THE CHALLENGE FOR 2025-2026:
DYNAMICS AND CONTROL OF A LUFFING CRANE

D4-6 (CONTROL PART)

This document is part of the bachelor course on Dynamics and Control of Mechanical Systems (4DB00).

DELIVERABLE 4

Introduce feedback

In this challenge, we aim to control the x-position of the load x_l by designing a controller to apply the appropriate input force F_A. In deliverable 4, we will first attempt to do this by analyzing several candidate P-controllers. In deliverable 5, we will attempt to do this by analyzing several candidate lead/lag controllers. In deliverable 6, you will use loop shaping to design a controller.
In module 4, you have practiced drawing Bode diagrams by hand and you have learned why feedback is often used to control dynamical systems. Additionally, you have learned how to simulate and how to determine stability of open-loop and closed-loop transfer functions. To complete the fourth deliverable, you will simulate several responses and investigate stability of the Luffing Crane in an open-loop and closed-loop configuration. From deliverable 3, we know that the plant has the following transfer function:

$$G_l(s) = \frac{5.1 \cdot 10^{-3} s^2 + 100s + 250}{5 \cdot 10^5 s^4 + 2.5 \cdot 10^5 s^3 + 6.4 \cdot 10^5 s^2 + 4.9 \cdot 10^4 s}$$

First, we will analyze the open loop behavior of the velocity of the load $V_{l,1}$ due to the input force F_A without the presence of a controller. Let this transfer function of this system be described by $H(s) = sG_l(s)$.

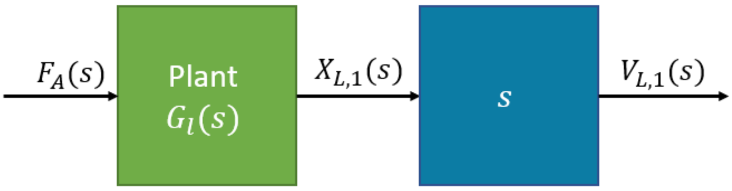

Figure 1: An open-loop configuration with an input F_A and output $V_{l,1}$.

Complete the following questions in Matlab:

a) Is the system $H(s)$ stable? Please explain how you came to this conclusion. **Hint:** Note the pole-zero cancellation.

b) Generate the impulse response of the transfer function $H(s)$ on the time interval $[0,\ 91]$ s, by utilizing the `impulse` function in Matlab. What happens to the response in the long term? What is the physical significance of this? **Pro tip:** Saving Matlab figures instead of taking screenshots removes the grey background around the axes, making documents with these figures much easier on the eyes. In addition, various file formats can be chosen to save the figures in, which can have significant advantages (such as .svg vector images for use in Office, or .eps for use in LaTeX).

Next, we will add a sensor to measure the x-position of the load $x_l(t)$, and a feedback controller $C(s)$ to determine the forcing to be applied by the actuator $F_A(t)$. The controller will calculate the forcing needed to try and best match the x-position of the load with a *desired* position, called a reference $r(t)$. The controllers input is an error signal $e(t)$, which is the difference between the reference and the measured x-position of the load.
Note that, in contrast to D4.a and D4.b, here we will again use the plant transfer function $G_l(s)$ (force to

displacement). In the following, for brevity, we will simply refer to this transfer function as $G(s)$ A typical closed-loop interconnection between a plant and a controller is depicted in Figure 2, where in this case the Luffing Crane is the plant $G(s) = G_l(s)$. In the figure, the output of the controller is $u(t) = F_A(t)$, and the final output signal is $y(t) = x_l(t)$. In addition to this, there are also (generally unknown) external disturbances $d(t)$ that disturb the plant, as well as (generally unknown) sensor noise $n(t)$ that will negatively influence the measured signal.

For deliverable 4 we will neglect the disturbance $d(s) = 0$ and the sensor noise $n(s) = 0$.

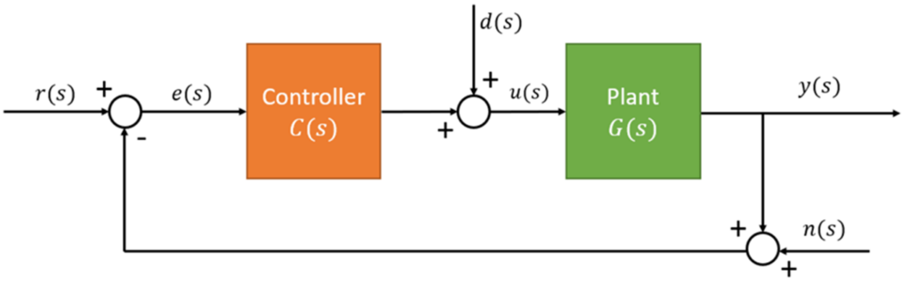

Figure 2: A typical closed-loop configuration of a plant (the Luffing Crane) and a controller.

