Topic: U-substitution in definite integrals

Question: Use a u-substitution with $u = \cos t$ to evaluate the definite integral.

$$\int_0^{\frac{\pi}{2}} (1 - 2\cos^2 t + \cos^4 t)\sin t \ dt$$

Answer choices:

A $\frac{8}{15}$

B $\frac{9}{16}$

C $\frac{7}{9}$

D $\frac{13}{15}$

Solution: A

Use u-substitution, letting

$$u = \cos t$$

$$du = -\sin t \ dt$$

$$-du = \sin t \ dt$$

Plugging these into the integral, we get

$$\int_{t=0}^{t=\frac{\pi}{2}} (1 - 2u^2 + u^4)(-du)$$

$$-\int_{t=0}^{t=\frac{\pi}{2}} u^4 - 2u^2 + 1 \ du$$

$$-\left(\frac{1}{5}u^5 - \frac{2}{3}u^3 + u\right)\Bigg|_{t=0}^{t=\frac{\pi}{2}}$$

Back-substituting for u before we evaluate over the interval, we get

$$\left(-\frac{1}{5}\cos^5 t + \frac{2}{3}\cos^3 t - \cos t\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$\left(-\frac{1}{5}\cos^5 \frac{\pi}{2} + \frac{2}{3}\cos^3 \frac{\pi}{2} - \cos \frac{\pi}{2}\right) - \left(-\frac{1}{5}\cos^5 0 + \frac{2}{3}\cos^3 0 - \cos 0\right)$$

$$(-0 + 0 - 0) - \left(-\frac{1}{5}(1) + \frac{2}{3}(1) - (1)\right)$$

$$\frac{1}{5}-\frac{2}{3}+1$$

$$\frac{3}{15}-\frac{10}{15}+\frac{15}{15}$$

$$\frac{8}{15}$$

Topic: U-substitution in definite integrals

Question: Use u-substitution to simplify the definite integral. Do not solve it.

$$\int_0^2 x^2\sqrt{x^3}\ dx$$

Answer choices:

A $\quad 3\int_0^2 \sqrt{u}\ du$

B $\quad \frac{1}{3}\int_{-2}^2 \sqrt{u}\ du$

C $\quad \frac{1}{3}\int_0^8 \sqrt{u}\ du$

D $\quad 3\int_0^8 \sqrt{u}\ du$

Solution: C

U-substitution allows us to simplify integrals that we wouldn't otherwise be able to evaluate.

When we make a substitution in our integral, it means we also need to change our limits of integration to match the new variable. This just means that we have to take our old limits of integration with respect to x, and plug each of them into our equation for u to find new limits of integration with respect to u.

If we change the limits of integration, then we can evaluate the definite integral without back-substituting. Remember, you don't absolutely have to change the limits of integration when you make a substitution. However, if you don't change the limits, it means you must back-substitute to put the function back in terms of x instead of u at the end of the problem before you evaluate over the interval.

We'll set

$$u = x^3$$

$$du = 3x^2\ dx$$

$$dx = \frac{du}{3x^2}$$

Since we're going to make a u-substitution, we'll also find limits of integration with respect to u instead of x. Next, we can solve for our u-substitution limits. Plugging our upper and lower limits of integration into our equation for u, we get

$$u = (0)^3$$

$$u = 0$$

and

$$u = (2)^3$$

$$u = 8$$

The old limits of integration with respect to x were $[0,2]$ and the new limits of integration with respect to u are $[0,8]$.

Making the substitution and attaching new limits of integration, we get

$$\int_0^2 x^2\sqrt{x^3}\ dx$$

$$\int_0^8 x^2\sqrt{u}\ \frac{du}{3x^2}$$

$$\int_0^8 \sqrt{u}\ \frac{du}{3}$$

$$\frac{1}{3}\int_0^8 \sqrt{u}\ du$$

Topic: U-substitution in definite integrals

Question: Use u-substitution to simplify the definite integral. Do not solve it.

$$\int_2^5 \frac{x+1}{x^2+2x}\ dx$$

Answer choices:

A $\quad \frac{1}{2}\int_8^{35} \frac{1}{u}\ du$

B $\quad \frac{1}{2}\int_0^{5} \frac{1}{u}\ du$

C $\quad 2\int_2^{5} \frac{1}{u}\ du$

D $\quad 2\int_8^{35} \frac{1}{u}\ du$

Solution: A

U-substitution allows us to simplify integrals that we wouldn't otherwise be able to evaluate.

When we make a substitution in our integral, it means we also need to change our limits of integration to match the new variable. This just means that we have to take our old limits of integration with respect to x, and plug each of them into our equation for u to find new limits of integration with respect to u.

If we change the limits of integration, then we can evaluate the definite integral without back-substituting. Remember, you don't absolutely have to change the limits of integration when you make a substitution. However, if you don't change the limits, it means you must back-substitute to put the function back in terms of x instead of u at the end of the problem before you evaluate over the interval.

We'll set

$$u = x^2 + 2x$$

$$du = 2x + 2\ dx$$

$$dx = \frac{du}{2(x+1)}$$

Since we're going to make a u-substitution, we'll also find limits of integration with respect to u instead of x. Next, we can solve for our u-substitution limits. Plugging our upper and lower limits of integration into our equation for u, we get

$$u = (2)^2 + 2(2)$$

$$u = 8$$

and

$$u = (5)^2 + 2(5)$$

$$u = 35$$

The old limits of integration with respect to x were $[2,5]$ and the new limits of integration with respect to u are $[8,35]$.

Making the substitution and attaching new limits of integration, we get

$$\int_2^5 \frac{x+1}{x^2+2x}\ dx$$

$$\int_8^{35} \frac{x+1}{u}\left[\frac{du}{2(x+1)}\right]$$

$$\int_8^{35} \frac{1}{u}\left(\frac{du}{2}\right)$$

$$\frac{1}{2}\int_8^{35} \frac{1}{u}\ du$$