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DEPARTMENT OF MATHEMATICS

OPTIMAL LOAN PORTFOLIO

A CASE STUDY OF CAL BANK LIMITED, GHANA

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DECLARATION

We honestly declare that this project is of our own and a true account of our research except for reference to other people's work, which have duly been cited.

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DEDICATION

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ABSTRACT

Ghana's banking sector has emerged from severe financial and reputational damage resulting from economic and government debt which necessitated the institution of the Financial Sector Reforms in the 1980s. The recent financial sector is a stiff and a competitive one that has arisen from the ashes of severe financial and reputational damage of the post-independence era. It has overcome several problems but few still exists; one of which is the huge portfolio of bad loans.

This project sought to design a linear programming model to optimize the loans given out to people of different sectors of the economy using the financial loan records of CAL Bank Ghana Limited and also, to determine the sectors of the banks that yielded the highest/best return on loan portfolios.

Per data taken from CAL Bank Limited, which included the loan types, the respective interest rates and the probability of bad debt, we modeled a Linear Programming Problem which was solved using the Quantitative Manager for Windows. It was observed from the solution that, for an amount of four hundred and eleven million, five hundred and eighty-two thousand Ghana cedis (GH¢411,582,000) to be disbursed as loans by the bank in 2012, a maximum profit of seventy-one million, six hundred thousand, two hundred and eighty Ghana cedis and eight pesewas (GH¢71,600,280.08) could be realized by the bank.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER ONE

INTRODUCTION

1.0 INTRODUCTION

The business activity of accepting and safeguarding monies owned by individuals and entities, and then lending out these monies in order to earn profits is termed as banking. A bank is any financial institution that receives, transfers, pays, exchanges, lends, invests, or safeguards money for its clients. The banks perform financial intermediation by pooling savings and channelling them into investments through maturity and risk transformations, thereby keeping the economy`s growth engine reviving. The vehicle to the development of every country or business firm is its financial strength which is partly driven by the banks.

Before the introduction of financial sector reforms, the financial system of the Ghanaian economy was dominated by state-owned banks which enjoyed monopoly over the entire banking sector in terms of their operations and branch network. Major developments in the country, in the lives of its people and in the financial sector necessitated the introduction of several banks (both local and foreign). Some of these banks are the Cal Bank, United Bank for Africa (UBA), UT Bank, Ecobank, uniBank, Stanbic Bank, just to mention a few.

The Bank of Ghana (BOG), is the Central Bank and it is principally tasked with the implementation and control of Ghana`s monetary policies. It uses the financial markets to stabilize the country`s money supply and ensure that the banking sector effectively attends to the needs of its subscribers. It encourages the establishment of new financial institutions by pursuing moderate policies with regard to entry into the banking system.

Rural banks were also first established in Ghana in 1976 to provide banking services to the rural population, providing credit to small-scale farmers and businesses and supporting development projects. The ARB Apex Bank is the umbrella bank for Rural Community Banks and supervises a hundred and thirty-seven of such banks throughout Ghana (as of January 2013). Gradually, the erstwhile monopolistic financial sector has been replaced by a stiff, competitive and ever dynamic sector survived by only the most innovative ones.

Lending and borrowing are inevitable aspects of human existence. People borrow money from friends, family, 'susu' groups, etc., but financial institutions have established themselves as the main lending outfits over the years. Loan portfolios are the major assets of banks, thrifts and other lending institutions. Loan portfolios are the loans that have been made or bought and are been held for repayment. The loan portfolio is listed as an asset on the lender's or investor's balance sheet. The value of a loan portfolio depends not only on the interest rate earned on the loans but also on both the principal and the average creditworthiness of the loans. After successfully raising the required regulatory capital, the next critical action for banks is to optimize the allocation of capital across the business value chain, from service delivery to lending. In order to deliver value to their customers, banks must transform their operations, including people, products, processes and technology for greater efficiency. The more banks respond to growing consumer expectations, the more they will be required to develop risk profiles around the new business direction which will be driven largely by technology. It is thus imperative that banks optimize their loan portfolios to create a win-win situation in delivering quality service to their customers and fulfilling the aim of every business (profit-driven) entity.

1.1 BACKGROUND

Ghana's banking sector has emerged from severe financial and reputational damage resulting from economic recession and government debt in the 1980s and 1990s, when Ghanaian banks and other financial institutions stopped lending to the private sector. The banking sector has seen major capital injection partly because of the political stability, attainment of micro and macroeconomic stability and the government's desire to make Ghana the "financial hub" of the Sub-region. The Central bank has promoted the enforcement of statutory requirements, more stringent supervision and increasing capital requirements. The Bank of Ghana had licensed thirty banks to operate in the country as of December 2013. In addition to the 30 banks, the sector also comprises a range of non-bank financial institutions, including several community banks established to mobilise rural savings.

In the post-independence period, extensive government intervention characterised the financial sector. Basically, all of the banks set up between the early 1960s and the late 1970s were wholly or majority owned by the public sector, while the government also acquired minority shares in the two already established foreign banks in the mid-1970s (Barclays Bank and Standard Chartered Bank). Prior to the economic and financial sector reforms, interest rates were administratively controlled by the Bank of Ghana (BOG) and a variety of controls were also imposed on the asset allocations of the banks, such as sectorial credit directives. The motivation for these policies was the belief that because of market imperfections and the nature of the financial system inherited from the colonial period, the desired pattern of investment could not be supported without extensive government intervention in financial markets.

Basically, policies were motivated by three objectives: to raise the level of investment and quicken the pace of development, to change the sectorial pattern of investment, and to keep interest rates both low and stable (Gockel, 1995, p117). Financial sector policies were characterised by severe financial repression, real interest rates were steeply negative and most of the credit was channelled to the public sector. To fill the perceived gaps not served by the commercial banks, especially for long-term finance, three Development Finance Institutions (DFIs) were set up: the National Investment Bank (NIB) in 1963 to provide long-term finance for industry; the Agricultural Development Bank (ADB) in 1965; and the Bank for Housing and Construction (BHC) in 1974, to provide loans for housing, industrial construction and companies producing building materials. The DFIs mobilised funds from deposits as well as from government and foreign loans and undertook commercial banking activities as well as development banking. The government did not nationalise the two foreign owned banks—Barclays Bank and Standard Chartered Bank (SCB)—which had been established in Ghana during the colonial period, but it did acquire 40 per cent equity stakes in the banks following an indigenisation decree enacted in 1975 (which was applied to all large scale industries).

However, after many decades of government domination of the banking sector, some problems became paramount. The service standards of the public sector banks began to decline. Their profitability declined and the efficiency of the staff became suspect.

Non-performing assets of these banks began to rise. In the early 1980s, as part of the structural and economic reforms and financial sector liberalisation, the government allowed the setting up of new banks in the private sector. The new generation private banks have now established themselves in the system and have set new standards of service and efficiency. These banks have also given tough but healthy competition to the remaining public sector banks.

1.1.1 FINANCIAL SECTOR REFORMS IN GHANA

Prior to 1987, there were several financial service sector restrictions that served to undermine private sector confidence in the Ghanaian banking system as a whole. The banking sector had been battered by exposure to wanton political influence, weak and incompetent management, insufficient capital, obsolete information and accounting systems and poor internal controls. In addition, the banks had a huge portfolio of Non-Performing Loans (NPL) and exposure to a few clients, mainly state-owned enterprises. The portfolios of the banks were not adequately diversified and, as a result, they attracted very few corporate and private clients. As a result of the non-performing nature of the banks, broad-based reform measures were required, not only to restructure the financial sector, but also to encourage the development of the financial market by deepening financial intermediation, creating new financial instruments for the people to invest in and establishing new financial institutions.

The broad-based Structural Adjustment Programme was introduced by the Ghana government in 1986 to restore fiscal and monetary discipline and realign prices by removing all controls. As a component of the Structural Adjustment Programme, the Financial Sector Liberalization Programme, captioned as the ‘Financial Sector Adjustment Program (FINSAP)’, was launched in 1987 to address the deterioration in performance of the financial sector. During that time, most banks in Ghana had become technically insolvent due to years of mismanagement and government interference in the administration of credit. The reform measures, aimed at restructuring the banks, included the removal of restrictions on entry into the formal financial sector. In addition to this, all direct state controls were removed to lay the foundation for banks to design and structure their own credit policies and to set their own interest rates.

The core essence of the reform exercise was to ultimately, enhance the competitiveness and efficiency of Ghana's financial institutions. The financial sector reforms programme can be conveniently grouped into three stages. The first phase of reforms was from 1987 to 1991; the second from 1992 to 1995; and the third phase of reforms has been from 1995 to 2003.

The first phase of the reforms, which spanned the period from 1987 to 1991, involved a review of the legal and regulatory environment and amendment of the existing banking acts and laws aimed at restructuring the banking sector to make them economically viable and efficient. The process also involved the revitalization of the financial sector through the creation of new financial institutions. The first phase started with a deregulation of interest rates. The process of interest rate liberalization was gradual, starting with the abolishing of the maximum and minimum deposits in September 1987. In February 1988, the minimum lending rates for commercial banks were also abolished, followed by the granting of operational rights to commercial banks to determine their own rates by March 1989. Consequently, the year 1990 started with a near complete liberalization of the interest rates system in Ghana. The deregulation of interest rates was, in part, meant to encourage competition among the banks leading to rates being determined by market forces. During this phase of reforms, the existing banking law was also revised in 1989.

The basic features of the legal reforms included the strengthening of risk exposure limits, the strengthening of capital adequacy ratios, the strengthening and standardization of accounting systems for all banks, the imposition of stringent reporting requirements, and improvement of onsite and offsite supervision of banks by the Central Bank. The management and financial restructuring of the banks also featured prominently in the financial sector reforms programme. Financial restructuring included the recapitalization of the banks with equity injection where liquidity was low, and the removal of non-performing assets from the balance sheets of distressed banks. Non-performing loans in the balance sheets of banks were substituted with government guaranteed interest-bearing bonds issued by the central bank of Ghana. A recovery trust for non-performing assets (NPART) was established in 1990. The performance of the NPART appeared quite impressive as

¢13 billion (old cedis) were recovered out of a total ¢18 billion (old cedis) outstanding in non-performing loans by the close of 1995.

