Central Limit Theorem

Random Sampling and sampling Bias:

Selection Bias → Selecting a set of samples

Participation Bias

Survivorship bias

Language bias

A sample should be representative of population (should include all the categories)

Laws of Large Nos:

The No of trails increase the accuracy of test gets better

Drug trails, stock market analysis quality control management

Central Limit Theorem, Sampling Distribution:

- **Parameter:** A numerical value that describes a characteristic of a population (e.g., population mean μ, population standard deviation σ).
- **Statistic:** A numerical value that describes a characteristic of a sample (e.g., sample mean $\bar{x}$, sample standard deviation s)
- The parameter is what we want to know (but usually can't measure directly).
- The statistic is what we calculate from a sample to estimate the parameter.

Population Distribution:

The distribution of a variable for all individuals in the population.

📌 Example: If you measured the height of every adult in India, the distribution of those heights is the population distribution.

Sample Distribution:

The distribution of values in a **single sample** taken from the population.

📌 Example: If you randomly select 100 adults from India and plot their heights, that's a sample distribution.

Sampling Distribution of the Mean:

- The distribution of the **sample means** from many samples of the same size taken from the population.
- 📌 Important: According to the **Central Limit Theorem**, this distribution **approaches normality** as the sample size increases, even if the population distribution is not normal.

Sample size should be >=30 then it is normal distribution

Standard Error:

Standard Error (SE)=σ/n

Population distribution mean = μ

population distribution variance = σ^2

population distribution std = σ

Standard Error (SE) measures how much the **sample mean (x̄)** is expected to vary from the **true population mean (μ)**. It tells us how **precise** our estimate of the population mean is based on a sample.

Key Points:

- **Smaller SE** → More precise estimate of the population mean.
- **Larger sample size (n)** → Smaller SE.
- SE is used to construct **confidence intervals** and perform **hypothesis testing**.

Z-Table and Z-Score

- A **Z-score** tells you **how many standard deviations** a data point is from the **mean**.
- The **Z-table** (or standard normal table) shows the **probability** (or area under the curve) to the **left** of a given Z-score in a **standard normal distribution**.

- **Z-Score Formula:**
  - $Z=X-\mu/\sigma$

  - 68.3 =1
  - 95.5 =2
  - 99.7 = 3

What is a Confidence Interval?

A **confidence interval (CI)** is a range of values, derived from sample data, that is likely to contain the **true population parameter** (like the mean) with a certain level of confidence.

Formula for Confidence Interval of the Mean (when population standard deviation is known):

📊 **Confidence Levels and Z-Scores**

Confidence Level	Z-Score (Two-Tailed)	Area in Each Tail
90%	±1.645	5%
95%	±1.960	2.5%
98%	±2.326	1%
99%	±2.576	0.5%

🔍 **How to Use This Table**

- If you're constructing a **confidence interval**, you use the Z-score to calculate the margin of error:

Margin of Error= $Z^{*}\sigma/\sqrt{n}$

- For a **95% confidence level**, you'd use **±1.960** as your Z-score.

Hypothesis Testing

Any claim that can be tested

Null(h_o) vs Alternate Hypothesis(h_a): Actual statement vs the counterfeit statement

4 Steps of Hypothesis Testing

Formulate Hypothesis → Finalize the right test→ Conduct a test→ Make a decision

Ho Housing inflation Z test T test Anova yes /no

is 10%

Ha Housing inflation

is >10%

Z Test, Rejection Region:

Rejection Region:

The rejection region is the range of values for the test statistic (like a Z-score or t-score) that leads you to reject the null hypothesis.

It's determined by:

- The significance level (α), such as 0.05 or 0.**01**
- **The type of test (one-tailed or two-tailed)**

Z- Test : used when comparing sample and population means, assuming the population variance is known, and the sample size is large[>30]

Ex: comparing the average height of a sample of basketball players to the known average height of basketball players nationally

✅ **If you're using a Z-test for hypothesis testing:**

1. Set your confidence level

- For **90% confidence**, the **critical Z-score** is **±1.645**.

2. Calculate the test statistic (Z-score) using:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- $\mu 0$ = population mean under the null hypothesis
- σ = population standard deviation
- n = sample size

T-Test:

Use when the population variance is unknown

Types include one sample t-test, independent two-sample t-test and paired sample t-test

Example: comparing the average test scores of two different classes.

Chi-square Test:

Used for categorical data to test relationship between variables

P-values:

The p-value is the probability of obtaining a test statistic (like a Z-score or t-score) as extreme as, or more extreme than, the one observed in your sample, assuming the null hypothesis is true.

🔍 **What Does It Tell You?**

- A **small p-value** (typically ≤ 0.05) indicates **strong evidence against the null hypothesis**, so you **reject** it.
- A **large p-value** (> 0.05) suggests **weak evidence against the null**, so you **fail to reject** it.