In this deliverable we will first investigate the following three P gain controllers.

$$P_1 = 80, \qquad P_2 = 400, \qquad P_3 = 700$$

Complete the following questions in Matlab:

c) Consider a closed-loop interconnection as depicted in Figure 2, where the controller is described by $C(s) = P$ where P is as a constant, and the plant (Crane) is described by the transfer function $G(s)$. Select $C(s) = P_1$, and determine if the closed-loop system between reference r and output y is stable. Please explain how you came to this conclusion. Repeat this for $C(s) = P_2$ and $C(s) = P_3$.

d) Select $C(s) = P_1$, and generate the step response of the closed-loop transfer function. Utilize the `step()` function in Matlab for this purpose and consider the time interval $t \in [0,\ 240]$s. Repeat this for $C(s) = P_2$ and $C(s) = P_3$ and give all plots in a single figure. Also, calculate the final value (i.e., in the limit $t \to \infty$) of the response for each case. If there is no final value for any of these cases, explain why. What influence does increasing P have on the final value?

e) Plot the poles and zeros of the closed-loop transfer functions in the complex plane by using `pzmap()` for each controller $C_i(s)$ as calculated in a). What is the effect of increasing the gain P? In your answer, mention how the position of the poles affect the frequency of oscillation, and settling time in the step-responses computed in b). Suppose we would like to design a controller with a maximum of 30% overshoot. Based on your findings, explain whether or not you would recommend the use of a (purely) proportional controller $C(s) = P$ for this application. **Hint**: In matlab $(L/(1+L))$ doesn't by default cancel common poles and zeros. `minreal(L/(1+L))` usually works at cancelling poles and zeros, but isn't perfect because of numerical precision. It's recommended to use `feedback(L, 1)` instead.

For your plot show the interval $[-0.24,\ -5.5 \cdot 10^{-3}]$ and $[-1.2,\ 1.2]$ for real and imaginary axis respectively. Additionally, provide a zoomed in version of this plot where the real axis shows the interval $[-0.05,\ -0.031]$ and the imaginary axis shows the interval $[-0.74,\ 0.74]$.

f) Select a PD controller given by $C(s) = 390s + 360$. Generate the step response of the closed-loop system from r to y on the time interval $[0,\ 79]$ s. Compare this step response to the one calculated in d) for $C(s) = P_2$ by plotting them in the same graph.

g) Analyze the result from f). In your answer, mention the final value, frequency of oscillation, and settling time and the closed-loop pole locations of both systems. Plot the location of the poles and zeros for both cases on a single graph where the real axis shows the interval $[-0.22,\ -0.025]$ and the imaginary axis shows the interval $[-1.2,\ 1.2]$.

DELIVERABLE 5

Analyze performance

In module 5 you have learned how to study stability, performance and robustness of the closed-loop interconnection between a controller $C(s)$ and a plant $G(s)$, by using the Bode and Nyquist diagrams of the open-loop system $L(s) = G(s)C(s)$. To complete the fifth deliverable, you will analyze these properties for a number of controllers that were designed by a coworker.
We will first investigate the following three lead/lag filters:

$$C_1(s) = \frac{770s+350}{4.3s+1}, \qquad C_2(s) = \frac{770s+210}{2.5s+1}, \qquad C_3(s) = \frac{770s+140}{1.7s+1}.$$

For this purpose, consider the closed-loop interconnection as depicted in Figure 2, where the controller is described by $C_i(s)$ (with $i = 1,2,3$) and the plant by the transfer function $G(s)$. Therefore, we consider the three open-loop systems $L_i(s) = C_i(s)G(s)$, with $i = 1,2,3$.

Complete the following questions in Matlab:

a) Plot, in one figure, the Nyquist contours for the three open-loop systems $L_i(s)$—with $i = 1,2,3$. Utilize the `nyquist` function in Matlab for this purpose, consider the interval $[-2, 0.2]$ for the x-axis, the interval $[-1, 1]$ for the y-axis and include a legend.

b) Use the Nyquist contours from question **a)** to determine, for each controller, whether the closed-loop interconnection in Figure 2 is stable. Please explain briefly how you came to this conclusion by making reference to the Nyquist stability criterion. **Hint**: Note that there is a pole of $L_i(s)$ with real part = 0. We will define this pole as stable. A consequence of this pole is that the Nyquist contour will diverge to infinity. However, the Nyquist contour is still closed - it will just do a 180 degrees clockwise loop around at infinity.