The second phase of the financial sector reforms spanned the period from 1991 to 1995. This era of reforms included reforms to the Bank of Ghana Law aimed at providing more supervisory control to the central bank. A second discount house, namely the Security Discount Company (SDC), was also established in 1991 with the main objective of assisting to optimize the allocation of resources within the banking sector and to make it feasible for resources to be mobilized to support the sector. This was followed, in 1993, by the enactment of the Non-Banking Financial Institutions (NBFI) Law. The NBFI Law was enacted to bring non-banking financial institutions under the supervision of the Bank of Ghana. Institutions covered were discount houses, finance houses, acceptance houses, building societies, leasing and hire-purchase companies, savings and loans companies, and credit unions.

In addition to this, the central bank also encouraged the establishment of rural banks as a way of making up for the inability of commercial banks to reach the rural areas and to provide support for the agricultural sector. The Bank of Ghana owns shares in most of the rural banks and also supervises them. The year 1992 witnessed the taking off of the divestiture programme with an initial progress made on divesting government shares of commercial banks.

The third phase of the reforms continued with the restructuring of financial services sector. In March 1995, there was a restructuring and merger of the Social Security Bank and the National Savings and Credit Bank with 21 per cent of shares divested through public offer and 40 per cent of shares sold to a strategic investor. The Ghana Commercial Bank, the largest bank was also targeted for divestiture with an initial 30 per cent of shares floated but later increased 42 per cent due to oversubscription of the initial offer. As part of the restructuring initiative, the central bank acquired shares in a number of commercial banks, an exercise envisaged to be a temporary measure. The divestiture programme was later stalled with the suspension of sale of the shares of the Ghana Commercial Bank by the Bank of Ghana. This was in response to public outcries over possible foreign domination of the country's banking sector. The incident resulted in significant loss of opportunity to stimulate competition and promote a more efficient banking system.

The year 2002 also witnessed the coming into effect of the new Central Bank Law. This law establishes and guarantees the independence of the Bank of Ghana. It also confirms the Bank's principal objective to be the pursuit of price stability and makes room for the enhancement of its operational efficiency and strengthening of its supervisory role. The recently introduced Financial Sector Strategic Plan (FINSSIP) provides for the medium-term direction of financial sector reform as from the year 2003. The emphasis of FINSSIP is on regulatory and judicial reform, institutional capacity building, protection of private property rights, and competition. The government also raised the minimum capital requirement for banks in 2006 in a bid to enhance banks' ability to withstand possible future deterioration of asset quality. The financial sector reforms are still on-going, and among the key measures taken to continue with reforms in the sector are the passing of the Credit Reporting Law which requires all banks to submit credit details to a reporting bureau and the elimination of the secondary reserve requirements to stimulate banking activities.

Other recent reforms in the financial sector include an upgrading of the payments system, strengthening of the central bank's supervisory capabilities and the enactment of the Insurance Act of 2005. Another key policy measure has been the enactment of the Anti-Money Laundering Act of 2007. This act provides a framework for criminalizing money laundering and establishes a Financial Intelligence Centre. It also facilitates the easing of restrictions on the acquisition of capital market instruments by both residents and non-residents. One of the outcomes from the reform measures have been the establishment of many bank and non-bank financial institutions. From the pre-liberalization era of two foreign and five state-owned banks with virtually no Non-Bank Financial Institutions (NBFIs), the sector has widened substantially.

The new banks are trying to revolutionise access to banking services, denied the population by the imperialist banks. Several banks have already made determined effort to roll out the use of internet banking, smartcard technology, mobile phone banking and the use of biometric technology to cover all their operation areas. The Bank of Ghana has increasingly exercised its power as a regulator in line with internationally accepted norms, and has implemented a series of tough supervisory measures. More recently, there is growing introduction of new products by the banks onto the market. Hitherto, banks that served the interest of the few elite and concentrated on investment banking, now facing an increasing competition from these new banks are now opening their doors to the poor in the Ghanaian society. The new banks are now serving all sectors of the Ghanaian society and not an elite few.

An important point to note is that the rural sector in a country like Ghana can grow only if cheaper credit (sub sized credits) is available to farmers for their short and medium term needs. This was one of the major reasons why governments, in the past, directed credits to that sector. While the commercial banks cater to the banking needs of the people in the cities and towns, there is another category of banks that looks after the credit and banking needs of the people living in the rural areas, particularly the farmers. Rural banks (micro finance institutions) have been sponsored by many commercial banks. These banks take care of the farmer-specific needs of credit and other banking facilities. They are owned, managed and patronized by the local people. Some of these banks also operate agencies to cater for communities that are located far from the bank's facilities. Savings mobilized through rural banks are invested in small-scale agricultural activities, cottage industries, transportation and trading. Rural banks also provide commercial banking services such as giving loans to people within the community in which they operate.

1.1.2 LOANS

In banking, a loan is a sum of money or consideration that an individual ,group or other legal entity borrows from another individual, group or legal entity (the latter being a financial institution) with the condition that it be returned or repaid at a later date and in most times with interest. The money received is known as the principal. Many Ghanaians depend on loans for various activities be it business or for personal purposes like marriage, building projects, ward’s education etc., and who better to provide these loans than financial institutions? The loan is generally provided at a cost, referred to as interest on the debt, which provides an incentive for the lender to engage in the loan.

Typically, the money is paid back in regular instalments, or partial repayments in an annuity, where each instalment is the same amount. This is done until the principal (initial amount taken) plus the interest has been amortized. Loan portfolios are the major assets of banks. The value of a loan portfolio depends on both the principal and interest owed and the average creditworthiness of the loans. Banks today are free to determine their interest rates within the given limits prescribed by the Central Bank. But due to the stiff competition in the financial sector, banks are not able to optimize their returns on these loan disbursements.

1.2 PROBLEM STATEMENT

The past few years have seen a phenomenal growth in the Ghanaian banking sector. Ghana's financial sector, according to the Bank of Ghana, is well capitalised, very liquid, profitable and recording strong asset growth. This has led to a very high competition among the financial institutions such that although the banks set their own interest rates, they are unable to get the best out of certain products they offer especially on loans.

Due to the risks involved in loan disbursements, some financial institutions even go bankrupt due to poor decisions. Most of those who also manage to stay above water do not get the best out of such loan disbursements. This does not ensure efficiency and profitability in the banking sector. These banks consequently cannot expand their capital-base in order to competitively keep being in business.

1.3 OBJECTIVES

With the singular truth of bad Loan Portfolio Management by most banks in Ghana, this study seeks to

i. Design a linear programming model to optimize the loans given out to people of different sectors of the economy using the financial loan records of CAL Bank Ghana Ltd.

ii. Serve as a scientific method for providing executives with an analytical and objective basis for decision making with regards to managing their loan portfolios.

iii. Help the Board of Directors of banks

a) to invest the bank's funds profitably for the benefit of shareholders and the protection of depositors

b) to grant loans on a sound and collectible basis

c) to serve the legitimate credit needs of their communities

d) determine the sectors of the banks that yield the best return on loan portfolios

e) and explore ways of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of banks.

1.4 METHODOLOGY

We are to find a mathematical way of helping banks in a keenly competitive sector of the economy broaden their capital base by optimizing returns on their huge portfolio of loans.

We model the bank's allocation of loans as a Linear Programming Problem (LPP) based strictly on the Bank's Loan Policy and its previous history on loan disbursements. Linear programming models deal with optimization problems that can be modelled with a linear objective function subject to a set of linear constraints. The model has three basic components; the objective function which is to be optimized (maximised or minimized), the constraints (limitation) and the non-negativity constraints. Linear programming is a mostly used mathematical optimization technique because of the simplicity of its solution algorithms.

The modelled LPP is then solved using the renowned Simplex Method. The Simplex method is an iterative procedure for solving Linear Programming Problems in a finite number of steps. The Simplex method was considered an appropriate method for solving the LPP developed as a result of its practical superiority and advantages over the other methods. The Simplex algorithm passes from vertex to vertex on the boundary of the feasible polyhedron, repeatedly increasing the objective function until either an optimal solution is found or, it is established that no solution exists.

Data for the study was obtained from CAL Bank Ghana Limited; a home born modern day bank which aspires to be a financial service institution of preference through delivery of quality service, using innovative technology and skilled personnel to achieve sustainable growth and enhanced stakeholder value. Results were tabulated and out of this, the LPP was modelled and solved using the Simplex Method.

1.5 JUSTIFICATION

The recent financial sector is a stiff, competitive one that has arisen from the ashes of the severe financial and reputational damage of the post-independence era. It has overcome several problems, but one of the existing few is the huge portfolio of bad loans. This stifles the tremendous potential banks have on the development of the economy and the people of Ghana. It does not allow banks to broaden their capital base in order to withstand the competition from most foreign banks with huge capital bases. Most Ghanaian banks wither away gradually and those who are able to stay in business are not able to carry out community development projects.

A more scientific way is thus needed to ensure efficient distribution of funds they have available for loans. This will ensure constant growth of these banks. The model proposed by this study will help the banks to get the best out of their portfolio of loans given the risks involved. When banks run efficiently, they are able to allocate a larger amount of their funds for social services in the community in which they operate. The banks will also know which part of the economy they should disburse their capital in the form of loans. This will help them avoid the problem of bad loans and in effect, maximize their profit. This study is thus, justified.

1.6 ORGANIZATION

The research is organized into five chapters. Chapter one which is the introduction, gives the background information of the study, statement of the problem, objectives of the study, scope and significance of the study, description of the methodology used and the organisation of the study.

Chapter two looks at review of related literature, which covers application of linear programming to portfolio selection, types of loan portfolio, portfolio segmentation, risk associated with loans and risk diversification.

Chapter three describes the methodology used for the study. It looks at the method of data collection, organizational profiles of the selected banks for the research.

Chapter four discusses and analyses the data collected.

Chapter five summarizes the various findings, conclusions and recommendations.

1.7 SUMMARY

In this chapter, a brief history of the Ghanaian banking system and its financial reforms has been reviewed. Some of their social responsibilities were given. The objectives of the work and the methodology used were also presented.

