Laboratory 2: RC Circuit Analysis

Msc. Aerospace Engineering, Técnico, University of Lisbon

Circuit Theory and Electronics Fundamentals

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Contents

1 Introduction

In this laboratory assignment it will be analysed a circuit composed by four elementary meshes. The circuit includes seven resistors R_i, a voltage source V_s, a current controlled voltage source V_d, a voltage controlled current source I_b and a capacitor C. The circuit described is portrayed in Figure-1.

Throughout the report it is presented a theoretical analysis, a simulation of the circuit and its analysis as well as a comparison of the obtained results.

In Section 2, it is applyed the nodes methods for t<0 and V_s=0, it is determined the natural, forced ad total solutions for voltage v_6, such as the frequency responses for v_s, v_c and v_6, in order to do a theoretical analysis of the circuit, using the Octave maths tool. In Section 3, it is executed an analysis of the circuit using the Ngspice tool to simulate it. In this section it is presented a simulation of the operating point for t¡0 and for v_s=0, the natural and total responses on node 6 and the frequency response in the same node. In Sectione 4, reults obtained with both Octave and Ngspice are displayed side-by-side, in order to compare the results. Lastly, in Section 5, it is performed a conclusion, bearing in mind the results from both the theoretical analysis and the simulation, from Section 2 and Section 3, respectively.

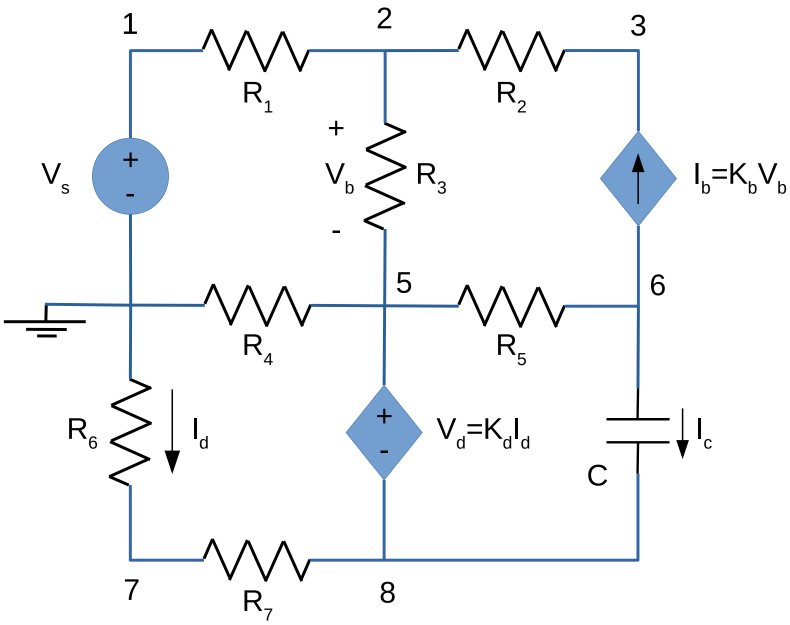

Figure 1: Circuit

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, in terms of its time and frequency responses.

2.1 Operating Point for t<0

In this section the circuit is analysed for t<0, according to the Figure 2, using the Node Method. Since the voltage source has a constant value $v_s(t) = V_s, t < 0$, and assuming the circuit is stabilized, one can conclude that the the voltage in the capacitor is constant, therefore, the current that passes through it is $i_c = \frac{dv_c}{dt} = 0$, and the capacitor can be replaced by an open circuit.

