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Laboratory 1: Circuit Analysis Methods

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Circuit Theory and Electronics Fundamentals

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Contents

1 Introduction

The laboratory assignment presented has of its purpose the study of a circuit structured in four elementary meshes, through which exist seven resistors R_i, a voltage source V_a, a current controlled voltage source V_c, a current source I_d and a voltage controlled current source I_b. The circuit can be seen in Figure-1.

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Throughout the report it is presented a theoretical analysis, a simulation of the circuit and its analysis and a comparison of results.

In Section 2, the are applied both mesh and nods methods, to do a theoretical analysis of the circuit, using the Octave maths tool. In Section 3, it is executed an analysis of the circuit using the Ngspice tool to simulate it. Lastly, in Section 4, it is performed a comparison between the results from both the theoretical analysis and the simulation, from Section 2 and Section 3, respectively.

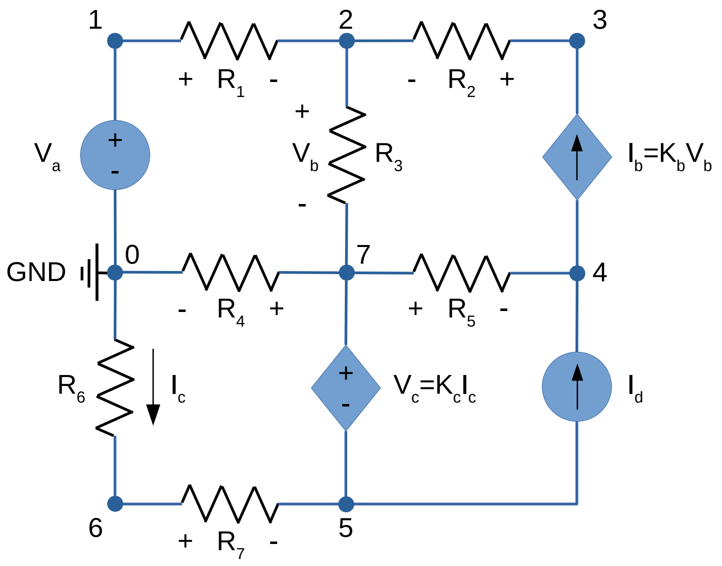

Figure 1: Circuit.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. This analysis was made with two different methods: the **Mesh Method** (Subsection 2.1) and the **Node Method** (Subsection 2.2), both based on circuit analysis basic principles:

- **Kirchoff Voltage Law (KVL):** This law states that the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero. This means, that the sum of all the potential differences in a closed loop must be zero. This law is directly related with the Energy Conservation principle: if we circulate the loop in one direction, we will end up just where we have started. Therefore, there cannot be any potential difference between the start and the end node, since they are the same node and energy cannot be lost. Notice, that for this assumption to be true, polarities and signs of the elements must be taken in account:

$$\sum_{i=1}^{n} V_i = 0$$

- **Kirchoff Current Law (KCL):** This law states that the algebraic sum of all the currents entering a node is equal to algbraic sum of all the currents leaving that same node. This law is also known as the Charge Conservation law: when a current enters a node it as no other option besides leaving the node, since that cannot be any current loss:

$$\sum_{i=1}^{n} I_i = 0$$

- **Ohm's Law:** Ohm's Law states that the current passing through a conductor is propotional to the voltage drop between the two terminals of that conductor:

$$V = RI,$$

where R represents the proportionality constant (resistance).

2.1 Mesh Method

This method consists on assigning currents to the circuit meshes and solving the circuit to find the values of each mesh current, using KVL at each individual mesh that is not connected to any (independent???) current source and Ohm's Law. In doing so we end up with a set of independent equations. In this point, the will be more unknows than equations. So, it is necessary to find more equations in order to have a solvable system. This last equations can be found in the relations between the current sources and the currents assigned to the loops. Now, we can solve the system obtained by using its matricial form. The procedure for this particular circuit is shown bellow:

Assuming the currents representend in Figure 2:

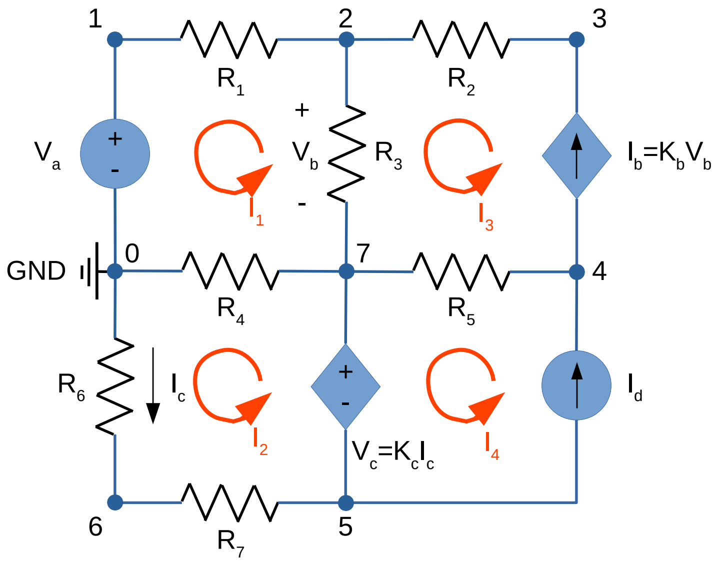

Figure 2: Mesh Currents Identification

$$\begin{cases} \text{Mesh 1: } R_1 I_1 + V_a + R_4(I_1 - I_2) + R_3(I_1 - I_3) = 0 \\ \text{Mesh 2: } R_4(I_2 - I_1) + R_6 I_2 + R_7 I_2 - K_c I_2 = 0 \\ \text{Current b: } I_3 = I_b = K_b V_b \\ \text{Current d: } I_4 = I_d \\ \\ \text{Ohm's Law: } V_b = R_3(I_3 - I_1) \end{cases}$$

We can now present this same system in matrix form:

$$
\begin{bmatrix} R_1+R_3+R_4 & -R_4 & -R_3 & 0 \\ -R_4 & R_4+R_6+R_7-K_c & 0 & 0 \\ R_3K_b & 0 & 1-K_bR_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \begin{bmatrix} -V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}
$$

2.2 Node Method

$$
\begin{cases}
\text{Node 1: } V_1 = V_a \\
\text{Node 2: } G_1(V_1 - V_2) + G_2(V_3 - V_2) - G_3(V_2 - V_7) = 0 \\
\text{Node 3: } -G_2(V_3 - V_2) + I_b = 0 \\
\text{Node 4: } G_5(V_7 - V_4) + I_d - I_b = 0 \\
\text{Node 5: } V_c = V_7 - V_5 \\
\text{Node 6: } G_6(V_0 - V_6) - G_7(V_6 - V_5) = 0 \\
\text{Node 7: } G_4(V_7 - V_0) + G_5(V_7 - V_4) + I_x - G_3(V_2 - V_7) = 0 \\
\\
\text{Current x: } I_x = I_d - I_c \\
\text{Current c: } I_c = G_7(V_6 - V_5) \\
\text{Voltage b: } V_b = V_2 - V_7
\end{cases}
$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & 0 & 0 & 0 & G_3 \\ 0 & G_2+K_b & -G_2 & 0 & 0 & 0 & -K_b \\ 0 & -K_b & 0 & -G_5 & 0 & 0 & G_5+K_b \\ 0 & 0 & 0 & 0 & -1 & K_cG_6 & 1 \\ 0 & 0 & 0 & 0 & G_7 & -G_6-G_7 & 0 \\ 0 & G_3 & 0 & G_5 & 0 & -G_6 & -G_3-G_4-G_5 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_d \\ 0 \\ 0 \\ I_d \end{bmatrix}
$$

2.3 Theoretical Analysis Results

Name	Value [A or V]
I_b	-2.263725e-04
I_d	1.011815e-03
I_{R1}	2.161226e-04
I_{R2}	-2.263725e-04
I_{R3}	-1.024993e-05
I_{R4}	1.194589e-03
I_{R5}	-1.238187e-03
I_{R6}	9.784660e-04
I_{R7}	9.784660e-04
V_1	5.125627
V_2	4.903891
V_3	4.446215
V_4	8.768409
V_5	-2.982745
V_6	-1.975719
V_7	4.934963

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 2 shows the simulated operating point results for the circuit under analysis.

Name	Value [A or V]
gib[i]	-2.26373e-04
id[current]	1.011815e-03
r1[i]	2.161226e-04
r2[i]	-2.26373e-04
r3[i]	-1.02499e-05
r4[i]	1.194589e-03
r5[i]	-1.23819e-03
r6[i]	9.784660e-04
r7[i]	9.784660e-04
v(1)	5.125627e+00
v(2)	4.903891e+00
v(3)	4.446215e+00
v(4)	8.768409e+00
v(5)	-2.98275e+00
v(6)	-1.97572e+00
v(7)	4.934963e+00
v(8)	-1.97572e+00

Table 2: Operating point. A variable followed by [i] or [current] is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

As we can see, the simulation results are similar to the ones we obtained in the section 2, concerning both the numerical values and the directions. Note that, unlike the table 1, in the simulation results we present an extra voltage at node 8, V_8, that is a ”dummy” node used to compute the dependent voltage source.

4 Conclusion

After the theoretical analysis and the simulation, it can be concluded that the objective of the work, the study of the circuit presented in Figure-1, has been accomplished.

There were performed a theoretical analysis, applying both mesh and nodes methods, using the Octave maths tool, and a circuit simulation, using the Ngspice tool, with which it is clear a seamless match of the theoretical and the simulation results. The achievement of the equality in results comes from the components of the circuit, which are all linear and, therefore, both models have to present the same results.