In the next chapter, we shall review some literature in the area of Portfolio Management and how Linear Programming is applied to it.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

In this chapter, we shall review journals, various writings and research concerning portfolio management and linear programming for bank portfolio management.

Portfolio management is the art and science of making decisions about investment mix and policy, matching investments to objectives, asset allocation for individuals and institutions, and balancing risk against performance (according to the investopedia). It is all about strengths, weaknesses, opportunities and threats in the choice of debt vis-à-vis equity, domestic vs. international, growth vis-à-vis safety and many other tradeoffs encountered in the attempt to maximize return at a given appetite for risk.

The work of Nobel Laureate Harry Markowitz in the early 1950s triggered a revolution in the profession of investment management. The doctoral thesis written by Markowitz (1952) at the University of Chicago dealt with portfolio selection and in it, he developed the basic portfolio model. The concepts of *efficient portfolio* and *efficient frontier* served as catalysts for the development of modern finance. The basic idea is that investors should rationally choose portfolios that offer the highest return for the least amount of risk. This is done by diversifying among several securities, reducing risk as compared to the combined risks of the constituents. Each portfolio can be classified along the axes of risk and return. Any portfolio that has a minimal amount of risk for a given amount of return is called efficient, and the line that connects these portfolios in a risk-return graph is called the efficient frontier. Although Markowitz focused on securities, his novel theories have found their way into many industries and environments. Optimal credit portfolio selection in banking and the optimization of energy distribution are just two examples (Koch 2012). Because of this work, Markowitz is often referred to as the "father of modern portfolio theory", and much subsequent research had been based on this effort (Sharpe, 1963, Fama, 1965 and Melnik, 1970).

The basic model, developed by Markowitz, derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of risk under a reasonable set of assumptions and derived the formula for computing the variance of the portfolio. This portfolio variance formulation indicated the importance of diversification for reducing risk, and showed how to properly diversify. The Markowitz model is based on certain assumptions. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return (Markowitz, 1952 and 1959).

2.1 THE EFFICIENT FRONTIER

The Full Variance Model developed by Markowitz is based on the assumption that the purpose of portfolio management is to minimize variance for every possible combination of the expected yield (Best and Grauer, 1991:980). It has been argued by Blume (1970) and Hodges and Brealey (1972) that the concept of the efficient frontier is basic to the understanding of portfolio theory. Assume that in the market place, there are a fixed number of common stocks in which a businessman can invest. Each of the securities has its own expected yield and standard deviation; others have the same standard deviation but vary in expected yield. The investor will select the security that offers the highest yield for a chosen level of risk exposure as presented by the standard deviation. It is assumed that investors try to minimize risk by minimizing the deviation from the expected yield, and this is done by means of portfolio diversification.

In 1993, Terri Gollinger and John Morgan, at the time working with Mellon Bank in Pittsburgh, published the pioneering article “Calculation of an Efficient Frontier for a Commercial Loan Portfolio” in the *Journal of Portfolio Management*. This article takes Markowitz’s portfolio theory to the banking sector and to the allocation and optimization of loan portfolios in particular. In their approach, the industry sectors take the place of securities in the Markowitz model, and the industry Zeta-scores are used as a proxy for risk. This Zeta-score represents the likelihood of a company going bankrupt in the next two years. Just as an investor searches for an optimal combination of risk and return in creating a portfolio of securities, a bank wants to extend loans to those industries that minimize risk for a given level of return (Koch, 2012).

2.2 LINEAR PROGRAMMING IN FINANCIAL MANAGEMENT

The use of linear and other types of mathematical programming techniques has received extensive coverage in the banking literature. Chambers and Chames (1961), as well as Cohen and Hammer (1967, 1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while Waterman and Gee (1963) and Fortson and Dince (1977) proposed less elegant formulations which were better suited for the small medium-sized banks. Several programming models have also been proposed for managing a bank's investment security portfolio, including those by Booth (1972). Baldirer et al., (1981) used linear programming model to solve fundamental issues facing senior bank management of Central Carolina Bank and Trust Company in structuring the bank's balance sheet of approximately $360 million.

2.3 LINEAR PROGRAMMING FOR BANK PORTFOLIO MANAGEMENT

James Tobin’s portfolio theory can be applied to bank portfolio management in that, a bank would maximize the rates of return of its portfolio of assets, subject to the expected degree of risk and liquidity. Chambers and Chames (1961), Cohen and Hammer (1967), Booth and Dash (1979) and others apply the linear programming model to the management of bank funds. By and large, banks do try to maximize the returns of their portfolio subject to legal policy, bounding and total asset constraints which denote riskiness and liquidity of the portfolio of assets. In a direct way, banks conform to the portfolio choice theory; balancing yield and liquidity against security. Although the computer cannot replace a manager, linear programming can serve as a useful guide.

Various portfolio theories have been propounded for the management of bank funds. Ronald I Robinson secondary reserves (1961) proposed four priorities of the use of banks funds. These include primary reserves (or protective investment), loans and advances (customer credit demand) and investment account (open market investment for income) in descending order of priority. His assessment has been fully supported in other works by Sheng-Yi and Yong (1988). A bank has to place primary reserves at the top of the priority in order to comply with the minimum legal requirement, to meet any immediate withdrawal demand by depositors and to provide a means of clearing cheques and credit obligations among banks. Secondary reserves include cash items from banks, treasury bills and other short-term securities. Bank should have to satisfy customers' loan demand before allocating the balance of the funds in the investment market.

Loans and investment are in fact complementary. According to Robinson (1961), investment should be tailored to the strength, seasonality and character of loan demand. He reiterated that banks that experience sharp seasonal fluctuations in loan demand need to maintain more liquidity in their investment programmme. Moreover, during a boom when loan demand is high and credit-worthy customers are available, banks should allocate more funds to loans and less funds to investment, and vice versa during recession when loan demand is low.

According to Robinson, (1961) the crucial banking problem is to resolve the conflict between safety and profitability in the employment of bank funds. The conflict is essentially the problem between liquidity and the size of the earning assets. Robinson suggested that where there is a conflict between safety and profitability, it is better to err on the side of safety. The best practice is identifying procedures that can bring out the optimal mixture of management of banks funds.

2.4 PORTFOLIO RISK AND REWARD

Banking is both a risk-taking and profit-making business, and bank loan portfolios should return profits commensurate with their risk. Although this concept is intellectually sound and almost universally accepted by bankers and examiners alike, banks have had difficulty implementing it. Over the years, volatility in banks' earnings usually has been linked to the loan portfolio. While there are many contributing factors including market forces, anxiety for income, poor risk measurement, and weak risk management, a common underlying factor has been banks' tendency to underestimate or underprice credit risk. Because bank managements and boards are responsible for serving their communities, achieving acceptable shareholder returns, and protecting the interests of depositors, they need to ensure that the loan portfolio provides consistent, reasonable returns. Individual credits and portfolio segments should be priced to provide reasonable shareholder while maintaining adequate capital and allowance levels.

The price (index rate, spread, and fees) charged for an individual credit should cover funding costs, overhead/administrative expenses, the required profit margin (generally expressed as a return on assets or equity), and risk. Funding costs are relatively easy to measure and incorporate into loan pricing. Measuring overhead and administrative costs is more complicated because banks traditionally have not had strong cost accounting systems. Additionally, common services with differing or ambiguous values to each user (What, for example, is the dollar value of loan review?) can be difficult to measure. The required profit margin is a straightforward concept and is usually derived from the strategic plan.

This leaves “risk,” which is the crux of the pricing dilemma. The methods used to incorporate risk into loan pricing decisions range from simple “pro rata” allocations of existing loan loss reserves and capital to complex estimations of default frequency and probability, loss levels, and loss volatility. Recent developments in credit and portfolio risk measurement and modeling are improving banks’ ability to measure and price risk more precisely and are facilitating the management of capital and the allowance for loan and lease losses. However, these methods require accurate risk measurement at the individual loan level and robust portfolio risk MIS. Even with these developments, the loan pricing decision is clouded. Banks often incorporate other revenues attributed to the lending “relationship” into the loan pricing decision. The lending relationship has been, and continues to be, used to win other business with the customer.

2.4.1 RISKS ASSOCIATED WITH LENDING

Risk is the potential that events, expected or unexpected, may have an adverse impact on the bank’s earnings or capital. There are about nine (9) categories of risk for bank supervision purposes. These risks are *credit, interest rate, liquidity, price, foreign exchange, transaction, compliance, strategic,* and *reputation*. Banks with international operations are also subject to *country risk* and *transfer risk*. These risks are not mutually exclusive; any product or service may expose the bank to multiple risks.

A key challenge in managing risk is to understand the interrelationships of the nine risk factors. Often, risks will be either positively or negatively correlated to one another. Actions or events will affect correlated risks similarly. For example, reducing the level of problem assets should reduce not only credit risk but also liquidity and reputation risk. When two risks are negatively correlated, reducing one type of risk may increase the other. For example, a bank may reduce overall credit risk by expanding its holdings of one- to four-family residential mortgages instead of commercial loans, only to see its interest rate risk soar because of the interest rate sensitivity and optionality of the mortgages. Lending can expose a bank's earnings and capital to all of the risks.

2.4.2 RISK IDENTIFICATION

Effective risk identification starts with the evaluation of individual credits. Rating the risk of each loan in timely credit evaluations is fundamental to loan portfolio management. Some banks apply risk ratings to relationships, others prefer to rate each facility, and still others rate both relationships and facilities. Risk ratings should also be applied to off-balance-sheet exposures like letters of credit and unfunded commitments that the bank is obligated to fund unless there is a default. These evaluations allow the prompt detection of changes in portfolio quality, enabling management to modify portfolio strategies and intensify the supervision of weaker credits in a timely manner.

After each loan has been risk rated, the ratings of individual credits should be reviewed, and they should be analyzed in the context of the portfolio segment and the entire portfolio. The analysis should ensure that ratings are consistently applied and should consider trends, migration data, and weighted average risk ratings. Risk ratings, when used in conjunction with other information (such as exception levels, past-due trends, and loan growth), can produce an instructive picture of asset quality and credit risk. Risk ratings can help the bank's portfolio managers in other ways as well; when they set underwriting standards, asset diversification goals, and pricing levels, for example.

