THE CHALLENGE FOR 2025-2026:
DYNAMICS AND CONTROL OF A LUFFING CRANE

D1-3 (DYNAMICS PART)

This document is part of the bachelor course on Dynamics and Control of Mechanical Systems (4DB00).

Information about the challenge

To demonstrate how the theory from this course can be applied in a real-life application, you will work on a challenge that shows the entire process of modelling and control design for a mechanical system. In order to successfully complete this challenge, it is required to complete 6 deliverables that correspond to the 6 modules of this course. A schematic overview of these deliverables is given below.

Deliverable 1: Construct a non-linear model	$\longrightarrow$	*Complete:* `answerform_1.docx` *Deadline:* 26th November at 17:00 *Percentage of grade:* 20%
Deliverable 2: Linearize the model	$\longrightarrow$	*Complete:* `answerform_2.docx` *Deadline:* 3rd December at 17:00 *Percentage of grade:* 15%
Deliverable 3: Create a transfer function	$\longrightarrow$	*Complete:* `answerform_3.docx` *Deadline:* 10th December at 17:00 *Percentage of grade:* 15%
Deliverable 4: Introduce feedback	$\longrightarrow$	*Complete:* `answerform_4.docx` *Deadline:* 17th December at 17:00 *Percentage of grade:* 15%
Deliverable 5: Analyze performance	$\longrightarrow$	*Complete:* `answerform_5.docx` *Deadline:* 7th January at 17:00 *Percentage of grade:* 15%
Deliverable 6: Design a controller	$\longrightarrow$	*Complete:* `answerform_6.docx` *Deadline:* 14th January at 17:00 *Percentage of grade:* 20%

Grading

The grade for the challenge is determined by the following equation:

$$Grade = 0.2D_1 + 0.15D_2 + 0.15D_3 + 0.15D_4 + 0.15D_5 + 0.2D_6, \tag{1}$$

where $0 \leq D_i \leq 10$ indicates the grade received for deliverable i.

Please submit each answer form in PDF format as a group (1 answer form per delivarble per group).

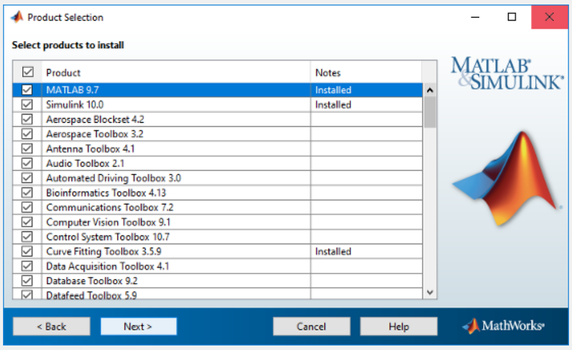
It should be noted that the *copying of work* from other groups is not allowed and will be regarded as fraud. Please be aware of the TU/e Code of Scientific Conduct that can be found at: `https://assets.w3.tue.nl/w/fileadmin/content/universiteit/Over_de_universiteit/integriteit/2019-01-31%20TUe%20Code%20of%20Scientific%20Conduct%20ENG.pdf`.
MATLAB and Simulink will be needed to complete this Challenge so it is important that you have Matlab installed on your laptop with the following toolboxes: Simulink, Symbolic Math, and the Control Systems toolbox.

Download the latest version of Matlab:

Please follow these steps in order to download and install the latest version of Matlab:

Step 1: Connect to the WiFi-network at the TU/e campus, or have an active VPN connection.

Step 2: Follow the steps described on this page:
`https://tue.topdesk.net/tas/public/ssp/content/detail/service?unid=55dd8069030e4a2f861399f0c492dcf0&from=7154af22-93da-4c07-8bf1-bd02a0cd3d79`
When you see the following window, please select the Simscape, Simscape Multibody, Simulink, Symbolic Math, and the Control Systems toolboxes (any number of additional toolboxes can be selected as well):



Getting to know the problem

Heavy duty lifting cranes have been an important piece of technology for the advancement of mankind. From transferring goods between ships and shore, to building high rise buildings; all would not have been possible without the use of cranes. It should therefore be no surprise that the improvement of these cranes can be a profitable venture. For this challenge, you will aim to improve a so called Luffing crane—as shown in Figure 1—which is used in the construction of high-rise buildings.

