Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{x+2}{(x-1)^2}\ dx$$

Answer choices:

A $\int \frac{1}{(x+1)^2} + \frac{3}{(x-1)}\ dx$

B $\int \frac{3}{(x-1)^2} + \frac{1}{(x-1)}\ dx$

C $\int \frac{3}{(x-1)^2} - \frac{1}{(x-1)}\ dx$

D $\int \frac{1}{(x-1)^2} - \frac{3}{(x-1)}\ dx$

Solution: B

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{x+2}{(x-1)^2}\ dx = \int \frac{A}{(x-1)^2} + \frac{B}{(x-1)}\ dx$$

Using partial fractions decomposition containing a linear factor, we have

$$\frac{x+2}{(x-1)^2} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)}$$

Now we'll solve for constants.

$$\frac{(x+2)(x-1)^2}{(x-1)^2} = \frac{A(x-1)^2}{(x-1)^2} + \frac{B(x-1)^2}{(x-1)}$$

$$x+2 = A + B(x-1)$$

$$x+2 = A + Bx - B$$

$$x+2 = Bx + A - B$$

$$x+2 = Bx + (A-B)$$

Equating coefficients on both sides, we get

[1] $B = 1$

[2] $A - B = 2$

We already know the value of B. Plugging [1] into [2] to solve for A, we get

$$A - 1 = 2$$

$$A = 3$$

Plugging the values for both constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{x+2}{(x-1)^2}\ dx = \int \frac{3}{(x-1)^2} + \frac{1}{(x-1)}\ dx$$

Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{4x^2+10x+8}{(x+1)^3}\ dx$$

Answer choices:

A $\displaystyle \int \frac{2}{(x+1)} + \frac{2}{(x+1)^2} + \frac{4}{(x+1)^3}\ dx$

B $\displaystyle \int \frac{2}{(x+1)^3} + \frac{3}{(x+1)^2} + \frac{4}{(x+1)}\ dx$

C $\displaystyle \int \frac{2}{(x+1)^3} - \frac{2}{(x+1)^2} + \frac{4}{(x+1)}\ dx$

D $\displaystyle \int \frac{2}{(x+1)^3} + \frac{2}{(x+1)^2} + \frac{4}{(x+1)}\ dx$

Solution: D

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{4x^2+10x+8}{(x+1)^3}\ dx = \int \frac{A}{(x+1)^3} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}\ dx$$

Using partial fractions decomposition containing a linear factor, we have

$$\frac{4x^2+10x+8}{(x+1)^3} = \frac{A}{(x+1)^3} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$$

Now we'll solve for constants.

$$\frac{(4x^2+10x+8)(x+1)^3}{(x+1)^3} = \frac{A(x+1)^3}{(x+1)^3} + \frac{B(x+1)^3}{(x+1)^2} + \frac{C(x+1)^3}{(x+1)}$$

$$4x^2+10x+8 = A + B(x+1) + C(x+1)^2$$

$$4x^2+10x+8 = A + Bx + B + C\left(x^2+2x+1\right)$$

$$4x^2+10x+8 = A + Bx + B + Cx^2 + 2Cx + C$$

$$4x^2+10x+8 = Cx^2 + Bx + 2Cx + A + B + C$$

$$4x^2+10x+8 = Cx^2 + (B+2C)x + (A+B+C)$$

Equating coefficients on both sides, we get

[1] $C = 4$

[2] $B + 2C = 10$

[3] $A+B+C=8$

We already know the value of C. Plugging [1] into [2] to solve for B, we get

$$B+2(4)=10$$

$$B=2$$

Plugging the values for B and C into [3] to solve for A, we get

$$A+2+4=8$$

$$A=2$$

Plugging the values for each of the three constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{4x^2+10x+8}{(x+1)^3}\ dx=\int \frac{2}{(x+1)^3}+\frac{2}{(x+1)^2}+\frac{4}{(x+1)}\ dx$$

Topic: Repeated linear factors

Question: Rewrite the integral using partial fractions. Do not solve it.

$$\int \frac{x^3 - 2x^2 + 2x - 3}{(x-2)^4}\ dx$$

Answer choices:

A $\displaystyle \int \frac{1}{(x-2)^4} + \frac{4}{(x-2)^3} + \frac{4}{(x-2)^2} + \frac{1}{(x-2)}\ dx$

B $\displaystyle \int \frac{1}{(x-2)^4} - \frac{6}{(x-2)^3} + \frac{4}{(x-2)^2} - \frac{1}{(x-2)}\ dx$

C $\displaystyle \int \frac{1}{(x-2)^4} + \frac{6}{(x-2)^3} + \frac{4}{(x-2)^2} + \frac{1}{(x-2)}\ dx$

D $\displaystyle \int \frac{1}{(x-2)} + \frac{6}{(x-2)^2} + \frac{4}{(x-2)^3} + \frac{1}{(x-2)^4}\ dx$

Solution: C

First, factor the denominator. Since we have a repeated factor, we need to include all factors of a lesser degree.

$$\int \frac{x^3-2x^2+2x-3}{(x-2)^4}\ dx = \int \frac{A}{(x-2)^4} + \frac{B}{(x-2)^3} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)}\ dx$$

Using partial fractions decomposition containing a linear factor, we have

$$\frac{x^3-2x^2+2x-3}{(x-2)^4} = \frac{A}{(x-2)^4} + \frac{B}{(x-2)^3} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)}$$

Now we'll solve for constants.

$$\frac{(x^3-2x^2+2x-3)(x-2)^4}{(x-2)^4} = \frac{A(x-2)^4}{(x-2)^4} + \frac{B(x-2)^4}{(x-2)^3} + \frac{C(x-2)^4}{(x-2)^2} + \frac{D(x-2)^4}{(x-2)}$$

$$x^3-2x^2+2x-3 = A + B(x-2) + C(x-2)^2 + D(x-2)^3$$

$$x^3-2x^2+2x-3 = A + Bx - 2B + C\left(x^2-4x+4\right) + D\left(x^3-6x^2+12x-8\right)$$

$$x^3-2x^2+2x-3 = A + Bx - 2B + Cx^2 - 4Cx + 4C + Dx^3 - 6Dx^2 + 12Dx - 8D$$

$$x^3-2x^2+2x-3 = Dx^3 + Cx^2 - 6Dx^2 + Bx - 4Cx + 12Dx + A - 2B + 4C - 8D$$

$$x^3-2x^2+2x-3 = Dx^3 + (C-6D)x^2 + (B-4C+12D)x + (A-2B+4C-8D)$$

Equating coefficients on both sides, we get

[1] $D = 1$

[2] $C - 6D = -2$

[3] $B - 4C + 12D = 2$

[4] $A - 2B + 4C - 8D = -3$

We already know the value of D. Plugging [1] into [2] to solve for C, we get

$$C - 6(1) = -2$$

$$C = 4$$

Plugging the values for C and D into [3] to solve for B, we get

$$B - 4(4) + 12(1) = 2$$

$$B = 6$$

Plugging the values for B, C and D into [4] to solve for A, we get

$$A - 2(6) + 4(4) - 8(1) = -3$$

$$A = 1$$

Plugging the values for each of the four constants back into the partial fractions decomposition, and putting the decomposition back into the integral, we get

$$\int \frac{x^3 - 2x^2 + 2x - 3}{(x-2)^4}\ dx = \int \frac{1}{(x-2)^4} + \frac{6}{(x-2)^3} + \frac{4}{(x-2)^2} + \frac{1}{(x-2)}\ dx$$