c) For each open-loop system $L_i(s)$—with $i = 1,2,3$—determine the crossover frequency ω_c in $\frac{\text{rad}}{\text{s}}$, the phase margin *PM* in degrees, the gain margin *GM* in dB and the modulus margin *MM* in dB. You may round off your answer to one decimal place.
If there are multiple frequencies satisfying $|L(j\omega)| = 1$ or $\angle L(j\omega) = -180°$—that is, if there are multiple frequencies at which these margins could be calculated—please consider the one that is *most critical* in terms of stability. *Note that:* the `margin()` command in Matlab will also consider this most critical frequency when calculating the *PM* and *GM*.

Now we will consider the closed-loop system as shown in Figure 2. We will also investigate performance and robustness of the different controllers.

Complete the following questions in Matlab:

d) Determine the step response of the closed-loop system for each controller $C_i(s)$ with $i = 1,2,3$. Add all plots on the same figure and plot over a duration of 100 seconds.

e) For the closed-loop response for each controller in question d), describe how well it is able to track the step reference signal. Compare each controller with respect to the following criteria: overshoot, steady state error, and settling time. Please explain, per criteria, which controller you would choose.

f) After some experimentation, your Crane suffers a bit of damage to the damper d_W, resulting in a 90% decrease in the damping coefficient. From a practical perspective, it is therefore important that closed-loop performance is not significantly affected by such small changes. Adjust, **for this question only**, the system parameters to the values stated above, and repeat the simulations in question d. The change in damping results in a new general equation of the transfer function:

$$G(s) = \frac{5.1 \cdot 10^{-3} s^2 + 10s + 250}{5 \cdot 10^5 s^4 + 7 \cdot 10^4 s^3 + 6.2 \cdot 10^5 s^2 + 4.9 \cdot 10^4 s}$$

g) Explain the difference in step responses, how is the closed-loop affected by the change in parameter (d_W) with respect to question d)? Comment on the robustness of the controllers.

As shown earlier in Figure 2, sensor noise may be present in the system that can influence the measured x-position of the load $x_l(t)$. Sensor noise is virtually always present, although for many applications it may be negligible. However, especially when trying to control systems to a high bandwidth, sensor noise can really limit the achievable performance. To determine the effect of noise such as this on the tracking performance of the system, the complementary sensitivity can be used:

$$T_i = \frac{L_i}{1+L_i} = \frac{C_i G}{1+C_i G} \tag{1}$$

From this equation, a high value means noise has a large effect on the output of the system, and a low value means noise has a small effect on the output of the system.

Complete the following question in Matlab:

h) For the **original value** d_W analyze the sensitivity functions S_i and explain what we can say about robustness of the closed-loop systems and conclude which controller is the most robust according to the modulus margin in S_i. In your answer, refer to the open-loop transfer functions L_i and the robustness margins that you calculated in question 5c).

i) For the **original value** d_W analyze the complementary sensitivity functions T_i, and explain what we can say about (high-frequency) noise rejection of the closed-loop system and conclude which controller is most suitable for rejecting high-frequency noise.

j) From question h) and i) make a bode plot of $S_2(s)$ and $T_2(s)$, i.e., the closed-loop transfer functions under the controller $C_2(s)$. Verify whether the identity $S_2 + T_2 = 1$ holds in this case. What can we say about the trade-off between tracking performance and noise rejection? In your answer, make a connection with the identity, high-gain feedback and the bode sensitivity integral.

DELIVERABLE 6

Design a controller

General note: It is highly advised to pass formative assessment 6 before working on this deliverable.

In module 6, you have learned how to design several types of controllers by appropriately shaping the open-loop system $L(s) = C(s)G_l(s)$. For this module, the transfer function of the Luffing Crane $G(s)$ from force F_A to the x-position of the load x_l, which has been parameterized in terms of the damping coefficient for the load, d_W, is given by:

$$G(s) = \frac{5.1 \cdot 10^{-3} s^2 + (0.2 + 0.049 \cdot d_W)s + 250}{5 \cdot 10^5 s^4 + (5 \cdot 10^4 + 100 \cdot d_W)s^3 + 6.2 \cdot 10^5 s^2 + 4.9 \cdot 10^4 s}. \tag{2}$$

To complete this last deliverable, you will design a feedback controller that enhances closed-loop performance for the given closed-loop system as illustrated in Figure 2. The feedback controller that will be used is given by the form

$$C_1(s) = K \frac{\frac{\alpha}{\omega_c} s + 1}{\frac{1}{\alpha \omega_c} s + 1}, \tag{3}$$

with the unknown, to-be-designed parameters K, ω_c, and α.