Figure 1: Luffing cranes are often used in the construction of high-rise buildings.

As can be seen from this figure, a typical crane consist of a basis on which the crane beam is mounted. A so-called “jib” is attached to the crane beam with a rotary joint, such that the angle between these beams can be adjusted. Furthermore, in some crane designs a “hoist block” is introduced, which is able to adjust the “hoist point”, as depicted above, along the jib. Finally, the length of the “chain” can be used to adjust the height of the “load”.

To position the load at a desired location, the crane operator is able to independently control the orientation of the jib, the position of the hoist block and the length of the chain. The operation of such a crane does, however, come with several challenges. For example, the load often starts swaying due to the movements of the crane and due to external forces such as air friction and wind. These effects will often limit the operating conditions and speed at which such a crane can be operated safely (as can be seen here: `https://www.youtube.com/watch?v=pn8fpf6ziTo`).

Challenge:
Several companies have observed that the operation of these cranes can become more time-efficient, if the operator would be able to *directly* control the position of the load. They have asked you to develop a controller that automatically adjusts the orientation of the jib, the position of the hoist block, and the length of the chain in order to reduce load sway. In this way, the operator will only have to focus on the actual position of the load. This type of control solution is currently used in indoor applications and is called sway control: `https://www.youtube.com/watch?v=UddjevEfPvU`.

System Description

To determine whether such a control solution is feasible for luffing cranes that are used outside, it is decided to first investigate a simple version of the problem in two dimensions. Figure 2 shows a schematic drawing of this two dimensional version of the crane, together with the terminology that will be used throughout the challenge.

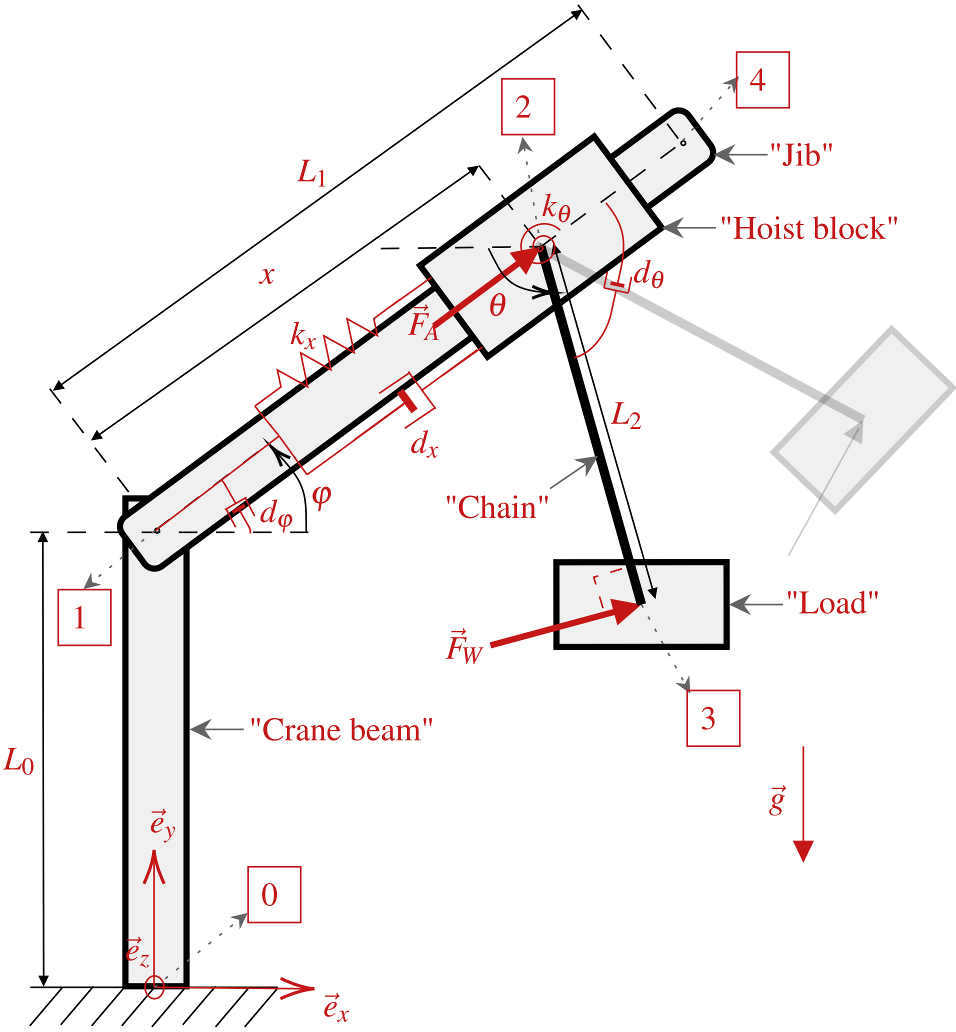


Figure 2: Schematic drawing of crane system.

