Integración de Bases de Conocimiento: Cambio de Creencias

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The Problem

Belief Revision: Two examples

(Gärdenfors 1988)

Beliefs:

- I gave my wife a diamond ring
- Diamonds scratch glasses
- My window is made of glass

Actions:

- I game my wife a diamond ring
- My wife tries to scratch the windows with the ring
- Nothing happens

Belief Revision: Two examples

(Gärdenfors & Rott 1995)

Beliefs:

- The bird caught in the trap is a swan
- The bird caught in the trap comes from Sweden
- Sweden is part of Europe
- All European swans are white

Consequences:

- The bird caught in the trap is white

New information:

- The bird caught in the trap is black

Which sentence(s) would you give up?

Definición del problema

Dado un corpus de conocimiento (una base de datos, una especificación algebraica, un conjunto de fórmulas lógicas, un conjunto de oraciones en lenguaje natural, etc.) que tenga asociada una noción de *consistencia/ coherencia/compatibilidad interna,* buscaremos determinar como transformar dicho corpus cuando nueva información *aparece* preservando la noción de *consistencia /coherencia/compatibilidad*.

Motivaciones originales

El Problema de aceptación de condicionales

“Si dos personas argumentan ‘Si p entonces q?’ y ambos dudan acerca de p ellos agregarán p hipotéticamente a su stock de conocimiento y sobre esa base evaluarán q". Ramsey (1931).

Posteriormente, Stalnaker reformuló la propuesta de Ramsey de la siguiente manera:
“En primer término, agregue el antecedente (hipotéticamente) a su stock de creencias; en segundo término realice los ajustes necesarios para mantener consistencia (sin modificar las creencias hipotéticas del antecedente); finalmente considere si el consecuente es o no verdadero” Stalnaker (1968,p. 102)

Motivaciones originales

El Problema de la Derogación

- A es un cuerpo de normas formado por
  {a, b, a&b → c}
- El legislador quiere derogar c...*Problema! No basta con eliminar c! (Alchourron&Bulygin 1975)*

- *Elimino a?, Elimino b?, Elimino* a&b → c? *Todos!?*
- Principio de Economía *(Todos, no!)*

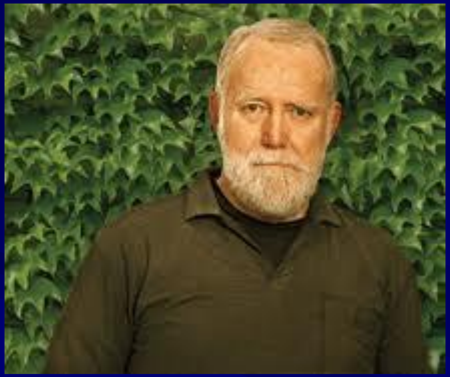
Un poco de historia

con ayuda de mi amigo Raúl Carnota.
(AGM Theory and Artificial Intelligence. 2011)

The Beginning

Philosophy

Philosophy

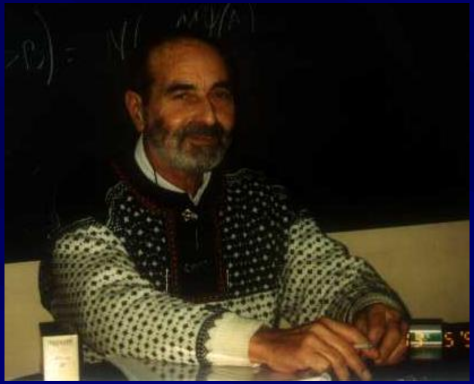


1977 *"Rational Conceptual Change*"
William L. Harper

1977, "*Subjunctives, Dispositions and Chances*"
Issac Levy

1971, "*Explanation and understanding*"
George Henrik von Wright

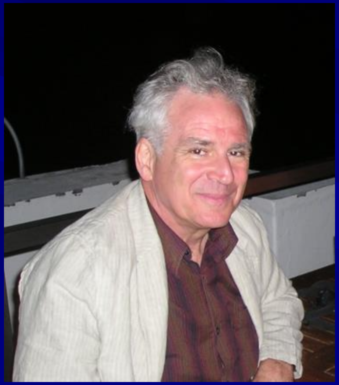
Philosophy



Deontic Logic

Philosophy of law

Carlos Alchourrón



Epistemology

Cognitive Science

Peter Gärdenfors

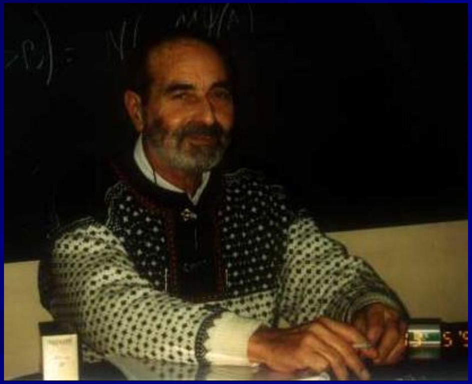
Modal Logic

Deontic Logic

David Makinson

Philosophy: Alchourrón and Makinson

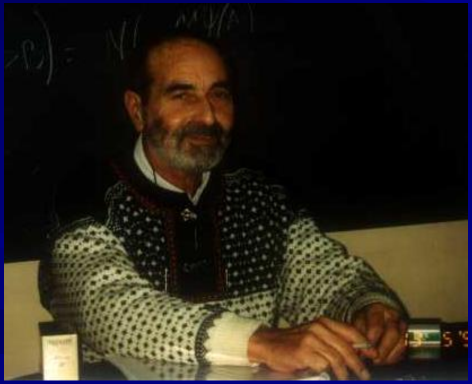
- To analyse the logical structure of the derogation procedure of a norm contained in a legal code.
- To find the general principles that any derogation should satisfy.
- To define a family of all the possible derogations.



"*Hierarchies of Regulations and their Logic*" 1981

Philosophy: Alchourrón and Makinson

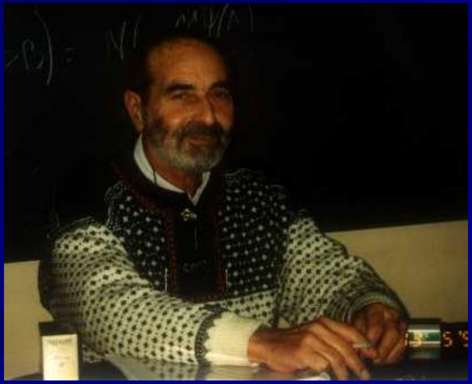
– Given a code A, create a partial order between the norms of A and induce an order on the set of parts of A.
The maximal sets of A that did not involve the standard were called "*remainders*"



"*Hierarchies of Regulations and their Logic*" 1981

Philosophy: Alchourrón and Makinson

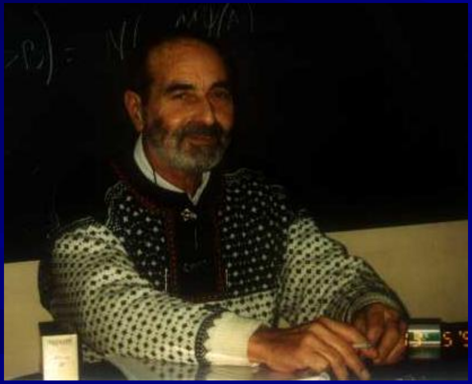
- The problem was not limited only to a set of norms.
  - The set A might be an arbitrary set of formulae and the problem now was how to eliminate one of the formulae or one of the consequence of the set.



"On the Logic of Theory Change: Contraction functions and their associated Revision functions", 1982

Philosophy: Alchourrón and Makinson

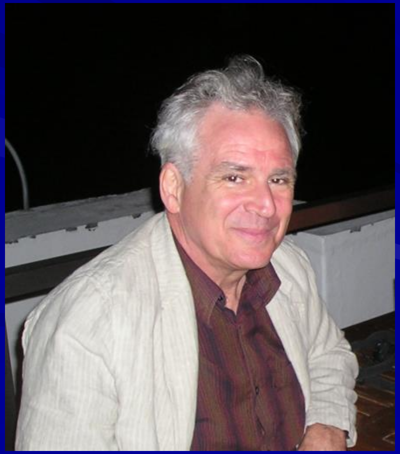
- In the paper two different ways to contract a theory by means of remainder sets was analyzed.
- *Maxichoice* and *Full Meet*



“*On the Logic of Theory Change:*
Contraction functions and their associated Revision functions”, 1982

Philosophy: Gärdenfors

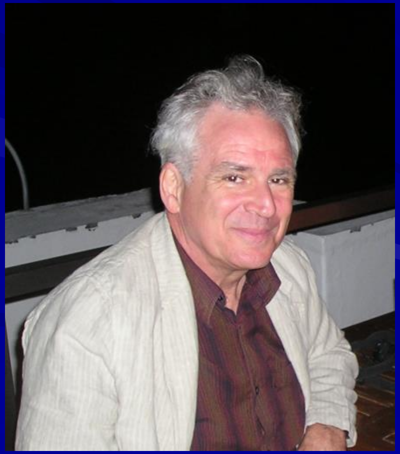
- He was looking for a model for *Explanations*
- Gärdenfors thought that *Explanations* can be expressed as different types of conditional sentences
- Gärdenfors receives an important influence from the philosophers Levi and Harper, leading him to make a thorough study of epistemic conditionals.



"*Conditional and Change of Belief*" , 1978
"*A pragmatic approach to explanations*", 1980

Philosophy: Gärdenfors

– He looking for a semantic for the epistemic conditionals.
– This semantic must be based on belief states and belief changes.