Complete the following questions in Matlab:

a) Design the feedback parameters (ω_c, α, K) for the feedback controller C_1 with $d_W = 100 \, \frac{\text{N·s}}{\text{rad}}$ such that the following design requirements are met (and motivate your choices):

- The feedback interconnection between $C_1(s)$ and $G_l(s)$—as depicted in Figure 2—is stable.
- The open-loop $L_1(s) = C_1(s)G(s)$ has the crossover frequency (bandwidth) $f_c = 0.025$Hz.
- The open-loop $L_1(s) = C_1(s)G(s)$ has a modulus margin (MM) of at most 6 dB.

b) For the open-loop system $L_1(s) = C_1(s)G(s)$—where $C_1(s)$ is the controller that you designed in question (6a)—determine the phase margin *PM* in degrees, gain margin *GM* in dB and the modulus margin *MM* in dB. Explain whether or not these margins are sufficiently large.

c) Perform a closed-loop step-response simulation. Plot of the x-position of the load x_l and the tracking error e over a time-interval of $t \in [0, 100]s$. What is the settling time when $|e(t)| < 0.025$ m?

d) To analyze the performance of the controller under plant perturbation, change the damping to $d_W = 10 \, \frac{\text{N·s}}{\text{rad}}$ and redo question 6b) and 6c) and evaluate the effect of this damping decrease on the Nyquist plots by plotting them together. Use the interval $[-2.5, \ 0.5]$ for the real axis and $[-2, \ 2]$ for the imaginary axis, include a legend and show the unit circle.

Resonance frequencies are known as bandwidth limiting factors. In the upcoming question you will use a notch-filter to deal with this phenomena. The new controller structure becomes:

$$C_2(s) = K\frac{\frac{\alpha}{\omega_c}s+1}{\frac{1}{\alpha\omega_c}s+1}\cdot\left(\frac{\omega_p}{\omega_z}\right)^2\frac{s^2+2\beta_z\omega_z s+\omega_z^2}{s^2+2\beta_p\omega_p s+\omega_p^2} \tag{4}$$

with the unknown parameters K, ω_c, α, ω_z, ω_p, β_z and β_p. Note that the parameters K, ω_c and α could be different from $C_1(s)$.

Complete the following questions in Matlab:

e) Design the feedback parameters K, ω_c, α, ω_z, ω_p, β_z and β_p for feedback controller C_2 for the initial damping coefficient $d_W = 100\ \frac{\mathrm{N\cdot s}}{\mathrm{rad}}$ such that the following design requirements are met (and motivate your choices):

- The feedback interconnection between $C_2(s)$ and $G(s)$—as depicted in Figure 2—is stable.
- The open-loop $L_2(s) = C_2(s)G(s)$ has the crossover frequency (bandwidth) $f_c = 0.06$ Hz.
- The open-loop $L_2(s) = C_2(s)G(s)$ has a modulus margin of at most 6dB.

f) Perform a step-response simulation using controller C_2. Give a plot of the output x_l and the tracking error e over a time-interval of $t \in [0,100]$ together with the response of controller C_1 obtained by question 6c). What is the settling time of the error for $|e(t)| < 0.025$ m with controller C_2?

g) Create a Bode diagram of the sensitivity function $S_1(s)$ and $S_2(s)$. What are the benefits, and possible downsides of feedback controller $C_2(s)$ compared to $C_1(s)$?

h) Perform a simulation of the closed-loop system using controller C_1 and C_2. Use a reference $r(t) = A_r\sin(\omega_r t)$ over a time-window of $t \in [0,1500]$ given input frequency $\omega_r = 0.01\ \frac{\mathrm{rad}}{\mathrm{s}}$ and amplitude $A_r = 0.1$ m (all other signals are omitted). Plot both error signals e_1 and e_2 calculated by $e_i = r - y_i$ in the same figure. What are your observations and does it match your expectations based on the sensitivity functions.

Hint: use the Matlab command `y = lsim(sys,u,t)` *to simulate custom input signals.*

i) A disturbance $d(t) = A_d\sin(\omega_d t)$ is acting onto the system with a given amplitude $A_d = 100$ N and frequency $\omega_d = 0.8\ \frac{\mathrm{rad}}{\mathrm{s}}$. Evaluate the effect of disturbance $d(t)$ onto the error signal $e(t)$ by time-domain simulations with $r(t) = 0$ for $t \in [0,50]$. Plot the error $e_1(t)$ and $e_2(t)$ in the same figure and compare the results. Which controller C_1 or C_2 is better for suppressing this disturbance and why?