2.5 CREDIT RISK AND ASSET PRICES

2.5.1 BOOM AND BUST CYCLES

The close relationship between macroeconomic cycles and boom and bust cycles in bank lending and asset prices has been described as a stylized fact by several authors dealing with financial stability. Two recent examples are Borio (2002) and Goodhart et al., (2004). Borio (2002) provided evidence about the cyclical co-movements between credit, asset prices and the macro-economy. Goodhart et al., (2004) analyze this dependency in the context of banking system liberalization and banking regulation during the last two decades. While these authors focus mainly on the past two decades, Bordo et al., (2001) point out that financial accelerator mechanisms and boom and bust cycles in a long term perspective were the rule rather than the exception. When banking systems were liberalized after the break down of the Bretton Woods System in the early 1970s banks were suddenly confronted with volatile exchange rates and interest rates, tighter margins and increased competition from financial markets. Due to this disintermediation process, banks often lost their biggest and safest borrowers in industry to the capital market. As a consequence banks began to increase lending to smaller and also riskier borrowers such as small and medium sized enterprises and persons. Banks also increasingly engaged in mortgage lending to households. Since such a larger and more dispersed pool of borrowers makes information acquisitions and monitoring more costly (compared to a small pool of large industry customers), the weight of collateralized lending increased. Goodhart et al., (2004) pointed out that this increasing weight of collateral as basis for bank lending automatically accentuates financial accelerator mechanisms described in the literature by Bernanke and Gilchrist (1999) or Kyotaki and Moore (1997).

Borio (2002) described a stylized pattern of such an accelerator mechanism or a financial cycle as we have observed it repeatedly in the past. The buildup of imbalances that trigger a crisis usually starts with booming economic conditions. This boom is accompanied by a climate of overly optimistic risk assessment, the gradual weakening of financing and credit constraints and hiking asset prices (in particular property and real estate prices). In this climate, financial and real imbalances are building up. At some point an essentially unpredictable trigger like an asset price drop or the interruption of an investment boom causes a sudden run down of financial buffers and once these buffers are exhausted and the contraction exceeds a certain threshold a full scale financial crisis occurs.

2.5.2 DATA ON ASSET PRICES AND DEFAULT

Since loan quality and asset prices both depend on the general macroeconomic conditions, these variables tend to be highly correlated. For instance, in the terminology of quantitative risk management, probabilities of default (credit risk) of individual borrowers are high at the same time when asset prices (market risk) are depressed. The distinction between market and credit risk, which has been common standard in the regulatory and supervisory community, has frequently been criticized by economists in the past (see for instance Hellwig (1995)). Academic research on quantitative risk management as well as risk management practitioners are currently undertaking substantial efforts to include these dependencies into their risk models and the integration of credit and market risk is an active field of research.

Following the work of Borio et al., (1994) the Bank for International Settlements (BIS) has constructed an aggregate API for several of the major industrial countries (Arthur (2004)). The aggregate application programming interface (API) of the BIS is a geometric weighted average of equity, commercial and residential real estate, the most important asset classes used in collateralized bank lending. The weights represent estimates of the shares of those assets in the total private sector wealth. While the aggregate API provides only a broad brush perspective on the risk of collateral values – and therefore LGD in a collateralized loan portfolio – we think that these data provide an excellent starting point to explore the order of magnitude by which credit risk measures are underestimated when collateral values (and thus recovery rates) are taken to be fixed or independent from credit risk. According to Crouhy et al., (2000), this assumption is currently used in most of the standard portfolio credit risk models used in the banking industry.

2.6 PROBABILITY OF LOSS ON LOAN PORTFOLIO

According to Klaus Rheinberger and Martin Summer in their credit risk portfolio models, three parameters drives loan losses: The Probability of Default by individual obligors (PD), the Loss Given Default (LGD) and the exposure at default (EAD). While the standard credit risk models focus on modelling the PD for a given LGD, a growing recent literature has looked closer into the issue of explaining LGD and of exploring the consequences of dependencies between PD and LGD. This literature is surveyed in Altman et al; (2003). Most of the papers on the issue of dependency between PD and LGD have been written for US data and usually find strong correlations between these two variables. The first papers investigating the consequences of these dependencies for credit portfolio risk analysis were Frye (2000a) and Frye (2000b) using a credit risk model suggested by Finger [1999] and Gordy (2000). The authors used a different credit risk model in the tradition of actuarial portfolio loss models and focus directly on two risk factors: an aggregate PD and an aggregate API as well as their dependence. The authors used this approach because their interest was to investigate the implications of some stylized facts on asset prices and credit risk that have frequently been found in the macroeconomic literature for the risk of collateralized loan portfolios. The authors also believe that the credit risk model we use gives us maximal flexibility with assumptions about the distribution of systematic risk factors.

There are a variety of models that try to capture the dependence between PD and LGD. These models are developed in the papers of Jarrow (2001), Jokivuolle and Peura (2003), Carey and Gordy (2003), Hu and Perraudin (2002), Bakshi et al. (2001), G¨urtler and Heithecker (2005) and Altman et al., (2004). Most of these papers look at bond data but some also cover loans. There is a literature that looks in some detail into the determinants of LGD. Acharya et al., (2003) investigated defaulted bonds, Duellmann and Trapp (2004) look into recoveries of US corporate credit exposures, Grunert and Weber (2005) investigated recoveries of German bank loans and Schuermann (2004) summarizes existing knowledge about recoveries. While these papers show a nuanced picture of the determinants of recoveries that consists of many microeconomic and legal features such as the industry sector in which exposures are held or the seniority of a claim, all papers find that macroeconomic conditions play a key role.

2.7 PORTFOLIO SEGMENTATION AND RISK DIVERSIFICATION

Risk diversification is a basic tenet of portfolio management. Concentrations of credit risk occur within a portfolio when otherwise unrelated loans are linked by a common characteristic. If this common characteristic becomes a common source of weakness for the loans in concentration, the loans could pose considerable risk to earnings and capital.

Managing the loan portfolio includes managing any concentrations of risk. By segmenting the portfolio into pools of loans with similar characteristics, management can evaluate them in light of the bank's portfolio objectives and risk tolerances and, when necessary, develop strategies for reducing, diversifying, or otherwise mitigating the associated risks. For many banks, "portfolio segmentation" has customarily meant dividing the loan portfolio into broad categories by loan types such as commercial and industrial loans, real estate loans, and consumer loans. As segmentation techniques became more sophisticated, banks identified industry concentrations. Although these divisions are a good starting point, the full benefits of portfolio segmentation can be realized only if the bank is able to form segments using a broader range of risk characteristics.

The defining characteristics of some useful segments are readily discernible - borrower's industry, geographic area, collateral, tenor, facility structure, and risk rating, for example. Other examples are: loans to consumers working for the same employer (or in the same industry), loans to commercial companies that are dependent on the same suppliers (or that sell to the same customers), loans with a common purpose or source of repayment (including guarantor support), loans to affiliated borrowers, and loans to industry sectors that are likely to react in a similar manner to a change - for instance, the trucking and airline industries because of their sensitivity to the price of oil. Because loans have multiple characteristics, it would not be unusual for a loan to be included in more than one portfolio segment. A construction loan, for example, might be included in a real estate concentration report, a geographic concentration report, and a report of non-amortizing loans.

Segmenting a portfolio and diversifying risk require comprehensive Management Information Systems (MIS). The MIS data base should include both on- and off-balance-sheet credit exposure. If a bank lacks adequate data on each loan or does not possess a system to "slice and dice" the data for analysis, management's ability to manage the loan portfolio is compromised. But identifying the concentration is only half the job; understanding the dynamics of the concentration and how it will behave in different economic scenarios is the other half. As information about a particular concentration is refined, its effect on the portfolio risk profile can be better evaluated. For example, a bank with a concentration in the communications industry should be able to monitor developments within that industry, track the performance of the portfolio segment, and relate developments in that segment to the overall portfolio. Banks should not be taking risks in industries or products they do not understand. Examiners must use their judgment to determine whether a bank's Management Information System (MIS) is sufficient to support effective concentration measurement and management.

There are a variety of techniques banks can use to manage portfolios and control concentration risk. The most common tool is setting exposure limits, or ceilings, on concentrations. Diversifying away from a limit can be accomplished by reducing certain exposures or increasing the borrower base. The reduction of exposures begins with a reassessment of individual borrowers' needs and requires considerable discipline. Nonetheless, it can be a useful tool to diversify risk over a larger customer base.

A bank can change the distribution of its assets by increasing the geographic diversification of borrowers; altering the bank's product mix (for example, by reducing commercial lending and increasing consumer lending); or changing the risk profile of the bank's target market (for example, by turning from middle-market, non-investment-grade customers to well-capitalized, investment-grade customers). Asset sales can also be used to manage concentrations. Banks sell whole loans, sell a portion of a loan into syndication, sell participations in a loan, and securitize certain types of loans. Each of these approaches entails risk/reward trade-offs that must be evaluated in light of the bank's strategic objectives.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

In this chapter, we discuss the manner in which our data was obtained and critically treat the topic of Linear Programming, with emphasis on the Simplex Algorithm as applied in this study.

3.1. THE SCOPE AND DATA SOURCES

The objective of this project is to offer a technique that could be applied in the selection of an optimal loan portfolio using a linear programming model. Linear programming approach is selected because it solves efficiently resource allocation problems in loans. Questionnaires were designed for the bank loan officer to obtain information on the portfolio management, percentage of default risk on loan application and the selection of optimal loan portfolio. The main sampling technique was purposive sampling procedure. The main branch was served with a questionnaire.

The questionnaire focused on how the bank adjust the loan condition to reflect the risk of lending and how to select a loan facility that would maximize profit with minimum resources and minimize risk on the loan portfolio in order to obtain an optimal portfolio.

Financial statement of the bank was used to determine the profitability of the bank and to find its lending management practices. The analysis of the financial statement will reveal the overall profit of the bank from loan and other financial activities. Quantitative data from primary and secondary sources were collected and analyzed using the linear programming model.

The simplex method works by iteratively improving the value of the cost function. This is accomplished by finding variable in the problem that can be increased at the expense of decreasing another variable in such a way as to effect an overall improvement in the cost function.