The objective is to control the x-position of the load, where it is desired to move the load—at a constant velocity—from an initial position x_0 to a desired final position x_f. The position of the load must be controlled by utilizing an actuator that applies a force $\vec{F}_A$ to the hoist block. Figure 3 shows the reference trajectory for the load position.

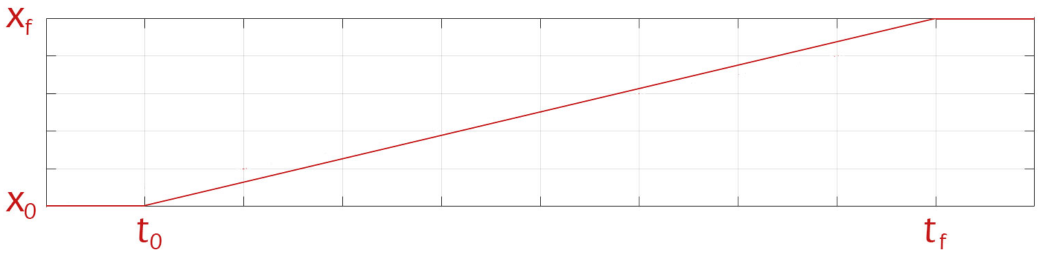


Figure 3: Reference trajectory for the load position.

To model the system, the following definitions and assumptions can be used:

- $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$ & $\boxed{4}$ in Figure 2 are specific points on the crane.
- The origin is defined at point 0 and a right-handed coordinate system is considered.
- Gravity is described by $\vec{g} = -g\vec{e}_y$.
- The crane beam is a rigid, homogeneous and (infinitely) thin rod of length L_0 and mass m_0. Length L_0 is defined as the distance between points 0 and 1.
- The jib is a rigid, homogeneous and (infinitely) thin rod of length L_1 and mass m_1. Length L_1 is defined as the distance between points 1 and 4.
- The chain between the hoist block and load is assumed to be a massless, rigid, homogeneous, and (infinitely) thin rod of length L_2. Note that, thus, the chain is assumed to allow for both tensile and compressive stresses, as opposed to a real chain which only allows for tensile stresses. Length L_2 is defined as the distance between points 2 and 3.
- The hoist block is a point mass of mass m_2 centered in point 2
- The load is a point mass of mass m_3 centered in point 3.
- $0 \leq x \leq L_1$ is the position of the hoist block, which is measured from point 1 to point 2.
- φ is the absolute angle of the jib, which rotates around point 1.
- At the rotational joint at point 1, a rotational damper is positioned with damping constant d_φ.
- θ is the absolute angle of the chain, which rotates around point 2.
- Torsional flexibility at point 2 is modelled as a torsional spring. The spring has a spring stiffness k_θ. The unstreched angular position of the spring is θ_{ref} (this means when $\theta - \varphi = \theta_{\text{ref}}$ (note that the spring is defined between the jib and the chain, not the chain and the horizontal x-axis), the torsional spring will produce no torque).
- At the rotational joint at point 2, a rotational damper is positioned with damping constant d_θ. Note that the corresponding damping moment is proportional to $\dot{\theta} - \dot{\varphi}$.
- The hoist block is fixed to the jib by a spring between points 1 and 2. This spring has a spring constant k_x and an unstretched length L_x.
- A linear damper is positioned between the hoist block and jib with damping constant d_x.
- The force $\vec{F}_A$ describes an external actuator force that acts on the hoist block. The force vector points along the jib towards point 4 and is of magnitude F_A.