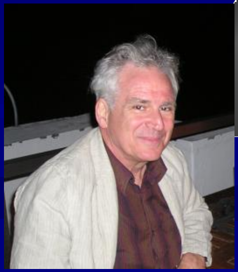


"An epistemic approach to conditionals", 1981
"Rules for rational changes of beliefs" 1982

Unity makes strength

AGM

1985



"On the Logic of Theory Change:
Partial Meet Contractions and Revision Functions", 1985
– Journal of Symbolic Logic

The Beginning

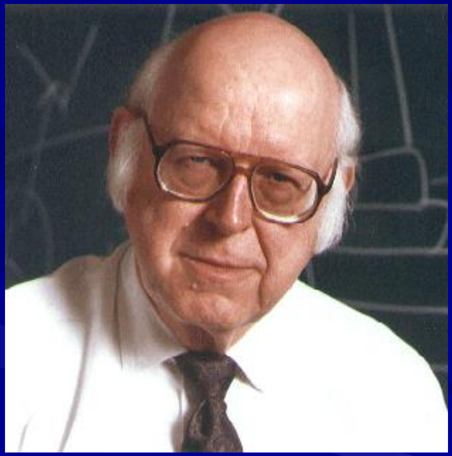
Belief Revision in
Artificial Intelligence

The AI crisis in the ´80

- Allen Newell pointed out three indicators:

"A first indicator comes from our continually giving to representation a somewhat magical role.

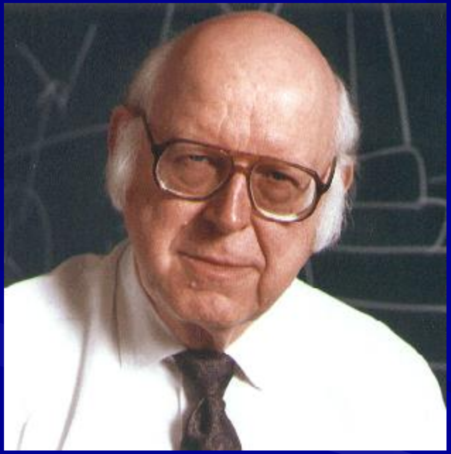
What is indicative of underlying difficulties is our inclination to treat representation like a homunculus, as the focus of real intelligence".



"The Knowledge Level" 1981

The AI crisis in the ´80

"A second indicator is the great theorem-proving controversy of the late sixties and early seventies. Everyone in AI has some knowledge of it, no doubt, for its residue is still very much with us. It needs only brief recounting."



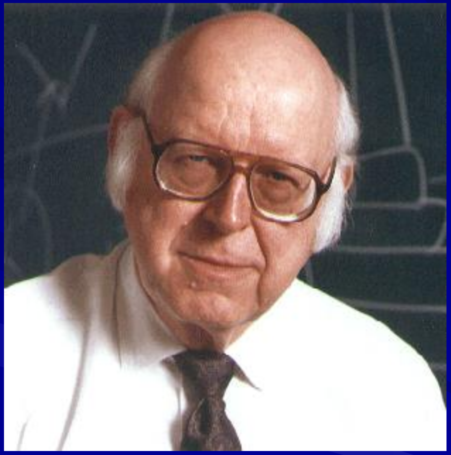
"The Knowledge Level" 1981

The AI crisis in the ´80

- The results of a questionnaire promoted in 79/80 by Brachman & Smith which was sent to the AI community

“The main result was overwhelming diversity -*a veritable jungle of opinions. There is no consensus on any question of substance.”*

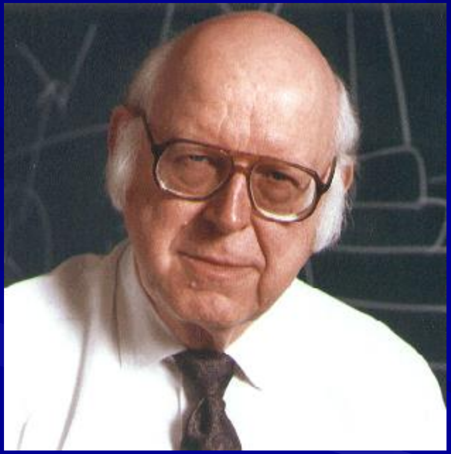
“As one [of the respondents] *said, “Standard practice of representation of knowledge is the scandal of AI.”*



“The Knowledge Level” 1981

The AI crisis in the ´80

- *Knowledge Representation and Reasoning* (KRR) must be a priority in the AI agenda.
- He postulates the existence of a "Knowledge Level"



"…there exists a distinct computer system level, lying immediately above the symbol level, which is characterized by knowledge as the medium and the principle of rationality as the law of behavior."

"The Knowledge Level" 1981

KRR

– Newell's work had an enormous influence on AI researchers: Brachman, Levesque, Moore, Halpern, Moses, Lifschitz, Vardi, Fagin, Ullman, Shapiro, Borgida, Winslett, etc.

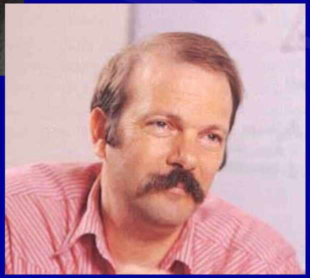
Looking for a model in AI

- *"On the semantics of updates in databases"2nd ACM SIGACT-SIGMOD symposium on Principles of database systems Georgia Março 21 - 23, 1983. Fagin, Ullman, Vardi*

"The ability of the database user to modify the content of the database, the so-called update operation, is fundamental to all database management systems".

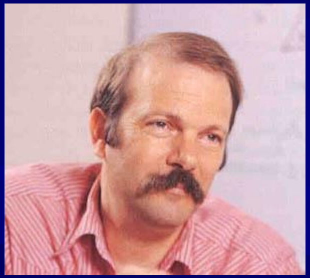
"First we consider the problem of updating arbitrary theories by inserting into them or deleting from them arbitrary sentences"

"when replacing an old theory by a new one we wish to minimize the change in the theory"



TARK (Theoretical Aspects of Rationality and Knowledge)

- Originally planned as a little workshop.
- Attended 40 researchers and other 250 integrate the *email list.*



"…included computer scientists, mathematicians, philosophers and linguists" […] *"given the evident interest in the area by groups so diverse, it seemed appropriate a conference, particularly one that could increase the knowledge of the workers of one field about the work developed in other fields"*

Moshe Vardi

Looking for a model in AI

- Reasoning About Knowledge: An Overview (Halpern). TARK 86 (March 19-22 California)

Keynote

"Most of the work discussed above has implicitly or explicitly assumed that the messages received are consistent. The situation gets much more complicated if messages may be inconsistent."

*"This quickly leads into a whole complex of issues involving **belief revision** and reasoning in the presence of inconsistency. Although **I won't attempt to open this can of worms here**, these are issues that must eventually be considered in designing a knowledge base"*

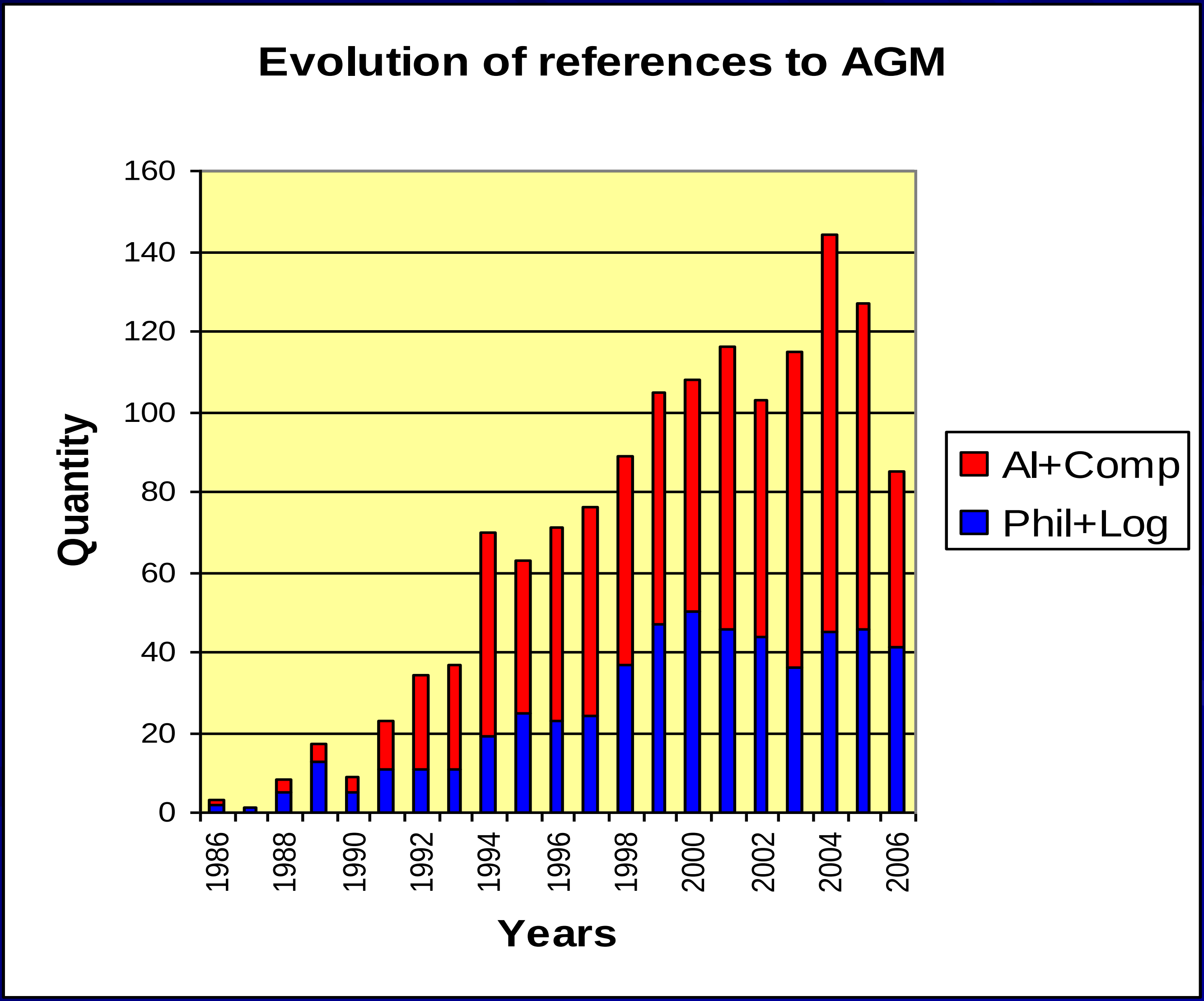
Motivaciones originales (IA)