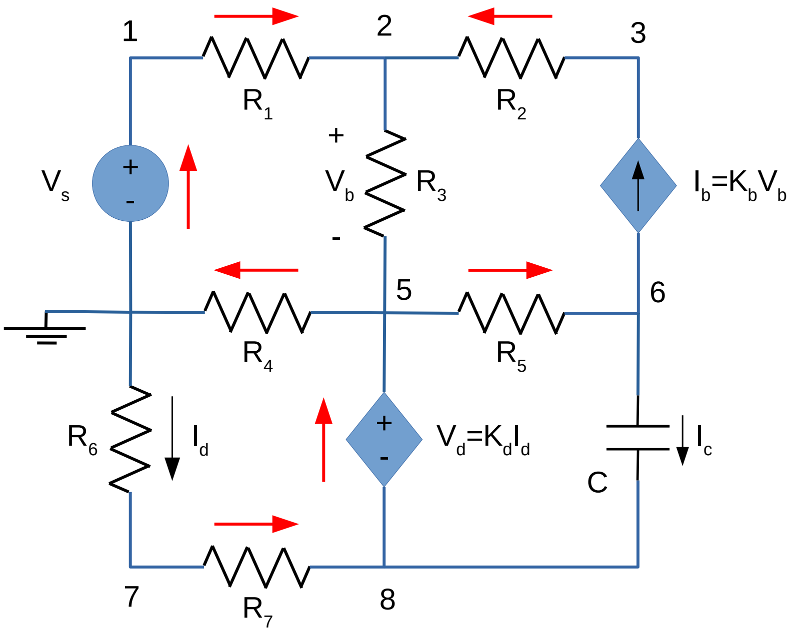

Figure 2: Currents Direction Identification

The system solved is shown below:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5 & G_7 & -G_7 \\ 0 & -K_b & 0 & K_b+G_5 & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The nodal voltages and the branch currents found by Octave to the system above are displayed in Table 1, respectively.

The results obtained show the behaviour of the circuit before V_s is oscilating.

Name	Value [V]
V_1	5.12562725920
V_2	4.90389094213
V_3	4.44621543662
V_4	0.00000000000
V_5	4.93496307069
V_6	5.63581571238
V_7	-1.97571911988
V_8	-2.98274501015

Name	Ampere [A]
I_1	0.00021612262
I_2	-0.00022637255
I_3	-0.00001024993
I_4	0.00119458862
I_5	-0.00022637255
I_6	0.00097846600
I_7	0.00097846600
I_b	-0.00022637255
I_c	-0.00000000000
I_{V_d}	0.00097846600
I_{V_s}	0.00021612262

Table 1: Results for the Circuit at $t < 0$

2.2 Node Analysis for $t = 0$ and $v_s = 0$

Throughout this section, it is computed the equivalent resistance, R_{eq}, as seen from the capacitor terminals. To achieve that, we make $v_s = 0$ and replace the capacitor with the voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8, respectively.

The goal of this step is to determine the initail conditions of the circuit and ensure the continuity of the voltage. The reasons to do it are further developed in Section 3.2. The matricial form of the system solved is shown below:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2 + K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_d * G_6 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -G_6 - G_7 & G_7 \\ G_4 & G_3 & 0 & -G_3 - G_4 & 0 & G_6 + G_7 & -G_7 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{bmatrix}$$

In Table 2, the results from the matricial system are presented.

Name	Value [V]
V_1	0.00000000000
V_2	0.00000000000
V_3	0.00000000000
V_4	0.00000000000
V_5	0.00000000000
V_6	8.61856072253
V_7	0.00000000000
V_8	0.00000000000

Table 2: Nodal voltages, expressed in Volts, when V_s=0

By creating a Thèvenin equivalent circuit, as seen from the voltage source's terminals (V_x) and solving this matricial system, one can compute the current I_x passing on the voltage source V_x, using the formula:

$$I_x = G_5 * (V_5 - V_6) - K_b * (V_2 - V_5). \tag{1}$$

In doing so, one can compute the equivalent resistance, R_{eq}, by dividing the voltage V_x over the current I_x.

$$R_{eq} = \frac{V_x}{I_x}. \tag{2}$$

Name	**Value**
I_x	-0.00278375998 A
V_x	8.61856072253 V
R_{eq}	3096.01431108000 Ohm

Table 3: Equivalent resistance, voltage V_x and current I_x, when v_s=0

2.3 Natural Solution

In this section the Thévenin Equivalent circuit is solved. The circuit consists of a single V-R-C loop where a current I_x circulates. The voltage source v_s drives its input, and the output voltage v_x is taken from the capacitor terminals. Applying the Kirchhoff Voltage Law (KVL), a single equation for the single loop in the circuit can be written as

$$Ri_x + v_x = v_s. \tag{3}$$

Since $v_x = v_6 - v_8$, assuming voltage v_8 as the ground, one obtains that $v_x = v_6$.