Secondary data was collected from published statement of the bank for the 2011 financial year to assess the banks financial standing. The financial statement would be used to assess the portfolio, outstanding loan at the end of each period, the percentage of loans to total inflows and provision on bad debts (CAL Bank, Annual Report 2011).

An analytical tool known as Quantitative Manager (QM) for Windows, which is a Management Scientific software, in linear programming was used to determine the optimum loan portfolio between the different loan types.

3.1.1 RESEARCH DESIGN

Based on the objectives of the study, both quantitative and qualitative methods were employed. The quantitative method involved the use of interviews to collect primary data from the Credit Manager. The interviews were conducted to provide data on interest rates on different portfolios.

On the other hand, the qualitative data were collected from secondary sources. These included: the amount of money to be disbursed in 2012, the probability of bad debt on each loan type.

In this study, the entire population of CAL Bank in Ghana was used. The data was collected from the Head Office of CAL Bank Limited and formulated as a Linear Programming Problem (LPP).

3.2 LINEAR PROGRAMMING

Simplistically, linear programming is the optimization of an outcome based on some set of constraints using a linear mathematical model. It is the problem of maximizing or minimizing a linear function over a convex polyhedron specified by linear and non-negativity constraints. Linear programming is a special case of mathematical programming (mathematical optimization). The linear programming theory falls within Convex Optimization Theory and is also considered to be an important part of Operations Research. The feasible region of a Linear Programming (LP) problem is a convex polyhedron, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists.

Linear programming is extensively used in business and economics, but may also be used to solve certain engineering problems. Linear programming is a considerable field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems. Certain special cases of linear programming, such as network flow problems and multi-commodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Likewise, linear programming is heavily used in microeconomics and company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory, such as *duality, decomposition,* and the importance of *convexity* and its generalizations.

3.2.1 REPRESENTATION OF LINEAR PROGRAMMING PROBLEMS

3.2.1.1 CANONICAL FORM

Linear programs are problems that can be expressed in canonical form as

$$\begin{array}{ll} Maximize & c^T x \\ subject\ to & Ax \leq b \\ and & x \geq 0 \end{array}$$

where x represents the vector of variables (to be determined), $\boldsymbol{c}$ and $\boldsymbol{b}$ are vectors of (known) coefficients, $\boldsymbol{A}$ is a (known) matrix of coefficients, and $()^T$ is the matrix transpose.

The expression to be maximized or minimized is called the *objective function* ($c^T x$ in this case). The inequalities $A\boldsymbol{x} \leq \boldsymbol{b}$ and $\boldsymbol{x} \geq \boldsymbol{0}$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions.

3.2.1.2 STANDARD FORM

Standard Form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts:

- A linear function to be maximized:
  - $f(x_1, x_{2)}) = c_1 x_1 + c_2 x_2$
- Problem constraints of the following form:
  - $a_{11}x_1 + a_{12}x_2 \leq b_1$
  - $a_{21}x_1 + a_{22}x_2 \leq b_2$
  - $a_{31}x_1 + a_{32}x_2 \leq b_3$
- Non-negative variables:
  - $x_1 \geq 0, x_2 \geq 0$

The problem is usually expressed in *matrix form*, and then becomes:

$$max\{c^T x | Ax \leq b; x \geq 0\}$$

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

3.2.1.3 AUGMENTED (SLACK) FORM

Linear programming problems must be converted into augmented form before being solved by the simplex algorithm. This form introduces non-negative slack variables to replace inequalities with equalities in the constraints. The problems can then be written in the following block matrix form:

$$Maximize\ Z:$$

$$\begin{bmatrix} 1 & -c^T & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} Z \\ X \\ X_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} \quad ; X, X_s \geq 0$$

where the X_s are the newly introduced slack variables, and Z is the variable to be maximized.

3.3 ALGORITHMS FOR SOLVING THE LP PROBLEM

3.3.1 SIMPLEX ALGORITHM

The simplex algorithm was developed by George Dantzig in 1947 and solves LP problems by constructing a feasible solution at a vertex of the polytope and then walks along a path on the edges of the polytope to vertices with non-decreasing values of the objective function until an optimum is reached for sure. The simplex algorithm has been proven to solve "random" problems efficiently, i.e. in a cubic number of steps, which is similar to its behavior on practical problems.

However, the simplex algorithm has poor worst-case behavior: Klee and Minty (1973) constructed a family of linear programming problems for which the simplex method takes a number of steps exponential in the problem size. In fact, for some time it was not known whether the linear programming problem was solvable in polynomial time, i.e. of complexity class P.

3.3.2 CRISS-CROSS ALGORITHM

Like the simplex algorithm of Dantzig, the criss-cross algorithm is a basis-exchange algorithm that pivots between bases. However, the criss-cross algorithm need not maintain feasibility, but can pivot rather from a feasible basis to an infeasible basis. The criss-cross algorithm does not have polynomial time-complexity for linear programming. In the worst case, both algorithms visit all 2^D corners of a (perturbed) cube in dimension D, known as the Klee-Minty cube.

3.3.3 CONIC SAMPLING ALGORITHM OF SERANG

Like other basis-exchange algorithms, Serang's conic sampling algorithm moves between vertices; but where the simplex algorithm moves along edges by removing and adding one basis at a time, the conic sampling method exchanges multiple bases at a time, and is not restricted to moving along edges of the polytope. Starting at a current vertex, the conic sampling method chooses a random vector that improves the objective value without violating any adjacent constraints. The algorithm then travels along this vector until a limiting constraint is encountered. From this point, the algorithm projects the objective vector orthogonal to this limiting constraint, and moves along this orthogonal projection until a new constraint is reached. This advancement and projection is repeated until a vertex is reached. Then, a new random vector is chosen. This process is repeated until no vector exists that can improve the objective without violating any local constraints, implying optimality. Essentially, the conic sampling method can be thought of as a vertex sampling method that randomly samples from the collection of vertices with improved objective value. If the vertices with superior objective value are sampled in a roughly uniform manner, then the expected runtime is logarithmic in the number of vertices (and thus polynomial). Sampling the vertices in this manner can permit large, beneficial jumps through the interior, and yield a substantial runtime improvement over the simplex method, especially when the number of constraints, and thus the number of potential vertices, is large; however, the tightest existing upper bound on the worst-case complexity of the conic sampling method is still exponential.

3.3.4 INTERIOR POINT ALGORITHMS

3.3.4.1 ELLIPSOID ALGORITHM, FOLLOWING KHACHIYAN

This is the first worst-case polynomial-time algorithm for linear programming. To solve a problem which has n variables and can be encoded in L input bits, this algorithm uses $O(n^4L)$ pseudo-arithmetic operations on numbers with $O(L)$ digits. Khachiyan's algorithm and his long standing issue was resolved by Leonid Khachiyan in 1979 with the introduction of the ellipsoid method. The convergence analysis has (real-number) predecessors, notably the iterative methods developed by Naum Z. Shor and the approximation algorithms by Arkadi Nemirovski and D. Yudin.

3.3.4.2 PROJECTIVE ALGORITHM OF KARMARKAR

Khachiyan's algorithm was of landmark importance for establishing the polynomial-time solvability of linear programs. The algorithm was not a computational break-through, as the simplex method is more efficient for all but specially constructed families of linear programs. However, Khachiyan's algorithm inspired new lines of research in linear programming. In 1984, N. Karmarkar proposed a projective method for linear programming. Karmarkar's algorithm improved on Khachiyan's worst-case polynomial bound (giving $O(n^{3.5}L)$). Karmarkar claimed that his algorithm was much faster in practical LP than the simplex method, a claim that created great interest in interior-point methods.

3.3.4.3 PATH-FOLLOWING ALGORITHMS

In contrast to the simplex algorithm, which finds an optimal solution by traversing the edges between vertices on a polyhedral set, interior-point methods move through the interior of the feasible region. Since then, many interior-point methods have been proposed and analyzed. Early successful implementations were based on *affine scaling* variants of the method. For both theoretical and practical purposes, barrier function or path-following methods have been the most popular since the 1990s.

The current opinion is that the efficiency of good implementations of simplex-based methods and interior point methods are similar for routine applications of linear programming. However, for specific types of LP problems, it may be that one type of solver is better than another (sometimes much better).

We use the renowned Simplex Algorithm of Dantzig for the purpose of this study by virtue of its efficiency and simplicity in solving various problems in industry.

3.4 THE SIMPLEX ALGORITHM

The Simplex Method is "a systematic procedure for generating and testing candidate vertex solutions to a linear program." (Gill et al. (1981) p. 337) It begins at an arbitrary corner of the solution set. At each iteration, the Simplex Method selects the variable that will produce the largest change towards the minimum (or maximum) solution. That variable replaces one of its compatriots that is most severely restricting it, thus moving the Simplex Method to a different corner of the solution set and closer to the final solution. In addition, the Simplex Method can determine if no solution actually exists. Note that the algorithm is greedy since it selects the best choice at each iteration without needing information from previous or future iterations.

The simplex algorithm operates on linear programs in standard form, that is, linear programming problems of the form;

$$minimize \qquad \boldsymbol{c}.\boldsymbol{x}$$

$$subject\ to \qquad \boldsymbol{Ax} = \boldsymbol{b}, \boldsymbol{x_i} \geq \boldsymbol{0}$$

with $x = (x_1, \dots, x_n)$, the variables of the problem,

$c = (c_1, \dots, c_n)$, the coefficients of the objective function,

A, a $p \times n$ matrix

and $b = (b_1, \dots, b_n)$, constants with $b_j \geq 0$.

There is a straightforward process to convert any linear program into one in standard form so this results in no loss of generality.

In geometric terms, the feasible region $\boldsymbol{Ax} = \boldsymbol{b}, \boldsymbol{x_i} \geq \boldsymbol{0}$ is a (possibly unbounded) convex polytope. There is a simple characterization of the extreme points or vertices of this polytope, namely $x = (x_1, \dots, x_n)$ is an extreme point if and only if the column vectors A_i where $x_i \neq 0$ are linearly independent. In this context such a point is known as a *basic feasible solution* (BFS).