- The force $\vec{F}_W$ [N] describes several non-linear friction effects that are caused by swaying of the load. These effects can be linearized to obtain a (damping) force of the form $F_W = -d_W \dot{\theta}$, with damping coefficient d_W.
- The moment of inertia for a rigid, homogeneous and (infinitely) thin rod of length L and mass m is, with respect to its center of mass, given by $J = \frac{1}{12} m L^2$.

Parameter values

Throughout this challenge, the following set of parameter values can be considered, unless stated otherwise (note that the value for the spring constant changes *after* deliverable 2):

Table 1: Parameters and their values.

Parameter	**Symbol**	**Value**
Gravitational acceleration	g	$9.81 \ \frac{\text{m}}{\text{s}^2}$
Length of the crane beam	L_0	30 m
Length of the jib	L_1	25 m
Length of the chain	L_2	10 m
Mass of the crane beam	m_0	$1 \cdot 10^4$ kg
Mass of the jib	m_1	$1 \cdot 10^4$ kg
Mass of the hoist block	m_2	2000 kg
Mass of the load	m_3	500 kg
Damping coefficient between point 1 and 2	d_x	$200 \ \frac{\text{N} \cdot \text{s}}{\text{m}}$
Damping coefficient for the load (for all problems except **D5.f**)	d_W	$2000 \ \frac{\text{N} \cdot \text{s}}{\text{rad}}$
Damping coefficient for the load (for problem **D5.f**)	d_W	$200 \ \frac{\text{N} \cdot \text{s}}{\text{rad}}$
Torsional damper between crane beam and jib	d_φ	$2 \ \frac{\text{N} \cdot \text{m} \cdot \text{s}}{\text{rad}}$
Torsional spring between jib and chain	k_θ	$200 \ \frac{\text{N} \cdot \text{m}}{\text{rad}}$
Rest angle of spring between jib and chain	θ_{ref}	0 rad
Torsional damper between jib and chain	d_θ	$10 \ \frac{\text{N} \cdot \text{m} \cdot \text{s}}{\text{rad}}$
Spring constant between point 1 and 2 **for deliverables 1-2**	k_x	$7000 \ \frac{\text{N}}{\text{m}}$
Spring constant between point 1 and 2 **for deliverables 3-6**	$k_x(D3 - D6)$	$0 \ \frac{\text{N}}{\text{m}}$
Unstretched length of the linear spring beween point 1 and 2	L_x	12.5 m

Deliverable 1

Construct a non-linear model

In module 1 you have learned how the equations of motion can be derived for a mechanical system by utilizing the method of Lagrange. To complete this first deliverable, you will compute these equations of motion for the Luffing Crane and, in addition, simulate the response of your model using Matlab.

For this purpose, let us introduce the generalized coordinates

$$\underline{q}(t) = \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix}, \tag{2}$$

which are the position of the hoist block and the absolute angle of the chain, respectively. Furthermore, the angle of the jib is regarded as a prescribed displacement, i.e. $s_{pd}(t) = \varphi(t)$.

Before constructing the equations of motion, it is first required to derive expressions for the kinetic energy T, the potential energy V and the column of non-conservative generalized forces Q_{nc} of the system.

Consider the Crane as depicted in Figure 2 for the following questions:

For these questions, please transform any product between vectors into a scalar expression and simplify your answer as much as possible. Note that your expression may not have to contain all variables and parameters. Use parametric notation, i.e., do not fill in any of the parameter values yet.

a) Express the position r_i and velocity $\dot{r}_i$ vector for points $i = 3$, and $i = 4$ in terms of the variables x, θ, φ, and the system parameters.

b) Express the kinetic energy T in terms of the variables x, θ, φ, and the system parameters.

c) Express the potential energy V in terms of the variables x, θ, φ, and the system parameters. Define the potential energy such that $V = 0$ on the (global) x-axis.

d) Express the generalized forces $\underline{Q}^{nc}$ that follow from the non-conservative forces acting on the system in terms of the variables x, θ, F_A, φ, and the system parameters. *Hint: Remember to take all non-conservative forces into account.*

Next, you will compute the equations of motion and simulate the dynamic response of the Luffing Crane in Matlab. We highly advice you to watch Web lecture C.1 if you have little, or no, experience with Matlab.

Complete the following question in Matlab:

e) Compute the parametric (i.e., do not fill in the parameter values!) Equations of Motion (EoMs) using T, V and $\underline{Q}^{nc}$ as determined in the previous exercises. Simplify each EoM as much as possible and rewrite each EOM ensuring that a zero appears on the right side of the equal sign.