Because $v_x = v_6$ is the voltage between capacitor C's plates, it is related to the current i_x by

$$i_x = C\frac{dv_6}{dt}. \tag{4}$$

Hence, Equation (3) can be rewritten as

$$RC\frac{dv_6}{dt} + v_6 = v_s. \tag{5}$$

To find the natural solution we set $v_s = 0$. As learned in the theory classes the natural solution is of the form

$$v_{6n}(t) = v_{6_0}e^{-\frac{t}{RC}}, \tag{6}$$

where R is the equivalent resistance computed in Section 2.2.

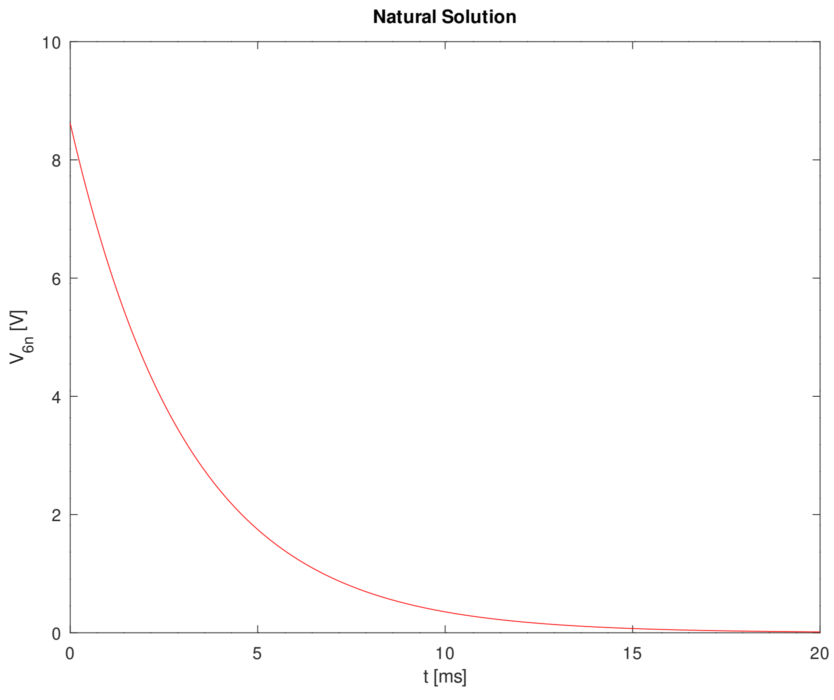

Figure 3: Natural Solution - v_6

2.4 Forced Solution

In this section we obtain the forced solution, v_{6f}, for $f = 1kHz$. To achieve the wanted result we use a phasor voltage source, $v_s = 1$, wich as the complex value of $v_s = e^{(-(\pi/2)*j)}$. It is also needed to replace the capacitor, C with its impedance, Z_C, wich is given by: $Z_C = 1/(j*\omega*C)$, where $\omega = 2*\pi*f$ is the angular frequency. In order to facilitate writing the equations we use G_C as This procedure allow us to solve the circuit using the Node Method, by solving the following system

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_3 & 0 & -G_3-G_4-G_5 & G_5+G_C & G_7 & -G_C-G_7 \\ 0 & -K_b & 0 & K_b+G_5 & -G_5-G_C & 0 & G_C \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

that give us the complex amplitudes in each node:

Name	Value [V]
V_1	$1.000000e^{-1.570796j}$
V_2	$0.956740e^{-1.570796j}$
V_3	$0.867448e^{-1.570796j}$
V_4	$0.000000e^{0.000000j}$
V_5	$0.962802e^{-1.570796j}$
V_6	$0.583850e^{1.717262j}$
V_7	$0.385459e^{1.570796j}$
V_8	$0.581928e^{1.570796j}$

Table 4: Complex Amplitudes in each node

2.5 Total Solution

Finally, in order to obtain the total solution we set $v_s = sin(w * t)$ and solve the linear differencial equation 5 whose solution is a superposition of the natural solution v_{6n} and the forced solution v_{6f}:

$$v_6(t) = v_{6n}(t) + v_{6f}(t). \tag{7}$$

The outcome is displayed in the next plot:

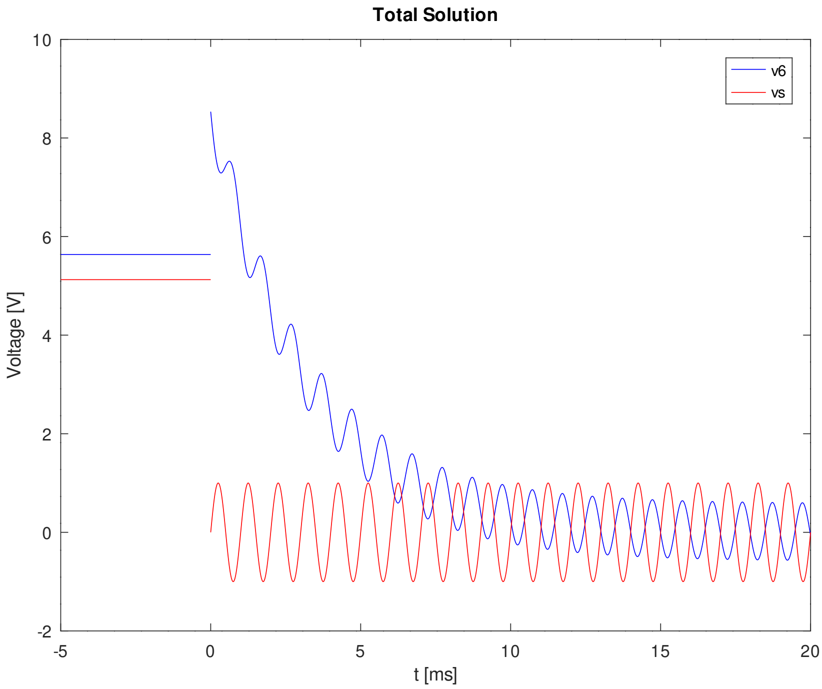

Figure 4: Total solution - v_6

2.6 Frequency Response

To finish the analysis on this circuit we study its frequency response for frequency range 0.1Hz to 1MHz. To do that we represent bellow the plots of both the magnitude and the phase of $V_c(f) = V_6(f) - V_8(f)$, $V_s(f)$ and $V_6(f)$. To find this quantities we just solve the system presented in Section 2.4 in loop, determining the transfer functions for each of the frequency values.

The tranfer function of each quantity is given by:

$$\begin{cases} H_6 = \frac{\bar{v_6}}{\bar{v_s}} \\ H_C = \frac{(\bar{v_6} - \bar{v_8})}{\bar{v_s}} \\ H_s = \frac{\bar{v_s}}{\bar{v_s}} \end{cases}$$

To do the plot we need to create a logharitmicaly spaced vector of frequencies. In doing so we end up with a vector of magnitudes and a vector of phases for each of the node voltages. With this procedure coming up with the plots consists on ploting each one of the vector's magnitude and phase against frequency.

So the magnitude is obtained by:

$$\begin{cases} \text{Magnitude } \bar{v_6} = abs(H_6) \\ \text{Magnitude } \bar{v_C} = abs(H_C) \\ \text{Magnitude } \bar{v_s} = abs(H_s) \end{cases}$$

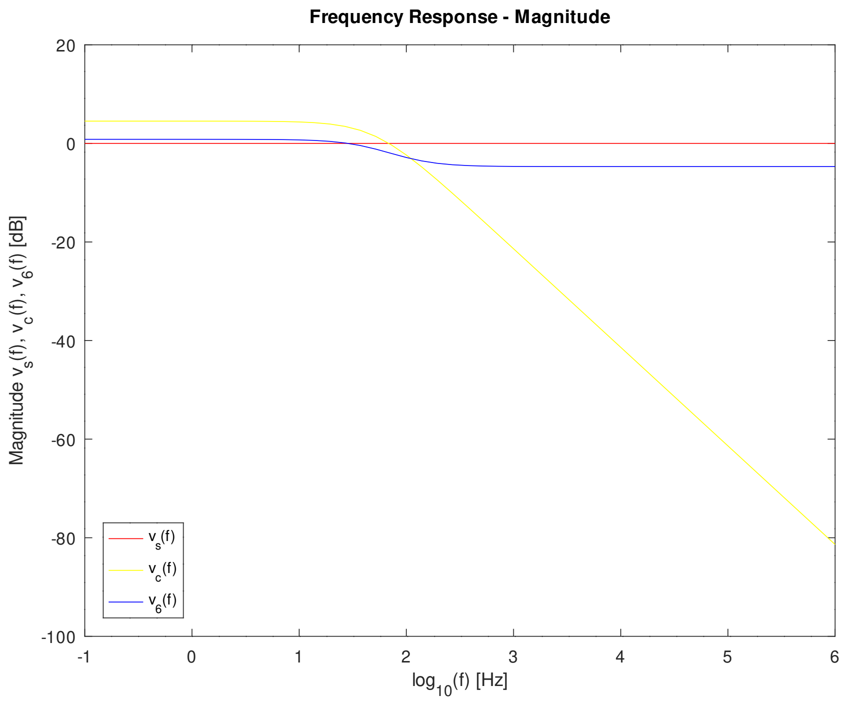

Figure 5: Magnitude of the frequency response of $v_s(f)$, $v_c(f)$ and $v_6(f)$

Analysing the plot above one understands easely why $\bar{v_s}$'s magnitude is zero. Its magnitude is 1, but since the plots are represented on a logarithmic scale is value in that scale is 0. Looking for $\bar{v_C}$'s magnitude one realizes that the highest the frequency the lowest the magnitude. When the frequency tends to infinity, it eventualy tends to zero, because the capacitor can not keep up its charging an discharging rithym with the rate of frequency oscilation.

Phase can be computed from the formulas bellow:

$$\begin{cases} \text{Phase } \bar{v_6} = angle(H_6) \\ \text{Phase } \bar{v_C} = angle(H_C) \\ \text{Phase } \bar{v_s} = angle(H_s) \end{cases}$$

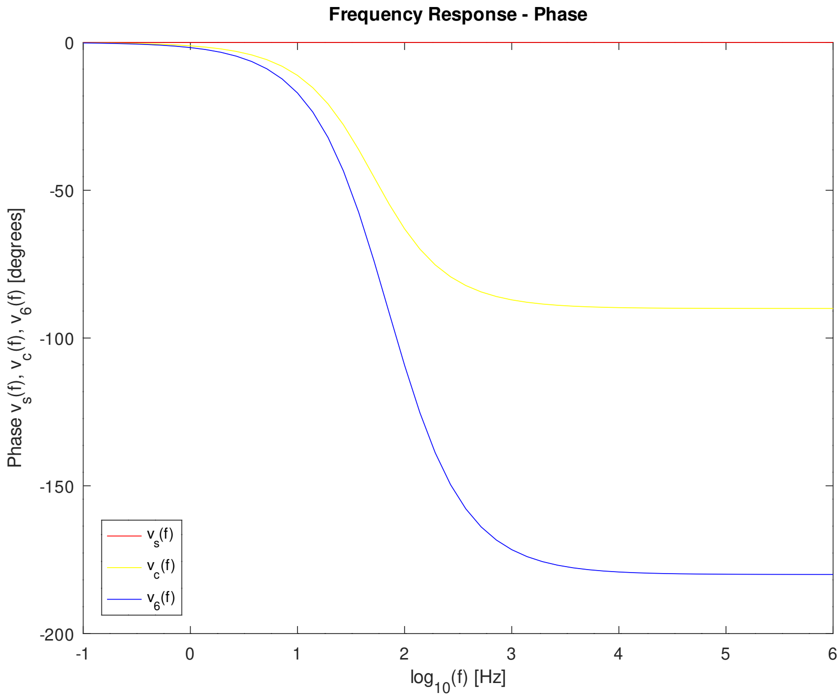

Figure 6: Magnitude of the frequency response of $v_s(f)$, $v_c(f)$ and $v_6(f)$

From this plot one observes that the phase of $\bar{v_s}$ is constant and equal to zero since it is the input source, and therefore, all other phases depend on its. The capacitor's phase it's initialy zero degrees and tends to -90 degrees as the frequency tends to infinity. This is justified by the value of its impedance wich as a phase of exactly -90 degrees. The phase varies continuously since the capacitor tries to keep up with the variation of ω.

3 Simulation Analysis

3.1 Operating Point Analysis for t<0

Table 5 shows the simulated operating point results for the circuit under analysis, when t<0.

Name	Value [A or V]
c[i]	0.000000e+00
gib[i]	-2.26373e-04
r1[i]	2.161226e-04
r2[i]	-2.26373e-04
r3[i]	-1.02499e-05
r4[i]	1.194589e-03
r5[i]	-2.26373e-04
r6[i]	9.784660e-04
r7[i]	9.784660e-04
v(1)	5.125627e+00
v(2)	4.903891e+00
v(3)	4.446215e+00
v(4)	-1.97572e+00
v(5)	4.934963e+00
v(6)	5.635816e+00
v(7)	-1.97572e+00
v(8)	-2.98275e+00

Table 5: Operating point. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

As we can see, the simulation results are similar to the ones we obtained in the section 2, concerning both the numerical values and the directions. As expected, the current through the capacitor is $i_c = 0$.

3.2 Analysis for t=0

Table 6 shows the simulated operating point results for the circuit under analysis, when t=0.

Name	Value [A or V]
gib[i]	6.327120e-18
r1[i]	-6.04063e-18
r2[i]	6.327120e-18
r3[i]	2.864858e-19
r4[i]	1.289989e-18
r5[i]	-2.78376e-03
r6[i]	1.301043e-18
r7[i]	2.625374e-18
v(1)	0.000000e+00
v(2)	6.197538e-15
v(3)	1.898958e-14
v(4)	-2.62707e-15
v(5)	5.329071e-15
v(6)	8.618561e+00
v(7)	-2.62707e-15
v(8)	-5.32907e-15

Table 6: Operating point. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

When t=0, the voltage v_s drops to the value $v_s = 0$, resulting in a discontinuity. However, because $i_c = \frac{dv_c}{dt}$, the voltage in the capacitor must be continuous for $t = 0-, t = 0, t = 0+$, otherwise, the current i_c would be infinite. Taking this into account, we replaced the capacitor with a voltage source $V_x = V_6(0-) - V_8(0-)$ that ensures this continuity. Note that, as whe can see in the table, the values of V_6 and V_8 in the nodes suffered a discontinuity in order to maintain v_c.

The values obtained in simulation for t=0 are also similar to the ones we get from theoretical analysis in subsection 2.2.

3.3 Analysis for t>0 - Natural Solution

Figure 7 shows the plot of the natural solution for v_6, when t>0.

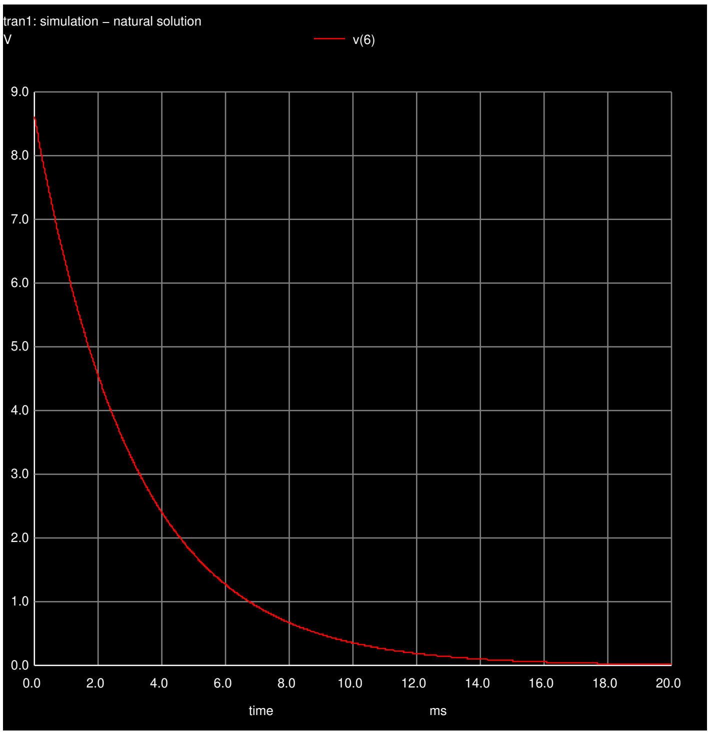

Figure 7: Natural solution

To simulate the natural solution, we imposed the boundary conditions for continuity obtained in last section, V_6 and V_8, with the voltage source $v_s = 0$. As we can see, the plot is similar to the one obtained in subsection 2.3, showing a negative exponential.

3.4 Analysis for t>0 - Total Solution

Figure 7 shows the plot of the total (natural+forced) solution for v_6 and v_s, when t>0. Here we defined $v_s(t) = sin(2 * \pi * f * t)$ as the voltage source. The plot is similar to the one obtained in subsection 2.5.