"La mayor parte de los trabajos ... suponen, implícita o explícitamente que los mensajes recibidos (por el agente) son consistentes. La situación se hace mucho más complicada si los mensajes pueden ser inconsistentes. Esto nos lleva rápidamente a todo el conjunto complejo de cuestiones involucradas en revisión de creencias y razonamiento en presencia de inconsistencia. <u>Pese a que no intentaré abrir esa lata de gusanos aqu</u>í, estas son cuestiones que deben en algún momento ser consideradas al diseñar una base de conocimientos, por ejemplo, pues siempre está presente la posibilidad de adquirir del usuario información inconsistente" *(TARK, Halpern 1986)*

- In 1988, Gärdenfors e Makinson presented the AGM model on TARK 88

- The can was opened …

Impacto de la Teoría de Cambio hoy 2700 referencias



Volvamos a un ejemplo:

- “Juan ha nacido en Puerto Carreño” (α),
- “José ha nacido en Puerto Ayacucho” (β),
- “Dos personas son compatriotas si han nacido en el mismo país” (δ).

Deseamos “incorporar” o “adicionar”

al conjunto la siguiente oración:

- “Juan y José son compatriotas”(σ).

Notar que se deduce que:

- “Juan y José han nacido en el mismo país”(σ’).

Adición

Podemos entonces definir una operación de adición que toma una oración y un conjunto y retorna el mínimo conjunto que incluye al anterior y alberga la nueva oración.

Esta operación de adición ejemplifica el caso más simple de la *noción de cambio* en un conjunto de oraciones.

Contracción:

Supongamos que σ es información errónea, y deseamos descartarla de nuestro conjunto. Es decir, deseamos que quede indeterminado si Juan y José son compatriotas o no.

Notemos que este tipo de cambio es diferente a la aceptación de que Juan y José *no* son compatriotas. Cabe preguntarse si la operación de “descarte”de información debe comportarse como la dual de la operación de la “adición”:

Si luego de descartar información procedemos a adicionarla, ¿deberíamos obtener el conjunto original o no? Una forma de establecer que el descarte sea mínimo podría ser exigir que la información abandonada pueda ser totalmente recuperada.

Revisión:

Supongamos que al consultar un Atlas descubrimos con sorpresa que:

- Puerto Carreño queda en Colombia (ε) y
- Puerto Ayacucho queda en Venezuela (ϕ).

Revisión (cont.):

Si adicionaramos (ε) y (ϕ) al conjunto $\{\alpha,\beta,\delta,\sigma\}$ resultaría un conjunto con información contradictoria:

Juan y José son compatriotas pero Puerto Carreño y Puerto Ayacucho no pertenecen al mismo país.

La simple adición no refleja satisfactoriamente la idea de *actualización consistente*.

Revisión (cont):

Si deseamos compatibilizar la información original con la más reciente, algún subconjunto del conjunto original debe ser “abandonado” o tal vez algo de la nueva información deba ser “relegado”.

- Podría ser errónea la información del lugar de nacimiento de Juan o José.
- El Atlas podría ser incorrecto (anterior al tratado).
- Podríamos suponer que es incorrecto que Juan y José son compatriotas.

Consistencia y Preferencia

Notar que tanto en contracción como en revisión utilizamos:

- Una noción lógica de consistencia.
- Una noción extralógica de selección de una sentencia o un conjunto de sentencias que eran preservadas o abandonadas.

Algunas cuestiones

Lenguaje de representación. En el ejemplo lenguaje natural. Sin embargo aparecen la nociones de "consistencia" en la información, contradicción, información derivada y vínculo entre los elementos del conjunto.

Necesitamos como mínimo un lenguaje formal que permita definir estas nociones, siendo el lenguaje de la lógica clásica un candidato natural (lo que requiere la aceptación de importantes idealizaciones).

También podría pensarse que la información sea codificada en otra lógica, ya sea modal, multivaluada, graduada, etc..

Algunas cuestiones (cont.)

Representación del corpus de información:

- una única fórmula en el lenguaje (la conjunción de las sentencias independientes) o
- un conjunto (quizás infinito) de fórmulas.

En el caso de ser conjunto,

- éste debe ser cerrado bajo la noción de consecuencia lógica o
- ser una simple enumeración de hechos “desnudos”?

Esta segunda opción implica la necesidad de calcular luego de alguna manera las consecuencias de estos hechos y tomar el compromiso de diferenciar o no entre información explícita e implícita.

Algunas cuestiones **(cont.)**

¿Puede el corpus actualizarse “espontáneamente” o requiere de un “estímulo” externo? es decir, ¿son estos internamente estables?

En caso de considerar un corpus de información que sufre cambios en respuesta a estímulos externos: ¿Debe haber diferencia entre el lenguaje de representación del corpus y el de la información que desata el cambio?

Aún si el lenguaje fuera el mismo, ¿deben ser homogéneas las representaciones? ¿Ambas fórmulas o ambas conjuntos de fórmulas?

Algunas cuestiones **(cont.)**

¿Los valores epistémicos asignados a las expresiones del lenguaje serán solamente aceptación, rechazo, indeterminación, o se deben considerar grados de aceptabilidad? ¿Que tipo de información puede representarse en el corpus? ¿Pueden ser representada información de cómo modificar el mismo corpus?

¿Debe relacionarse la noción de pertenencia de una sentencia al corpus y la validez de dicha sentencia? ¿Podemos contener en el corpus información falsa? ¿Las expresiones del lenguaje serán interpretadas como verdaderas o falsas o ni verdaderas ni falsas, o tendremos grados de verdad, inconsistencia, etc.?

Algunas cuestiones (cont.)

Por otro lado, parece fundamental definir operaciones que respondan a la noción de mínimo cambio, o máxima preservación del corpus de información. Es decir, se requiere alguna forma de "calcular el valor" de la información a ser descartada. ¿Existe un orden de preferencia que representa su credibilidad, solidez o valor informacional entre expresiones del lenguaje? ¿Este está incluído en el corpus de información o es intrínseco a la operación de cambio? ¿El mínimo cambio debe ser cuantitativo o cualitativo? ¿Cuántas y cuáles son las distintas formas en que un corpus de información puede ser modificado? ¿Son independientes o interdefinibles?

Algunas cuestiones **(cont.)**

El vínculo entre el corpus original y el actualizado: ¿es relacional o funcional? La función o relación: ¿Es sólo entre el corpus original, la nueva información y corpus o deben considerarse otro parámetros más? ¿Las operaciones de cambio deben tener en cuenta la historia de los cambios producidos, o cada nueva operación se produce independientemente de las anteriores? ¿El proceso de actualización del corpus debe mantener la interpretación de las expresiones del lenguaje, o es pensable que un cambio altere las proposiciones asociadas a las expresiones del corpus?

Dominio de la charla:

Modelo AGM:

- *Teorías estables*,
- *Conjuntos de fórmulas* (posiblemente infinitos) de un *lenguaje supraclasical*, *cerrados por un operador de consecuencia lógica*, que incluye las *tautologías*, la regla de *introducción de disyunción en las premisas*, y *compacidad*.
- No existe *distinción entre creencias explícitas y las derivadas*.
- La información que desencadena el cambio se representada por una fórmula, privilegiada respecto de la teoría existente.

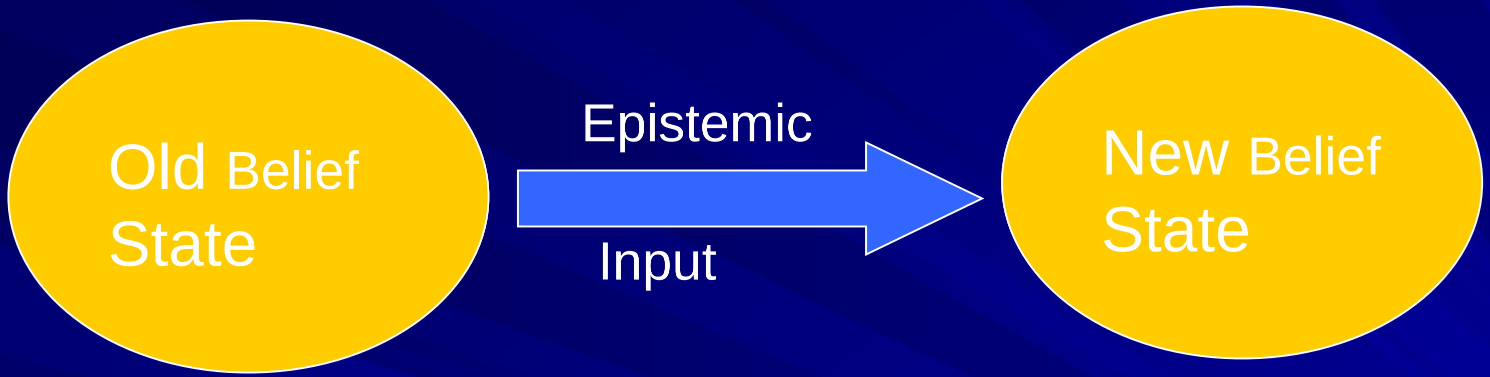
Dominio de la charla:

En AGM hay tres actitudes epistémicas: aceptado, rechazado, ignorado. Eso da lugar a seis cambios posibles:

1. De ignorado a aceptado.
2. De ignorado a rechazado.
3. De aceptado a rechazado.
4. De rechazado a aceptado.
5. De aceptado a ignorado.
6. De rechazado a ignorado.