To verify whether or not these equations are correct, you will now compare time trajectories that follow from these equations of motion to measured signals that are provided in the Matlab file called

'`MeasuredSignals.mat`'. Note that the data is stored in a Matlab struct and it contains one field with a time vector `t`, one field with $x(\mathtt{t})$, and one field with $\theta(\mathtt{t})$.

Complete the following questions in Matlab:

f) Generate a time trajectory for the EoMs that are computed in question e, by using these EoMs in the Simulink environment as explained in this video. Please consider the initial position and velocity $\underline{q}(0) = \begin{bmatrix} 11.5 & \frac{1}{3}\pi \end{bmatrix}^\top$ and $\underline{\dot{q}}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$ in combination with $F_A(t) = 0$, $s_{pd}(t) = \frac{1}{12}\pi$, and a simulation time of 110 seconds (note that the same settings have been used to obtain the provided measured signals).
Add the following 2 plots:

- A plot with time on the x-axis and the position of the hoist block $x(t)$ on the y-axis. The x-axis should show the interval $\begin{bmatrix} 0 & 110 \end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix} 11.15 & 12.26 \end{bmatrix}$ m. In addition, include a title, appropriate labels for both axes, and a legend.
- A plot with time on the x-axis and the absolute angle of the chain $\theta(t)$ on the y-axis. The x-axis should show the interval $\begin{bmatrix} 0 & 110 \end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix} 0.95 & 2.05 \end{bmatrix}$ rad. In addition, include a title, appropriate labels for both axes, and a legend.

  Pro tip: Saving Matlab figures instead of taking screenshots removes the grey background around the axes, making documents with these figures much easier on the eyes. In addition, various file formats can be chosen to save the figures in, which can have significant advantages (such as .svg vector images for use in Office, or .eps for use in LaTeX).

  Hint: To simulate the system, the EoMs can be rewritten as:

$$\underline{M}(\underline{q}(t))\underline{\ddot{q}}(t) = \underline{F}(\underline{q}(t), \underline{\dot{q}}(t), t),$$

  where $\underline{\ddot{q}}(t) = \begin{bmatrix} \ddot{x}(t) \\ \ddot{\theta}(t) \end{bmatrix}$ *is a vector containing the second derivative of the generalized coordinates. Furthermore, note that* $\underline{M} \in \mathbb{R}^{2\times 2}$ *is a matrix, and* $\underline{F}$ *represents a 2-dimensional vector.*

g) Compare the simulated output responses to the provided measured signals by adding the measured data to the plots in question f (note that these measured signals are obtained using the same simulation conditions). Add the combined plots to the document and include a legend in each plot.
Are there any differences between the time trajectories of questions f and g? If so, can you explain them?

DELIVERABLE 2

Linearize the model

In module 2 you have learned how the equilibrium positions of a system can be determined by using the equations of motion. In addition, it is explained how stability of these positions can be analyzed and you have learned to linearize the equations of motion around a given equilbrium point. To complete this second deliverable, you will construct a linear model that describes the non-linear equations of motion around one of its stable equilibrium positions.

For this purpose, let us first identify the equilibrium positions ($\underline{q}_0$) and assess their stability.

Use the expression for V that was derived in deliverable 1 for the following questions:

a) Numerically calculate all equilibrium positions in the domain $0 \leq x \leq L_1$, $-\pi \leq \theta \leq \pi$ for the input $F_A = 0$ and the prescribed displacement $s_{pd}(t) = \dot{s}_{pd}(t) = \ddot{s}_{pd}(t) = 0$. You may round off your answers to 3 decimal places.

b) For each equilibrium position, determine whether or not it is stable.

Next, you will linearize the equations of motion around a stable equilibrium position and simulate the resulting linear system in Matlab. These linear equations—that describe the system behavior around an equilibrium point—are of the form

$$\underline{M}\ddot{\underline{q}}_1(t) + \underline{D}\dot{\underline{q}}_1(t) + (\underline{K} + \underline{K}^Q)\underline{q}_1(t) = \underline{Q}(t),$$

with $\underline{q}(t) = \begin{bmatrix} x(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ \theta_1(t) \end{bmatrix} + \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix} = \underline{q}_1(t) + \underline{q}_0$.