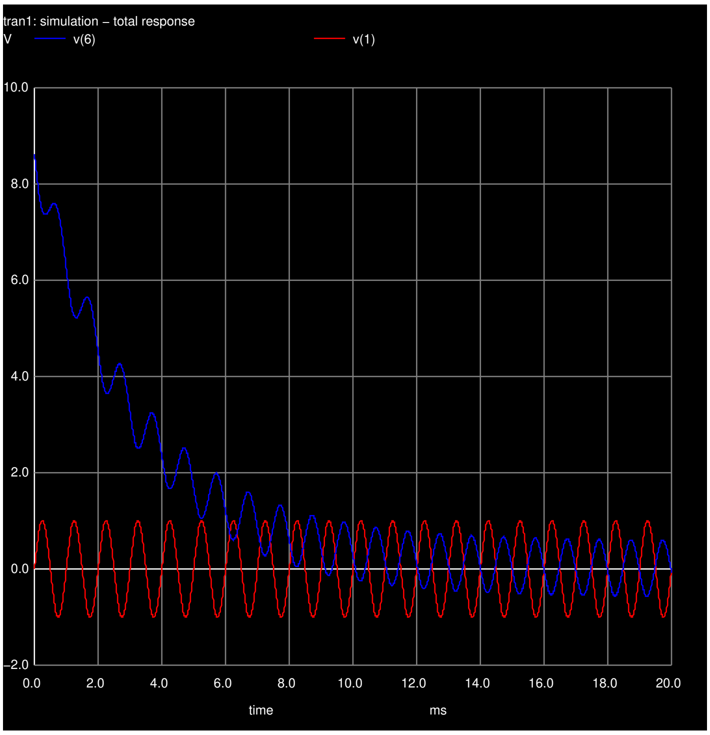

Figure 8: Total solution of the voltage source and the capacitor

3.5 Frequency Response

In this section we simulated the frequency response of v_s, v_6 and $v_c = v_6 - v_8$ from frequency $0.1Hz$ to $1MHz$. The plot 9 shows the magnitude response while plot 10 shows the phase response. The plots are similar to the ones obtained in subsection 2.6

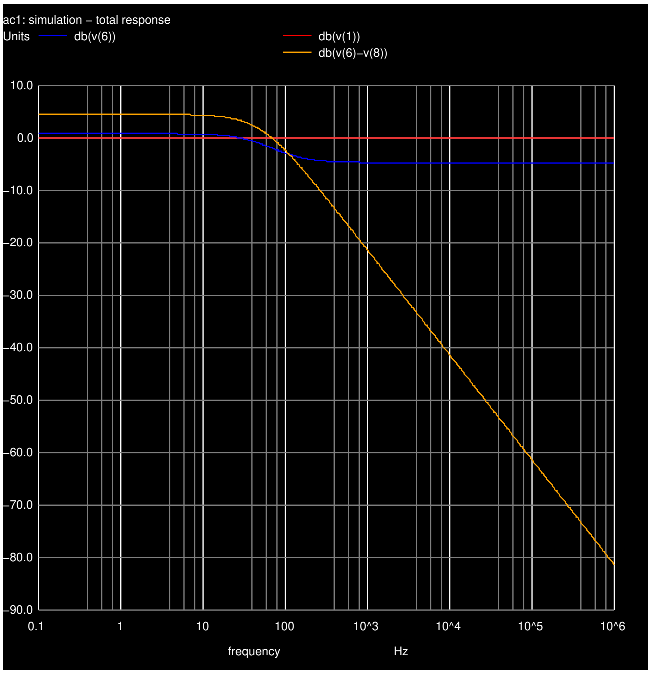

Figure 9: Magnitude in frequency response

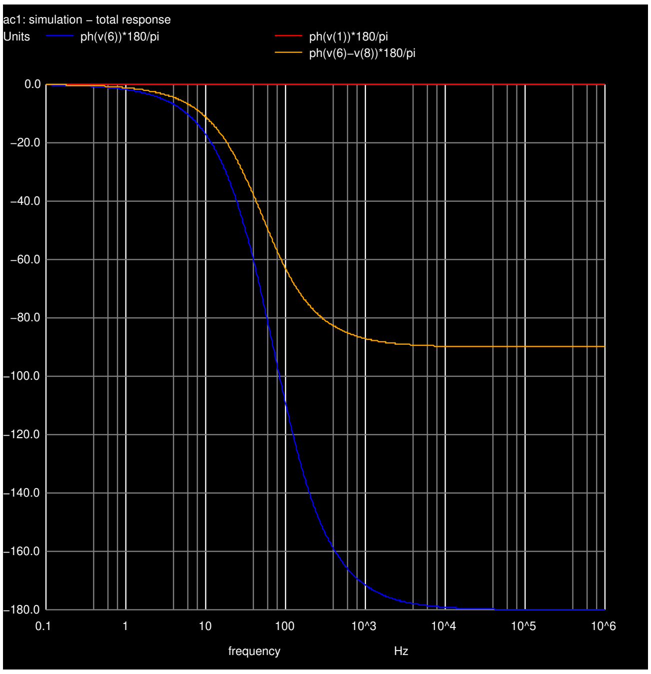

Figure 10: Phase in frequency response

4 Comparison between Octave and Ngspice results

4.1 Analysis for t<0

Table 7 shows, on the left and center, the nodal voltages and the branch currents, found by Octave, and, on the right, the simulated operating point results, by Ngspice, all for the circuit under analysis, when t<0.

Name	Value [V]
V_1	5.12562725920
V_2	4.90389094213
V_3	4.44621543662
V_4	0.00000000000
V_5	4.93496307069
V_6	5.63581571238
V_7	-1.97571911988
V_8	-2.98274501015

Name	Ampere [A]
I_1	0.00021612262
I_2	-0.00022637255
I_3	-0.00001024993
I_4	0.00119458862
I_5	-0.00022637255
I_6	0.00097846600
I_7	0.00097846600
I_b	-0.00022637255
I_c	-0.00000000000
I_{V_d}	0.00097846600
I_{V_s}	0.00021612262

Name	Value [A or V]
c[i]	0.000000e+00
gib[i]	-2.26373e-04
r1[i]	2.161226e-04
r2[i]	-2.26373e-04
r3[i]	-1.02499e-05
r4[i]	1.194589e-03
r5[i]	-2.26373e-04
r6[i]	9.784660e-04
r7[i]	9.784660e-04
v(1)	5.125627e+00
v(2)	4.903891e+00
v(3)	4.446215e+00
v(4)	-1.97572e+00
v(5)	4.934963e+00
v(6)	5.635816e+00
v(7)	-1.97572e+00
v(8)	-2.98275e+00

Table 7: Results for the Circuit at $t < 0$. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4.2 Analysis for t=0

Table 8 shows, on the left, the nodal voltages and the branch currents, found by Octave, and, on the right, the simulated operating point results, by Ngspice, both for the circuit under analysis, when V_s=0, respectively.

Name	Value [V]
V_1	0.00000000000
V_2	0.00000000000
V_3	0.00000000000
V_4	0.00000000000
V_5	0.00000000000
V_6	8.61856072253
V_7	0.00000000000
V_8	0.00000000000

Name	Value [A or V]
gib[i]	6.327120e-18
r1[i]	-6.04063e-18
r2[i]	6.327120e-18
r3[i]	2.864858e-19
r4[i]	1.289989e-18
r5[i]	-2.78376e-03
r6[i]	1.301043e-18
r7[i]	2.625374e-18
v(1)	0.000000e+00
v(2)	6.197538e-15
v(3)	1.898958e-14
v(4)	-2.62707e-15
v(5)	5.329071e-15
v(6)	8.618561e+00
v(7)	-2.62707e-15
v(8)	-5.32907e-15

Table 8: Results for the Circuit at $t = 0$. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

5 Conclusion

After the theoretical analysis, the simulation and the results' comparison, it can be concluded that the objective of the work, the study of the RC circuit presented in Figure-1, has been accomplished.

There were performed a theoretical analysis, in which it was applyedthe nodes method, for t¡0 and for v_s=0, and it was determined the natural, forced ad total solutions for voltage v_6, such as the frequency responses for v_s, v_c and v_6, using the Octave maths tool. Furthermore, a circuit simulation, using the Ngspice tool, with which it was simulated the operating point for t¡0 and for v_s=0, the natural and total responses on node 6 and the frequency response in the same node. With these, it is clear a match of the theoretical and the simulation results. The achievement of the equality in results comes from the fact that the models used to solve and analyse the circuit are similar in both the theoretical analysis and the simulation.