TÉCNICO LISBOA

Laboratory 1: Circuit Analysis Methods

Msc. Aerospace Engineering, Técnico, University of Lisbon

Circuit Theory and Electronics Fundamentals

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1 Introduction

The laboratory assignment presented has for its purpose the study of a circuit structured in four elementary meshes, through which exist seven resistors R_i, a voltage source V_a, a current controlled voltage source V_c, a current source I_d and a voltage controlled current source I_b. The circuit can be seen in Figure-1.

Throughout the report it is presented a theoretical analysis, a simulation of the circuit and its analysis as well as a comparison of the obtained results.

In Section 2, both mesh and nodes methods are applied, in order to do a theoretical analysis of the circuit, using the Octave maths tool. In Section 3, it is executed an analysis of the circuit using the Ngspice tool to simulate it. Lastly, in Section 4, it is performed a comparison between the results from both the theoretical analysis and the simulation, from Section 2 and Section 3, respectively.

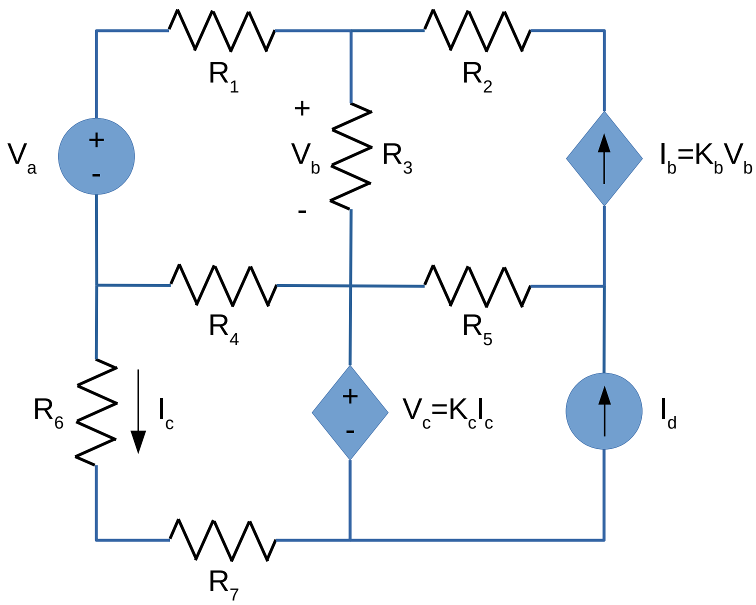

Figure 1: Circuit

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. This analysis was made with two different methods: the **Mesh Method** (Subsection 2.1) and the **Node Method** (Subsection 2.2), both based on circuit analysis basic principles:

- **Kirchhoff Voltage Law (KVL):** This law states that the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero. This means, that the sum of all the potential differences in a closed loop must be zero. This law is directly related with the Energy Conservation principle: if one circulates the loop in one direction, one will end up just where has started. Therefore, there cannot be any potential difference between the start and the end node, since they are the same node and energy cannot be lost. Notice, that for this assumption to be true, polarities and signs of the elements must be taken in account:

$$\sum_{i=1}^{n} V_i = 0$$

- **Kirchohff Current Law (KCL):** This law states that the algebraic sum of all the currents entering a node is equal to algebraic sum of all the currents leaving that same node. This law is also known as the Charge Conservation law: when a current enters a node it as no other option besides leaving the node, since that cannot be any current loss:

$$\sum_{i=1}^{n} I_i = 0$$

- **Ohm's Law:** Ohm's Law states that the current passing through a conductor is propotional to the voltage drop between the two terminals of that conductor:

$$V = RI,$$

  where R represents the proportionality constant (resistance).

2.1 Mesh Method

This method consists on assigning currents to the circuit meshes and solving the circuit to find the values of each mesh current, using KVL at each individual mesh that is not connected to any current source and Ohm's Law. In doing so, one ends up with a set of independent equations. At this point, there will be more unknows than equations, because some meshes contain current sources. So, it is necessary to find more equations in order to have a solvable system. This last equations can be found in the relations between the current sources and the currents assigned to the loops ($I_3 = I_b$ and $I_4 = I_d$). Finally, one can solve the system obtained by using its matricial form. The procedure for this particular circuit is shown bellow:

Assuming the currents representend in Figure 2:

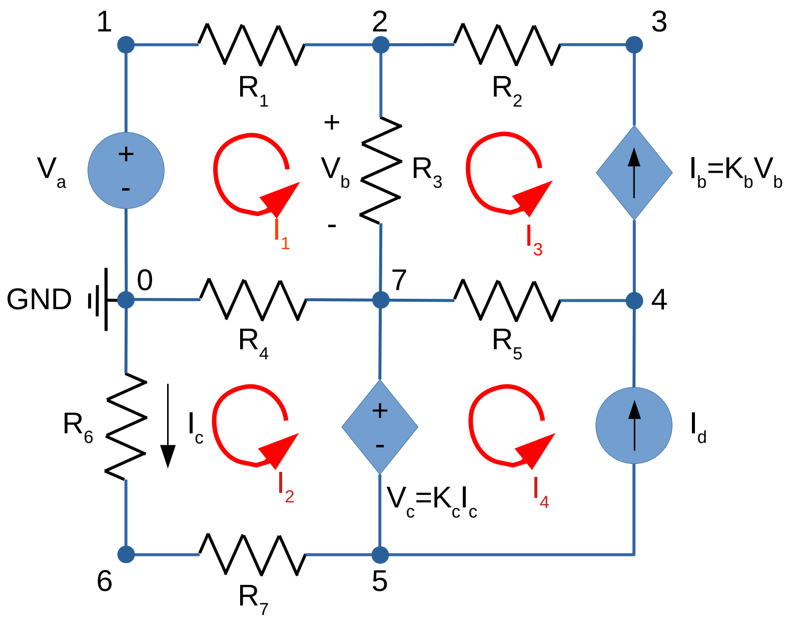

Figure 2: Mesh Currents Identification

$$
\begin{cases}
\text{Mesh 1: } R_1 I_1 + V_a + R_4(I_1 - I_2) + R_3(I_1 - I_3) = 0 \\
\text{Mesh 2: } R_4(I_2 - I_1) + R_6 I_2 + R_7 I_2 - K_c I_2 = 0 \\
\text{Current b: } I_3 = I_b = K_b V_b \\
\text{Current d: } I_4 = I_d \\
\\
\text{Ohm's Law: } V_b = R_3(I_3 - I_1)
\end{cases}
$$

We now can present this same system in its matricial form:

$$
\begin{bmatrix}
R_1 + R_3 + R_4 & -R_4 & -R_3 & 0 \\
-R_4 & R_4 + R_6 + R_7 - K_c & 0 & 0 \\
R_3 K_b & 0 & 1 - K_b R_3 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\ I_2 \\ I_3 \\ I_4
\end{bmatrix}
\begin{bmatrix}
-V_a \\ 0 \\ 0 \\ I_d
\end{bmatrix}
$$

Solving the system above, one can determine the current in each mesh, and from there compute each of the currents and voltages in the circuit.

In order to do this, in the next expression it is presented how to compute the current in each resistor from the mesh method's currents.

$$
\begin{cases}
I_{R_1} = -I_1 \\
I_{R_2} = I_3 \\
I_{R_3} = I_3 - I_1 \\
I_{R_4} = I_2 - I_1 \\
I_{R_5} = I_3 - I_4 \\
I_{R_6} = I_2 \\
I_{R_7} = I_2
\end{cases}
$$

2.2 Node Method

The Node Method consists on applying KCL in nodes not connected to voltage sources. In doing so, one can find a set of linnearly independent equations that represent the circuit. However, the number of unknows will be greater then the number of equations, because some nodes are connected to voltage sources, where the current values is at start, unknown. In order to find more equations, one can find relations between the currents imposed by the current sources and the currents in each branch (applying KCL to node 5, one gets $I_x = I_d - I_c$). Beyond, this relations it is also necessary to use Ohm's Law.

The identification of the directions of the currents is shown bellow in Figure 3:

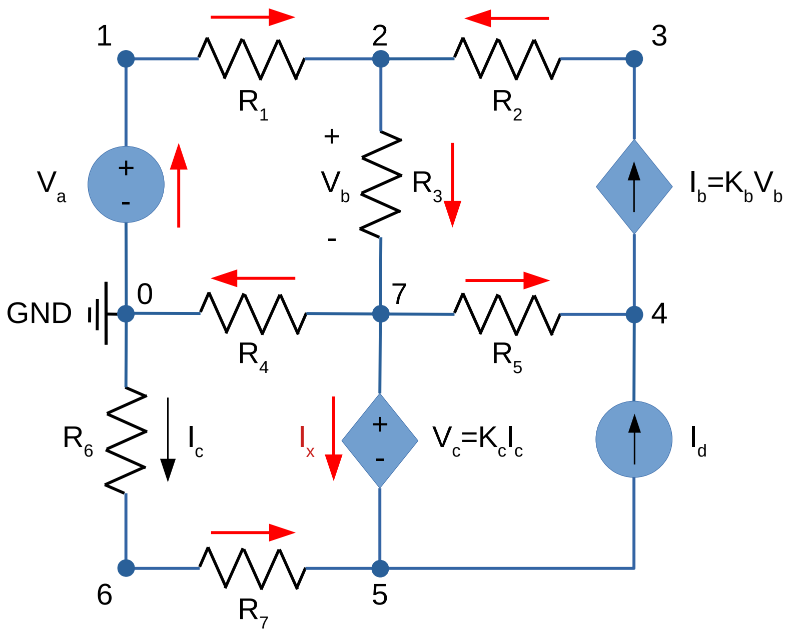

Figure 3: Currents Direction Identification

Now one can set a system of linnearly independent equations:

$$
\begin{cases}
\text{Node 1: } V_1 = V_a \\
\text{Node 2: } G_1(V_1 - V_2) + G_2(V_3 - V_2) - G_3(V_2 - V_7) = 0 \\
\text{Node 3: } -G_2(V_3 - V_2) + K_b V_b = 0 \\
\text{Node 4: } G_5(V_7 - V_4) + I_d - K_b V_b = 0 \\
\text{Node 5: } V_c = V_7 - V_5 \\
\text{Node 6: } G_6(V_0 - V_6) - G_7(V_6 - V_5) = 0 \\
\text{Node 7: } G_4(V_7 - V_0) + G_5(V_7 - V_4) + I_x - G_3(V_2 - V_7) = 0 \\
\\
\text{Current x: } I_x = I_d - I_c \\
\text{Ohm's Law: } I_c = G_6(0 - V_6) \\
\text{Voltage b: } V_b = V_2 - V_7
\end{cases}
$$

Writing the equations above in matricial form, one gets:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & 0 & 0 & 0 & G_3 \\ 0 & G_2+K_b & -G_2 & 0 & 0 & 0 & -K_b \\ 0 & -K_b & 0 & -G_5 & 0 & 0 & G_5+K_b \\ 0 & 0 & 0 & 0 & -1 & K_cG_6 & 1 \\ 0 & 0 & 0 & 0 & G_7 & -G_6-G_7 & 0 \\ 0 & G_3 & 0 & G_5 & 0 & -G_6 & -G_3-G_4-G_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_d \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

Solving this system, one obtains the voltage drops between each branch and the ground branch. Therefore, it is possible complete the circuit analysis.

Note: The results obtained in the Mesh Method must be the same as the results obtained in the Node Method.

The circuit analysis results are shown in the next subsection (2.3).

2.3 Theoretical Analysis Results

In this subsection it is presented the results obtained from solving the matrix equations presented in both Subsection 2.1 and Subsection 2.2

Name	Value [A or V]
I_b	-2.263725e-04
I_d	1.011815e-03
I_{R1}	2.161226e-04
I_{R2}	-2.263725e-04
I_{R3}	-1.024993e-05
I_{R4}	1.194589e-03
I_{R5}	-1.238187e-03
I_{R6}	9.784660e-04
I_{R7}	9.784660e-04
V_1	5.125627
V_2	4.903891
V_3	4.446215
V_4	8.768409
V_5	-2.982745
V_6	-1.975719
V_7	4.934963

Table 1: Current in each resitor, expressed in Ampere, and nodal voltages, expressed in Volts

3 Simulation Analysis

3.1 Operating Point Analysis

Table 2 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
gib[i]	-2.26373e-04
id[current]	1.011815e-03
r1[i]	2.161226e-04
r2[i]	-2.26373e-04
r3[i]	-1.02499e-05
r4[i]	1.194589e-03
r5[i]	-1.23819e-03
r6[i]	9.784660e-04
r7[i]	9.784660e-04
v(1)	5.125627e+00
v(2)	4.903891e+00
v(3)	4.446215e+00
v(4)	8.768409e+00
v(5)	-2.98275e+00
v(6)	-1.97572e+00
v(7)	4.934963e+00
v(8)	-1.97572e+00

Table 2: Operating point. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

As we can see, the simulation results are similar to the ones we obtained in the section 2, concerning both the numerical values and the directions. Note that, unlike the table 1, in the simulation results we present an extra voltage at node 8, V_8, that is a ”dummy” node used to compute the dependent voltage source.

4 Conclusion

After the theoretical analysis and the simulation, it can be concluded that the objective of the work, the study of the circuit presented in Figure-1, has been accomplished.

There were performed a theoretical analysis, applying mesh and nodes methods, using the Octave maths tool, and a circuit simulation, using the Ngspice tool, with which is clear a seamless match of the theoretical and the simulation results. The achievement of the equality in results comes from the components of the circuit, which are all linear and, therefore, both analysis have to present the same results. Furthermore, it comes from the fact that the models used to solve and analyse the circuit are similar in both the theoretical analysis and the simulation.