NOTE: $\underline{q}_1(t)$ are the generalized coordinates of the linearized system and will be referred to as *relative* coordinates (relative with respect to the equilibrium position as $\underline{q}_1(t) = \underline{q}(t) - \underline{q}_0$.

Complete the following questions in Matlab:

c) Linearize the equations of motion—that are computed in question e of deliverable 1— around a stable equilibrium position (also use the prescribed displacement as defined in Question 2a). For the set of linear equations, add the symbolic expressions for $\underline{M}$, $\underline{D}$, $\underline{K}$, $\underline{K}^Q$ and $\underline{Q}$.

Note that: If you found multiple stable equilibria in questions a and b, please consider the equilibrium "closest (i.e., magnitude of difference) to" $\underline{q} = [12.5, \frac{1}{2}\pi]$.

d) Generate a time trajectory with the linear equations as obtained from question c. In addition, simulate a time trajectory for the nonlinear system. Please consider the initial position and velocity $\underline{q}(0) = \begin{bmatrix} 11.5 & \frac{1}{3}\pi \end{bmatrix}^\top$ and $\underline{\dot{q}}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$ in combination with $F_A(t) = 0$, $s_{pd}(t) = 0$, and a simulation time of 110 seconds.
Add the following 2 plots:

- A plot with time on the x-axis and the position of the hoist block $x(t)$, for each performed simulation, on the y-axis. In other words, your plot should contain *both* responses (nonlinear system, and linearized system around a stable equilibrium point). The x-axis should show the interval $\begin{bmatrix} 0 & 110 \end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix} 11.42 & 13.08 \end{bmatrix}$ m. In addition, include a title, appropriate labels for both axes, and a legend.
- A plot with time on the x-axis and the absolute angle of the chain $\theta(t)$, for each performed simulation, on the y-axis. In other words, your plot should contain *both* responses (nonlinear system, and linearized system around a stable equilibrium point). The x-axis should show the interval $\begin{bmatrix} 0 & 110 \end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix} 0.95 & 2.05 \end{bmatrix}$ rad. In addition, include a title, appropriate labels for both axes, and a legend.

e) Describe how each response of the linearized system compares to the response of the nonlinear system. What *conclusions* can be drawn from this?

Deliverable 3

Frequency response analysis

In module 3 you have learned how to determine the eigenfrequencies and eigenmodes of a system, by utilizing the expressions for the linear equations of motion that describe the behavior of a system around an equilibirum point.
During deliverable 3, we will again use the stable equilibrium position from question c of deliverable 2 to obtain a set of linear equations using Matlab. With these linear equations of motion, you can determine the eigenfrequencies and eigenmodes of the system around this equilibrium point.
In addition, we will assume that $s_{pd}(t) = (\dot{s}_{pd}(t) = \ddot{s}_{pd}(t)) = 0$.

For questions a–b we will consider the undamped linear response of the system. This response can be obtained by changing all damping values to $d_\bullet = 0$, before linearizing the equations of motion around the stable equilibrium position $\underline{q}_0$ from question c of deliverable 2.
Please verify that the resulting linear equations of motion satisfy $\underline{D} = 0$.
Furthermore, **remember to change parameter k_x to 0**, as indicated in Table 1.

Complete the following questions in Matlab:

a) Determine the eigenfrequencies ω_1 and ω_2, with $0 \leq \omega_1 \leq \omega_2$, and the corresponding eigenvectors $\underline{u}_1$ and $\underline{u}_2$ of the undamped system. Do this, by using the `eig` function in Matlab. Normalize $\underline{u}_1$ and $\underline{u}_2$ such that their first entries (related to the first DOF) equal 1.

For the remainder of this deliverable, we will again consider the original system *with* damping. For this reason, you may return the damping values to those as found in Table 1 and please verify that the linear equations of motion used in the following questions satisfy $\underline{D} \neq 0$.