En AGM, los puntos 1 y 2 se denominan expansiones, los puntos 3 y 4 son referidos como revisiones, y los puntos 5 y 6 reciben el nombre de contracciones. Estas son funciones que tienen por dominio pares ordenados teorías-fórmulas y por rango teorías, basadas en el “principio de mínimo cambio”.

Belief Change – AGM Model



What is a belief state? Set of sentences, largely consistent, and closed under a (classical) consequence operation Cn.

What sort of input? A (self-consistent?) sentence.

Nature of the state transformation? Expansion, Contraction, Revision.

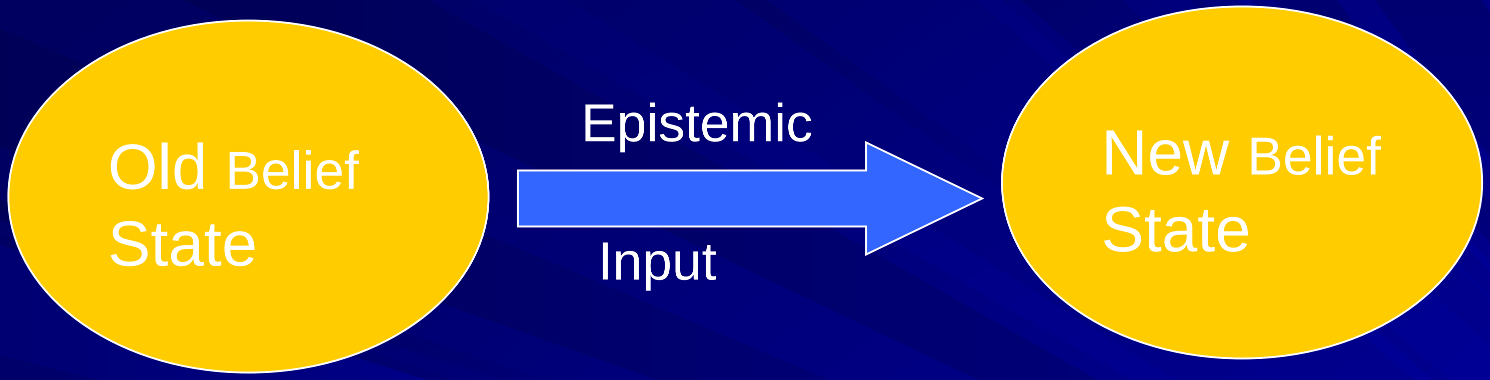
Why change the state? Old knowledge is erroneous (static environment).

An alternative model: Update

In 1992, Katsuno and Mendelzon presented a type of operator of change that they called update. Whereas revision operators are intended to capture the change yielded by evolving knowledge about a static situation, update operators are intrended to mirror the change in knowledge produced by an evolving situation.

"We make a fundamental distinction between two kinds of modifications to a knowledge base. The first one, *update* consists of bringing the knowledge base up to date, when the world described by it changes. ...The second kind of modification, *revision*, is used when we are obtaining new information about a static world. ...We claim the AGM postulates describe only revisions."

Belief Change – KM Model



What is a belief state? A sentence.

What sort of input? Another sentence.

Nature of the state transformation? Erase, Update.

Why change the state? Old knowledge is outdated (dynamic environment).

Updates vs. Revision

The difference lies in *the source* of incorrect beliefs

Belief *revision* assumes that the world is unchanging: an agent's change in belief occurs when that agent discovers something new about the static world or discovers an error in his beliefs about the static world.

Belief *updates* involve a change in belief prompted by actions in a dynamic world. This process behaves differently than revision.

Updates vs. Revision

Suppose:

I believe that Reema lives in Rochester.

Types of changes:

1. I learn that she has just moved to Cairo.

   (Update)

2. I learn that I have been mistaken and that she lives in Cairo.

   (Revision)

Updates vs. Revision

Suppose:

the agent knows that there is either a book on the table or a magazine on the table, but not both.

Types of changes:

1. The agent is told that there is a book on the table. He concludes that there is no magazine on the table.
   (Revision)
2. The agent is told that after the first information was given, a book has been put on the table. In this case he should not conclude that there is no magazine on the table.
   (Update)

Static Constraint: Inferential Coherence

Minimum synchronic conditions for inferential coherence of a belief state:

- **Maxim 1**. An agent S's beliefs should be logically consistent.
- **Maxim 2**: If proposition ϕ is inferable from S's beliefs, then S should believe ϕ.

Diachonic coherence

Economic constraints:

- **Maxim 3**: The amount of information lost in a belief change should be kept minimal.
- **Maxim 4**: In so far as some beliefs are considered more important than others, one should retract the least important to restore equilibrium.

Coherence of choice

Dynamic Constraint:

- **Maxim 5**: In so far as choices are to be made when performing a belief change, these choices should be coherent.

(i.e., preference orderings should be respected)

Menu example

Modeling Belief States

- Logical model of rational belief change
  - Let *X* and *Y* denote *sets* of well-formed formulae (*wffs*) in a propositional language, $\mathcal{L}$, and ϕ and ψ denote arbitrary formulas in $\mathcal{L}$.

    e.g., $X = \{\psi, \phi \rightarrow \psi\}$

Important: We will interpret these sets of *wffs* as sets of beliefs held by an ideal agent. This is the motivation for considering the non-classical extensions and alternatives to propositional logic.

Modeling Belief States

- Logical model of rational belief change
  - Let *X* and *Y* denote *sets* of well-formed formulae (wffs) in a propositional language, $\mathcal{L}$, and ϕ and ψ denote arbitrary formulas in $\mathcal{L}$.
  - A set *X* of *wffs* is *inconsistent* if there exists a *wff* ϕ such that $X \models \phi$ and $X \models \neg\phi$. If there is no such *wff*, then *X* is *consistent*.
  - *Inference operation*: Let $Cn(X) = \{\phi : X \models \phi\}$.
  - Let *K* and *H* denote *logical theories*, e.g., $K = Cn(X)$, for some set of *wffs* *X*.

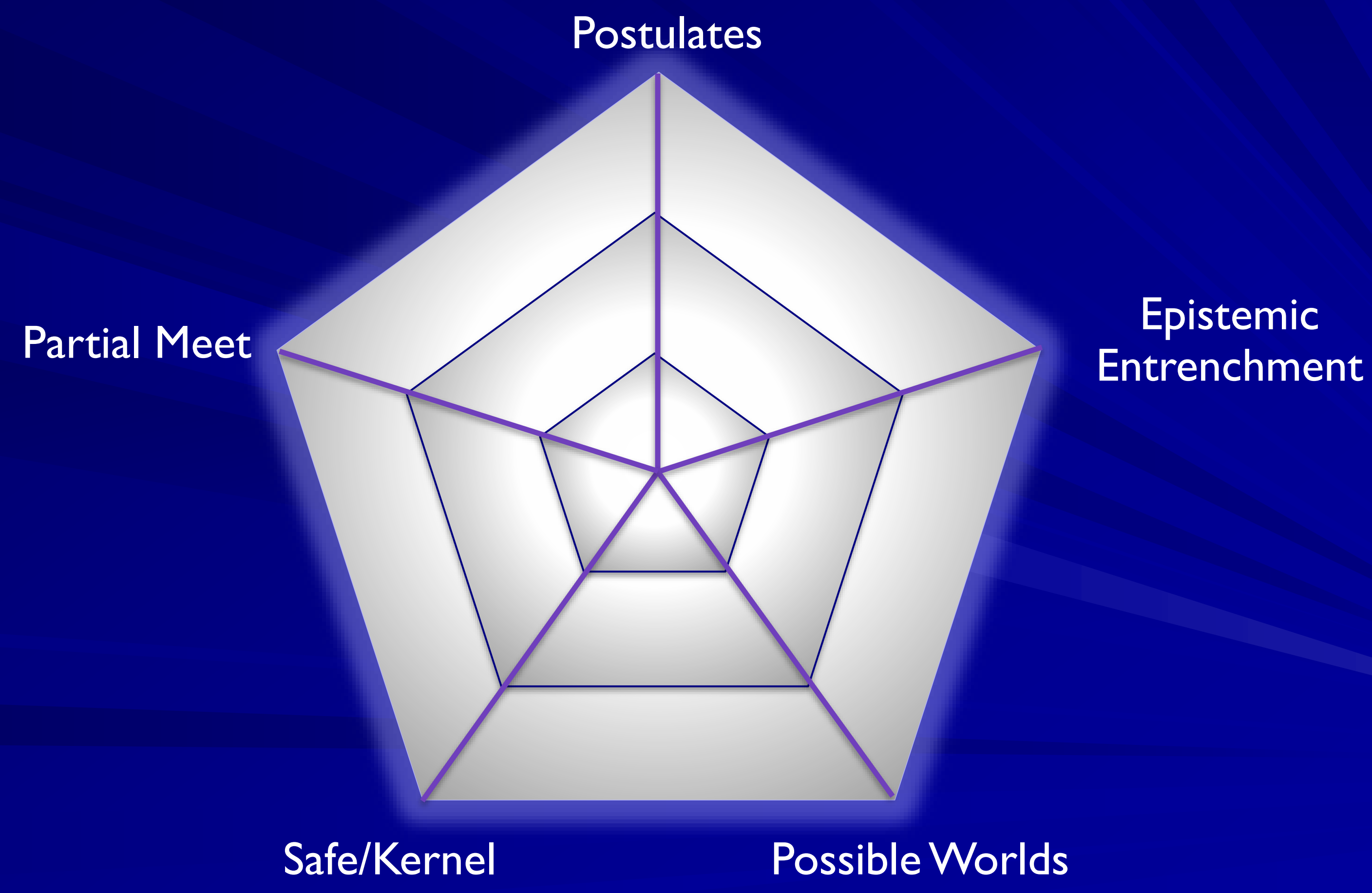
AGM

- Alchourrón, Gärdenfors and Makinson (1985) proposed a set of *rationality postulates* for expansion, contraction and revision operators.
- A *belief change operator* is a 2-place function from a *logical theory*, K, and target formula, ϕ, to a replacement theory:

$$2^{\mathcal{L}} \times \mathcal{L} \rightarrow 2^{\mathcal{L}}$$

AGM

5 different equivalent presentations



Expansion: the + operator

The expression $K + \phi$ denotes the replacement theory resulting from an expansion of K by ϕ.

