 1, & n = 0 \\ T(n-1) + 2^n, & n \ge 1 \end{cases}$

$$T(n) = T(n-1) + 2^n = T(n-2) + 2^{n-1} + 2^n = T(n-n) + 2^1 + 2^2 + \dots + 2^{n-1} + 2^n =$$
$$= 1 + (2^{n+1} - 2) = 2^{n+1} - 1$$

c) $T(n) = \begin{cases} 1 & , n = 1 \\ 2T([\frac{n}{2}]) + 1 & n \ge 2 \end{cases}$

$$T(n) = 2T\left(\left[\frac{n}{2}\right]\right) + 1 = 2\left(2T\left(\left[\frac{n}{4}\right]\right) + 1\right) + 1 = 4T\left(\left[\frac{n}{4}\right]\right) + 2 + 1 =$$
$$= 2^m T(2^{m-m}) + \sum_{i=0}^{m-1} 2^i = 2^m + 2^m - 1 =$$
$$= 2^{m+1} - 1 = 2^{\log n} \cdot 2 - 1 = 2n - 1$$

d) $T(n) = \begin{cases} 1 & , n = 1 \\ aT([n/a]) + n & , n \ge 2,\ a \ge 2 \end{cases}$

$$n = a^m \rightarrow m = \log_a n$$

$$T(n) = aT(a^{m-1}) + a^m = a(aT(a^{m-2}) + a^{m-1}) + a^m =$$
$$= a^2 T(a^{m-2}) + a^m + a^m = a^2(aT(a^{m-3}) + a^{m-2}) + a^m + a^m =$$
$$= a^3 T(a^{m-3}) + a^m + a^m + a^m = a^m T(a^{m-m}) + m a^m =$$
$$= n + \log_a n \cdot an = n + n \log_a n$$