Integración de Bases de Conocimientos: Cambio de Creencias en bases

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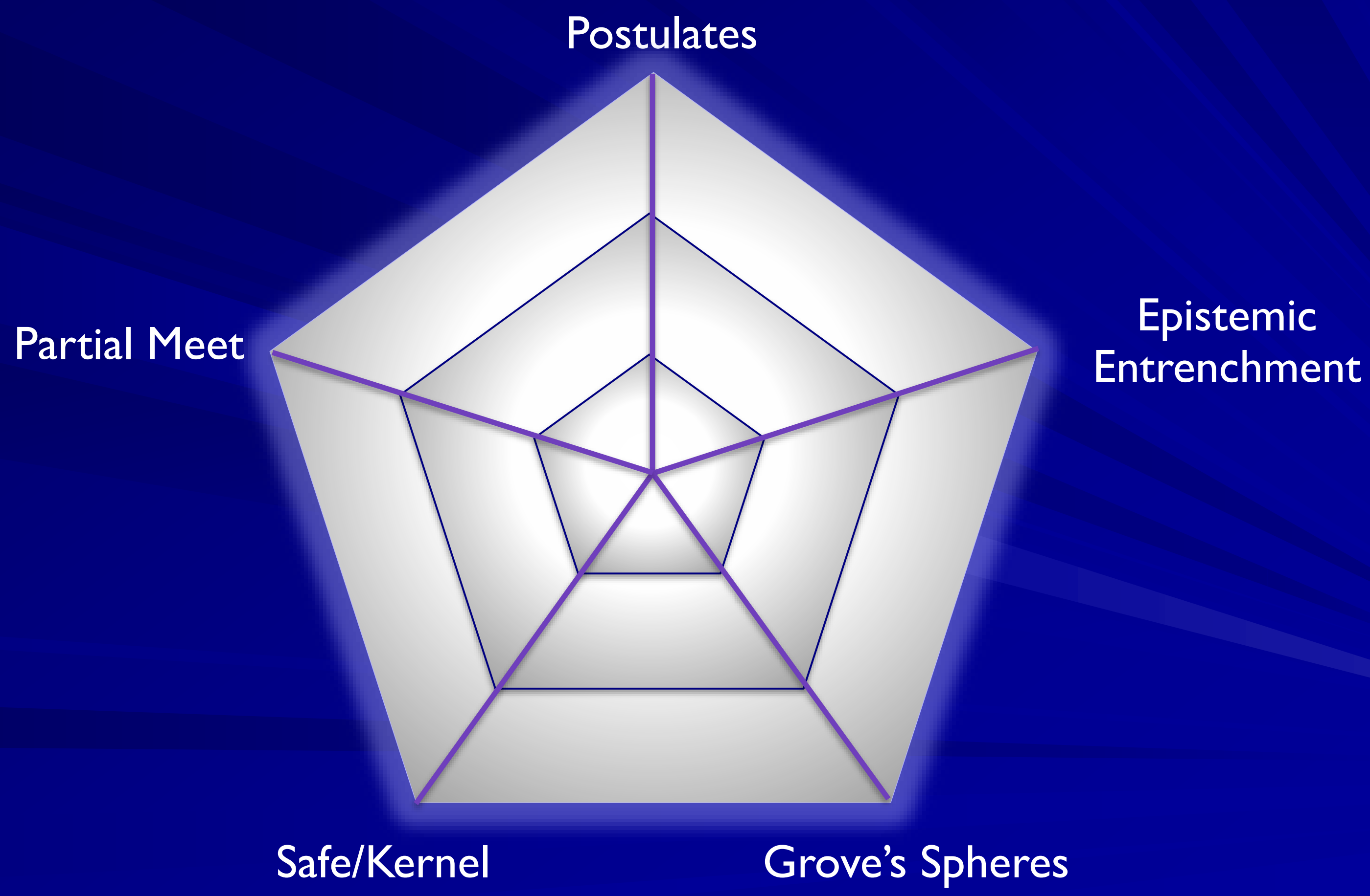
AGM

5 different equivalent presentations on Belief Sets



General Notion of Logic:

Definition: Given a language $\mathcal{L}$, a logic is a set (denoted by $L \subseteq \mathcal{L}$), equipped with a consequence operator (denoted by Cn) where Cn: $\wp(L) \rightarrow \wp(L)$ satisfies the Tarki's basic conditions:

1. $A \subseteq Cn(A)$.
2. If $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$.
3. $Cn(Cn(A)) \subseteq Cn(A)$.

Theorem: For every logic (L,Cn), if Cn satisfies these three properties then there exists a contraction function satisfying (-1)-(-5).

Addition properties

4. If $\varphi \in Cn(X)$ then there exists finite set $X' \subseteq X$ such that $\varphi \in Cn(X')$.
5. If $\varphi \in Cn(X \cup \{\psi\})$ and $\varphi \in Cn(X \cup \{\chi\})$ then $\varphi \in Cn(X \cup \{\psi \vee \chi\})$
6. $\psi \in Cn(X \cup \{\varphi\})$ if and only if $\varphi \to \psi \in Cn(X)$.

Theorem: For every logic (*L*,*Cn*), if *Cn* satisfies these six properties then there exists a contraction function satisfying (-1)-(-6).

Example:

Let (*L*,*Cn)* be a logic where:

L = {a, b}.

Cn(∅) = ∅, *Cn*(a) = *Cn*({a, b})={a, b}, *Cn*(b) = {b}.

Note that b belongs all closed set by *Cn* (except ∅).

If we find to contract b from {a, b} the result can not be other than ∅, if we want to satisfy the postulate of success. But then, the postulate of recovery is violated.

Belief Bases

"*in real life, when we perform a contraction or derogation, we never do it to the theory itself (in the sense of a set of propositions closed under consequence) but rather on some finite or recursive or at least recursively enumerable base for the theory*" Makinson

Belief Bases

- In AGM, a belief state is modeled as a deductively closed theory. Thus, there is no distinction between ‘basic beliefs’ and ‘derived beliefs’. Possibility infinite sets.
- Inconsistency leads to trivialization.
- Highest priority to incoming information.
- No distinction between explicit and implicit belief.

  Thus, retracting a formula ϕ does not entail retracting all formulas derived from ϕ.

Ideal Theories vs. Real World

- Ideal Belief Revision theories assume:
  - No reasoning limits (time or storage)
    - All derivable beliefs are acquirable (deductive closure)
  - All belief credibilities are known and fixed
- Real world
  - Reasoning takes time, storage space is finite
    - Some implicit beliefs might be currently inaccessible
  - Source/belief credibilities can change

Two Traditions in Belief Bases

à la Dalal: This case is associated with a coherentist epistemic representation in which the corpus of beliefs is considered as a whole and none of the parts has a structural feature that differentiates it from the others. Thus belief bases are a mere expressive resource,and the belief change operation use the whole theory to perform the change, i.e.; if $Cn(B_1) = Cn(B_2)$, then $Cn(B_1 - \alpha) = Cn(B_2 - \alpha)$. This principle is known as *irrelevance of syntax*.

Two Traditions in Belief Bases

à la Hansson: This case is associated with a foundacionist epistemic representation. In this case, the belief change is performed in the belief base.

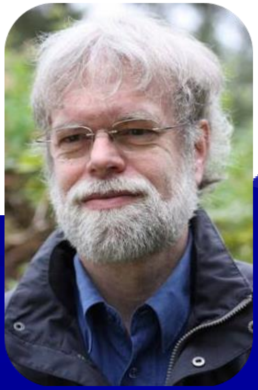
Example: α: Paris is the capital of France.

β: There is milk in the fridge.

$\alpha, \beta \in B \Rightarrow \alpha \leftrightarrow \beta \in Cn(B)$

Contract by β must give up β and $\alpha \leftrightarrow \beta$.

In the rest of this part, we will assume Hansson's approach.



Hansson's Approach

- B finite set of formulas.
- $B+\alpha = B \cup \{\alpha\}$.
- Epistemic attitudes:
  - $\alpha \in Cn(B)$: α (implicitly) believed.
  - $\alpha \in B$: α explicitly believed.
  - $\alpha \in Cn(B) \setminus B$: a merely derived.

Filtering Condition

"If β has been retracted from a base B in order to bar derivations of α from B, then the contraction of Cn(B) by α should not contain any sentences which were in Cn(B) "just because" β was in Cn(B)."

(Fuhrmann)

Example (Hansson):

- α: Paris is the capital of France.
- β: There is milk in the fridge.
- $\alpha, \beta \in B$ then $\alpha \leftrightarrow \beta \in Cn(B)$
- When we revise by $\neg\beta$, we must choose between giving up α and $\alpha \leftrightarrow \beta$.
- In the belief base approach, $\alpha \leftrightarrow \beta$ is automatically chosen and α is remains in the revised base ("**Disbelief Propagation**")

Advantages of the use of bases

Expressivity

- $B1 = \{\alpha, \beta\}$, $B2=\{\alpha, \alpha\leftrightarrow\beta\}$.
- $Cn(B1) = Cn(B2)$
- $B1*\neg\alpha = \{\neg\alpha, \beta\}$
- $B2*\neg\alpha = \{\neg\alpha, \alpha\leftrightarrow\beta\}$
- $\beta \in Cn(B1*\neg\alpha)$, but $\beta \notin Cn(B2*\neg\alpha)$.

Inconsistency Tolerance

- $B1 = \{p, \neg p, q1, q2, q3\}$
- $B2 = \{p, \neg p, \neg q1, \neg q2, \neg q3\}$
- $Cn(B1) = Cn(B2)$, but $Cn(B1-p) \neq Cn(B2-p)$

Why belief bases?

Implementations should use belief bases.

Define models with belief bases reduces the implementation gap.

The implementation Gap

- The theoretical model

- The implementation

How to construct belief bases functions?

Expansion: $B + \alpha = B \cup \{\alpha\}$.

Revision: $(B - \neg\alpha) \cup \{\alpha\}$ (via Levi identity).

How to construct belief bases functions?

Contraction: Current strategies:

- Replicate the constructive models of beliefs sets and obtain their axiomatic characterization in order to determine its “behavior”.
- Construct base functions from functions in belief sets.

Axioms: Simple Translations

Success If $\nvdash \alpha$, then $B-\alpha \nvdash \alpha$.

Inclusion $B-\alpha \subseteq B$.

(note that is different from $Cn(B-\alpha) \subseteq Cn(B)$)

Vacuity If $A \nvdash \alpha$, then $B \subseteq B - \alpha$,

Conjunctive Factoring $B - \alpha \wedge \beta = \begin{cases} B - \alpha \text{ or} \\ B - \beta \text{ or} \\ B - \alpha \cap A - \beta \end{cases}$

Axioms: About sentences behaviour

Extensionality If $\vdash \alpha \leftrightarrow \beta$, then $B - \alpha = B - \beta$

Uniformity If it holds for all subsets B' of B that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$.

Axioms: Minimal Change

Relevance If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set B' such that $B-\alpha \subseteq B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

Core retainment If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set B' such that $B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

Weak Relevance If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set B' such that $B-\alpha \subseteq B' \subseteq Cn(B)$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

Relative Closure $B \cap Cn(B-\alpha) \subseteq B-\alpha$.

Failure (If $\vdash \alpha$, then $B-\alpha = B$

Partial Meet Base Contraction

Construction

- $B \perp \alpha$: maximal subsets of B that fail to imply α
- γ: function that selects some elements of $B \perp \alpha$
- $B -_{\gamma} \alpha = \bigcap g(B \perp \alpha)$

Partial Meet Base Contraction

Axiomatic

Success If $\nvdash \alpha$, then $B-\alpha \nvdash \alpha$.

Inclusion $B-\alpha \subseteq B$.

(note that is different from $Cn(B-\alpha) \subseteq Cn(B)$.

Uniformity If it holds for all subsets B' of B that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$.

Relevance If $\beta \in B$ and $\beta \notin B-\alpha$ then there is some set B' such that $B-\alpha \subseteq B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

Kernel Base Contraction

Construction

- $B \perp\!\!\!\perp \alpha$: minimal subsets of B that imply α
- σ: function that selects at list one element of each set in $B \perp\!\!\!\perp \alpha$
- $B -_{\sigma} \alpha = B \setminus \sigma(B \perp\!\!\!\perp \alpha)$

Kernel Base Contraction

Axiomatic

Success If $\nvdash \alpha$, then $B - \alpha \nvdash \alpha$.

Inclusion $B - \alpha \subseteq B$.

Uniformity If it holds for all subsets B' of B that $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B - \alpha = B - \beta$.

Core retainment If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set B' such that $B' \subseteq B$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

Contraction - Example

- $B=\{p, p\vee q, p\leftrightarrow q\}$
- $B \perp\!\!\!\perp (p\wedge q)=\{\{p, p\leftrightarrow q\},\{p\vee q,\ p\leftrightarrow q\}\}$
- $B\perp(p\wedge q)=\{\{p,\ p\vee q\},\{p\leftrightarrow q\}\}$
- $B-(p\wedge q)=\{p\}$ can be constructed as kernel but not partial meet contraction.

Basic Related AGM-Contraction

$$B - \alpha = (Cn(B) \div \alpha) \cap B$$

Axiomatic

Success If $\nvdash \alpha$, then $B - \alpha \nvdash \alpha$.

Inclusion $B - \alpha \subseteq B$.

Extensionality If $\vdash \alpha \leftrightarrow \beta$, then $B - \alpha = B - \beta$

Weak Relevance If $\beta \in B$ and $\beta \notin B - \alpha$ then there is some set B' such that $B - \alpha \subseteq B' \subseteq Cn(B)$ and $\alpha \notin Cn(B')$ but $\alpha \in Cn(B' \cup \{\beta\})$.

New Operators for Belief Bases

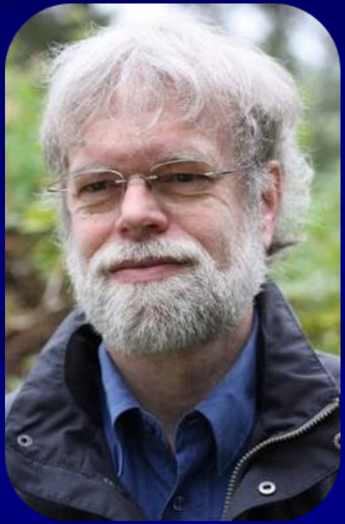
Belief Bases can recognize between different inconsistent bases:

$B_1 = \{\alpha, \neg\alpha\}$

$B_2 = \{\alpha, \beta, \alpha \rightarrow \neg\beta\}$

then new operators can be defined for inconsistent belief bases.

Hansson proposed:



Consolidation Makes a belief base consistent.

$$B! = B - \perp$$

Semi-Revision Non-prioritized revision, new info may be rejected.

$$B?\alpha = (B + \alpha)!$$

External Revision (Reversed Levi Identity)

$$B \pm \alpha = (B + \alpha) - \neg\alpha$$

Consolidation

- If a belief base is inconsistent, then it can be made consistent by removing enough of its more dispensable elements. This operation is called consolidation. The consolidation of a belief base A is denoted A!. A plausible way to perform consolidation is to contract by falsum (contradiction), i.e. $A! = A \div \perp$.
- Unfortunately, this recipe for consolidation of inconsistent belief bases does not have a plausible counterpart for inconsistent belief sets. The reason is that since belief revision operates within classical logic, there is only one inconsistent belief set. Once an inconsistent belief set has been obtained, all distinctions have been lost, and consolidation cannot restore them.

Consolidation

- Consolidation – making a base consistent
- Consolidation of a base **B** (**B**!) such that
  - **B**! is consistent
  - **B**! $\subseteq$ **B**
  - If $\mathbf{p} \in$ **B****B**!, then **X** such that **X** $\subseteq$ **B**, **X** is consistent and **X** $\cup$ $\{\mathbf{p}\}$ is inconsistent

Revision of Belief Bases

Levi Identity

- $B \underline{+} \alpha = (B - \neg\alpha) + \alpha$

Reversed Levi Identity (Hansson)

- $B\underline{+}\alpha = (B+\alpha) - \neg\alpha$
  - Intermediate state may be inconsistent.

Let $B = \{p\rightarrow r, p\rightarrow \neg r\}$.

(a) internal revision: $B\perp\neg p = \{\{p\rightarrow r\},\{p\rightarrow \neg r\}\}$, hence,

for $\gamma_1(B\perp \neg p) = \{\{p\rightarrow r\}\}$, we have $B \square_{\gamma 1} p = \{p\rightarrow r, p\}$.

(b) external revision: $B+p \perp\neg p = \{\{p\rightarrow r,p\}, \{p\rightarrow \neg r, p\}\}$, hence,

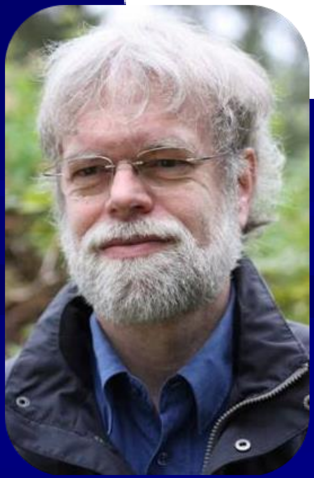
for $\gamma_2(B+p\wedge \neg p)=\{\{p\rightarrow \neg r, p\}\}$, we have $B\underline{+}_{\gamma 2}\ p = \{p\rightarrow \neg r, p\}$.

Local Change

Main idea: Not all of the agent's belief are relevant.

Example: I believe it is always raining in Holland.
I see it is a sunny day in Amsterdam $\Rightarrow$ revision

The revision can be done without checking consistency with irrelevant beliefs.



Local Change

A compartment of $B =$ formulas of B that contribute for (dis)proving some formula in A.

$C_A(B) = Cn(c(A,B))$

$C_A(B)$ gives the classical consequences of those beliefs in B which are relevant for the formulas in A.

- C_A satisfies:
  - $B \subseteq B' \Rightarrow C_A(B) \subseteq C_A(B')$ (monotony)
  - $\alpha \in C_A(B) \Rightarrow$ there is $B' \subset B$, B' finite, s.t. $\alpha \in C_A(B')$ (compactness)
  - $C_A(C_A(B)) = C_A(B)$ (iteration)
- C_A does not satisfy
  - $B \not\subseteq C_A(B)$ (inclusion)

Local Change

Suppose I am at home and I hear on the radio that my friend Carol has been murdered yesterday night and that there were no traces of doors or windows having been forced. I talked to her yesterday on the phone and she was home with her flatmates Ann and Bill. I know that no one else, except for Ann, Bill and Carol has the keys to their apartment. I conclude that **Ann or Bill must have done it**. But I have known Ann for quite some time and cannot believe that she would be able to murder anyone. I believe **Ann did not do it**. For similar reasons, I believe **Bill did not do it**. This is clearly inconsistent with my belief that one of them did it. So I decide to visit my friend Paul to ask what he thinks. In front of his place I see **the lights are on**. I know that **if the lights are on, then Paul is home**. I get out of the car and Paul's neighbor, inferring that I am coming to visit him tells me **Paul is not home**. This is all very confusing, but I am sure of one thing: I do not believe **I am asleep**!

Local Change

Let p stand for the proposition "Paul is at home",
q for "The lights are on",
a for "Ann is the murderer",
b for "Bill is the murderer" and
r for "I am asleep".

My belief base B after talking to Paul's neighbor contains:

$$\{\neg p, q, q \rightarrow p, a \vee b, \neg a, \neg b, \neg r\}.$$

I am interested in whether Paul is at home, that is, the relevant beliefs are:

$$c(p, B) = \{q, q \rightarrow p, \neg p\}.$$

Even though this set is inconsistent we have that:

$$r \notin C_r(B) = Cn(c(r, B)) = Cn(\{\neg r\}).$$

Local Contraction

The local partial meet contraction operator based on a local inference operator C_A and a selection function γ is the operator $\dot{-}_{C_A,\gamma}$ such that:

$B \dot{-}_{C_A,\gamma} \alpha = \bigcap \gamma(B \perp_{C_A} \alpha)$.

Postulates:

- If $\alpha \notin C_A(\emptyset)$, then $\alpha \notin C_A(B \dot{-} \alpha)$ (success)
- $B \dot{-} \alpha \subseteq B$ (inclusion)
- If $\beta \in B \setminus (B \dot{-} \alpha)$, then there is some B' such that $B \dot{-} \alpha \subseteq B' \subseteq B$, $\alpha \notin C_A(B')$ and $\alpha \in C_A(B' \cup \{\beta\})$ (relevance)
- If for all subsets B' of B, $\alpha \in C_A(B')$ if and only if $\beta \in C_A(B')$, then $B \dot{-} \alpha = B \dot{-} \beta$ (uniformity)

Challenges for Belief Bases

- Complete the theoretical models
- Understand better the concept of minimal change.
- Understand how the model works at the supplementary level.
- Implement (outside the toys problems) belief base change for resource-bounded agents.

Ontologies

- An ontology in computer science is an explicit, formal specification of the terms of a domain of application, along with the relations among these terms.
- An ontology provides a (structured) vocabulary which forms the basis for the representation of general knowledge.
- Ontologies have found extensive application in Artificial Intelligence and the Semantic Web, as well as in areas such as software engineering, bioinformatics, and database systems

Ontologies: Description Logics

Research in ontologies in Artificial Intelligence has focussed on description logics (DL), a (decidable) fragment of first order logic.

Two components,

TBox , for expressing concepts.
Characterises a domain of application.

ABox , that contains assertions about specific individuals and instances.
Contains information on a specific instance of a domain

Ontologies and Belief Change

- a viewpoint on ontology evolution:
  – Addressing the problem of ontology evolution using techniques from belief change
- In particular:
  – AGM theory of contraction
  – In ontologies represented using some DL or OWL flavor

Ontologies and belief change

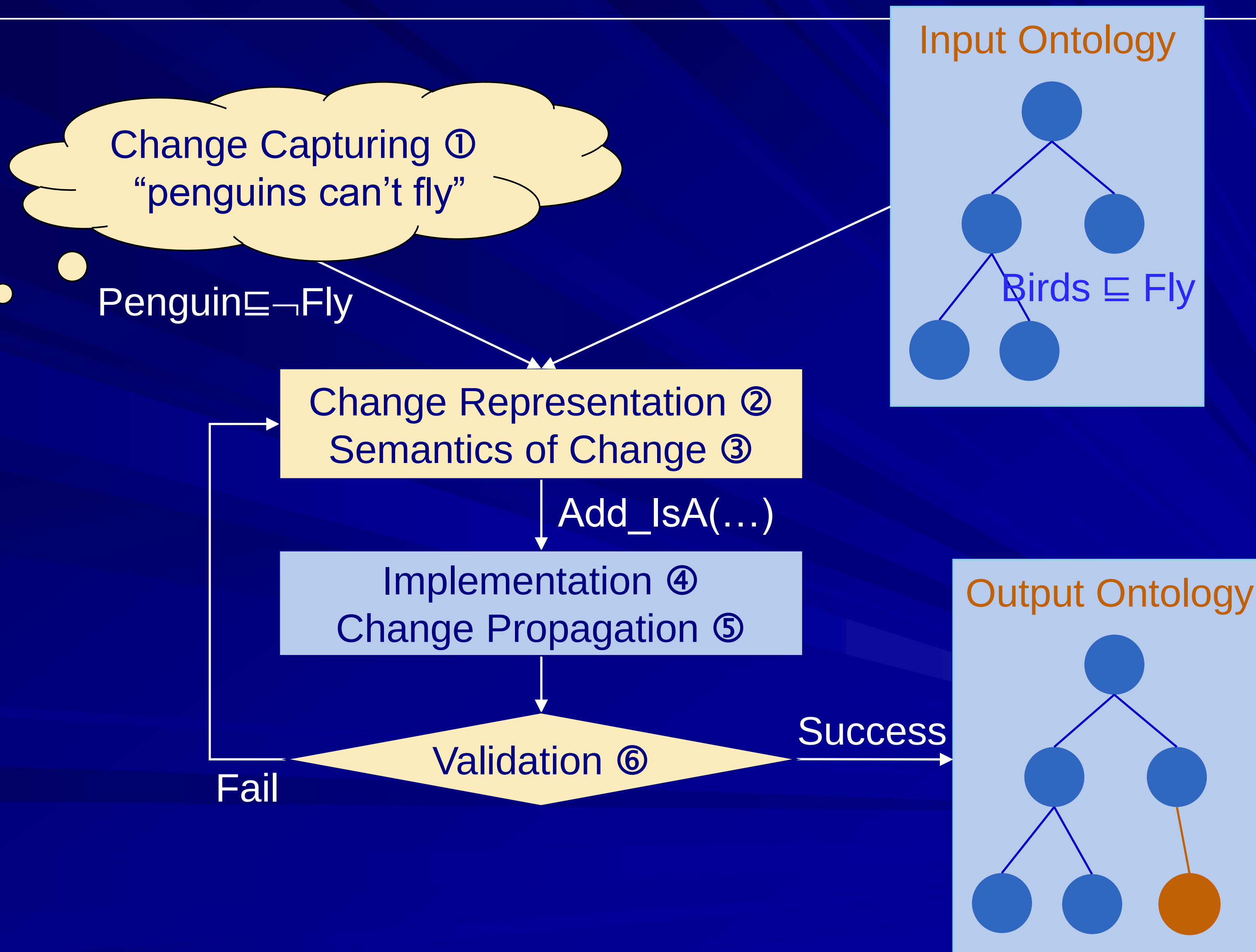
Crucially, an ontology will be expected to evolve,

- either as domain information is corrected and refined, or
- in response to a change in the underlying domain.

In a description logic, such change may come in two different forms:

- the background knowledge, traditionally stored in the TBox, may require modification, or
- the ground facts or data, traditionally stored in the ABox, may be modified.

Example


Change Capturing ①
"penguins can't fly"
Penguin⊑¬Fly
Input Ontology
Birds ⊑ Fly
Change Representation ②
Semantics of Change ③
Add_IsA(…)
Implementation ④
Change Propagation ⑤
Validation ⑥
Fail
Success
Output Ontology

The problem

The problem is that not every tarskian logic admits an AGM contraction.

In fact, there are logics which don’t admit any AGM contraction.

Example

Assume a logic $< L, Cn >$ with:

$$L = \{a, b\}$$
$$Cn(\emptyset) = \emptyset$$
$$Cn(\{a\}) = \{a\}$$
$$Cn(\{b\}) = \{a, b\}$$
$$Cn(\{a, b\}) = \{a, b\}$$
$$K = \{a, b\}$$

If $b \in K - a$ then by closure $Cn(\{b\}) = \{a, b\} \subseteq K - a$ but that contradicts success.

But if $K - a = \emptyset$ then $K - a \cup \{a\} = \{a\} \neq K$

AGM Compliance (Flouris et al.)

Definition (AGM Compliance)

A logic L is **AGM compliant** iff there is an operator of AGM contraction in L.

Definition (Decomposability)

A logic is L is **decomposable** iff:

$$\forall X, K \subseteq L : Cn(\emptyset) \subset Cn(X) \subset Cn(K) \Big(\exists Z \subseteq L : Cn(X \cup Z) = Cn(K) \Big) \tag{3}$$

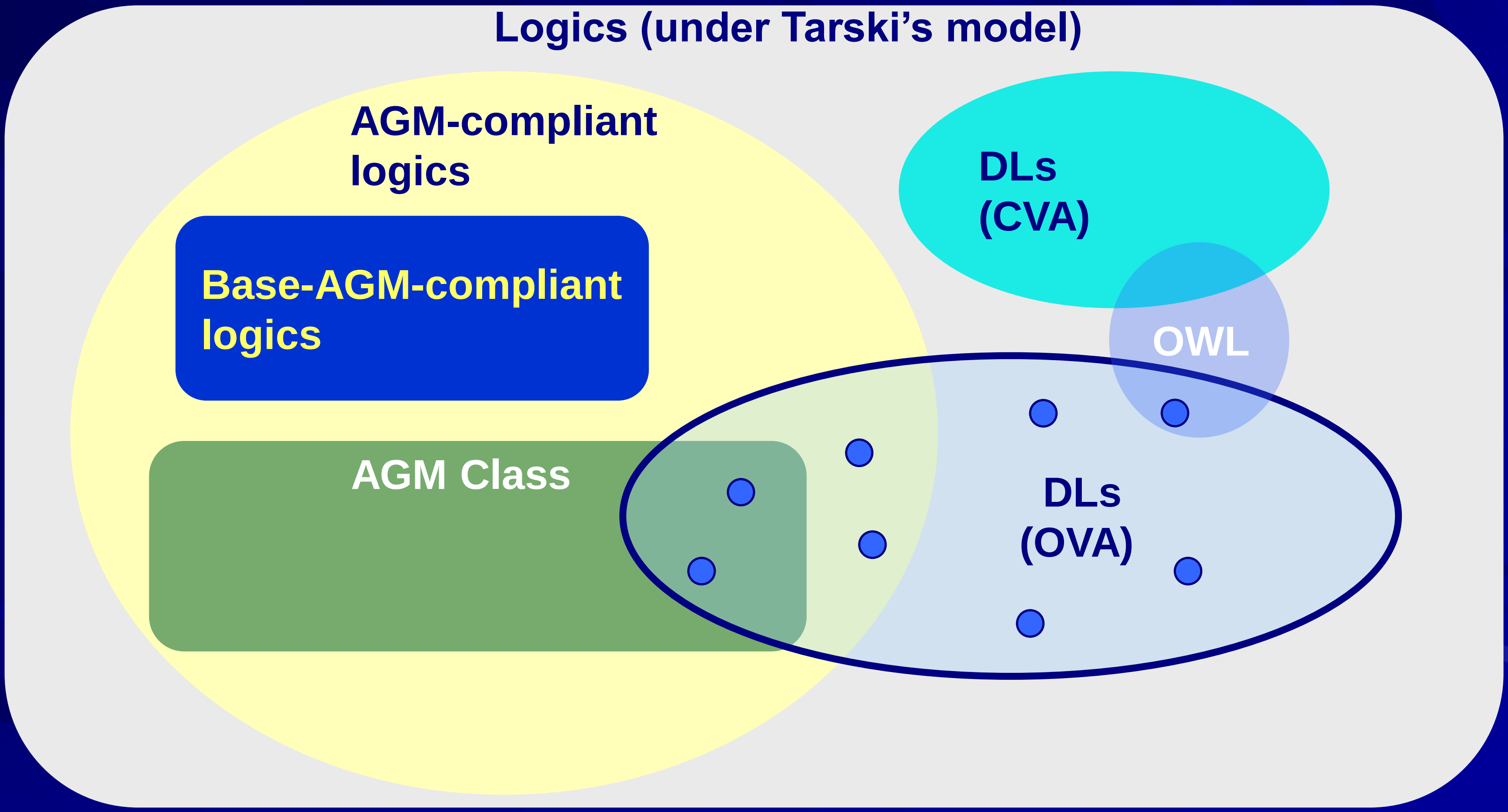
Theorem

A logic is **AGM compliant** *iff it is* **decomposable**

AGM-Compliance and DLs

- two types of DLs
  - CVA (Closed Vocabulary Assumption): allows the description of the ontological signature using DL axioms
  - OVA (Open Vocabulary Assumption): ignores the signature because it cannot be described using DL axioms
- DLs (CVA): non-AGM-compliant
- DLs (OVA): some are AGM-compliant, some are not
  - Introduced results, heuristics, rules of thumb
- OWL (different flavors, CVA or OVA, annotation features, owl:imports): all non-AGM-compliant

The big picture



Partial List of DLs (OVA)