$$+ : 2^{\mathcal{L}} \times \mathcal{L} \rightarrow 2^{\mathcal{L}}$$

Expansion postulates

(+1) If K is a theory, then $K + \phi$ is a theory.

The expansion operator applied to a theory yields a theory.

(+2) $\phi \in (K + \phi)$.

Expansion always succeeds: the target formula ϕ is always included in the expanded theory.

Expansion postulates

(+3) $K \subseteq (K + \phi)$.

An expanded theory includes the original theory.

(+4) If $\phi \in K$, then $(K + \phi) = K$.

Expanding a theory K with a formula that is already in K does not change K.

Expansion postulates

(+5) If $K \subseteq H$, then $(K + \phi) \subseteq (H + \phi)$.

Expansion by the same formula ϕ on a theory K that is a subset of a theory H preserves the set-inclusion relation between K and H.

(+6) $(K + \phi)$ is the smallest theory satisfying (+1) to (+5).

The expanded theory is the smallest possible and does not include wffs admitted by an operation which does not satisfy postulates (+1) to (+5). The set of theories satisfying (+1) to (+5) is closed under set intersection.

Expansion postulates

Remarks:

One way to expand a theory K is simply to add the target formula and close this set under logical consequence, that is to replace K by $Cn(K \cup \{\phi\})$.

Thm 3: $K + \phi = Cn(K \cup \{\phi\})$.

Note: this is the only AGM operation which guarantees a unique replacement theory.

Contraction: the - operator

The expression *K* - ϕ denotes the replacement theory resulting from a contraction of *K* by ϕ.

$$- : 2^{\mathcal{L}} \times \mathcal{L} \rightarrow 2^{\mathcal{L}}$$

Belief Change -- Contraction

- K - x is the result of contracting K by x
- Models belief suspension; so $x \notin K - x$ if possible
- Belief in x is lost; does not mean that belief in $\neg x$ is gained.
- Quality – no capricious gain in information. We must ensure that $K - x \subseteq K$.
- Categorial Matching: we must ensure that K – x is closed under Cn.
- Quantity – Information loss should be minimised. Not as easy!
- What should be $Cn(a, a \to b) - b$?

  $Cn(a)$? or $Cn(a \to b)$? or $Cn(a \vee b)$?
- Choice Problem! An extra-logical mechanism is necessary.

Contraction postulates

(-1) If K is a theory, then $K - \phi$ is a theory.

The contraction operator applied to a theory yields a theory.

(-2) $(K - \phi) \subseteq K$.

The contracted theory is a subset of the original theory.

Contraction postulates

(-3) If $\phi \notin K$ then $(K - \phi) = K$.

If the target formula ϕ to be contracted is not in the original theory, then the replacement theory is identical to the original theory.

(-4) $\phi \notin (K - \phi)$ only if ϕ is not a tautology.

The target formula ϕ is always removed from a theory by contraction unless ϕ is a tautology.

Contraction postulates

(-5) If $\phi \in K$ then $K \subseteq ((K\text{-}\phi)+\phi)$.

The Recovery Postulate.

(-6) If $\phi \equiv \psi$, then $(K - \phi) = (K - \psi)$.

Logically equivalent formulas give rise to identical contractions.

Contraction postulates

(-7) $(K - \phi) \cap (K - \psi) \subseteq (K - \phi \wedge \psi)$.

The formulas that are in both the theory contracted by the target formula ϕ and the theory contracted by the target formula ψ are in the theory contracted by the target conjunction, $\phi \wedge \psi$. It is important to note that contracting by a conjunction is not the same as iterative contractions by each conjunct. Contracting by a conjunction entails removing the joint truth of the two formulas, which may be achieved by retracting one of the conjuncts.

Contraction postulates

(-8) If $\phi \notin (K - \phi \wedge \psi)$, then $(K - \phi \wedge \psi) \subseteq K - \phi$.

If the target formula of a contraction operation is a conjunction succeeds in removing one of the conjuncts, ϕ, then every formula that is removed by a contraction with *that conjunct* (i.e., ϕ) alone is also removed by the contraction with the conjunction.

Contraction postulates

Remarks:

While Expansion guarantees a unique replacement theory, note that the contraction postulates do not determine a unique replacement theory.

This property will be illustrated with a series of examples. Notice that this feature introduces the need for extra-logical constraints to guide our choice among candidate replacement theories.

Recovery is problematic

Example: I have read in a book about Cleopatra that she had both a son and a daughter. My set of beliefs therefore contains both α and β, where α denotes that Cleopatra had a son and β that she had a daughter. I then learn from a knowledgeable friend that the book is in fact a historical novel. After that I contract $\alpha \vee \beta$ from my set of beliefs, i.e., I do not any longer believe that Cleopatra had a child. Soon after that, however, I learn from a reliable source that Cleopatra had a child. It seems perfectly reasonable for me to then add $\alpha \vee \beta$ to my set of beliefs without also reintroducing either α or β.

Recovery is problematic (cont.).

Example: I previously entertained the two beliefs, "x is divisible by 2" (α) and "x is divisible by 6" (β). When I received new informaction that induced me to give up the first of these beliefs (α), the second (β) had to go as well (since α would otherwise follow from β).

Then I received new information that made me accept the belief "x is divisible by 8" (δ). Since α follows from δ, $(K - \alpha) + \alpha$ is a subset of $(K - \alpha) + \delta$, then by recovery I obtain that "x is divisible by 24" (γ) contrary to the intuition.

Revision: the * operator

The expression $K * \phi$ denotes the replacement theory resulting from an revision of K by ϕ.

$$* : 2^{\mathcal{L}} \times \mathcal{L} \rightarrow 2^{\mathcal{L}}$$

Belief Change -- Revision

- $K * x$ is the result of accommodating x into K, even if x is not consistent with K
- Models belief accommodation, so $x \in K*x$.
- $K*x$ must be consistent if possible.
- Quality – no capricious gain in information. We must ensure that $K * x \subseteq K+ x$.
    - Does not say much if x is inconsistent with K.
- Categorial Matching: ensure that $K * x$ is closed under Cn.
- Quantity – Information loss should be minimised. Not as easy!
- What should be $Cn(a, a \rightarrow b) * \neg b$?

  $Cn(a, \neg b)$? or $Cn(\neg a, \neg b)$? or $Cn(\neg b)$?
- Choice Problem again!

Revision postulates

(*1) If K is a theory, then $K * \phi$ is a theory.

The revision operator applied to a theory yields a theory.

(*2) $\phi \in (K * \phi)$.

Revision always succeeds: the target formula ϕ is always included in the expanded theory.

Revision postulates

(*3) $(K * \phi) \subseteq (K + \phi)$.

A revision never incorporates formulas that are not in the expansion of the original theory by the same target formula.

Revision postulates

(*4) If $\neg\phi \notin K$, then $(K + \phi) \subseteq (K * \phi)$.

If the negation of a target formula is not in a theory, then the result of expanding the theory by that target formula is a subset of the result of revising the theory by the target formula. When taken with (*3), it follows that if the target formula is consistent with the original theory, then a revision is identical with the expansion, that is

$$(K + \phi) = (K * \phi).$$

Revision postulates

(*5) $K * \phi = \perp$ if and only if $\models \neg\phi$.

Given that a theory is consistent, if we attempt to revise the theory by a contradiction the replacement theory is inconsistent. This is the only case where revision applied to a consistent theory yields an inconsistent theory.

(*6) If $\phi \equiv \psi$, then $(K * \phi) = (K * \psi)$.

Logically equivalent formulas give rise to identical revisions.

Revision postulates

(*7) $(K * \phi \wedge \psi) \subseteq ((K * \phi) + \psi)$.

When revising a theory by a target formula that is a conjunction we may retain every formula in the original theory by (1) first revising the original theory by one conjunct and then (2) expand the revised theory by the other conjunct. Compare:

$$(K * \phi \wedge \psi) \subseteq (K + (\phi \wedge \psi)) = ((K + \phi) + \psi), \text{ by } (*3).$$

Since (*3) gives us $(K * \phi) \subseteq (K + \phi)$, (*7) gives us a tighter upper-bound on $(K * \phi \wedge \psi)$ than (*3).

Revision postulates

(*8) If $\neg\psi \notin K * \phi$, then $((K * \phi)+\psi) \subseteq (K * \phi \wedge \psi)$.

