Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1^{n-1}}{2^n}$$

Answer choices:

A The series is convergent and $r = \frac{1}{4}$.

B The series is divergent and $r = 1$.

C The series is convergent and $r = \frac{1}{2}$.

D The series is divergent and $r = 2$.

Solution: C

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{1^{n-1}}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1^n 1^{-1}}{2^n}$$

$$\sum_{n=0}^{\infty} 1^{-1} \cdot \frac{1^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{1}\left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

Alternatively, instead of manipulating the series directly into standard form, we could have written out the first few terms,

$$\sum_{n=0}^{\infty} \frac{1^{n-1}}{2^n}$$

$$\frac{1^{0-1}}{2^0}+\frac{1^{1-1}}{2^1}+\frac{1^{2-1}}{2^2}+\frac{1^{3-1}}{2^3}+\frac{1^{4-1}}{2^4}+\dots$$

$$\frac{1^{-1}}{1}+\frac{1^0}{2}+\frac{1^1}{4}+\frac{1^2}{8}+\frac{1^3}{16}+\dots$$

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots$$

$$1+\left(\frac{1}{2}\right)^1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4+\dots$$

Either way, we can identify $a=1$ and $r=1/2$. We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$\left|\frac{1}{2}\right|=\frac{1}{2}<1$$

we can say that $|r|<1$ and therefore that the series converges.

Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{8^{n-1}}{2^n}$$

Answer choices:

A The series is convergent and $r = \frac{1}{2}$.

B The series is divergent and $r = 2$.

C The series is convergent and $r = \frac{1}{8}$.

D The series is divergent and $r = 4$.

Solution: D

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{8^{n-1}}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{8^n 8^{-1}}{2^n}$$

$$\sum_{n=0}^{\infty} 8^{-1} \cdot \frac{8^n}{2^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{8}\left(\frac{8}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{1}{8}(4^n)$$

Alternatively, instead of manipulating the series directly into standard form, we could have written out the first few terms,

$$\sum_{n=0}^{\infty} \frac{8^{n-1}}{2^n}$$

$$\frac{8^{0-1}}{2^0}+\frac{8^{1-1}}{2^1}+\frac{8^{2-1}}{2^2}+\frac{8^{3-1}}{2^3}+\frac{8^{4-1}}{2^4}+\dots$$

$$\frac{8^{-1}}{1}+\frac{8^0}{2}+\frac{8^1}{4}+\frac{8^2}{8}+\frac{8^3}{16}+\dots$$

$$\frac{1}{8}+\frac{1}{2}+\frac{8}{4}+\frac{64}{8}+\frac{512}{16}+\dots$$

$$\frac{1}{8}(1+4+16+64+256+\dots)$$

Either way, we can identify $a=1/8$ and $r=4$. We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$|4|=4\geq 1$$

we can say that $|r|\geq 1$ and therefore that the series diverges.

Topic: Geometric series test

Question: Use the geometric series test to say whether the geometric series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{3^n}$$

Answer choices:

A The series is convergent and $r = -\frac{1}{3}$.

B The series is divergent and $r = -1$.

C The series is convergent and $r = \frac{1}{3}$.

D The series is divergent and $r = 1$.

Solution: A

We need to get the series into standard form for a geometric series to make sure the series is geometric. Since the index starts at $n = 0$, standard form is

$$\sum_{n=0}^{\infty} ar^n$$

so we'll rewrite the series as

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{3^n}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-1)^{-1}}{3^n}$$

$$\sum_{n=0}^{\infty} (-1)^{-1} \cdot \frac{(-1)^n}{3^n}$$

$$\sum_{n=0}^{\infty} \frac{1}{(-1)} \left(\frac{-1}{3}\right)^n$$

$$\sum_{n=0}^{\infty} -\left(-\frac{1}{3}\right)^n$$

Alternatively, instead of manipulating the series directly into standard form, we could have written out the first few terms,

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{3^n}$$

$$\frac{(-1)^{0-1}}{3^0}+\frac{(-1)^{1-1}}{3^1}+\frac{(-1)^{2-1}}{3^2}+\frac{(-1)^{3-1}}{3^3}+\frac{(-1)^{4-1}}{3^4}+\dots$$

$$\frac{(-1)^{-1}}{1}+\frac{(-1)^{0}}{3}+\frac{(-1)^{1}}{9}+\frac{(-1)^{2}}{27}+\frac{(-1)^{3}}{81}+\dots$$

$$-1+\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\dots$$

$$-1\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\frac{1}{81}-\dots\right)$$

Either way, we can identify $a=-1$ and $r=-1/3$. We'll use the geometric series test to determine whether this geometric series converges or diverges. Since

$$\left|-\frac{1}{3}\right|=\frac{1}{3}<1$$

we can say that $|r|<1$ and therefore that the series converges.