It can be shown that for a linear program in standard form, if the objective function has a minimum value on the feasible region then it has this value on (at least) one of the extreme points. This in itself reduces the problem to a finite computation since there are a finite number of extreme points, but the number of extreme points is unmanageably large for all but the smallest linear programs.

It can also be shown that if an extreme point is not a minimum point of the objective function then there is an edge containing the point, so that the objective function is strictly decreasing on the edge moving away from the point. If the edge is finite then the edge connects to another extreme point where the objective function has a smaller value, otherwise the objective function is unbounded below on the edge and the linear program has no solution. The simplex algorithm applies this insight by walking along edges of the polytope to extreme points with lower and lower objective values. This continues until the minimum value is reached or an unbounded edge is visited, concluding that the problem has no solution. The algorithm always terminates because the number of vertices in the polytope is finite; moreover since we jump between vertices always in the same direction (that of the objective function), we hope that the number of vertices visited will be small.

The solution of a linear program is accomplished in two steps. In the first step, known as Phase I, a starting extreme point is found. Depending on the nature of the program this may be trivial, but in general it can be solved by applying the simplex algorithm to a modified version of the original program. The possible results of Phase I are that, either a basic feasible solution is found or that the feasible region is empty. In the latter case the linear program is called *infeasible*. In the second step, Phase II, the simplex algorithm is applied using the basic feasible solution found in Phase I as a starting point. The possible results from Phase II are either an optimum basic feasible solution or an infinite edge on which the objective function is unbounded below.

The simplex algorithm for solving the standard minimization problem is summarized below:

STEP 1

Convert to a system of equations by introducing slack variables to turn the constraints into equations, and rewriting the objective function in standard form.

STEP 2

Write down the initial tableau.

STEP 3

Select the pivot column: Choose the negative number with the largest magnitude in the bottom row (excluding the rightmost entry). Its column is the pivot column. (If there are two candidates, choose either one.) If all the numbers in the bottom row are zero or positive (excluding the rightmost entry), then you are done: the basic solution maximizes the objective function (see below for the basic solution).

STEP 4

Select the pivot in the pivot column: The pivot must always be a positive number.

For each positive entry **b** in the pivot column, compute the ratio **a/b**, where **a** is the number in the *Answer column* in that row. Of these test ratios, choose the smallest one. The corresponding number **b** is the pivot.

STEP 5

Use the pivot to clear the column by using Gaussian Elimination, and then re-label the pivot row with the label from the pivot column. The variable originally labeling the pivot row is the departing or exiting variable and the variable labeling the column is the entering variable.

STEP 6

Go to Step 3.

3.4.1 THE SIMPLEX TABLEAU

The simplex tableau is a tabular representation of the augmented matrix of the system of equations obtained from the objective function and the set of constraints. In developing a tabular approach we adopt these notations:

c_j = Objective function coefficients for variable j

b_i = Right-hand side coefficients (value) for constraint i

a_{ij}= coefficients of variable j in constraint i

c_b= Objective function coefficients of the basic variables.

$(c_j - z_j)$ = the net evaluation per unit of variable jth–variable.

$[A]$ matrix = the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

From Table 3.1, the top row of the table represents the c_j, the objective function coefficients.

The next row gives the headings for the various columns which are then followed by constraint coefficients. The z_j row and the $(c_j - z_j)$ row, which provides the current value of the objective function and the net contribution per unit of the jth variable respectively, are presented.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variable, with a set of constraints.

Table 3.1 General Form of The Simplex Tableau

Decision variables						Slack variables					
c_j		c_1	c_2	…	c_n	0	0	…	0	Solution	Objective function coefficients
c_n	Basic variable	x_1	x_2	…	x_n	s_1	s_2	…	s_m		Headings
0	s_1	a_{11}	a_{12}	…	a_{1n}	1	0	…	0		Constraint coefficients
…	s_2	a_{21}	a_{22}	…	a_{2n}	0	1	…	0		
0	…	…	…	…	…	…	…	…	…		
	s_m	a_{m1}	a_{m2}	…	a_{mn}	0	0	…	1		
	Z_j	Z_1	Z_2	…	Z_{mn}	Z_{11}	Z_{12}	…	Z_{1m}	Current value of objective function	
	$C_j - Z_j$	$C_1 - Z_1$	$C_2 - Z_2$	…	$C_n - Z_m$	$C_{11} - Z_{11}$	$c_{12} - Z_{12}$	…	$C_{1N} - Z_{1N}$		Reduced cost(net contribution /unit)

Let a linear program be given by a canonical tableau. The simplex algorithm proceeds by performing successive pivot operations which each give an improved basic feasible solution; the choice of pivot element at each step is largely determined by the requirement that this pivot improve the solution.

3.4.2 PIVOT OPERATIONS

The geometrical operation of moving from a basic feasible solution to an adjacent basic feasible solution is implemented as a pivot operation. First, a non-zero *pivot element* is selected in a non-basic column. The row containing this element is multiplied by its reciprocal to change this element to 1, and then multiples of the row are added to the other rows to change the other entries in the column to 0. The result is that, if the pivot element is in row r, then the column becomes the r-th column of the identity matrix. The variable for this column is now a basic variable, replacing the variable which corresponded to the r-th column of the identity matrix before the operation. In effect, the variable corresponding to the pivot column enters the set of basic variables and is called the *entering variable*, and the variable being replaced leaves the set of basic variables and is called the *leaving variable*. The tableau is still in canonical form but with the set of basic variables changed by one element.

3.4.3 ENTERING VARIABLE SELECTION

Since the entering variable will, in general, increase from 0 to a positive number, the value of the objective function will decrease if the derivative of the objective function with respect to this variable is negative. Equivalently, the value of the objective function is decreased if the pivot column is selected so that the corresponding entry in the objective row of the tableau is positive.

If there is more than one column so that the entry in the objective row is positive then the choice of which one to add to the set of basic variables is somewhat arbitrary and several entering variable choice rules have been developed.

If all the entries in the objective row are less than or equal to 0 then no choice of entering variable can be made and the solution is in fact optimal. It is easily seen to be optimal since the objective row now corresponds to an equation of the form

$$Z(x) = Z_B + nonnegative\ terms\ corresponding\ to\ nonbasic\ variables$$

Note that by changing the entering variable choice rule so that it selects a column where the entry in the objective row is negative, the algorithm is changed so that it finds the maximum of the objective function rather than the minimum.

3.4.4 LEAVING VARIABLE SELECTION

Once the pivot column has been selected, the choice of pivot row is largely determined by the requirement that the resulting solution be feasible. First, only positive entries in the pivot column are considered since this guarantees that the value of the entering variable will be nonnegative. If there are no positive entries in the pivot column then the entering variable can take any nonnegative value with the solution remaining feasible. In this case the objective function is unbounded below and there is no minimum.

Next, the pivot row must be selected so that all the other basic variables remain positive. A calculation shows that this occurs when the resulting value of the entering variable is at a minimum. In other words, if the pivot column is c, then the pivot row r is chosen so that b_r/a_{cr} is the minimum over all r so that $a_{cr} > 0$. This is called the ***minimum ratio test***.

If there is more than one row for which the minimum is achieved then a dropping variable choice rule can be used to make the determination.

3.4.5 SIMPLEX METHOD WITH MIXED CONSTRAINTS

Some Linear Programming problem may consists of a mixture of $\leq, =$, and $\geq$ signs in the constraints and for which we may wish to maximize or minimize the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing with problem with mixed constraints.

STEP1: Ensure that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by

$$Max\,W = -Min\,(-Z)$$

STEP2: For each constraints involving 'greater or equal to', we convert to 'less than or equal to', that is, constraints of the form

$$a_{21}x_1 + a_{22}x_2 \ldots + a_{2n}x_n \geq b_2$$

Is multiplied by negative one to obtain

$$-a_{21}x_1 - a_{22}x_2 \ldots - a_{2n}x_n \leq -b_2$$

STEP 3: Replace constraints

$$a_{21}x_1 + a_{22}x_2 \ldots + a_{2n}x_n = b_2$$

by $$a_{21}x_1 + a_{22}x_2 \ldots + a_{2n}x_n \leq b_2$$

and $$a_{21}x_1 + a_{22}x_2 \ldots + a_{2n}x_n \geq b_2$$

where the latter is written as

$$-a_{21}x_1 - a_{22}x_2 \ldots - a_{2n}x_n \leq -b_2$$

STEP 4: Form the initial simplex tableau

STEP 5: If there exist no negative entry appearing on the RIGHT HAND SIDE column of the initial tableau, proceed to obtain the optimum basic feasible solution

STEP 6: If there exist a negative entry on the Right Hand Side column of the initial tableau,

a) Identify the most negative at the Right Hand Side; this row is the pivot row.
b) Select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element.
c) Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation.

STEP 7: Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

3.4.6 SIMPLEX METHOD SOLUTIONS

The simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of three special cases has occurred.

These special cases are:

a) Alternative/Multiple optimal solutions
b) Unbounded solutions
c) Infeasible solutions

3.4.6.1 ALTERNATIVE OPTIMAL SOLUTIONS

The simplex method provides a clear indication of the presence of alternative or multiple, optimal solutions upon its termination. These alternative optimal solutions can be recognized by considering the $(C_j - Z_j)$ row. Assume that we are maximizing and remember that when all $(C_j - Z_j)$ values are all negative, we know that an optimal solution has been obtained. Now, the presence of an alternative optimal solution will be indicated by the fact that for some variable not in the basis, the corresponding $(C_j - Z_j)$ value will equal zero.

Thus, this variable can be entered into the basis, the appropriate variable can be removed from the basis, and the value of the objective function will not change. In this manner, the various alternative optimal solutions can be determined.

3.4.6.2 UNBOUNDED SOLUTIONS

In the case of an unbounded solution, the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity. Specifically, for a maximization problem we will encounter a simplex tableau having a non-basic variable whose $(C_j - Z_j)$ row value is strictly greater than zero. And for this same variable, all of the elements in its column will be zero or a negative value (i.e. every coefficient in the pivot column will be either negative or zero). Thus, in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators. Negative numbers in the denominators cannot be considered since this will result in the introduction of a basic variable at a negative level.