Complete the following questions in Matlab:

b) Simulate the *nonlinear* equations of motion using the input signal $F_A(t) = 1500\sin(\omega_2 t)$, where ω_2 denotes the highest eigenfrequency obtained in question a above, by utilizing Simulink. Please consider the initial position and velocity $\underline{q}(0) = \begin{bmatrix}12.5 & \frac{1}{2}\pi\end{bmatrix}^\top$, $\underline{\dot{q}}(0) = \begin{bmatrix}0 & 0\end{bmatrix}^\top$, prescribed displacement $s_{pd}(t) = 0$ and a simulation time of 130 seconds.
Add the following 2 plots to the document:

- A plot with time on the x-axis and the position of the hoist block $x(t)$ on the y-axis. The x-axis should show the interval $\begin{bmatrix}0 & 130\end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix}11.5 & 22.5\end{bmatrix}$ m. In addition, include a title, and appropriate labels for both axes.
- A plot with time on the x-axis and the absolute angle of the chain $\theta(t)$ on the y-axis. The x-axis should show the interval $\begin{bmatrix}0 & 130\end{bmatrix}$ s and the y-axis the interval $\begin{bmatrix}1.38 & 1.821\end{bmatrix}$ rad. In addition, include a title, and appropriate labels for both axes.

c) Repeat this simulation experiment with the input signal $F_A(t) = 1500\sin(0.8\omega_2 t)$ and the input signal $F_A(t) = 1500\sin(1.3\omega_2 t)$. After the initialization of the experiments, do you observe a difference between the amplification of these sinusoidal input signals and the input signal considered in question b? If so, can you explain this difference?

By applying the Laplace transformation to the linear equations of motion, we obtain the expression

$$\begin{bmatrix}X_1(s)\\ \Theta_1(s)\end{bmatrix} = \begin{bmatrix}G_1(s)\\ G_2(s)\end{bmatrix} F_A(s) = \underline{G}(s)F_A(s),$$

where $\underline{G}(s)$ is a single input, multiple output (SIMO) transfer function. Please note that the signals $X_1(s)$, $\Theta_1(s)$ and $F_A(s)$ are the Laplace transformed signals $x_1(t)$, $\theta_1(t)$ and $F_A(t)$, respectively.

Complete the following questions in Matlab:

d) Compute the damped transfer functions $G_1(s)$ and $G_2(s)$ as defined above using the linear equations of motion around the stable equilibrium position $\underline{q}_0$ from question c of deliverable 2, i.e., *with* damping. Add the bode diagrams of these transfer functions to the document.

e) Determine the gains $|G_1(j\omega_2)|$ and $|G_2(j\omega_2)|$. Are these values similar to the amplification that you observed in question b?

In order to design a position controller, we would like to create a SISO transfer function that describes the effect of the input force $F_A(t)$ on the x-position of the load

$$\begin{aligned} x_l(t) &= x(t) + L_2 \sin\left(\theta(t) - \frac{\pi}{2}\right) \\ &= x(t) - L_2\cos(\theta(t)), \end{aligned}$$

see Figure 2, the horizontal position of point 3 with respect to point 0 (given that as prescribed displacement $s_{pd}(t) = \varphi(t) = 0$).
To obtain such a transfer function, we require a linear approximation that describes this function around the operating point $\underline{q}_0 = [x_0, \theta_0]^\top$. A first-order Taylor approximation can be used for this purpose. Since the Taylor approximation is not part of this course, we will now provide you with the resulting approximation

$$x_l(t) \approx x_1(t) + L_2\theta_1(t) + x_0,$$

where $\underline{q}_1(t) = [x_1(t), \theta_1(t)]^\top$ denotes the generalized relative coordinates of the linearized dynamics. Then, let us define a *variation* in the x-position of the load with regards to the operating point $\underline{q}_0$, which is given by $x_{l,1}(t) = x_l(t) - x_0$, in order to obtain the equation

$$x_{1,1}(t) \approx x_1(t) + L_2\theta_1(t).$$

By applying the Laplace transformation to this equation for $x_{l,1}(t)$, we obtain an expression of the form

$$X_{l,1}(s) = G_l(s)F_A(s),$$

where $X_{l,1}(s)$ is the Laplace transformed signal $x_{l,1}(t)$ and $G_l(s)$ is a SISO transfer function.

To complete this deliverable, you will now compute the SISO transfer function $G_l(s)$.

Complete the following questions:

f) Express $G_l(s)$ in terms of the variables $G_1(s)$, $G_2(s)$ and the system parameters. Note that your expression does not have to contain *all* variables and parameters.

g) Add the bode diagram of $G_l(s)$ to the document.