AGM-compliant DLs	Non-AGM-compliant DLs
✓ $ALCO^{\neg,\sqcap}$ ✓ $ALC^{\neg,\sqcap}$ with no Abox ✓ ALCO with no axioms involving role terms ✓ ALC with empty Abox and no axioms involving role terms ✓ All DLs with more operators (but no more connectives) than the above DLs ………	✓ SH, SHI, SHIN, SHOIN, SHOIN(D), $SHOIN^{+}$, $SHOIN^{+}$(D), SHIQ, SHIF, SHIF(D), $SHIF^{+}$, $SHIF^{+}$(D) ✓ FL_0, FL^{-} with role axioms ✓ All DLs between ALH and ALHCIOQ ✓ OWL DL, OWL Lite without annotations and all flavors of OWL with annotations ………

Where does this problem come from?

There are some evidences associating the problem with the recovery postulate. The main evidence is this:

Theorem

Every tarskian logic admits a contraction operator that satisfies the AGM postulates without the recovery postulates.

So a possible solution should be to replace the recovery postulate.

Relevance

Hansson has proposed the postulate of relevance:

Definition (Relevance)

$K - a$ satisfies **relevance** iff:

$$\forall b \in K \setminus K - a \left(\exists K' : K - a \subseteq K' \subseteq K \wedge a \in Cn(K' \cup \{b\}) \setminus Cn(K') \right) \tag{4}$$

Result (Riveiro-Wassermann)

Theorem (Weak Existence)

Every tarskian compact logic admits the contraction operator that satisfies AGM postulates with relevance instead of recovery

Theorem (Weak Rationality)

For propositional logic the AGM postulates are equivalent to this new set of postulates

Theorem (Representation)

For every belief set K closed under compact and tarskian logical consequence, - is a partial meet contraction operation over K if and only if - satisfies the postulates (K-1)-(K-4), (relevance) and (K-6).

Example

Assume a description logic $< L, Cn >$ that admits the connective $\sqsubseteq$ between concepts and roles, and the constructor $\forall$:

- Roles = {enrolledAt, haveClassAt}
- Concept = {SpecialStudent}
- $SS = SpecialStudent$, $e = enrolledAt$, $h = haveClassAt$
- $K = Cn(\{h \sqsubseteq e\}) = Cn(\{h \sqsubseteq e, \forall h.SS \sqsubseteq \forall e.SS\})$
- By inclusion, success and closure we have:
  $K - (\forall h.SS \sqsubseteq \forall e.SS) = Cn(\emptyset)$
- Recovery is not satisfied: $Cn\big(\{\forall h.SS \sqsubseteq \forall e.SS\}\big) \neq K$
- Relevance is satisfied: Let $K' = Cn(\emptyset)$ and consider the 2 options for β: $h \sqsubseteq e$ and $\forall h.SS \sqsubseteq \forall e.SS$, in both cases $\forall h.SS \sqsubseteq \forall e.SS \in Cn(K' \cup \beta)$.

Horn Belief Change

Address belief change in the expressively weaker language of *Horn clauses*, where a Horn clause can be written as a rule in the form $a_1 \wedge a_2 \wedge \cdots \wedge a_n \rightarrow a$ for $n \geq 0$, and where a, a_i $(1 \leq i \leq n)$ are atoms.

An agent's beliefs are represented by a Horn clause knowledge base, and the input is a conjunction of Horn clauses.

Horn Belief Change

This topic is interesting for several reasons. It sheds light on the theoretical underpinnings of belief change, in that it weakens the assumption that the underlying logic contains propositional logic. As well, Horn clauses have found extensive use in artificial intelligence and database theory, in areas such as logic programming, truth maintenance systems, and deductive databases.

Logic Program Revision

- The problem:
  - A LP represents consistent incomplete knowledge;
  - New *factual* information comes.
  - **How to incorporate the new information?**
- The solution:
  - Add the new facts to the program;
  - If the union is consistent this is the result;
  - Otherwise *restore consistency* to the union.
- The new problem:
  - **How to restore consistency to an inconsistent program?**

Simple revision example - 1

P: $\quad$ flies(X) ← bird(X), not ab(X). $\quad$ bird(a) ←.

ab(X) ← penguin(X). bird(X) ← penguin(X).

- So flies(a) is true. Next, we learn penguin(a).

P ∪ {penguin(a)} is consistent, flies(a) is false, not ab(a) is defeated. Nothing needs to be done.

- We learn instead ¬flies(a). flies(a) is rebutted.

P ∪ {¬flies(a)} is inconsistent. What to do?

Since the inconsistency rests on the assumption not ab(a), revise that assumption, e.g. by adding the fact ab(a), thereby obtaining a new program P'.