Topic: Hydrostatic force

Question: A rectangular tank is 3 m tall, 4 m wide, 6 m long, and full of water. If hydrostatic pressure at the bottom of the tank is $6{,}000$ Pa, find the hydrostatic force on the bottom of the tank.

Answer choices:

A $432{,}000$ N

B $144{,}000$ N

C $78{,}000$ N

D $60{,}000$ N

Solution: B

The formula we use to calculate hydrostatic force on a horizontal surface is

$$F = PA$$

where P is the hydrostatic pressure at the bottom of the tank and A is the surface area of the bottom of the tank.

We've been told that hydrostatic pressure at the bottom of the tank is $6{,}000$ Pa, but before we can plug into our formula, we need to find the surface area of the bottom of the rectangular tank. Since we know that that the area of a rectangle is $A = l \cdot w$, we can plug in the given length and width and get

$$A = 6 \text{ m} \cdot 4 \text{ m}$$

$$A = 24 \text{ m}^2$$

Plugging everything we know into the force equation, we get

$$F = 6{,}000 \text{ Pa} \cdot 24 \text{ m}^2$$

$$F = 6{,}000 \text{ kg/ms}^2 \cdot 24 \text{ m}^2$$

$$F = 144{,}000 \text{ kg m/s}^2$$

$$F = 144{,}000 \text{ N}$$

Topic: Hydrostatic force

Question: A rectangular tank is 4 m tall, 3 m wide, and full of water. Assuming the density of water is $\rho = 1{,}000$ kg/m^3, find the hydrostatic force on the end of the tank.

Answer choices:

A $\quad 130{,}500$ N

B $\quad 180{,}000$ N

C $\quad 235{,}200$ N

D $\quad 720{,}000$ N

Solution: C

To find hydrostatic force on one end of the tank, we'll use the modified force equation

$$F = WAd$$

where W is the weight of the liquid, A is the area of the surface and d is the depth of the liquid.

Since weight is density $\times$ gravity, weight is

$$W = \left(\frac{1{,}000\ \text{kg}}{\text{m}^3}\right)\left(\frac{9.8\ \text{m}}{\text{s}^2}\right)$$

$$W = \frac{9{,}800\ \text{kg}}{\text{m}^2\text{s}^2}$$

Since we're looking for force against a vertical surface and force at deeper depths is greater than force at shallower depths, we can't use the area of the entire surface in our force equation. Instead, we have to divide the surface into small horizontal strips so that we can assume that the force against each strip is roughly the same throughout the strip.

If we divide the end of the tank into tiny slices of equal depth, then each strip is 3 m wide and Δx tall, and sitting at a depth of x_i. The area of one strip is $A_i = 3 \cdot \Delta x$. The force against one strip is

$$F = WAd$$

$$F_i = (9{,}800)(3\Delta x)(x_i)$$

In order to solve for the force against the end of the tank, instead of against a small strip of it, we need to sum together the force against all of the slices, and take the limit as the number of slices approaches infinity, $n \to \infty$. Let's put this all together and see how it looks.

$$F = \lim_{n \to \infty} \sum_{i=1}^{n} (9{,}800)(3\Delta x)(x_i)$$

We need to remember that taking the limit as $n \to \infty$ of the sum of the force against all of the slices is the same as taking the integral of our force equation over the interval of the depth, $[0,4]$. Remember, when we move this into an integral, x_i becomes x, and Δx becomes dx. Let's put this all together and see how it looks.

$$F = \int_0^4 (9{,}800)(3\ dx)x$$

$$F = 29{,}400 \int_0^4 x\ dx$$

$$F = 29{,}400 \left(\frac{x^2}{2} \right) \Big|_0^4$$

$$F = 14{,}700x^2 \Big|_0^4$$

$$F = 14{,}700(4)^2 - 14{,}700(0)^2$$

$$F = 235{,}200$$

The hydrostatic force on the end is $F = 235{,}200$ N.

Topic: Hydrostatic force

Question: Find the hydrostatic force on the bottom of a tank, which is filled to the top with water, if the tank is 8 m long, 1 m wide, and 5 m tall. Assume the density of water is $\rho = 1{,}000$ kg/m^3.

Answer choices:

A $F = 392{,}000$ N

B $F = 196{,}000$ N

C $F = 98{,}000$ N

D $F = 49{,}000$ N

Solution: A

The formula we use to calculate hydrostatic force on a horizontal surface is

$$F = PA$$

where P is the hydrostatic pressure at the bottom of the tank and A is the surface area of the bottom of the tank.

The formula we use to calculate hydrostatic pressure is

$$P = \rho g d$$

where ρ is fluid pressure, g is gravity and d is depth. If we're dealing with water, and not some other liquid, we can simplify the formula, knowing that the density of water is $\rho = 1{,}000$ kg/m^3.

Plugging in water's given density, the gravitational constant, and the depth of the water, we get

$$P = \left(1{,}000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(5\ \text{m})$$

$$P = 49{,}000\ \frac{\text{kg}}{\text{ms}^2}$$

$$P = 49{,}000\ \text{Pa}$$

This is the hydrostatic pressure at the bottom of the tank, but before we can plug into our formula, we need to find the surface area of the bottom of the rectangular tank. Since we know that that the area of a rectangle is $A = l \cdot w$, we can plug in the given length and width and get

$$A = 8 \text{ m} \cdot 1 \text{ m}$$

$$A = 8 \text{ m}^2$$

Plugging everything we know into the force equation, we get

$$F = 49{,}000 \text{ Pa} \cdot 8 \text{ m}^2$$

$$F = 392{,}000 \text{ kg/ms}^2 \cdot \text{m}^2$$

$$F = 392{,}000 \text{ kg m/s}^2$$

$$F = 392{,}000 \text{ N}$$