✅ Typical Thresholds (α levels):

Significance Level (α)	Decision Rule
0.10	Reject if $p \leq 0.10$
0.05	Reject if $p \leq 0.05$
0.01	Reject if $p \leq 0.01$

✅ **Decision Rule in Hypothesis Testing:**

- If the **p-value is less than** your chosen **significance level (α)** (e.g., 0.05 or 5%):
  - **Reject the null hypothesis**
  - This means the result is **statistically significant**
- If the **p-value is greater than or equal to α**:
  - **Fail to reject the null hypothesis**
  - This means there is **not enough evidence** to support the alternative hypothesis

One-Tailed vs Two-Tailed Test:

📊 **Visual Summary:**

Feature	One-Tailed Test	Two-Tailed Test
Direction	One direction (>, <)	Both directions (≠)
Rejection region	One tail	Both tails
Common α = 0.05	Entire 5% in one tail	2.5% in each tail
Use case	Specific direction	Any difference

✅ **Conclusion Rules in Hypothesis Testing**

🔹 **Using Z-Score:**

- Compare your **calculated Z-score** to the **critical Z-score** (based on your significance level α).
- **If**:
- $|Zcalculated|>Zcritical|Zcalculated|>Zcritical$

→ **Reject the null hypothesis**

🔹 **Using p-value:**

- Compare your **p-value** to the **significance level (α)**.
- **If**:

$p\text{-value}<\alpha p\text{-value}<\alpha$

→ **Reject the null hypothesis**

🧠 **Why Both Methods Work:**

Both approaches are just two sides of the same coin:

- **Z-score** tells you how far your result is from the null hypothesis in standard deviations.
- **p-value** tells you the probability of getting such a result (or more extreme) if the null hypothesis were true.

They always lead to the **same conclusion**.

Type 1 and Type 2 Errors:

🧪 **Hypothesis Testing Recap**

- **Null Hypothesis (H_0):** No effect or no difference
- **Alternative Hypothesis (H_1):** There is an effect or difference

❌ **Type I Error (False Positive) second word is prediction (False Prediction)**

- **What it is:** Rejecting the null hypothesis when it is actually **true**
- **You think there's an effect, but there isn't**
- **Probability of this error: α** (significance level, e.g., 0.05)

🔹 **Example:**

You conclude that a new drug works better than the old one, but in reality, it doesn't.

❌ **Type II Error (False Negative)**

- **What it is:** Failing to reject the null hypothesis when it is actually **false**
- **You miss a real effect**
- **Probability of this error: β**
- **Power of the test = 1 − β** (probability of correctly rejecting a false null)

🔹 **Example:**

You conclude that a new drug doesn't work better, but in reality, it does.

✅ **Null Hypothesis (H_0):**

The new medicine is **not better** than the current one.

✅ **Alternative Hypothesis (H_1):**

The new medicine **is better** than the current one.

❌ **Type I Error (False Alarm):**

You **think the new medicine works better**, but it actually **doesn't**.

- You **rejected** the null hypothesis when it was **true**.
- Like a **false positive**.
- This happens when your test result falls in the **red rejection zone** in the diagram.

❌ **Type II Error (Missed Opportunity):**

You **think the new medicine doesn't work better**, but it actually **does**.

- You **did not reject** the null hypothesis when it was **false**.
- Like a **false negative**.
- This happens when your result is **not extreme enough** to fall in the rejection zone, even though the medicine is actually better.

Yes — the significance level (α) defines the size of the rejection region, and it represents the chance of making a Type I error.

🧠 **So in simple terms:**

- **Type I Error: You say someone is guilty (reject H_0) when they are actually innocent (H_0 is true).**
  → Wrongly punishing the innocent.
- **Type II Error: You say someone is innocent (fail to reject H_0) when they are actually guilty (H_0 is false).**
  → Letting a criminal go free.

TAKEAWAYS

Type 1 Error: Occurs when we incorrectly reject a true null hypothesis.

Type 2 Error: Occurs when we fail to reject a false null hypothesis.

Type 1 error is also known as a "false positive" while Type 2 error is a "false negative".

Beta (ß): The probability of making a Type 2 error (false negative).

Statistical Power: The probability of correctly rejecting a false null hypothesis,

equal to 1-ß.

Balancing Type 1 and Type 2 errors is crucial in statistical analysis.

📈 What is Statistical Power?

Statistical power is the probability that a hypothesis test will correctly reject a false null hypothesis. In other words, it's the ability of a test to detect a real effect when one actually exists.

✅ In Simple Terms:

Power = 1 − β

Where:

β (beta) is the probability of a Type II error (failing to reject a false null hypothesis).

So, higher power means lower chance of missing a real effect.

TAKEAWAYS

1. Statistical power, denoted as 1-ß, represents the probability of correctly rejecting a false null hypothesis in a hypothesis test.
2. Effect size quantifies the magnitude of the difference or relationship between two groups or variables in a study.

🧪 What is A/B Testing?

A/B Testing is a method used to compare two versions of something to determine which one performs better. It's widely used in marketing, product design, and web development.

🔍 How It Works:

1. **You have two versions**:
   - **A** = the current version (control)
   - **B** = the new version (variant)
2. **Split your audience randomly**:
   - Half see **Version A**
   - Half see **Version B**
3. **Measure performance**:
   - This could be clicks, purchases, sign-ups, etc.
4. **Analyse the results**:
   - Use statistical tests (like a Z-test or t-test) to see if the difference is **significant**.

A/B Testing Using Z Test: