Calculus 2 Workbook Solutions

Volume of revolution

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MATH

DISKS, HORIZONTAL AXIS

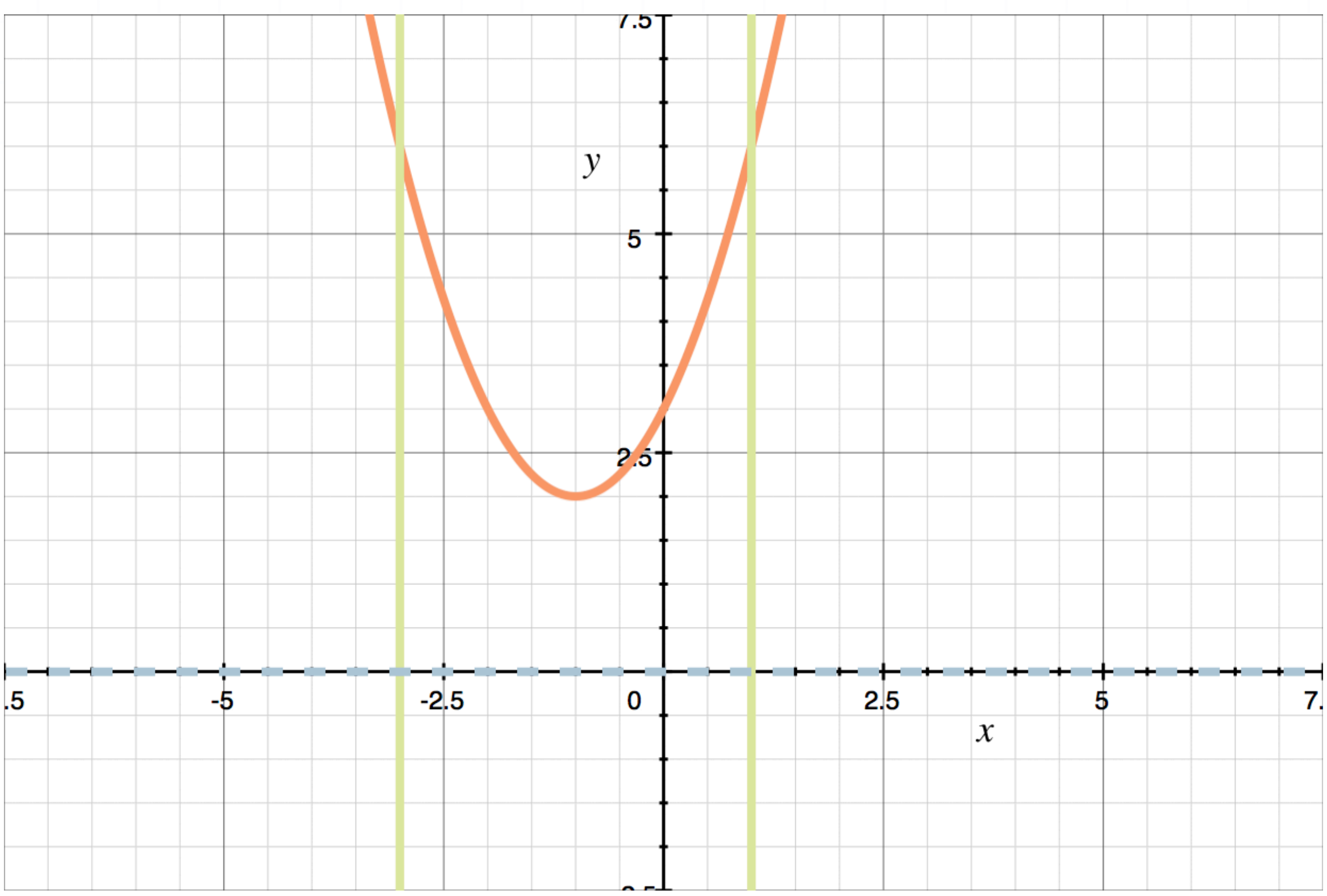
1. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = x^2 + 2x + 3$$

$x = -3$ and $x = 1$

Solution:

A sketch of the region and the axis of revolution $y = 0$ is



The volume given by disks is

$$V=\int_a^b \pi\left[f(x)\right]^2\ dx$$

$$V=\int_{-3}^{1}\pi(x^2+2x+3)^2\ dx$$

$$V=\int_{-3}^{1}\pi(x^4+2x^3+3x^2+2x^3+4x^2+6x+3x^2+6x+9)\ dx$$

$$V=\pi\int_{-3}^{1}x^4+4x^3+10x^2+12x+9\ dx$$

Integrate, then evaluate over the interval.

$$V=\pi\left(\frac{1}{5}x^5+x^4+\frac{10}{3}x^3+6x^2+9x\right)\Bigg|_{-3}^{1}$$

$$V=\pi\left(\frac{1}{5}(1)^5+1^4+\frac{10}{3}(1)^3+6(1)^2+9(1)\right)$$

$$-\pi\left(\frac{1}{5}(-3)^5+(-3)^4+\frac{10}{3}(-3)^3+6(-3)^2+9(-3)\right)$$

$$V=\pi\left(\frac{1}{5}+1+\frac{10}{3}+6+9\right)$$

$$-\pi\left(\frac{1}{5}(-243)+81+\frac{10}{3}(-27)+6(9)-27\right)$$

$$V=\pi\left(\frac{1}{5}+\frac{10}{3}+16\right)-\pi\left(-\frac{243}{5}+81-90+54-27\right)$$

$$V=\pi\left(\frac{1}{5}+\frac{10}{3}+16\right)-\pi\left(-\frac{243}{5}+18\right)$$

$$V=\pi\left(\frac{3}{15}+\frac{50}{15}+\frac{240}{15}\right)-\pi\left(-\frac{729}{15}+\frac{270}{15}\right)$$

$$V=\frac{293}{15}\pi+\frac{459}{15}\pi$$

$$V=\frac{752\pi}{15}$$

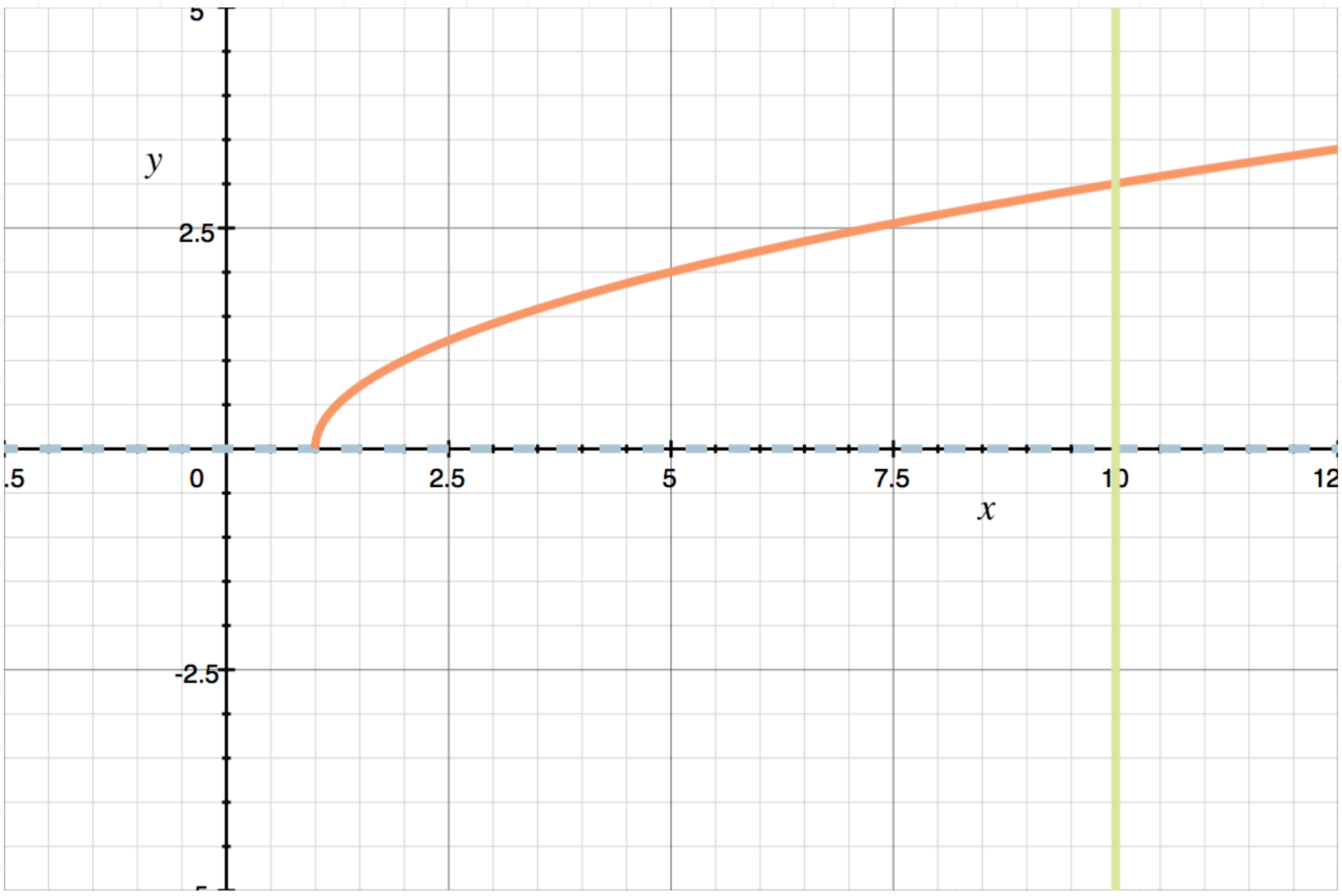
■ 2. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y=\sqrt{x-1}$$

$x=1$ and $x=10$

Solution:

A sketch of the region and the axis of revolution $y=0$ is



The volume given by disks is

$$V = \int_a^b \pi \left[f(x)\right]^2 \ dx$$

$$V = \int_1^{10} \pi \left(\sqrt{x-1}\right)^2 \ dx$$

$$V = \pi \int_1^{10} x - 1 \ dx$$

Integrate, then evaluate over the interval.

$$V = \pi \left(\frac{1}{2}x^2 - x\right)\Big|_1^{10}$$

$$V = \pi \left(\frac{1}{2}(10)^2 - 10\right) - \pi \left(\frac{1}{2}(1)^2 - 1\right)$$

$$V=\pi(50-10)-\pi\left(\frac{1}{2}-1\right)$$

$$V=40\pi+\frac{1}{2}\pi$$

$$V=\frac{81\pi}{2}$$

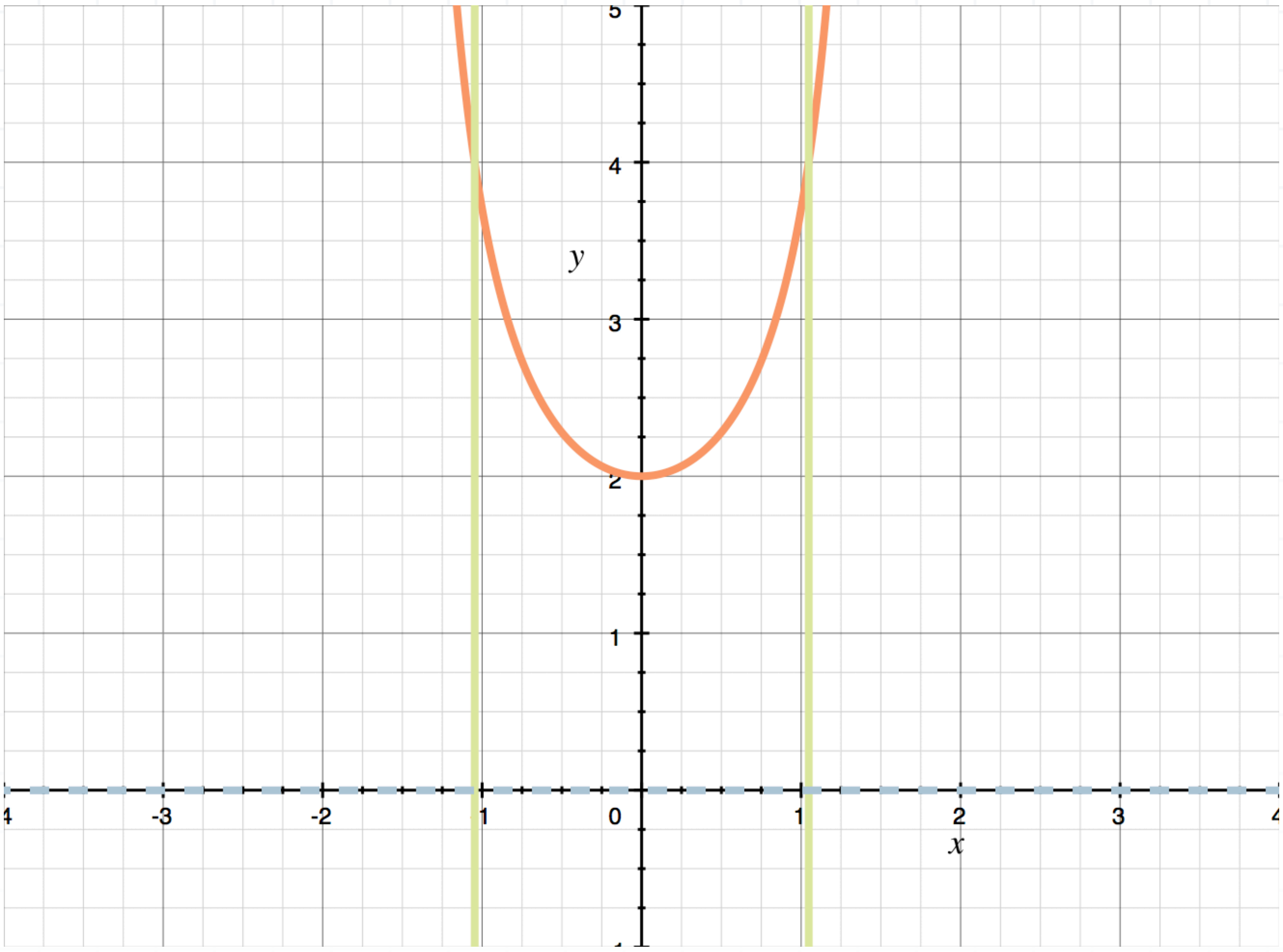
■ 3. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y=2\sec x$$