Zeros in the denominator will produce a ratio having an undefined value and would indicate that the entering basic variable should be increased indefinitely (i.e. infinitely) without any of the current basic variables being driven from the basis. Therefore, if we have an unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value. Generally, arriving at an unbounded solution indicates that the problem was originally mis-formulated within the constraint set and needs reformulation.

3.4.6.3 INFEASIBLE SOLUTION

An indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables, which should be driven to zero by the simplex method, will be present as a positive basic variable in the solution that appears to be optimal. For example, assuming one wish to solve a maximization problem in which artificial variables are required. Then, at some iteration one achieve a solution in which all the $(C_j - Z_j)$ values are zero or negative, but which has one or more artificial variables as positive basic variables.

When an infeasible solution is indicated the management science analyst should carefully reconsider the construction of the model, because the model is either improperly formulated or two or more of the constraints are incompatible. Reformulation of the model is mandatory for cases in which the no feasible solution condition is indicated.

3.5 DEGENERACY

A linear program is said to be degenerate if one or more basic variables have a value zero. This occurs whenever there is a tie in the minimum ratio prior to reaching the optimal solution. This may result in cycling, that is the procedure could possibly alternate between the same set of non-optimal basic feasible solutions and never reach the optimal solution.

In order to overcome this problem, the following steps may be used to break the tie between the key row tie

1) Select the rows where the ties are found for determining the key row.
2) Find the coefficient of the slack variable and divide each coefficient by the coefficients in the key column in order to break the tie. If the ratios at this stage do not break the tie, find the similar ratios for the coefficient of the decision variables.
3) Compare the resulting ratio column by column
4) Select the row which has the smallest ratio and this now becomes the key row.

3.6 DUALITY

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. In matrix form, we can express the primal problem as:

$$maximize \quad c^T x \quad\quad subject\ to\ Ax \leq b,\ x \geq 0$$

with the corresponding symmetric dual problem,

$$minimize \quad b^T y \quad\quad subject\ to\ A^T y \geq c,\ y \geq 0$$

There are two ideas fundamental to duality theory.

- the dual of a dual linear program is the original primal linear program.
- every feasible solution for a linear program gives a bound on the optimal value of the objective function of its dual.

The strong duality theorem states that if the primal has an optimal solution, x^*, then the dual also has an optimal solution, y^*, such that

$$c^T x^* = b^T y^*$$

The weak duality theorem states that the objective function value of the dual at any feasible solution is always greater than or equal to the objective function value of the primal at any feasible solution.

A linear program can also be unbounded or infeasible. Duality theory tells us that if the primal is unbounded then the dual is infeasible by the weak duality theorem. Likewise, if the dual is unbounded, then the primal must be infeasible. However, it is possible for both the dual and the primal to be infeasible.

3.7 SENSITIVITY ANALYSIS

Suppose that you have just completed a linear programming solution which has a major impact. How much will the result change if your basic data is slightly wrong? Will it give a completely different outcome, or change the outcome only slightly? These are the kind of questions addressed by sensitivity analysis. It allows us to observe the effect of changes in the parameters in the LP problem on the optimal solution. It is also useful when the values of the problem parameters are not known. This sort of examination of impact of the input data on output results is very crucial.

There are several ways to approach sensitivity analysis. If your model is small enough to solve quite quickly, you can simply change the initial data and solve the model again to see what results you get. At the extreme, if your model is very large and takes a long time to solve, you can apply formal methods of classical sensitivity analysis. The classical methods rely on the relationships between the initial tableau and any later tableau to quickly update the optimum solution when changes are made to the coefficient of the original tableau.

3.8 SUMMARY

Linear Programming and the Simplex method were discussed in this chapter. In the next chapter, the data collected from the bank will be analyzed.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

This chapter presents data collection and analysis of the study. A linear programming model is proposed to solve the problem. The data to be analyzed was taken from Cal Bank Limited.

4.1.0 PROFILE OF CAL BANK LIMITED

4.1.1 ORGANIZATIONAL PROFILE

CAL Bank commenced operations in July 1990, and is considered to be one of the most innovative banks in Ghana. The Bank mobilizes resources in world financial markets, and channels them to the Ghanaian market. In this way, CAL Bank supports the development of the national economy, focusing particularly on the manufacturing and export sectors.

With its highly skilled professional staff, CAL Bank plays an important role in the Ghanaian financial sector by providing wholesale banking services to corporate clients with sound financial bases and competent management. Emphasis is placed on the economic viability and technical feasibility of each project, as well as the marketability of the client's products and services.

4.1.2 VISION

The vision of CAL Bank Limited is to be a leading financial services group creating sustainable value for shareholders. (Source: CAL Bank Limited)

4.1.3 MISSION

CAL Bank Limited aspires to be a financial services institution of preference through delivery of quality service, using innovative technology and skilled personnel to achieve sustainable growth and enhanced stakeholder value.

4.1.4 CORE VALUES

CAL Bank differentiates its self from the other banks through its values. The core values shape the way the bank is run and interact as a team. The Bank has five core values - all of which reflect the company's belief that its people are as critical to its success as its customers. The bank is;

- Passionate about service
- Efficient and effective
- Proactive innovators
- Result focused
- Flexible and decisive

4.1.5.0 PRODUCTS AND SERVICES OFFERED

4.1.5.1 DOMESTIC OPERATIONS SERVICES

CAL Bank offers divers range of domestic operations to its high valued clients including: Personal Banking (current account, flexisave account, kiddysave account, student account, premium account). Electronic Products and Services (CAL Alert, CAL SMS Banking, Ezwich, VISA Electron Card).Business Banking (Corporate Current account, Corporate Savings Account, Corporate Premium Account).Wealth Management (CAL Investplus, Fixed Deposit, Call Account).International Remittances (MoneyGram, Vigo, Western Union).Services (Corporate and Institutional Banking, Corporate Finance, Private Banking and Retail & Business Banking and provision of advisory services). (Source: CAL Bank Limited)

4.1.5.2 INTERNATIONAL OPERATIONS SERVICES

CAL Bank offers quality and satisfactory foreign exchange services to its existing and potential clients, which includes:

Establishment of Letters of Credit, Bills for Collection, Inward Remittance, Payment and Transfer of funds (Advance Payment, open Account payment and Direct Funds Transfer) Correspondent Banking Operations, Clean Collections and Foreign teller Services, Advisory Services.

4.1.5.3 TREASURY SERVICES

CAL Bank's Treasury is at the forefront of innovation in the domestic, foreign exchange and money markets. This is done by providing services required to meet clients' foreign exchange needs on cost-effective basis, facilitate international trade transactions and also ensure effective management of investments. Treasury also facilitates foreign payments through the establishment of Letters of Credit, Collections and Direct transferred on behalf of clients and it also liaises with other functional departments to meet the foreign exchange needs of clients.

4.1.5.4 RETAIL AND BUSINESS BANKING

CAL Bank's Retail and Business Banking provides innovative banking solutions to both individuals and Small and Medium Scale Enterprises (SME's). They offer advisory services to individuals and businesses on investments as well as introduce our clients to the Bank's range of range of well-tailored transactional and investment products.

To effectively deliver these unique services, the SME clients have been further categorized into four tiers; Platinum, Gold, Silver and Bronze, according to the size of their turnovers, customer growth, business structure, net interest and non-interest income contribution. The bank has several credit facilities and business loans, namely; Pharma Scheme, School Scheme, Fuel Scheme, Working Capital Financing, Auto Loans, Contract/LPO Financing.

4.1.5. 5 CORPORATE AND INSTITUTIONAL BANKING

CAL Bank's Corporate and Institutional Banking is responsible for managing the financial solutions to corporate, multinationals, large domestic companies. Some of the special products offered include: the provision of overdrafts, medium and long term loan financing, foreign currency loans, international trade finance (such as Letters of Credit and Guarantees), account and liquidity management services, payment solution and collection services.

4.1.5.6 CORPORATE FINANCE

CAL Bank's Corporate Finance operations focuses on excellence; the hallmark of a service oriented bank. Services provided include;

Company valuations and advisory services on merge acquisitions and corporate divestitures, loan syndications, project analysis and evaluations, sourcing for local and foreign debit equity financing, target market identification and identification of potential investors, research/sector studies investment analysis. (Source: CAL Bank Limited)

The bank continuously leverages its expertise in investment banking to provide a comprehensive array of corporate advisory services. The bank's investment professionals bring extensive capital market experience to structuring innovative financing solutions to fit clients' needs.

For this study, data was collected from CAL Bank Limited with focus on the loan records for the 2011 financial year. Analysis would be based on the amount allocated to each portfolio, associated bad debts and the interest rate on each type of loan. Table 4.1 presents different loan types given out by the bank, the interest charged on each loan type, and the probability of bad debt associated with each loan type.

Table 4.1 Loan Types, Interest Rates and Probability of Bad Debt.

Loan Types	Interest rates (%)	Probability of bad debt
Agriculture, Forestry & Fishing	0.30	0.07
Mining and Quarrying	0.28	0.02
Manufacturing	0.30	0.1
Construction	0.31	0.09
Electricity, Gas and Water	0.28	0.01
Commerce and Finance	0.28	0.01
Transport, storage and Communications	0.30	0.02
Services	0.30	0.01
Personal	0.29	0.01

The amount to be given out as loans in 2012 financial year is four hundred and eleven million, five hundred and eighty two thousand Ghana cedis (GH¢411,582,000.00).

4.2.0 PORTFOLIO SELECTION

The bank has GH¢411,582,000.00 to invest as loans.

Decision: How much to invest in each of the nine (9) investment options?

Objective: Maximize interest earned

4.2.1 THE MODEL OF THE PROBLEM

The variables of the model can be defined as follows:

$x_1 =$ Agriculture, Forestry and Fishing

$x_2 =$ Mining and Quarrying

$x_3 =$ Manufacturing

$x_4 =$ Construction

$x_5 =$ Electricity, Gas and Water

$x_6 =$ Commerce and Finance

$x_7 =$ Transportation, Storage and Communication

$x_8 =$ Services

$x_9 =$ Personal

The Bank's policy is to allocate:

- at least 20% of the total fund to Agriculture, Forestry and Fishing, Construction, Commerce and Finance and Personal.
- at most 10% of the total fund to Construction.
- at most 25% of the total fund to Mining and Quarrying, and Construction
- at most 17% of the total fund to Commerce and Finance, Transportation, Storage and Communication, and Service.
- at most 12% of the fund to Manufacturing and Electricity, Gas and Water.
- at most 9% of the total fund to Personal Loans
- at most 29% of the total fund to Agriculture, Forestry and Fishing, and Mining and Quarrying.
- at least 2% of the total fund to Transport, Storage and Communication.
- at least 14% of the total fund to Manufacturing, Commerce and Finance.
- at least 2.5% to Manufacturing.
- The bank also has a stated policy specifying that the overall ratio for bad debt on loan must not exceed 0.04.

The objective of CAL Bank is to maximize its net return comprised of the difference between the revenue from interest and lost funds due to bad debts. Since bad debts are not recoverable, both as principal and interest, the objective function may be written as:

$$\begin{aligned} Maximize\ Z = {} & 0.3(0.93x_1\ ) + 0.28(0.98x_2) + 0.3(0.99x_3) \\ & + 0.31(0.81\ x_4)\ \ 0.28(0.99\ x_5) + 0.28(0.99\ x_6) + 0.3(0.98\ x_7) \\ & + 0.3(0.9x_8) + 0.29(0.99\ x_9) - 0.07x_1 - 0.02x_2 - 0.1x_3 - 0.09x_4 \\ & - 0.01x_5 - 0.01\ x_6 - 0.02x_7 - 0.01x_8 - 0.01\ x_9 \end{aligned}$$

This function simplifies to:

$$Maximize\ Z = 0.209\, x_1 + 0.2544\, x_2 + 0.197\, x_3 + 0.183\, x_4 + 0.2672\, x_5 + 0.2672\, x_6 + 0.274\, x_7 + 0.287\, x_8 + 0.2771\, x_9$$

4.2.2 CONSTRAINTS

1. Total funds

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \leq 411{,}582{,}000$$

2. Agriculture, Forestry and Fishing, Construction, Commerce and Finance, and Personal Loans

$$x_1 + x_4 + x_6 + x_9 \geq 82{,}316{,}400$$

3. Construction

$$x_5 \leq 41{,}158{,}200$$

4. Mining and Quarrying, and Construction

$$x_2 + x_4 \leq 102{,}895{,}500$$

5. Commerce and Finance, Transport, Storage and Communication, and Services

$$x_6 + x_7 + x_8 \leq 69{,}968{,}940$$

6. Manufacturing and Electricity, Gas and Water

$$x_3 + x_5 \leq 49{,}389{,}840$$

7. Personal

$$x_9 \leq 28{,}810{,}740$$

8. Agriculture, Forestry and Fishing, and Mining and Quarrying

$$x_1 + x_2 \leq 119{,}358{,}800$$

9. Transport, Storage and Communication

$$x_7 \geq 8{,}231{,}640$$

10. Manufacturing, Commerce and Finance

$$x_3 + x_6 \geq 57{,}621{,}480$$

11. Manufacturing

$$x_3 \geq 10{,}289{,}550$$

12. Bad Debt

$$0.03x_1 - 0.02x_2 + 0.6x_3 + 0.05x_4 - 0.03x_5 - 0.03x_6 - 0.02x_7 - 0.03\,x_8 - 0.03x_9 \leq 0$$

Solving the above Linear Programming Problem using the QM for Windows, the final tableau is given below;

Table 4.2 The Final Tableau of the Simplex Method

(Untitled) Solution

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9		RHS	Dual
Maximize	0.209	0.2544	0.197	0.183	0.2672	0.2672	0.274	0.287	0.2771			
Constraints 1	1.	1.	1.	1.	1.	1.	1.	1.	1.	$\leq$	411,582,000	0.
Constraints 2	1.	0.	0.	1.	0.	1.	0.	0.	1.	$\geq$	82,316,400	0.
Constraints 3	0.	0.	0.	1.	0.	0.	0.	0.	0.	$\leq$	41,158,200	0.
Constraints 4	0.	1.	0.	1.	0.	0.	0.	0.	0.	$\leq$	102,895,500	0.
Constraints 5	0.	0.	0.	0.	0.	1.	1.	1.	0.	$\leq$	69,968,940	0.0945
Constraints 6	0.	0.	1.	0.	1.	0.	0.	0.	0.	$\leq$	49,389,840	0.2772
Constraints 7	0.	0.	0.	0.	0.	0.	0.	0.	1.	$\leq$	28,810,740	0.2574
Constraints 8	1.	1.	0.	0.	0.	0.	0.	0.	0.	$\leq$	119,358,800	0.1402
Constraints 9	0.	0.	0.	0.	0.	0.	1.	0.	0.	$\leq$	8,231,640	0.0228
Constraints 10	0.	0.	1.	0.	0.	1.	0.	0.	0.	$\leq$	57,621,480	0.0198
Constraints 11	0.	0.	1.	0.	0.	0.	0.	0.	0.	$\leq$	10,289,550	0.1389
Constraints 12	0.03	-0.02	0.06	0.05	-0.03	-0.06	-0.02	-0.03	-0.03	$\leq$	0	0.9829
Solution	34,543,500	84,815,300	10,289,550	18,080,200	39,100,290	47,331,930	8,231,640	14,405,370	28,810,740		71,600,280.08	

Table 4.2 also shows that the dual value linking to Constraint 6 is 0.2772, which indicates that the optimal profit will increase by 0.2772 for each increase in the total amount to be disbursed. It is therefore advisable to increase the amount available.

The optimal Solution is;

x_1 = 34,543,500

x_2 = 84,815,300

x_3 = 10,289,550

x_4 = 18,080,200

x_5 = 39,100,290

x_6 = 47,331,930

x_7 = 8,231,640

x_8 = 14,405,370

x_9 = 28,810,740

With maximum profit of GH¢71,600,280.08

4.3 SUMMARY

This chapter was devoted for data collection and analysis of the study.

In the next chapter, which is the last chapter of the study, we shall consider the summary of the findings, conclusions and recommendations of the study.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter presents the summary of the results, conclusions and recommendations of the study.

5.1 SUMMARY

The thesis was conducted in order to analyze the possibility of improving the profit on loans given out by the CAL Bank Limited. Data on loan types, corresponding interest rates and probability of bad debt on each loan type were gathered. The problem was modeled as a linear programming problem. The Quantitative Manager for Windows (QMW) software was used to solve the problem.

5.2 CONCLUSION

It was observed that out of the total amount of four hundred and eleven million, five hundred and eighty-two thousand Ghana cedis (GH¢411,582,000) to be disbursed as loan in 2012 financial year, thirty-four million, five hundred and forty-three thousand, five hundred Ghana cedis (GH¢34,543,500) should be allocated to Agriculture, Forestry and Fishing sector.

The Mining and Quarrying sector should receive an amount of eighty-four million and eight hundred and fifteen thousand, three hundred Ghana cedis (GH¢84,815,300.00). Also Manufacturing should be given ten million two hundred and eighty-nine thousand five hundred fifty (GH¢10,289,550.00), eighteen million eighty thousand two hundred Ghana cedis (GH¢18,080,200.00) should be allocated to the Construction sector. The Electricity, Gas and Water sectors should be allocated thirty-nine million seven hundred and one hundred thousand two hundred and ninety Ghana cedis (GH¢39,100,290.00).

The Commerce and Finance sector should be allocated forty-seven million, three hundred and thirty one thousand, nine hundred and thirty Ghana cedis (GH¢47,331,930.00).

Transport, Storage and Communication sector should be allocated eight million, two hundred and thirty-one thousand, six hundred and forty Ghana cedis (GH¢8,231,640.00). The Services sector should be allocated fourteen million, four hundred and five thousand, three hundred and seventy Ghana cedis (GH¢14,405,370.00). For Personal loans, twenty eight million, eight hundred and ten thousand, seven hundred and forty Ghana cedis (GH¢28,810,740.00) should be allocated.

With these allocations, CAL Bank would be able to make a maximum profit of seventy-one million, six hundred thousand, two hundred and eighty Ghana cedis, eight pesewas (GH¢71,600,280.08) on loans.

5.3 RECOMMENDATIONS

One of the most successful and important application of quantitative analysis to solving business problems has been in the areas of given out loans. From the conclusions it was realized that using quantitative methods to give out loans help banks to increase their profits. It is therefore recommended that CAL Bank Limited should adopt this model in their allocation of funds reserved for loans.

Secondly, it is recommended that banks be educated to employ quantitative method to find an appropriate quantitative model to help them disburse funds of the banks more efficiently.

Finally, Management should evaluate the adequacy of credit Management information systems (MIS). All evaluations of MIS should assess timeliness, accuracy, level of detail, clarity of report format, and distribution channels.

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APPENDIX

KWAM NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

RESEARCH QUESTIONNAIRE

This questionnaire seeks to collect data for the study 'Optimum Loan Portfolio' Case study of CAL Bank Limited, Ghana.

The data is solely needed for academic purpose and would be conducted in a highly confidential manner. Thank you.

Please tick the appropriate one.

1. Which of the following sectors did the bank invest in 2011?

A. Agriculture, Forestry and Fishing [ ]   B. Mining and Quarrying [ ]   C. Manufacturing [ ]

D. Construction [ ]   E. Electricity, Gas and Water [ ]   F. Commerce and Finance [ ]

G. Transportation, Storage and Communication [ ] H. Services [ ]   I. Personal [ ]

2. How much was proposed for investment in loans for 2011?

   ………………………………………………………………………………………..

3. What are the respective interest rates?

A. Agriculture, Forestry and Fishing…….   B. Mining and Quarrying …….

C. Manufacturing…….   D. Construction……..   E. Electricity, Gas and Water……..

F. Commerce and Finance ……..   G. Transportation, Storage and Communication ………

H. Services ………   I. Personal ………

4. Does CAL Bank make provision for bad debt on loans to be given out?

Yes [ ]   No [ ]

5. What is the probability of bad debt on each portfolio?

A. Agriculture, Forestry and Fishing…….   B. Mining and Quarrying …….

C. Manufacturing…….   D. Construction……..

E. Electricity, Gas and Water……..   F. Commerce and Finance ……..

G. Transportation, Storage and Communication ………

H. Services ………   I. Personal ………