Topic: Intersection of polar curves

Question: Find the points at which $r = \sin\theta$ and $r = \cos\theta$ intersect.

Answer choices:

A $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

B $\left(-\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

C $\left(\frac{\sqrt{2}}{2}, \frac{5\pi}{4}\right)$

D $\left(\frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right)$

Solution: A

To find points of intersection, we'll set the curves equal to one another to solve for θ,

$$\sin\theta = \cos\theta$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

then find the associated values of r.

$$r = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$r = \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

The polar curves intersect at

$$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) \text{ and } \left(-\frac{\sqrt{2}}{2}, \frac{5\pi}{4}\right)$$

But we notice that these are actually identical points in space, so we'll state just a single intersection point at

$$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$$

Topic: Intersection of polar curves

Question: Find the points at which $r = \cos\theta$ and $r = \cos(2\theta)$ intersect.

Answer choices:

A $(1,0)$

B $(1,2\pi)$

C $(1,0)$, $\left(-\frac{1}{2},\frac{2\pi}{3}\right)$, and $\left(-\frac{1}{2},\frac{4\pi}{3}\right)$

D $(1,0)$ and $(1,\pi)$

Solution: C

To find points of intersection, we'll set the curves equal to one another to solve for θ,

$$\cos\theta = \cos(2\theta)$$

$$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

then find the associated values of r.

$$r = \cos(0) = 1$$

$$r = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$r = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

The polar curves intersect at

$$(1,0), \left(-\frac{1}{2}, \frac{2\pi}{3}\right) \text{ and } \left(-\frac{1}{2}, \frac{4\pi}{3}\right)$$

Topic: Intersection of polar curves

Question: Find the points at which $r = 3\cos\theta$ and $r = 1 + \cos\theta$ intersect.

Answer choices:

A $\left(1, \frac{\pi}{2}\right)$ and $\left(-1, \frac{3\pi}{2}\right)$

B $\left(1, \frac{\pi}{2}\right)$ and $\left(1, \frac{3\pi}{2}\right)$

C $\left(\frac{3}{2}, \frac{\pi}{3}\right)$ and $\left(\frac{3}{2}, \frac{5\pi}{3}\right)$

D $\left(\frac{3\sqrt{3}}{2}, \frac{\pi}{3}\right)$ and $\left(\frac{3\sqrt{3}}{2}, \frac{5\pi}{3}\right)$

Solution: C

To find points of intersection, we'll set the curves equal to one another to solve for θ,

$$3\cos\theta = 1 + \cos\theta$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

then find the associated values of r.

$$r = 3\cos\left(\frac{\pi}{3}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

$$r = 3\cos\left(\frac{5\pi}{3}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

The polar curves intersect at

$$\left(\frac{3}{2}, \frac{\pi}{3}\right) \text{ and } \left(\frac{3}{2}, \frac{5\pi}{3}\right)$$