Topic: Area between left and right curves

Question: Find the area between the curves.

$$x = 3y^2$$

$$x = y^2 + 2$$

Answer choices:

A $\frac{8}{3}$

B $\frac{4}{3}$

C $-\frac{4}{3}$

D $-\frac{8}{3}$

Solution: A

In order to calculate the area between two curves, we need to follow these steps:

1. Decide whether the curves are
    a. upper and lower curves, or
    b. left and right curves.
2. Find points of intersection.
3. Determine which curve has the larger value between each point of intersection.
4. Plug everything into the appropriate formula.

Since the curves we're given are both expressed for x in terms of y, it means these are left and right curves.

To find points of intersection, we'll set the curves equal to each other.

$$3y^2 = y^2 + 2$$

$$2y^2 = 2$$

$$y^2 = 1$$

$$y = \pm 1$$

These two points of intersection define the endpoints of our interval in terms of y, which means our next step is to determine which curve has a larger x-value on the y-interval $[-1,1]$.

We can do this by picking a y-value within the interval and plugging it into both functions. Whichever curve returns a larger value we'll call $f(y)$, and whichever curve returns a lower value we'll call $g(y)$.

Plugging $y = 0$ into both functions, we get

$$x = 3y^2$$

$$x = 3(0)^2$$

$$x = 0$$

and

$$x = y^2 + 2$$

$$x = (0)^2 + 2$$

$$x = 2$$

Since $x = y^2 + 2$ gives a larger value, we'll say

$$g(y) = 3y^2$$

and

$$f(y) = y^2 + 2$$

Now we can plug these functions and the interval we found earlier into the formula for area between left and right curves.

$$\int_{a}^{b} f(y) - g(y)\ dy$$

$$\int_{-1}^{1} y^2 + 2 - 3y^2\ dy$$

$$\int_{-1}^{1} -2y^2 + 2\ dy$$

$$\frac{-2y^3}{3} + 2y \Big|_{-1}^{1}$$

$$\left[\frac{-2(1)^3}{3} + 2(1)\right] - \left[\frac{-2(-1)^3}{3} + 2(-1)\right]$$

$$\frac{-2}{3} + 2 - \frac{2}{3} + 2$$

$$-\frac{4}{3} + \frac{12}{3}$$

$$\frac{8}{3}$$

Topic: Area between left and right curves

Question: Find the area between the curves.

$$x = 2y^2 + 1$$

$$x = y^2 + 5$$

Answer choices:

A $-\dfrac{32}{3}$

B $\dfrac{16}{3}$

C $\dfrac{32}{3}$

D $-\dfrac{16}{3}$

Solution: C

In order to calculate the area between two curves, we need to follow these steps:

1. Decide whether the curves are

    a. upper and lower curves, or

    b. left and right curves.

2. Find points of intersection.

3. Determine which curve has the larger value between each point of intersection.

4. Plug everything into the appropriate formula.

Since the curves we're given are both expressed for x in terms of y, it means these are left and right curves.

To find points of intersection, we'll set the curves equal to each other.

$$2y^2 + 1 = y^2 + 5$$

$$y^2 = 4$$

$$y = \pm 2$$

These two points of intersection define the endpoints of our interval in terms of y, which means our next step is to determine which curve has a larger x-value on the y-interval $[-2,2]$.

We can do this by picking a y-value within the interval and plugging it into both functions. Whichever curve returns a larger value we'll call $f(y)$, and whichever curve returns a lower value we'll call $g(y)$.

Plugging $y = 1$ into both functions, we get

$$x = 2y^2 + 1$$

$$x = 2(1)^2 + 1$$

$$x = 3$$

and

$$x = y^2 + 5$$

$$x = (1)^2 + 5$$

$$x = 6$$

Since $x = y^2 + 5$ gives a larger value, we'll say

$$g(y) = 2y^2 + 1$$

and

$$f(y) = y^2 + 5$$

Now we can plug these functions and the interval we found earlier into the formula for area between left and right curves.

$$\int_a^b f(y) - g(y)\ dy$$

$$\int_{-2}^{2} y^2 + 5 - \left(2y^2 + 1\right) \ dy$$

$$\int_{-2}^{2} -y^2 + 4 \ dy$$

$$\frac{-y^3}{3} + 4y \Big|_{-2}^{2}$$

$$\left[\frac{-(2)^3}{3} + 4(2)\right] - \left[\frac{-(-2)^3}{3} + 4(-2)\right]$$

$$\left(\frac{-8}{3} + 8\right) - \left(\frac{8}{3} - 8\right)$$

$$\left(\frac{16}{3}\right) - \left(\frac{-16}{3}\right)$$

$$\frac{32}{3}$$

Topic: Area between left and right curves

Question: Find the area between the curves.

$$x = y^2 + y + 3$$

$$x = 2y^2 + 2y + 1$$

Answer choices:

A $\frac{2}{3}$

B $\frac{9}{2}$

C $-\frac{9}{2}$

D $-\frac{2}{3}$

Solution: B

In order to calculate the area between two curves, we need to follow these steps:

1. Decide whether the curves are

    a. upper and lower curves, or

    b. left and right curves.

2. Find points of intersection.

3. Determine which curve has the larger value between each point of intersection.

4. Plug everything into the appropriate formula.

Since the curves we're given are both expressed for x in terms of y, it means these are left and right curves.

To find points of intersection, we'll set the curves equal to each other.

$$2y^2 + 2y + 1 = y^2 + y + 3$$

$$y^2 + y - 2 = 0$$

$$(y + 2)(y - 1) = 0$$

$y = -2$ and $y = 1$

These two points of intersection define the endpoints of our interval in terms of y, which means our next step is to determine which curve has a larger x-value on the y-interval $[-2,1]$.

We can do this by picking a y-value within the interval and plugging it into both functions. Whichever curve returns a larger value we'll call $f(y)$, and whichever curve returns a lower value we'll call $g(y)$.

Plugging $y = 0$ into both functions, we get

$$x = y^2 + y + 3$$

$$x = (0)^2 + 0 + 3$$

$$x = 3$$

and

$$x = 2y^2 + 2y + 1$$

$$x = 2(0)^2 + 2(0) + 1$$

$$x = 1$$

Since $x = y^2 + y + 3$ gives a larger value, we'll say

$$f(y) = y^2 + y + 3$$

and

$$g(y) = 2y^2 + 2y + 1$$

Now we can plug these functions and the interval we found earlier into the formula for area between left and right curves.

$$\int_{a}^{b} f(y) - g(y)\ dy$$

$$\int_{-2}^{1} y^2 + y + 3 - \left(2y^2 + 2y + 1\right)\ dy$$

$$\int_{-2}^{1} -y^2 - y + 2\ dy$$

$$\frac{-y^3}{3} - \frac{y^2}{2} + 2y\Big|_{-2}^{1}$$

$$\left[\frac{-(1)^3}{3} - \frac{(1)^2}{2} + 2(1)\right] - \left[\frac{-(-2)^3}{3} - \frac{(-2)^2}{2} + 2(-2)\right]$$

$$\left(\frac{-1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - \frac{4}{2} - 4\right)$$

$$-\frac{2}{6} - \frac{3}{6} + \frac{12}{6} - \frac{16}{6} + \frac{12}{6} + \frac{24}{6}$$

$$\frac{9}{2}$$