So long as a formula ψ is consistent with a revised theory K by another formula, ϕ, then the resulting theory from applying the two step procedure mentioned in (*7) is a subset of revising K by the conjunction of the two formula in question, $\phi \wedge \psi$. Together, (*7) and (*8) entail that the two step process in (*7) is identical to the conjunction as a whole, that is

$$((K * \phi)+\psi) = (K * \phi \wedge \psi)$$

given that ψ is consistent with the revised theory in the first step.

Revision postulates

Remarks:

Like Contraction, the revision postulates do not determine a unique replacement theory.

While we defined the revision operator, *, the contraction operator, -, and the expansion operator, +, independently of one another, we may nevertheless define these operators in terms of one another.

Success is controversial

- **Example**: Yesterday, the Pope called me to wish me good luck in this talk.
- **Example**: Yesterday, my mother called me to wish me good luck in this talk.
- **Example**: One day when you return back from work, your son tells you, as soon as you see him: “A dinosaur has broken our Ming's dynasty vase in the living-room”. You probably accept one part of the information, namely that the vase has been broken, while rejecting the part of it that refers to a dinosaur.

The Levi Identity

Thm 8: Given that the contraction function satisfies postulates (-1) to (-4) and (-6), and the expansion function satisfies (+1) to (+6), the revision function as defined by the *Levi Identity*

$$K * \phi = ((K - \neg\phi) + \phi)$$

satisfies (*1) to (*6). Furthermore, if (+7) is satisfied, then (*7) is satisfied; if (+8) is satisfied, then (*8) is satisfied.

The Harper Identity

Thm 9: Given that the revision function * satisfies (*1) to (*6), the contraction function - as defined by the *Harper Identity*

$$K - \phi = K \cap (K * \neg\phi)$$

satisfies (-1) to (-6). Furthermore, if (*7) is satisfied, then (-7) is satisfied and if (*8) is satisfied, then (-8) is satisfied.

Definiciones Explícitas

Remainder

El conjunto $K\perp a$, ‘K menos a', como la familia de subconjuntos maximales B incluído en K tales que $a \notin Cn(B)$. Así $B \in K\perp a$ si:

1) $B \subseteq K$.

2) $B \nvdash a$.

3) $\forall B' \subseteq K$ tal que $B' \nvdash a$ resulta que $B \not\subset B'$.

Observación:

$$\text{Si } K = Cn(K) \text{ entonces } \forall B \in K\perp a : B = Cn(B)$$

Meet y Choice

Se define la función ~ de meet contraction como:

$$K \sim \alpha = \begin{cases} \cap K \perp \alpha \text{ si } K \perp \alpha \neq \emptyset \\ K \end{cases}$$

Dada una función de selección τ tal que para todo a : τ (K,a) $\in$ (K$\perp$a) se define la función choice contraction como:

$$K \backslash \alpha = \begin{cases} \tau(K, \alpha) \ \text{ si } K \perp \alpha \neq \emptyset \\ K \end{cases}$$

Meet contraction es pobre

Proposición: Sea K cualquier teoría , y a cualquier proposición. Entonces $K\sim a = Cn(\neg a) \cap K$ excepto en el caso límite de que $a \notin K$ en cuyo caso será $K\sim a=K$.

Observación: K partir de una meet contraction la revisión asociada usando Levi resulta:

$$K{*}a = (K\sim(\neg a))+a = Cn((Cn(a) \cap K)\cup\{a\}) = Cn(a)$$

Choice Contraction es exagerada

Proposición: Sea K una teoría que contenga una proposición a, y sea w cualquier proposición. Entonces $(a \vee w) \in (K \backslash a)$ o $(a \vee \neg w) \in (K \backslash a)$.

Observación: A partir de una choice contraction la revisión asociada usando Levi resulta que para cualquier proposición w :

$$w \in K{*}a \text{ o } \neg w \in K{*}a.$$

Partial Meet Contraction

DEFINITION: (AGM85) Let K be a belief set. Let $K\perp\alpha$ be the set of all maximal subsets of K that does not imply α. Let γ be a selection function that satisfies: $\gamma(K\perp\alpha)$ is a non-empty subset of K if $K\perp\alpha$ is non-empty, in which case $\gamma(K\perp\alpha) = \{K\}$.

The partial meet contraction on K that is generated by γ is the operation $\sim_\gamma$ such that for all sentence α:

$$K \sim_\gamma \alpha = \cap\, \gamma(K\perp\alpha)$$

An operation - on K is a partial meet contraction if and only if there is a selection function γ for K such that for all sentences α:

$$K - \alpha = K \sim_\gamma \alpha\ .$$

Safe/Kernel Contraction.

- En el caso de las funciones Partial Meet, la contracción de K-α se basa en los conjuntos maximales de K que no implican α.
- Otra idea, en cierta forma dual a ésta, es pensar en construcciones basadas en los conjuntos minimales de K que implican α.
- Si elimina al menos un elemento de cada uno de estos conjuntos minimales, el conjunto resultante no contendrá α.

Safe/Kernel Contraction.

Definición: Sea K una teoría y α una sentencia. El conjunto $K \perp\!\!\!\perp \alpha$ es el conjunto definido por las siguientes condiciones. $H \in K \perp\!\!\!\perp \alpha$ sii

(1) $H \subseteq K$

(2) $\alpha \in Cn(H)$

(3) No existe un conj. H' tal que $H' \subset H$ y $\alpha \in Cn(H')$

(Es decir, $K \perp\!\!\!\perp \alpha$ es el conjunto de todos los subconjuntos de H minimales que implican α).

Safe/Kernel Contraction.

Definición: Sea K una teoría, una función de incisión para K es una función σ tal que para toda fórmula α:

(1) $\sigma(K \perp\!\!\!\perp \alpha) \subseteq \cup(K \perp\!\!\!\perp \alpha)$.

(2) Si $\emptyset \neq H \in K \perp\!\!\!\perp \alpha$, entonc. $H \cap \sigma(K \perp\!\!\!\perp \alpha) \neq \emptyset$.

Definición: Sea σ una función de incisión para K, la Kernel Contraction »_{σ} será:

$$K \text{»}_{\sigma} \alpha = Cn(K \setminus \sigma(K \perp\!\!\!\perp \alpha)).$$

Safe/Kernel Contraction.

Definición: Una función de incisión $\boldsymbol{\sigma}$ para K es Smooth si y solo si para todo H $\subseteq$ K sucede que si H $\vdash \beta$ y $\beta \in \boldsymbol{\sigma}(K \perp\!\!\!\perp \alpha)$ entonces H $\cap$ $\boldsymbol{\sigma}(K \perp\!\!\!\perp \alpha) \neq \emptyset$. Una Kernel Contraction se denomina Smooth si y solo si está basada sobre una función de incisión Smooth

Teorema: Si K es un conjunto de creencias. Entonces »$_{\sigma}$ es una smooth kernel contraction si y sólo si es una partial meet contraction para K.

Entrenchment

Def.: An *epistemic entrenchment* relation $\leq_e$ is defined on formula of $\mathcal{L}$, where

$$\phi \leq_e \psi$$

is interpreted to express that *ψ is as epistemically entrenched as ϕ* and satisfies the 5 postulates, (EE1) through (EE5). Let $\phi <_e \psi$ stand for ψ is strictly more entrenched than ϕ, and $\phi =_e \psi$ for ψ and ϕ are equally entrenched.

Entrenchment postulates

(EE1) If $\phi \leq_e \psi$ and $\psi \leq_e \chi$, then $\phi \leq_e \chi$.

The epistemic entrenchment relation is transitive.

(EE2) If $\phi \vdash \psi$, then $\phi \leq_e \psi$.

A formula is at most as entrenched as the formulas it logically entails. If ϕ entails ψ and we wish to retract ψ, we need to retract ϕ also to avoid deriving ψ in the replacement theory. On the other hand, ϕ should be at most as entrenched as ψ so that ϕ may be retracted without necessarily retracting ψ.

Entrenchment postulates

(EE3) For all ϕ, ψ, either $\phi \leq_e \phi \wedge \psi$ or $\psi \leq_e \phi \wedge \psi$

Retracting the conjunction $\phi \wedge \psi$ is achieved by either retracting ϕ or retracting ψ. Thus, the conjunction is at least as entrenched as one of the conjuncts

From (EE2), we have the opposite relations $\phi \wedge \psi \leq_e \phi$ and $\phi \wedge \psi \leq_e \psi$. From (EE2) and (EE3), together, we have $\phi \wedge \psi =_e \phi$ or $\phi \wedge \psi =_e \psi$. In other words, a conjunction is as entrenched as its least entrenched conjunct.

Entrenchment postulates

(EE4) When $K \neq \perp$, then $\phi \notin K$ iff $\phi \leq_e \phi \wedge \psi$ or $\psi \leq_e \phi \wedge \psi$.

Formulas not in the theory are the least entrenched and, if the theory is consistent, vice versa.

(EE5) If $\phi \leq_e \psi$ for all ϕ, then $\vdash \psi$

The most entrenched formulas are the tautologies.

Correspondence Results

(C$^-$) $\psi \in (K - \phi)$ iff $\psi \in K$ and either $\phi <_e \phi \vee \psi$ or $\vdash \phi$.

(C$^-$) specifies what formulas are retained in a contraction given an epistemic entrenchment relation. Only formulas that are in the original theory *K* can be included in the contracted theory. In addition, if the target formula is a tautology, then all formulas are retained. Otherwise, if the target formula ϕ is less entrenched than the disjunction $\phi \vee \psi$, then ψ is retained.

Correspondence Results

($C^{\leq e}$) $\phi \leq_e \psi$ iff $\phi \notin K\text{-}\ \phi \wedge \psi$ or $\vdash (\phi \wedge \psi)$

An epistemic entrenchment relation can be constructed from a contraction function by **($C^{\leq e}$)**. If a conjunct ϕ is not retained in the contracted theory, then it cannot be more entrenched than the other conjunct. Note that both conjuncts can be absent from the contracted theory, in which case the two conjuncts are equally entrenched, $\phi =_e \psi$.

Correspondence Results

Thm: Given an epistemic entrenchment ordering $\leq_e$ that satisfies (EE1) to (EE5), condition **($\mathbf{C^-}$)** uniquely determines a contraction function which satisfies the AGM contraction postulates (-1) to (-8) and condition **($\mathbf{C}^{\leq_e}$)**.

Correspondence Results

Thm: Given a contraction function which satisfies the AGM contraction postulates (-1) to (-8), condition **($C^{\leq e}$)** uniquely determines an epistemic entrenchment ordering $\leq_e$ that satisfies (EE1) to (EE5) and condition **(C^-)**.

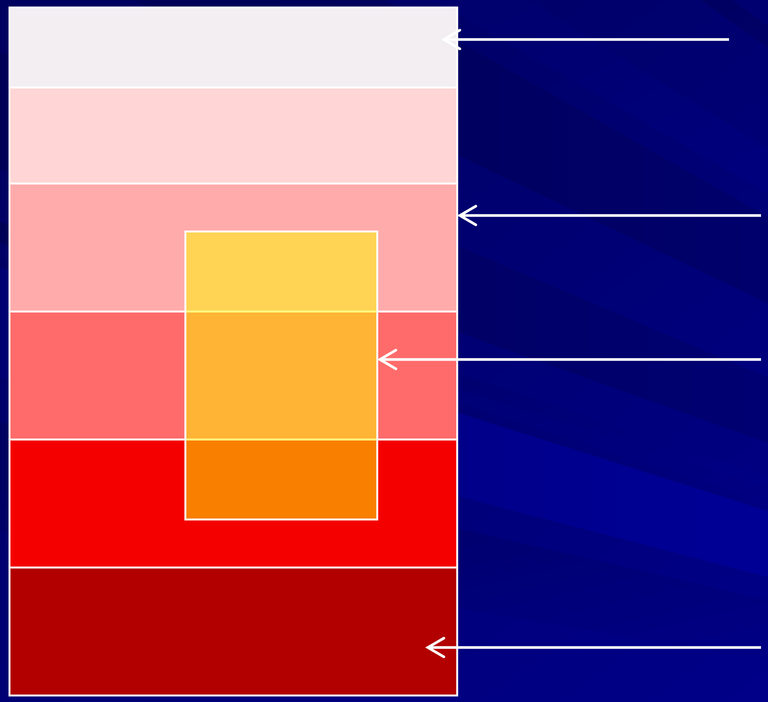
Remarks

- Rationality postulates generate a set of candidate theory change functions
- An entrenchment relation allows us to pick a specific function among the class.

Remarks

- The entrenchment ordering provides a constructive way to choose a specific contraction operator from the set of all possible operators.

Other Construction (Grove 88)



Least plausible worlds/interpretations

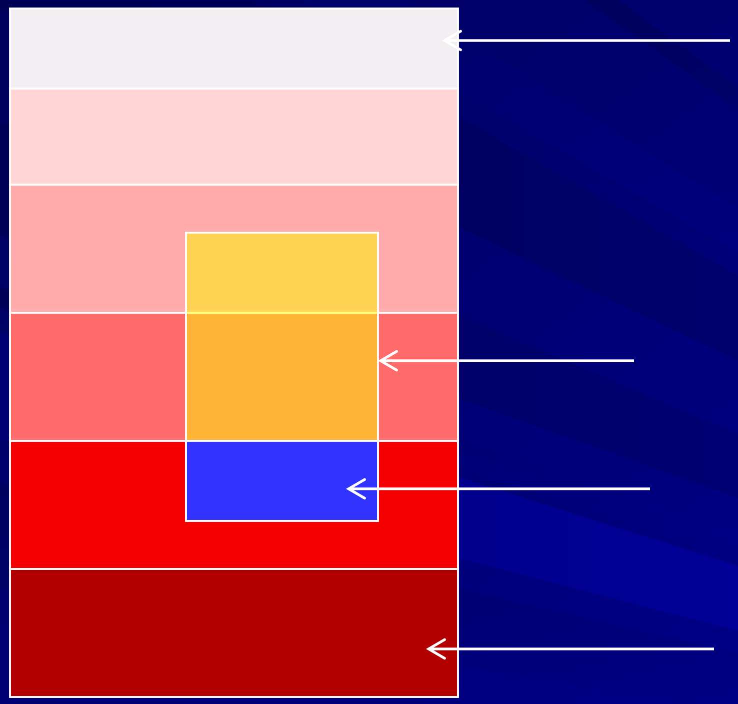
Set of all worlds Ω

Worlds satisfying the evidence: [x]

Most plausible worlds/interpretations: [K]

- A sentence x is represented by the set [x] of worlds that satisfy it.
- theory K: by the set [K] of worlds that satisfy every sentence in K.
- Total preorder over Set of all worlds Ω, [K]
- System of Spheres!

Other construction cont...(Grove 88)



Least plausible worlds/interpretations

Worlds satisfying the evidence: [x]

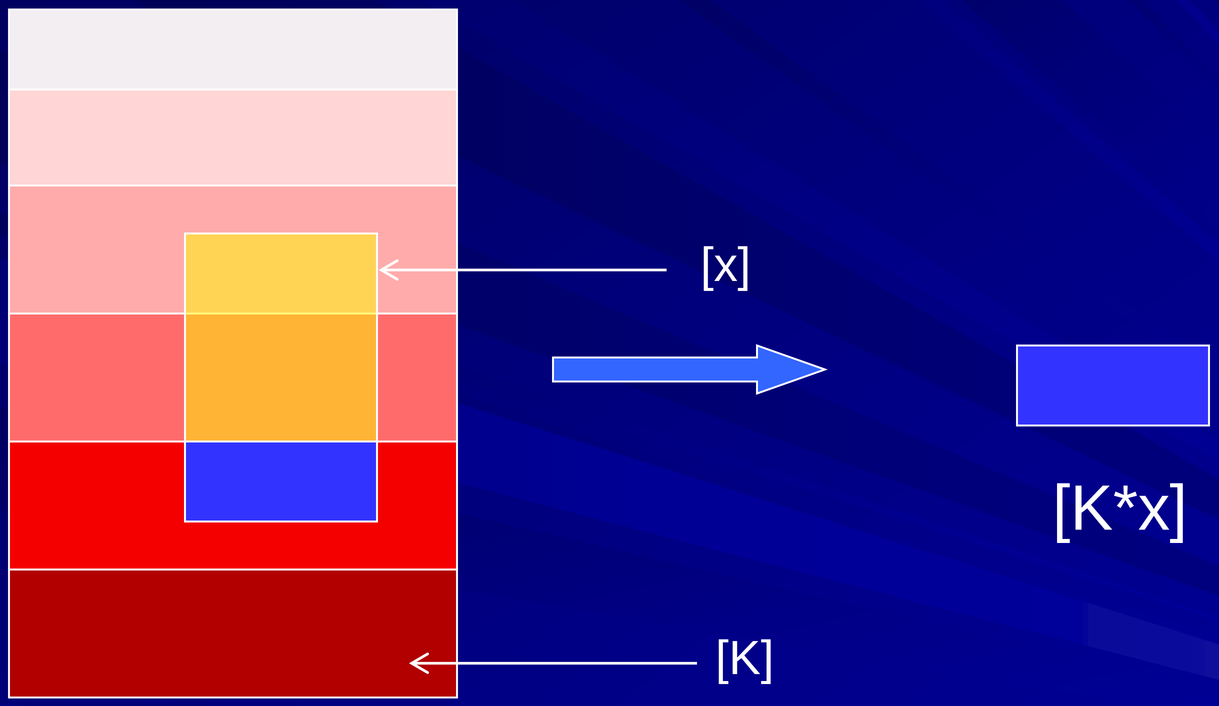
Most plausible among evidential-worlds: min$_{\leq}$ [x]

Most plausible worlds/interpretations: [K]

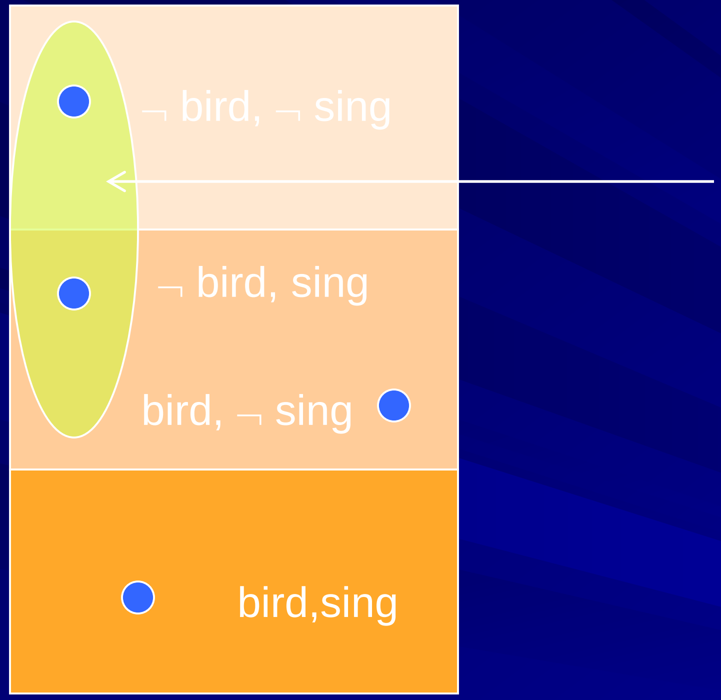
Theorem (Grove 88): Operation * is AGM rational iff there is an <u>appropriate</u> plausibility ordering $\leq$ such that [K*x] = min$_{\leq}$ [x] for all x.

Revision operation * and plausibility ordering $\leq$ are inter-definable.

Belief Revision - Semantics



Example



$K = Cn(bird,sing)$

$K^*{}_{\neg\, bird} = Cn(\neg\, bird,sing)$

Similarly,

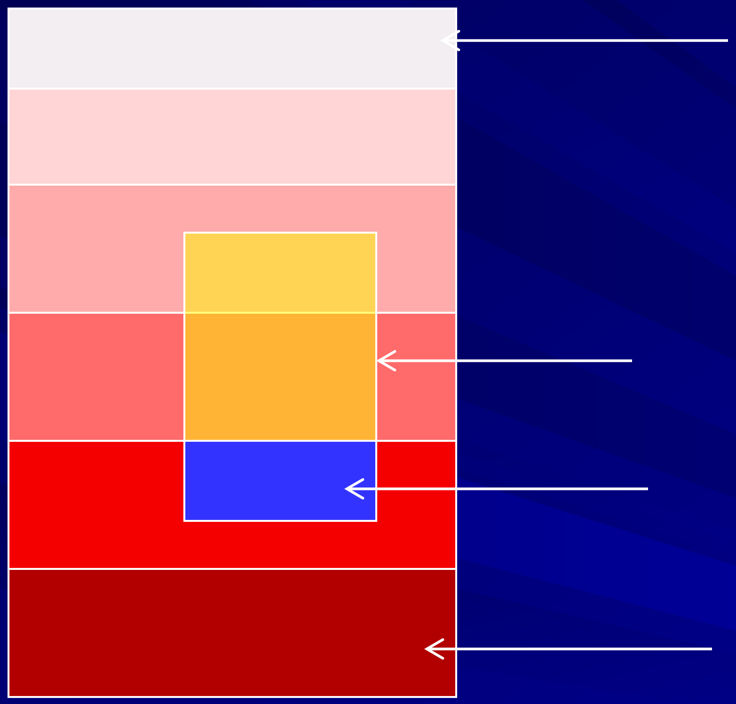
$K^*{}_{\neg\, sing} = Cn(bird, \neg\, sing)$

Knowledge: Tweety is a bird that sings.

Evidence: Tweety is not a bird.

Resultant Knowledge: Tweety is a non-bird that sings.

Constructing Contraction



Least plausible worlds/interpretations

Worlds falsifying x : [¬x]

Most plausible in [¬x] :

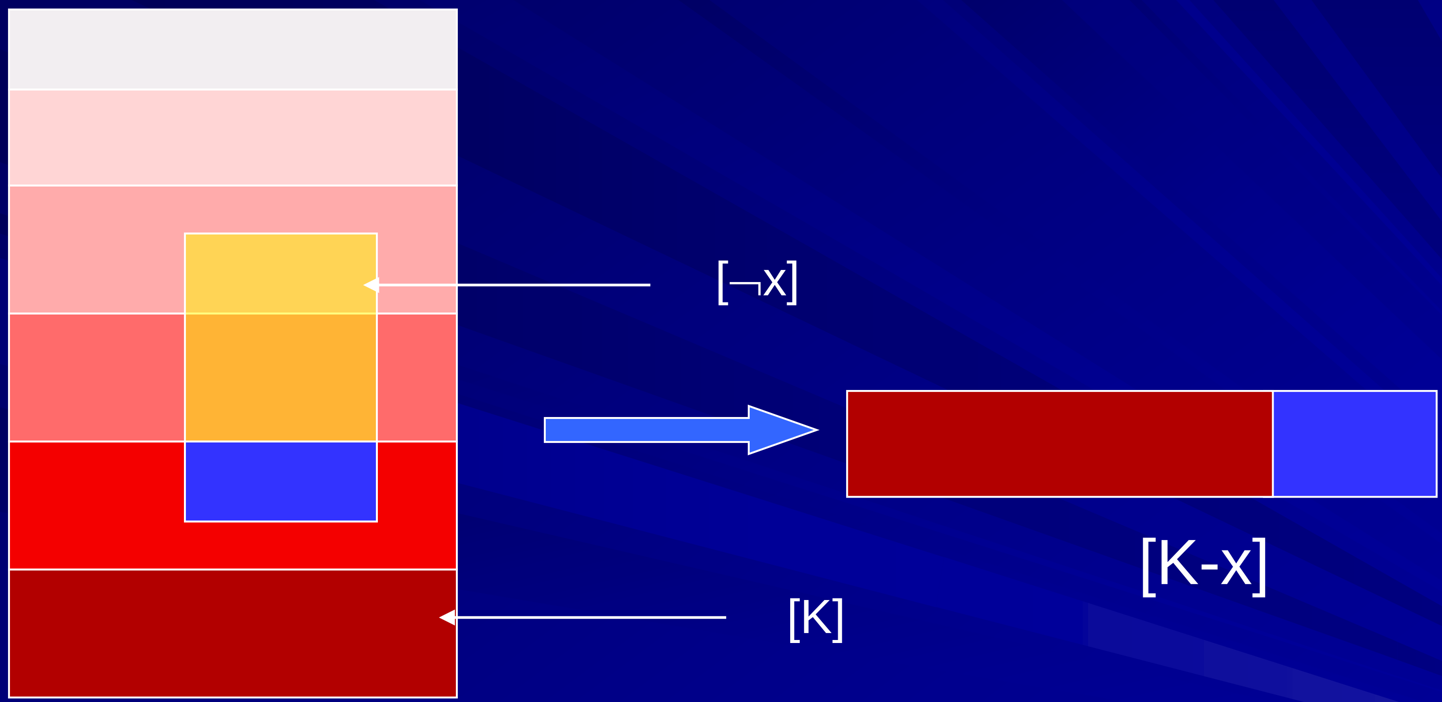
$\min_{\leq}$ [¬ x]

Most plausible worlds/interpretations: [K]

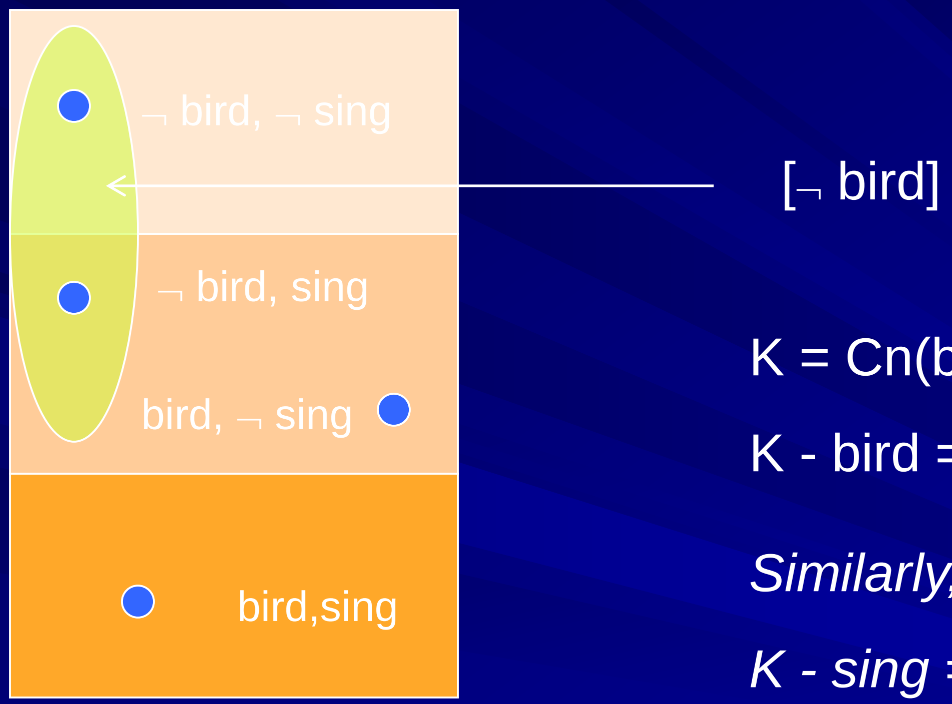
[K – x] is identified with [K] ∪ min≤ [¬ x].

Note: K-x ∈ K translates to: [K] ∈ [K-x]. In order to suspend belief in x, some worlds in [¬ x] need to be entertained. The best among them are chosen for this purpose.

Belief Contraction - Semantics



Example



K = Cn(bird,sing)

K - bird = Cn(sing)

Similarly,

K - sing = Cn(bird)

Knowledge: Tweety is a bird that sings.

Suspend: Tweety is a bird.

Resultant Knowledge: Bird or not, Tweety sings.

Resumen AGM

Cinco presentaciones:

1. Postulados.
2. Conjuntos maximales que no implican α.
3. Conjuntos minimales que implican α.
4. Orden epistémico entre sentencias.
5. Orden entre modelos.

Conclusions

- This talk was based on

“AGM 25 Years: Twenty-Five Years of Research in Belief Change”

Eduardo Fermé and Sven Ove Hansson.

Journal of Philosophical Logic 40, (2) : 113-114. 2011

… plus some updates

Conclusions

- This talk was an attempt to summarize (in 3 hours) major developments in twenty-eight years of AGM-inspired research.
- In preparing the talk we benefited from the help of more than fifty colleagues who answered our queries and provided us with information.
- I also received many suggestions and material from friends and colleagues that help me to improve the presentation.
- In addition to its academic excellence, the belief revision community is a remarkably generous one.
- This bodes well for the future of belief revision research.