2.7 $f(n) = 3n^2 - n + 4$

$g(n) = 4\log n + 5$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n\log n + 5}{3n^2 - n + 4} = \lim_{n\to\infty} \frac{n^2\left(\frac{\log n}{n} + \frac{5}{n^2}\right)}{n^2\left(3 - \frac{1}{n} + \frac{4}{n^2}\right)} = 0$$

$$f(n) + g(n) = O(f) = O(n^2)$$

2.13. (a, d, g, h)

a) $T(n) = \begin{cases} O(1) & n = 0 \\ T(n-1) + O(1) & n \geq 1 \end{cases}$

$$T(n) \leq T(n-1) + C \leq T(n-2) + 2C \leq T(n-n) + nC = O(n)$$

d) $T(n) = \begin{cases} O(1) & n \leq a \\ aT(n-a) + O(1) & n > a \end{cases}, \quad a > 1$

$$T(n) \leq aT(n-a) + C \leq a(aT(n-2a) + C) + C = a^2T(n-2a) + Ca + C$$

$$\leq a^2(aT(n-3a) + C) + Ca + C = a^3T(n-3a) + a^2C + aC + C \leq$$

$$\leq a^{\frac{n}{a}}T\left(n - \frac{na}{a}\right) + C\sum_{i=0}^{\frac{n}{a}-1} a^i \leq a^{\frac{n}{a}}C + \frac{a^{\frac{n}{a}} - 1}{a-1}C$$

$$T(n) = O\left(2^{\frac{n}{a}}\right)$$

g) $T(n) = \begin{cases} O(1) & n = 1 \\ T([n/a]) + O(1) & n \geq 2,\ a \geq 2 \end{cases}$

$$T(n) = aT\left(\left[\frac{n}{a}\right]\right) + C = aT(a^{m-1}) + C \leq a(aT(a^{m-2}) + C) + C =$$

$$= a^2T(a^{m-2}) + aC + C \leq a^2(aT(a^{m-3}) + C) + aC + C =$$

$$= a^3T(a^{m-3}) + a^2C + aC + C \leq a^mT(a^{m-m}) + a\sum_{i=0}^{m-1} a^i =$$

$$= a^mC + C\frac{a^m - 1}{a-1} = nC + \frac{n-1}{a-1}C = O(n)$$

h) $T(n) = \begin{cases} O(1) & n = 1 \\ aT\left(\left[\frac{n}{a}\right]\right) + O(n) & n \geq 2,\ a \geq 2 \end{cases}$

$$n = a^m \Rightarrow m = \log_a n$$

$$T(n) \leq aT(a^{m-1}) + a^m \cdot C \leq a(aT(a^{m-2}) +$$

$$+ a^{m-1} C) + a^m \cdot C = a^2 T(a^{m-2}) + a^m C + a^m C$$

$$a^m T(a^{m-m}) + m a^m \cdot C = nC + n \log_a n \cdot C \leq O(n \log$$