$x=-\dfrac{\pi}{3}$ and $x=\dfrac{\pi}{3}$

Solution:

A sketch of the region and the axis of revolution $y=0$ is



The volume given by disks is

$$V = \int_{a}^{b} \pi \left[f(x)\right]^2 \ dx$$

$$V = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \pi(2\sec x)^2 \ dx$$

$$V = 4\pi \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec^2 x \ dx$$

Integrate, then evaluate over the interval.

$$V = 4\pi \tan x \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$V = 4\pi \tan \frac{\pi}{3} - 4\pi \tan \left(-\frac{\pi}{3} \right)$$

$$V = 4\pi\sqrt{3} - 4\pi\left(-\sqrt{3}\right)$$

$$V = 4\sqrt{3}\pi + 4\sqrt{3}\pi$$

$$V = 8\sqrt{3}\pi$$

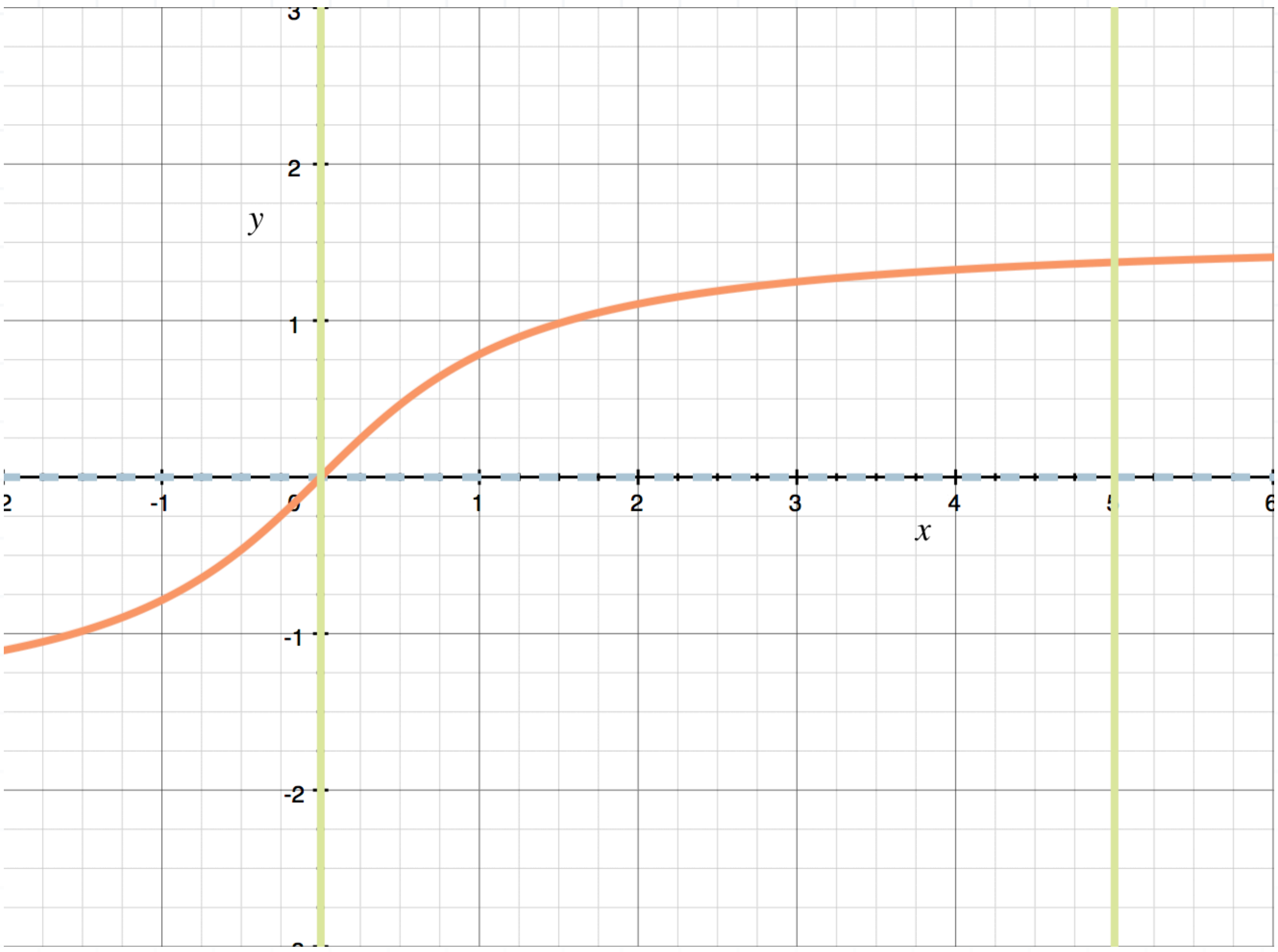
■ 4. Set up the integral that approximates the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis. Do not evaluate the integral.

$$y = \arctan x$$

$x = 0$ and $x = 5$

Solution:

A sketch of the region and the axis of revolution $y = 0$ is



The volume given by disks is

$$V = \int_a^b \pi \left[f(x)\right]^2 \ dx$$

$$V = \int_0^5 \pi \arctan^2 x \ dx$$

$$V = \pi \int_0^5 \arctan^2 x \ dx$$

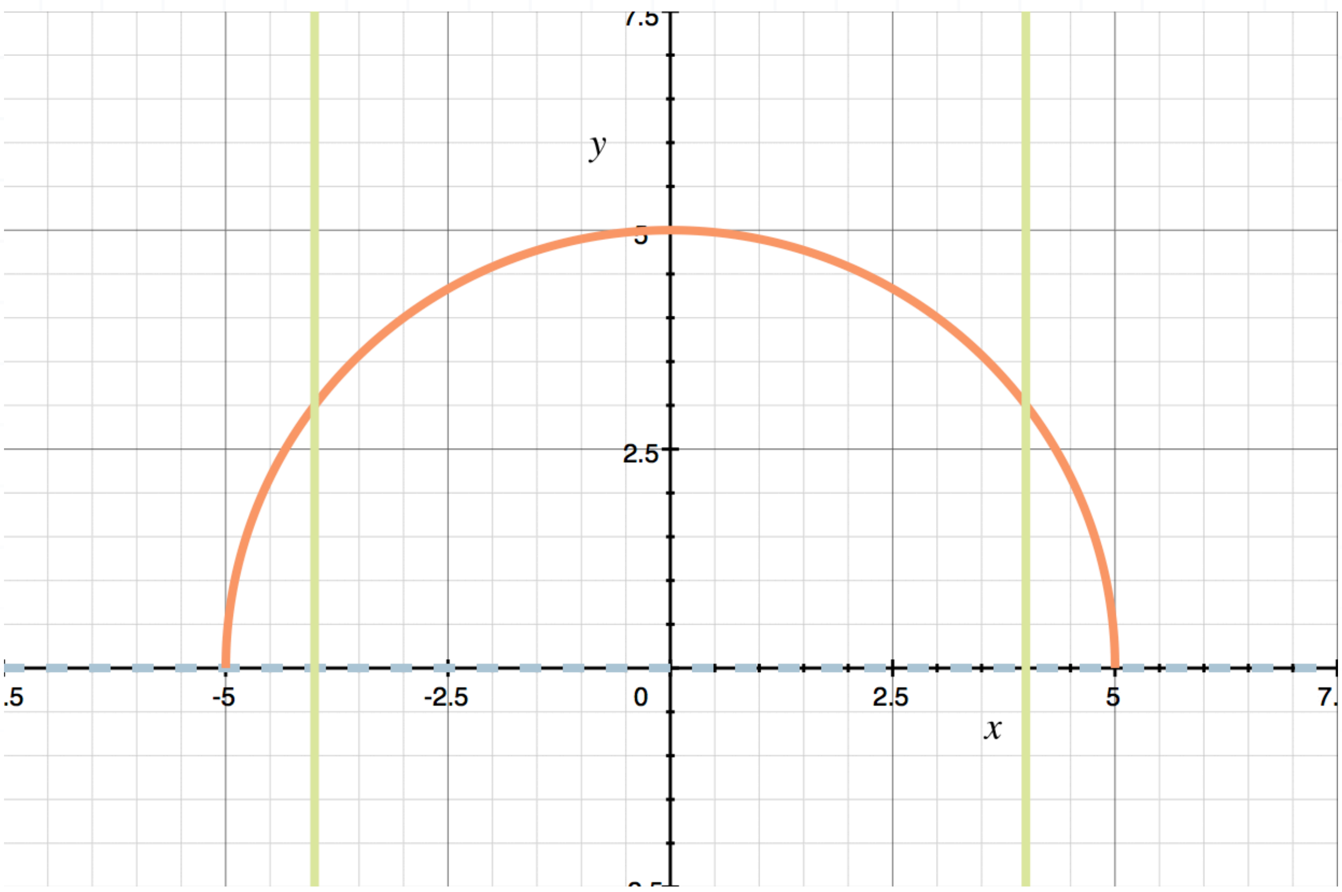
■ 5. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$y = \sqrt{25 - x^2}$$

$x = -4$ and $x = 4$

Solution:

A sketch of the region and the axis of revolution $y = 0$ is



The volume given by disks is

$$V = \int_a^b \pi \left[f(x) \right]^2 \, dx$$

$$V = \int_{-4}^{4} \pi \left[\sqrt{25 - x^2} \right]^2 \, dx$$

$$V = \pi \int_{-4}^{4} 25 - x^2 \, dx$$

Integrate, then evaluate over the interval.

$$V=\pi\left(25x-\frac{1}{3}x^3\right)\Big|_{-4}^{4}$$

$$V=\pi\left(25(4)-\frac{1}{3}(4)^3\right)-\pi\left(25(-4)-\frac{1}{3}(-4)^3\right)$$

$$V=\pi\left(100-\frac{64}{3}\right)-\pi\left(-100+\frac{64}{3}\right)$$

$$V=\pi\left(\frac{300}{3}-\frac{64}{3}+\frac{300}{3}-\frac{64}{3}\right)$$

$$V=\frac{472\pi}{3}$$

DISKS, VERTICAL AXIS

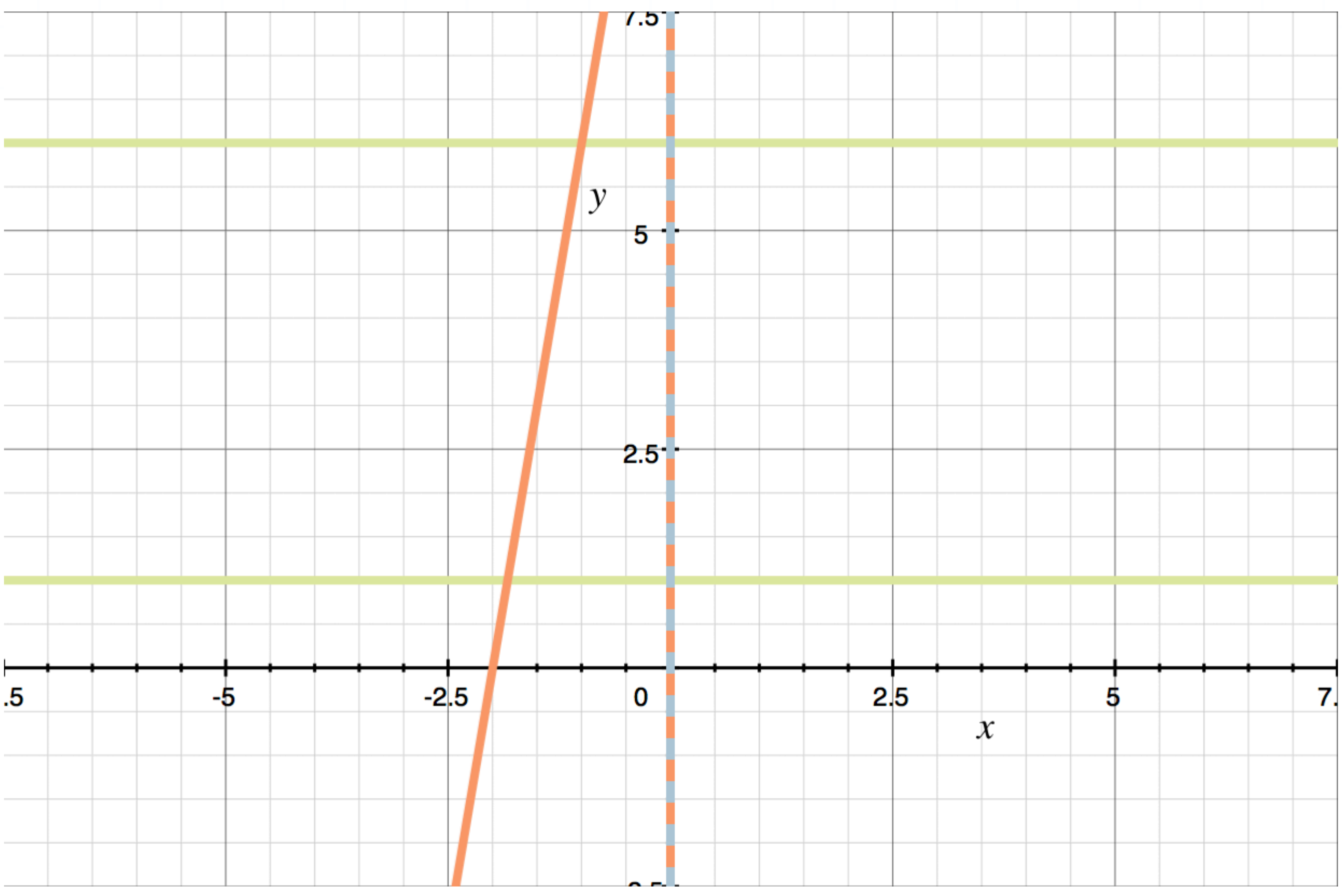
■ 1. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$$x = \frac{1}{6}y - 2 \text{ and } x = 0$$

$$y = 1 \text{ and } y = 6$$

Solution:

A sketch of the region and the axis of revolution $x = 0$ is



The volume given by disks is

$$V=\int_{c}^{d}\pi\left[f(y)\right]^{2}\ dy$$

$$V=\int_{1}^{6}\pi\left[\frac{1}{6}y-2\right]^{2}\ dy$$

$$V=\pi\int_{1}^{6}\frac{1}{36}y^{2}-\frac{2}{3}y+4\ dy$$

Integrate, then evaluate over the interval.

$$V=\pi\left(\frac{1}{108}y^{3}-\frac{1}{3}y^{2}+4y\right)\Big|_{1}^{6}$$

$$V=\pi\left(\frac{1}{108}(6)^{3}-\frac{1}{3}(6)^{2}+4(6)\right)-\pi\left(\frac{1}{108}(1)^{3}-\frac{1}{3}(1)^{2}+4(1)\right)$$

$$V=\pi\left(2-12+24\right)-\pi\left(\frac{1}{108}-\frac{1}{3}+4\right)$$

$$V=\pi\left(2-12+24-\frac{1}{108}+\frac{1}{3}-4\right)$$

$$V=\pi\left(10-\frac{1}{108}+\frac{1}{3}\right)$$

$$V=\pi\left(\frac{1{,}080}{108}-\frac{1}{108}+\frac{36}{108}\right)$$

$$V=\frac{1{,}115\pi}{108}$$

2. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$$x = \frac{3}{7}\sqrt{y} + 2 \text{ and } x = 0$$

$$y = 2 \text{ and } y = 5$$

Solution:

A sketch of the region and the axis of revolution $y = 0$ is

The volume given by disks is

$$V = \int_c^d \pi \left[f(y)\right]^2 \ dy$$

$$V=\int_{2}^{5}\pi\left[\frac{3}{7}\sqrt{y}+2\right]^2\ dy$$

$$V=\pi\int_{2}^{5}\frac{9}{49}y+\frac{12}{7}\sqrt{y}+4\ dy$$

Integrate, then evaluate over the interval.

$$V=\pi\left(\frac{9}{98}y^2+\frac{8}{7}y^{\frac{3}{2}}+4y\right)\Bigg|_{2}^{5}$$

$$V=\pi\left(\frac{9}{98}(5)^2+\frac{8}{7}(5)^{\frac{3}{2}}+4(5)\right)-\pi\left(\frac{9}{98}(2)^2+\frac{8}{7}(2)^{\frac{3}{2}}+4(2)\right)$$

$$V=\pi\left(\frac{225}{98}+\frac{8\sqrt{125}}{7}+20-\frac{36}{98}-\frac{8\sqrt{8}}{7}-8\right)$$

$$V=\pi\left(\frac{27}{14}+\frac{8\sqrt{125}-8\sqrt{8}}{7}+12\right)$$

$$V=\pi\left(\frac{27}{14}+\frac{16\sqrt{125}-16\sqrt{8}}{14}+\frac{168}{14}\right)$$

$$V=\frac{16\sqrt{125}-16\sqrt{8}+195}{14}\pi$$

$$V=\frac{80\sqrt{5}-32\sqrt{2}+195}{14}\pi$$

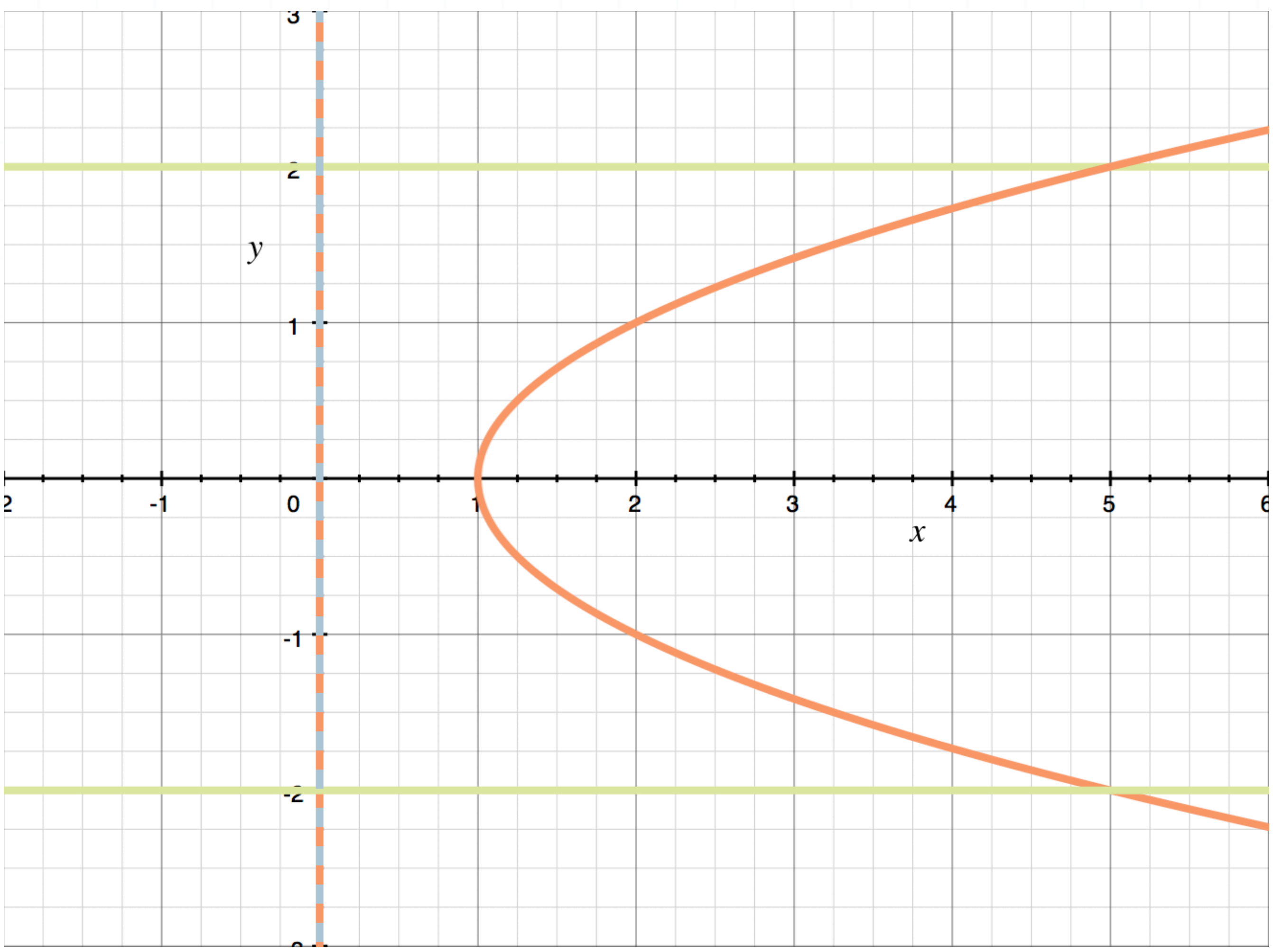
3. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = y^2 + 1$ and $x = 0$

$y = -2$ and $y = 2$

Solution:

A sketch of the region and the axis of revolution $x = 0$ is



The volume given by disks is

$$V = \int_c^d \pi \left[f(y)\right]^2 \, dy$$

$$V = \int_{-2}^{2} \pi \left[y^2 + 1\right]^2 \ dy$$

$$V = \pi \int_{-2}^{2} y^4 + 2y^2 + 1 \ dy$$

Integrate, then evaluate over the interval.

$$V = \pi \left(\frac{1}{5}y^5 + \frac{2}{3}y^3 + y \right) \Bigg|_{-2}^{2}$$

$$V = \pi \left(\frac{1}{5}(2)^5 + \frac{2}{3}(2)^3 + 2 \right) - \pi \left(\frac{1}{5}(-2)^5 + \frac{2}{3}(-2)^3 - 2 \right)$$

$$V = \pi \left(\frac{32}{5} + \frac{16}{3} + 2 \right) - \pi \left(-\frac{32}{5} - \frac{16}{3} - 2 \right)$$

$$V = \pi \left(\frac{32}{5} + \frac{16}{3} + 2 + \frac{32}{5} + \frac{16}{3} + 2 \right)$$

$$V = \pi \left(\frac{64}{5} + \frac{32}{3} + 4 \right)$$

$$V = \pi \left(\frac{192}{15} + \frac{160}{15} + \frac{60}{15} \right)$$

$$V = \frac{412\pi}{15}$$

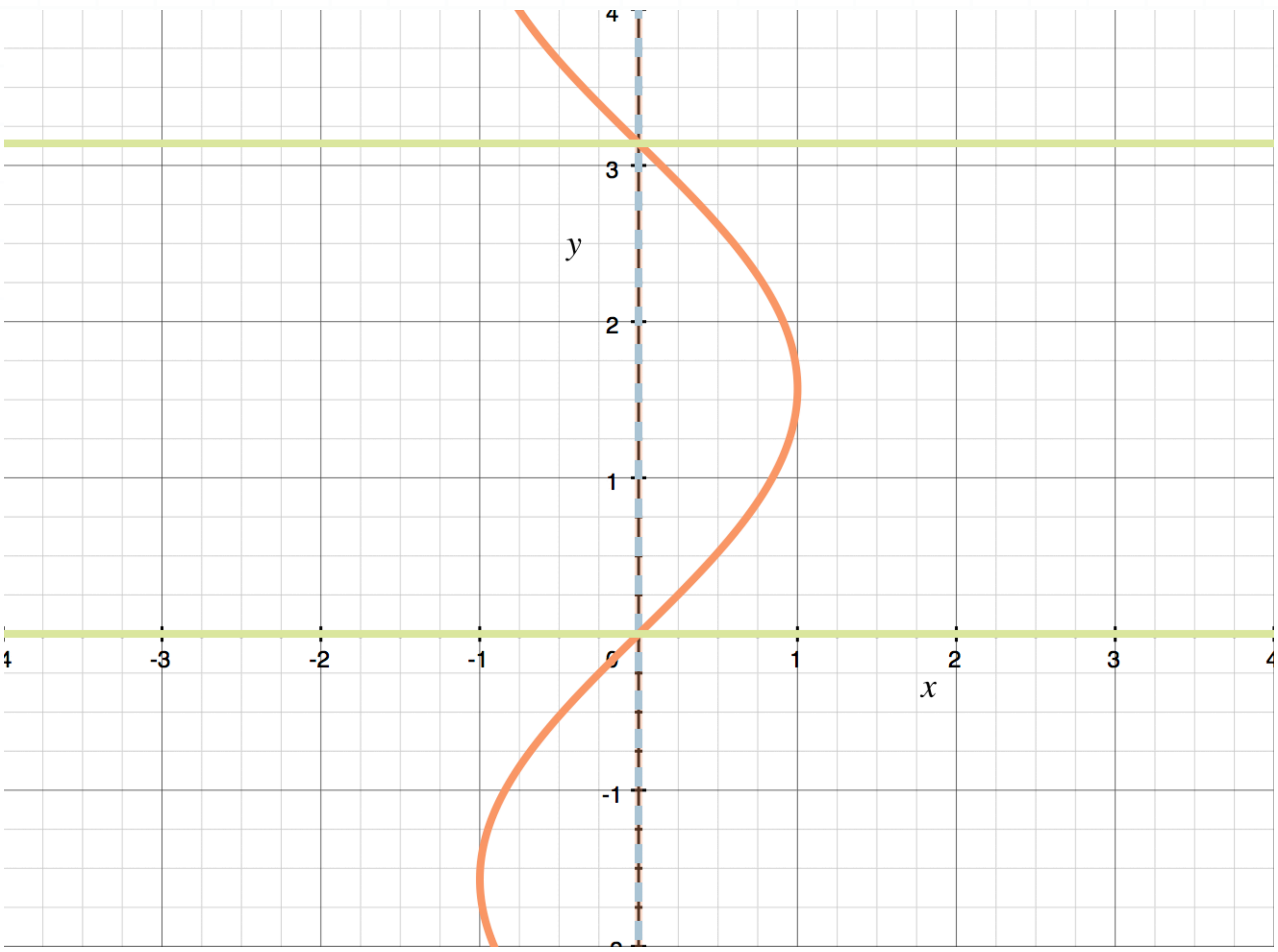
4. Use disks to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis. Set up the integral, but do not evaluate it.

$$x = \sin y$$

$y = 0$ and $y = \pi$

Solution:

A sketch of the region and the axis of revolution $x = 0$ is



The volume given by disks is

$$V = \int_c^d \pi \left[f(y)\right]^2 \ dy$$

$$V = \int_0^\pi \pi \left[\sin y\right]^2 \ dy$$

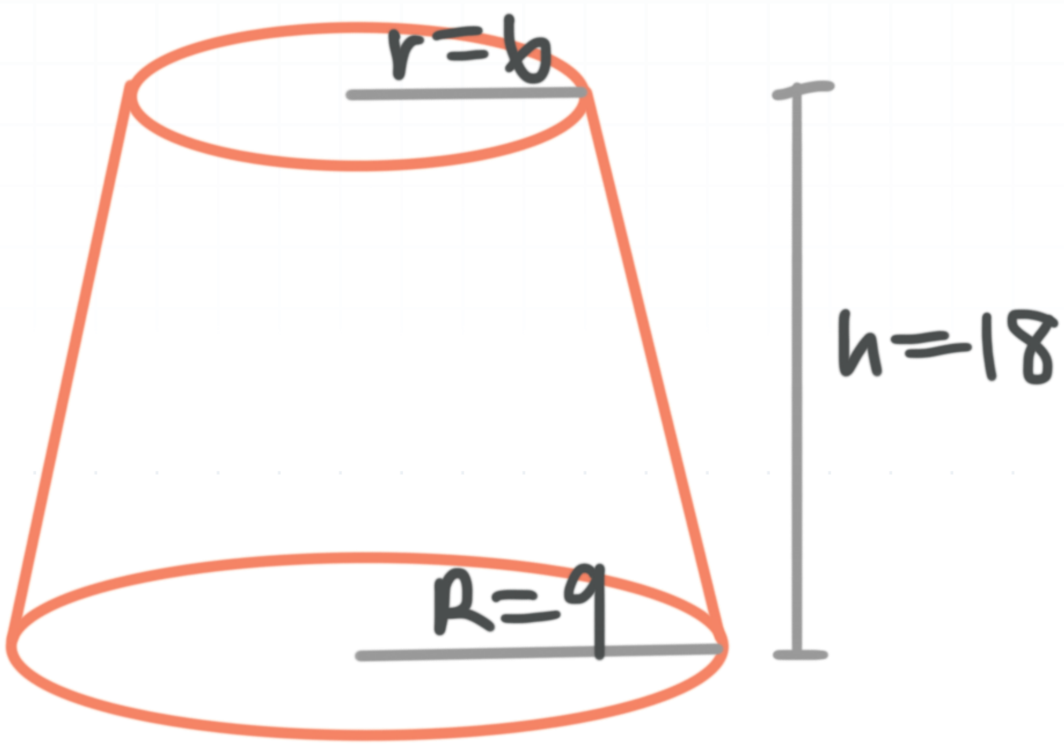
$$V = \pi \int_0^\pi \sin^2 y \ dy$$

DISKS, VOLUME OF THE FRUSTUM

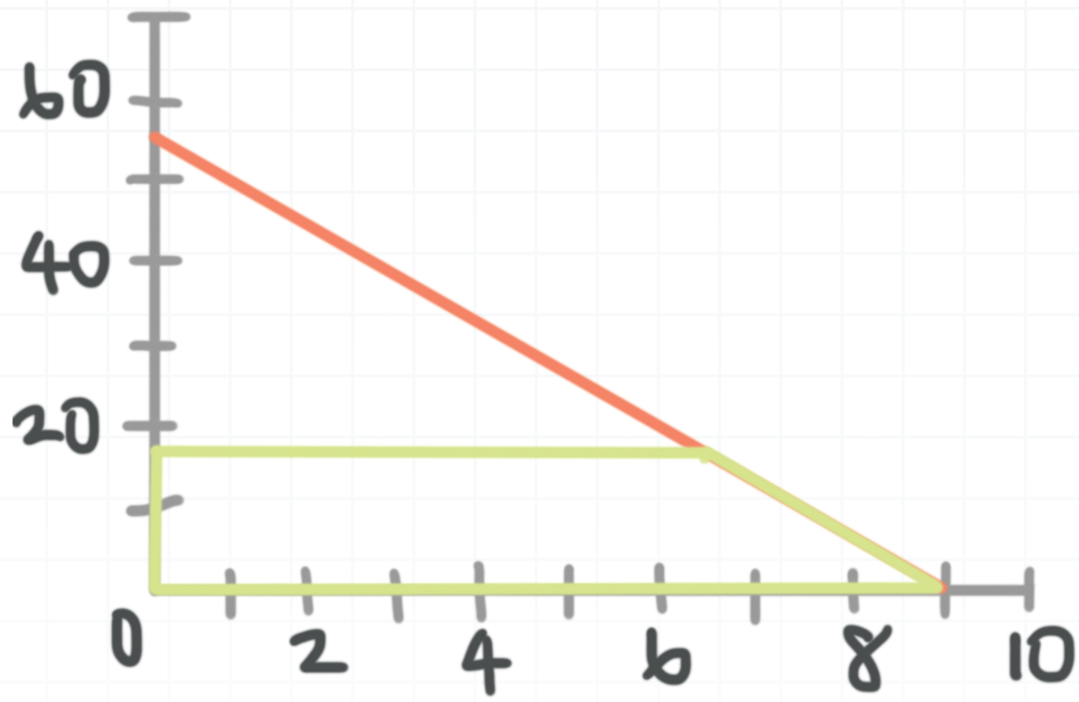
1. Use disks to find the volume of the frustum of a right circular cone with height $h = 18$ inches, a lower base radius $R = 9$ inches, and an upper radius of $r = 6$ inches.

Solution:

A sketch of the frustum is



We could create this frustum by rotating this green region about the y-axis.



The slope of the line contains $(9,0)$ and $(6,18)$. The slope that connects the points is

$$m = \frac{18-0}{6-9} = -\frac{18}{3} = -6$$

Then the line that gives the slant height is

$$y = -6x + 54$$

$$y - 54 = -6x$$

$$x = \frac{y-54}{-6}$$

$$x = 9 - \frac{1}{6}y$$

Then the volume of the frustum, using disks, is given by

$$V = \int_c^d \pi \left[f(y)\right]^2 \ dy$$

$$V = \int_0^{18} \pi \left[9 - \frac{1}{6}y\right]^2 \ dy$$

$$V = \pi \int_0^{18} 81 - 3y + \frac{1}{36}y^2 \ dy$$

Integrate, then evaluate over the interval.

$$V = \pi \left(81y - \frac{3}{2}y^2 + \frac{1}{108}y^3\right) \Big|_0^{18}$$

$$V = \pi \left(81(18) - \frac{3}{2}(18)^2 + \frac{1}{108}(18)^3\right) - \pi \left(81(0) - \frac{3}{2}(0)^2 + \frac{1}{108}(0)^3\right)$$

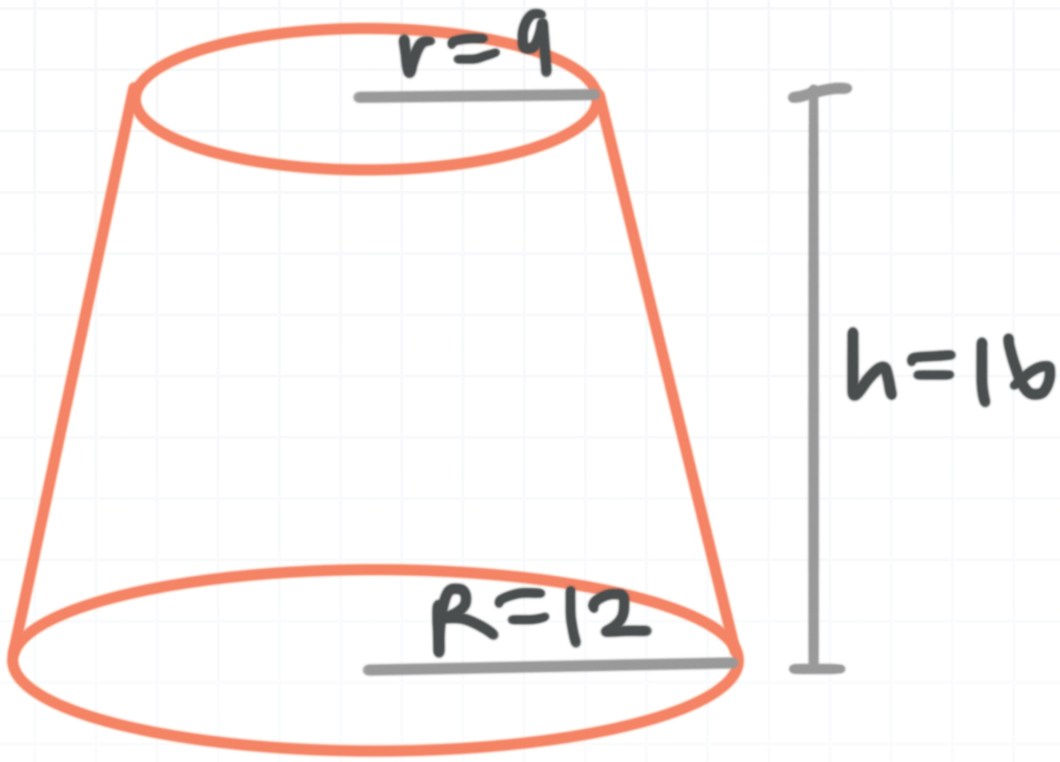
$$V = \pi(1{,}458 - 486 + 54)$$

$$V = 1{,}026\pi$$

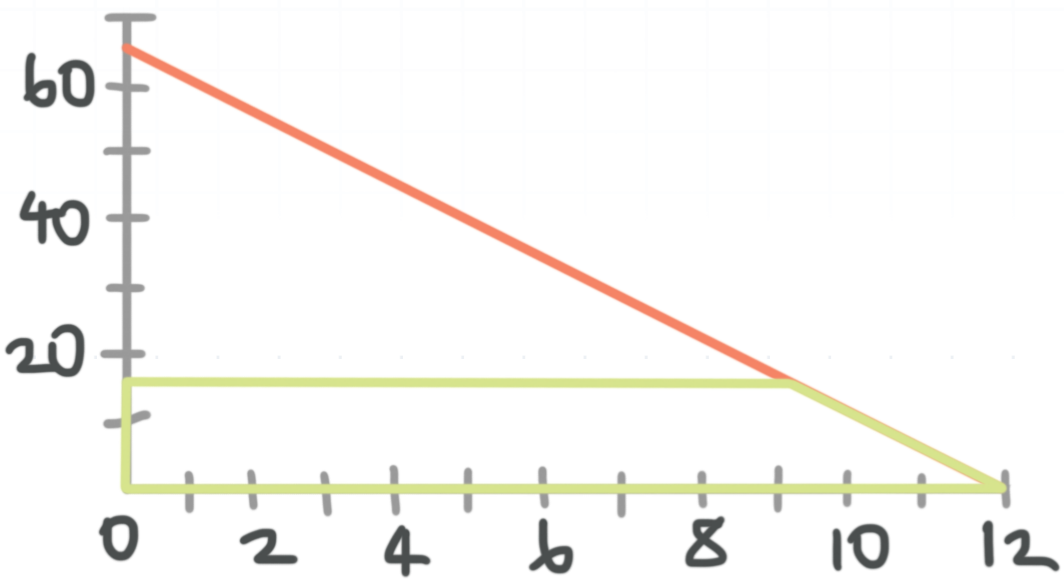
■ 2. Use disks to find the volume of the frustum of a right circular cone with height $h = 16$ inches, a lower base radius $R = 12$ inches, and an upper radius of $r = 9$ inches.

Solution:

A sketch of the frustum is



We could create this frustum by rotating this green region about the y-axis.



The slope of the line contains $(12,0)$ and $(9,16)$. The slope that connects the points is

$$m = \frac{16-0}{9-12} = -\frac{16}{3}$$

Then the line that gives the slant height is

$$y = -\frac{16}{3}x + 64$$

$$y - 64 = -\frac{16}{3}x$$

$$3y - 192 = -16x$$

$$x = -\frac{3}{16}y + 12$$

Then the volume of the frustum, using disks, is given by

$$V = \int_c^d \pi \left[f(y) \right]^2 \ dy$$

$$V = \int_0^{16} \pi \left[-\frac{3}{16}y + 12 \right]^2 \ dy$$

$$V = \pi \int_0^{16} \frac{9}{256}y^2 - \frac{9}{2}y + 144 \ dy$$

Integrate, then evaluate over the interval.

$$V = \pi \left(\frac{3}{256}y^3 - \frac{9}{4}y^2 + 144y \right) \Big|_0^{16}$$

$$V = \pi \left(\frac{3}{256}(16)^3 - \frac{9}{4}(16)^2 + 144(16) \right) - \pi \left(\frac{3}{256}(0)^3 - \frac{9}{4}(0)^2 + 144(0) \right)$$

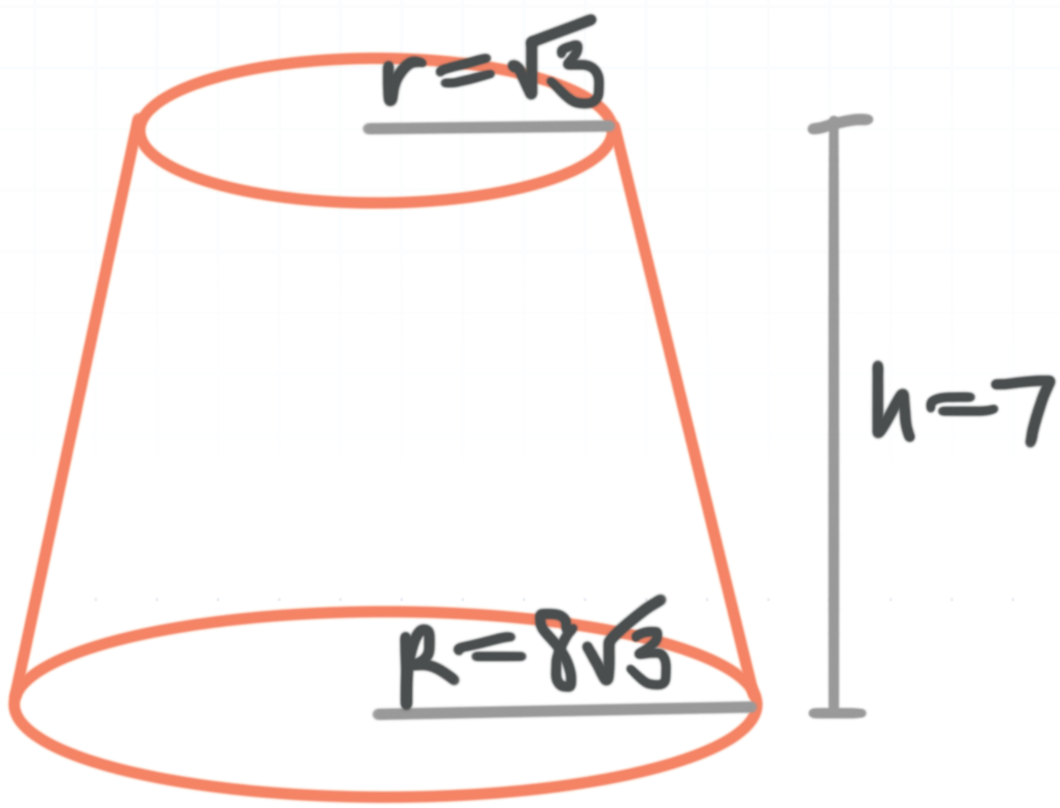
$$V = \pi(48 - 576 + 2{,}304)$$

$$V = 1{,}776\pi$$

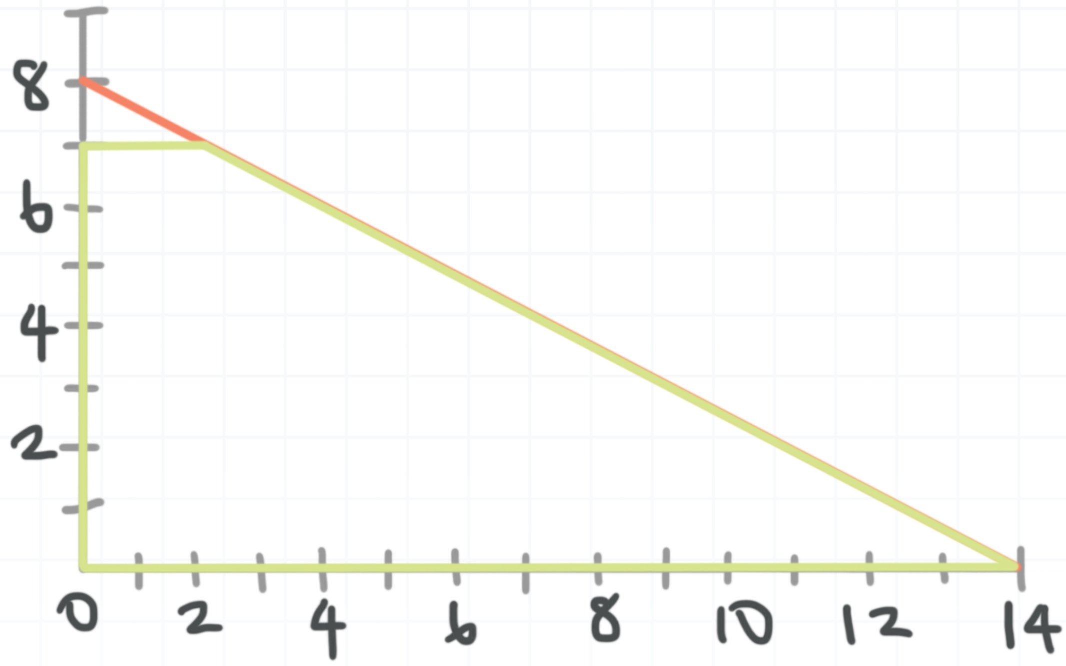
3. Use disks to find the volume of the frustum of a right circular cone with height $h = 7$ inches, a lower base radius $R = 8\sqrt{3}$ inches, and an upper radius of $r = \sqrt{3}$ inches.

Solution:

A sketch of the frustum is



We could create this frustum by rotating this green region about the y-axis.



The slope of the line contains $\left(8\sqrt{3},0\right)$ and $\left(\sqrt{3},7\right)$. The slope that connects the points is

$$m=\frac{7-0}{\sqrt{3}-8\sqrt{3}}=-\frac{7}{7\sqrt{3}}=-\frac{1}{\sqrt{3}}$$

Then the line that gives the slant height is

$$y=-\frac{1}{\sqrt{3}}x+8$$

$$y-8=-\frac{1}{\sqrt{3}}x$$

$$8-y=\frac{1}{\sqrt{3}}x$$

$$x=8\sqrt{3}-\sqrt{3}y$$

Then the volume of the frustum, using disks, is given by

$$V=\int_c^d \pi\left[f(y)\right]^2\ dy$$

$$V=\int_0^7 \pi\left[8\sqrt{3}-\sqrt{3}y\right]^2\ dy$$

$$V=\pi\int_0^7 192-48y+3y^2\ dy$$

Integrate, then evaluate over the interval.

$$V=\pi(192y-24y^2+y^3)\Big|_0^7$$

$$V=\pi(192(7)-24(7)^2+7^3)-\pi(192(0)-24(0)^2+0^3)$$

$$V=\pi(1{,}344-1{,}176+343)$$

$$V=\pi(1{,}344-1{,}176+343)$$

$$V=511\pi$$

WASHERS, HORIZONTAL AXIS

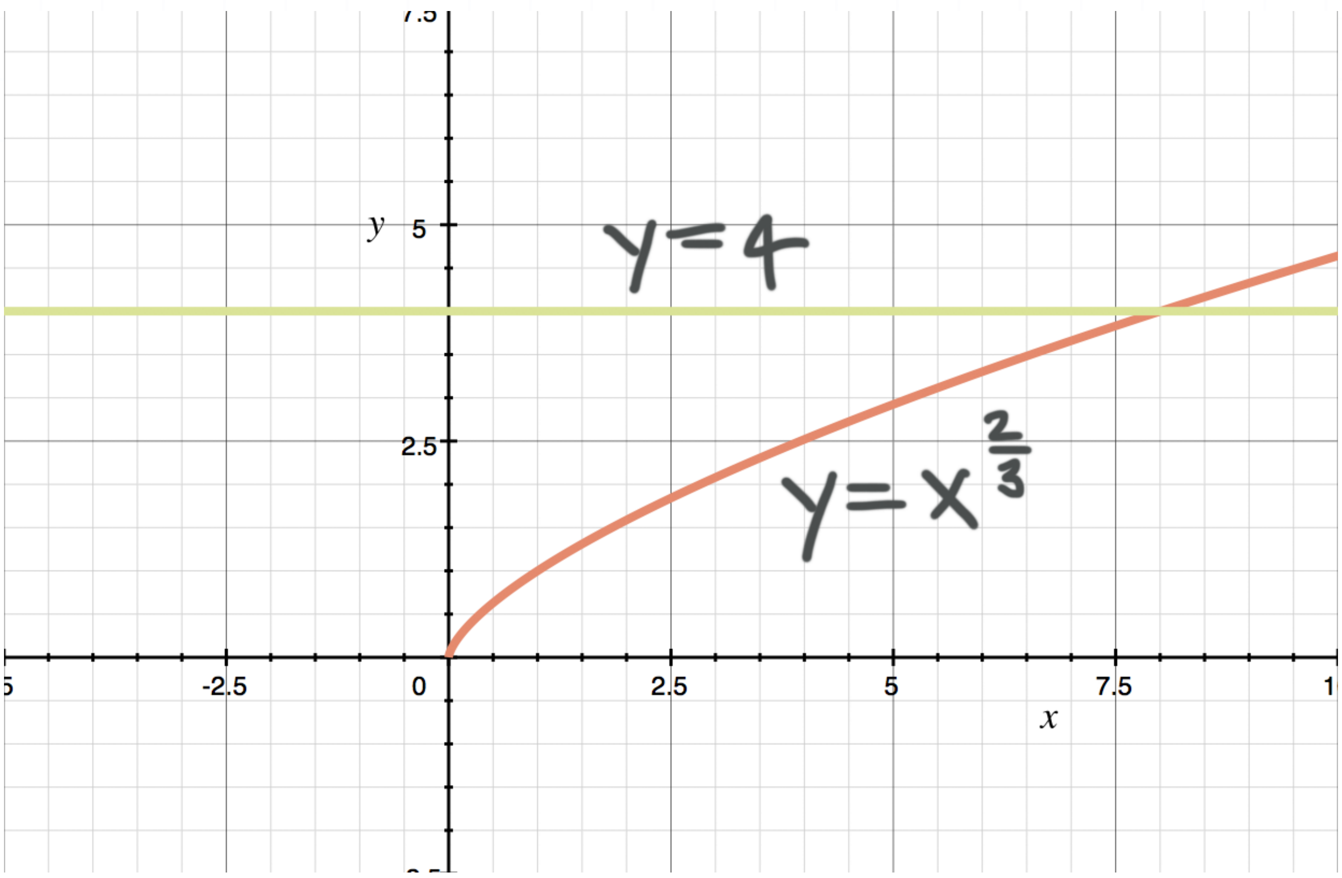
1. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y = x^{\frac{2}{3}}$ and $y = 4$

$x = 0$ and $x = 8$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_a^b \pi \left[f(x)\right]^2 - \pi \left[g(x)\right]^2 \ dx$$

$$\int_0^8 \pi \left[4\right]^2 - \pi \left[x^{\frac{2}{3}}\right]^2 \ dx$$

$$\pi \int_0^8 16 - x^{\frac{4}{3}} \ dx$$

Integrate, then evaluate over the interval.

$$\pi \left(16x - \frac{3}{7}x^{\frac{7}{3}}\right)\Big|_0^8$$

$$\pi \left(16(8) - \frac{3}{7}(8)^{\frac{7}{3}}\right) - \pi \left(16(0) - \frac{3}{7}(0)^{\frac{7}{3}}\right)$$

$$\pi \left(128 - \frac{3}{7}(2)^7\right)$$

$$\frac{896\pi}{7} - \frac{384\pi}{7}$$

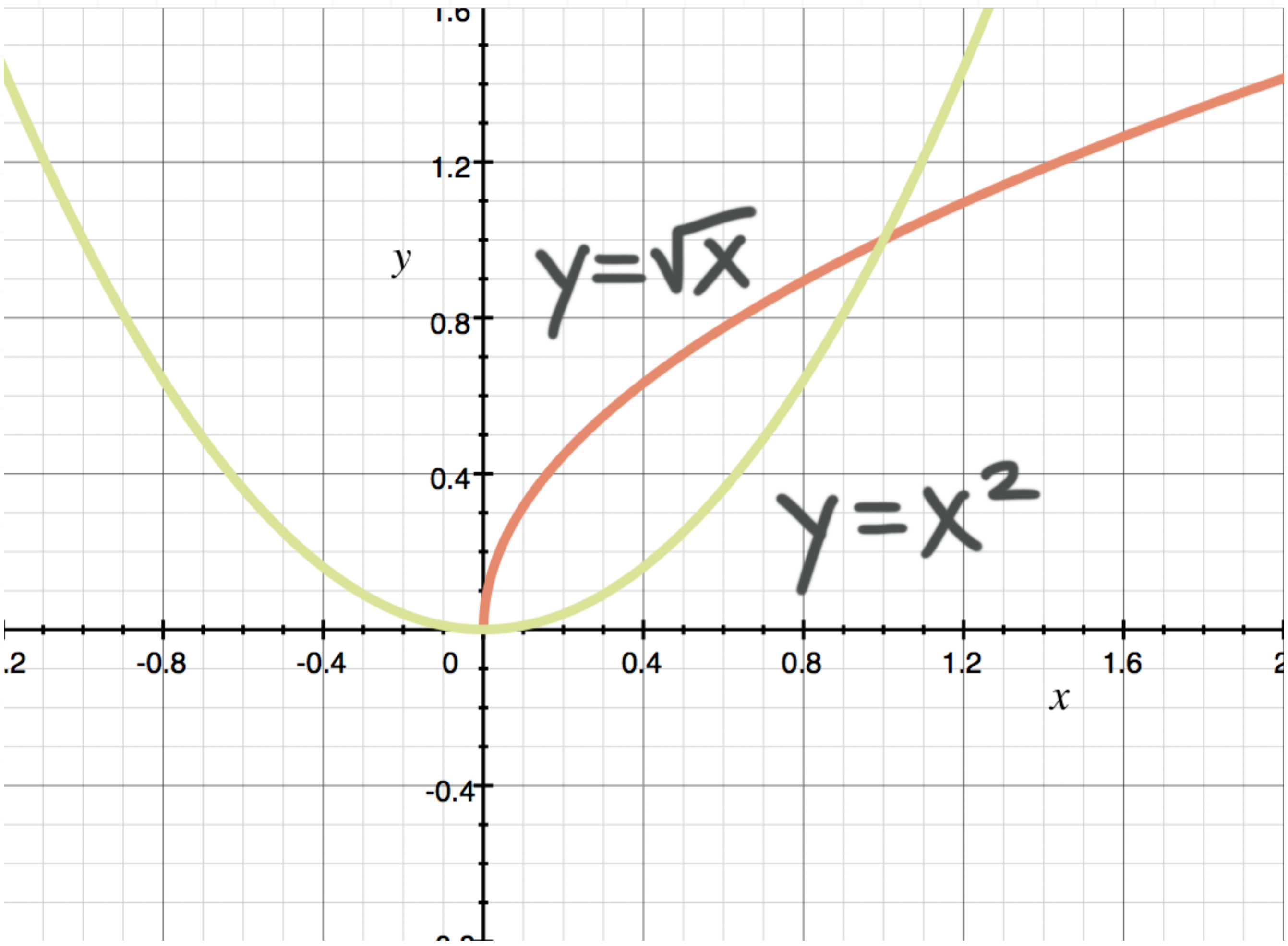
$$\frac{512\pi}{7}$$

■ 2. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y = x^2$ and $y = \sqrt{x}$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_a^b \pi\left[f(x)\right]^2-\pi\left[g(x)\right]^2\ dx$$

$$\int_0^1 \pi\left[\sqrt{x}\right]^2-\pi\left[x^2\right]^2\ dx$$

$$\pi\int_0^1 x-x^4\ dx$$

Integrate, then evaluate over the interval.

$$\pi\left(\frac{1}{2}x^2-\frac{1}{5}x^5\right)\Big|_0^1$$

$$\pi\left(\frac{1}{2}(1)^2-\frac{1}{5}(1)^5\right)-\pi\left(\frac{1}{2}(0)^2-\frac{1}{5}(0)^5\right)$$

$$\pi\left(\frac{1}{2}-\frac{1}{5}\right)$$

$$\pi\left(\frac{5}{10}-\frac{2}{10}\right)$$

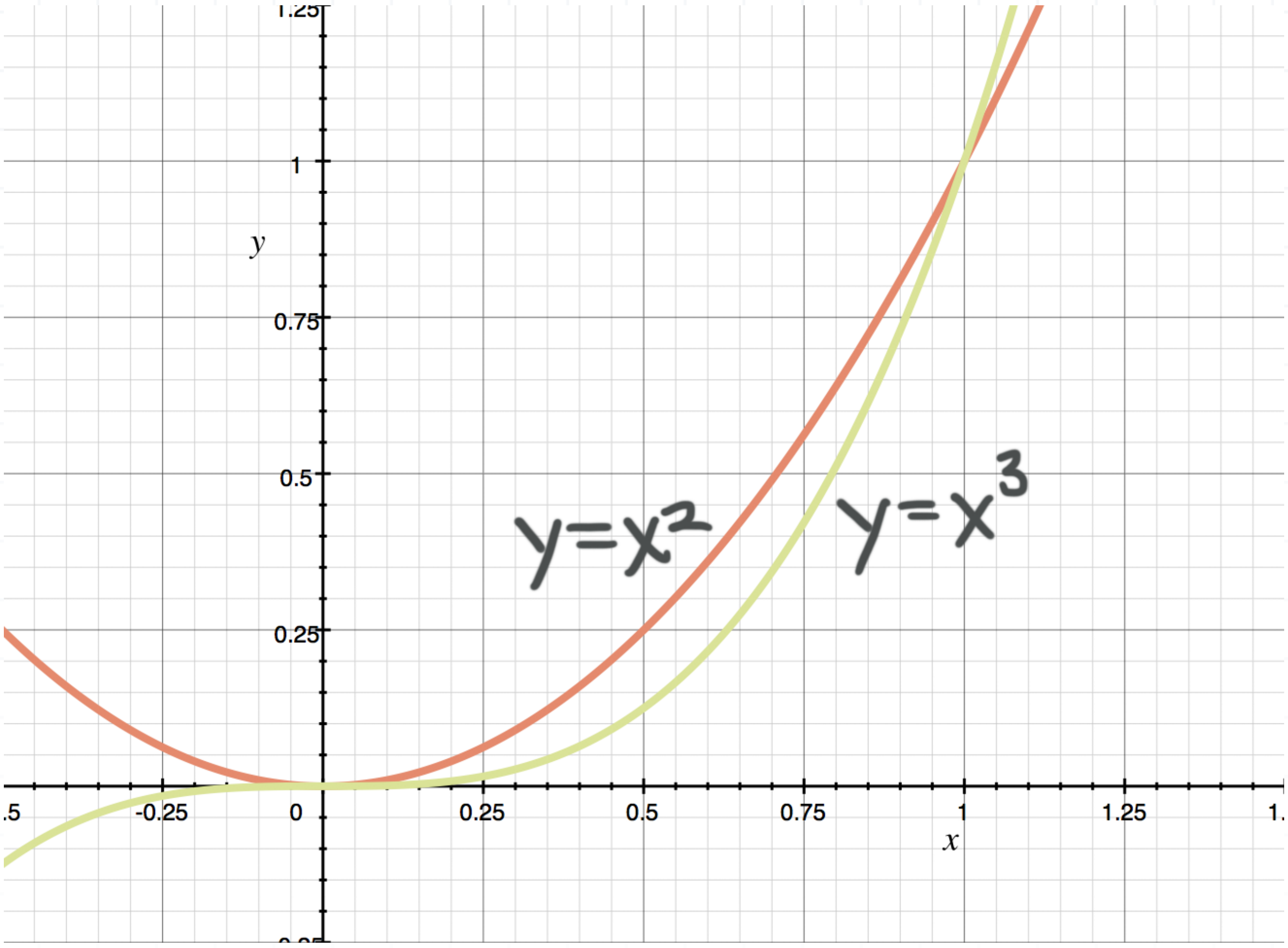
$$\frac{3\pi}{10}$$

■ 3. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$y=x^2$ and $y=x^3$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_a^b \pi \left[f(x)\right]^2 - \pi \left[g(x)\right]^2 \ dx$$

$$\int_0^1 \pi \left[x^2\right]^2 - \pi \left[x^3\right]^2 \ dx$$

$$\pi \int_0^1 x^4 - x^6 \ dx$$

Integrate, then evaluate over the interval.

$$\pi \left(\frac{1}{5}x^5 - \frac{1}{7}x^7 \right) \Bigg|_0^1$$

$$\pi\left(\frac{1}{5}(1)^5-\frac{1}{7}(1)^7\right)-\pi\left(\frac{1}{5}(0)^5-\frac{1}{7}(0)^7\right)$$

$$\pi\left(\frac{1}{5}-\frac{1}{7}\right)$$

$$\pi\left(\frac{7}{35}-\frac{5}{35}\right)$$

$$\frac{2\pi}{35}$$

WASHERS, VERTICAL AXIS

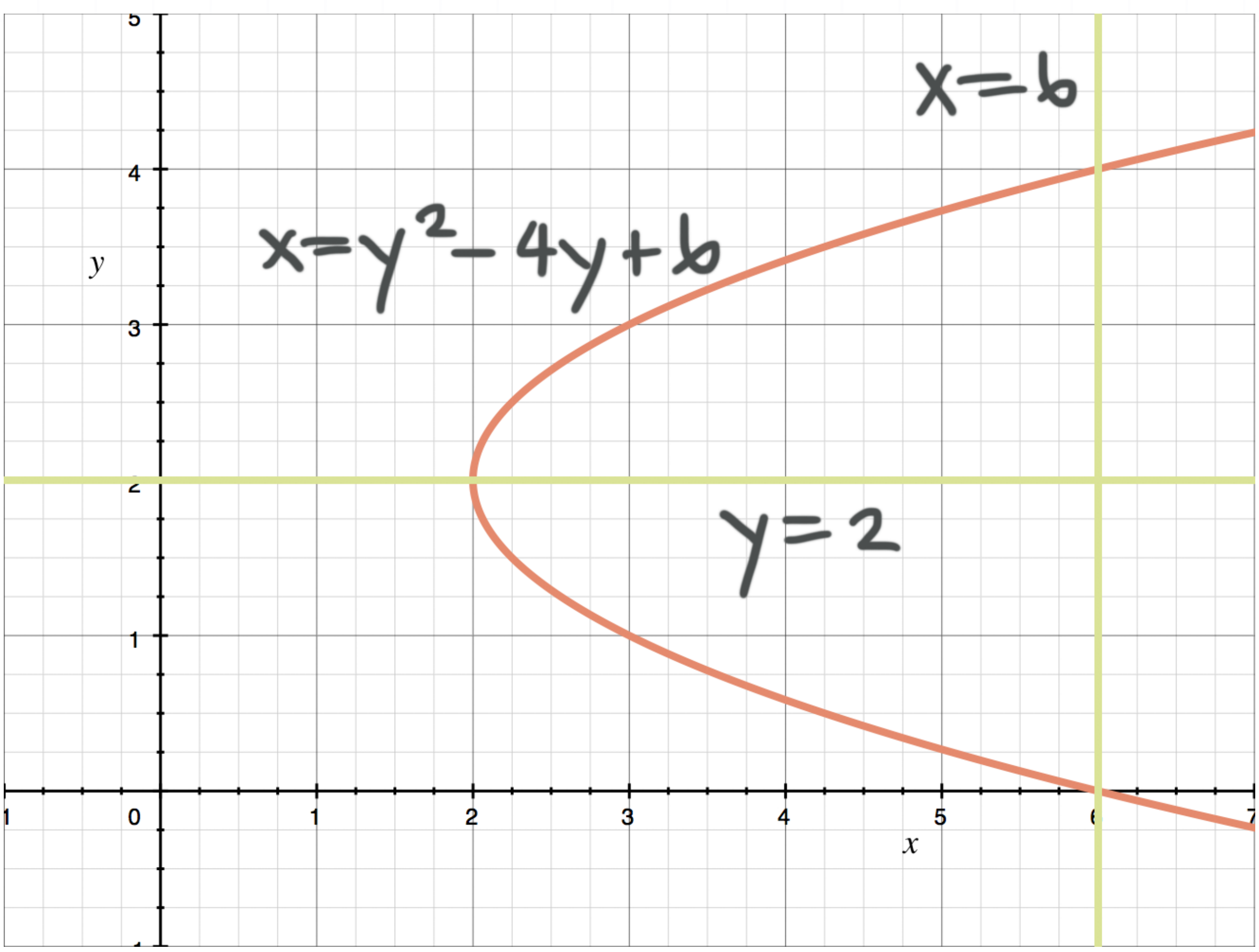
■ 1. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = y^2 - 4y + 6$ and $x = 6$

$y = 2$ and $y = 4$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_{c}^{d} \pi \left[f(y)\right]^2 - \pi \left[g(y)\right]^2 \ dy$$

$$\int_{2}^{4} \pi \left[6\right]^2 - \pi \left[y^2 - 4y + 6\right]^2 \ dy$$

$$\int_{2}^{4} 36\pi - \pi(y^4 - 4y^3 + 6y^2 - 4y^3 + 16y^2 - 24y + 6y^2 - 24y + 36) \ dy$$

$$\pi \int_{2}^{4} 36 - y^4 + 8y^3 - 28y^2 + 48y - 36 \ dy$$

$$-\pi \int_{2}^{4} y^4 - 8y^3 + 28y^2 - 48y \ dy$$

Integrate, then evaluate over the interval.

$$-\pi \left(\frac{1}{5}y^5 - 2y^4 + \frac{28}{3}y^3 - 24y^2 \right) \Bigg|_{2}^{4}$$

$$-\pi \left(\frac{1}{5}(4)^5 - 2(4)^4 + \frac{28}{3}(4)^3 - 24(4)^2 \right) + \pi \left(\frac{1}{5}(2)^5 - 2(2)^4 + \frac{28}{3}(2)^3 - 24(2)^2 \right)$$

$$-\pi \left(\frac{1}{5}(1{,}024) - 2(256) + \frac{28}{3}(64) - 24(16) \right) + \pi \left(\frac{1}{5}(32) - 2(16) + \frac{28}{3}(8) - 24(4) \right)$$

$$-\pi \left(\frac{1{,}024}{5} - 512 + \frac{1{,}792}{3} - 384 \right) + \pi \left(\frac{32}{5} - 32 + \frac{224}{3} - 96 \right)$$

$$-\frac{1{,}024\pi}{5} - \frac{1{,}792\pi}{3} + 896\pi + \frac{32\pi}{5} + \frac{224\pi}{3} - 128\pi$$

$$-\frac{992\pi}{5}-\frac{1{,}568\pi}{3}+768\pi$$

$$-\frac{2{,}976\pi}{15}-\frac{7{,}840\pi}{15}+\frac{11{,}520\pi}{15}$$

$$\frac{704\pi}{15}$$

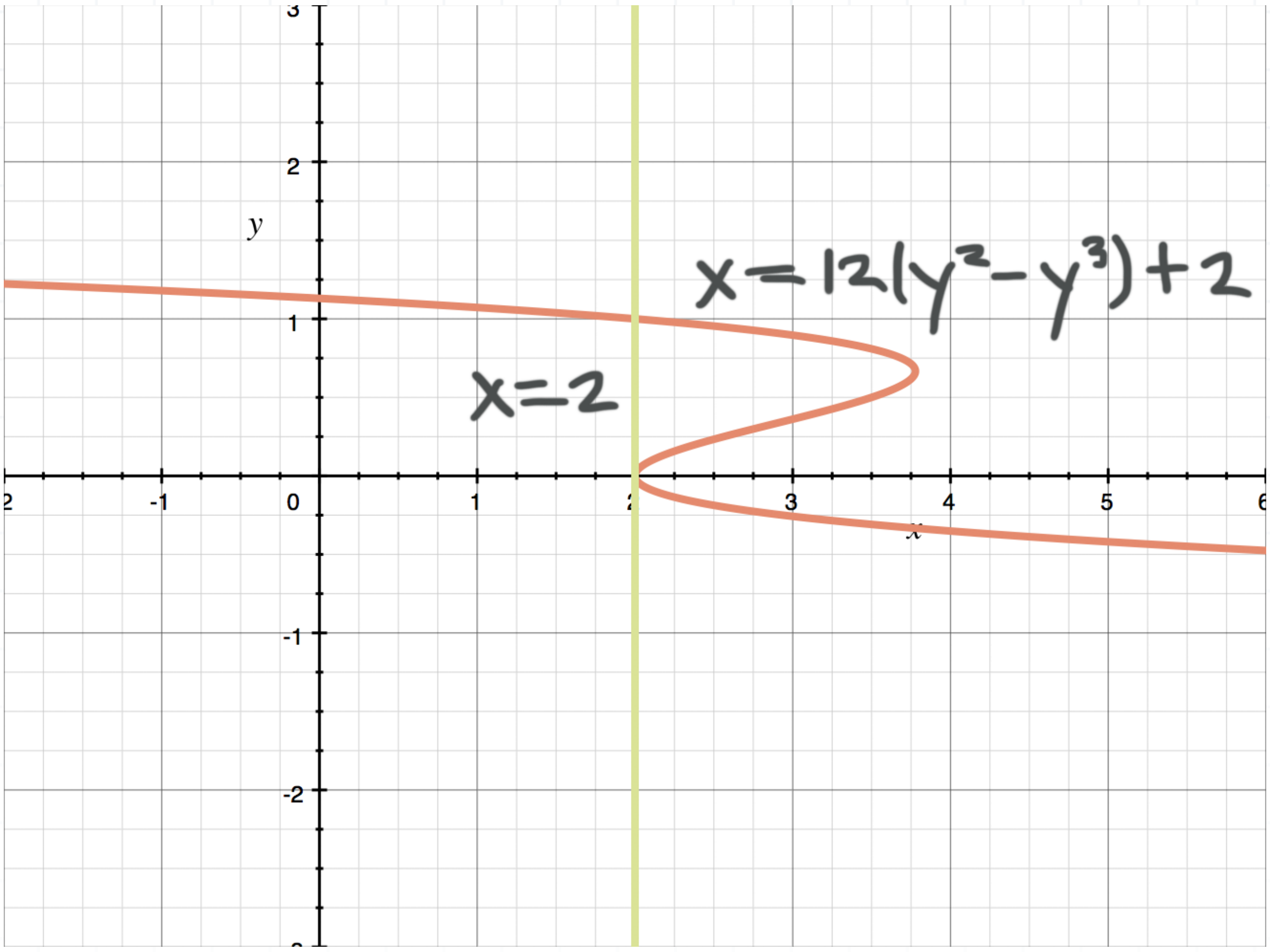
2. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x=12(y^2-y^3)+2$ and $x=2$

$y=0$ and $y=1$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_c^d \pi \left[f(y)\right]^2 - \pi \left[g(y)\right]^2 \ dy$$

$$\int_0^1 \pi \left[12(y^2 - y^3) + 2\right]^2 - \pi \left[2\right]^2 \ dy$$

$$\int_0^1 \pi \left[12y^2 - 12y^3 + 2\right]^2 - 4\pi \ dy$$

$$\int_0^1 \pi(144y^4 - 144y^5 + 24y^2 - 144y^5 + 144y^6 - 24y^3 + 24y^2 - 24y^3 + 4) - 4\pi \ dy$$

$$48\pi \int_0^1 3y^6 - 6y^5 + 3y^4 - y^3 + y^2 \ dy$$

Integrate, then evaluate over the interval.

$$48\pi\left(\frac{3}{7}y^7 - y^6 + \frac{3}{5}y^5 - \frac{1}{4}y^4 + \frac{1}{3}y^3\right)\Big|_0^1$$

$$48\pi\left(\frac{3}{7}(1)^7 - (1)^6 + \frac{3}{5}(1)^5 - \frac{1}{4}(1)^4 + \frac{1}{3}(1)^3\right)$$

$$-48\pi\left(\frac{3}{7}(0)^7 - (0)^6 + \frac{3}{5}(0)^5 - \frac{1}{4}(0)^4 + \frac{1}{3}(0)^3\right)$$

$$48\pi\left(\frac{3}{7} - 1 + \frac{3}{5} - \frac{1}{4} + \frac{1}{3}\right)$$

$$48\pi\left(\frac{180}{420} - \frac{420}{420} + \frac{252}{420} - \frac{105}{420} + \frac{140}{420}\right)$$

$$4\pi\left(\frac{47}{35}\right)$$

$$\frac{188\pi}{35}$$

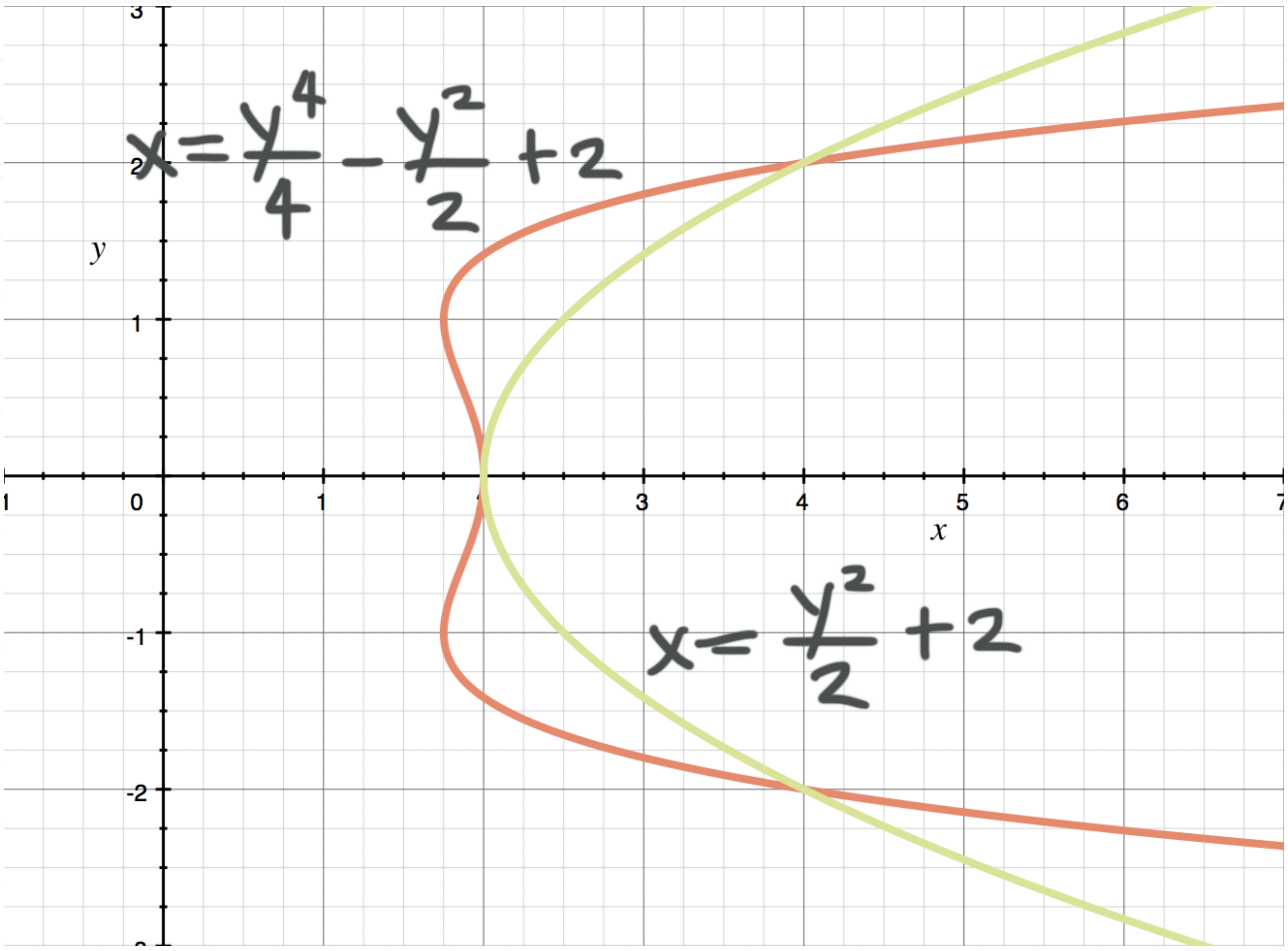
■ 3. Use washers to find the volume of the solid that's formed by rotating the region enclosed by the curves about the y-axis.

$x = \frac{y^4}{4} - \frac{y^2}{2} + 2$ and $x = \frac{y^2}{2} + 2$

$y = -2$ and $y = 2$

Solution:

A sketch of the region is



The volume given by washers is

$$\int_c^d \pi\left[f(y)\right]^2 - \pi\left[g(y)\right]^2\ dy$$

$$\int_{-2}^{2} \pi\left[\frac{y^2}{2}+2\right]^2 - \pi\left[\frac{y^4}{4}-\frac{y^2}{2}+2\right]^2\ dy$$

$$\pi\int_{-2}^{2} \frac{y^4}{4}+2y^2+4-\left(\frac{y^8}{16}-\frac{y^6}{8}+\frac{y^4}{2}-\frac{y^6}{8}+\frac{y^4}{4}-y^2+\frac{y^4}{2}-y^2+4\right)\ dy$$

$$\pi\int_{-2}^{2}\frac{y^4}{4}+2y^2+4-\frac{y^8}{16}+\frac{y^6}{8}-\frac{y^4}{2}+\frac{y^6}{8}-\frac{y^4}{4}+y^2-\frac{y^4}{2}+y^2-4\ dy$$

$$\pi\int_{-2}^{2}-\frac{y^8}{16}+\frac{y^6}{4}-y^4+4y^2\ dy$$

$$-\pi\int_{-2}^{2}\frac{y^8}{16}-\frac{y^6}{4}+y^4-4y^2\ dy$$

Integrate, then evaluate over the interval.

$$-\pi\left(\frac{y^9}{144}-\frac{y^7}{28}+\frac{y^5}{5}-\frac{4y^3}{3}\right)\Bigg|_{-2}^{2}$$

$$-\pi\left(\frac{(2)^9}{144}-\frac{(2)^7}{28}+\frac{(2)^5}{5}-\frac{4(2)^3}{3}\right)+\pi\left(\frac{(-2)^9}{144}-\frac{(-2)^7}{28}+\frac{(-2)^5}{5}-\frac{4(-2)^3}{3}\right)$$

$$\pi\left(-\frac{512}{144}+\frac{128}{28}-\frac{32}{5}+\frac{32}{3}-\frac{512}{144}+\frac{128}{28}-\frac{32}{5}+\frac{32}{3}\right)$$

$$\pi\left(-\frac{64}{9}+\frac{64}{7}-\frac{64}{5}+\frac{64}{3}\right)$$

$$\pi\left(-\frac{6{,}720}{945}+\frac{8{,}640}{945}-\frac{12{,}096}{945}+\frac{20{,}160}{945}\right)$$

$$\frac{9{,}984\pi}{945}$$

$$\frac{3{,}328\pi}{315}$$

CYLINDRICAL SHELLS, HORIZONTAL AXIS

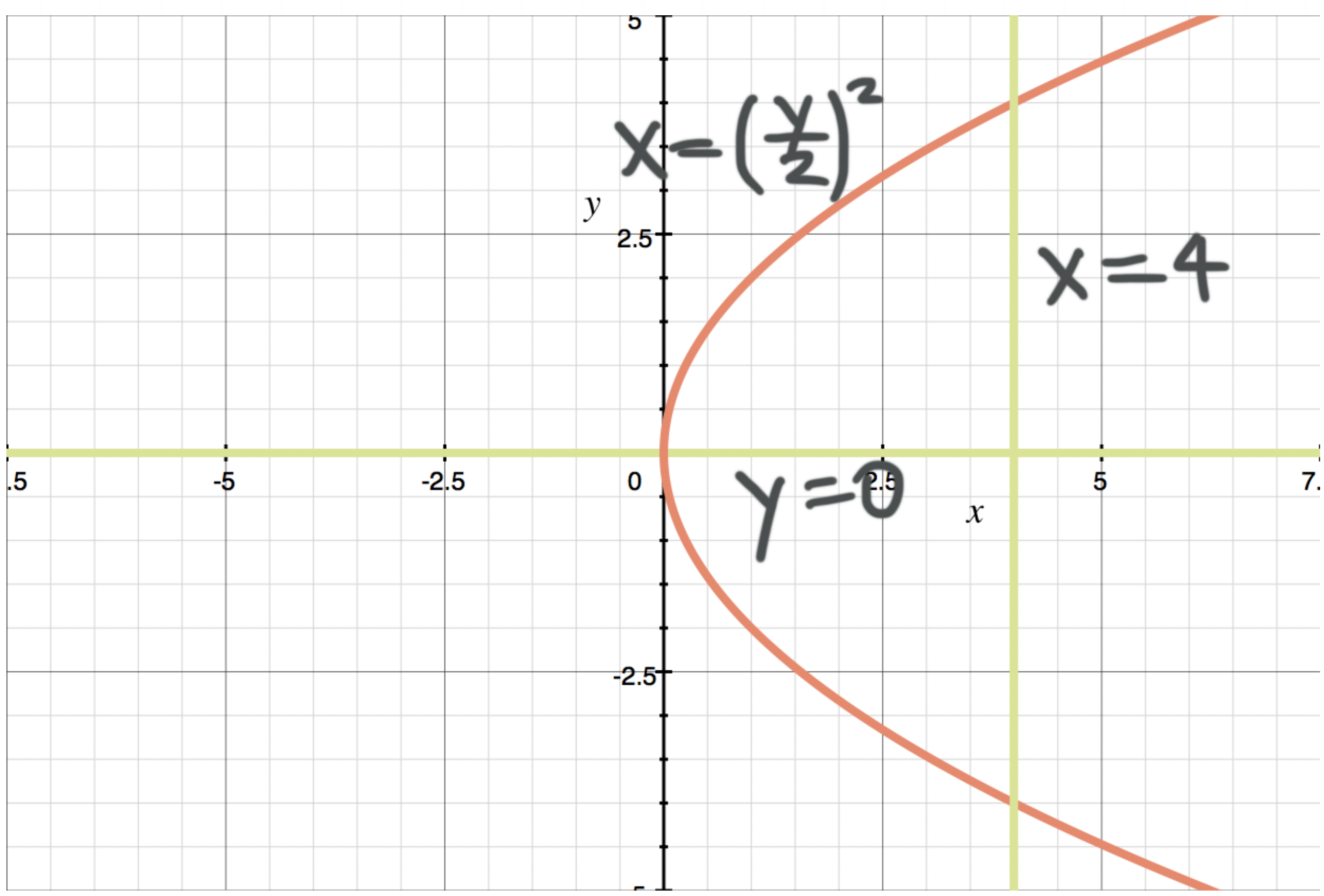
■ 1. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$$x = \left(\frac{y}{2}\right)^2 \text{ and } x = 4$$

$$y = 0$$

Solution:

A sketch of the region is



The volume given by cylindrical shells is

$$\int_c^d 2\pi y \left[f(y) - g(y)\right] \ dy$$

$$\int_0^4 2\pi y \left[4 - \left(\frac{y}{2}\right)^2\right] \ dy$$

$$2\pi \int_0^4 y \left(4 - \frac{y^2}{4}\right) \ dy$$

$$2\pi \int_0^4 4y - \frac{y^3}{4} \ dy$$

Integrate, then evaluate over the interval.

$$2\pi \left(2y^2 - \frac{y^4}{16}\right)\Big|_0^4$$

$$2\pi \left(2(4)^2 - \frac{4^4}{16}\right) - 2\pi \left(2(0)^2 - \frac{0^4}{16}\right)$$

$$2\pi(32 - 16)$$

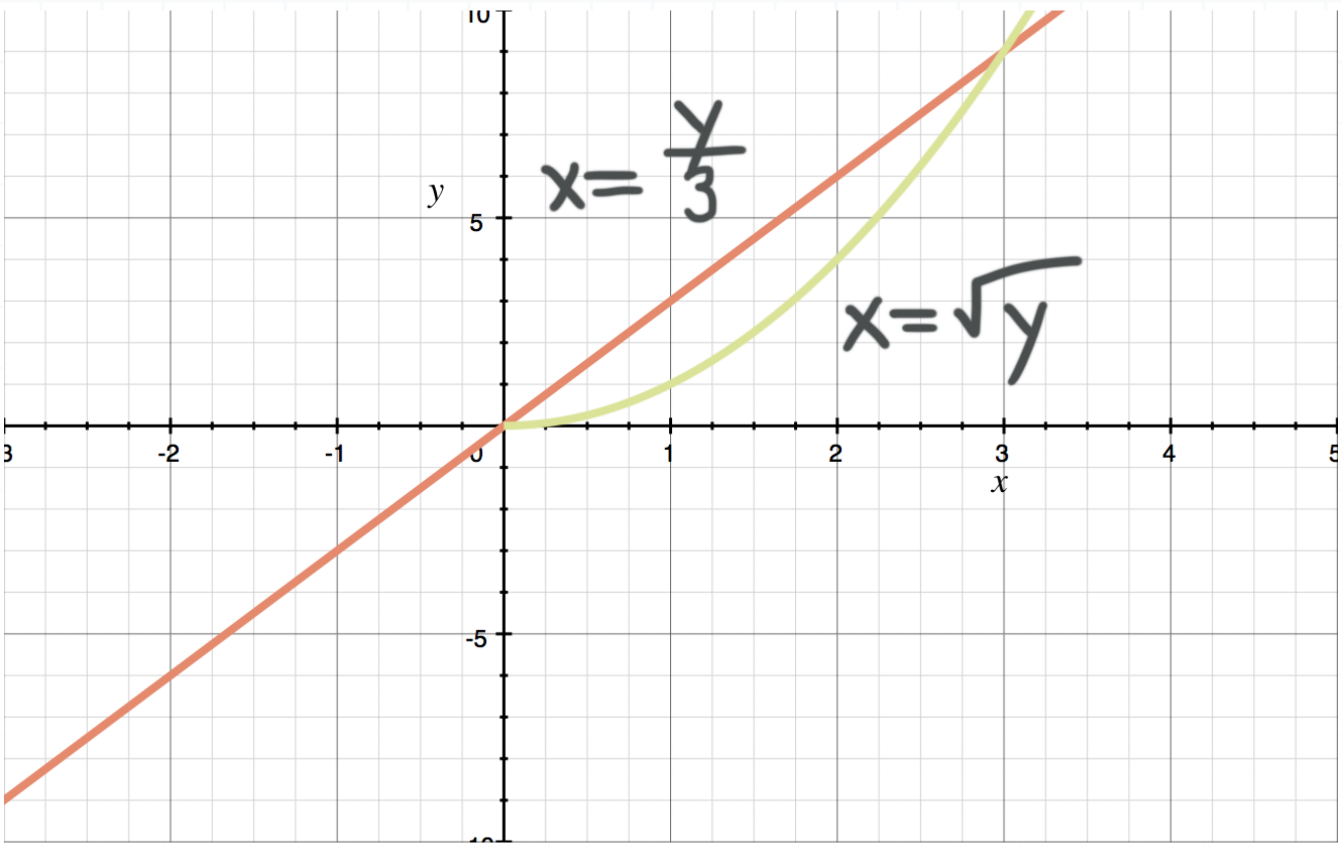
$$32\pi$$

■ 2. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$x = \dfrac{y}{3}$ and $x = \sqrt{y}$

Solution:

A sketch of the region is



The volume given by cylindrical shells is

$$\int_{c}^{d} 2\pi y\left[f(y)-g(y)\right]\ dy$$

$$\int_{0}^{9} 2\pi y\left[\sqrt{y}-\frac{y}{3}\right]\ dy$$

$$2\pi\int_{0}^{9} y^{\frac{3}{2}}-\frac{y^2}{3}\ dy$$

Integrate, then evaluate over the interval.

$$2\pi\left(\frac{2}{5}y^{\frac{5}{2}}-\frac{y^3}{9}\right)\Big|_0^9$$

$$2\pi\left(\frac{2}{5}(9)^{\frac{5}{2}}-\frac{9^3}{9}\right)-2\pi\left(\frac{2}{5}(0)^{\frac{5}{2}}-\frac{0^3}{9}\right)$$

$$2\pi\left(\frac{486}{5}-81\right)$$

$$2\pi\left(\frac{486}{5}-\frac{405}{5}\right)$$

$$\frac{162\pi}{5}$$

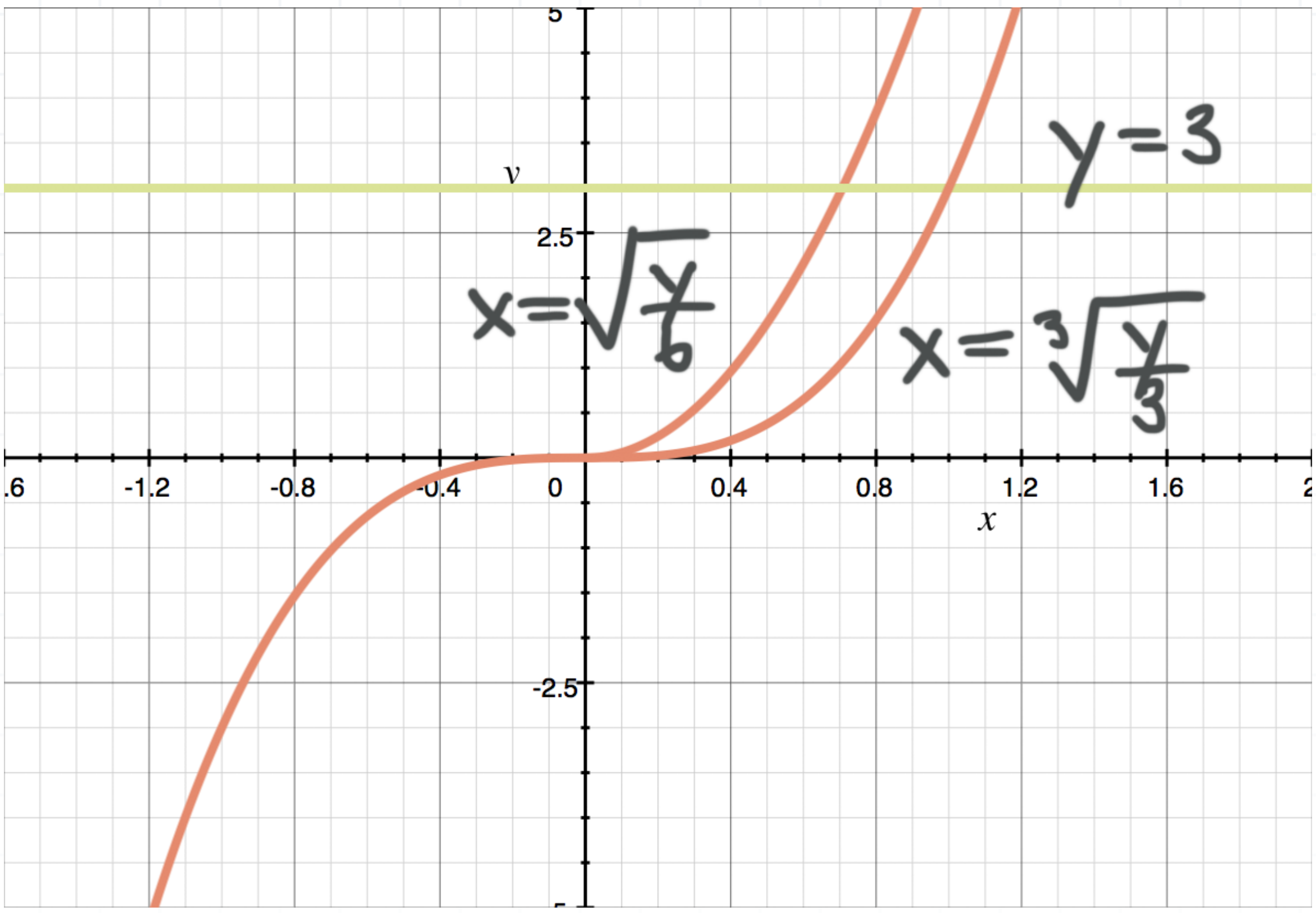
■ 3. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$x=\sqrt[3]{\frac{y}{3}}$ and $x=\sqrt{\frac{y}{6}}$

$$y=3$$

Solution:

A sketch of the region is



The volume given by cylindrical shells is

$$\int_c^d 2\pi y\left[f(y)-g(y)\right]\ dy$$

$$\int_0^3 2\pi y\left(\sqrt[3]{\frac{y}{3}}-\sqrt{\frac{y}{6}}\right)\ dy$$

$$2\pi\int_0^3 \frac{\sqrt[3]{9}}{3}y^{\frac{4}{3}}-\frac{\sqrt{6}}{6}y^{\frac{3}{2}}\ dy$$

Integrate, then evaluate over the interval.

$$2\pi\left(\frac{\sqrt[3]{9}}{7}y^{\frac{7}{3}}-\frac{\sqrt{6}}{15}y^{\frac{5}{2}}\right)\Bigg|_0^3$$

$$2\pi\left(\frac{\sqrt[3]{9}}{7}(3)^{\frac{7}{3}}-\frac{\sqrt{6}}{15}(3)^{\frac{5}{2}}\right)-2\pi\left(\frac{\sqrt[3]{9}}{7}(0)^{\frac{7}{3}}-\frac{\sqrt{6}}{15}(0)^{\frac{5}{2}}\right)$$

$$2\pi\left(\frac{\sqrt[3]{9}}{7}(3)^{\frac{7}{3}}-\frac{\sqrt{6}}{15}(3)^{\frac{5}{2}}\right)$$

$$2\pi(1.3115584)$$

$$8.241$$

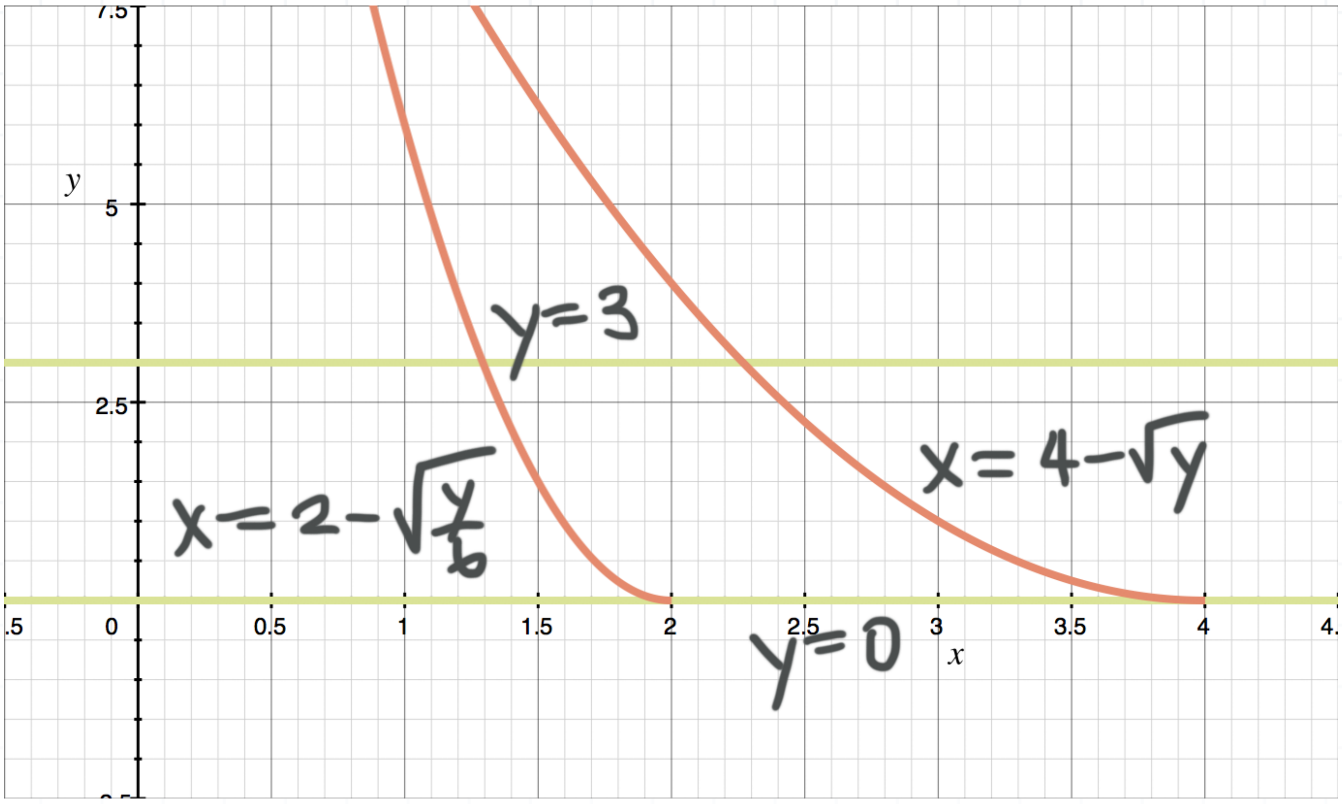
■ 4. Use cylindrical shells to find the volume of the solid that's formed by rotating the region enclosed by the curves about the x-axis.

$x=4-\sqrt{y}$ and $x=2-\sqrt{\frac{y}{6}}$

$y=0$ and $y=3$

Solution:

A sketch of the region is



The volume given by cylindrical shells is

$$\int_c^d 2\pi y\left[f(y)-g(y)\right]\ dy$$

$$\int_0^3 2\pi y\left[\left(4-\sqrt{y}\right)-\left(2-\sqrt{\frac{y}{6}}\right)\right]\ dy$$

$$2\pi\int_0^3 y\left(2-\sqrt{y}+\sqrt{\frac{y}{6}}\right)\ dy$$

$$2\pi\int_0^3 2y-y^{\frac{3}{2}}+y\frac{\sqrt{y}}{\sqrt{6}}\ dy$$

$$2\pi\int_0^3 2y-y^{\frac{3}{2}}+\frac{\sqrt{6}}{6}y^{\frac{3}{2}}\ dy$$

Integrate, then evaluate over the interval.

$$2\pi\left(y^2-\frac{2}{5}y^{\frac{5}{2}}+\frac{\sqrt{6}}{15}y^{\frac{5}{2}}\right)\Bigg|_0^3$$

$$2\pi\left(3^2-\frac{2}{5}(3)^{\frac{5}{2}}+\frac{\sqrt{6}}{15}(3)^{\frac{5}{2}}\right)-2\pi\left(0^2-\frac{2}{5}(0)^{\frac{5}{2}}+\frac{\sqrt{6}}{15}(0)^{\frac{5}{2}}\right)$$

$$2\pi\left(9-\frac{18\sqrt{3}}{5}+\frac{27\sqrt{2}}{15}\right)$$

$$18\pi\left(1-\frac{2\sqrt{3}}{5}+\frac{3\sqrt{2}}{15}\right)$$

$$2\pi(5.31020)$$

$$33.365$$

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